Implementation Cycles

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The paper describes an artificial economy in which firms in different sectors make inventions at different times but innovate simultaneously to take advantage of high aggregate demand. In turn, high demand results from simultaneous innovation in many sectors. The economy exhibits multiple cyclical equilibria, with entrepreneurs' expectations determining which equilibrium obtains. These equilibria are Pareto ranked, and the most profitable equilibrium need not be the most efficient. While an informed stabilization policy can sometimes raise welfare, if large booms are necessary to cover fixed costs of innovation, stabilization policy can stop all technological progress.

I. Introduction

At least since Keynes (1936), economists have suspected that an autonomous determinant of agents' expectations can lead them to do business in a way that makes these expectations come true. Several recent studies confirmed this suspicion by exhibiting economies with multiple self-fulfilling rational expectations equilibria. Most notably, Azariadis (1981), Cass and Shell (1983), Grandmont (1983), and Farmer and Woodford (1984), among others, investigate such equilibria in overlapping generations models, while Diamond (1982) and Diamond and Fudenberg (1982) study them in a model with search-mediated trade. None of these studies, however, focuses on

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Keynes's specific concern about the influence of the state of long-term expectation, or business confidence, on businessmen's plans to undertake or postpone investment projects. A model addressing this concern is presented in this paper.[1] In the model, entrepreneurs hold partly arbitrary but commonly shared expectations about the future path of the economy and independently choose a pattern of investment that fulfills these expectations. Expectations influence the cyclical behavior of macroeconomic variables, the efficiency properties of the economy, and, in some cases, long-run development as well.

Specifically, the theory describes the possibilities for both cyclical and noncyclical implementation of innovations occurring despite the steady arrival of inventions.[2] The model is one of a multisector economy, in which each sector receives ideas about cheaper means for producing its output. Such inventions arrive to each sector at a constant rate. When a firm in a sector invents a low-cost technology, it can start using it at any time after the invention. Although the inventing firm can profit from becoming the lowest-cost producer in its sector, such profits are temporary. Soon after the firm implements its invention ("innovates"), imitators enter and eliminate all profits. For this reason, the firm would like to get its profits when they are the highest, which is during a general boom. Expectations about the date of arrival of this boom determine whether the firm is willing to postpone innovation until the boom comes.

[1] Several recent theoretical studies are related to the current discussion. Weitzman (1982) and Solow (1985) discussed multiple Pareto-ranked equilibria in economies with increasing returns; my work sheds doubt on the importance they attribute to the increasing returns assumption. Rotemberg and Saloner (1984, 1985) study oligopolies that intensify exogenously started booms through price wars or inventory buildups. Their work shares with mine the "unsmoothing" character of private agents' response to macroeconomic fluctuations, though they do not model endogenous cycles. Finally, Judd (1985) discusses a completely different mechanism that leads to innovation cycles. In his model, there is an infinite supply of possible innovations, but when firms introduce too many new products within a short time, they compete for the same consumer resources and reduce profits from each particular product. Furthermore, when imitation leads to price reductions for recently introduced products, consumers substitute toward these products, making entry by yet newer products even less profitable. As a result, after a period of innovation, entrepreneurs wait until the secular growth of the economy renders further innovation profitable. Judd's mechanism is almost the opposite of mine: innovations in his model repel rather than attract other innovations.

[2] Because of the essential role played by innovation in this paper, the theory of business cycles it presents might (incorrectly) be thought to be Schumpeterian. Schumpeter (1939) thought the innovation process to be essentially autonomous and completely independent of market demand. His inventors create markets rather than adapt to enter good markets. In contrast, Schmookler (1962, 1966) believed that innovation occurs in markets in which demand is substantial and profits from innovation can be great. My theory, then, is more Schmooklerian than Schumpeterian. Schmookler, however, insisted that expectations are adaptive and that innovation takes place in the markets in which demand has proven to be high. My work, in contrast, emphasizes foresight as a determinant of the timing of innovation.

If all firms owning inventions share the expectations about the timing and size of the general boom, they can time their innovations to make this boom a reality. When firms in different sectors all anticipate an imminent boom, they put in place the inventions they have saved. By innovating simultaneously, firms give a boost to output and fulfill the expectation of a boom. When, on the other hand, firms expect a boom only in the distant future, they may choose to delay implementation of inventions. When firms in different sectors postpone innovation, the economy stays in a slump. A firm in a given sector affects the fortunes of firms in other sectors by distributing its profits, which are then spent on output of firms in all sectors. In turn, they benefit when profits from other sectors are spent on their own products. By waiting to innovate, all firms contribute to the general prosperity of a boom, which affords them profits that are worth waiting for.

When expectations drive investment, the economy can fluctuate without fluctuations in invention. Any of a number of cycles of different durations can be an equilibrium, depending on agents' anticipations about the length of the slump. The longer is the slump, the bigger is the boom that follows it. One possible equilibrium outcome is the immediate implementation of inventions, in which case output grows without a cycle. When the economy does fluctuate, it falls behind its productive potential as firms postpone innovation but catches up in a boom. Productivity grows in spurts. An economy with these features is described in Sections II–III.

The possibility of a cyclical equilibrium sheds doubt on a frequently articulated view that a market economy smooths exogenous shocks. Inventions here can be interpreted as shocks hitting the economy, which are essentially identical each period. But these shocks can be "saved." If they are, the stock of technological knowledge grows steadily but is embodied into technology periodically. The economy follows a cyclical path when a much smoother path is available.

When expectations are autonomous, the economy can end up in any of its several perfect-foresight stationary cyclical equilibria. These equilibria are Pareto ranked, so the economy can settle into a very bad equilibrium. But expectations need not be truly autonomous; they may reflect agents' preferences over equilibria. For example, some equilibria may generate higher profits for all firms whose actions affect the equilibrium path. If the myriad of firms in the economy could coordinate on production plans supporting this equilibrium, it is arguably the natural outcome to expect. The question then arises whether the most profitable equilibrium is the one preferred by consumers. Section V shows that if innovation does not require fixed costs, the acyclical equilibrium is both the most profitable and the most efficient. In contrast, Section VI supplies an example in which, with contemporaneously incurred fixed costs, the most profitable equilibrium is cyclical, but the most efficient one is acyclical. The example raises the possibility of a disagreement between workers and firm owners over the preferred path of the economy.

In the model discussed in Sections II–VI, long-term development of the economy is independent of which equilibrium obtains. In the long run, all good ideas are put to use, with or without business cycles. Since cycles are inefficient, a countercyclical fiscal policy that could steer the economy to the steady growth equilibrium would be desirable. An example of such a policy, presented in Section VII, is a progressive tax system (or a tax surcharge during booms) that reduces the profitability of innovation during booms and thus eliminates cycles and raises welfare.

In general, however, long-term development might rely on the cycle, and an ignorant countercyclical policy might be harmful. For example, if each innovator must incur a fixed cost in the period prior to innovation (e.g., he must build a plant), large sales during booms may be necessary to enable the entrepreneur to cover his fixed costs. Innovations introduced during slumps may lose money, and even steady growth equilibria may fail to sustain innovation because of an insufficient level of aggregate demand. In this case, only a cyclical equilibrium is compatible with implementation of inventions. In an alternative equilibrium (called "stone-age"), firms never expect a boom and do not innovate at all.

If the government understands that long-term growth can be sustained only with fluctuations, it will forgo stabilization policy. An attempt to eliminate the cycle with aggregate demand management at best will be wasteful and at worst will steer the economy into the stone-age equilibrium. The success of countercyclical policy should be judged in the light of the possibility that an ignorant policy can entail substantial welfare losses if it blocks technological progress.

While focusing on the role of expectations and on coordination, I depart significantly from a down-to-earth theory of investment. Capital in the model is a stock of knowledge embodied into a technology that uses no durable assets. Investment constitutes taking available ideas that are not being used and adding them to the stock of utilized knowledge. The cycles are implementation cycles rather than cycles in physical investment. Knowledge, however, is a very imperfect proxy for a physical asset since it does not offer the same opportunities for physical smoothing of consumption. Section VIII discusses the consequences of introducing capital into the model and considers additional assumptions that must be made to preserve the results. It also discusses additional extensions and presents conclusions.

II. Setup of the Model

A. *The Consumer*

The household side of the economy consists of one representative consumer, who lives for an infinite number of discrete periods. The consumer's preferences are defined each period over a list of N goods, which is constant over time. The lifetime utility function is given by

$$\sum_{t=1}^{\infty} \rho^{t-1} \frac{\left(\prod_{j=1}^{N} x_{tj}^{\lambda}\right)^{1-\gamma}}{1-\gamma}, \tag{1}$$

where $\lambda = 1/N$ and x_{tj} is consumption of good j in period t.

I use Cobb-Douglas preferences within a period to abstract from substitution between different goods.[3] In a model addressing macroeconomic questions, equilibrium in each sector should be determined by aggregate demand and not by prices in other sectors, a property guaranteed by Cobb-Douglas preferences. The infinitely lived consumer formulation assures that the results are not driven by the restricted market participation property of overlapping generations models. All the results I present also hold in a finite-horizon economy.

Assume that in period t each good j is sold at the price p_{tj} on a separate market and that the consumer's income is y_t. If the interest rate paid at time t is denoted by r_{t-1}, the consumer's budget constraint is

$$\sum_{t=1}^{\infty} \frac{y_t - \sum_{j=1}^{N} p_{tj} x_{tj}}{D_{t-1}} = 0, \tag{2}$$

where $D_t = (1 + r_1) \ldots (1 + r_t)$ and $D_0 = 1$.

The assumption of one lifetime budget constraint for the representative consumer relies on perfect capital markets. In particular, it means that an entrepreneur can borrow against the profits (known with certainty) that will be earned from his yet unmade invention. We can think of inventors selling the claims to profits from future inventions in a competitive stock market so that all these claims are traded from the start. The model thus allows for consumers with heterogeneous wealth levels (specifically, inventors and noninventors) and for capital market transactions between them. Once these transactions

[3] Cycles of the type I discuss will be easier to sustain, the less substitutability there is between different goods.

are completed, we can think in terms of a representative consumer with a lifetime budget constraint.

Maximization of (1) subject to (2) yields consumption expenditures c_t in period t such that

$$c_t^{\gamma} = \left(\sum_{i=1}^{N} p_{ti}x_{ti}\right)^{\gamma} = \rho^{t-1}D_{t-1} \cdot (\Pi\, p_{jt}^{\lambda})^{\gamma-1} \cdot \frac{1}{\alpha}, \tag{3}$$

where α is the Lagrange multiplier on (2). In addition, we get constant expenditure shares for various goods:

$$x_{ti}p_{ti} = \lambda c_t. \tag{4}$$

Assume that physical storage is impossible. Competitive interest rates adjust to make $y_t = c_t$ so that the consumer wants neither to borrow nor to save. Equilibrium interest rates are given by

$$1 + r_t = \frac{1}{\rho} \cdot \left(\frac{y_{t+1}}{y_t}\right)^{\gamma} \cdot \frac{\left(\prod_{j=1}^{N} p_{jt+1}^{\lambda}\right)^{1-\gamma}}{\left(\prod_{j=1}^{N} p_{jt}^{\lambda}\right)^{1-\gamma}}. \tag{5}$$

Finally,

$$y_t = \Pi_t + L, \tag{6}$$

where Π_t are aggregate profits in period t, L is the inelastic labor supply, and wage at each t is taken to be one without loss of generality. Throughout, I measure prices and interest rates in wage units.

B. Market Structure and Innovations

Prior to period 1, output of each good j can be produced by firms with constant returns to scale technologies in which output is equal to the labor input. Every period, firms play a Bertrand price game, without capacity limitations. The Bertrand assumption is equivalent to assuming competition whenever no innovation takes place while letting the innovator be a dominant firm in its market. In period 1, then, equilibrium prices all equal unity.

Each period, one firm in each of n sectors generates an invention. These inventions are made in a very strict order. In the first period, firms in sectors $1, \ldots, n$ get ideas; in the second period, firms in sectors $n + 1, \ldots, 2n$ invent, and so on. In period $T^* = N/n$, firms in the last n sectors invent, and in period $T^* + 1$, the next round of inventions begins with sectors $1, \ldots, n$. This order is permanent for all rounds of invention.

An invention in each sector is a technology that produces one unit of output with $1/\mu$ the labor it took to produce this output with the best technology known up to then. The rate of technological progress μ exceeds one and is the same for all goods and for all times. Thus ideas from the first round permit a unit of output to be produced with $1/\mu$ units of labor, those from the second round with $1/\mu^2$ units, and so forth.[4]

In any period from the date it gets the invention, the firm can enter the market and implement it. Assume that the firm can postpone innovation without the danger that another firm implements it first (until, of course, the next idea arrives to the sector). When it innovates, the firm enters a Bertrand market, in which it becomes the lowest-cost producer. Equilibrium price equals the marginal cost of inefficient firms, but the innovator captures the whole market. He does not want to lower the price since demand is unit elastic, and he cannot raise it without losing all his sales. In the period after innovation, imitators enter and compete away all profits, with the price falling to the marginal cost of the efficient technology, or $1/\mu$ times the old price.

C. *The Decision to Innovate*

Consider what happens to a firm that innovates when aggregate demand is y_t and the marginal cost of an inefficient producer in its sector is w_{ti}. It gets revenue λy_t for output $\lambda y_t/w_{ti}$, obtained at a unit cost w_{ti}/μ and a total cost $\lambda y_t/\mu$. Its profits are

$$\pi_t = \lambda y_t - \left(\frac{\lambda y_t}{w_{ti}}\right) \cdot \left(\frac{w_{ti}}{\mu}\right) = \frac{\lambda(\mu - 1)y_t}{\mu}. \tag{7}$$

Independence of profits of the unit cost level of inefficient firms is a special feature of the Cobb-Douglas and constant unit cost assumptions; it does not buy any important results. Each firm innovating in period t will make π_t. I use the notation $m \equiv \lambda(\mu - 1)/\mu$, so that $\pi_t = my_t$.

Importantly, I assume that a firm owning an invention chooses its date of innovation (and hence the only date on which it makes profits) to maximize the present value of profits. It may be argued that, in a representative consumer economy, the firm should do what that consumer wants, and hence, if profit maximization leads to an inefficiency, it is an inappropriate objective for the firm. To deal with this

[4] Because prices are rapidly falling, it is important to keep in mind that restrictions on the speed of innovation are necessary to keep lifetime utility finite. These restrictions will be discussed later in the paper.

objection, the economy I describe can be replicated as in Hart (1982) so that owners of firms do not consume their firms' output. Suppose that there are two representative consumers on two identical islands, each laboring on his own island and consuming the fruit of his own labor but owning firms and saving on the other island. Suppose that the two islands are in the same equilibrium so that interest rates, incomes, and profits are the same on both. Then any firm that the representative consumer owns cannot affect the prices he faces by altering its date of innovation. In that case, the owner's objective is profit maximization since the firm's choices enter their problem only through the budget constraint and capital markets are perfect. After replication, all the issues I study remain. Having one representative consumer and a profit-maximizing firm is a simplifying but perfectly legitimate abstraction.

III. Construction of a Periodic Equilibrium

The principal decision of the firm holding an invention is to determine when to innovate. In this section, I show that firms in different sectors, receiving ideas at different times, may all choose to innovate in the period of high profits and high aggregate demand, that is, when other firms innovate. This synchronization of innovations gives rise to a multiplicity of perfect-foresight equilibria. One of them is always the steady growth acyclical equilibrium, in which inventions are implemented immediately. From the set of equilibria with fluctuating output, I focus on constant period cycles.

In a perfect-foresight equilibrium, firms form expectations about the path of interest rates and of aggregate demand, and these expectations are fulfilled by the chosen timing of innovations. Firms are assumed to be small so that each firm ignores its own impact on the behavior of aggregate variables. Similarly, when a firm makes its decisions, it cares only about aggregate data, and not about what is happening in any sector other than its own.[5]

Suppose we look for cycles of period $T \leq T^*$, in which inventions accumulated in periods $1, \ldots, T$ are implemented together in period T, called a T-boom. Innovations are imitated in period $T + 1$, which is also the first period of the next cycle. (I speak in terms of periods $1, \ldots, T$, but this should be interpreted as modulo T.)

[5] I could alternatively assume a continuum of infinitesimal sectors. The essential assumption is that firms take the behavior of aggregate variables as given and ignore their own impact on those variables. It is therefore misleading to interpret this model as a game between sectors.

The conditions for the existence of a perfect-foresight cyclical equilibrium of period T are twofold. First, it must be the case that if firms inventing in periods $1, \ldots, T-1$ expect the boom to take place in period T, they choose to innovate in period T rather than in the period in which they get their ideas or any period prior to T. Second, if firms with inventions expect a boom in period T, they must prefer not to wait past period T to innovate; in particular, they should not want to wait until the next boom. I take up these two conditions in order.

To find conditions for postponement, fix T and consider first the periods of no innovation. Prior to the boom, there are no profits earned in the economy, and hence in periods $1, \ldots, T-1$ income is L. Since prices do not change either, interest rates are given by $1 + r_1 = \ldots = 1 + r_{T-2} = 1/\rho$. Next consider a T-boom. Since a firm will never sit on a new idea until the next idea comes to its sector and makes the first one obsolete, T must satisfy

$$T < T^* = \frac{N}{n}. \tag{8}$$

Since profits are the same in each innovating sector, we have

$$\Pi_T = nTmy_T, \tag{9}$$

which combined with (6) yields

$$\pi_T = \frac{mL}{1 - nTm} = my_T. \tag{10}$$

Note that the condition $T < 1/nm$ is implied by (8). We also apply (5) to get

$$1 + r_{T-1} = \left(\frac{1}{\rho}\right)(1 - nTm)^{-\gamma} \tag{11}$$

since prices do not change from period $T-1$ to period T.

If a firm getting its idea in period 1 is willing to wait until period T to implement it, so will firms getting their ideas in periods $2, \ldots, T-1$ since the interest rate is positive throughout and income stays constant at L. For the same reason, if a firm that gets the idea in period 1 wants to postpone implementing it beyond period 1, it will not want to implement it until time T.

We can now calculate the condition under which the firm that gets its idea in period 1 is willing to delay innovation until period T. That condition is $\pi_T/D_{T-1} > \pi_1$ or

$$\rho^{T-1}(1 - nTm)^{\gamma-1} > 1. \tag{12}$$

Expression (12) can be interpreted as follows. Profits in this model are proportional to output, while the discount factor is proportional to output raised to the power γ. The more concave is the consumer's utility, the higher must be the interest rate to keep him from wanting to borrow in the period prior to the boom. Discounted profits are thus proportional to output raised to the power $1 - \gamma$. Also, $(1 - nTm)$ is the share of wages in income, and hence discounted profits are proportional to $(1 - nTm)^{\gamma - 1}$ since the wage bill is constant. The more firms innovate, the higher is the share of profits and hence the higher is income, and the more profitable it is to innovate at that time. This effect, however, is mitigated by declining marginal utility of income and by discounting. In particular, with logarithmic utility, there can be no delay since the discount rate is proportional to profits. I therefore assume throughout that $0 \leq \gamma < 1$.[6] But as long as $\gamma < 1$ and (12) is satisfied, interest rates do not rise by enough prior to the boom to offset firms' preference for getting their profits during that boom.

When (12) holds, if all firms but one inventing in periods $1, \ldots, T$ are willing to wait until period T to innovate, that last firm is also willing to wait until period T. We now need to find under what conditions, when all but one of the firms innovate in period T, the last one does not want to wait beyond that. For even when a firm waits until the boom, it may want to wait one more period because of price declines in period $T + 1$ and the resulting possibility of negative interest rates (in wage units) at time T.

Two influences can keep the firm from postponing innovation beyond a boom. First, even despite price declines, discounting may be sufficient to render postponement unprofitable. Second, by the time a firm might want to innovate in the future, the next invention may have arrived in its sector, thus preventing it from profiting from its own idea. I first provide the condition under which a firm does not want to wait beyond a boom even without a danger that its invention will be surpassed, and then I deal with that danger.

Observe first that if the firm does not want to wait until $2T$ to innovate, it will not want to wait until any period before $2T$ either. For if it did, it would also want to postpone innovation from then until $2T$, by (12). Observe also that the firm unwilling to wait until $2T$ would not choose to wait beyond $2T$ since delaying innovation from $2T$ to $2T + t \leq 3T$ is just like delaying innovation from T to $T + t \leq 2T$ (or even worse if the next idea may be coming into the sector). All we need to find, then, is the condition under which the firm prefers entering at time T to entering at $2T$.

[6] In contrast, Grandmont (1983) requires high γ's to generate a cycle in an overlapping generations model.

To get this, apply (5) to obtain

$$1 + r_T = \frac{1}{\rho} \cdot \frac{(1 - nTm)^{\gamma}}{\mu^{nT\lambda(1-\gamma)}}. \tag{13}$$

The power of μ appears in (13) since imitators drive the price of the goods whose production was innovated last period to $1/\mu$ of their period T levels (and profits to zero). After imitation, the pattern of interest rates in periods $T + 1, \ldots, 2T - 1$ repeats that in periods $1, \ldots, T - 1$, and profits in period $2T$ are again given by (7). To prevent delay until $2T$, then, we need

$$\rho\mu^{n\lambda(1-\gamma)} < 1. \tag{14}$$

Condition (14) excludes the possibility that firms want to postpone their innovation indefinitely; it is also equivalent to the transversality condition for the consumer's problem, guaranteeing that lifetime utility (1) is finite in equilibrium.

The inverse of the left-hand side of (14) raised to the power T is the discount factor between periods T and $2T$. That inverse also equals the interest rate that would prevail in a steady growth equilibrium. Inequality (14) says that, looking from period T, period $2T$ profits (which in wage units are the same) should be discounted. The problem is that prices fall after period T, and hence the period T interest rate may be negative. Nevertheless, by assuming (14) we insist that, on average, the future be discounted at a positive rate, which happens only if technological progress is not too fast.[7]

When (14) holds, no firm wants to postpone innovation beyond a boom even when the next innovation in its sector does not arrive until after the next boom. When (14) fails, a firm wishes to postpone innovation until the boom just prior to the arrival of the next idea into its sector. In this case, only the cycle of length $T^* = N/n$ can be sustained as a periodic perfect-foresight equilibrium, and this cycle always exists when (14) is false. (Proof: Equation [12] reduces to $\rho^{[N/n] \quad 1}\mu^{1-\gamma} > 1$, which is true whenever $\rho\mu^{n/N[1-\gamma]} > 1$.) Because failure of (14) leads to the conclusion of infinite lifetime utility, the usefulness of this case is unclear, and I ignore it from now on.

The main arguments of this section can be summarized in the following proposition.

[7] An alternative interpretation of (14) can be made if we take as numeraire the price of a good whose production is innovated in period T. Then by period $2T$, the real price of each good whose production has not been innovated at time T rises by a factor of μ, as do the real wage, aggregate income, and profits. Condition (14) says that the real discount rate between periods T and $2T$ actually exceeds μ. As before, it amounts to saying that the force of time preference dominates the rate of increase of real income and profits.

PROPOSITION 1. Suppose that the pace of innovation is slow enough that (14) holds. Then for every T satisfying (8) and (12), there exists a perfect-foresight cyclical equilibrium in which all accumulated inventions are implemented simultaneously every T periods.

This result has a simple economic interpretation. If firms can receive profits in only one period, they would like to do so at the time of high aggregate demand. The latter obtains when profits are high, and profits are high when many firms innovate. The rise in interest rates in the period prior to the boom is not sufficient to offset this preference for synchronization.

The paths of utility and of the interest rate in wage units over a T-cycle are shown in figure 1. Over time, the magnitude of the cycle stays constant thanks to Cobb-Douglas preferences, which imply that profits in each boom are the same and each round of cost reductions has the same effect on interest rates. A detrended output series exhibits a cyclical pattern with both booms and recessions.

IV. The Multiplicity of Equilibria

Proposition 1 suggests that, for a given set of parameter values, there may be several periodicities T for which there exists a cycle. In particular, it is obvious that $T = 1$ always works, so the 1-cycle—a steady growth equilibrium in which ideas are implemented as soon as they are received—always exists.

To study the multiplicity of perfect-foresight equilibria of a constant period, define the left-hand side of (12) as the function

$$f(T) \equiv \rho^{T-1}(1 - nmT)^{\gamma - 1}.$$

Remember that $mLf(T)$ is the present value that a firm inventing in period 1 attaches to its invention in a T-cycle. We are interested in T's between one and N/n, for which $f(T) > 1$ when parameters satisfy (14). To describe this set, it is useful to start with two calculations. The proofs of these and subsequent claims are collected in Appendix A.

LEMMA 1. The function $f(T)$ attains a minimum at a positive T_M under (14). Furthermore, $f(T)$ is decreasing for $T < T_M$ and increasing for $T > T_M$.

LEMMA 2. Under (14), $f(N/n) < f(1)$.

The lemmas imply that $f(T)$ attains its minimum somewhere to the right of $T = 1$. It may decrease all the way to N/n or reach a minimum before N/n and rise afterward, but not all the way to $f(1)$. In fact, from the restrictions imposed so far, we cannot ascertain either the sign of $f'(N/n)$ or whether $f(N/n)$ is greater or less than unity. The possibilities for the set of T's that keep $f(T)$ above one are then as follows. It can include all T's between one and N/n, only low T's, or both low

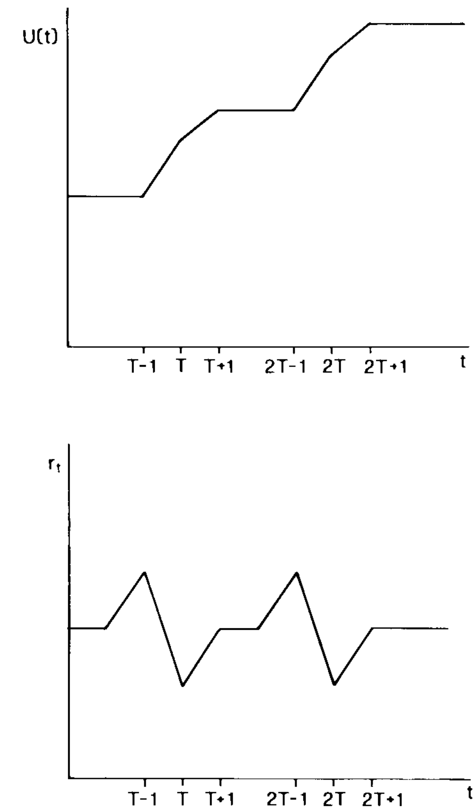

FIG. 1.—Paths of utility and interest over the cycle

T's and high T's, with a break in the middle. The three possibilities are shown in figure 2. They demonstrate the general multiplicity of equilibria in this model.

Multiple equilibria arise naturally when expectations govern the timing of investment. In a 1-cycle, agents always expect a constant but mild boom and promptly innovate to make it come true. In a longer-period cycle, agents expect a low level of aggregate demand for a time and correctly anticipate the moment of a big boom. Compared with short cycles, long cycles have longer (and deeper, after detrending) slumps but also wider spread booms.[8] In addition, there can be equilibria with variable periods of the cycle. As long as firms do not want to wait until the next cycle's boom, the period of the cycle today

[8] In contrast, Diamond and Fudenberg (1982) exhibit a business cycle that at every stage provides agents with a lower utility flow than does the good stationary equilibrium.

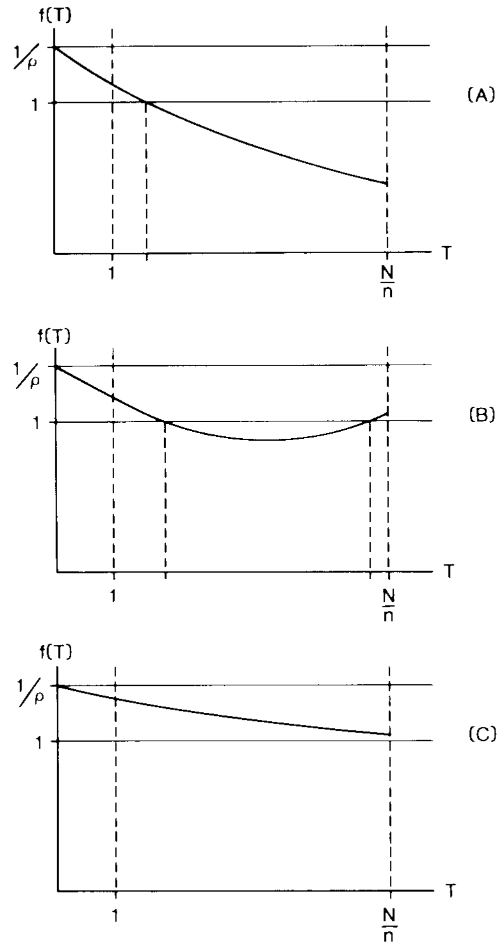

FIG. 2.—Multiplicity of cyclical equilibria. *A*, only short cycles are feasible; *B*, short and long cycles are feasible; *C*, all cycles are feasible.

does not affect the period of the cycle in the future. Expectations can support any one of these equilibria, as long as beliefs are common to all the market participants.

It is worth noting that, when (12) and (14) hold with a strict inequality, none of these equilibria is sensitive to small perturbations of the aggregate demand process. For example, suppose that a firm in some sector makes a mistake and innovates in a slump. If the impact of this mistake on aggregate demand in the boom is negligible, (12) will continue to hold and other firms will stay with their planned timing of innovation. Equilibria are thus invariant to small exogenous fluctuations of demand.

V. Coordination, Profitability, and Efficiency

When several T-cycles qualify as equilibria, it becomes an issue which one of them should occur. An almost persuasive view argues that expectations are completely autonomous in this model, and therefore any discussion of equilibrium selection is unwarranted. Alternatively, one might ask which equilibrium firms will prefer and then maintain that the most profitable equilibrium is the most plausible one.

To do this, consider a firm in period 1, contemplating its own profits in various equilibria. If its profits are the highest in the T^*-cycle, all firms receiving ideas up until T^* would also prefer a boom at T^* to any earlier boom. Moreover, they know they cannot have a bigger boom than the T^*-boom even after T^* since a new round of inventions precludes delay by some firms. In this case, the T^*-cycle is the most profitable for all firms receiving ideas up until T^*, and in this sense it is focal. (Note that not *all* firms prefer a boom at T^*; if $T^* = 3$, firms getting inventions in period 4 might well prefer the 2-cycle.) Alternatively, if firms getting inventions in period 1 prefer a boom right then to a boom in any future period, it is plausible to expect them to be able to coordinate on immediate innovation. In this case, steady growth is a focal outcome.[9]

In my example, discounted profits in the T-cycle for a firm getting an invention in period 1 are given by $mLf(T)$. By lemmas 1 and 2, this quantity is the highest in the 1-cycle, which I therefore consider to be a plausible outcome. Though this weakens the case for my model as a predictor of cycles, the next section will present a generalization in which the T^*-cycle may well be the most profitable.

The next obvious question is one of the consumer's preference between equilibria. Though it is intuitively appealing that the consumer should prefer the 1-cycle to all others, this proposition is not trivial to show. True, in the 1-cycle, the consumer gets price reductions the soonest, and the production set of the economy expands at the fastest technologically feasible rate. As a result, if we compare a T-cycle with the 1-cycle, in all periods other than T-booms, the consumer is clearly better off in the 1-cycle. In T-booms, however, high profits may compensate for higher prices. Take, for example, period T^* in the T^*-cycle. In that period, innovation occurs in all sectors and, hence (by a standard result in tax theory), there is no static distortion since the markup is the same on all goods. In period T^* in the 1-cycle, we have the same production set and a distortion since only n out of N prices exceed the marginal cost. Hence period T^* welfare is higher in

[9] In this model, as well as in the extension with fixed costs from Sec. VI, cycles of period T, with $1 < T < T^*$, cannot be the most profitable for all firms receiving inventions between times 1 and T.

the T^*-cycle. In fact, for extremely high rates of innovation (i.e., rates of innovation that require unreasonably small discount rates ρ for [14] to hold), the consumer prefers the T^*-cycle. For example, let $T^* = 2$, $\gamma = 0$, so that (12) and (14) reduce to $\rho\mu > 1$ and $\rho\mu^{1/2} < 1$. We calculate that $U(1) - U(2) = (1 + \rho\mu^{1/2})[2\mu/(\mu + 1)] - (1 + \rho\mu)$. By setting $\rho = 1/4$ and $\mu = 9$, we satisfy (12) and (14), while $U(1) - U(2) = -.05 < 0$. Nonetheless, the following proposition holds.

PROPOSITION 2. Assume (14) and that $\mu \leq T$. Then the consumer's lifetime utility is higher in the 1-cycle equilibrium than it is in the T-cycle equilibrium.

The restriction $\mu \leq T$ is economically meaningless; however, it is weaker than the restriction $\mu < 2$. If we think of a period as a year, a "reasonable" value for the size of innovation in an average sector cannot be nearly that high.

The consumer thus prefers immediate innovation and will end up with it if firms can coordinate the timing of innovation to settle on the most profitable equilibrium. On the other hand, if expectations are truly autonomous and lead to a cyclical outcome, the consumer's welfare falls short even of its second-best potential attained through immediate innovation. Of course, no equilibrium in this model is efficient.

To pinpoint the sources of inefficiency in the model, consider its deviations from the Walrasian paradigm. First, firms do not act as price takers, and, in particular, firms recognize the effect of their innovation today on tomorrow's price. Second, the innovator's output improves the productive opportunities of imitators since the latter cannot imitate until innovation has taken place. This externality can be described with missing markets and turns out not to matter as long as imitators earn zero profits.[10] Absence of price taking is thus the culprit of inefficiency. In fact, it can be shown that if firms act as price takers in a similar model generalized to allow for decreasing returns (so that there are profits in equilibrium), we could not obtain a delay of innovation even with production externalities introduced by imitation.

[10] Because of constant returns, imitators always earn zero profits (since they always play the Bertrand game against at least the innovator). Hence they cannot afford to pay anything for the right to imitate. Even if we open personalized markets for imitation rights (following Arrow [1970]) but let the innovator set the price, he could sell nothing at any positive price; nor will he give away his idea at zero price. As a result, personalized markets will clear at a small positive price, at which both supply and demand equal zero in all periods. Put another way, in making his decisions the innovator can ignore these markets, and therefore their absence is irrelevant.

VI. The Case of Fixed Costs

Suppose that innovation requires a one-time expenditure of F units of labor at the time the innovation is implemented. Imitation, as before, comes free in the period after innovation. This change raises the relative desirability of innovation during a large boom and allows for the most profitable equilibrium to be cyclical.

Adaptation of the earlier analysis yields

$$\pi_T = \frac{mL - F}{1 - nTm} = my_T - F, \tag{10'}$$

for which we need to assume that

$$mL - F \geqslant 0. \tag{15}$$

The condition for preference for delay from period 1 to period T then is

$$\rho^{T-1}(1 - nTm)^{\gamma - 1}\left(\frac{L}{L - nTF}\right)^{\gamma} > 1. \tag{12'}$$

Fixed costs improve the possibilities for existence of T-cycles because, with fixed costs, aggregate demand is lower in a T-boom, and therefore r_{T-1} is lower. In fact, the ratio of profits in period T to what they would have been if a firm innovated in period 1 is the same with or without fixed costs. The essential difference fixed costs make is that they lower the interest rate in period $T - 1$ and thus raise *discounted* T-boom profits. (Note that, as long as $mL - F \geqslant 0$, $F < \lambda L$ and $nTF < nT\lambda L < L$.) The condition that a firm not wish to wait until the next boom remains (14), so we sum up with the following proposition.

PROPOSITION 3. Whenever there exists a T-cycle in a model without fixed costs, there also exists a T-cycle if fixed costs are low enough ([15] holds). Furthermore, with fixed costs, a T-cycle exists for some parameter values that do not admit a T-cycle without fixed costs (e.g., logarithmic preferences).

Proposition 3 shows that the multiplicity of equilibria is at least as big a problem now as it was without fixed costs. Moreover, since the relative profitability of long-period equilibria rises with F, a firm obtaining an invention in period 1 may now prefer the T^*-cycle, even when (14) holds.

LEMMA 3. If F is just below mL, the T^*-cycle is the most profitable, provided that

$$\rho^{(N/n)-1} \cdot \left[\mu - \left(\frac{n}{N}\right)(\mu - 1)\right] > 1. \tag{16}$$

It should be noted that condition (16) implies the existence of a T^*-cycle. For parameter values satisfying (14) and (16) simultaneously, firms may very well end up in a T^*-cycle. At the same time, if F is just below mL, the consumer prefers the 1-cycle. The reason is that profits are virtually equal to zero, and hence "high" profits at T^* cannot offset the lower path of prices of the 1-cycle. In this special case, the consumer's preference for immediate innovation is clear-cut. Fixed costs thus introduce the possibility that firms will "choose" an equilibrium that the consumer dislikes. If expectations accommodate this selection, we end up with a perfect-foresight equilibrium whose efficiency may be substantially lower than that of a less profitable equilibrium.

VII. Public Stabilization Policy and Long-Run Growth

In an economy that develops in a T-cycle, fiscal policy can eliminate fluctuations. Such a policy can raise welfare when steady growth is the socially preferred and feasible equilibrium. Consider first the economy without fixed costs, discussed in Sections II–III. Let the government step in in the first period after a boom and introduce a progressive income tax schedule $\tau(y_t)$, to be applied to all income. The proceeds of the tax are thrown into the sea.

In this economy, income and profits in period t are reduced by a factor of $[1 - \tau(y_t)]$, and interest rates are again given by (5), where the income is after-tax. The firm should now maximize its owner's discounted after-tax profits. Under these modifications, it does not pay a firm to postpone innovation until period T if τ satisfies

$$\rho^{T-1}\left\{\frac{1-\tau\left(\dfrac{L}{1-nTm}\right)}{(1-nTm)[1-\tau(L)]}\right\}^{1-\gamma} \leq 1. \tag{17}$$

Set $\tau(L) = \tau(L/(1 - nm)) = 0$ and, for each $T > 1$, let $\tau(L/(1 - nTm))$ be the tax rate satisfying (17) with equality. In this case, profits are lower in the boom, so even though the interest rate prior to the boom is also lower, since $\gamma < 1$, discounted profits from investing in the boom fall. The economy is thus stabilized on the 1-cycle; the government collects no taxes but stands ready to implement its policy. A progressive tax (or tax surcharge) here resembles an automatic stabilizer (Baily 1978). Furthermore, at the time of the announcement, the policy has an infinite multiplier, as income jumps from L to $L/(1 - nm)$ without any government expenditure. Finally, the policy raises both welfare and profits and hence should receive universal support.[11]

Even if the government sets its tax variables suboptimally but still stabilizes the economy on steady growth, the amount of harm such a policy can do is limited. If the government sets $\tau(L/(1 - nm)) > 0$, it collects revenues, buys goods, and throws them into the sea. Still, such a policy can only waste what is collected; it cannot arrest technological progress.

This, however, is not the case in a more general model. What makes the cases studied in Sections II–VI special is that long-run development of the economy is independent of the particular cyclical path that it follows. Eventually, all good ideas are put to use, with or without business cycles. While fiscal policy stabilizes growth, it has no consequences for development.

An alternative possibility is that the cycle is essential for development. This would be the case if, for example, firms could not cover their fixed costs when they expect other firms to innovate as soon as they invent. Thus aggregate demand during steady growth is too low to sustain it as an equilibrium. Only in a boom of a cyclical equilibrium might aggregate demand be high enough to enable firms to cover fixed costs. In addition, there will exist a stone-age equilibrium in which, because firms do not expect other firms to innovate, innovation is unprofitable and never takes place. Cyclical synchronization of innovations is thus essential for implementation of inventions. Appendix B presents an example of such an economy.

If cyclical growth is essential for innovation, stabilization policy can do more harm than good. The reason is that if taxes render innovation unprofitable even during booms, there may be no times when the firm can earn a profit from implementing its invention. As a result, the economy will settle in the stone-age equilibrium. Too aggressive a fiscal policy, while getting rid of the cycle, can actually endanger technological progress.

VIII. Conclusion

The examples discussed in this paper have attempted to illustrate the impact of entrepreneurs' expectations about the future path of macroeconomic variables on their decisions to undertake or postpone investment projects. An economy in which aggregate demand spillovers favor simultaneous implementation of projects in different sectors was shown to exhibit cyclical equilibria, with duration of slumps

[11] One potential disadvantage of this policy is that it is not subgame perfect. When it comes to a boom, the government would prefer not to tax and waste the income.

governed largely by expectations. In some examples, business cycles were either the most profitable or even unique outcomes. Furthermore, although countercyclical policy stabilizes the economy in some cases, an aggressive intervention can interfere with long-run development.[12]

The model I discussed can be amplified to study the nature of cyclical equilibria in a somewhat more realistic context. For example, if inventions come in different sizes (e.g., different μ's relative to the same F), firms with big ideas need not wait for the boom, even when firms with small ideas do. Alternatively, a very large unanticipated innovation (or some other shock) can have a large enough impact on aggregate demand so as to trigger a boom prior to its otherwise anticipated time.[13] Finally, if inventions arrive into some sectors more often than they do into others, economywide equilibrium can consist of overlapping cycles of different periodicities, as emphasized by Schumpeter (1939).

Some extensions of the model are suggestive of ways of getting to a unique equilibrium. For example, suppose some periods (such as the Christmas season) are characterized by an especially high marginal utility of consumption and therefore by low interest rates preceding them. The present value of profits earned from innovating in such periods might be especially high, resulting in their selection as booms. Although in the model without fixed costs cyclical equilibria with seasonal booms might be focal, they are not unique. With fixed costs, however, such equilibria can be made unique.

In evaluating the usefulness of this model, it might be fruitful to recall four conditions that seem to be responsible for cyclical equilibria. First, there must be a constantly replenished supply of pure profit opportunities. Second, these opportunities cannot be exploited forever without pure profits' being eliminated by entry. Third, profits in different sectors of the economy must spill over into higher demand in other sectors. Fourth, this spillover must be significant at the moment profits are received: intertemporal smoothing of consumption should not make the moment of receipt of income irrelevant for demand. As this discussion suggests, I do not regard innovation to be the critical part of the story: it is simply an extremely convenient way to model temporary pure profit opportunities. Furthermore, I consider the first three conditions to be quite appropriate for a market economy.

[12] This paper has not considered micro interventions, such as patent policy. For example, a policy granting each innovator a permanent patent ensures immediate implementation of inventions. Such a policy, however, is socially very costly.

[13] Farrell and Saloner (1984) studied a different mechanism for such a domino effect; some results similar to theirs hold in my model.

Absence of capital, however, is a critical assumption that cannot be eliminated without substituting an alternative. For suppose we add capital to the model. Then in a period of a slump, when the consumers realize that they will be better off in the future, they will attempt to dissave and to consume now, thereby reducing the future capital stock and smoothing out consumption between periods. In this case, there will be no general boom, and implementation cycles will be impossible in equilibrium. As I specified the model, physical dissaving is not feasible because there is no capital.[14] When all the adjustment to fluctuations in income occurs through interest rates, the incentives for firms not to wait for the boom are insufficient to eliminate cycles.

With capital, we need additional assumptions to accommodate implementation cycles. First, borrowing constraints can restrict opportunities for consumption smoothing. The results of this paper can be developed in an overlapping generations model of capital with the conclusion that, if entrepreneurs cannot borrow against future profits, cyclical equilibria are feasible. An alternative formulation, which is perhaps a fruitful subject for future research, is to consider a model with durable irreversible investment as in Arrow (1968). The effect of durable capital should be to limit the amount of physical dissaving that the economy can do. As a result, durable capital might accommodate implementation cycles, though I have not verified this possibility. Finally, the economy may be subject to uninsurable and unanticipated shocks, in which case the mechanism discussed in this paper will work to accentuate cycles, though it will not cause them.

Appendix A

Proof of Lemma 1

Setting $f'(T) = 0$, we obtain $T_M = (1/nm) + [(1 - \gamma)/\ln \rho]$. Taking logs of both sides of (14), we get that $\ln \rho + n\lambda(1 - \gamma)\ln \mu < 0$, so

$$\frac{1}{\ln \rho} > -\frac{1}{n\lambda(1 - \gamma)\ln \mu}.$$

Therefore,

$$T_M = \frac{1}{nm} + \frac{1 - \gamma}{\ln \rho} > \frac{1}{nm} - \frac{1 - \gamma}{n\lambda(1 - \gamma)\ln \mu}$$
$$= \frac{1}{n\lambda}\left(\frac{\mu}{\mu - 1} - \frac{1}{\ln \mu}\right) > 0,$$

[14] With fixed costs standing in for capital, as in Sec. VII, physical dissaving again is not feasible.

where the last inequality follows since $\ln \mu > (\mu - 1)/\mu$ for $\mu > 1$. The sign of the derivative of $f(T)$ is the sign of $(1 - nTm)\ln \rho + nm(1 - \gamma)$, which is negative for $T < T_M$ and positive for $T > T_M$.

Proof of Lemma 2

First, it can be verified that

$$\mu\left(1 - \frac{n}{N}\right) + \frac{n}{N} < \mu^{1-(n/N)} \quad \text{when } \mu > 1 \text{ and } 0 < \frac{n}{N} < 1. \tag{A1}$$

Now the claim of the lemma amounts to asserting that

$$X \equiv \left[\mu - \left(\frac{n}{N}\right)(\mu - 1)\right]^{1-\gamma} \cdot \rho^{(N/n)-1} < 1.$$

But (14) implies that $\rho < \mu^{-n/N(1-\gamma)}$, so

$$\begin{aligned} X &< \left[\mu - \left(\frac{n}{N}\right)(\mu - 1)\right]^{1-\gamma} \cdot [\mu^{-(1-\gamma)n/N}]^{(N-n)/n} \\ &= \left\{\left[\mu - \left(\frac{n}{N}\right)(\mu - 1)\right] \cdot \mu^{-(N-n)/N}\right\}^{1-\gamma}. \end{aligned}$$

The last expression is smaller than one provided that $\mu - (n/N)(\mu - 1) < \mu^{1-(n/N)}$, which is just (A1).

Proof of Lemma 3

Applying (10′), we can compute that

$$\pi_{T^*} = \rho^{(N/n)-1} \cdot \mu^{1-\gamma} \cdot \left(\frac{L}{L - NF}\right)^{\gamma} \cdot (mL - F)$$

and

$$\pi_1 = \frac{mL - F}{1 - nm}.$$

Therefore,

$$\begin{aligned} \frac{\pi_{T^*}}{\pi_1} &= \mu^{1-\gamma} \cdot \left(\frac{L}{L - NF}\right)^{\gamma} \cdot (1 - nm) \cdot \rho^{(N/n)-1} \\ &\approx \mu(1 - nm)\rho^{(N/n)-1} \end{aligned}$$

when $F \approx mL$. But $\mu(1 - nm) = \mu - (n/N)(\mu - 1)$, so when (16) holds, $\pi_{T^*}/\pi_1 > 1$, as claimed.

Proof of Proposition 2

Notation

Denote $\alpha = n/N$ so that $T^* = 1/\alpha$. Let $L(T)$ be the equilibrium *lifetime* utility of the agent in a T-cycle equilibrium for $T = 1, \ldots, T^*$. I will prove later that to compare $L(1)$ with $L(T)$ it is enough to look at the first T periods. Accordingly, denote by $U(T)$ the total utility attained over the first T periods in a T-cycle equilibrium and by $U(1)$ the total utility attained over the first T periods in a 1-cycle equilibrium. Denote by $v(T)$ utility in a T-boom. Also, for each of the first T periods of the 1-cycle equilibrium define $f_t = \Pi_{j=1}^{N} x_{tj}^{\lambda}$, where x_{tj} is equilibrium consumption of good j in period t in the 1-cycle, and define g_t as the corresponding quantity for a T-cycle. Thus

$$U(1) = \sum_{i=1}^{T} \rho^{i-1}(f_t^{1-\gamma})$$

and

$$U(T) = \sum_{i=1}^{T} \rho^{i-1}(g_t^{1-\gamma}).$$

The proof is long and tedious, so I outline the steps first. Step 1 proves that, when welfare in a 1-cycle equilibrium is compared with that in the T-cycle equilibrium, it is enough to look at the first T periods of the individual's life. Steps 2 and 3 restrict the parameter space to the cases that are most favorable to T-cycle welfare: step 2 shows that it is sufficient to look at $\gamma = 0$, and step 3 shows that, for $\gamma = 0$, it suffices to look at the highest permissible ρ, which by (14) is $\rho = \mu^{-n/N}$. Step 4 shows that $v(t)$ is never greater than μ^{α}, the value it attains when $T = T^*$. Step 5 shows that $U(1) > U(T)$ as long as $\mu < T$ and completes the proof.

Step 1

$L(1) > L(T)$ iff $U(1) > U(T)$. For both the 1-cycle equilibrium and the T-cycle equilibrium, the history of periods $T + 1, \ldots, 2T$ is the same as the history of periods $1, \ldots, T$, except nT prices are $1/\mu$ of their old levels. This makes the utility in period $T + x$ (where $x = 1, \ldots, T$) equal to $\rho^T \cdot \mu^{nT\lambda(1-\gamma)}$ times the utility in period x. Extending this argument to future periods, we get

$$L(1) = \frac{U(1)}{1 - \rho^T \mu^{nT\lambda(1-\gamma)}}$$

and

$$L(T) = \frac{U(T)}{1 - \rho^T \mu^{nT\lambda(1-\gamma)}}.$$

Inequality (14) ensures that these expressions are finite, and thus the assertion is proved.

Step 2

Other things equal, if $U(1) > U(T)$ for $\gamma = 0$, then $U(1) > U(T)$ for $\gamma > 0$. Observe that $\partial U(1)/\partial f_t = (f_t)^{-\gamma} \cdot \rho^{t-1}$ and $\partial U(T)/\partial g_t = (g_t)^{-\gamma} \cdot \rho^{t-1}$. In the 1-cycle, income (in wage units) stays constant over time, but prices are falling. Thus

$$f_i > f_j \quad \text{for } i > j. \tag{A2}$$

Also, before period T, the individual enjoys both higher income and lower prices in the 1-cycle than he does in the T-cycle. Thus

$$g_i < f_i \quad \text{for } i < T. \tag{A3}$$

If $g_T < f_T$ also, as may be the case if $T < T^*$, we are done with proving that $U(1) > U(T)$. The question is what happens when

$$g_T > f_T, \tag{A4}$$

so we assume from now on that this is the case. Our maintained hypothesis is that

$$\sum_{i=1}^{T} \rho^{i-1} f_i > \sum_{i=1}^{T} \rho^{i-1} g_i.$$

Now suppose that $\gamma > 0$ and apply the mean value theorem to find that

$$U(T) - U(1) = \sum_{i=1}^{T} \rho^{i-1} h_i^{-\gamma}(g_i - f_i),$$

where

$$f_T < h_T < g_T \tag{A5}$$

and

$$g_i < h_i < f_i \quad \text{for } i < T. \tag{A6}$$

Now combine (A5), (A2), and (A6) to show that $h_T > f_T > f_i > h_i$ for $i < T$. The last inequality implies that, for any $i < T$, we have

$$h_i^{-\gamma} > h_T^{-\gamma}. \tag{A7}$$

Using (A7) and (A3), we then obtain that

$$\sum_{i=1}^{T} \rho^{i-1} h_i^{-\gamma}(g_i - f_i) < \sum_{i=1}^{T} \rho^{i-1} h_T^{-\gamma}(g_i - f_i)$$
$$= h_T^{-\gamma}\left(\sum_{i=1}^{T} \rho^{i-1} g_i - \sum_{i=1}^{T} \rho^{i-1} f_i\right) < 0$$

by assumption. This proves the claim.

Step 3

If $\gamma = 0$ and $U(1) > U(T)$ for some ρ_1, then $U(1) > U(T)$ for all $\rho < \rho_1$. We are taking step 2 into account and also assuming that

$$\sum_{t=1}^{T-1} \rho_1^{t-1}(f_t - g_t) > \rho_1^{T-1}(g_T - f_T).$$

Multiply both sides by ρ^{T-1}/ρ_1^{T-1} to obtain

$$\sum_{t=1}^{T-1} \frac{\rho^{T-1}}{\rho_1^{T-t}}(f_t - g_t) > \rho^{T-1}(g_T - f_T).$$

But $\rho^{T-1}/\rho_1^{T-t} < \rho^{t-1}$ when $\rho < \rho_1$. Hence

$$\sum_{t=1}^{T-1} \rho^{t-1}(f_t - g_t) > \sum_{t=1}^{T-1} \frac{\rho^{T-1}}{\rho_1^{T-t}}(f_t - g_t) > \rho^{T-1}(f_T - g_T).$$

Q.E.D.

Since (14) imposes an upper bound on ρ in terms of μ and since we are taking $\gamma = 0$, we assume from now on that $\rho = \mu^{-\alpha}$.

Step 4

Utility in a T-boom, $v(T)$, is bounded above by μ^{α}. A calculation reveals that

$$
\begin{aligned}
v(T) &= \frac{1}{1 - T\alpha[(\mu - 1)/\mu]}\, \rho^{T-1} \\
&= \rho^{T-1}\, \frac{\mu}{(T^* - T)\alpha(\mu - 1) + 1} \\
&= \frac{\mu(T^* - T)\alpha}{(T^* - T)\alpha(\mu - 1) + 1}\, \mu^{\alpha}.
\end{aligned}
$$

The last equality follows since $\rho = \mu^{\alpha}$. By the mean value theorem,

$$\mu^{(T^* - T)\alpha} = 1 + (\mu - 1)(T^* - T)\alpha \cdot y^{-T\alpha}$$

for some $1 < y < \mu$. Then $y^{-T\alpha} < 1$, and so

$$\mu^{(T^* - T)\alpha} < 1 + (\mu - 1)(T^* - T)\alpha.$$

Using the last expression for $v(T)$ implies that $v(T) < \mu^{\alpha}$.

Step 5

$U(1) > U(T)$ for $\mu < T$. When $\rho = \mu^{-\alpha}$ and $\gamma = 0$, we can compute that

$$U(1) = \frac{1}{1 - \alpha[(\mu - 1)/\mu]} \cdot T.$$

Applying the mean value theorem to $f(x) \equiv 1/x$ for x between one and $1 - \alpha[(\mu - 1)/\mu]$, we obtain

$$U(1) = T\left\{1 - \alpha\, \frac{\mu - 1}{\mu} \left[\frac{-1}{(1 - Y)^2}\right]\right\}$$

for some $0 < Y < \alpha[(\mu - 1)/\mu]$. But then $U(1) > T\{1 + \alpha[(\mu - 1)/\mu]\}$.

When $\rho = \mu^{-\alpha}$, $\gamma = 0$, and $v(T) < \mu^{\alpha}$, we have

$$U(T) \leqslant 1 + \mu^{-\alpha} + \ldots + \mu^{(-T+1)\alpha} + \mu^{\alpha} < (T - 1) + \mu^{\alpha}.$$

The last inequality follows since each of the $T - 2$ middle terms is less than one. Applying the mean value theorem for $f(x) = x^{\alpha}$ for x between one and μ, we obtain $(T - 1) + \mu^{\alpha} = (T - 1) + 1 + (\mu - 1)\alpha Z^{\alpha - 1}$ for some $1 < Z < \mu$. But since $Z^{\alpha - 1} < 1$, we have $U(T) < T + \alpha(\mu - 1)$.

Putting the bounds on $U(1)$ and $U(T)$ together, we get

$$U(1) - U(T) > T + T\alpha\, \frac{\mu - 1}{\mu} - T - \alpha(\mu - 1) = \alpha(\mu - 1)\left(\frac{T}{\mu} - 1\right) > 0$$

by assumption. This completes the proof.

Appendix B

This Appendix describes an economy that grows either in cycles or with no innovation at all. Suppose that an innovating firm must incur a fixed cost in the period prior to innovation. If the innovation reduces the unit cost from that of the currently *used* technology by a factor of μ, then this fixed cost is F.

If, however, a round of innovation has been skipped and the innovation improves the currently used technology by a factor of μ^2, then the fixed cost is $2F$ (similarly $3F$ for μ^3, etc.). This technology captures the notion that more dramatic innovations are costlier to implement. In other respects, the economy is the same as that of Section II of the text.

Calculation of equilibria generally follows Section III, except now y_t exceeds c_t by the amount of fixed-cost investment needed for period $t + 1$ innovation. If an innovation requires a fixed cost aF, the cost to the firm is $aF(1 + r_t)$ since aF must be saved in period t and savings earn interest. Thus fixed costs are a limited form of capital. With savings in the model, interest rate expressions must be modified to allow for divergence of income from consumption.

Consider an economy in which $T^* = 2$ and $\gamma = 0$. Thus half of all sectors receive an invention each period. I will present an example in which (1) equilibria in which each sector implements every nth round of invention as soon as it arrives, while skipping the first $n - 1$ rounds, do not exist (in particular, for $n = 1$, steady growth equilibrium does not exist); (2) the 2-cycle exists; (3) the stone-age equilibrium, in which entrepreneurs expect no innovation to take place—and none does—also exists.

Requirement 1 amounts to the condition that an innovating firm's profits be negative in an equilibrium in which every k periods $N/2$ sectors reduce costs by a factor of μ^k, followed next period by the same reduction in the other $N/2$ sectors. For $k = 1$, this is the steady growth equilibrium. The condition that ensures that 1 is satisfied is

$$\frac{\mu^k - 1}{\mu^k} \cdot \frac{1}{N} \left\{ \frac{L - (N/2)Fk}{1 - [(\mu^k - 1)/2\mu^k]} \right\} - \frac{Fk}{\rho\mu^{k/2}} < 0, \quad \text{for } k = 1, 2 \ldots . \tag{B1}$$

To satisfy requirement 2, we calculate the 2-cycle. The interest rate before the boom is $1 + r_1 = 1/\rho$, and the interest rate before the slump is $1 + r_2 = 1/\rho\mu$ since all prices fall after the boom. Profits are given by

$$\pi_1 = m(L - NF) - F(1 + r_2) = \frac{1}{N}\frac{\mu - 1}{\mu}(L - NF) - F(1 + r_2) \tag{B2}$$

and

$$\pi_2 = m\mu L - F(1 + r_1) = \frac{1}{N}(\mu - 1)L - F(1 + r_1) \tag{B3}$$

in the slump and boom, respectively. We are seeking parameters for which

$$\frac{\pi_2}{1 + r_1} > 0 > \pi_1 \tag{B4}$$

or

$$\rho\left[\frac{1}{N}(\mu - 1)L\right] - F > 0 > \frac{1}{N}\left(\frac{\mu - 1}{\mu}\right)(L - NF) - \frac{F}{\rho\mu}. \tag{B5}$$

The last condition for the 2-cycle is (14) from Section III, which here reduces to

$$\rho\mu^{1/2} < 1. \tag{B6}$$

Finally, for the stone-age equilibrium to exist (requirement 3), it must be unprofitable for a firm to innovate alone in a period, or

$$\frac{\mu - 1}{\mu}\frac{1}{N}L - \frac{F}{\rho} < 0. \tag{B7}$$

Let $L = 100$, $\mu = 2$, $\rho = .7$, and $NF = 60$. For these parameter values, conditions (B1) and (B5)–(B7) can be shown to hold. In this case, equilibria with innovation but without synchronization do not exist, and bunching is necessary for technological progress.

Macroeconomic stabilization policy of the type described in Section VII will not work in this economy. To establish this, let τ_1 and τ_2 be the income tax rates for busts and booms, respectively, and observe that, as before, taxation reduces consumption and income. Because the consumer's utility is linear in income, after-tax interest rates will not be affected by the imposition of taxes. However, a firm pays for its plant at pretax interest rates, which increase as a result of imposition of the tax. These interest rates are given by

$$1 + R_1 = \frac{1}{\rho(1 - \tau_2)} \tag{B8}$$

and

$$1 + R_2 = \frac{1}{\rho\mu(1 - \tau_1)}. \tag{B9}$$

In this example, taxation strictly raises the cost of capital. As a result, if innovation was not profitable in a slump before taxes were introduced, it will not be profitable with taxes. Nor will taxes make innovation in a steady growth equilibrium profitable, thereby permitting such equilibrium to reappear. Furthermore, a firm may no longer be able to break even if it innovates in a boom. Profits in a 2-boom are now given by

$$\pi_2 = \left[\frac{1}{N}(\mu - 1)L - F(1 + R_1)\right] \cdot (1 - \tau_2). \tag{B10}$$

For the parameter values from the example and $\tau_2 > .15$, π_2 is negative. This means that a tax rate of 15 percent or higher on a 2-boom's income sends the economy into the stone-age equilibrium, which exists regardless of the level of the tax.

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