Quantitative Easing in Joseph's Egypt with Keynesian Producers

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Abstract

This paper considers monetary policy to avoid liquidity traps when households can accumulate real wealth. As in the biblical story of Joseph managing seven fat years to prepare for seven lean years, the flexible-price allocation uses savings to turn current income into future consumption. With nominal rigidities, the economy can remain in a recession even when policy equates nominal bonds' real return with the natural rate of interest. *Josephean Quantitative Easing*, purchases of assets backed by real wealth, can destroy all recessionary equilibria by putting a floor under future consumption. This requires no commitment to a time-inconsistent plan.

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> "Accordingly, let Pharaoh find a man of discernment and wisdom, and set him over the land of Egypt. And let Pharaoh take steps to appoint overseers over the land and organize the land of Egypt in the seven years of plenty. Let all the food of these good years that are coming be gathered, and let the grain be collected under Pharaoh's authority as food to be stored in the cities. Let that food be a reserve for the land for the seven years of famine which will come upon the land of Egypt, so that the land may not perish in the famine."
>
> Genesis 41:33-36 in Berlin and Brettler (2004)

1 Introduction

The biblical story of Joseph accumulating grain during seven fat years to smooth consumption into seven prophetically foreseen lean years is one of the oldest cases of macroeconomic management following a shock to desired savings. Such shocks are the standard driving force behind the modern theory of liquidity traps (Krugman, 1998; Eggertsson and Woodford, 2003), but Joseph possessed two advantages over those models' monetary policy makers. First, Pharaoh granted Joseph command and control over the Egyptian agricultural sector, so there was no need to accommodate the market failures that underlie new Keynesian models. Second, Joseph possessed a *storage technology* for moving consumption from the present into the future. In contrast, standard liquidity trap models prominently feature a fallacy of composition: Individuals believe they can intertemporally substitute unlimited amounts of consumption using bonds, but bonds are in zero net supply. With these advantages, Joseph guided Egypt to the prosperous outcome recorded in the Bible. Since liquidity traps arise because households' desired savings exceeds the supply of bonds when income equals potential output, one might reasonably speculate that the storage technology was the key to Joseph's success. That is, adding storage or another form of capital accumulation to the standard new Keynesian model can remove both the fallacy of composition and the threat of liquidity traps. Indeed, Krugman (1998) addresses this possibility by arguing that adjustment costs make avoiding a liquidity trap using capital accumulation infeasible.

In this paper, I add potentially costly storage to a New Keynesian model with nominal rigidities, and I characterize the monetary policies that can be used to avoid a liquidity trap and successfully accumulate assets during fat years for later consumption. Since the Joseph story is familiar from both the Bible and Broadway, I use it throughout the paper as a mnemonic device. The model's shock to desired savings qualitatively resembles the productivity sequence in the Joseph story, but it has one year of plenty and an infinite horizon of famine instead of the biblically specified seven years for each phase. This *Egyptian scenario* is the supply-side analogue of the preference shock employed by Eggertsson and Woodford (2003) and Christiano, Eichenbaum, and Rebelo (2011). Because production requires elastically-supplied labor, the economy can fall into a liquidity trap. Although the original Joseph story featured literal storage prominently, here it represents wealth accumulation in general. This can be achieved by accumulating inventories, running a current account surplus, or investing in productive capital.

Without storage, monetary policy can avoid a recession if inflation expectations are high enough for some positive nominal rate of interest to clear the bond market when output equals its flexible-price level. With storage, such inflation expectations are *necessary but not sufficient* for this. The economy has a continuum of equilibria in which monetary policy is consistent with the flexible-price output. A static deflationary coordination game underlies this multiplicity: If firms with flexible prices expect deflation, they choose low prices. This lowers both real aggregate consumption and marginal cost and thereby confirms their expectations. When there is no storage, households' optimal bond purchases remove this indeterminacy: the Euler equation determines the level of current consumption given the rational anticipation that consumption will equal its flexible-price level when the shock to desired savings has passed. With storage, future consumption is a free variable. This allows the static coordination game's multiplicity to manifest itself in a dynamic setting. Because storage bounds the real interest rate from below, it cannot be lowered any further to lift the economy out of such a recession.

This multiplicity implies that avoiding a recession in the economy's initial fat year might require a policy to raise current savings and thereby increase future consumption. Both Eggertsson and Woodford (2003) and Werning (2012) advocate lifting expectations of future consumption by committing to low future interest rates which lead consumption to overshoot its long-run level. A monetary authority that exchanges nominal bonds for assets *backed by storage* achieves the same goal without requiring commitment. The natural non-negativity constraint on storage prevents households from offsetting the resulting real wealth accumulation. After the shock to desired savings has passed, unwinding the monetary authority's position increases consumption; and the rational expectation of this keeps consumption and output at their flexible-price levels when the propensity to save is still high. A substantial fraction of assets currently on the Federal Reserve's balance sheet are securities backed by claims to accumulated capital, namely housing; and the Bank of Japan and European Central Bank are both purchasing assets backed by loans to the private sector. Therefore, it seems reasonable to label the balance-sheet expansion in the model a form of *quantitative easing*. Instead of increasing "aggregate demand," it removes recessionary outcomes by shrinking the equilibrium set. Although the liabilities on Pharoh's balance sheet offsetting Joseph's accumulation of grain went unrecorded, one might reasonably consider such an accumulation of real assets by the sovereign to be a prototype for this form of quantitative easing. Hence, I label it *Josephean*.

Unlike the closed economies in typical liquidity trap models, small open economies can intertemporally substitute consumption by running a current account surplus, investing the proceeds abroad, and repatriating them in the future. Krugman (1998) dismissed the possibility that such trade-facilitated intertemporal substitution could lift an economy out of a liquidity trap based on an analysis that takes the shortcut (his word) "that one can ignore the effect of the current account on the future investment income of the country."[1] This paper shows that accounting for the country's future investment income is crucial for designing appropriate monetary policy in a liquidity trap. In an international context, Josephean quantitative easing (JQE) resembles a monetary authority accumulating foreign reserves to implement an export-promoting competitive devaluation of its currency. However, JQE does not operate through the real exchange rate. Instead, it (possibly) improves outcomes by putting a floor on expectations of *future* domestic consumption, just as it does in a closed economy. The foreign country experiences a current account deficit that reverses itself when the possibility of a liquidity trap has passed in the home country. Such unstable international capital flows are not an undesirable side effect of JQE; they are its goal.

Although early models of liquidity traps featured a fallacy of composition, recently Correia, Farhi, Nicolini, and Teles (2013) and Christiano, Eichenbaum, and

[1] See (Krugman, 1998, Page 164).

Rebelo (2011) have examined them in models with capital accumulation. Indeed, this paper's model is nearly a special case of that in Correia, Farhi, Nicolini, and Teles (2013). Those authors characterize the Pigouvian taxes that allow a competitive equilibrium to coincide with the optimal allocation. This paper complements theirs by showing how policy can make the flexible-price allocation the *unique* equilibrium using JQE. (Since firms' markups might be part of a preexisting scheme that grants monopoly rights to induce innovation, I consider only the modest goal of implementing the flexible-price allocation instead of the more ambitious aspiration of achieving a completely distortion-free allocation.) Christiano, Eichenbaum, and Rebelo (2011) quantitatively examined the government spending multiplier under the common assumptions that an interest-rate rule that satisfies the Taylor principle governs the nominal interest rate (subject to the zero-lower-bound) and that equilibrium sequences converge to the unique steady state with active monetary policy (Benhabib, Schmitt-Grohé, and Uribe, 2001). In the present model, such an imposition of local determinacy indeed eliminates equilibrium multiplicity. When the interest-rate rule and its inflation target are appropriately chosen, the unique equilibrium implements the flexible-price allocation. However, this criterion eliminates a continuum of other less desirable equilibria merely because they induce the monetary authority to drive the economy into the zero lower bound permanently. Aruoba, Cuba-Borda, and Schorfheide (2014) document that such a equilibrium replicates the Japanese experience since 1995 well. Furthermore, neither market-clearing nor individual optimality requires inflation to equal the monetary authority's target in the long run (Cochrane, 2011). Therefore there are neither theoretical nor empirical grounds for removing such outcomes from consideration ex-ante. I show that an appropriate choice of JQE can remove them from the equilibrium set ex-post.

The shocks to desired savings in liquidity-trap models are usually interpreted as stand-ins for the balance-sheet repair that follows financial-market turmoil. Eggertsson and Krugman (2012) expand on this by explicitly modeling the financial turmoil as a "Fisher-Minsky-Koo" moment, in which a contraction of consumer credit and debt deflation reduce aggregate demand. Fornaro (2013) shows that consumer debt forgiveness then can be Pareto improving: Borrowers' consumption increases while savers' consumption remains the same. I anticipate that adding capital accumulation to that environment can determine the potential of unconventional monetary policy to mitigate such a liquidity trap (holding fixed the dysfunctional consumer-credit market) by encouraging real wealth accumulation. However, that extension lies beyond this paper's scope.

The remainder of this paper proceeds as follows. The next section contains the model's primitive assumptions, and Section 3 presents its flexible-price allocation. Section 4 adds nominal rigidities and characterizes the resulting recessionary equilibria. These can be divided into two classes, *liquidity traps* and *confidence recessions*. Section 5 shows how JQE can destroy these equilibria, and it places this paper's results in the context of previous theoretical characterizations of QE. Section 6 develops the interpretation of the model as a small open economy that stores consumption by trading the aggregate good with a large foreign sector. Section 7 offers concluding remarks on the relevance of JQE for current monetary policy at the Bank of Japan and the European Central Bank. The model of the text embodies a linear storage technology. An appendix demonstrates that this paper's key results, the existence of confidence recessions and JQE's ability to eliminate both them and liquidity traps, are robust to allowing the marginal cost of storage to increase with its quantity.

2 Primitive Assumptions

The model features three key features of New Keynesian economies, monopolistic competition so that goods' prices are set by specific agents rather than by a Walrasian auctioneer, nominal rigidities which generate a Phillips curve trading off inflation and output, and a market for nominal bonds with an interest rate set by a monetary authority subject to the zero lower bound. Additionally, the monetary authority can issue nominal bonds and invest the proceeds within the storage technology. This access to a technology for intertemporal transformation of goods does *not* distinguish the monetary authority from the economy's households. Henceforth I anthropomorphize this authority and name him "Joseph".

The presentation of the model's primitive assumptions follows the conventional preferences-technology-trading opportunities road map. A single representative household populates the model economy. Its preferences over streams of consumption goods and time spent at work are

$$\begin{aligned} U(\{C_t\},\{N_t\}) &= \sum_{t=0}^{\infty} \beta^t \left(\ln C_t - \theta N_t\right), \text{ with} \\ C_t &= \left(\int_0^1 C_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}. \end{aligned}$$

Here N_t is time spent at work, $C_t(j)$ is the consumption of good j (with $j \in [0,1]$) in year t, and $\varepsilon > 1$ is the elasticity of substitution between any two of the differentiated goods. It is well-known that quasi-linear preferences like these feature an infinite Frisch elasticity of labor supply. I adopt it here for algebraic convenience. Since there is no uncertainty in this economy and I restrict attention to deterministic equilibria, risk-aversion plays no role in this analysis. I (implicitly) set the elasticity of intertemporal substitution to one only to avoid unnecessary parameter proliferation.[2]

Without storage, the natural non-negativity constraint on time at work would be irrelevant because the marginal rate of substitution between consumption and leisure grows without bound as consumption goes to zero. Since storage creates the possibility of consumption without work, I make this constraint explicit with

$$N_t \geq 0. \tag{1}$$

Replacing (1) with positive lower bound on hours worked, which is perhaps more realistic, would leave this paper's results unchanged.

The technology for producing each of the differentiated goods is the same: one unit of labor yields A_t units of the good in question. To make a liquidity trap possible, I assume that $A_0 = A^H$ and $A_t = A^L < A^H$ for all $t \geq 1$. This is the Egyptian scenario mentioned above.

The economy's other technology is that for storage. To have S units of the *aggregate* good available next year, one must invest $S/(1-\delta)$ units of the aggregate good today. Here, δ is the depreciation rate on storage. So that this technology

[2] One might generalize this paper's analysis to account for the possibility that a sunspot variable influences equilibrium inflation and (possibly) quantities. I ignore this possiblity to maintain the paper's focus on the economy's liquidity trap.

cannot be used to transfer resources from the future into the present, I require

$$S_t \geq 0. \tag{2}$$

Although it is natural to assume that $\delta \geq 0$, the analysis below only requires

$$\beta(1-\delta) < 1. \tag{3}$$

This more general bound on δ will be helpful when interpreting "storage" as investment abroad with a positive real return.

Trade occurs in a labor market, product markets, and financial markets. The labor market is perfectly competitive with nominal wage W_t. Product markets conform to the familiar monopolistic competition framework. Each product's monopolist chooses its nominal price taking as given all other products' prices, aggregate income, and the household's demand system for all of the differentiated products. The functions $P_t(\cdot)$ and $Y_t(\cdot)$ give all of the monopolists' nominal prices in year t and their corresponding quantities sold.

The model's nominal rigidity resembles the sticky information setup of Mankiw and Reis (2002). Each year, half of the economy's producers set their nominal prices for the current and next years. Unlike in a sticky-price model, the two years' prices may be different from each other. Mankiw and Reis adopt the Calvo (1983) assumption that the timing of each monopolist's opportunity to set prices is stochastic. To keep the algebra simple, I adopt a version of Fisher's (1977) timing assumption: it deterministically arrives every two years. Mankiw and Reis assert that switching from sticky prices to sticky information improves the Phillips curve's empirical performance. The use of sticky information here has a more modest motivation: By eliminating intertemporal trade-offs in price setting, I focus the analysis on intertemporal substitution and the obstacles to its efficient execution.

Joseph sets the interest rate for nominal bonds subject to the zero lower bound. Before setting the interest rates for bonds purchased in t that mature in $t+1$, he observes storage brought into the year S_t, the nominal wage W_t, producers' price choices and real outputs $P_t(\cdot)$ and $Y_t(\cdot)$, and households' consumption C_t. Joseph collects this information, the rationally anticipated path for A_t, and the complete histories of consumption, storage, and nominal wages and prices through year $t-1$ into the information set Ω_t and inputs it into the interest rate rule $i_t = \rho(\Omega_t)$. Joseph selects this rule at the beginning of time and thereafter follows its prescriptions absolutely. Wicksellian models (like this one) place the zero lower bound in the monetary policy rule as a stand-in for the analogous no-arbitrage condition that would come out of an explicit specification for money demand. Accordingly, I henceforth require $\rho(\Omega_t) \geq 0$ for all possible Ω_t.

To undertake JQE, Joseph issues nominal bonds, uses the proceeds to acquire the aggregate good, and directly invests the goods acquired in the storage technology. It is the restriction to investing in assets that directly contribute to real national wealth that distinguishes JQE from general quantitative easing, not the direct use of the storage technology *per se*. (Extending the model to have Joseph invest in privately-issued assets backed by stored goods changes nothing.) Let Q_{t+1} denote the amount of the aggregate good available in $t+1$ from Joseph's storage investments during t, and use B_{t+1} to represent the nominal redemption value of the bonds Joseph issued in t to finance that storage. In contrast with the Pigouvian policy maker in Correia, Farhi, Nicolini, and Teles (2013), Joseph has access to no other tax instruments. Therefore, given $Q_0 = B_0 = 0$, the sequences Q_t and B_t must satisfy the feasibility constraint

$$Q_{t+1} = (1-\delta)\left(\left(\frac{B_{t+1}}{1+i_t} - B_t\right)/P_t + Q_t\right). \tag{4}$$

3 The Flexible-Price Allocation

The equilibrium allocation when producers face no nominal rigidities serves as a baseline for the subsequent analysis. Whether the household or the government undertakes storage is a matter of indifference when prices are flexible, so I assume for this section that $Q_{t+1} = B_{t+1} = 0$ for all $t \geq 0$.

Begin the construction of a flexible-price equilibrium with the household's purchases of differentiated goods, and let Y_t denote the quantity of the aggregate good created.

$$Y_t \equiv \left(\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

The household's optimal allocation of nominal consumption expenditures across differentiated goods has the familiar form:

$$Y_t(j) = Y_t \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon}$$

with P_t the aggregate price index

$$P_t \equiv \left(\int_0^1 P_t(j)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}} dj.$$

By construction, $P_t Y_t$ is the household's total nominal expenditure on goods.

Given the household's initial holdings of nominal bonds and the aggregate good from storage; utility maximization requires choosing sequences of aggregate consumption, hours worked, the values of all assets subject to the budget constraint

$$P_t C_t + \frac{B_{t+1}}{1+i_t} + P_t \frac{S_{t+1}}{1-\delta} \leq W_t N_t + B_t + P_t S_t + D_t, \tag{5}$$

and the non-negativity constraints in (1) and (2). Here, D_t is the dividend earned from the household's ownership of the producers,

$$D_t = P_t(C_t + S_{t+1}/(1-\delta) - S_t) - W_t N_t;$$

and i_t is the interest rate given by $\rho(\cdot)$.[3]

In an equilibrium, the sequences for C_t, N_t, B_{t+1}, and S_{t+1} solve the household's utility maximization problem given the sequences for D_t, W_t, P_t, and i_t. If we denote the Lagrange multipliers on the year t budget constraint and the non-negativity constraints on storage and labor with $\beta^t \lambda_t / P_t$, $\beta^t \lambda_t \nu_t$, and $\beta^t \lambda_t \upsilon_t$; the utility maximization problem yields familiar conditions for optimal labor supply, optimal bond purchases, and optimal storage.

$$\theta C_t = \frac{W_t}{P_t} + \upsilon_t \tag{6}$$

$$1 = \beta(1+i_t) \frac{P_t C_t}{P_{t+1} C_{t+1}} \tag{7}$$

$$1 = \beta(1-\delta) \frac{C_t}{C_{t+1}} + \nu_t \tag{8}$$

[3] Producers are entirely equity financed. Since they face unlimited liability, D_t may be negative.

These, together with the transversality condition

$$\lim_{t\to\infty} \beta^t \frac{S_{t+1} + B_{t+1}/P_{t+1}}{C_{t+1}} = 0$$

are necessary and sufficient for the household's utility maximization.

In the flexible-price baseline, producers always set the optimal monopoly price, so

$$P_t = \left(\frac{\varepsilon}{\varepsilon - 1}\right) \frac{W_t}{A_t}. \tag{9}$$

The only remaining requirements for a flexible-price equilibrium are bond market clearing, $B_{t+1} = 0$ for all $t \geq 0$, and the aggregate resource constraint

$$A_t N_t = C_t + S_{t+1}/(1-\delta) - S_t.$$

From Walras's law, this guarantees labor-market clearing.

There are many flexible-price equilibria, but they all share a single allocation of consumption, storage, and hours worked. Since I repeatedly reference this allocation's values below, I denote the associated values of C_t, S_t, and N_t with $\tilde{C}_t$, $\tilde{S}_t$, and $\tilde{N}_t$. The tilde should bring flexibility to mind. To reduce the number of cases under review, I henceforth suppose that the economy starts with no consumption available from storage: $S_0 = 0$.

3.1 Mild Famines

When A^L/A^H is sufficiently close to one, the flexible-price allocation does not use storage. In this sense, the foreseen famine is "mild." To begin this case's equilibrium analysis, suppose that indeed $\tilde{S}_t = 0$ for all $t \geq 1$. The optimal price-setting condition in (9) determines W_t/P_t. Substituting this into (6) and imposing the resource constraint gives

$$\tilde{C}_t = \left(\frac{\varepsilon - 1}{\varepsilon}\right) \frac{A_t}{\theta}, \text{ and} \tag{10}$$

$$\tilde{N}_t = \tilde{C}_t / A_t. \tag{11}$$

Setting $\tilde{S}_{t+1} = 0$ is consistent with (8) for $t \geq 1$, because $\tilde{C}_t$ is constant from $t = 1$ onwards and $\beta(1-\delta) < 1$. For the household also to choose $\tilde{S}_1 = 0$, we requre that

C_0/C_1 is not too large.

$$\frac{\beta A^H}{A^L} < \frac{1}{1-\delta}. \tag{12}$$

The inequality in (12) puts a lower bound on A^L which defines a "mild" famine. When it holds good, $\tilde{S}_t = 0$ and expressions for $\tilde{C}_t$ and $\tilde{N}_t$ in (10) and (11) together give the unique flexible-price equilibrium allocation.

Completing the construction of an equilibrium requires finding an interest rate rule for Joseph and sequences of nominal prices and wages that are consistent with this allocation. For the rule, consider

$$i_t = \max\left\{0, \pi^\star \beta^{-1} \frac{\tilde{C}_{t+1}}{\tilde{C}_t} \left(\frac{\pi_t}{\pi^\star}\right)^\phi - 1\right\}. \tag{13}$$

This is a censored inflation targeting rule with a time-varying intercept. In it, $\pi^\star$ is the target inflation rate, $\pi_0 \equiv P_0 \pi^\star$, and $\pi_t \equiv P_t/P_{t-1}$ for $t \geq 1$.[4] When $\pi_t = \pi^\star$, the underlying non-censored rule tracks the nominal interest rate consistent with the flexible-price allocation. This is the "natural" interest rate. Otherwise ϕ regulates the response of i_t to deviations from the inflation target. If $\phi > 1$, the rule satisfies the "Taylor principle."

To construct an equilibrium using (13), set $\pi^\star \geq \beta A^H/A^L$ and $P_t = \pi^{\star\, t}$. With these values, (13) gives

$$i_t = \pi^\star \beta^{-1} \frac{A_{t+1}}{A_t} - 1.$$

This satisfies (7) for all $t \geq 0$; so this sequence of allocations, prices, and interest rates forms a flexible-price equilibrium.

Since inflation never deviates from its target, any value of ϕ is consistent with achieving $\pi_t = \pi^\star$ always. However, this result only holds if $\pi^\star \geq \beta A^H/A^L$, which in turn guarantees that the equilibrium avoids the zero lower bound. If instead $\pi^\star < \beta A^H/A^L$, then (13) sets i_0 to zero if $\pi_0 = \pi^\star$. With this interest rate, clearing the nominal bond market (given $C_t = \tilde{C}_t$ always) requires π_1 to exceed $\pi^\star$. For reasons familiar from Benhabib, Schmitt-Grohé, and Uribe (2001), the evolution of subsequent inflation depends on ϕ. If $\phi < 1$, then inflation temporarily overshoots

[4] Both with flexible prices and sticky price plans, there is no loss to Joseph's economy from setting $\pi^\star \neq 1$ as long as all equilibria considered are deterministic.

$\pi^\star$. If instead $\phi = 1$, then inflation remains permanently at π_1. Finally, with $\phi > 1$ the Taylor principle induces Joseph to raise the nominal interest rate more than one-for-one with inflation. Bond-market clearing then requires inflation to rise *even further*, leading to an explosive inflation sequence. Regardless of its implications for inflation, the interest rate rule has no influence on flexible price allocations.

3.2 Severe Famines

Now, suppose that (12) does not hold, so A^L is low enough to induce storage. As in the case of a mild famine, $\upsilon_0 = 0$ and Equation (10) determines $\tilde{C}_0$.

To determine $\tilde{C}_t$ for $t \geq 1$, hypothesize that $\tilde{S}_1 > 0$ and $\tilde{S}_t = 0$ for $t \geq 2$. The first assumption and (8) give us

$$\tilde{C}_1 = \beta(1-\delta)\tilde{C}_0. \tag{14}$$

Since the economy faces a severe famine, this exceeds the value of C_1 consistent with setting $N_1 > 0$. So that this value of $\tilde{C}_1$ is also consistent with the household setting S_2 to zero, assume

$$1 \geq \beta^2(1-\delta)^2\frac{A^H}{A^L}. \tag{15}$$

That is, the rate of return from saving across two years is not too large. Although none of the results below depend on this particular limit on the duration of storage and its attendant vacation, I henceforth assume that (15) holds good to keep the analysis simple. With this, $\tilde{S}_t = 0$ and (10) characterizes $\tilde{C}_t$ for all $t \geq 2$.

Given the sequences for $\tilde{C}_t$ and $\tilde{S}_t$ in hand, the budget constraints determine $\tilde{N}_t$ for $t = 0$ and $t \geq 2$. The consumption sequence and (7) determine equilibrium real interest rates. To decompose these into nominal interest rates and inflation; use the interest rate rule in (13). If the given value of $\pi^\star > \beta\tilde{C}_0/\tilde{C}_1$, then $i_0 > 0$ and $\pi_t = \pi^\star$ for all t. Otherwise, $i_0 = 0$ and $\pi_t > \pi^\star$ always.

Figure 1 summarizes this section's results with (qualitative) plots of the flexible-price allocation and its associated real interest rate over time. In each panel, the blue line with circles corresponds to the case of a mild famine, while the orange line with squares gives analogous values for a severe famine. The upper-right panel plots productivity for the two cases, which share a common value for A^H. The upper-left panel gives consumption, which begins at $\tilde{C}_0$ in both cases. With the mild famine, it falls to $\tilde{C}_1$ and stays there forever. With a severe famine, the household carries wealth into year 1, so $\tilde{C}_1 > \tilde{C}_2$ in this case. Regardless, consumption reaches its long-run value in year 2. The lower-left panel gives the associated gross *real* interest rates, which do not depend on the particular interest rate rule employed by Joseph. With a mild famine, this equals $A^L/(\beta A^H)$. Making the foreseen famine worse by reducing A^L reduces this until it reaches $1-\delta$. For even lower values of A^L, the household's use of the storage technology keeps this "natural" interest rate from falling. With a mild famine, the real interest rate reaches its long-run value, $1/\beta$, in year 1. In the case of a severe famine, the real interest rate remains below $1/\beta$ in year 1 because consumption is still higher than its long-run value.[5] Finally, the lower-right panel gives hours worked in the two cases. Under a mild famine, the ratio of consumption to wages is constant. Since these preferences satisfy standard balanced-growth restrictions, this means that wage changes' income and substitution effects exactly offset to leave hours worked constant. The case of a severe famine shows the Lucas and Rapping (1969) theory of intertemporal substitution and labor supply in action. Temporarily high real wages in year 0 induce the household to expand labor supply, accumulate savings, and raise consumption in future years. The future consumption boom lowers hours worked for the one year that it lasts.

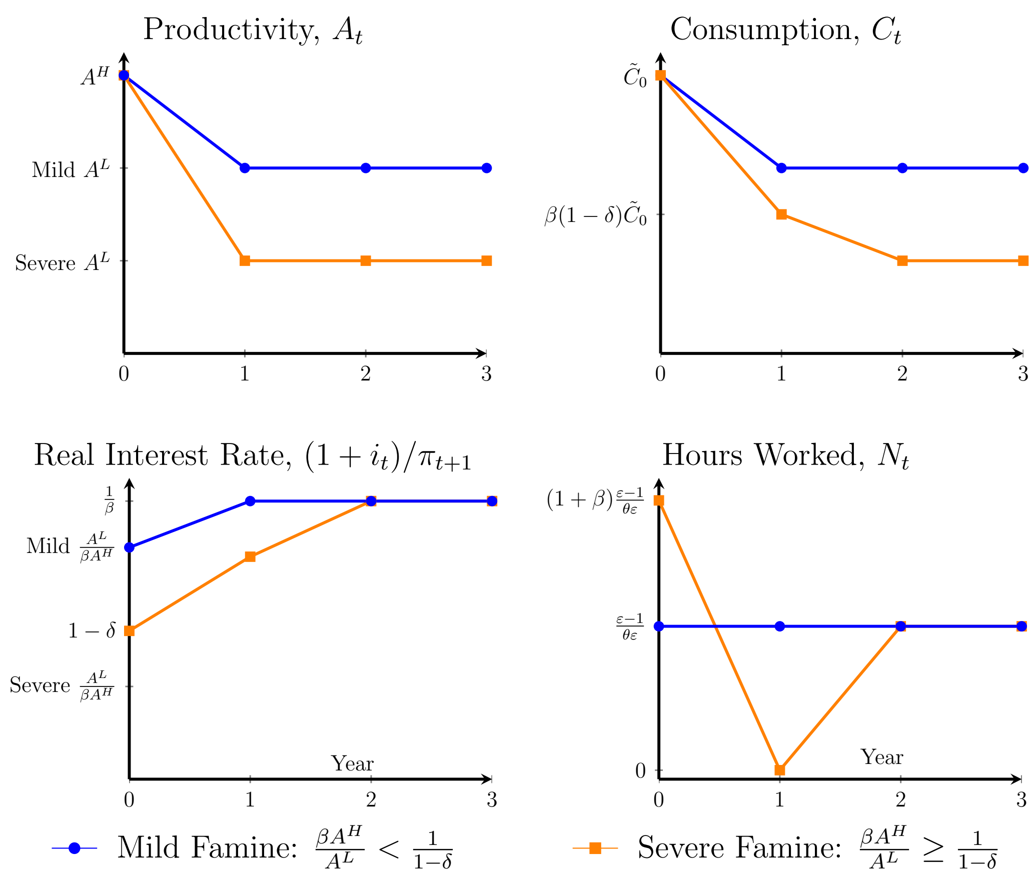
4 Equilibria with Nominal Rigidities

This section shows how nominal rigidities can interfere with implementing the flexible-price allocation, which requires reexamining producers' optimal pricing decisions and appropriately redefining equilibrium. For this section, I continue to hold $B_{t+1} = Q_{t+1} = 0$, so the resulting equilibria are *without JQE*.

Denote the price chosen by a firm in year $t-j$ that will apply in year t with P_t^j; so P_t^0 is the price chosen by producers with a current price choice, and P_t^1 is the

[5] As drawn, $(1+i_1)/\pi_2$ in the case of a severe famine is less than $(1+i_0)/\pi_1$ with a mild famine. This is possible, but not necessary.

Figure 1: The Flexible-Price Allocation and Real Interest Rate



price for t chosen by producers that set their price plans in $t-1$.

Since there is no uncertainty, the optimal price choices are

$$P_t^0 = \left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{W_t}{A_t} \;\; \forall t \geq 0 \text{ and} \tag{16}$$

$$P_t^1 = \left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{W_t}{A_t} \;\; \forall t \geq 1. \tag{17}$$

The right-hand sides of (16) and (17) are identical, but they apply to different years. The preset price, P_0^1, is one of the economy's initial conditions; which I normalize to one. With these firm-level prices, the aggregate price index is

$$P_t = \left(\frac{1}{2}P_t^{0\;1-\varepsilon} + \frac{1}{2}P_t^{1\;1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}} \tag{18}$$

The definition of an equilibrium with nominal rigidities simply adds the prices from pre-existing plans to the economy's initial conditions; replaces (9) with (16), (17) and (18); and accounts for the effects of pricing distortions on aggregate output in the aggregate resource constraint.

$$\frac{A_t N_t}{\frac{1}{2}\left(\frac{P_t^0}{P_t}\right)^{-\varepsilon} + \frac{1}{2}\left(\frac{P_t^1}{P_t}\right)^{-\varepsilon}} = C_t + S_{t+1}/(1-\delta) - S_t. \tag{19}$$

Here, total output is written as a linear function of hours worked, with productivity dependent on differentiated goods' producers' relative prices. This is at its maximum when all goods' nominal prices equal each other.[6]

4.1 The Fundamental Multiplicity

Generically, this economy has multiple equilibrium allocations. When π_1 indexes the equilibrium set, Joseph can guide the economy to a desired outcome by appropriately managing inflation expectations. (The tools of an inflation-targeting regime (Bernanke and Mishkin, 1997) could be useful for this task.) In other cases, π_1 is constant across equilibria and instead they differ in the expected (and realized) value of C_1. (This paper shows how JQE can manage these expectations of future consumption.) Both cases can be understood as specific instances of a *fundamental*

[6]To show this, use (18), the fact that $x^{\frac{\varepsilon}{\varepsilon-1}}$ is convex if $\varepsilon > 0$, and Jensen's inequality.

multiplicity that arises in the model when the only dynamic considerations come from firms' preset prices. That is, the economy has neither a bond market nor a storage technology.

In year 0, half of the firms have nominal prices fixed at $P_0^1 = 1$, while the other half can choose their nominal prices. Additionally, the household provides labor and spends all of its income on consumption. Given $P_0^1 = 1$, equilibrium in this alternative economy requires C_0, N_0, W_0, P_0, and P_0^0 to satisfy the optimal labor supply condition in (6) with υ_0 set to zero, the optimal pricing condition in (17), the price aggregation rule in (18), and the resource constraint in (19) with S_t and S_{t+1} set to zero.

Four conditions restrict five unknowns. To show mechanically that this under determination indeed results in equilibrium multiplicity, select any

$$C_0 > 2^{\frac{1}{1-\varepsilon}} \tilde{C}_0. \tag{20}$$

The optimal labor supply condition then determines the real wage W_0/P_0. Together, the conditions for optimal flexible prices and price aggregation imply that the price level solves

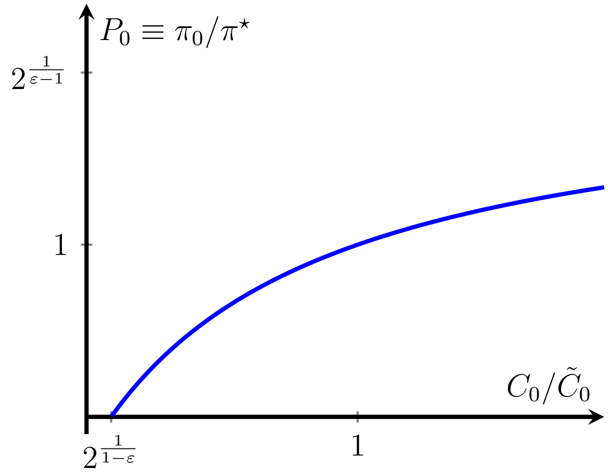
$$P_0 = \left(\frac{1}{2} + \frac{1}{2} \left(P_0 \frac{C_0}{\tilde{C}_0} \right)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}. \tag{21}$$

This can be interpreted as the economy's Phillips curve, positively connecting the consumption gap, $C_0/\tilde{C}_0$, with inflation, $\pi_0/\pi^\star \equiv P_0$. The assumed lower bound for C_0 guarantees that the solution for P_0 is real. Because firms' choices of current prices do not constrain their choices of future prices, it does not have the familiar dependence on expected future inflation.

Figure 2 presents this Phillips curve graphically. It begins arbitrarily close to the point $(2^{\frac{1}{1-\varepsilon}}, 0)$, crosses through the 45° line at $(1, 1)$, and asymptotes to $2^{\frac{1}{\varepsilon-1}}$ as $C_0/\tilde{C}_0$ goes to infinity. Any point on this Phillips curve is consistent with equilibrium: Given C_0 and P_0 from such a point, W_0 and P_0^0 can be obtained immediately from (6) and (17). The resource constraint then determines N_0.

Equilibrium multiplicity in the full model can be better understood in light of the fundamental multiplicity by considering the Euler equation for optimal nominal bond purchases, (7). Given i_0, π_1, and C_1, this determines C_0 and thereby selects one of many points on the Phillips curve of Figure 2. In the "standard" analysis of the

Figure 2: The Phillips Curve



liquidity traps with discretionary monetary policy, $i_0 = 0$ and C_1 is assumed to equal $\tilde{C}_1$. Then, inflation expectations determine current macroeconomic performance, as in Krugman (1998). For the present model, Subsection 4.3 covers this case in detail by assuming that the famine is mild. With storage, C_1 becomes endogenous; *even if we assume that it equals its flexible-price value given* S_1. In this case – which is covered in Subsection 4.4 given a severe famine – the economy can have multiple equilibria with constant (across equilibria) values of i_0 and π_1.

Economically, the fundamental equilibrium multiplicity reflects a coordination failure (Cooper and John, 1988). Producers with flexible prices must coordinate on an expectation of real marginal cost $(W_0/(A_0P_0))$. Increasing this expectation raises their prices and lowers the economy's average markup over marginal cost, thereby boosting economic activity. This raises marginal cost through (6), so firms' expectations of higher marginal cost are fulfilled.[7] The lower bound on C_0 in (20) reflects a limit to the amount of damage nominal rigidity can do to this economy: Equilibrium cannot feature lower consumption than that which would occur if P_0^1/P_0^0 were driven to infinity so that half of the economy's goods are effectively not available.

[7] A host of macroeconomic models feature equilibrium multiplicity. Among those, the one most closely related to this fundamental multiplicity is that of Shleifer (1986). That model also omits external effects of production and derives equilibrium multiplicity from a static coordination failure. However, its coordination failure concerns technological development, which is arguably more relevant for medium-run fluctuations than are this model's short-run pricing decisions.

4.2 The Flexible-Price Allocation Replicator

An equilibrium can exhibit the familiar Keynesian connection between disinflation and output in the initial year, because adjustments of P_0^0 influence consumption and marginal cost and thereby change the average markup of producers with fixed nominal prices. However, as Cochrane (2013) noted, this is not a *necessary* feature of equilibrium in a new Keynesian economy. When Joseph follows an interest rate rule like (13), then there always exists an equilibrium that implements the flexible-price allocation. As in the examples considered by Cochrane in the standard three-equation model, this requires inflation to overshoot $\pi^\star$ when this target is too low.

I call this equilibrium the flexible-price allocation replicator. Begin its construction by selecting $\pi^\star \geq \beta$; assigning the interest rate rule in (13) to Joseph with the given value of $\pi^\star$; and setting C_t, N_t, and S_{t+1} to $\tilde{C}_{t+1}$, $\tilde{N}_{t+1}$, and $\tilde{S}_{t+1}$ respectively. Select $P_0^0 = 1$ and π_1 to satisfy (7) given i_0, C_0, and C_1. If $\pi^\star \geq \beta \tilde{C}_0/\tilde{C}_1$, then $\pi_1 \leq \pi^\star$. Otherwise, $\pi_1 > \pi^\star$. In either case, inflation progresses thereafter according to $\pi_t/\pi^\star = (\pi_{t-1}/\pi^\star)^\phi$.

This equilibrium construction restates a basic piece of intuition from Krugman (1998) and Eggertsson and Woodford (2003): It is possible to implement the flexible-price allocation if short-run inflation expectations are high enough. To the extent that central banks facing actual liquidity traps have set inflation targets that are both credible and inappropriately low, the flexible-price replicator deliniteates the problem of escaping a liquidity trap rather than solving it.

4.3 Mild Famines

Although liquidity traps driven by low inflation expectations and a mild anticipated famine are not this paper's focus, they provide a necessary baseline for the results when the famine is severe. Its characterization begins with the following intuitive proposition, which allows us to restrict the analysis of this subsection to equilibria with $S_1 = 0$.

Proposition 1. *There exists no equilibrium in which both $S_1 > 0$ and $\pi_1 < 1/(1-\delta)$.*

Proof. Suppose otherwise. Since $S_1 > 0$, we can rearrange (7) and (8) to get

$$\frac{1 + i_0}{\pi_1} = (1 - \delta).$$

Therefore

$$1 + i_0 = \pi_1(1 - \delta) < 1$$

This violates the zero-lower bound on interest rates, and so contradicts the equilibrium definition. □

Intuitively, a liquidity trap caused by a mild famine decreases C_0 and so reduces the household's incentive to save. Even with consumption at its higher flexible-price level, this incentive is insufficient to induce positive storage, so $S_1 = 0$.

To construct a liquidity trap equilibrium that resembles the "discretionary" equilibrium of Eggertsson and Woodford (2003), presume that consumption, storage, and hours worked equal $\tilde{C}_t$, $\tilde{S}_t$, and $\tilde{N}_t$ for $t \geq 1$. Next, fix a value for $\pi^\star \in [\beta, \beta 2^{\frac{1}{\varepsilon-1}})$. (The upper bound on $\pi^\star$ helps ensure that the lower bound in (20) does not constrain C_0 in a recession with a high inflation target that is not credible. See Footnote 8 below for details.) With this, select $\pi_1 \in [\beta, betaA^H/A^L)$. Given this value for π_1, C_0 can be determined from

$$C_0 = \frac{\pi_1 \tilde{C}_1}{\beta \max\left\{1, \pi^\star \beta^{-1} \left(\frac{A^L}{A^H}\right) P_0(C_0)^\phi\right\}} \tag{22}$$

In (22), $P_0(C_0)$ is the Phillips curve from Section 4.1. Since its left-hand side strictly increases with C_0 while its right-hand side weakly decreases with C_0, this uniquely determines C_0. The assumed upper bound on π_1 guarantees that $C_0 < \tilde{C}_0$. To ensure that C_0 also exceeds its lower bound in (20), assume that[8]

$$\frac{A^L}{A^H} > 2^{\frac{1}{1-\varepsilon}}. \tag{23}$$

Since $S_0 = 0$; N_0, W_0, P_0^0, and P_0 can be obtained from C_0 following the logic of Section 4.1.

The determination of π_t for $t \geq 2$, combines the interest rate rule with the

[8]For this demonstration, suppose first that $\pi^\star \leq \beta A^H/A^L$. In this case, $C_0 = \pi_1 \tilde{C}_1/\beta$; so (23) and the presumption that $\pi_1 > \beta$ immediately imply (20). If instead $\pi^\star > \beta A^H/A^L$, use $P_0(C_0) \leq 1$, to show that $1 + i_0 \leq \pi^\star \beta^{-1} A^L/A^H$. Therefore, we have

$$\frac{C_0}{\tilde{C}_0} = \frac{\pi_1 \tilde{C}_1}{\beta(1+i_0)\tilde{C}_0} \geq \frac{\pi_1 A^H}{\beta \pi^\star \beta^{-1} A^L} \frac{A^L}{A^H} = \frac{\pi_1}{\pi^\star} \geq \frac{\beta}{\pi^\star} > 2^{\frac{1}{1-\varepsilon}}$$

Here, the final inequality comes from the assumed upper bound on $\pi^\star$.

first-order condition that characterizes the household's demand for bonds to get

$$\pi_{t+1} = \beta \max\left\{1, \pi^\star \beta^{-1} \left(\frac{\pi_t}{\pi^\star}\right)^\phi\right\} \tag{24}$$

for all $t \geq 1$. This difference equation and the presumed value of π_1 yield the desired inflation sequence. I summarize this equilibrium construction with a

Proposition 2. *Suppose that*

- $\beta A^H/A^L < 1/(1-\delta)$,
- $\pi^\star \in [\beta, \beta 2^{\frac{1}{\varepsilon-1}})$, *and*
- *(23) holds ;*

and select $\pi_1 \in [\beta, \beta A^H/A^L)$. Then there exists a equilibrium in which π_t equals the given value of π_1 for $t = 1$, $S_{t+1} = 0$ for all $t \geq 0$, $C_0 < \tilde{C}_0$ and $C_t = \tilde{C}_t$ for all $t \geq 1$.

Proof. The only requirement that the proposed equilibrium does not satisfy by construction is (8). To verify that the value of ν_0 required to satisfy this (given C_0 and C_1) is not negative, use (22) and the proposition's first stated assumption to get

$$\frac{\beta C_0}{C_1}(1-\delta) = \frac{\pi_1(1-\delta)}{1+i_0} \leq \frac{\beta A^H/A^L(1-\delta)}{1+i_0} < 1$$

So $\nu_0 = 1 - (1-\delta)\beta C_0/C_1 > 0$. $\square$

In the equilibria of Proposition 2, short-run inflation expectations that are rational but too low cause a real recession. Whether or not $i_0 = 0$ as in other models' liquidity traps depends on $\pi^\star$ and ϕ. If $\pi^\star \leq \beta A^H/A^L$, then i_0 must equal zero. If instead $\pi^\star > \beta A^H/A^L$, then the initial deflation can force i_0 to hit the zero lower bound if ϕ is large enough. These equilibria can be unambiguously labelled liquidity traps. However, i_0 can exceed zero if both $\pi^\star > \beta A^H/A^L$ and ϕ is small.

With the caveat that i_0 might be positive, I will hereafter refer to the equilibria of Proposition 2 as liquidity traps. Clearly, their multiplicity (indexed by π_1) arises from the fundamental multiplicity described above. Their traditional interpretation labels C_0 *aggregate demand.* In this story, monetary policy that is made too tight by the zero lower bound and inappropriately low inflation expectations lowers aggregate demand through (7), and this brings about an accompanying deflation. Indeed, the equilibrium construction does lead from the determination of C_0 in the bond market to the value of P_0 required to support that outcome. In this sense, the equilibrium of Proposition 2 conforms to the familiar pattern of other new Keynesian models of liquidity traps.

Implicitly, Proposition 2 embodies the now conventional policy prescription for avoiding a liquidity trap: Somehow convince the public that π_1 will exceed the inverse of the natural rate of interest $(\beta A^H/A^L)$. This would allow Joseph to set nominal bonds' real return to the value required by the flexible price allocation with a positive nominal interest rate. If we set $\pi^\star > \beta A^H/A^L$, $\phi > 1$ and *presume* that $\pi_t = \pi^\star$ in the long run (for large t), then the inflation target's presumed long-run credibility and the Taylor principle mathematically guarantee that $\pi_1 = \pi^\star$. In light of Cochrane's (2011) extensive critique of this scheme's economic foundations (or lack thereof), I choose not to adopt it as a useful resolution of equilibrium indeterminacy. Nevertheless, it is worth understanding how such an "active" monetary policy selects from the equilibrium set if only because it is commonly embodied in applied work.

4.4 Severe Famines

Although storage occurs in the flexible-price allocation when the expected famine is severe $(\beta A^H/A^L > 1/(1-\delta))$, it need not do so when there are nominal rigidities. Intuitively, the possibility of storage is irrelevant when bonds offer a higher real rate of return. To see this more formally, note that Proposition 1's preconditions did not exclude the case of a severe famine. Therefore, it guarantees that $S_1 = 0$ in all equilibria with $\pi_1 < 1/(1-\delta)$; even if $\tilde{S}_1 > 0$. The following corollary guarantees that such liquidity traps exist.

Corollary 2.1. *Suppose that*

- $1/(1-\delta) < \beta A^H/A^L$,
- $\pi^\star \in [\beta, \beta 2^{\frac{1}{\varepsilon-1}})$, *and*
- *(23) holds;*

and select $\pi_1 \in [\beta, 1/(1-\delta))$, *then the equilibrium of Proposition 2 exists.*

Proof. The construction preceding Proposition 2 goes through without modification if $1/(1-\delta) < \beta A^H/A^L$. To verify that the value of ν_0 required to satisfy (8) is not negative, use (22) and the upper bound for π_1 to get

$$\frac{\beta C_0}{C_1}(1-\delta) = \frac{\pi_1(1-\delta)}{1+i_0} < \frac{1}{1+i_0} \leq 1$$

So again, $\nu_0 = 1 - (1-\delta)\beta C_0/C_1 > 0$. □

Clearly, achieving the flexible-price allocation requires $\pi_1 \geq 1/(1-\delta)$. Such fortunate inflation expectations are consistent with achieving the flexible-price allocation, but setting $\pi_1 \geq 1/(1-\delta)$ opens up the possibility of falling into a confidence recession in which $C_0 < \tilde{C}_0$, $N_0 < \tilde{N}_0$, and $S_1 < \tilde{S}_1$ even though $(1+i_0)/\pi_1 = (1-\delta)$. To construct one, select

$$C_1 \in \left[\frac{A^L}{\theta}\frac{\varepsilon-1}{\varepsilon}, \tilde{C}_1\right). \tag{25}$$

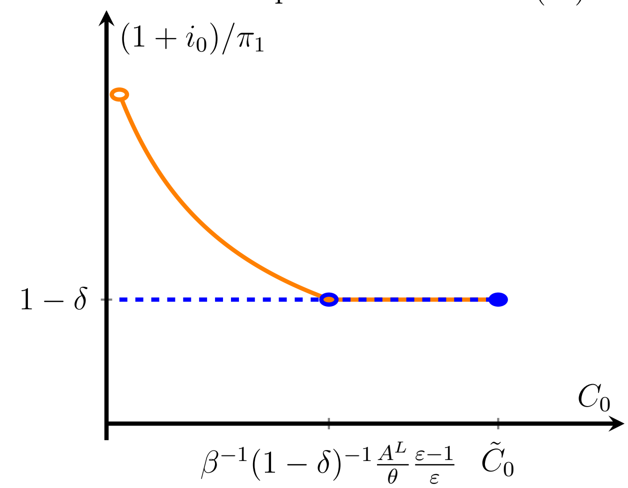
The lowest value for C_1 in this range is that from the flexible-price allocation without storage. Proceeding, suppose that $(1+i_0)/\pi_1 = (1-\delta)$. (Below, this will be confirmed for i_0 given by (13) and some $\pi_1 \leq \pi^\star$.) With this supposition and the given value of C_1, (7) determines

$$C_0 = \beta^{-1}(1-\delta)^{-1}C_1.$$

The assumption in (23) guarantees that the lowest possible value of C_0 exceeds its lower bound from (20) because $\beta(1-\delta) < 1$. Applying Section 4.1's analysis then determines values for W_0, P_0^0, and P_0 consistent with (6), (17), and (18). The optimal labor supply condition requires that $N_1 = 0$ whenever C_1 exceeds its lower bound in (25), so the resulting requirement that year 0 production equals $C_0 + S_1 = C_0 + C_1/(1-\delta)$ determines N_0.

The equilibrium construction continues by feeding $\pi_0 \equiv P_0\pi^\star$ into the interest rate rule (13) to yield i_0. With this in hand, the unique value of π_1 consistent with both bond-market clearing and the household's non-negative choice for S_1 is $(1+i_0)/(1-\delta) \geq 1/(1-\delta)$. With this, (**??**) and (**??**) yield i_t and π_t for $t \geq 2$. The upper bound on the return from storage in (15) allows us to set the remaining values of C_t, S_t, and N_t to $\tilde{C}_t$, $\tilde{S}_t$, and $\tilde{N}_t$ respectively. Again, I summarize this equilibrium with a

Figure 3: The Intertemporal-Substitution (IS) Curve



Proposition 3. *Suppose that $\beta A^H/A^L < 1/(1-\delta)$, and select C_1 from the interval in (25). Then there exists an equilibrium with the given value of C_1 and $C_0 < \tilde{C}_0$. In this equilibrium, $(1+i_0)/\pi_1 = (1-\delta)$. Furthermore, C_0 and N_0 are strictly increasing with the chosen value for C_1.*

In these confidence recessions, households' choices of storage are strategic complements: One household's optimal choice of S_1 increases with all other households' choices. This complementarity arises from the fundamental multiplicity combined with the endogeneity of C_1.[9]

Figure 3 summarizes these results with an intertemporal-substitution (IS) curve, which gives the combinations of real interest rates and consumption consistent with the equilibria of Corollary 2.1 and Proposition 3. An empty orange circle denotes the limiting equilibrium as C_0 is driven to its lower bound in (20), while a solid blue circle marks the equilibrium that implements the flexible-price allocation.[10] If bonds' real return exceeds $1-\delta$, then $S_1 = 0$. Over this range, the IS curve inherits its shape from the Euler equation (7). These equilibria are the model's liquidity

[9]One might hypothesize that the equilibrium multiplicity demonstrated by Proposition 3 arises from the anticipation of different paths for inflation and nominal interest rates. To show that this is incorrect, set ϕ to zero. In this special case of extremely passive interest-rate policy, the equilibria of Proposition 3 share common inflation and interest-rate sequences: $\pi_t = \pi^\star$ and i_t always equals the "natural" interest rate in the intercept of (13). This justifies the claim at the start of Section 4.1 that confidence recessions can share interest-rate and inflation sequences.

[10]Because Proposition 3's interval for C_1 is open to the right, it does not include this flexible-price equilibrium. However, this equilibrium is indeed the limit as $C_1 \to \tilde{C}_1$.

traps from Corollary 2.1. Corollary 2.1 says that Joseph could choose any of these equilibria if he had complete control over π_1.

If instead bonds' real return equals $(1-\delta)$, Proposition 3 tells us that any

$$C_0 \in [\beta^{-1}(1-\delta)^{-1}\frac{A^L}{\theta}\frac{\varepsilon-1}{\varepsilon}, \tilde{C}_0)$$

is consistent with equilibrium. Therefore, the IS curve becomes horizontal. All points on this horizontal segment to the left of the blue dot represent confidence recessions. In all but one of these, the transitional equilibrium point denoted by a blue circle filled with orange where the IS curve's horizontal segment begins, $S_1 > 0$. The transitional equilibrium is both a liquidity trap and a confidence recession. In it, the non-negativity constraint on storage does not bind. Nevertheless, the household chooses $S_1 = 0$.

The IS curve's horizontal segment suggests that Joseph might not be able to avoid a confidence recession *even if monetary policy could somehow determine* π_1. Indeed, this is the case if either $\pi^\star \leq 1/(1-\delta)$ (so the zero lower bound binds even when the equilibrium implements the flexible-price allocation) or $\phi = 0$ (so i_0 does not respond to any initial recession). In these cases, all of the equilibria on the IS curve's horizontal segment share the same value of π_1, so Joseph cannot guarantee that the best of them is implemented by appropriately guiding short-run inflation expectations.[11] If both $\pi^\star > 1/(1-\delta)$ and $\phi > 0$, then the only equilibrium with $\pi_1 = \pi^\star$ implements the flexible-price allocation. In this special case, Joseph can indeed guide the economy to the flexible-price allocation if he (somewhat magically) could set $\pi_1 = \pi^\star$. Given *any* interest rate rule, properly-implemented JQE can accomplish this goal

5 Josephean Quantitative Easing

Incorporating JQE into the analysis requires only relaxing the assumption that $B_1 = Q_1 = 0$ and modifying the resource constraint to account for Joseph's storage.

$$\frac{A_t N_t}{\frac{1}{2}\left(\frac{P_t^0}{P_t}\right)^{-\varepsilon} + \frac{1}{2}\left(\frac{P_t^1}{P_t}\right)^{-\varepsilon}} = C_t + (S_{t+1} + Q_{t+1})/(1-\delta) - S_t - Q_t$$

[11] If $\phi > 1$

I maintain the assumption that Joseph sets $B_t = Q_t = 0$ for $t \geq 2$ to mimic the flexible-price allocation's absence of storage after the famine's first year.

Given total storage, its decomposition between S_{t+1} and Q_{t+1} is of no consequence to the household. Nevertheless, Joseph might prefer public storage because setting $B_1 > 0$ and Q_1 to the resulting real goods accumulated can impact the equilibrium *set* through two channels. First, the accumulation of a primary surplus and its offsetting liabilities allows the fiscal theory of the price level to determine P_1. In turn, this requires Joseph's real cost of funds (the real return on nominal bonds) to equal the real return on his storage investments. In the liquidity traps proven to exist by Corollary 2.1, the real return on nominal bonds exceeds the cost of storage. Therefore, these are inconsistent with even a small amount of JQE. I summarize this first channel in the following

Proposition 4. *If $B_1 > 0$, then in any equilibrium,*

$$\frac{1+i_0}{\pi_1} = (1-\delta). \tag{26}$$

Furthermore, there exists no equilibrium with consumption and prices equal to those from an equilibrium proven to exist by Corollary 2.1.

Proof. To prove that Equation (26) must hold in an equilibrium with $B_1 > 0$, use (4) for year 0, $Q_1 = (1-\delta)B_1/((1+i_0)P_0)$, to eliminate Q_1 from the same equation for year 1, $B_1/P_1 = Q_1$. Remove B_1 from the resulting equation and rearrange. For the second assertion, note that in the referenced equilibria we have

$$1 - \beta\frac{C_0}{C_1}(1-\delta) = \nu_0 > 0 = 1 - \beta\frac{C_0}{C_1}\frac{1+i_0}{\pi_1},$$

which contradicts Equation (26). □

In theory, even a small amount of JQE can substitute for inflation-expectations management by other (unmodeled) means, such as the communications protocols of an inflation-targeting regime. In practice, its efficacy at this task depends on whether or not households expect (4) to hold in year 1. If Joseph could recover any capital loss incurred from deflation by taxing households, then deflation might occur in equilibrium. In that case, nominal bonds' real rate of return would exceed the real rate of return on storage, so Proposition 4's conclusions would not hold.

With this potentially important caveat in place, we can proceed to consider the second channel for JQE to influence the equilibrium set: The primary surplus Q_1 places a floor on C_1. This in turn bounds C_0 from below and thereby eliminates confidence recessions with consumption beneath the bound. Unsurprisingly, this channel's efficacy depends on the magnitude of B_1, not just its sign.

To develop this in more detail, define

$$\underline{C}_0 \equiv (1-\delta)^{-1}\beta^{-1}\frac{A^L}{\theta}\frac{\varepsilon-1}{\varepsilon} \text{ and } \underline{P}_0 \equiv P_0(\underline{C}_0).$$

These are the initial consumption and price level in the worst equilibrium of Proposition 3; which is the transitional equilibrium in Figure 3's IS curve. (The underlines indicate that these are lower bounds.) With this notation, we can state

Proposition 5. *Define* $\underline{B}_1 \equiv \beta\underline{C}_0\underline{P}_0 \max\{1, \pi^\star(1-\delta)\underline{P}_0^\phi\}$. *For each* $B_1 \in [\underline{B}_1, \pi^\star\tilde{C}_1]$, *there exists a threshold* $\bar{C}_0(B_1)$ *for* C_0 *such that*

1. *there is no equilibrium with* $C_0 < \bar{C}_0(B_1)$;
2. *any equilibrium of Proposition 3 with* $C_0 \geq \bar{C}_0(B_1)$ *has a corresponding equilibrium with the given value of* B_1 *and the same sequences for* C_t *and* N_t;
3. $\bar{C}_0(\bar{B}_1) = \underline{C}_0$;
4. $\bar{C}_0(B_1)$ *is strictly increasing in* B_1; *and*
5. $\bar{C}_0(\pi^\star\tilde{C}_1) = \tilde{C}_0$.

Proof. From Proposition 4, we know that the lowest price level in any equilibrium with $B_1 > 0$ is $\underline{P}_0$, so the largest amount of the aggregate good that Joseph can take into year 1 with the funds raised by issuing nominal bonds with redemption value B_1 is $(1-\delta)B_1/(\underline{P}_0 \max\{1, \pi^\star(1-\delta)\underline{P}_0^\phi\})$. The definition of $\underline{B}_1$ ensures that this equals $\beta(1-\delta)\underline{C}_0$ – the lowest equilibrium value of C_1 consistent with $B_1 > 0$, when $B_1 = \underline{B}_1$.

Define

$$\Upsilon(C, B) \equiv \beta(1-\delta)C - \frac{(1-\delta)B}{P_0(C)\max\{1, (1-\delta)\pi^\star P_0(C)^\phi\}},$$

and with this define $\bar{C}_0(B_1)$ implicitly from $\Upsilon(\bar{C}_0(B_1), B_1) = 0$. That is, $Q_1 = C_1$ if $C_0 = \bar{C}_0(B_1)$. We know that $\Upsilon(\underline{C}_0, B_1) \leq 0$, because issuing $B_1 \geq \underline{B}_1$ bonds facing the same price level and nominal interest rate as the worst equilibrium of Proposition 3 and investing the proceeds yields $Q_1 \geq \beta(1-\delta)\underline{C}_0$. Alternatively $\Upsilon(\tilde{C}_0, B) \geq 0$, because the assumed upper bound on B_1 keeps $Q_1 \leq \beta(1-\delta)\tilde{C}_0$ when the price level and interest rate are those from the flexible-price allocation. Furthermore, $\Upsilon(C, B)$ is strictly increasing with C. t Therefore, there exists exactly one value of $\bar{C}_0(B_1)$ that satisfies its definition. With this in hand the proposition's third and fifth conclusions can be directly verified by substitution into $\Upsilon(C, B)$, and the fourth conclusion follows from noting that increasing B strictly decreases this function.

Proposition 4 immediately implies that no equilibrium exists with $C_0 < \underline{C}_0$. The first conclusion's demonstration only requires us to demonstrate the same if $C_0 \in [\underline{C}_0, \bar{C}_0(B_1))$. For this, assume the opposite.

- Since $P_0(C_0)$ is strictly increasing, we know that $P_0 < P_0(\bar{C}_0)$ and $1 + i_0 \leq \max\{1, (1-\delta)\pi^\star P_0(\bar{C}_0(B_1))^\phi\}$. Therefore, $Q_1 > \beta(1-\delta)\bar{C}_0(B_1) > \beta(1-\delta)C_0$. From Proposition 4, (7), and (8), we know that $C_1 = \beta(1-\delta)C_0$, so $Q_1 > C_1$.

- The upper bound on the return to storage in (15) can be rewritten as $1 \geq \beta(1-\delta)\tilde{C}_1/\tilde{C}_2$. Since in the hypothesized equilibrium $C_1 < \tilde{C}_1$ and because (6) requires that $C_2 \geq \tilde{C}_2$ in any equilibrium; we know that $1 > \beta(1-\delta)C_1/C_2$. From (8) and the complementary slackness condition, we therefore can conclude that $S_2 = 0$.

Because $Q_1 > C_1$ and $Q_2 = 0$ by assumption, the resource constraint requires $S_2 > 0$. Therefore, these two conclusions of assuming that $C_0 < \bar{C}_0(B_1)$ contradict each other.

All that remains to be demonstrated is the Proposition's second conclusion. Begin this by adopting the original equilibrium's sequences for C_t, N_t, W_t, P_t^0, P_{t+1}^1, P_t, D_t, and i_t. Then, set $Q_1 = (1-\delta)B_1/(P_0(1+i_0))$. Since $C_0 \geq \bar{C}_0(B_1)$, $Q_1 \leq \beta(1-\delta)\bar{C}_0(B_1) \leq C_1$. Therefore, we can set $S_1 = C_1 - Q_1$ without violating the non-negativity constraint on storage. To complete the candidate equilibrium, set $S_t = B_t = Q_t = 0$ for all $t \geq 2$. The sequences for C_t, N_t, B_{t+1}, and S_{t+1} solve the household's utility maximization problem given the sequences for D_t, W_t, P_t, and i_t; because the household is indifferent between directly accumulating C_1 and indirectly doing so by purchasing bonds with the same rate of return. Firms' original pricing decisions remain optimal; and B_{t+1}, Q_{t+1}, and i_t satisfy (4). Therefore, the candidate is indeed an equilibrium. $\square$

To summarize, the two channels for JQE allow Joseph to destroy all liquidity traps and confidence recessions by setting $B_1 = \pi^\star \tilde{C}_1$. A slightly smaller balance-sheet expansion eliminates some confidence recessions, but leaves those with C_0 slightly below $\tilde{C}_0$ in place. Finally, a very small balance sheet expansion eliminates liquidity traps by equating nominal bonds' real return with that of storage, but it leaves room for households to coordinate on a confidence recession with too little saving. Regardless, JQE requires no commitment to time-inconsistent interest-rate or balance-sheet policies.

Ricardian equivalence with policy commitment provides the point of departure for most theoretical discussions of QE. For example, Eggertsson and Woodford (2003) "argue that the possibility of expanding the monetary base through central bank purchases of a variety of types of assets does little if anything to expand the set of feasible paths for inflation and real activity that are consistent with equilibrium under some (fully credible) policy commitment."[12] Nothing in this paper contradicts this assertion. Here, JQE potentially improves economic outcomes by *shrinking* the set of feasible paths that are consistent with equilibrium.

Previous models of liquidity traps featuring policy-relevant QE have either featured frictions that impede private borrowing and lending (Cúrdia and Woodford, 2011; Gertler and Karadi, 2011), financial markets segmented by asset maturity (Chen, Cúrdia, and Ferrero, 2012), or limited commitment that can be overcome somewhat by manipulating the maturity structure of the monetary authority's balance sheet (Bhattari, Eggertsson, and Gafarov, 2014). In all of those approaches, QE can potentially improve a given equlibrium outcome. In contrast, JQE has no impact on an equilibrium if it would have occurred anyways. Instead, it guides households' expectations towards the flexible-price allocation. The quality of that substitution depends on how close B_1 is to $\pi^\star \tilde{C}_1$.

With segmented financial markets, the monetary authority can influence assets' relative prices by changing their relative supplies. Accordingly, empirical investigations of QE have concentrated on measuring its impact on asset prices. The present economy is Ricardian, but it would be incorrect to conclude from that fact

[12] See Page 143 of Eggertsson and Woodford (2003).

alone that JQE does not influence asset prices. When it is successful, in the sense that it eliminates a confidence recession that would have otherwise occurred, long-dated interest rates fall because both C_0 and C_1 rise while all C_t for $t \geq 2$ remain unchanged. Although Eggertsson and Woodford (2003) emphasize that forward-guidance can expand current economic activity by reducing long-dated real interest rates, the analogous reduction in this model is a consequence of such an economic expansion; not its cause.

6 An Open Economy Interpretation

Although I have developed the analysis of JQE in a closed economy, the most empirically-relevant interpretation of the model's linear storage technology is as a representation of using international trade to achieve intertemporal substitution. For this, suppose that the economy is small relative to a large foreign sector. The aggregate good can be shipped either to or from the foreign sector at the iceberg transportation cost τ. The real rate of return available in the foreign sector is r^f. Then, if we define δ with

$$(1-\delta) = (1-\tau)^2(1+r^f);$$

we can interpret storage as shipping aggregate goods abroad, selling them, investing the proceeds in foreign bonds, and repatriating the proceeds in the next year by shipping the aggregate good back home. Two notable differences exist between this open economy extension and the original closed economy. First, the restriction that $S_t \geq 0$ should be interpreted as a limit on uncollateralized international borrowing. Second, the possibility of importing the good eliminates all equilibria in which $P_0 < (1-\tau)$. Thus, greater openness to trade improves the worst possible equilibrium but does not necessarily eliminate the possibility of falling into a liquidity trap. With this caveat, any equilibrium of the closed economy has a doppelganger in the small-open-economy extension.[13]

[13] If we suppose that the foreign sector uses a currency subject to no inflation with a price-level of 1, then we can introduce a market for the exchange of home and foreign currencies. If the aggregate good is shipped in either direction, then the price of foreign currency in units of home currency is $e_t = P_t/(1-\tau)$. Therefore, international trade never occurs unless purchasing-power parity holds up to the constant iceberg transportation cost. If there is no international trade, then equilibrium only requires $e_t \in [P_t(1-\tau), P_t/(1-\tau)]$. One might conclude that JQE depreciates

In the open economy, JQE mimics the monetary mechanics of a sterilized competitive devaluation (swap interest-bearing domestic liabilities for foreign assets), and the resulting current account surplus matches the conventional Mundell-Fleming model's predictions. This paper is not the first to notice the strong resemblance between sterilized interventions and quantitative easing. For example, Rajan (2014) labels such interventions (tongue in cheek) as "Quantitative External Easing" (QEE). He reports

> Indeed, some advanced economy central bankers have privately expressed their worry to me that QE "works" primarily by altering exchange rates, which makes it different from QEE only in degree rather than in kind.[14]

It is inconceivable that these anonymous central bankers had JQE in mind when confiding with Rajan, but from this model's perspective QE and QEE are indeed cut from the same cloth. Nevertheless, changes to the real exchange rate play no role in JQE's effectiveness. Instead, it works by coordinating home-country households' savings decisions and thereby enabling them to substitute consumption intertemporally using international trade. As noted in the introduction, the foreign sector's initial current-account deficit and its eventual reversal are not side effects of JQE. Together, they are its goal.

If the foreign sector itself also faces a Keynesian shortfall in aggregate demand, then JQE can easily turn into a beggar-thy-neighbor affair. However, without foreign-sector inefficiencies it results in a Pareto-efficient allocation of *world* resources. This suggests that the international monetary policy cooperation advocated by Rajan (2014) can indeed improve worldwide macroeconomic performance when these two possibilities can be distinguished. Further investigation of this point is certainly worthwhile, but it requires an explicit model of the foreign sector that lies beyond this paper's scope.

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the currency if it eliminates an equilibrium with $S_1 = 0$ and $e_t < P_t/(1-\tau)$ that otherwise would have occurred. However, such a depreciation is not logically necessary and so cannot be said to cause the home country's initial current-account surplus.

[14] Page 6 of Rajan (2014)

7 Conclusion

A shock to the demand for real assets leads households to accumulate goods in storage for consumption, just as in the biblical Joseph story, when prices are flexible. Price stickiness (modeled here with sticky price plans) can disrupt this outcome and send the economy into a recession *even when nominal bonds' real return is consistent with the flexible-price allocation.* In this sense, conventional interest-rate policy and forward guidance that manipulates inflation expectations cannot necessarily guide the economy to its potential. QE that purchases *real* assets, JQE, can fill this gap. JQE puts a floor on future national wealth and consumption, and the expectation of high future consumption raises current consumption and output. Ironically, the full solution to the "paradox of thrift" coordinates an *increase* in savings.

Although the Federal Reserve has begun to end its balance-sheet expansion, QE's relevance for global monetary policy has not abated. Under the heading of "Quantitative and Qualitative Easing," the Bank of Japan currently purchases about ¥60 to ¥70 Trillion per month of securities. About ¥50 Trillion of these are Japanese sovereigns, while the remainder are a wide variety of *private* assets. The analysis of JQE suggests that reversing these amounts could improve Japanese economic performance by increasing the economy's real wealth accumulation. To date, the ECB's quantitative easing has more closely resembled JQE. Its *Targeted Long-Term Refinancing Operations* directly extend credit to banks that themselves expand credit to private non-financial borrowers, and it just began direct purchases of securities backed by Euro-area non-financial private assets.[15] This paper provides a possible justification for the ECB's quantitative easing: Increasing European households' real wealth boosts their expectations of future consumption and thereby improves current economic performance.

[15] See the Introductory Statement to Mario Draghi's press conference on 4 September 2014.

Appendix: Increasing Marginal Costs of Storage

This appendix replaces the linear storage technology employed in the text with a concave technology that is represented by a convex cost function. So that the profits associated with the resulting scarce storage opportunities are properly accounted, I take the storage technology out of the households' hands and add banks to the model. There is a unit mass of banks, each of which can produce S units of the aggregate good next year by investing $\Xi(S)$ units of the aggregate good in the storage technology. This input-requirement/cost function is twice differentiable everywhere, and satisfies $\Xi(0) = 0$, $\Xi'(0) > 0$, and $\Xi''(S) > 0$. As did the households' investments in the model's text, banks' investments must satisfy $S_{t+1} \geq 0$. This technology's analogue to (3) is

$$\frac{\beta}{\Xi'(0)} < 1. \tag{A1}$$

This ensures that storage is not worthwhile when consumption is constant. Finally, the aggregate resource constraint with flexible prices and this technogy is

$$A_t N_t = C_t + \Xi(S_{t+1}) - S_t.$$

Banks finance their inputs by issuing nominal bonds. In the next year, they use the proceeds from selling storage technology's output to retire them. Any remaining proceeds are returned to the representative household as dividends. Just like the economy's firms, these dividends can be negative because banks face unlimited liability. Banks in year t choose S_{t+1} to maximize real dividends in period $t+1$, $S_{t+1} - \Xi(S_{t+1})(1 + i_t)/\pi_{t+1}$. If we use ω_t to denote the nonnegativity constraint's Lagrange multiplier, then the first-order necessary condition for this problem is

$$1 + \omega_t = \Xi'(S_{t+1})(1 + i_t)/\pi_{t+1}. \tag{A2}$$

If $\omega_t > 0$, then the cost of storage investment exceeds its benefit, so $S_{t+1} = 0$. In the special case with $\Xi'(S) \equiv 1/(1 - \delta)$, this can be combined with (7) to get (8), with $\nu_t = \omega_t(1 - \delta)\beta C_t/C_{t+1}$.

I The Flexible-Price Allocation

With the text's linear storage technology, famines were classified into severe and moderate depending on whether or not the flexible price allocation set $\tilde{S}_1 > 0$. With the more general convex cost of storage, it is useful to divide famines into three categories; severe, intermediate, and moderate. In a severe famine, $\tilde{S}_1 > 0$ and $\tilde{N}_1 = 0$. That is the household saves in order to take a vacation during the famine's first year. In an intermediate famine, $\tilde{S}_1 > 0$ but $\tilde{N}_1 > 0$. The household uses storage to reallocate hours worked from year 1 to year 0 and thereby save on its utility cost, but the consumption profile is the same as that in a mild famine, when $S_1 = 0$ and the storage technology is irrelevant.

I.1 Mild Famines

To define a mild famine, replace (12) with

$$\frac{\beta A^H}{A^L} \leq \Xi'(0). \tag{A3}$$

When this holds, the flexible-price allocation is exactly the same as that in the text.

I.2 Severe Famines

To define a severe famine, first denote the flexible-price allocation's consumption for years $t \geq 1$ with a *mild* anticipated famine using

$$C^\star \equiv \frac{A^L}{\theta}\frac{\varepsilon - 1}{\varepsilon}.$$

In a severe famine, the marginal cost of increasing storage at $C^\star$ is less than its benefit when $C_1 = C^\star$. That is

$$\frac{\beta A^H}{A^L} > \Xi'(C^\star) \tag{A4}$$

(As in the text, $\tilde{C}_0$ depends only on A^H, ε, and θ.) This guarantees that $\tilde{S}_1 \geq \tilde{C}_1$. Suppose for the moment that $\tilde{S}_1 = \tilde{C}_1$ as in the text. With this, combining banks' profit maximization condition with (7) gives

$$\beta \tilde{C}_0 = \tilde{C}_1 \Xi'(\tilde{C}_1). \tag{A5}$$

This implicitly defines $\tilde{C}_1^s$. With this in hand, replacing (15) with

$$\frac{\beta \tilde{C}_1}{\tilde{C}_2} < \Xi'(0) \tag{A6}$$

guarantees that indeed $S_2 = 0$.[16] Aside from this modification to $\tilde{C}_1$ and the attendant change to $\tilde{N}_0$, the flexible-price allocation with a severe famine is the same as that in the text.

I.3 Intermediate Famines

With the linear technology of the text, $\Xi'(0) = \Xi'(C^\star)$, so either (A3)or (A4) must hold. The assumption that $\Xi''(S) > 0$ creates a third case.

$$\Xi'(0) < \frac{\beta A^H}{A^L} \leq \Xi'(C^\star) \tag{A7}$$

This says that the marginal benefit of storage when $C_0 = \tilde{C}_0$ and $C_1 = C^\star$ exceeds its marginal cost when there is no storage but is less than its marginal cost when storage equals or exceeds $C^\star$. In this case, the flexible-price allocation's consumption profile equals that from a mild famine. To retrieve $\tilde{S}_1$, use banks' profit maximization condition.

$$\frac{\beta \tilde{C}_0}{\tilde{C}_1} = \Xi'(\tilde{S}_1) \tag{A8}$$

With $\tilde{S}_1^i$ in hand, the resource constraint immediately yields $\tilde{N}_0$ and $\tilde{N}_1$. All of the allocation's other quantities equal those from a mild famine.

[16] Together, (A4) and (A6) require that

$$\beta^2 A^H / A^L \in \left(\beta \Xi'(C^\star), \Xi'(0)\Xi'(C^\star)\right).$$

This interval is non-empty because (A1) guarantees that $\beta < \Xi'(0)$ and (6) and the convexity of $\Xi(\cdot)$ together ensure that $\Xi(\tilde{C}_1) \geq \Xi(C^\star)$. Therefore, the assumptions in (A4) and (**??**) can be simultaneously satisfied.

II Equilibria with Nominal Rigidities

Much of the text's analysis of equilibria with nominal rigidities applies to the model with $\Xi(S)'' > 0$ with little or no modification. Section 4.1's characterization of the Phillips curve has nothing to do with the storage technology, and adapting the flexible-price allocation replicator of Section 4.2 to this economy is a simple exercise. As in the text, adding nominal rigidities introduces a possible production inefficiency into the resource constraint.

$$\frac{A_t N_t}{\frac{1}{2}\left(\frac{P_t^0}{P_t}\right)^{-\varepsilon} + \frac{1}{2}\left(\frac{P_t^1}{P_t}\right)^{-\varepsilon}} = C_t + \Xi(S_{t+1}) - S_t. \tag{A9}$$

II.1 Mild Famines

With this change, the analogues to Propositions 1 and 2 are

Proposition A1. *There exists no equilibrium in which both $S_1 > 0$ and $\pi_1 < \Xi'(0)$.*

Proof. Suppose otherwise. Since $S_1 > 0$, we can rearrange (A2) to get

$$1 + i_0 = \frac{\pi_1}{\Xi'(S_1)} < \frac{\pi_1}{\Xi'(0)} < 1$$

This violates the zero-lower bound on interest rates. □

Proposition A2. *Suppose that*

- $\beta A^H / A^L < \Xi'(0)$,
- $\pi^\star \in [\beta, \beta 2^{\frac{1}{\varepsilon-1}})$, *and*
- *(23) holds ;*

and select $\pi_1 \in [\beta, \beta A^H/A^L)$. Then there exists a equilibrium in which π_t equals the given value of π_1 for $t = 1$, $S_{t+1} = 0$ for all $t \geq 0$, $C_0 < \tilde{C}_0$ and $C_t = \tilde{C}_t$ for all $t \geq 1$.

Proof. The proposed equilibrium is that constructed in the text, and the only requirement that the proposed equilibrium does not satisfy by construction is (A2).

To verify that the value of ω_0 required to satisfy this is not negative, use the upper bound for π_1 and proposition's first stated assumption to get

$$\frac{1+i_0}{\pi_1}\Xi'(0) > \frac{1+i_0}{\beta A^H/A^L}\Xi'(0) > 1.$$

So $\omega_0 = \frac{1+i_0}{\pi_1}\Xi'(0) - 1 > 0$. □

The analogue to Corollary 2.1 applies to both intermediate and severe famines

Corollary A2.1. *Suppose that*

- $\Xi'(0) < \beta A^H/A^L$,
- $\pi^\star \in [\beta, \beta 2^{\frac{1}{\varepsilon-1}})$, *and*
- *(23) holds;*

and select $\pi_1 \in [\beta, \Xi'(0))$, *then the equilibrium of Proposition A2 exists.*

Proof. The proposed equilibrium is from construction preceding Proposition 2, and this goes through without modification if $\Xi'(0) < \beta A^H/A^L$. To verify that the value of ω_0 required to satisfy (A2) is not negative, use the upper bound for π_1 to get

$$\frac{1+i_0}{\pi_1}\Xi'(0) > 1+i_0 \geq 1.$$

So again, $\omega_0 = \frac{1+i_0}{\pi_1}\Xi'(0) - 1 > 0$. □

II.2 Intermediate Famines

With an intermediate famine, a type of recessionary equilibrium arises that does not appear in the model of the text, a *storage recession*. To construct one, set $C_1 = \tilde{C}_1$ and select

$$C_0 \in [\frac{\tilde{C}_1\Xi'(0)}{\beta}, \tilde{C}_0]. \tag{A10}$$

The lower end of this interval is the largest C_0 from an equilibrium of Corollary A2.1, and this exceeds $2^{\frac{1}{1-\varepsilon}}\tilde{C}_0$. Therefore, we can apply Section 4.1 to find values for P_0^0, P_0, and W_0 consistent with any such C_0, (6), (17), and (18). Continuing, combine (7) with (A2) to yield

$$\frac{\beta C_0}{\tilde{C}_1} = \Xi'(S_1)$$

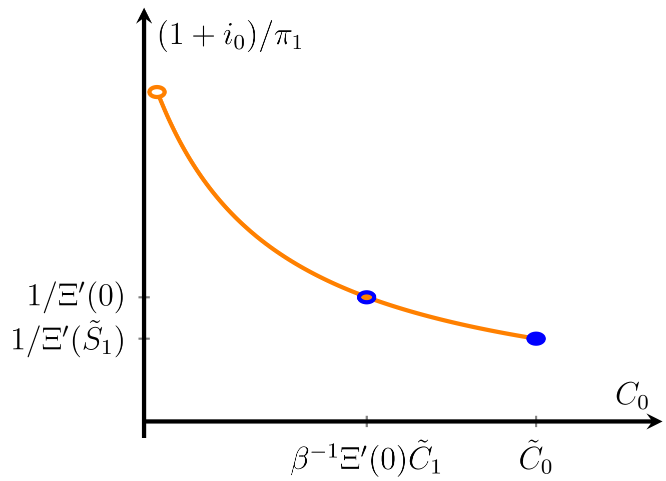

Figure A1: The IS Curve with an Intermediate Famine

This implicitly defines S_1 as an increasing function of C_0. This can be no greater than $\tilde{S}_1$ because $C_0 \leq \tilde{C}_0$. To complete the equilibrium allocation's construction, set $C_t = \tilde{C}_t$, $S_t = 0$, and $N_t = \tilde{N}_t$ for $t \geq 2$.

To get this equilibrium allocation's accompanying nominal interest rates and prices, plug $\pi_0 \equiv \pi^\star P_0$ into (13) to get i_0; and use this and the real interest rate implied by S_1 to set π_1.

$$\pi_1 = (1+i_0)\Xi'(S_1) \tag{A11}$$

Equations (13) and (24) then give i_t and π_t for $t \geq 2$. I summarize this with a

Proposition A3. *Suppose that $\beta A^H/A^L > \Xi'(0)$, and select C_0 from the interval in (A10). Then there exists an equilibrium with the given value of C_0 and $C_1 = \tilde{C}_1$. Furthermore, S_1 is strictly increasing with the chosen value for C_0.*

Economically, a storage recession occurs when a high real interest rate (supported by a lack of real investment) resolves the fundamental multiplicity with a low value of C_0.

Figure A1 plots the IS curve for the case with an intermediate famine. As in Figure 3, the empty orange circle indicates the limit of the liquidity trap equilibria as C_0 is driven to its lower bound of $2^{\frac{1}{1-\varepsilon}}\tilde{C}_0$, and the solid blue circle denotes the equilibrium that implements the flexible-price allocation. For real interest rates above $1/\Xi'(0)$, the economy is in a liquidity trap, and below that but above $1/\Xi'(\tilde{S}_1$ it is in a storage recession. The blue circle filled with orange is the transitional equilibrium that falls into both of these categories. This IS curve is kinked at the analogous transitional equilibrium of Figure 3. Here it is not because all of the equilibria share a common value of $\tilde{C}_1$ and differ only in assets' real interest rates.

II.3 Severe Famines

Since both Corollary 2.1 and Proposition 3 apply when the anticipated famine is severe, either a liquidity trap or a storage recession is possible in this case. To construct a confidence recession, begin by selecting

$$C_1 \in \left[C^\star, \tilde{C}_1\right). \tag{A12}$$

Then set $S_1 = C_1$ and $C_0 = \beta^{-1} C_1 \Xi'(C_1)$. Given C_0, retrieve P_0 from the Phillips curve and then get W_0 and P_0^0 from (6) and (17). The resource constraint in (A9) then determines N_0, and $N_1 = 0$. The interest-rate rule in (13) gives i_0; and setting $\pi_1 = (1 + i_0)\Xi'(C_1)$ ensures that both (7) and (A2) are satisfied. For $t \geq 2$; setting $S_t = 0$, $C_t = \tilde{C}_t$; $N_t = \tilde{N}_t$, π_t using (24), and i_t using (13) completes the equilibrium construction. The upper bound on the return to storage in (A6) ensures that this corner solution for S_2 maximizes bank profits given C_0 and C_1. With this construction in hand, I can state the following analogue to Proposition 3.

Proposition A4. *Suppose that $\Xi'(C^\star) < \beta A^H / A^L$, and select C_1 from the interval in (A12). Then there exists an equilibrium with the given value of C_1 and $C_0 < \tilde{C}_0$. In this equilibrium, C_0 and N_0 are strictly increasing with the chosen value for C_1.*

Figure A2 plots the IS curve for this model that is the direct analogue of that in Figure 3. As in the earlier IS curves, the empty orange circle indicates the limit attaned from driving C_0 to its lower bound, and the solid blue dot marks the equilibrium that implements the flexible-price allocation. Blue circles are at the *two* transitional equilibria. The left equilibria is both a liquidity trap and a storage recession, while the right one is both a storage recession and a confidence recession. The IS curve is differentiable at the first one but not at the second. Unlike the one in Figure 3, the IS curve has no horizontal segment. The discussion in the text emphasized the fact that Joseph might not be able to guide the economy to the flexible-price allocation even if he (somewhat magically) could set π_1 directly. Here with $\Xi''(\cdot) > 0$, such extremely-effective guidance of inflation expectations does

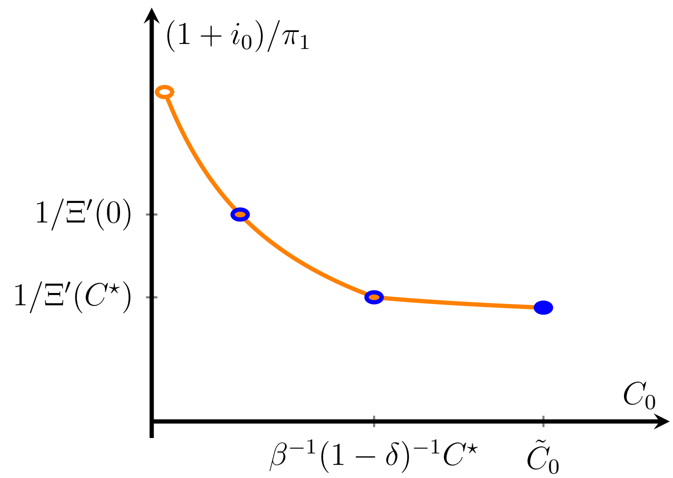

Figure A2: The IS Curve with a Severe Famine

indeed destroy all recessionary equilibria. Of course, the sensitivity of economic outcomes with respect to π_1 depends inversely on $\Xi''(\cdot)$. If this is very small, then the IS curve is nearly horizontal and very small changes in π_1 can have large impacts on C_0. To the extent that actual central bankers can only influence π_1 imprecisely and indirectly, the ability of JQE to eliminate recessionary outcomes remains of interest.

III Josephean Quantitative Easing

In the text, Joseph invested directly in the storage technology. Here, banks make this investment decision; so the specification of JQE must be suitably modified. Joseph issues nominal bonds with nominal redemption value B_1 and uses the proceeds to make loans to banks *at the same nominal interest rate.* Banks invest the proceeds of their borrowing, both from Joseph and from the private sector, and repay the loans in the next period with the proceeds of the storage technology. Since $\Xi''(\cdot) > 0$, the banks will have profits following a positive investment in storage. These are returned to the representative household as dividends.

Of course, neither the household nor the banks care about the fraction of a given investment in storage that Joseph intermediates. However, it is relevant for the equilibrium analysis because Joseph's intermediation choice places a lower bound on total borrowing. To denote that it is a lower bound, I use $\underline{B}_t$ to indicate the year t redemption value of bonds issued by Joseph in year $t-1$. I continue to use B_t to represent the household's *total* bond holdings, so the bonds directly issued by banks are worth $B_t - \underline{B}_t \geq 0$ on redemption.

The text separated the influence of JQE into two channels, which corresponded to the elimination of liquidity traps by setting $\underline{B}_1 > 0$ and the elimination of confidence recessions by setting $\underline{B}_1 = \pi^\star \tilde{C}_1$. The presence of storage recessions (which requires $\Xi''(\cdot) > 0$), makes these two channels less distinct. Therefore, I place the results analogous to Propositions 4 and 5 within the following single proposition.

Proposition A5. *Suppose that $\Xi'(0) < \beta A^H / A^L$. For each*

$$\underline{B}_1 \in \left(0, \Xi^{-1}(\tilde{S}_1) \max\{1, \pi^\star / \Xi'(\tilde{S}_1)\}\right],$$

there exists a threshold $\bar{C}_0(\underline{B}_1)$ for C_0 such that

1. *there is no equilibrium with $C_0 < \bar{C}_0(\underline{B}_1)$;*
2. *any equilibrium of Proposition A3 or A4 with $C_0 \geq \bar{C}_0(\underline{B}_1)$ has a corresponding equilibrium with some $B_1 \geq \underline{B}_1$ and the same sequences for C_t and N_t;*
3. *$\bar{C}_0(\underline{B}_1)$ is strictly increasing in $\underline{B}_1$; and*
4. *$\bar{C}_0(\Xi^{-1}(\tilde{S}_1) \max\{1, \pi^\star / \Xi'(\tilde{S}_1)\}) = \tilde{C}_0$.*

Proof. Define

$$B^\star \equiv P_0(\beta^{-1} C^\star \Xi'(C^\star)) \Xi^{-1}(C^\star) \max\{1, \pi^\star P_0(\beta^{-1} C^\star \Xi'(C^\star))^\phi / \Xi'(\tilde{S}_1)\}$$

If the anticipated famine is intermediate, then $B^\star$ equals the upper bound of the admissible interval for $\underline{B}_1$. If instead it is severe, then it lies in this interval's interior.

To define $\bar{C}_0(\underline{B}_1)$, first define

$$\Upsilon(C, B) \equiv \begin{cases} \frac{\beta C}{C^\star} - \Xi'\left(\Xi^{-1}\left(\frac{B}{\max\left\{1, \pi^\star P_0(C)^\phi / \Xi'(C^\star)\right\}}\right)\right) & \text{if } B \leq B^\star \\ \frac{\beta C}{\theta(C)} - \Xi'\left(\Xi^{-1}\left(\frac{B}{\max\left\{1, \pi^\star P_0(C)^\phi / \Xi'(\tilde{S}_1)\right\}}\right)\right) & \text{otherwise;} \end{cases}$$

where $\theta(C)$ definitionally satisfies $\theta(C)\Xi'(\theta(C)) = C$. If $B < B^\star$, then $\Upsilon(\beta^{-1} C^\star \Xi'(0), B) < 0$ and $\Upsilon(\beta^{-1} C^\star \Xi'(C^\star, B) \geq 0$. If instead $B \geq B^\star$, then $\Upsilon(\beta^{-1} C^\star \Xi'(C^\star), B) \leq 0$ and

$\Upsilon(\tilde{C}_0, B) \geq 0$. In either case, $\Upsilon(C, B)$ is strictly increasing with C, so we can define $\bar{C}_0(\underline{B}_1)$ with $\Upsilon(\bar{C}_0(\underline{B}_1), \underline{B}_1) = 0$.

Since $\Upsilon(C, B)$ is strictly decreasing with B, the proposition's third assertion is easily proven; and its fourth assertion can be easily verified by noting that the proposed threshold satisfies the definition given here. To demonstrate the first assertion, suppose that it is not true. That is, there exists an equilibrium with $C_0 < \bar{C}_0(\underline{B}_1)$. This requires $P_0 < P_0(\bar{C}_0(\underline{B}_1))$ and $1+i_0 \leq \max\{1, \pi^\star P_0(\bar{C}_0(\underline{B}_1))^\phi / \Xi'(\tilde{S}_1)\}$. Both of these changes raise the real purchasing power of Joseph's bond issuance, so $S_1 > \Xi^{-1}\left(B_1/\max\{1, \pi^\star P_0(\bar{C}_0(\underline{B}_1))^\phi/\Xi'(\tilde{S}_1)\}\right)$. Therefore,

$$\Xi'(S_1) > \Xi'\left(\Xi^{-1}\left(B_1/\max\{1, \pi^\star P_0(\bar{C}_0(\underline{B}_1))^\phi/\Xi'(\tilde{S}_1)\}\right)\right) = \frac{\beta \bar{C}_0(\underline{B}_1)}{\tilde{C}_1}$$

However, equilibrium and $S_1 > 0$ require that $\Xi'(S_1) = \beta C_0/C_1$. Therefore, we can conclude that $C_0 > \bar{C}_0(\underline{B}_1)$, a contradiction.

The final assertion to be proven is the second. This proceeds by construction, exactly paralleling the analogous demonstration for Proposition 5. □

Note that the portion of this proof that demonstrates JQE's effectiveness in destroying recessionary equilibria is somewhat different from the analogous demonstration from the text. Since $\Xi''(\cdot) > 0$, the proof proceeds by showing that JQE bounds the marginal cost of storage from below. If I allowed $\Xi''(S)$ to equal zero while still maintaining the convexity of $\Xi(\cdot)$, then this proof strategy would not be available. In this alternative, adapting the proof of the text (which bounds C_1 from below) is straightforward.

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