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FINC 520, Spring 2009
Homework 2, due Friday, April 17.

1. Suppose that $\{x_t\}$ is a covariance stationary process. Then, by Wold's theorem, x_t has the following orthogonal decomposition:

$$x_t = \sum_{j=0}^{\infty} d_j \varepsilon_{t-j} + \eta_t,$$

where ε_t is

$$\varepsilon_t = x_t - P\left[x_t | x_{t-1}, x_{t-2}, ...\right].$$

Prove

$$x_t - P\left[x_t | x_{t-k}, x_{t-k-1}, ...\right] = \varepsilon_t + d_1 \varepsilon_{t-1} + ... + d_{k-1} \varepsilon_{t-(k-1)},$$

for $k = 2, ...$.

2. Suppose x_t and y_t are each $AR(1)$ processes:

$$x_t = \rho x_{t-1} + \varepsilon_t, \; y_t = \upsilon_t + \gamma \upsilon_{t-1}.$$

Show how to construct an ARMA representation for $x_t + y_t$.