1. Suppose that \( \{x_t\} \) is a covariance stationary process. Then, by Wold’s theorem, \( x_t \) has the following orthogonal decomposition:

\[
x_t = \sum_{j=0}^{\infty} d_j \varepsilon_{t-j} + \eta_t,
\]

where \( \varepsilon_t \) is

\[
\varepsilon_t = x_t - P[x_t|x_{t-1}, x_{t-2}, \ldots].
\]

Prove

\[
x_t - P[x_t|x_{t-k}, x_{t-k-1}, \ldots] = \varepsilon_t + d_1 \varepsilon_{t-1} + \ldots + d_{k-1} \varepsilon_{t-(k-1)},
\]

for \( k = 2, \ldots \).

2. Suppose \( x_t \) and \( y_t \) are each \( AR(1) \) processes:

\[
x_t = \rho x_{t-1} + \varepsilon_t, \quad y_t = \nu_t + \gamma \nu_{t-1}.
\]

Show how to construct an ARMA representation for \( x_t + y_t \).