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Chaos, sunspots and automatic stabilizers☆

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Abstract

We consider a real business cycle model with an externality in production. Depending on parameter values, the model has sunspot equilibria, cyclical and chaotic equilibria, and equilibria with deterministic or stochastic regime switching. We study the implications of this model environment for automatic stabilizer tax systems. Stabilization is desirable because the efficient allocations are characterized by constant employment and output growth. We identify an automatic stabilizer income tax-subsidy schedule with two properties: (i) it specifies the tax rate to be an increasing function of aggregate employment, and (ii) earnings are subsidized when aggregate employment is at its efficient level. The first feature eliminates inefficient, fluctuating equilibria, while the second induces agents to internalize the externality. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

There is considerable interest in business cycle models with multiple, self-fulfilling rational expectations equilibria. These models offer a new source of impulses to business cycles – disturbances to expectations – and they offer new mechanisms for propagating and magnifying the effects of existing shocks, such as shocks to monetary policy, to government spending, and to technology. Although initial versions of these models appear to rely on empirically implausible parameter values, recent vintages are based on increasingly plausible empirical foundations.[1] The models also offer a new perspective on macroeconomic stabilization policy. Most mainstream equilibrium models suggest that, at best, the gains from macroeconomic stabilization are small.[2] In models with multiple equilibria, institutional arrangements and policy rules designed to reduce fluctuations in output may produce very large gains.[3]

This paper examines the potential gains from output stabilization in a particular business cycle model with multiple equilibria. We consider a version of the one-sector, external increasing returns model studied by Baxter and King (1991), Benhabib and Farmer (1994), and Farmer and Guo (1994, 1995). We adopt a particular parameterization of this model which allows us to obtain an analytic characterization of the global set of competitive equilibria. This set is remarkably rich, and includes sunspot equilibria, regime switching equilibria like those studied in Hamilton (1989), and equilibria which appear chaotic. There are equilibria with very poor welfare properties in this set. We obtain a closed form expression for the efficient allocations, despite the lack of convexity in the aggregate resource constraint due to the externality. We show that the efficient allocations are unique and display no fluctuations. In this sense, output stabilization is desirable in our model economy. The policy problem is to design tax rules which stabilize the economy on the efficient allocations.

We analyze some pitfalls in the design of such a tax system. For example, we show that a system which stabilizes the economy on the wrong output growth rate could actually reduce welfare.[4] This illustrates the dangers in the

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[1] See Benhabib and Farmer (1996) and Harrison (1998).

[2] See Kydland and Prescott (1980) and Sargent (1979, p. 393) for classic statements of the proposition that the gains are actually negative. Researchers who incorporate frictions like price rigidities do see some role for activist policy. But, the welfare gains tend to be small.

[3] For recent work on the implications of multiple equilibrium models for policy design, see Bryant (1981), Diamond and Dybvig (1983), Grandmont (1986), Guesnerie and Woodford (1992), Shleifer (1986), Woodford (1986b,1991), and the articles in the symposium summarized in Woodford (1994).

[4] This possibility has been discussed by Guesnerie and Woodford (1992, pp. 383–388), Shleifer (1986) and Woodford (1991, p. 103) in other contexts.

traditional approach to policy design, which tends to focus on minimizing output variance.[5]

We also display a tax system which supports the efficient allocations. We show that such a tax system must be an *automatic stabilizer*. That is, it must specify that the tax rate rise and fall with aggregate economic activity. Since the unique equilibrium under this tax regime displays no fluctuations, the tax rate that is realized in equilibrium is actually constant. If the tax system instead fixed the tax rate at this constant, and did not commit to varying the rate with the level of economic activity, then there would be multiple equilibria. Although one of these equilibria is the efficient one, there would be no guarantee of it being realized.[6]

Why does an automatic stabilizer tax system have the potential to stabilize fluctuations in our model economy? Absent tax considerations, if everyone believes the return to market activity is high, then they become more active and the externality causes the belief to be fulfilled. By undoing the effects of the externality, a tax rate that rises with increased market activity can prevent beliefs like this from being confirmed.[7]

The outline of the paper is as follows. The model is presented in Section 2. Section 3 establishes a characterization result for the set of competitive equilibria. Sections 4 and 5 analyze the deterministic and stochastic equilibria of the model, respectively. Section 6 considers the impact of an automatic stabilizer tax policy and reports the socially optimal allocations. Section 7 concludes.

2. The model

We accomplish two things in this section. In the first subsection, we describe the preferences, technology and shocks in the economy. This section also states our functional form and parameter assumptions. We discuss the competitive decentralization in the second subsection.

2.1. Preferences, technology and shocks

We only consider non-fundamental shocks, i.e., shocks which have no impact on preferences or technology. The date t realization of these shocks is

[5] An influential example is the analysis of Poole (1970), who argues that the appropriate choice of monetary policy regime depends on whether shocks emanate from financial markets or investment decisions. The criterion driving the policy design in Poole's analysis is minimization of output variance.

[6] This part of our analysis also illustrates potential dangers in the standard practice of focusing exclusively on local uniqueness properties of equilibria. With the constant tax regime discussed in the text, the efficient equilibrium is determinate, so that a local analysis would falsely conclude that it is unique.

[7] Although we emphasize the potential stabilizing role of simple tax rules, tax rules can also be destabilizing (see Schmitt-Grohe and Uribe, 1997).

summarized in the vector, s_t. For simplicity, we only consider environments in which the number of possible values of s_t is finite for each t. Let $s^t = (s_0, s_1, \dots, s_t)$ denote a history of realizations up to and including date t, $t = 0,1,2 \dots$. The probability of history s^t is denoted $\mu_t(s^t)$, $t = 0,1,\dots$. To simplify the notation, from here on we delete the subscript t on μ. We adopt this notational convention for all functions of histories. The probability of s^{t+1} conditional on s^t is denoted $\mu(s^{t+1}|s^t) \equiv \mu(s^{t+1})/\mu(s^t)$.

For each history, s^t, the preferences of the representative household over consumption and leisure are given by

$$\sum_{j=t}^{\infty} \sum_{s^j|s^t} \beta^{j-t}\mu(s^j)u[c(s^j),n(s^j)], \tag{1}$$

where $\beta \in (0,1)$ is the discount rate, $s^j \mid s^t$ denotes histories, s^j, that are continuations of the given history, s^t, and $c(s^j)$, $n(s^j)$ denote consumption and labor, respectively, conditional on history s^j. We assume

$$u(c,n) = \log c + \sigma \log(1-n), \tag{2}$$

where $\sigma > 0$.

Since the production technology is static, we can describe it without the s^t notation. Production occurs at a large number of locations. A given location which uses capital, K, and labor, N, produces output, Y using the following production function:

$$Y = f(y,K,N) = y^{\gamma}K^{\alpha}N^{(1-\alpha)}, \quad 0 \leqslant \gamma,\alpha \leqslant 1. \tag{3}$$

Here, y denotes the average level of production across all locations. We assume

$$\alpha = 1 - \gamma, \tag{4}$$

with $\gamma = 2/3$. With this value of γ, the model implies that labor's share of income in the competitive decentralization described below is 2/3, which is close to the value estimated using the national income and product accounts (Christiano, 1988).

The relation between the economywide average level of output and the economywide average stock of capital, k, and labor, n, is obtained by solving $y = f(y, k, n)$ for y:

$$y = k^{\alpha/(1-\gamma)}n^{(1-\alpha)/(1-\gamma)} = kn^{\gamma/(1-\gamma)} = kn^2 \tag{5}$$

given Eq. (4) and our assumed value of γ.

Finally, the resource constraint for this economy is

$$c(s^t) + k(s^t) - (1-\delta)k(s^{t-1}) \leqslant k(s^{t-1})n(s^t)^2 = y(s^t). \tag{6}$$

2.2. Decentralization

In what follows we describe the household and firm problems, and our competitive equilibrium concept. In addition, we introduce a government which has the power to tax and to transfer resources.

2.2.1. Households

At each s^t and t, the representative household faces the following sequence of budget constraints:

$$c(s^j) + k(s^j) - (1-\delta)k(s^{j-1})$$
$$= [1-\tau(s^j)][r(s^j)k(s^{j-1}) + w(s^j)n(s^j)] + T(s^j), \text{ all } s^j \mid s^t, \quad j \geqslant t, \tag{7}$$

where $r(s^j)$ and $w(s^j)$ denote the market rental rate on capital and the wage rate, respectively. Also, $\tau(s^j)$ is the tax rate on income, $T(s^j)$ denotes lump-sum transfers from the government, and $k(s^j)$ denotes the stock of capital at the end of period j, given history s^j. The household also takes $k(s^{t-1})$ as given at s^t. Finally, the household must satisfy the following inequality constraints:

$$k(s^j) \geqslant 0, \quad c(s^j) \geqslant 0, \quad 0 \leqslant n(s^j) \leqslant 1 \tag{8}$$

for all $s^j \mid s^t$ and $j \geqslant t$ and takes as given and known the actual future date-state contingent prices and taxes:

$$\{r(s^j), w(s^j), \tau(s^j), T(s^j); j \geqslant t, \text{all } s^j \mid s^t\}. \tag{9}$$

Formally, at each s^t and t, the household problem is to choose $\{c(s^j), n(s^j), k(s^j); j \geqslant t, \text{all } s^j \mid s^t\}$ to maximize Eq. (1) subject to Eqs. (7)–(9), and the initial stock of capital, $k(s^{t-1})$. The intertemporal Euler equations corresponding to this problem are

$$u_c(s^j) = \beta \sum_{s^{j+1}|s^t} \mu(s^{j+1}|s^j)u_c(s^{j+1})\{[1-\tau(s^{j+1})]r(s^{j+1}) + 1 - \delta\} \tag{10}$$

all $s^j \mid s^t, j \geqslant t$, and the intratemporal Euler equations are

$$\frac{-u_n(s^j)}{u_c(s^j)} = [1-\tau(s^j)]w(s^j), \quad \text{all } s^j \mid s^t, \ j \geqslant t. \tag{11}$$

Here, $u_c(s^j)$ and $u_n(s^j)$ denote the partial derivatives of u with respect to its first and second arguments, evaluated at $c(s^j), n(s^j)$. Finally, the household's transversality condition is

$$\lim_{T\to\infty} \beta^T \sum_{s^T|s^t} \mu(s^T|s^t)u_c(s^T)\{[1-\tau(s^T)]r(s^T) + 1 - \delta\}k(s^{T-1}) = 0. \tag{12}$$

The sufficiency of the Euler equations, (10) and (11), and transversality condition, (12), for an interior solution to the household problem may be established by applying the proof strategy for Theorem 4.15 in Stokey and Lucas with Prescott (1989).

2.2.2. Firms

The technology at each location is operated by a firm. Omitting the s^t notation, the representative firm takes y, r, and w as given and chooses K and N to maximize profits:

$$Y - rK - wN \tag{13}$$

subject to Eq. (3). The firm's first-order conditions for labor and capital are

$$f_N = w, \quad f_K = r, \tag{14}$$

where f_K and f_N are the derivatives of f with respect to its second and third arguments, respectively. We assume the firm behaves symmetrically, so that consistency requires $y = Y, k = K, n = N$. Imposing these, we obtain

$$f_N = (1 - \alpha)nk, \quad f_K = \alpha n^2. \tag{15}$$

2.2.3. Government

The income tax rate policy, $\tau(s^t)$, is specified exogenously, and we require that the following budget constraint be satisfied for each s^t:

$$\tau(s^t)[r(s^t)k(s^{t-1}) + w(s^t)n(s^t)] = T(s^t). \tag{16}$$

2.3. Equilibrium

We adopt the following definition of equilibrium.[8]

Definition 1. A sequence-of-markets equilibrium is a set of prices $\{r(s^t), w(s^t)$; all s^t, all $t \geq 0\}$, quantities $\{y(s^t), c(s^t), k(s^t), n(s^t)$; all s^t, all $t \geq 0\}$, and a tax policy $\{\tau(s^t), T(s^t)$; all $s^t, t \geq 0\}$ with the following properties, for each t, s^t:

- given the prices, the quantities solve the household's problem;
- given the prices and given $\{y(s^t) = k(s^{t-1})n(s^t)^2\}$, the quantities solve the firm's problem;
- the government's budget constraint is satisfied;
- the resource constraint is satisfied.

[8] It is easily verfied that the analysis would have been unaltered had we instead adopted the date 0, Arrow–Debreu equilibrium concept. In this case, households would have had access to complete contingent claims markets.

We find it useful to define an *interior* equilibrium. This is a sequence-of-markets equilibrium in which $a \le n(s^t) \le b$ for all s^t for some a and b satisfying $0 < a \le b < 1$.

3. Characterizing equilibrium

In the next two sections of the paper, we study deterministic equilibria in which prices and quantities do not vary with s_t and stochastic (sunspot) equilibria in which prices and quantities do vary with s_t. The analysis of these equilibria is made possible by a characterization result, which is the subject of this section.

Substituting Eqs. (14) and (15) into the household's intertemporal Euler equation, (10), we get

$$\frac{1}{\tilde{c}(s^t)} = \beta \frac{1}{\lambda(s^t)} \sum_{s^{t+1}|s^t} \mu(s^{t+1}|s^t) \frac{1}{\tilde{c}(s^{t+1})} \{[1-\tau(s^{t+1})]\alpha n(s^{t+1})^2 + 1 - \delta\}, \quad (17)$$

where

$$\tilde{c}(s^t) = \frac{c(s^t)}{k(s^{t-1})},\ \lambda(s^t) = \frac{k(s^t)}{k(s^{t-1})}. \quad (18)$$

Substituting Eq. (15) into the household intratemporal Euler equation (11), we get

$$\tilde{c}(s^t) = [1-\tau(s^t)]\frac{\gamma}{\sigma} n(s^t)[1-n(s^t)]. \quad (19)$$

The resource constraint implies that

$$\tilde{c}(s^t) = n(s^t)^2 + 1 - \delta - \lambda(s^t). \quad (20)$$

Combining the two Euler equations, (17) and (19), and the resource constraint, (20), our system collapses into a single equation in current and next period's employment:

$$\sum_{s_{t+1}} \mu(s^{t+1}|s^t) v[n(s^t), n(s^{t+1}); \tau(s^{t+1})] = 0, \quad \text{all } s^t,\ t \ge 0, \quad (21)$$

where v is

$$v(n, n'; \tau') = \frac{1}{n^2 + 1 - \delta - \lambda} - \frac{\beta[(1-\tau')\alpha(n')^2 + 1 - \delta]}{\lambda[(n')^2 + 1 - \delta - \lambda']} \quad (22)$$

with

$$\lambda = n^2 + 1 - \delta - (1-\tau)\frac{\gamma}{\sigma} n(1-n). \quad (23)$$

Here, a $'$ denotes next period's value of the variable. The transversality condition, (12), is equivalent to

$$\lim_{T\to\infty}\sum_{s^T}\beta^T\mu(s^T)\frac{\{[1-\tau(s^T)]\alpha n(s^T)^2+1-\delta\}}{[1-\tau(s^T)]\frac{\gamma}{\sigma}n(s^T)[1-n(s^T)]}=0. \tag{24}$$

The basic equilibrium characterization result for this economy is given in Proposition 1.

Proposition 1. Suppose that $\tau(s^t)\equiv 0$. If, for all s^t and $t\geq 0$,

$$\{n(s^t)\}$$

satisfies Eq. (21) and

$$a\leq n(s^t)\leq b,\quad \text{for some } 0<a\leq b<1$$

then $\{n(s^t)\}$ corresponds to an equilibrium.

Proof. To establish the result, we need to compute the remaining objects, prices and quantities, in an equilibrium and verify that they satisfy Eqs. (10), (11), (12), (14), and (6). A candidate set of objects is found in the obvious way. The sufficiency of the first-order and transversality conditions for household optimization and the sufficiency of the first-order conditions for firm optimization guarantee that these are an equilibrium.

The characterization result indicates that understanding the equilibria of the model requires understanding the v function. It is easily confirmed that $v=\omega$ defines a quadratic function in n' for each fixed n and ω.[9] (Later, we refer to ω as the *Euler error*.) Hence, for each n, ω there are two possible n' : $n'=f_u(n,\omega)$ and $n'=f_l(n,\omega)$, where

$$f_u(n,\omega)=\frac{1}{2}\{b(n,\omega)+\sqrt{b(n,\omega)^2-4c(n,\omega)}\},$$

$$f_l(n,\omega)=\frac{1}{2}\{b(n,\omega)-\sqrt{b(n,\omega)^2-4c(n,\omega)}\}, \tag{25}$$

Here,

$$b(n,\omega)=\frac{\varphi(n)q(n,\omega)}{\alpha+q(n,\omega)\varphi(n)},\qquad c(n,\omega)=\frac{1-\delta}{\alpha+q(n,\omega)\varphi(n)}, \tag{26}$$

[9] For other example models in which the Euler equation has two solutions for every initial condition, see Benhabib and Perli (1994) and Benhabib and Rustichini (1994).

$$\varphi(n) = \frac{n^2 + 1 - \delta - \dfrac{\gamma}{\sigma} n(1-n)}{\beta n(1-n)}, \tag{27}$$

$$q(n, \omega) = 1 - \frac{\gamma}{\sigma} n(1-n)\omega. \tag{28}$$

The function v has the shape of a saddle, as can be seen in Fig. 1(a). The intersection of v and the zero plane ($\omega = 0$) is depicted in Fig. 1(a) as the boundary between the light and dark region of v. This intersection defines the curves $f_u(\cdot,0)$ and $f_l(\cdot,0)$, which are shown in Fig. 1(b). We refer to these as the upper and lower branches of the function v. The lower branch intersects the 45-degree line at two points, which are denoted $\bar{n}^1$ and $\bar{n}^2$. These intersection

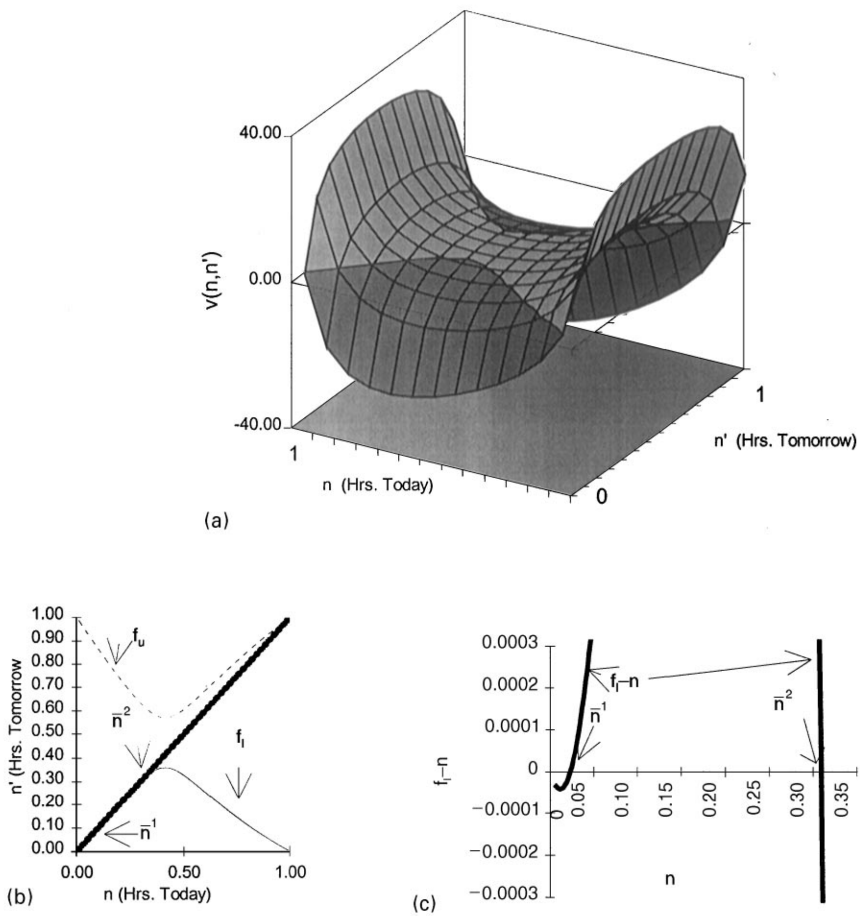

Fig. 1. (a) The $v(n, n')$ function, (b) contour: $v(n, n') = 0$, (c) close-up of Fig. 1(b).

points cannot be seen in Fig. 1(b), but can be seen in Fig. 1(c), which displays $n' - n$ for n near the origin. It is easy to see from Fig. 1(a) that with higher values of ω, f_l increases and f_u decreases. The figure also indicates that for these functions to be real-valued, ω must not be too big.

The branches in the figure are computed using our *baseline parameterization*, $\sigma = 2$, $\beta = 1.03^{-1/4}$, $\delta = 0.02$, $\alpha = 1/3$. Here, $\bar{n}^1 = 0.02$ and $\bar{n}^2 = 0.31$. The gross growth rates of capital (that is, λ) at these two points are 0.973 and 1.004, respectively.

4. Deterministic equilibria

We briefly discuss the set of deterministic equilibria. Since prices and quantities depend on t, but not on s_t, we can drop the history notation, and use the conventional time subscript notation instead. The set of deterministic equilibria is quite rich. For example, any constant sequence $\{n_t\}$, with $n_t = \bar{n}^1$ or $n_t = \bar{n}^2$, satisfies the conditions of the characterization result and so is an equilibrium. Similarly, any sequence with $n_0 \in (\bar{n}^1, \tilde{n})$ and $n_{t+1} = f_l(n_t, 0)$, $t \geq 0$ is also an equilibrium, with $n_t \to \bar{n}^2$. Here, $\tilde{n}$ satisfies $\tilde{n} > \bar{n}^2$ and $\bar{n}^1 = f_l(\tilde{n}, 0)$. Fig. 2 exhibits

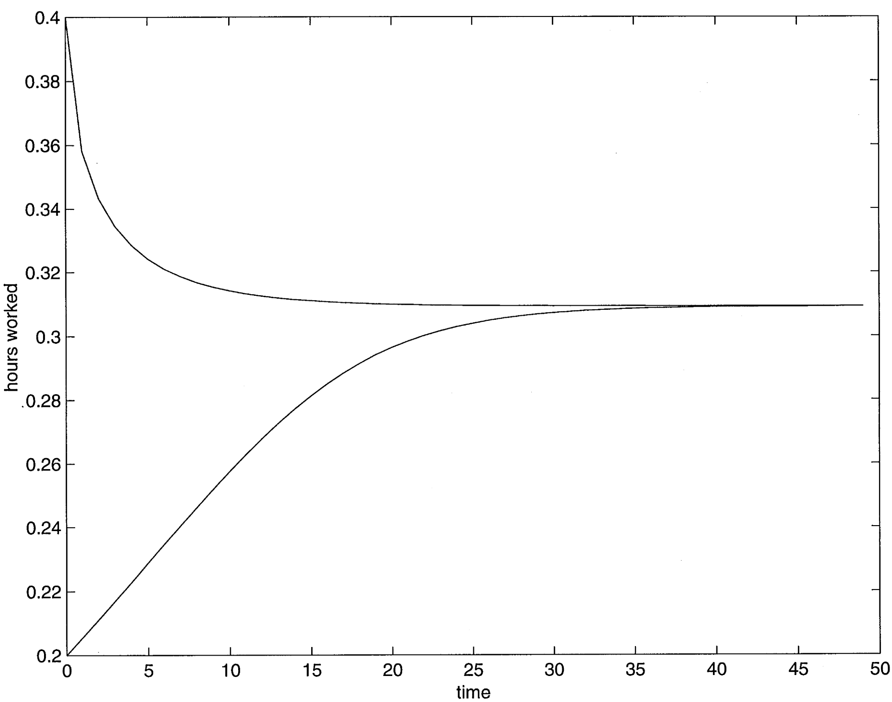

Fig. 2. Two equilibria on the lower branch.

two equilibrium paths, one starting with $n_0 = 0.4$ and other with $n_0 = 0.2$. Each path converges monotonically to $\bar{n}^2$.

Other deterministic equilibria are more exotic and display a variety of types of *regime switching*. For example, the equilibrium employment policy function could be time non-stationary, with employment determined by the lower branch for, say, six periods, followed by a single-period jump to the upper branch, followed by another six-period sojourn on the lower branch, and so on. The model has another type of regime switching equilibrium too, in which the employment policy function is discontinuous.

As an example of the latter, consider equilibria in which employment, n', is determined by the upper branch for n over one set of intervals in (0,1) and by the lower branch over the complement of these intervals. One example of this is given by

$$n' = f(n), \text{ where } f(n) \equiv \begin{cases} f_u(n, 0), & \text{for } n \le \bar{n}^1 \\ f_l(n, 0), & \text{for } \bar{n}^1 < n \le m^1 \\ f_u(n, 0), & \text{for } m^1 < n \le m^2, \\ f_l(n, 0), & \text{for } m^2 < n, \end{cases} \tag{29}$$

where $m^1 < \bar{n}^2$ and m^2 are a chosen set of numbers. By considering different values of σ, Eq. (29) defines a family of maps. This family includes maps which exhibit characteristics that resemble chaos. See Christiano and Harrison (1996) for an extended discussion.

5. Sunspot equilibria

In this section, we study equilibria of our model in which prices and quantities respond to s_t. We construct two equilibria to illustrate the possibilities. The first, which we call a *conventional sunspot equilibrium*, uses f_l only. This equilibrium is constructed near the deterministic steady state, $\bar{n}^2$, which, as noted above, has a continuum of deterministic equilibria which converge to it. Our choice of name reflects that this type of equilibrium is standard in the quantitative sunspot literature.[10] The second equilibrium considered, which we call a *regime switching sunspot equilibrium*, involves stochastically switching between f_l and f_u. Our principle interest in these equilibria has to do with their welfare properties.

[10] Because a continuum of other nonstochastic equilibria exists near the steady state equilibrium, $\bar{n}^2$, this equilibrium is said to be *indeterminate* (Boldrin and Rustichini, 1994, p. 327). For a general discussion of the link between indeterminate equilibria and sunspots, see Woodford (1986a). Examples of quantitative analyses that construct sunspot equilibria in the neighborhood of indeterminate equilibria include Benhabib and Farmer (1994,1996), Farmer and Guo (1994,1995) and Gali (1994a,b).

However, we find it useful to also display their business cycle properties. We display the business cycle properties of US data and of a standard real business cycle model as benchmarks. Our benchmark real business cycle model is the one analyzed in Christiano and Todd (1996).[11]

5.1. Conventional sunspot equilibrium

In this equilibrium, $s \in R$ is independently distributed over time, with $s = -0.06$ and $s = 0.06$ with probability 1/2 each. These values for s were chosen so that the standard deviation of output, after first logging and then filtering by the Hodrick and Prescott (1997) method ('HP-filter'), equals the corresponding empirical analog. Given any n, next period's hours worked, n', is computed by first drawing s and then solving

$$n' = f_l(n,s), \tag{30}$$

where f_l is defined in Eq. (25). We set the initial level of hours worked, n_0, to $\bar{n}^2$. Recall that $\bar{n}^2$ is the higher of the two deterministic steady states associated with the lower branch, f_l. That is, of the two solutions to $x = f_l(x,s)$, $\bar{n}^2$ is the larger of the two.

To establish that this stochastic process for employment corresponds to an equilibrium, it is sufficient to verify that the conditions of the characterization result are satisfied. The first condition is satisfied by construction, and the second is satisfied because $n(s^t)$ remains within a compact interval that is a strict subset of the unit interval. That is, let a be the smaller of the two values of n that solve $a = f_l(a, -0.06)$, and let $b > a$ be the unique value of n with the property $a = f_l(b, -0.06)$. Here, a and b are 0.0249 and 0.9509 after rounding. We verified that if $a \leq n \leq b$, then $a \leq n' \leq b$ for $n' = f_l(n, -0.06)$ and $n' = f_l(n, 0.06)$. Thus, $prob[a \leq n' \leq b | a \leq n \leq b] = 1$. It follows that $a \leq n(s^t) \leq b$ for all histories, s^t, with $\mu(s^t) > 0$. The conditions of the characterization result are satisfied, and so we conclude that $n(s^t)$ corresponds to an equilibrium.

The first-moment properties of this equilibrium are reported in Panel C of Table 1. They are similar to the corresponding properties of the US data (Panel A) and of the real business cycle model (Panel B). The second-moment properties of this equilibrium (see Table 2, Panel C) also compare favourably with the corresponding sample analogs, at least relative to the performance of the real business cycle model (see Table 2, Panel B). Four observations are worth stressing. First, consumption in both models is smooth relative to output, as in the data. The two models also perform similarly in terms of their implications for the volatility of investment. Second, the conventional sunspot equilibrium

[11] See Christiano and Harrison (1996) for a discussion, using our model, of the econometrics of conventional and regime switching sunspot equilibria.

Table 1
First-moment properties

n	c/y	k/y	i/y	Growth in k	Growth in y
Panel A: US data					
0.23	0.73	10.62	0.27	1.0047	1.0040
Panel B: Real business cycle model					
0.23	0.73	10.64	0.27	1.0040	1.0040
Panel C: Conventional sunspot					
0.309	0.745	10.46	0.255	1.0045	1.0046
Panel D: Regime switching sunspot					
0.094	5.17	298	− 4.17	0.989	4.74

Notes: Entries in the table are the mean of the indicated variable. US data results are taken from Christiano (1988). Results in Panel B are based on the real business cycle model in Christiano and Todd (1996). That model corresponds to the one in this paper, with $\sigma = 3.92, \gamma = 0, \delta = 0.021$, $\alpha = 0.344$, and a production function that has the form $Y = K^{\alpha}(zn)^{(1-\alpha)}$, with $z = z_{-1}\exp(\lambda)$ and $\lambda \sim \mathrm{IIN}(0.004, 0.018^2)$. See the text for a discussion of the entries in Panels C and D.

does somewhat better on the volatility of hours worked than does the real business cycle model. For example, the real business cycle model implies that productivity is about 65% more volatile than hours worked, whereas the conventional sunspot equilibrium implies that productivity is about as volatile as hours worked. In the data, productivity is about 30 percent less volatile than hours worked. Third, hours and productivity are procyclical in the real business cycle model and the convential sunspot equilibrium, as they are in the data. The conventional equilibrium's implication that productivity is procyclical reflects the increasing returns in the model. Fourth, the model inherits a shortcoming of standard real business cycle models in overpredicting the correlation between productivity and hours worked. In the data, this quantity is essentially zero.

Some of these properties can also be seen by examining the plots in Fig. 3. They are graphs of the logged and HP filtered data from the equilibrium described above. Consumption is smooth and investment is volatile in these graphs. In addition, hours worked and productivity are seen to be procyclical. Overall, this sunspot equilibrium compares quite well to the real business cycle model in its ability to mimic key features of postwar US. business cycles.

5.2. Regime switching sunspot equilibrium

For this equilibrium, $s = [s(1), s(2)] \in R^2$, with $s(1) \in \{u, l\}$ and $s(2) = \omega \in \{-0.06, 0.06\}$. That is, the first element of s indicates whether the economy

Table 2
Second-moment properties

x_t	σ_x/σ_y	Correlation of y_t with $x_{t+\tau}$				
		$\tau = 2$	$\tau = 1$	$\tau = 0$	$\tau = -1$	$\tau = -2$
Panel A: US data						
y	0.02	0.65	0.86	1.00	0.86	0.65
c	0.46	0.48	0.66	0.78	0.76	0.61
i	2.91	0.33	0.56	0.71	0.68	0.57
n	0.82	0.69	0.81	0.82	0.66	0.41
y/n	0.58	0.12	0.32	0.55	0.55	0.53
y/n, n	0.70	− 0.17	− 0.07	− 0.03	0.21	0.33
Panel B: Real business cycle model						
y	0.02	0.51	0.74	1.00	0.74	0.51
c	0.55	0.59	0.78	0.98	0.69	0.44
i	2.37	0.45	0.70	0.99	0.76	0.55
n	0.38	0.40	0.67	0.98	0.77	0.57
y/n	0.63	0.57	0.78	0.99	0.71	0.47
y/n, n	1.65	0.61	0.77	0.94	0.61	0.33
Panel C: Conventional sunspot						
y	0.02	0.35	0.63	1.00	0.63	0.35
c	0.33	0.58	0.72	0.87	0.44	0.13
i	3.13	0.26	0.57	0.99	0.66	0.40
n	0.51	0.22	0.54	0.98	0.66	0.42
y/n	0.52	0.46	0.69	0.98	0.57	0.27
y/n, n	1.02	0.49	0.68	0.91	0.44	0.11
Panel D: Regime switching sunspot						
y	0.78	− 0.07	− 0.07	1.00	− 0.07	− 0.07
c	0.06	0.25	0.30	0.35	− 0.42	− 0.35
i	na	na	na	na	na	na
n	0.54	− 0.11	0.11	0.99	− 0.01	− 0.03
y/n	0.47	− 0.03	− 0.02	0.99	− 0.13	− 0.12
y/n, n	0.88	0.01	0.02	0.96	− 0.19	− 0.17

Notes: See the text for a description of the equilibria. Prior to the analysis, all data have been logged and then HP filtered. Here, σ_x is the standard deviation of the logged, HP filtered variable, x_t. All but the first and last row of the 'σ_x/σ_y' column in each panel report σ_x/σ_y. The first row has σ_y, and the last, $\sigma_{y/n}/\sigma_n$. The correlations reported in the last row of each panel are corr[$(y/n)_t$, $n_{t-\tau}$]. Results in Panels A and B in this table are taken from Christiano and Todd (1996, Tables 2 and 3). For a description of the underlying model, see notes to Table 1.

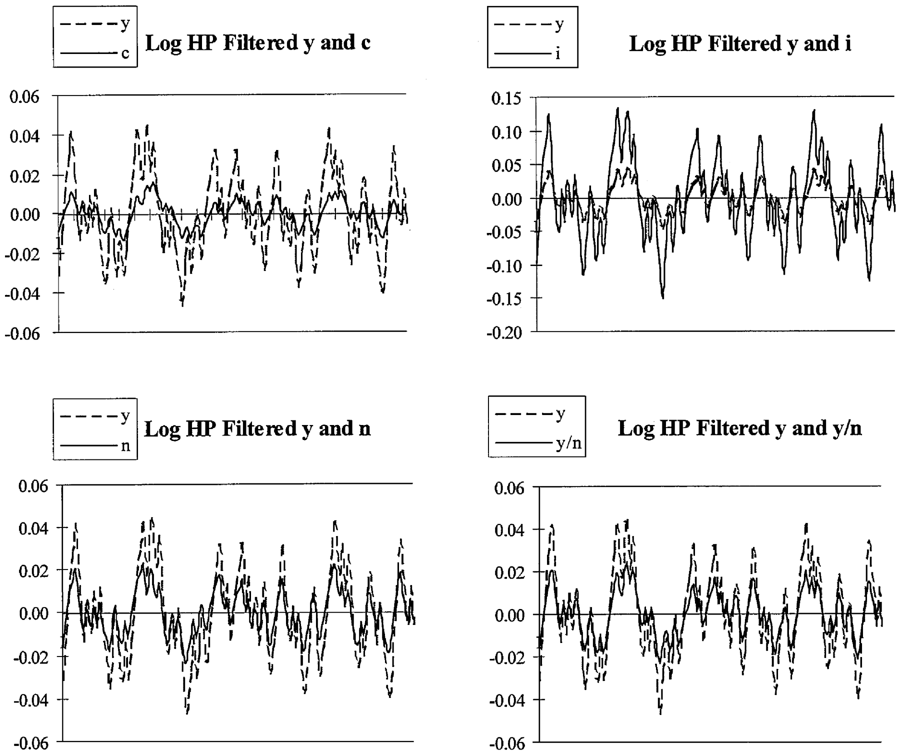

Fig. 3. Logged and HP filtered data from conventional sunspot equilibrium.

is on the lower or upper branch, and the second element corresponds to the conventional sunspot variable. Given a history, s^t, and an associated level of employment, $n(s^t)$, we draw s_{t+1} and solve for $n(s^{t+1})$ using

$$n(s^{t+1}) = f_{s_{t+1}(1)}(n(s^t), s_{t+1}(2)) \tag{31}$$

for $t = 1,2,\ldots$. The date zero level of employment is set to $\bar{n}^2$. When the value of $s(1)$ changes along a history, we say there has been a regime switch.

We constructed the probabilities, $\mu(s^{t+1})$, used to draw s_{t+1} as follows. First, $s_{t+1}(1)$ and $s_{t+1}(2)$ are independent random variables, and $s_{t+1}(2)$ has the same distribution as s in the previous equilibrium. The probabilities for $s_{t+1}(1) \in \{u,l\}$ were constructed to guarantee that $a \leq n(s^{t+1}) \leq b$ with probability one, for a, b such that $0 < a \leq b < 1$. We used the values of a and b from the conventional sunspot equilibrium. Also,

$$\text{prob}[s_{t+1}(1) = l] = \begin{cases} 0.9, & \text{for } \tilde{n}^1 < n(s^t) < \tilde{n}^2 \\ 1, & \text{otherwise} \end{cases}, \tag{32}$$

where $\tilde{n}^1 = 0.0370$ and $\tilde{n}^2 = 0.9279$. We verified numerically that, if $a \leq n(s^t) \leq b$, then $\text{prob}[a \leq n(s^{t+1}) \leq b] = 1$. It follows that, for all s^t such that $\mu(s^t) > 0$, $a \leq n(s^t) \leq b$.[12] This establishes the second of the two conditions of the characterization result. To establish the first condition, note that by Eq. (31),

$$v(n(s^t), n(s^{t+1})) = s_{t+1}(2), \quad \text{for all } s^t \tag{33}$$

and by construction of the Euler error, $s_{t+1}(2)$,

$$\sum_{s^{t+1}|s^t} \mu(s^{t+1}|s^t) s_{t+1}(2) = 0, \text{ for all } s^t. \tag{34}$$

This establishes that the conditions of the characterization result are satisfied, and we conclude that $n(s^t)$ corresponds to an equilibrium.

We now consider the dynamic properties of the regime switching sunspot equilibrium. First moment properties are reported in Panel D of Table 1, while second-moment properties are reported in Panel D of Table 2. Regime switching is the key to understanding the dynamics of this equilibrium. Periodically, the economy switches to the upper branch, f_u, where employment is very high. The economy typically stays on the upper branch only briefly. When it switches down again, employment drops to a very low level, near a. Employment then rises slowly until another switch occurs, when the economy jumps to the upper branch, and the process continues. The fact that the economy spends much time in the left region of the lower branch explains why average employment in this equilibrium is so low. This also explains why investment is, on average, negative. Regarding the second-moment properties, output is substantially more volatile than it is in the data. Also, output displays very little serial correlation. The positive serial correlation produced by sojourns on the lower branch is offset by the negative serial correlation associated with transient jumps to the upper branch. These observations are supported by the time series plots of the logged, HP filtered data from this equilibrium, presented in Fig. 4.

The regime switching equilibrium nicely illustrates a type of sunspot equilibrium that is possible. However, in contrast with the conventional sunspot equilibrium, the first and second moment properties of this equilibrium do not match the corresponding quantities in the data. In Christiano and Harrison (1996), we explore other strategies for determining if the data exhibit the type of regime-switching possible in our model. The findings there complement the results here in suggesting that this type of behaviour is probably empirically unimportant. This type of equilibrium is a novel aspect of dynamic equilibrium

[12] Our specification of $\tilde{n}^1$ and $\tilde{n}^2$ is crucial for guaranteeing the second condition of the characterization result. For example, with $\tilde{n}^1 = a$ and $\tilde{n}^2 = b$, histories, s^t, in which hours worked fluctuate between values that approach 0 and 1 occur with high probability. With $\mu(s^t)$ specified in this way, the second condition of the characterization result fails.

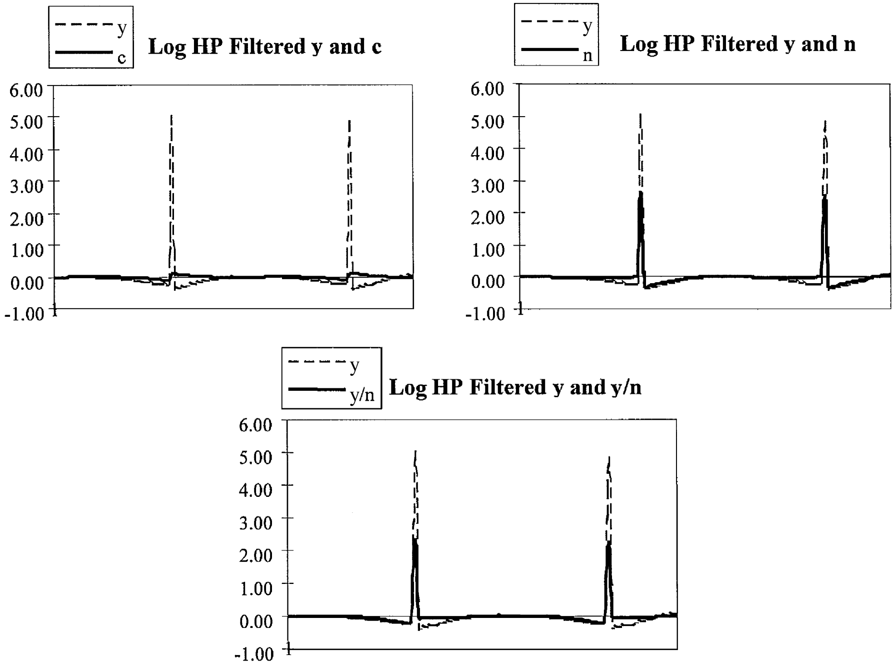

Fig. 4. Logged and HP filtered date from regime switching sunspot equilibrium.

models, and it would be interesting if it turned out to be empirically relevant in the context of other models.

5.3. *Welfare analysis*

We approximated the expected discounted utility for our equilibria using a Monte Carlo simulation method. For the conventional sunspot equilibrium and the regime switching sunspot equilibrium, the expected present discounted utilities are -378.21 (0.24) and -570.58 (1.77), respectively (numbers in parentheses are Monte-Carlo standard errors).[13] To understand the impact on utility

[13] For each equilibrium, we drew 1,000 histories, s^t, each truncated to be length 2,500 observations. Subject to the initial level of employment being $\bar{n}^2$ always, we computed consumption and employment along each history. For each equilibrium, we computed 1,000 present discounted values of utility, $v_1, \dots, v_{1000}$. Our Monte Carlo estimate of expected present discounted utility, v, is the sample average of these: $\bar{v} = \frac{1}{1000}\sum_{i=1}^{1000} v_i$. The fact that we use a finite number of replications implies that $\bar{v}$ is approximately normally distributed with mean v and standard deviation $\sigma_i/\sqrt{1{,}000}$, where σ_i is estimated by the standard deviation of $v_1, \dots, v_{1000}$. We refer to $\sigma_i/\sqrt{1{,}000}$ as the *Monte-Carlo standard error*.

Table 3
Percentage utility gain relative to constant equilibrium

Conventional sunspot I	Conventional sunspot II	Regime switching
0.9%	11.2%	− 289%

Notes: This is the constant percentage decrease in consumption required for households in the indicated equilibrium to be indifferent between that equilibrium and the constant equilibrium at $n = \bar{n}^2$. Let v denote the discounted utility associated with the constant employment level. Let $\bar{v}$ denote the discounted utility associated with one of the other equilibria. Then, the number in the table is $100[\exp((1-\beta)(\bar{v}-v)) - 1]$.

of variance in the Euler error, $s(2)$, we also computed expected utility for a high variance version of our conventional sunspot equilibrium. In this case, $s(2) \in \{-0.55, 0.55\}$. The expected present value of utility for this equilibrium is -363.35 (2.14). The present discounted level of utility associated with the constant employment deterministic equilibrium at $\bar{n}^2$ is -378.49. We refer to this equilibrium as the *constant employment equilibrium*.

To compare these welfare numbers, we converted them to consumption equivalents. That is, we computed the constant percentage increase in consumption required in the constant employment equilibrium to make a household indifferent between that equilibrium and another given equilibrium. The results are shown in Table 3. They indicate that going from the constant employment equilibrium to the regime switching sunspot equilibrium is equivalent to a 289% permanent drop in consumption. Going to the conventional sunspot equilibrium is equivalent to a 0.9% permanent rise in consumption, and going to the high variance version of that equilibrium is equivalent to an 11.2% rise in consumption.

An interesting feature of these results is that, despite concavity in the utility function, increasing volatility in $s(2)$ raises welfare. This reflects a trade-off between two factors. First, other things being the same, a concave utility function implies that a sunspot equilibrium is welfare-inferior to a constant, deterministic equilibrium (*concavity effect*). However, other things are not the same. The increasing returns means that by bunching hard work, consumption can be increased on average without raising the average level of employment (*bunching effect*). When the volatility of the model economy with initial employment $\bar{n}^2$ is increased by raising the volatility of $s(2)$, then the bunching effect dominates the concavity effect. When volatility is instead increased by allowing regime switches, then the concavity effect dominates. In interpreting these results, it is important to recognize that they say nothing about the nature of the efficient allocations. All of the equilibria that we consider are inefficient, because of the presence of the externality in production.

6. Policy analysis

We now consider the impact of various policies on the set of equilibria. We consider two procyclical tax policies that reduce the set of interior equilibria to a singleton in that output is a constant. We refer to the first as a *pure* stabilizer because it does not distort margins in equilibrium. The second tax policy introduces just the right distortions so that the equilibrium supports the optimal allocations. We show that, for a tax policy to isolate the efficient allocations as a unique equilibrium, it is necessary that the tax rate vary in the right way with the state of the economy. For example, under a constant tax rate policy, the equilibrium is not unique. Interestingly, the equilibria are isolated in this case, so that they would escape detection under the usual procedure of analyzing local equilibria.

6.1. A pure automatic stabilizer

In this section, we display a particular procyclical tax rate rule which reduces the set of equilibria to a singleton with $n_t = \bar{n}^2$ for all t (the constant employment equilibrium). The tax policy has the property that in equilibrium, the tax rate is always zero and thus does not distort any margins. Given our previous results for the constant employment equilibrium, this tax rate rule improves welfare relative to the regime switching sunspot equilibrium, but actually reduces welfare relative to the conventional sunspot equilibrium. The possibility that stabilization of a sunspot by government policy might reduce welfare should not be surprising, given that both the sunspot equilibrium and the $\bar{n}^2$ equilibrium are inefficient.

Consider the following tax rate:

$$\tau(n) = 1 - \frac{\bar{n}^2}{n}, \tag{35}$$

where n denotes economywide average employment and $\bar{n}^2$ is the higher of the two nonstochastic steady state employment levels (see Fig. 1(b)). Note that this tax rate is zero when aggregate employment is $\bar{n}^2$. It turns positive for higher levels of employment and negative for lower levels.

Let $\tilde{v}(n, n')$ denote Eq. (22) after substituting out for $\tau(n)$ from Eq. (35). It is easily verified that, for each value of n, there is at most one n' that solves $\tilde{v}(n, n') = 0$. This is given by

$$n' = f(n) = \frac{\bar{n}^2 - K(n)(1-\delta)}{\bar{n}^2[1 + \alpha K(n)]},$$

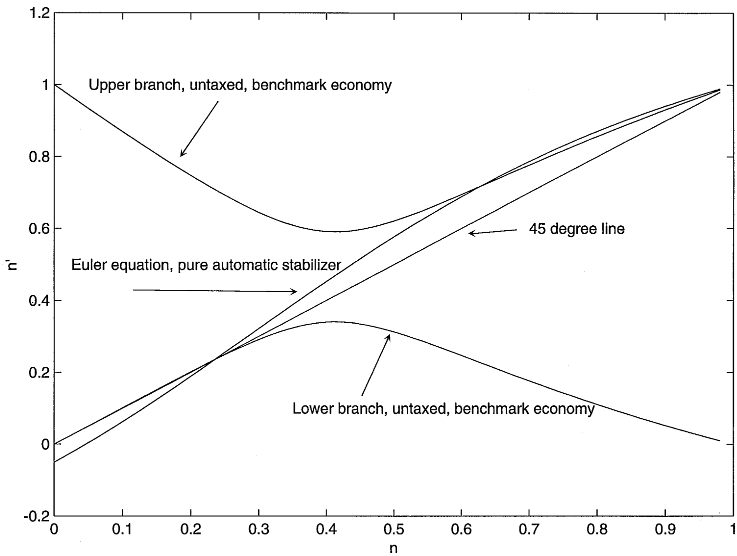

Fig. 5. Euler equation, $v(n, n') = 0$, for taxed and untaxed economies.

where

$$K(n) = \frac{\beta \bar{n}^2}{\lambda(n)}(1-n),\ \lambda(n) = n^2 + 1 - \delta - \frac{\gamma}{\sigma}\bar{n}^2(1-n).$$

The function, f, and its derivative, f', have the property that at $n = 1$,

$$f(1) = 1,\ f'(1) = \beta\left[\frac{\alpha\bar{n}^2 + 1 - \delta}{2 - \delta}\right] < \beta,$$

since $\alpha\bar{n}^2 < 1$. Fig. 5 shows f under our baseline parameter values. For convenience, the two branches of $v = 0, f_u$ and f_l, are also displayed.

There are three things worth emphasizing about f. First, it cuts the 45-degree line from below at $n = \bar{n}^2$, and it intersects the horizontal axis at a positive level of employment. This implies that there is no infinite sequence, n_t, $t = 0, 1, 2, \ldots$, with $n_0 < \bar{n}^2$ and $n_t = f(n_{t-1})$, such that $n_t > 0$ for all t. Since satisfaction of the Euler equation, $\tilde{v} = 0$, is a necessary condition for an interior solution to the household problem, it follows that there is no interior equilibrium with $n_o < \bar{n}^2$. Second, a sequence of employments, n_t, $t = 0, 1, \ldots$, which has the property $n_t = f(n_{t-1})$ and $n_o > \bar{n}^2$, has the property $n_t \to 1$ as $t \to \infty$. Appealing again to the necessity of the Euler equation, we conclude that there is no interior equilibrium with $n_o > \bar{n}^2$. Third, $n_t = \bar{n}^2$ for all t satisfies the Euler and transversality conditions and so corresponds to an interior equilibrium. Thus, the only deterministic interior equilibrium is the one that corresponds to $n_t = \bar{n}^2$ for

$t=0,1\ldots$. That sunspot equilibria are also ruled out follows from the fact that the Euler equation cuts the 45-degree line from below and from the arguments in Woodford (1986a). These remarks establish the following proposition.

Proposition 2. For the baseline parameterization and under the tax policy in Eq. (35), there is a unique interior equilibrium with $n_t = \bar{n}^2$ for all t.

Note that under the tax rate policy considered here, $\tau_t = 0$ in equilibrium. Evidently, the mere threat to change tax rates is enough to rule out other equilibria. This feature of fiscal (and monetary) policies designed to select certain equilibria can also be found in other models with multiple expectational equilibria. (See, for example, Boldrin, 1992, p. 215 and Guesnerie and Woodford, 1992, pp. 380–382).

6.2. Optimal allocations

The efficient allocations correspond to a fictitious planner's choice of investment, employment, and consumption to maximize discounted utility subject to the resource constraint. We reproduce the utility function here for convenience:

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\mu(s^t)\{\log[c(s^t)]+\sigma\log[1-n(s^t)]\}. \tag{36}$$

The resource constraint is

$$c(s^t)+k(s^t)-(1-\delta)k(s^{t-1})\le k(s^{t-1})[n(s^t)]^2,\quad \text{for all } t,s^t. \tag{37}$$

This problem simplifies greatly. Thus, using the change of variable in Eq. (18) and the identity

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\mu(s^t)\log k(s^{t-1})=\frac{1}{1-\beta}\left\{\log k_{\mathrm{o}}+\beta\sum_{t=0}^{\infty}\sum_{s^t}\mu(s^t)\beta^t\log\lambda(s^t)\right\} \tag{38}$$

the objective function can be written

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\mu(s^t)\left\{\log[n(s^t)^2+1-\delta-\lambda(s^t)]+\frac{\beta}{1-\beta}\log\lambda(s^t)\right.$$
$$\left.+\sigma\log[1-n(s^t)]\right\}+\frac{1}{1-\beta}\log k_o. \tag{39}$$

In Eq. (39), consumption has been substituted out using the (scaled) resource constraint after replacing the weak inequality in Eq. (37) by a strict equality.

Notice that the objective in Eq. (39) is separable across dates and states. This has two implications. First, the efficient allocations are insensitive to sunspots. Second, the efficient levels of employment and capital accumulation do not exhibit cycles. It is trivially verified that this result is independent of the curvature on leisure in the utility function, the degree of nonconvexity on labor in the production function, and the degree of homogeneity on capital in the resource constraint.[14] Thus, for example, increasing the gains from bunching production, by raising the power on labor above 2, and reducing the associated costs, by making utility linear in leisure, still does not imply that the efficient allocations exhibit cycles.

With our specification of preferences, optimizing (39) requires that the planner maximize, for each t, s^t,

$$\log[n^2 + 1 - \delta - \lambda] + \frac{\beta}{1-\beta}\log\lambda + \sigma \log[1-n] \tag{40}$$

by choice of n and λ, subject to

$$0 \le \lambda \le n^2 + 1 - \delta, 0 \le n \le 1. \tag{41}$$

The objective, Eq. (40), is not concave, because of the nonconcavity in the production function. However, for fixed n, Eq. (40) is strictly concave in λ, and its optimal value is readily determined to be $\lambda = \beta(n^2 + 1 - \delta)$. Substituting this into (40), the criterion maximized by the efficient allocations becomes

$$\frac{1}{1-\beta}\log(n^2 + 1 - \delta) + \sigma\log(1-n) \tag{42}$$

after constant terms are ignored. The constraint on this problem is $0 \le n \le 1$. There are two values of n that solve the first-order condition associated with maximizing Eq. (42), and the larger of the two is the global optimum. This is given by n^0, where

$$n^0 = \frac{1}{2}[\phi + \sqrt{\phi^2 - 4\xi}], \quad \phi = \frac{2}{2+\sigma(1-\beta)}, \quad \xi = \frac{\sigma(1-\beta)(1-\delta)}{2+\sigma(1-\beta)}. \tag{43}$$

With the baseline parameter values, $n^0 = 0.98$, which implies that the optimal value of λ is 1.94, or 94% per quarter. That equilibrium employment is so high reflects the fact that the efficient allocations internalize the externality in the production function.

[14] Lack of cycling in the efficient allocations also obtains for utility functions which are homogeneous of degree $\gamma \neq 0$ in consumption. See Appendix A for further discussion.

It is easily verified that the tax rate which supports n^0 as an equilibrium is $\tau = -2$. It is not surprising that this involves a subsidy, since the tax must in effect coax individuals into internalizing the positive externality associated with production. Consider first the case in which the tax rate is simply fixed at $\tau = -2$ for every n. Let $\tilde{v}\,(n, n')$ denote Eq. (22) after substituting out for $\tau = -2$. It effect, reducing τ from zero to -2 pushes the saddle in Fig. 1(a) down, so that the $\omega = 0$ plane now covers the seat of the saddle. The consequences can be seen in Fig. 6(a), which displays the values of n' that solve $\tilde{v}(n, n') = 0$ for $n \in (0, 1)$. Note the region of values for n for which there are no values of n' that solve $\tilde{v}(n, n') = 0$. In the other regions, there are generally two values of n' that solve this equation for each n. Interestingly, the unique intersection of these points with the 45-degree line, at n^0, is associated with a slope greater than one. As a result, the equilibrium associated with $n^0, n^0, n^0, \ldots$ is determinate. However, there is at least one other equilibrium, $\tilde{n}, n^0, n^0, \ldots$ (See Fig. 6(a) for $\tilde{n}$). Evidently, the constant tax rate policy does not guarantee a unique equilibrium.

One way to construct a tax regime that selects only the desirable equilibrium follows the strategy taken in the previous subsection. Thus, consider

$$\tau(n) = 1 - \frac{3n^0}{n}.$$

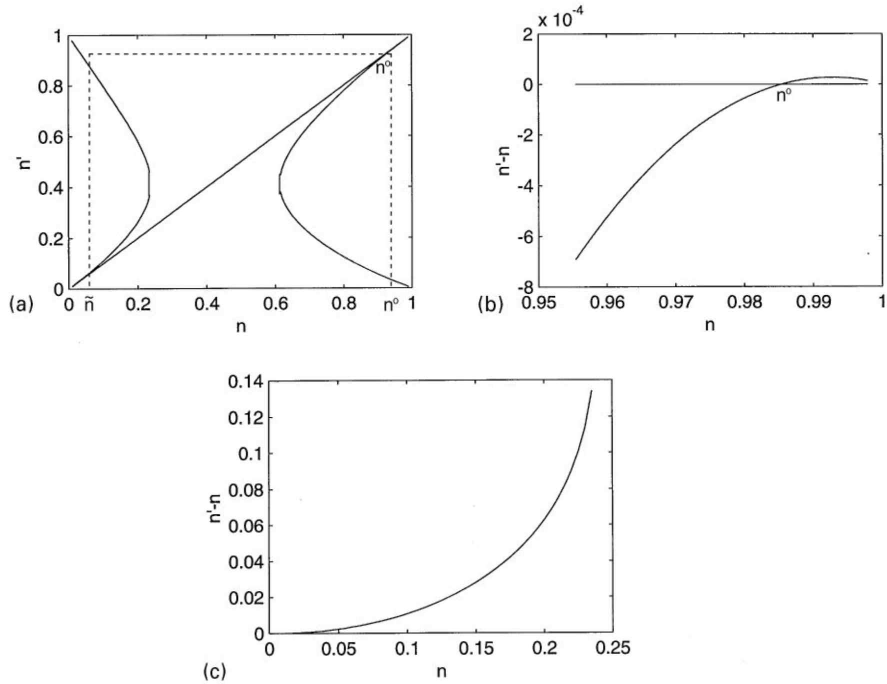

Fig. 6. (a) $v(n, n') = 0$, with income tax rate $= -2$, (b) $n' - n$ versus n, (c) $n' - n$ versus n.

Evidently, with this policy, $\tau(n^0) = -2$, so that there is an equilibrium associated with this tax policy which supports the efficient allocations. Also, it is easily verified that – following the same reasoning as in Section 6.1 – the Euler equation has only one branch. In addition, we found for the baseline parameter values that this branch is monotone, and it cuts the 45-degree line from below. It follows by the logic leading to Proposition 2 that there is a unique interior equilibrium.

7. Conclusion and directions for future research

We have studied a model environment in which the gains from adopting an automatic stabilizer tax system are potentially very large. An example was displayed in which the gains are equivalent to increasing consumption by a factor of 3! We showed, however, that for positive gains to be realized, it is important that the tax system be structured appropriately. In our model, the tax system has an important impact on the growth rate of the economy, and stabilization could be counterproductive if the economy were stabilized on the wrong growth path. Subject to this qualification, the environment analyzed here seems to rationalize the importance assigned by macroeconomists before the 1970s to devising automatic stabilizer tax systems.[15] Our analysis raises several questions that deserve further investigation.

First, how robust is our result that a properly constructed tax system necessarily eliminates fluctuations? We have shown that this is so under a particular homogeneity assumption on the resource constraint. But, standard models do not satisfy this condition. It is also of interest to investigate what happens when shocks to fundamentals are introduced, and a less extreme position is taken on the nature of production externalities.[16]

Second, to what extent does the business cycle behavior of the US economy reflect the stabilizing influence of automatic stabilizers? It is clear that, at least under a broad interpretation of automatic stabilizers, their role has been significant. For example, the government's commitment to defend the liquidity of the banking system, with the Federal Deposit Insurance Corporation as its backbone, has essentially eliminated the sort of financial panics that are thought to have contributed to recessions in the past. This policy works to stabilize the economy by a mechanism similar to the one studied in this paper.[17] It would be

[15] For a review, see Christiano (1984).

[16] For some steps in this direction, see Guo and Lansing (1996).

[17] The government's commitment to provide liquidity to banks during a bank run eliminates individuals' incentive to run on the bank, and thereby eliminates bank runs as equilibrium phenomena. Similarly, in our model the commitment to cut taxes in a recession reduces individuals' incentives to reduce labor effort then, and this eliminates recessions as equilibrium phenomena.

of interest to understand whether the US tax code might also have played a stabilizing role via this mechanism.

Third, would the US economy benefit from adjustments to the tax code designed to bring about further stabilization? The analysis of Lucas (1987) suggests that, without a different specification of household preferences, the answer is 'no'. Lucas adopts a utility function much like ours and argues that, given the observed volatility of measured consumption, the upper bound on the potential gains from stabilization is negligibly small. The case that further stabilization is desirable would have to be based on a very different model: one in which either the representative agent assumption is dropped, or preferences are very different, or both.

Finally, we emphasize that we have not rationalized 'automatic stabilizers' in the sense of tax rates that demonstrate significant procyclicality in equilibrium. It is important to draw attention to this, since some might think that this is precisely what automatic stabilizers are all about. In our model, an efficient tax system stabilizes fluctuations entirely and so our analysis sheds no light on the cyclical properties of an efficient tax system when the efficient allocations exhibit fluctuations in equilibrium. One way to investigate this further is to introduce shocks to fundamentals. However, we conjecture that an efficient tax regime would move procyclically with sunspot shocks, but would move very little with technology shocks. Assuming an efficient tax regime eliminates sunspot equilibria, the optimal 'automatic stabilizer' tax rate would then *not* be procyclical in equilibrium. An interesting question is what happens when the tax regime cannot respond differently to fluctuations due to sunspots and to fluctuations due to technology shocks. Possibly, under these circumstances an efficiently constructed tax regime would exhibit significant procyclicality in equilibrium.

Appendix A. Linearity of policy rules under homogeneity

In this appendix, we establish efficiency for a policy of the form, $k_{t+1} = \lambda^* k_t$ and $n_t = n^*$, where λ^*, n^* are fixed numbers. We do this for a class of economies in which the resource constraint is homogeneous in capital and in which preferences are homogeneous in consumption. Our result parallels that in Alvarez and Stokey (1995), except their environment does not explicitly allow for variable hours worked.

Consider the following planning problem:

$$\max_{\{k_t+1,\ n_t,\ c_t\}} \sum_{t=0}^{\infty} \beta^t u(c_t, n_t), \quad 0 < \beta < 1 \tag{A.1}$$

subject to the following feasibility constraints:

$$k_0 > 0 \text{ is given}, 0 \leq c_t \leq F(k_t, k_{t+1}, n_t), 0 \leq n_t \leq 1, k_{t+1} \geq 0,$$

$$\text{for } t = 0, 1, 2, \dots .$$

We assume that F is homogeneous:

$$F(k, k', n) = k^{\psi} f\left(\frac{k'}{k}, n\right), \text{ where } f(\lambda, n) \equiv F(1, \lambda, n),\ \lambda = \frac{k'}{k},\ \psi \geq 0. \tag{A.2}$$

In terms of λ and n, the constraints on the planner are:

$$B \equiv \{\lambda, n : 0 \leq n \geq\ \leq 1, 0 \leq \lambda, \text{ and } f(\lambda, n) \geq 0\}.$$

That is, the planner's feasible set is the set of infinite sequences, $\{\lambda_t, n_t\}_{t=0}^{\infty}$, such that λ_t,$n_t \in B$ for each $t \geq 0$. We place the following assumptions on f:

$$f : B \rightarrow R_+, \text{ continuous, decreasing in } \lambda, \text{ and increasing in } n. \tag{A.3}$$

Also, there exists a largest value of λ, $\bar{\lambda} > 0$, such that

$$f(\bar{\lambda}, 1) \geq 0, \quad \beta(\bar{\lambda})^{\psi\gamma} < 1, \tag{A.4}$$

and there exists $0 \leq \tilde{n} \leq 1$ such that

$$f(1, \tilde{n}) > 0. \tag{A.5}$$

We place the following assumptions on u:

$$u(c, n) = c^{\gamma} g(n)/\gamma,\ \gamma \neq 0,\ g(n) \geq 0,\ g \text{ is continuous and non–increasing.} \tag{A.6}$$

We have the following proposition.

Proposition. 3. If

(i) the functions F and u satisfy Eqs. (A.2), (A.3) and (A.6);
(ii) Eq. (A.4) holds when $\gamma > 0$, and Eq. (A.5) holds when $\gamma < 0$;

then, a policy of the following form solves Eq. (A.1):

$$k_{t+1} = \lambda^* k_t, \quad n_t = n^*, \quad t \geq 0, \quad \text{for fixed } (n^*, \lambda^*) \in B.$$

Proof. Write $u(c, n) = k^{\gamma\psi}(f(\lambda, n))^{\gamma} g(n)/\gamma$. Also,

$$k_t^{\gamma\psi} = \left(\prod_{j=0}^{t-1} \lambda_j^{\gamma\psi}\right) k_0^{\gamma\psi}, \quad t = 1, 2, \ldots.$$

Simple substitution establishes

$$v(k_0) = \max_{\{k_{t+1},\ n_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(F(k_t, k_{t+1}, n_t), n_t) = k_o^{\gamma\psi} w,$$

where

$$w = \max_{\{(\lambda_t, n_t) \in B\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left(\prod_{j=0}^{t-1} \lambda_j^{\gamma\psi} \right) \frac{(f(\lambda_t, n_t))^{\gamma} g(n_t)}{\gamma}. \tag{A.7}$$

We establish $-\infty < w < \infty$. When $\gamma < 0$, then u is bounded above by zero and so trivially, $w < \infty$. For the case $\gamma > 0$, consider the (infeasible!) policy of applying the entire time endowment both to labor effort and to leisure, and of applying all of output both to consumption and to investment. The value of this policy is $\bar{w} = (f(0,1))^{\gamma} g(0)/[\gamma(1 - \beta\bar{\lambda}^{\gamma\psi})]$. We have $w < \infty$, since $w \leq \bar{w} < \infty$. To establish $-\infty < w$ when $\gamma > 0$ note simply that u is bounded below by zero in this case. For the case $\gamma < 0$, note that the feasible policy, $\lambda_t = 1$, $n_t = \tilde{n}$, for $t \geq 0$ has return $k_0^{\gamma\psi}\tilde{w}$, where $\tilde{w} = f(1, \tilde{n})^{\gamma} g(\tilde{n})/[\gamma(1-\beta)]$, so that $-\infty < \tilde{w} \leq w$.

We have established that w is a finite scalar. By writing Eq. (A.7) out explicitly, one verifies that w satisfies the following expression:

$$w = \max_{(\lambda_0,\, n_0) \in B} \{ (f(\lambda_0, n_0))^{\gamma} g(n_0)/\gamma + \beta(\lambda_0)^{\gamma\psi} w \}. \tag{A.8}$$

Let λ^* and n^* denote values of λ_0 and n_0 that solve the above maximization problem. The result follows from the fact that these solve a problem in which the objectives and constraints are independent of k_0.

Remark 1. The proof for the class of utility functions $u(c, n) = \log(c) + g(n)$ is a trivial perturbation on the argument in the text.

Remark 2. When $\gamma > 0$, then the fixed point problem in Eq. (A.8) can be shown to be the fixed point of a contraction mapping. In this case, w in Eq. (A.7) is the only solution to Eq. (A.8), and the contraction mapping theorem provides an iterative algorithm for computing w, λ^*, and n^*.

Remark 3. When $\gamma < 0$, the mapping implicitly defined in Eq. (A.8) is not necessarily a contraction. Still, it may be possible to find w, λ^* and n^* by 'contraction iterations'. To see this, consider the case $f(\lambda, n) = (\bar{\lambda} - \lambda)$, $g(n) = 1$, $\psi = 1$, $\bar{\lambda} > 1$, $\gamma < 0$, so that $B = \{\lambda, n : 0 \leq \lambda \leq \bar{\lambda}, 0 \leq n \leq 1\}$ and Eq. (A.4) is satisfied. Then, define $T(w) = \max_{\lambda}(\bar{\lambda} - \lambda)^{\gamma}/\gamma + \beta\lambda^{\gamma} w$, $\lambda(w) = \arg\max_{\lambda}(\bar{\lambda} - \lambda)^{\gamma}/\gamma + \beta\lambda^{\gamma} w$. It is easy to verify: (i) for $w > 0$, $T(w) = \infty$, $\lambda(w) = 0$, (ii) $T(0) = \bar{\lambda}^{\gamma}/\gamma < 0$, $\lambda(0) = 0$, and (iii) for $w < 0$, $\mathrm{d}T(w)/\mathrm{d}w = \beta[\lambda(w)]^{\gamma}$, $d^2T(w)/dw^2 > 0$, $\lambda(w) = \bar{\lambda}/[1 + (\beta\gamma w)^{1/(\gamma-1)}]$, so that, as $w \to -\infty$, $\lambda(w) \to \bar{\lambda}$. From these observations it is easy to see that, although T is not a contraction (its derivative is not less than unity in absolute value everywhere), there is nevertheless only one w such that $w = Tw$, and also $w = \lim_{j\to\infty} T^j w_0$ for any $w_0 \leqslant 0$. See Alvarez and Stokey (1995) for a further discussion of iterative schemes for computing w in this case.

Remark 4. When $\gamma < 0$, there is an alternative to contraction iterations for finding w, λ^* and n^*. From Eq. (A.7),

$$w = \frac{f(\lambda^*, n^*)^{\gamma} g(n^*)}{\gamma(1 - \beta(\lambda^*)^{\gamma\psi})}.$$

The two first-order conditions associated with Eq. (A.8), together with the above expression, constitute three equations in the three objects sought.

References

Alvarez, F., Stokey, N.L., 1995. Dynamic programming with homogeneous functions. University of Chicago, manuscript.

Baxter, M., King, R., 1991. Productive externalities and business cycles. Discussion Paper 53, Institute for Empirical Macroeconomics, Federal Reserve Bank of Minneapolis, November.

Benhabib, J., Farmer, R.E.A., 1994. Indeterminacy and increasing returns. Journal of Economic Theory 63, 19–41.

Benhabib, J., Farmer, R.E.A., 1996. Indeterminacy and sector-specific externalities. Journal of Monetary Economics 37, 421–443.

Benhabib, J., Perli, R., 1994. Uniqueness and indeterminacy: on the dynamics of endogenous growth. Journal of Economic Theory 63, 113–142.

Benhabib, J., Rustichini, A., 1994. Introduction to the symposium on growth, fluctuations, and sunspots: Confronting the data. Journal of Economic Theory 63, 1–18.

Boldrin, M., 1992. Dynamic externalities, multiple equilibria, and growth. Journal of Economic Theory 58, 198–218.

Boldrin, M., Rustichini, A., 1994. Growth and indeterminacy in dynamic models with externalities. Econometrica 62, 323–342.

Bryant, J., 1981. Bank collapse and depression. Journal of Money, Credit and Banking 13, 454–464.

Christiano, L.J., 1984. A reexamination of the theory of automatic stabilizers. Carnegie-Rochester Conference Series on Public Policy 20, 209–210.

Christiano, L.J., 1988. Why does inventory investment fluctuate so much?. Journal of Monetary Economics 21, 247–280.

Christiano, L.J., Harrison, S.G., 1996. Chaos, sunspots and automatic stabilizers. National Bureau of Economic Research Working Paper 5703.

Christiano, L.J., Todd, R., 1996. Time to plan and aggregate fluctuations. Federal Reserve Bank of Minneapolis Quarterly Review 20, 14–27.

Diamond, D., Dybvig, P., 1983. Bank runs, deposit insurance, and liquidity. Journal of Political Economy 91, 401–419.

Farmer, R.E.A., Guo, J.T., 1994. Real business cycles and the animal spirits hypothesis. Journal of Economic Theory 63, 42–72.

Farmer, R.E.A., Guo, J.T., 1995. The econometrics of indeterminacy: An applied study. Carnegie-Rochester Conference Series on Public Policy 43, 225–271.

Gali, J., 1994a. Monopolistic competition, business cycles, and the composition of aggregate demand. Journal of Economic Theory 63, 73–96.

Gali, J., 1994b. Monopolistic competition, endogenous markups and growth. European Economic Review 38, 748–756.

Grandmont, J.M., 1986. Stabilizing competitive business cycles. In: Grandmont, J.M. (Ed.), Nonlinear Economic Dynamics. Academic Press.

Guesnerie, R., Woodford, M., 1992. Endogenous fluctuations. In: Laffont, J.J. (Ed.) Advances in Economic Theory. Sixth World Congress, Vol. 2. Cambridge University Press, Cambridge.

Guo, J.T., Lansing, K., 1999. Fiscal policy, increasing returns, and endogeneous fluctuations, University of California, Riverside, unpublished manuscript.

Hamilton, J., 1989. A new approach to the economic analysis of nonstationary time series and the business cycle. Econometrica 57, 357–384.

Harrison, S.G., 1998. Evidence on the empirical plausibility of externalities and indeterminacy in a two sector model. Barnard College, Columbia University, unpublished manuscript.

Hodrick, R.J., Prescott, E.C., 1997. Postwar US business cycles: An empirical investigation. Journal of Money, Credit, and Banking 29, 1–16.

Kydland, F., Prescott, E.C., 1980. A competitive theory of fluctuations and the feasibility and desirability of stabilization policy. In: Fischer, S. (Ed.), Rational expectations and economic policy. University of Chicago Press, Chicago.

Lucas, R.E., Jr., 1987. Models of Business Cycles. Basel Blackwell.

Poole, W., 1970. Optimal choice of monetary policy instruments in a simple stochastic macro model. Quarterly Journal of Economics 84, 197–216.

Sargent, T., 1979. Macroeconomic Theory, Academic Press.

Schmitt-Grohe, S., Uribe, M., 1997. Balanced-budget rules, distortionary taxes, and aggregate instability. Journal of Political Economy 105, 976–1000.

Shleifer, A., 1986. Implementation cycles. Journal of Political Economy 94, 1163–1190.

Stokey, N., Lucas, R.E. Jr. (with E. Prescott), 1989. Recursive methods in economic dynamics, Harvard University Press.

Woodford, M., 1986a. Stationary sunspot equilibria: The case of small fluctuations around a deterministic steady state. University of Chicago, September, unpublished manuscript.

Woodford, M., 1986b. Stationary sunspot equilibria in a finance constrained economy. Journal of Economic Theory 40, 128–137.

Woodford, M., 1991. Self-fulfilling expectations and fluctuations in aggregate demand. In: Mankiw, N.G., Romer, D. (Eds.) New Keynesian economics: Coordination failures sand real rigidities, Vol. 2. MIT Press, Massachusetts, pp. 77–110.

Woodford, M., 1994. Determinacy of equilibrium under alternative policy regimes. Economic Theory 4, 323–326.