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Julio J. Rotemberg, Michael Woodford

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Oligopolistic Pricing and the Effects of Aggregate Demand on Economic Activity

Julio J. Rotemberg

Massachusetts Institute of Technology

Michael Woodford

University of Chicago

We construct a dynamic general equilibrium model in which the typical industry colludes by threatening to punish deviations from an implicitly agreed-on pricing path. We use methods similar to those of Kydland and Prescott to calibrate linearized versions of both our model and an analogous perfectly competitive model. We then compute the two models' predictions concerning the economy's responses to a change in military spending. The responses predicted by the oligopolistic model are closer to the empirical responses estimated with postwar U.S. data than the corresponding predictions of the competitive model.

In this paper we argue that the effects of aggregate demand shocks on economic activity are a consequence of imperfect competition. The aggregate demand shock that we model explicitly is a change in government purchases. For our empirical analysis we concentrate on the effect of military purchases because they are likely to be the most nearly exogenous government purchases. In spite of this relatively

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narrow focus, we believe that a similar analysis would be appropriate for other kinds of aggregate demand shocks.

We model the consequences of imperfect competition for aggregate fluctuations by constructing a completely specified intertemporal general equilibrium model. This model is identical to those studied in the real business cycle literature, except that firms producing goods are modeled as oligopolistic using a variant of the Rotemberg and Saloner (1986) model of repeated oligopolistic interaction. We assign quantitative magnitudes to the parameters of this model on the basis of facts about the U.S. economy that, for the most part, are unrelated to the economy's response to government purchases. We solve the model and generate quantitative predictions regarding the effect of exogenous changes in government purchases. We then compare these predictions both to those of a similarly calibrated competitive model and to the estimated response of the economy to unpredictable changes in military purchases. We conclude that the fit between the theoretical model and the empirical observations improves when we assume that firms price oligopolistically.

Section I motivates our model by explaining the benefits of considering imperfectly competitive models when discussing the effects of changes in aggregate demand. Section II presents our model of oligopolistic interaction. Section III embeds this in a standard general equilibrium model that we then linearize around its steady state. Section IV discusses the calibration of this linearized model's parameters. Section V describes how we measure the responses of output, hours worked, and the real wage to changes in military spending in the postwar period; Section VI compares these estimated responses of the economy to those predicted by our model. Section VII shows the robustness of our empirical findings by analyzing prewar data. Section VIII presents conclusions and directions for future research.

I. The Markup and the Transmission of Product Demand Shocks to the Labor Market

Perfectly competitive models predict that aggregate demand shocks such as changes in government spending can increase employment *only* by increasing households' willingness to supply labor. Changes in government spending, or aggregate demand more generally, do not affect firms' demand for labor at a given real wage. This feature of the competitive model can be understood quite simply and has been noted before (Lindbeck and Snower 1987; Woodford 1991). Suppose that aggregate production possibilities are described by a function $Y_t = F(K_t, H_t, z_t)$, where Y_t , K_t , H_t , and z_t , respectively, represent output, capital, labor input, and an index of the state of technol-

ogy at time t ; F is a concave function of (K_t, H_t) . The capital stock K_t is predetermined at time t , and z_t is exogenous with respect to firms' decisions. Then perfect competition in product and labor markets implies that the aggregate demand for labor is given by

$$F_H(K_t, H_t, z_t) = w_t, \quad (1)$$

where w_t is the real wage. For given K_t and z_t , this demand curve slopes downward because F_{HH} is negative. The equilibrium level of employment (and hence output) and the real wage are then determined once we specify a labor supply function, which can be written in the Frisch form as

$$H_t = H(w_t, \lambda_t), \quad (2)$$

representing the solution to households' optimization problem. The variable λ_t represents the marginal utility of wealth at time t . It depends on the expected real return on savings, on expectations of future real wages, and so forth.

Aggregate demand in period t can change for a variety of reasons. Demand rises if the government wants to increase its purchases. Demand for investment can rise in response to changed perceptions about future returns on investment. Demand for consumer goods can rise because consumers become more impatient and want to consume more now or because foreigners temporarily want to buy more of our output. These changes do not affect either K_t or z_t , and so they cannot affect the demand for labor. Any effect they have on employment must come from an effect on the marginal utility of wealth λ_t and through this on labor supply.

Several authors have proposed mechanisms through which increases in government purchases raise the marginal utility of wealth. One is that lifetime wealth is lower because of the need to finance the additional government spending. A second is that the increase in real interest rates induced by increases in government purchases (because of the need to induce intertemporal substitution in consumption) also raises the current marginal utility of wealth for a given expectation of the future marginal utility of wealth (see Hall 1980; Barro 1981; Baxter and King 1988; Aiyagari, Christiano, and Eichenbaum 1989). Other changes in aggregate demand could well have similar effects, although the argument for negative wealth effects seems special to increases in government purchases.

In this paper we argue that the standard competitive model's inability to induce shifts in labor demand as a result of changes in aggregate demand is a weakness. We shall present evidence below that increases in military purchases raise output together with real product wages paid in the private sector. This cannot be reconciled with an unchang-

ing labor demand curve. Rather, it requires that the labor demand curve shift out.

In this paper we show that increases in aggregate demand can raise the demand for labor if the assumption of perfect competition is dropped. Suppose that producers have some market power and are able to set price above marginal cost. Then (1) must be replaced by

$$F_H(K_t, H_t, z_t) = \mu_t w_t, \quad (3)$$

where μ_t is the ratio of price to marginal cost (or markup) in period t . Variations in the markup now shift labor demand (the relationship between H_t and w_t) just as technology shocks do. Increases in demand, such as those that might be caused by increases in government purchases, can now raise output and employment even with constant labor supply as long as they lower markups. As a result, increases in demand can raise output together with real wages.

A variety of models have been proposed in which markups can vary in response to demand conditions.¹ The simple one that we explore in depth here is based on Rotemberg and Saloner (1986). The basic idea of their model is that a small number of firms within an oligopoly collude to keep their prices above marginal cost. This collusion is supported only by the threat to revert to lower prices in the future (to punish) if any member of the oligopoly deviates by cutting prices. An increase in the industry's current demand relative to the industry's future demand raises the gains from undercutting relative to the losses from the future punishment. To prevent an immediate breakdown of collusion, the optimal incentive-compatible collusive agreement involves a smaller markup in these circumstances. In this way an increase in aggregate demand lowers markups and increases labor demand. We now proceed to describe this mechanism formally.

II. Oligopolistic Price Setting

We embed the Rotemberg and Saloner (1986) model of oligopolistic pricing in a complete dynamic general equilibrium model. Apart from endogenizing the demand curve and the production costs faced by each oligopoly, our presentation here extends that model in a number of respects. In particular, we allow for a much more general class of stochastic processes for industry demand (see also Haltiwanger and Harrington 1988; Kandori 1991), we allow for time-

¹ For references and a comparison of some of the leading alternatives, see Rotemberg and Woodford (1991). That paper also presents reasons for our particular interest in the model developed here. See also the brief remarks in Sec. VIII.

varying production costs, and we assume that the goods produced by the different firms in each industry are not perfect substitutes. The latter extension implies a nontrivial modification of the structure of the optimal symmetric collusive agreement. It no longer belongs to the class of equilibria in which a failure to comply with the collusive agreement results in reversion to Bertrand competition.

We consider an economy in which a large number of differentiated goods are produced, each by a single firm. These firms (and goods) are grouped in I industries, each consisting of m firms. Here m is assumed to be small and I is large; the goods produced by firms belonging to the same industry are close substitutes, whereas those produced by firms in different industries are less good substitutes. We therefore assume strategic interaction among firms in the same industry, whereas industries interact with one another as monopolistic competitors (in a sense clarified below).

The demand for produced goods, whether by consumers, government agencies, firms for investment purposes, or firms as inputs in current production, is assumed to be a demand for a single composite good. (This allows us to give an obvious interpretation to "aggregate demand.") The total output of this composite good at t , Q_t , is given by an aggregator function $Q_t = f(x_t^1, \dots, x_t^I)$, where x_t^i denotes the quantity obtained of the i th industry's composite good. Each industry's composite good is in turn defined by an aggregator function $x_t^i = g(q_t^{i1}, \dots, q_t^{im})$, where q_t^{ij} denotes the quantity obtained in period t of the good produced by the j th firm in the i th industry. In the case of output used for investment purposes, we assume that a single capital good is produced instantaneously using the differentiated products as inputs (with the aggregator functions playing the role of production functions) and then accumulated for use in future production. As a result, past investment decisions have an effect on current production possibilities only through a single capital aggregate.

We also assume that f and g are increasing, concave, and symmetric functions of their arguments (i.e., they are invariant under permutation of the arguments) and that they are twice continuously differentiable and homogeneous of degree one. As a result, the demand for each firm's product depends only on relative prices and total purchases of the composite good Q ; it is independent of the composition of these purchases (private consumption, government, etc.). Further boundary conditions are given in the Appendix. Briefly, we assume that the composite goods produced by all industries are essential, in that demand for each industry's output remains positive no matter how extreme relative prices may be. By contrast, demand for a given firm's product is zero if its price is too high relative to those of other

firms in its industry. These assumptions are consistent with the idea that goods in the same industry are relatively good substitutes but those in different industries much less so.

The homogeneous aggregator functions imply that the demand q_t^{ij} for each firm ij 's product can be written as a function of total expenditures and prices in period t or, alternatively, as a function of purchases Q_t of the composite good and relative prices. We focus only on symmetric equilibria. Therefore, we restrict ourselves to the case in which all firms in industries other than i charge the price p_t when discussing strategic interactions among firms in industry i . We thus write firm ij 's demand at time t as

$$q_t^{ij} = Q_t D^j \left(\frac{p_t^{i1}}{p_t}, \dots, \frac{p_t^{im}}{p_t} \right),$$

where the demand functions D^j are the same for all industries. Furthermore, the functions D^j for $j = 1, \dots, m$ are all the same function after appropriate permutation of the arguments.

Each firm also has an identical production technology

$$q_t^{ij} = \min \left[\frac{F(K_t^{ij}, z_t H_t^{ij}) - \Phi z_t N_t}{1 - s_M}, \frac{M_t^{ij}}{s_M} \right], \quad (4)$$

where K_t^{ij} denotes the capital services, H_t^{ij} the hours of labor, and M_t^{ij} the materials inputs employed by firm ij in period t . Here $0 < s_M < 1$ is the share of materials costs in the value of gross output (in a symmetric equilibrium with equal prices for all produced goods), $\Phi > 0$ indicates the existence of fixed costs, and F is increasing in both arguments and is homogeneous of degree one, so that marginal cost is independent of the scale of operation. Our assumption that materials are used in fixed proportions implies that the numerator of the first term represents the production function for value added by firm ij .² This function is directly comparable to the production function assumed by authors such as Aiyagari et al. (1989). We explicitly consider materials inputs because they are important for calibrating the

² In a competitive model, an arbitrary production function $Q = G(K, H, M)$ implies the existence of a production function for value added $F(K, H)$ defined by

$$F(K, H) = \max_M [G(K, H, M) - M],$$

assuming a symmetric equilibrium in which all produced goods, including materials, have the same price. With imperfect competition, however, materials inputs are generally not employed efficiently so that there is no production function $F(K, H)$ for gross national product independent of the markup. We avoid this complication here by assuming fixed coefficients and thus obtain an analogue of the standard aggregate production function for competitive models in eq. (23) below.

markup of prices over marginal cost; the markup μ in equation (3) is a markup over the marginal cost of producing value added, and this is a larger quantity than the markup we focus on here, namely the markup over the total marginal cost of production. The difference is important if one wishes to use evidence about firms or sectors of the economy to calibrate the average value of μ for the aggregate economy.³

The process $\{z_t\}$ represents exogenous labor-augmenting technical change.⁴ Technological progress is assumed to take this form, as in King, Plosser, and Rebelo (1988), because it allows the existence of a stationary equilibrium without special functional form assumptions for the production function F . The term $\Phi z_t N_t$ represents fixed costs, as a result of which the technology involves increasing returns (in the sense that average costs decrease with scale, although marginal costs do not). The fixed costs are included so as to make the postulated average markup of prices over marginal costs consistent with the observed moderate level of profits in the U.S. economy.⁵ The size of the fixed costs is assumed to grow over time, with exogenous growth in the population N_t and in labor productivity z_t . Otherwise, a stationary equilibrium would not exist because the importance of fixed costs would change over time.⁶

All firms purchase factor inputs (capital services, hours, and materials) in economywide competitive markets. At a symmetric equilibrium, each firm produces the same output and uses the same share of total labor and capital inputs. Therefore, (4) implies that

$$(1 - s_M)Q_t = F(K_t, z_t H_t^l) - \Phi z_t N_t \quad (5)$$

and

$$s_M Q_t = M_t \quad (6)$$

where K_t is the predetermined aggregate capital stock at t , H_t^l are total production hours at t , and M_t are total materials inputs at t . The number of industries I is assumed to be so large that no single industry's pricing and production decisions are expected to affect factor prices. Thus each firm has the same marginal cost, which, when (5)

³ See further discussion in Sec. IV below.

⁴ We could easily consider the effects of technology shocks in our model by assuming that this process is stochastic, although we do not take that up here.

⁵ See Hall (1987) and further discussion in Sec. IV.

⁶ It would be more satisfactory to endogenize the growth of fixed costs, e.g., as a result of the creation of new industries as the economy grows. This will have little effect on our results as long as the endogenous growth of fixed costs occurs only in response to long-run profits rather than to short-term fluctuations in profits due to variations in government purchases of the kind analyzed here.

and (6) are used, equals

$$\frac{(1 - s_M)W_t}{z_t F_H(K_t, z_t H_t^p)} + s_M p_t, \quad (7)$$

where F_H is the partial derivative of F with respect to its second argument and W_t is the wage at t . Let γ_t represent the ratio of p_t to this marginal cost. Thus γ_t is the markup charged in all industries except possibly industry i . It is given by

$$\frac{1}{\gamma_t} = \frac{(1 - s_M)w_t}{z_t F_H(K_t, z_t H_t^p)} + s_M. \quad (8)$$

To simplify the presentation, we now model firms' pricing decisions directly as a choice of their markup of price over marginal cost; firm ij expects the demand for its output to be

$$q_t^{ij} = Q_t D^j \left(\frac{\gamma_t^{i1}}{\gamma_t}, \dots, \frac{\gamma_t^{im}}{\gamma_t} \right),$$

where γ_t^{ik} is the markup chosen by firm ik . Firm ij seeks to maximize its present discounted value of profits

$$E_0 \sum_{t=0}^{\infty} \alpha^t \rho_t \left(\frac{\gamma_t^{ij} - 1}{\gamma_t} \right) q_t^{ij}, \quad (9)$$

which can be written in this way because $1/\gamma_t$ equals marginal cost. Here the stochastic process $\{\rho_t\}$ represents a pricing kernel for contingent claims; thus a security whose payout in period $t + j$ in units of the period $t + j$ composite good is the random variable x_{t+j} has a period t value of $E_t[\rho_{t+j} x_{t+j}]/\rho_t$ in units of the period t composite good. This process is normalized so that $\rho_0 = 1$. The parameter α represents the probability that the game will be repeated in period $\tau + 1$ given that it is played at τ . With probability $1 - \alpha$ the game ends, because of the dissolution of the oligopoly or the renegotiation of the implicit agreement among its members in a way that is independent of the history of previous play.⁷ As we discuss below, the main role of this parameter is to reduce the size of possible punishments for deviations. We have dropped the value of the fixed costs from the firm's objective function since they are assumed to be independent of pricing and production decisions in industry i .

⁷ As shown below, the optimal collusive agreement is not dependent on the history of past play. Hence, if a new collusive agreement is negotiated, the history of play prior to that date will become irrelevant. For the purpose of studying the strategic interactions prior to such renegotiation, one may thus assume as well that the oligopoly ceases to exist at that time.

Before we consider dynamic aspects of the firms' strategic interaction, it is important to ensure certain properties for the one-shot pricing game between the mI firms. In particular, let

$$\pi(\gamma^i; \gamma) = \frac{\gamma^i - 1}{\gamma} D^j \left(\frac{\gamma^i}{\gamma}, \dots, \frac{\gamma^i}{\gamma} \right) \tag{10}$$

denote the single-period profits per unit of aggregate demand Q received by each firm in industry i , assuming that each charges a common markup γ^i but all firms in other industries charge γ . Similarly let

$$\pi^d(\gamma^i; \gamma) = \max_{\rho} \left(\rho - \frac{1}{\gamma} \right) D^j \left(\frac{\gamma^i}{\gamma}, \dots, \rho, \dots, \frac{\gamma^i}{\gamma} \right) \tag{11}$$

denote the maximum single-period profits per unit of aggregate demand that can be obtained by a deviating firm in industry i if all other firms in its industry charge a markup of γ^i but all firms in other industries charge γ . Here the deviating firm ij charges $\gamma^{ij} = \rho\gamma$, and ρ is the j th argument of D^j .

To ensure standard properties for the equilibria of the one-shot game, we make the following assumptions.

ASSUMPTION 1. The demand functions D^j satisfy $D^j_k(\rho, \dots, \rho) > 0$ for all $k \neq j$ and all $\rho > 0$, and

$$\sum_k D^j_k(1, \dots, 1) < -1.$$

ASSUMPTION 2. For any $\gamma \geq 1$, $\pi(\gamma^i; \gamma)$ is a *unimodal* function of γ^i ; that is, it has a unique maximum, $\gamma^{iM}(\gamma)$, and is a nondecreasing function for all γ^i below this maximizing value and a nonincreasing function for all γ^i above it.

ASSUMPTION 3. For any $\gamma \geq 1$, $\pi(\gamma^i; \gamma) - \pi^d(\gamma^i; \gamma)$ is a unimodal function of γ^i , reaching a maximum value of zero for some $\gamma^{iB}(\gamma) > 0$.

ASSUMPTION 4. For any $\gamma \geq 1$,

$$\left(\rho - \frac{1}{\gamma} \right) D^j(1, \dots, \rho, \dots, 1)$$

is a unimodal function of ρ .

Assumption 2 ensures that there is a unique fully collusive (monopoly) outcome for a single industry i , namely $\gamma^{iM}(\gamma)$, which is a continuous function of γ . Assumption 3 ensures that, similarly, there is a unique Bertrand equilibrium for a single industry i , given by $\gamma^{iB}(\gamma)$, which is also a continuous function of γ . We can then define a sym-

metric equilibrium for the economy as a whole, with perfect collusion in each industry, as a common markup γ^M such that $\gamma^M = \gamma^{iM}(\gamma^M)$. We may similarly define a symmetric Bertrand equilibrium for the economy as a whole as a common markup γ^B such that $\gamma^B = \gamma^{iB}(\gamma^B)$. With assumption 4, these equilibria are uniquely defined by

$$\gamma^M = \frac{\sum_k D_k^j(1, \dots, 1)}{1 + \sum_k D_k^j(1, \dots, 1)}$$

and

$$\gamma^B = \frac{D_j^i(1, \dots, 1)}{1 + D_j^i(1, \dots, 1)}.$$

Assumption 1 guarantees that both these equilibria exist and that $1 < \gamma^B < \gamma^M$.⁸ It also ensures that $\gamma^{iB}(\gamma) < \gamma^{iM}(\gamma)$ so that collusion raises prices. Finally, these assumptions also imply that $\gamma^{iM}(\gamma) > \gamma$ if and only if $\gamma < \gamma^M$ and similarly that $\gamma^{iB}(\gamma) < \gamma$ if and only if $\gamma > \gamma^B$.

Let us now consider an infinitely repeated pricing game for some industry i . The m firms are assumed to interact strategically in choosing their joint markups $\{\gamma_t^{i1}, \dots, \gamma_t^{im}\}$. However, they take as given the stochastic process $\{\gamma_t^j\}$ for the markup in other industries, as well as the stochastic process $\{Q_t\}$ for aggregate demand. In assuming that industry i does not regard its (collusively determined) pricing policies as having any effects on markups in other industries or on aggregate demand, we are assuming that the I industries are monopolistic competitors in the sense of Dixit and Stiglitz (1977), just as in the collusive one-shot game described above. The firms similarly take as given the stochastic process $\{\rho_t\}$ defining competitive asset prices.

The strategic interaction between the firms in industry i can be described by the following repeated game. In each period t , the m firms simultaneously choose their markups $\{\gamma_t^{i1}, \dots, \gamma_t^{im}\}$, which then determine period t sales and profits. At a symmetric equilibrium, all firms in industry i charge the markup γ_t^i at time t . The expected value at t of the present discounted value of future profits at the symmetric equilibrium plays an important role in what follows. For firms in

⁸ Note that the first part of assumption 1 again implies that goods produced within the same industry are relatively close substitutes, and the implied demand curve faced by an industry as a whole is relatively elastic. This latter condition, while familiar, is not essential for our analysis. If it fails, it is not possible to sustain full collusion as a noncooperative equilibrium of the repeated game under any circumstances. This actually would simplify the characterization of implicit collusion and would provide a stronger justification for focusing on imperfectly collusive equilibria as we do below.

industry i this present value is

$$X_t^i = E_t \left\{ \sum_{k=1}^{\infty} \left(\frac{\alpha^k \rho_{t+k}}{\rho_t} \right) \pi(\gamma_{t+k}^i; \gamma_{t+k}) Q_{t+k} \right\}. \quad (12)$$

We assume that, because of antitrust laws, the firms making up the oligopoly are unable to enforce any contractual penalties for breach of a collusive agreement. Whatever collusion occurs must be enforced solely through the threat that the other firms will refuse to collude in the future if a given firm cheats on the (implicit) collusive agreement at any point in time. We also assume perfect information; that is, each firm chooses its markup γ_t^i with knowledge of all aggregate state variables realized in period t or earlier and with knowledge of the complete history of prices charged by all firms in its industry in all periods *prior* to period t . Hence, firms may respond to deviations from the implicit agreement by other firms in their industry after a one-period delay.

This sort of repeated game is well known to admit of a very large set of Nash equilibria and, indeed, of a large set of subgame perfect Nash equilibria, even if $\{\gamma_t, Q_t\}$ are fixed over time. We further specify our equilibrium concept, as in Abreu (1986) and Rotemberg and Saloner (1986), and so are able to obtain a determinate equilibrium response to government purchase shocks in the cases considered below. We stipulate that the (implicit) collusive agreement between the firms in each industry i is the *optimal symmetric perfect equilibrium* of the oligopoly supergame for that industry. It is that subgame perfect equilibrium, among those that specify symmetric actions for all firms in the industry under all contingencies, that achieves the highest discounted present value of profits for each firm, with the stochastic processes $\{\gamma_t, Q_t\}$ taken as given.

We give details of the characterization of the optimal symmetric collusive agreement in the Appendix, following the analysis of a similar game in Abreu (1986). The maximum possible degree of collusion is sustained when the penalty for a firm's deviation from its equilibrium actions is made as severe as possible. For the equilibrium to be subgame perfect, this penalty must be credible in the sense that it must be consistent with equilibrium play after the deviation.

In the Appendix, we show that when some boundary conditions are assumed on the aggregator functions introduced above, the credible penalty can be one in which, after a deviation, a deviating firm earns a present discounted value of zero (in addition to having to pay its fixed costs). The most important assumption is that there exist a $\tilde{\gamma}$ smaller than one such that, when all firms in industry i charge a markup of $\tilde{\gamma}$, a deviating firm cannot sell positive quantities by charg-

ing any price in excess of marginal cost. This assumption requires that the goods produced by firms in the industry be relatively good substitutes. It is also necessary that the firms that charge $\tilde{\gamma}$ not be hurt too badly by this, so that the threat is credible. Specifically, we assume that

$$\pi(\tilde{\gamma}; \gamma_t)Q_t + X_t^i \geq 0 \quad (13)$$

so that firms that cut markups to $\tilde{\gamma}$ still expect a nonnegative present discounted value of profits. Condition (13) also implies that the goods within the same industry are good substitutes. With these assumptions, the optimal symmetric perfect equilibrium is one in which a deviating firm earns a present discounted value of zero after its deviation. Thus its loss after the deviation, relative to what it would have earned in the absence of a deviation, is X_t^i .

Given this loss, firms in industry i have an incentive to deviate if the one-period benefit from deviating, $\pi^d(\gamma_t^i; \gamma_t)Q_t - \pi(\gamma_t^i; \gamma_t)Q_t$, exceeds X_t^i . Thus equilibrium deviations are prevented if and only if

$$\pi^d(\gamma_t^i; \gamma_t)Q_t - \pi(\gamma_t^i; \gamma_t)Q_t \leq X_t^i \quad (14)$$

In the optimal symmetric equilibrium, γ_t^i is chosen to maximize $\pi(\gamma_t^i; \gamma_t)$, subject to the incentive compatibility constraint (14). Note that in this constrained maximization, γ_t , Q_t , and X_t^i are all unaffected by the choice of γ_t^i .

It is evident that the γ_t^i that solves this problem depends only on the values of γ_t and X_t^i/Q_t . We can characterize this solution as follows. For X_t^i/Q_t equal to zero, (14) is satisfied only if $\gamma_t^i = \gamma^{IB}(\gamma_t)$, so this is the solution. For positive values of X_t^i/Q_t , $\gamma^{IB}(\gamma_t)$ still satisfies (14), but, given assumption 3, so do all values in an interval around this value. Moreover, assumptions 1 and 3 imply that industry profits as well as the left-hand side of (14) are strictly increasing in the industry markup γ_t^i , for values of the markup near $\gamma^{IB}(\gamma_t)$. Therefore, for small enough positive values of X_t^i/Q_t , γ_t^i is the largest value consistent with (14), so that this condition must hold as an equality. As X_t^i/Q_t increases further, (14) becomes consistent with a markup of $\gamma^{IM}(\gamma_t)$. Beyond this point, $\gamma_t^i = \gamma^{IM}(\gamma_t)$ and (14) ceases to bind, so that X_t^i/Q_t ceases to affect the industry's markup.

In a symmetric equilibrium for the entire economy, each industry sets prices in this way and sets the same markup at t . Hence γ_t^i must equal γ_t at all times and under all contingencies. Thus one must have a common value $X_t^i = X_t$ for all i . Finally, the common markup γ_t depends only on X_t/Q_t in a time-invariant way; government purchases, technology shocks, or preference shocks that do not change the aggregator functions, for example, all affect the equilibrium

markup only through their effects on X_t/Q_t .⁹ We now describe how markups depend on X_t/Q_t . We showed in the previous paragraph that in each industry either $\gamma^{iB}(\gamma_t) \leq \gamma_t \leq \gamma^{iM}(\gamma_t)$ and (14) binds or $\gamma_t \leq \gamma^{iM}(\gamma_t)$ and (14) holds as an inequality. That is, either $\gamma^B \leq \gamma_t \leq \gamma^M$ and $X_t/Q_t = \phi(\gamma_t)$, where

$$\phi(\gamma) \equiv \max_{\rho} \frac{\rho\gamma - 1}{\gamma} D^j(1, \dots, \rho, \dots, 1) - \frac{\gamma - 1}{\gamma} D(1, \dots, 1), \quad (15)$$

or $\gamma_t = \gamma^M$ and $X_t/Q_t \geq \phi(\gamma_t)$. Differentiating this function, we obtain

$$\phi'(\gamma) = \frac{\gamma^2}{D(1, \dots, \rho(\gamma), \dots, 1) - 1}. \quad (16)$$

For $\gamma > \gamma^B$, the optimal ρ in (15) is less than one, so that (16) implies that $\phi(\gamma)$ is strictly increasing. Therefore,

$$\gamma_t = \gamma\left(\frac{X_t}{Q_t}\right) \equiv \min\left[\phi^{-1}\left(\frac{X_t}{Q_t}\right), \gamma^M\right], \quad (17)$$

where the value of ϕ^{-1} is selected that is no less than γ^B .¹⁰ Then (17) defines a unique markup γ_t as a function of X_t/Q_t . Furthermore, the function $\gamma(X/Q)$ is continuous and nondecreasing; it is strictly increasing for all $0 < X/Q < \phi(\gamma^M)$ and is differentiable at all points except $\phi(\gamma^M)$.

The intuition for the form of the dependence of γ on X_t/Q_t is simple. If the incentive constraint binds, the sustainable degree of collusion is an increasing function of the losses expected from the breakdown of collusion (X_t), relative to the size of potential current profits from undercutting the prices charged by the other firms in one's industry (proportional to current demand Q_t). Once the incentive constraint ceases to bind, the markup always equals the value γ^M associated with perfect collusion (derived above).

We can also bound the degree of variation of γ with X_t/Q_t . For any $\gamma > \gamma^B$, ρ in (15) is smaller than one, so that $D^j(1, \dots, \rho(\gamma), \dots, 1) - 1$ is larger than $\gamma\phi(\gamma)/(\gamma - 1)$. With (16), this implies that, on the interval $\gamma^B < \gamma < \gamma^M$,

$$\frac{X_t\gamma'}{Q_t\gamma} \leq \gamma - 1, \quad (18)$$

⁹ For a generalization of this result to the case of nonhomothetic preferences over the differentiated produced goods, see Rotemberg and Woodford (1991). One might also imagine variation in equilibrium markups resulting from stochastic variation in the aggregator functions, although that is not our concern here.

¹⁰ If there exists no $\gamma \geq \gamma^B$ such that $\phi(\gamma) = X_t/Q_t$, we may define $\phi^{-1} = \infty$.

which provides an upper bound on the elasticity of the markup with respect to X_t/Q_t .

For purposes of comparison with aggregate data, as well as comparison with the literature on competitive models, it is useful to rewrite the model above in terms of *value added* rather than gross output. If we define aggregate value added (GNP) as $Y_t = Q_t - M_t$ and the value added markup (i.e., ratio of price to marginal cost of an additional unit of value added) as

$$\mu_t = \frac{(1 - s_M)\gamma_t}{1 - s_M\gamma_t}, \quad (19)$$

equations (4) and (8) become

$$Y_t = F(K_t, z_t H_t^p) - \Phi z_t N_t \quad (20)$$

and

$$z_t F_H(K_t, z_t H_t^p) = \mu_t w_t. \quad (21)$$

We can similarly use (10) to simplify (12):

$$X_t = E_t \left\{ \sum_{j=1}^{\infty} \left(\frac{\alpha^j \rho_{t+j}}{\rho_t} \right) \left(\frac{\mu_{t+j} - 1}{\mu_{t+j}} \right) Y_{t+j} \right\}. \quad (22)$$

Finally, we can write the equilibrium markup at t , μ_t , as a time-invariant function

$$\mu_t = \mu \left(\frac{X_t}{Y_t} \right), \quad (23)$$

the form of which follows directly from (17). When $s_M \gamma^M < 1$,¹¹ the function $\mu(X_t/Y_t)$ is well defined for all $X_t/Y_t \geq 0$. For X_t/Y_t below the critical level that allows all firms to charge γ^M , it is a smooth function with elasticity $\epsilon_\mu = X_t \mu' / Y_t \mu$. Inequality (18) implies that this elasticity satisfies

$$0 < \epsilon_\mu \leq \mu - 1. \quad (24)$$

Equations (20)–(23) suffice to determine the evolution of $\{\mu_t, X_t, H_t^p, Y_t\}$ given processes for $\{z_t, K_t, w_t, \rho_t\}$. Technological progress $\{z_t\}$ is assumed to be exogenous, and determination of the other variables requires that we consider the labor supply, saving, and portfolio decisions of households. We take up these issues in the next section.

¹¹ In order for the characterization of equilibrium in the next section to be correct, it suffices that $\gamma_t < s_M^{-1}$ for all values of γ_t that actually occur in equilibrium. Since we consider only small fluctuations around a steady-state growth path, we need assume only that the steady-state value of γ is less than $1/s_M$. But this follows from our assumption below of a finite steady-state value of μ greater than one.

III. The Complete Dynamic General Equilibrium

We now present the complete general equilibrium model of the economy and discuss our method of analyzing the response to shocks to the level of government purchases. The economy consists of a large number of identical infinite-lived households. The representative household seeks to maximize

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t N_t U \left(\frac{C_t}{N_t}, \frac{H_t}{N_t} \right) \right\}, \quad (25)$$

where β denotes a constant positive discount factor, N_t denotes the number of members per household in period t , C_t denotes total consumption by the household in period t , and H_t denotes total hours worked by members of the household in period t , including both hours supplied to the private sector and hours supplied to (or conscripted by) the government. By normalizing the number of households at one, we can use N_t also to represent the total population, C_t to denote aggregate consumption, and so on. We assume, as usual, that U is a concave function, increasing in its first argument and decreasing in its second argument. (The class of utility functions is further specialized below.)

The additively separable preference specification (25) implies that consumption demand and labor supply by the household are given by time-invariant Frisch demand and supply curves of the form

$$\frac{C_t}{N_t} = C(w_t, \lambda_t), \quad (26a)$$

$$\frac{H_t}{N_t} = H(w_t, \lambda_t), \quad (26b)$$

where λ_t denotes the marginal utility of wealth in period t , and w_t is the real wage. In terms of these functions, the conditions for market clearing in the labor market and the product market, respectively, can be written as

$$H_t^p + H_t^g = N_t H(w_t, \lambda_t) \quad (27)$$

and

$$N_t C(w_t, \lambda_t) + [K_{t+1} - (1 - \delta)K_t] + G_t = Y_t, \quad (28)$$

where H_t^p and H_t^g represent hours supplied to the private and government sectors, respectively, G_t represents government purchases of produced goods, and δ is the constant rate of depreciation of the capital stock, satisfying $0 < \delta \leq 1$.¹² We explicitly distinguish between

¹² Equation (28) is the standard GNP accounting identity, except that we do not count value added by the government sector as part of either G_t or Y_t .

government purchases of privately produced goods and government purchases of hours because, as stressed by Wynne (1989), they should have different effects. In the competitive model, government purchases of goods tend to raise private value added as long as they reduce real wages. Purchases of hours do tend to raise real wages but, as a result, reduce private value added.

We assume that households, like firms, have access to a complete set of frictionless securities markets. It is then a consequence of optimal portfolio choice by the representative household that the stochastic asset pricing kernel be given by

$$\rho_t = \beta' \lambda_t. \tag{29}$$

Furthermore, one of the assets that must be priced using this kernel is physical capital (or claims thereto), and the price of a unit of capital must be one (where the composite produced good is again the numeraire). It follows that

$$1 = \beta E_t \left\{ \left(\frac{\lambda_{t+1}}{\lambda_t} \right) \left[\frac{F_K(K_{t+1}, z_{t+1} H_{t+1}^p)}{\mu_{t+1}} + (1 - \delta) \right] \right\}, \tag{30}$$

where we have used (29) for the pricing kernel. A rational expectations equilibrium is then a set of stochastic processes for the endogenous variables $\{Y_t, K_t, H_t^p, w_t, \mu_t, X_t, \lambda_t, \rho_t\}$ that satisfy (13), (20)–(23), and (27)–(30), given the exogenous processes $\{G_t, H_t^g, z_t, N_t\}$.¹³

We analyze the response to shocks to government purchases of goods and hours using essentially the method of King et al. (1988). This involves restricting our attention to the case of small stationary fluctuations of the endogenous variables around a steady-state growth path. Let us first consider the conditions under which stationary solutions to these equations are possible. Given the existence of trend growth in both $\{z_t\}$ and $\{N_t\}$, in equilibrium variables such as $\{Y_t, w_t, \dots\}$ will exhibit trend growth as well. However, a stationary solution for transformed (detrended) variables exists if the equilibrium conditions in terms of these transformed variables do not involve z_t or N_t . As in King et al., this requires only that the representative household's preferences be such that the Frisch demand and supply curve satisfy the following assumption.

ASSUMPTION 5. There exists a $\sigma > 0$ such that $H(w, \lambda)$ is homogeneous of degree zero in $(w, \lambda^{-1/\sigma})$ and $C(w, \lambda)$ is homogeneous of degree one in $(w, \lambda^{-1/\sigma})$.¹⁴

¹³ The variables $Y_t, H_t^p, w_t, \mu_t, X_t, \lambda_t,$ and ρ_t must be measurable with respect to information available at time t ; K_t must be measurable with respect to information available at time $t - 1$; and information available at time t consists of the realizations at time t or earlier of the variables $G_t, H_t^g, z_t,$ and N_t .

¹⁴ The family of utility functions U with this property is discussed in Sec. IV.

Given assumption 5, there exists an equilibrium in which the detrended endogenous state variables

$$\begin{aligned} \tilde{Y}_t &= \frac{Y_t}{z_t N_t}, & \tilde{X}_t &= \frac{X_t}{z_t N_t}, & \tilde{K}_t &= \frac{K_t}{z_t N_t}, \\ \tilde{H}_t^p &= \frac{H_t^p}{N_t}, & \tilde{w}_t &= \frac{w_t}{z_t}, & \tilde{\lambda}_t &= \lambda_t(z_t)^\sigma \end{aligned}$$

are stationary, given stationary processes for the exogenous variables

$$\tilde{G}_t = \frac{Y_t}{z_t N_t}, \quad \tilde{H}_t^g = \frac{H_t^g}{N_t}$$

and constant rates of growth

$$\frac{N_{t+1}}{N_t} = \gamma_N, \quad \frac{z_{t+1}}{z_t} = \gamma_z$$

for population and productivity.¹⁵ For example, equations (28) and (30) become, respectively,

$$C(\tilde{w}_t, \tilde{\lambda}_t) + [\gamma_z \gamma_N \tilde{K}_{t+1} - (1 - \delta) \tilde{K}_t] + \tilde{G}_t = \tilde{Y}_t$$

and

$$1 = \beta \gamma_z^{-\sigma} E_t \left\{ \left(\frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t} \right) \left[\frac{F_K(K_{t+1}, z_{t+1} H_{t+1}^p)}{\mu_{t+1}} + (1 - \delta) \right] \right\}.$$

The other equilibrium conditions similarly can be written so as to involve only the detrended state variables and so admit a stationary solution in terms of those variables.

Like King et al., we furthermore seek to characterize such a stationary equilibrium only in the case of small fluctuations of the detrended state variables around their steady-state values, that is, the constant values that they take in a deterministic equilibrium growth path in the case in which \tilde{G}_t and \tilde{H}_t^g are constant. In order to apply this method, we must first verify the existence of a steady-state growth path in the case of no fluctuations in the detrended government purchase variables. The relatively innocuous assumptions required in order to show the existence of a steady state are discussed in the Appendix.

We furthermore require that in the steady state the incentive compatibility constraint (14) binds for all industries, so that perfect collusion is not possible. Only in this case do small fluctuations in X_t/Y_t

¹⁵ In fact, the result holds if the growth rates γ_N and γ_z are stationary random variables rather than constants, but for simplicity, we ignore these possibilities here.

around its steady-state value result in fluctuations in the equilibrium markup. We show in the Appendix that if the goods produced by firms within an industry are close substitutes, (14) is binding at the steady state as long as the number of firms per industry exceeds $(1 + r)/[1 + r - \alpha(1 + g)]$, where r is the steady-state real rate of return and g is the economy's steady-state growth rate. The intuition for this condition is that (14) is binding if the benefits from deviating are large relative to the punishment that follows deviations. These benefits rise as the number of firms producing goods within an industry increases and as their goods become closer substitutes. On the other hand, the punishment becomes less severe as α falls and its importance rises with g for a given r . Given that g equals about .008 per quarter and r equals only about .015, the number of firms per industry would have to be very large if α were equal to one. If, instead, we let α equal .9, the punishment is smaller, so that the number of firms need equal only 10.

Given a steady state, we approximate a stationary equilibrium involving small fluctuations around it by the solution to a log-linear approximation to the equilibrium conditions. This linearization uses derivatives evaluated at the steady-state values of the state variables.¹⁶ In writing the log-linear equations, we use the notation \hat{Y}_t for $\log(\tilde{Y}_t/\bar{Y})$, \hat{w}_t for $\log(\tilde{w}_t/\bar{w})$, and so on, where \bar{Y} and \bar{w} denote steady-state values. The log-linear approximation to the Frisch consumption demand and labor supply functions (26) can be written

$$\hat{C}_t = \epsilon_{Cw}\hat{w}_t + \epsilon_{C\lambda}\hat{\lambda}_t, \quad (31a)$$

$$\hat{H}_t = \epsilon_{Hw}\hat{w}_t + \epsilon_{H\lambda}\hat{\lambda}_t, \quad (31b)$$

in terms of the familiar Frisch elasticities.

The log-linearized equilibrium conditions may then be written as

$$\hat{Y}_t = \mu_{sK}\hat{K}_t + \mu_{sH}\hat{H}_t^p, \quad (32)$$

$$\frac{s_K}{\epsilon_{KH}}(\hat{K}_t - \hat{H}_t^p) = \hat{\mu}_t + \hat{w}_t, \quad (33)$$

$$\hat{\mu}_t = \epsilon_{\mu}(\hat{X}_t - \hat{Y}_t), \quad (34)$$

$$\hat{X}_t = E_t \left\{ \hat{\lambda}_{t+1} - \hat{\lambda}_t + \left(\frac{r-g}{1+r} \right) \left(\frac{1}{\mu+1} \hat{\mu}_{t+1} + \hat{Y}_{t+1} \right) + \left(\frac{1+g}{1+r} \right) \hat{X}_{t+1} \right\}, \quad (35)$$

¹⁶ The method is the same as in King et al. This can be made rigorous and justified as an application of a generalized implicit function theorem, as shown in Woodford (1986). It should be understood that when we refer to small fluctuations around the steady-state values, we have in mind stationary random variables with a sufficiently small bounded support.

$$(1 - \theta^g)\hat{H}_t^p + \theta^g\hat{H}_t^g = \epsilon_{Hw}\hat{w}_t + \epsilon_{H\lambda}\hat{\lambda}_t, \tag{36}$$

$$s_C(\epsilon_{Cw}\hat{w}_t + \epsilon_{C\lambda}\hat{\lambda}_t) + s_I\left[\left(\frac{1+g}{g+\delta}\right)\hat{K}_{t+1} - \left(\frac{1-\delta}{g+\delta}\right)\hat{K}_t\right] + s_G\hat{G}_t = \hat{Y}_t, \tag{37}$$

$$E_t\left\{\hat{\lambda}_{t+1} - \hat{\lambda}_t + \left(\frac{r+\delta}{1+r}\right)\frac{s_H}{\epsilon_{KH}}(\hat{H}_{t+1}^p - \hat{K}_{t+1}) - \left(\frac{r+\delta}{1+r}\right)\hat{\mu}_{t+1}\right\} = 0. \tag{38}$$

Here we have reduced the set of equilibrium conditions from eight to seven by substituting out the variable p_t using (29). The remaining seven equations are log-linearized versions of (20)–(23), (27)–(28), and (30), respectively.¹⁷ The coefficients in the log-linear equation system have been written in terms of parameters presented in table 1. Column 1 of the table gives the formulas that, when evaluated at the steady-state values of the detrended state variables, allow us to compute the value of these parameters. These computations, as well as the descriptions given in the table, will be discussed in the next section.

The system of log-linear equations may be further simplified as follows. We can solve (33) and (36) for \hat{H}_t^p and \hat{w}_t as functions of \hat{K}_t , $\hat{\lambda}_t$, and $\hat{\mu}_t$. Substitution into (32) allows us to solve for \hat{Y}_t , and substitution into (34) then allows us to solve for \hat{X}_t , both again as functions of the same three state variables. We can then eliminate these four state variables from the remaining three equilibrium conditions, obtaining a system of difference equations of the form

$$\mathbf{A}\begin{pmatrix} E_t\hat{\mu}_{t+1} \\ E_t\hat{\lambda}_{t+1} \\ E_t\hat{K}_{t+1} \end{pmatrix} = \mathbf{B}\begin{pmatrix} \hat{\mu}_t \\ \hat{\lambda}_t \\ \hat{K}_t \end{pmatrix} + \mathbf{C}\begin{pmatrix} \hat{G}_t \\ \hat{H}_t^g \end{pmatrix} + \mathbf{D}\begin{pmatrix} E_t\hat{G}_{t+1} \\ E_t\hat{H}_{t+1}^g \end{pmatrix}. \tag{39}$$

We assume that the exogenous variables $\{\hat{G}_t, \hat{H}_t^g\}$ are subject to stationary fluctuations. Thus as shown, for example, by Blanchard and Kahn (1980), (39) has a unique stationary solution if and only if the matrix \mathbf{A} is nonsingular and the matrix $\mathbf{A}^{-1}\mathbf{B}$ has one eigenvalue with modulus less than one and two with moduli greater than one. Woodford (1986) shows that this is also the case in which the original nonlinear equilibrium conditions have a locally unique stationary so-

¹⁷ Equilibrium condition (13) need not be included here. If the inequality holds strictly in the steady state, then it also holds at all times in the case of bounded fluctuations around the steady-state values of the state variables, assuming that these fluctuations are of sufficiently small amplitude.

TABLE 1

CALIBRATED PARAMETERS

Parameter	Defined by (1)	Values (2)	Description (3)
g	$\gamma_L \gamma_N - 1$.008	Steady-state growth rate (per quarter)
s_C	C/\bar{Y}	.697	Share of private consumption expenditure in private value added
s_G	\bar{G}/\bar{Y}	.117	Share of government purchases of goods in private value added
s_I	$(g + \delta)(\bar{K}/\bar{Y})$.186	Share of private investment expenditure in private value added
δ		.013	Rate of depreciation of capital stock (per quarter)
θ^g	\bar{H}^g/\bar{H}	.17	Share of total hours supplied to government sector
s_H	$F_H \bar{H}^g/F$.75	Share of labor income in private value added
s_K	$F_K \bar{K}/F$.25	Share of capital in private value added
r	$(F_K/\mu) - \delta$ or $\gamma_2^g \beta^{-1} - 1$.015	Steady-state real rate of return (per quarter)
ϵ_{KH}	$F_K F_H / F_{KH} F$	1	Elasticity of substitution between capital and hours
$1/\sigma$		1, .33	Elasticity of consumption growth with respect to real return, with hours worked held constant
ϵ_{H^w}		6, 1.30	Intertemporal elasticity of labor supply
μ		1, 1.20	Steady-state markup (ratio of price to marginal cost)
ϵ_μ		0, .19	Elasticity of the markup with respect to X/\bar{Y}
α	$\bar{X}\mu/\bar{Y}\mu$.9	Probability that collusion will continue in next quarter

lution. Moreover, when perturbed by exogenous shocks whose support is sufficiently small, this solution involves only small fluctuations around the steady state. For the calibrated parameter values discussed in the next section and for all sufficiently nearby values, we find that there is indeed exactly one stable eigenvalue, and so we are able to derive a unique equilibrium response to the shocks to military spending with which we are concerned.¹⁸ We can approximate this unique response by calculating the solution to the log-linear system (39) using the formulas of Blanchard and Kahn (1980) or Hansen and Sargent (1980). Properties of this solution are reported below in the form of impulse response functions for two types of innovations in military purchases.

The resulting solution is a linear function of the exogenous variables. This means that we can decompose fluctuations in the state variables into the contributions from each of the shocks that affect our two exogenous variables. It also means that our analysis of the effect of military purchases would not be affected by the inclusion of other exogenous shocks. If we were to consider such shocks as random variation in the rate of growth of productivity or of population, it would be possible to decompose the predicted fluctuations in the state variables into those resulting from the variations in military purchases and those resulting from these other shocks. Moreover, *the predicted responses to innovations in military purchases would be the same as those calculated here.*

IV. Calibration of the Model's Parameters

The system of linear equations (39) that we use to solve for the response of the model to changes in military purchases depends on the parameters listed in table 1. We now tighten the predictions of our model by following Kydland and Prescott (1982) and obtaining realistic values for these parameters on the basis of empirical characteristics of the U.S. economy. The parameters of our model can be arranged into three groups. The first group consists of parameters describing the scale of various sectors and activities. In this group are such parameters as the economy's growth rate, the fraction of its output consumed by the government, and so forth. In the second group are

¹⁸ There is no theoretical necessity for this condition of "saddle-point stability" to be satisfied in this model. Since equilibrium does not correspond to the solution of a planning problem (in contrast to the case of the corresponding perfectly competitive model), equilibrium need not be unique. Neither are there any "turnpike theorems" to guarantee convergence to the steady state in the absence of exogenous shocks. On the possibility of complicated equilibrium dynamics in the presence of distortions, even in representative consumer economies, see Guesnerie and Woodford (1991).

the parameters that describe the representative individuals' preferences. Finally, in the third group are the parameters that capture the character of the imperfectly competitive interactions among firms.

Using the average quarterly growth rates for the period since World War II, we set g equal to .008 per quarter. We turn next to the share parameters. One important difference between our calibration and that of Kydland and Prescott is our treatment of consumer durables. In Kydland and Prescott, they are treated as capital. This means that their concept of output exceeds the usual measurement of GNP by the services of these durables. This imputation of output raises some measurement difficulties. Even more severe difficulties attend the measurement of the labor input that goes into producing this output.

When developing their theory, Kydland and Prescott assume that output is produced via a Cobb-Douglas production function of capital and labor. On the other hand, they ignore the time input of people who operate consumer durables whether they be lawn mowers or refrigerators. Kydland and Prescott treat the services of durables as being produced by the durables alone. An alternative would be to assume that some effort is spent in producing services from these durables. Unfortunately, since this labor input is not measured, it is unclear how employment, and thus productivity, ought to be measured if one follows this approach.

To avoid these difficulties, we treat consumer durables as consumed in the period in which they are purchased, thus blurring the distinction between durables and nondurables. We do this for simplicity and for two other reasons. First, Mankiw (1982) shows that the empirical behavior of consumer expenditure on durables closely approximates that of consumer expenditure on nondurables. Second, it is not reasonable to suppose that the services of durable goods are produced from durables and labor by imperfectly competitive firms. It is more reasonable to suppose that this production occurs in the home or, equivalently, by perfectly competitive firms.¹⁹ The construction of such a two-sector model is left for future research.

As a result, the shares s_C and s_G are the average ratios of consumer expenditure and government purchases over private value added (i.e., GNP minus the value added by the federal, state, and local governments). They thus equal .697 and .117, respectively. Since s_I

¹⁹ A similar problem arises with respect to housing. Some housing services are sold by firms using labor services that are counted in the conventional measurements of employment. Other housing services are produced by their owner-occupants. Because a significant fraction of housing services are provided by firms and because much of the labor input that is employed here is measured as conventional employment, we treat housing services as produced by imperfectly competitive firms.

is equal to $1 - s_C - s_G$, it equals .186. Our assumptions also imply that the capital/output ratio \bar{K}/\bar{Y} equals 9.0.²⁰ Since s_I equals $g + \delta$ times this capital/output ratio, the implied quarterly depreciation rate δ is .013. This compares with the quarterly depreciation of .025 assumed by Kydland and Prescott.

When federal, state, and local governments are combined, the ratio of total government employment to total employment has averaged .17 in the postwar era. We assume that hours per employee are the same in the government as in the rest of the economy so that θ^g equals .17 as well.

Several studies, including Hall (1987), have noted that average profits are close to zero in the U.S. economy. As a result, the industry studies of Hall (1987) and Morrison (1990) find a strong connection between the average markup and the degree of increasing returns. In particular, zero steady-state profits imply that

$$(\mu - 1)\bar{Y} = \Phi. \quad (40)$$

We use this relation to calibrate Φ . From (20) and (40), it follows that $\bar{Y} = F(\bar{K}, \bar{H}^p)/\mu$. Equation (21) then implies that $F_H \bar{H}^p/F$, which equals s_H , also equals the share of labor in value added $\bar{w}\bar{H}^p/\bar{Y}$. The homogeneity of F implies that $s_K = 1 - s_H$. Our treatment of durables also affects our estimate of the share of labor. Viewing durables as consumption implies that the share of labor s_H is .75. This estimate is obtained by assuming that a fraction s_H of proprietor's income consists of payments to labor. Thus s_H equals private employee compensation plus s_H times proprietor's income divided by private value added. Kydland and Prescott, who treat the entire services of durables as compensation to capital, obtain instead a labor share equal to .64.

The steady-state real interest rate r equals $1 - \delta + (F_K/\mu)$. The argument in the previous paragraph established that this equals $1 - \delta + [(1 - s_H)\bar{Y}/\bar{K}]$, which equals about .015 per quarter. Note that this is also the ratio of the net payments to capital over the capital stock itself.

The final parameter of technology is the elasticity of substitution ϵ_{KH} . As is standard practice, we assume that this equals one. If technical progress were neutral, such an elasticity would be required to explain the rough constancy of relative factor shares in spite of a secular increase in the cost of labor. (We have, however, assumed labor-augmenting technical progress, which implies constant factor shares for any elasticity of substitution.)

²⁰ This is four times the ratio of private total tangible assets minus durables in 1987 over that year's private value added.

We now turn to the preference parameters. Before discussing their numerical values, we show that assumption 5 implies that there are only two independent preference parameters. First, assumption 5 immediately implies that

$$\epsilon_{Hw} - \sigma\epsilon_{H\lambda} = 0, \quad (41a)$$

$$\epsilon_{Cw} - \sigma\epsilon_{C\lambda} = 1. \quad (41b)$$

Moreover, King et al. (1988) show that assumption 5 implies that the utility function U in (25) can be written in the form $(C_t/N_t)^{1-\sigma} \times Z(H_t/N_t)$, where Z is a decreasing function with $Z'' < 0$. This implies that

$$\frac{\hat{C}_t}{1 - \sigma} \frac{Z'(\hat{H}_t)}{Z(\hat{H}_t)} = -\hat{w}_t$$

and that $\hat{\lambda}_t = (1 - \sigma)\hat{C}_t^{-\sigma} Z(\hat{H}_t)$. Differentiating this latter expression and using the former, we get

$$\hat{\lambda}_t = -\sigma\hat{C}_t + (\sigma - 1) \frac{\hat{w}\hat{H}}{\hat{C}} \hat{H}_t.$$

Using (41a) and (41b), we get

$$\frac{\epsilon_{Cw}}{\epsilon_{Hw}} = \frac{1 - \sigma}{\sigma} \frac{\hat{w}\hat{H}}{\hat{C}}. \quad (42)$$

The three restrictions (41a), (41b), and (42) imply that there are only two independent parameters among ϵ_{Cw} , $\epsilon_{C\lambda}$, ϵ_{Hw} , $\epsilon_{H\lambda}$, and σ . To preserve comparability with earlier studies, we calibrate σ and ϵ_{Hw} . We then use (42) to derive ϵ_{Cw} and (41) to derive the other two elasticities. The first of our calibrated parameters, σ , gives the reciprocal of the percentage change in the growth rate of consumption when the real return changes by 1 percent, with hours worked in the two periods held constant. The hypothesis of constant hours in this definition is important because, if σ is different from one, the utility function is not separable between leisure and consumption. Raising σ raises $\partial^2 U / \partial C \partial H$, making consumption more complementary with employment. The second parameter is the elasticity of labor supply with respect to the real wage, with the marginal utility of wealth held constant.²¹ It thus corresponds to the change in labor supply with respect to a temporary change in the wage.

Calibrations of real business cycle models assume that $\sigma = 1$. This choice is based in part on the fact that several authors (e.g., Hansen

²¹ This Frisch elasticity is also often referred to as the "intertemporal elasticity of labor supply" (see, e.g., Card 1991).

and Singleton 1982) have obtained estimates of this parameter near one using "Euler equation" methods. Unfortunately the estimates from these methods are very sensitive to the instruments used as well as to the normalization adopted in the estimation procedure. Hall (1988) shows that the correlation between changes in consumption and expected real returns is small in practice and thus argues that the intertemporal substitutability of consumption is low (and σ quite high, possibly infinite).

The real business cycle approach to estimation of the elasticity of labor supply deserves special comment. The idea is to assume a functional form of the utility function so special that it is possible to derive the elasticity of labor supply from knowledge of the ratio of hours worked to hours available for working. It is hard to see why there is any general connection between the average fraction of time spent in market activities and the elasticity of these hours with respect to temporary changes in the wage. Indeed, small modifications of the assumed utility function eliminate any such connection, and no such connection exists for our class of utility functions.

This approach leads Aiyagari et al. (1989) to assume an ϵ_{Hw} of about six. This exceeds the vast majority of estimated labor supply elasticities whether the estimates are computed by carefully trying to keep the marginal utility of wealth constant or whether they incorporate the effect of varying participation rates. Typically, estimated labor supply elasticities for males are near zero (see Killingsworth 1983; Pencavel 1986; Card 1991). Estimates for female labor supply elasticities range more widely, but while many studies obtain estimates in the 0.5–1.5 range, hardly any obtain estimates above two (see Killingsworth 1983; Killingsworth and Heckman 1986).

In spite of these reservations, we use these parameter values when simulating the competitive version of our model. We do this to preserve comparison with the work of Aiyagari et al. and because these parameters make the competitive model predict accurately the output response to military spending.

The baseline simulations of our model are computed using substantially lower elasticities for both consumption and labor supply. Since we do not know the appropriate values for these elasticities, we have picked them to some extent with the objective of making the model perform well. In our baseline simulation, $\sigma = 3$ and $\epsilon_{Hw} = 1.3$. While the model seems to fit better with such relatively low elasticities, the qualitative features of the model's response are not sensitive to their precise values. Given that the real interest rate is $\gamma_z^\sigma \beta^{-1} - 1$ and that population growth averages about .0048, the implied β is .995.

There are three parameters that capture the competitive structure in which the firms operate. The first is the value of the average

markup μ , which also determines the size of fixed costs through (40). The second is ϵ_μ , the elasticity of the markup with respect to X/Y . This parameter is the main determinant of the extent to which the demand for labor shifts as military purchases change. Finally, there is α , the probability that the collusive agreement will remain in force the next period.

There are several sources of evidence regarding the size of the average markup. Given our zero-profit assumption (40), the average markup μ is also the ratio of average costs to marginal cost, or the inverse of the conventional index of returns to scale.²² Hence evidence about either market power or increasing returns is equally relevant to the calibration of μ .

One source of evidence is the time-series analysis using industry data based on the work of Hall (1987, 1988). This body of work measures the response of revenues (at base period prices) and costs to changes in demand. The extent to which output increases induced by changes in demand lead revenues to rise by more than costs measures the extent to which price exceeds marginal costs. Hall's estimates, using value added as a measure of output, indicate estimates of the markup μ of over 1.8 for all seven one-digit industries he considers. Subsequent work by Domowitz, Hubbard, and Petersen (1988) uses gross industry output and takes into account materials inputs as well as capital and labor. Their estimates of the average value of the markup of price over marginal cost, which corresponds to γ , range between 1.4 and 1.7 for 17 out of their 19 industries. Morrison (1990) directly estimates instead a flexible functional form cost function, again using data on gross industry output and materials inputs. Her estimates of γ and of the ratio of average to marginal cost range between 1.2 and 1.4 for 16 out of her 18 industries (as in the case of Domowitz et al., the remaining two estimates are larger).

These two sets of estimates do not contradict each other because, as long as the materials share s_M is positive, μ is smaller than γ , as can be seen from (19). For example, $\gamma = 1.3$ (a typical value for Morrison's industries) and a materials share of .5 (which is also typical for those industries) imply $\mu = 1.9$. Thus the estimates of both Domowitz et al. and Morrison need to be increased using (19).

There is an extensive literature concerning the measurement of returns to scale in regulated industries. This literature is summarized in Panzar (1989); it studies the extent to which average costs differ for different plants within the same industry. The returns to scale found in the telecommunications industry tend to be substantial, with

²² According to (40), there is actually a connection between the average markup and the size of fixed costs. However, our calculations do not hinge on the precise functional form (20).

most studies finding returns to scale on the order of 1.4. Those found for electric power generation seem to be somewhat sensitive to the exact specification. Christensen and Greene (1976) found that only half the 1970 plants were operating at a scale at which marginal cost was below average cost. By contrast, Chappell and Wilder (1986) found much more substantial returns to scale when taking into account the multiplicity of outputs of many electric utilities.

These findings are of only limited relevance to our analysis, for these studies seek to measure the degree of long-run returns to scale, that is, the rate at which average costs decline as one goes from a small plant to a larger one. However, constant returns in this sense is perfectly consistent with large gaps between short-run average costs and short-run marginal costs. This would happen in particular if plant size exceeded the size that minimizes costs. Firms would rationally make such capacity choices if, for instance, producing at an additional location or introducing an additional variety raised a firm's sales for any given price, as in the Chamberlinian model of monopolistic competition. It is of course the ratio of short-run average costs to short-run marginal costs that matters for our analysis, for it is this ratio that determines the effect of short-run changes in output on both labor productivity and the level of marginal costs.

A third source of evidence on ratios of price to marginal cost is even more indirect. It stems from estimates of the price elasticity of the demand for individual products produced by particular firms. Studies of this elasticity, at least for branded products, are very common in the marketing literature. Tellis (1988) surveys this literature and reports that the median measured price elasticity is just under two. This suggests that the markup of these firms would equal two even if they behaved like monopolistic competitors. Implicit collusion would lead to even higher markups. Moreover, these elasticities again lead to estimates of gross-output markups; the value added markups with which we are concerned would be significantly higher.

In our basic simulations we employ a value of μ equal to 1.2. This is an extremely conservative choice since it amounts to a γ of less than 1.1 when there is a materials share of .5. We use this value to show that even very conservative values of μ can rationalize the response of the economy to changes in aggregate demand. We also discuss below the effect on our results of changing the value of μ .

Unfortunately, we have little independent basis on which to base our value of ϵ_μ . The theory requires only that it be less than $\mu - 1$. In Rotemberg and Woodford (1991), we suggest methods for measuring this parameter. These methods require that one specify the average value of μ in advance. For a μ equal to 1.2, our basic estimate of ϵ_μ is .19, so that we use this value in our simulations.

Finally, we choose the parameter α so that it becomes plausible that the incentive compatibility condition (14) binds at the steady state. This requires that (A24) in the Appendix hold. For our choice of α , .9, (A24) holds if goods within an industry are close substitutes and the number of firms in each industry is close to 10. With higher values of α , a larger number of firms is required. The effect on our simulations of lowering α is very similar to that of lowering ϵ_μ so that essentially identical plots are obtained with an α of one and an ϵ_μ of .15.

V. Estimation of Impulse Responses

In this section we describe how we estimate the economy's response to changes in military spending. We compare these empirical responses to those predicted by the oligopolistic and competitive models in the next section. We focus on the effect of national defense purchases of goods and services and military employment on the value added produced by the private sector, the real product wage in this sector, and the hours worked in the private sector.

As in the other empirical studies of the connection between military spending and aggregate activity, we treat military spending as exogenous. An inspection of the data on the evolution of military spending on goods and services after World War II reveals that the three main increases in spending correspond to the Korean War, the Vietnam War, and the Reagan buildup. All three of these episodes appear to be responses to the perceived threat of communist regimes and thus can plausibly be treated as exogenous to the state of U.S. economic activity. Thus we treat innovations in military spending in an autoregressive model as the exogenous shocks to military spending that are uncorrelated with any other shocks.

We use real GNP minus the value added by the federal, state, and local governments as our measure of private value added. The real wage is obtained by dividing hourly earnings in manufacturing by the private value added deflator. This private value added deflator is constructed by dividing nominal value added produced in the private sector by constant-dollar value added in the private sector. Our private hours variable is computed by multiplying the number of people on private nonagricultural payrolls by the average hours worked by all nonagricultural employees. It thus is proportional to the variable that enters our theoretical model only if average hours per employee are the same in the government as in the private sector and if farm hours and employment behave like other private hours and employment.

We use two variables to capture changes in military purchases. The

first is the logarithm of real national defense purchases of goods and services. The second is the logarithm of the number of people employed by the military. This number is obtained by subtracting the civilian population from the total population inclusive of armed forces stationed overseas. From these two variables, we infer the separate evolution of military purchases of hours and of privately produced output.

We estimate the bivariate process for the logarithm of real military spending and of military hours in levels with trends because a Dickey-Fuller test rejects the presence of a unit root in these series. We then face the problem of choosing an orthogonal decomposition of the two innovations in this bivariate process. This decomposition does not affect the nature of the comparison between the estimated impulse response functions and the theoretical predictions of the calibrated models. The complete two-dimensional space of possible combinations of shocks is the same in any event, as is the identification of theoretical and empirical responses associated with particular shocks in that space. The choice of orthogonal decomposition is only a choice of basis vectors (and hence coordinates) for the two-dimensional space of possibilities, which matters only for the graphical presentation of our results.

We are not completely indifferent to the choice of basis, however, since we wish to direct attention mainly to a single type of innovation. In particular, we want to emphasize shocks to overall military purchases that are typical in the sense that they are accompanied by neither unusually large nor unusually small changes in military employment. As a result, we choose the orthogonalization in which the current innovations in total military purchases are entirely attributed to the current "shock to military purchases" rather than partially attributed to the shock that is orthogonal to it. As a result, the shock that is orthogonal to our shock to military purchases lowers military purchases of privately produced output whenever it raises military personnel. This latter shock thus changes the composition of military spending in a way that is not common to the Korean War, the Vietnam War, and the Reagan buildup. In all three episodes, military purchases of goods and services rose together with military employment.

Let m_t denote the logarithm of real military purchases and h_t^m the logarithm of military employment at t . Then our shock to military purchases, η_t^m , is the residual in a regression of m_t on lagged values of m and h^m , a constant, and a linear trend. We also run the regression explaining h_t^m with a constant, a linear trend, the current value of m_t , and lags of both h_t^m and m_t . The residual in this regression, η_t^h , is orthogonal to η_t^m . With data from 1947:4 to 1989:1, the resulting

estimates are

$$\begin{aligned}
 m_t = & .335 - 4.7e^{-5}t + 1.226m_{t-1} - .197m_{t-2} - .093m_{t-3} \\
 & (.138) \quad (9e^{-5}) \quad (.080) \quad (.115) \quad (.071) \\
 & + .639h_{t-1}^m - .841h_{t-2}^m + .177h_{t-3}^m + \eta_t^m, \\
 & (.086) \quad (.135) \quad (.092)
 \end{aligned} \tag{43}$$

$$R^2 = .982; \text{ Durbin-Watson} = 2.14;$$

$$\begin{aligned}
 h_t^m = & .377 - 2.1e^{-4}t + .307m_t + .058m_{t-1} - .400m_{t-2} + .040m_{t-3} \\
 & (.124) \quad (8e^{-5}) \quad (.073) \quad (.117) \quad (.103) \quad (.063) \\
 & + 1.290h_{t-1}^m - .677h_{t-2}^m + .338h_{t-3}^m + \eta_t^p, \\
 & (.089) \quad (.133) \quad (.082)
 \end{aligned} \tag{44}$$

$$R^2 = .988; \text{ Durbin-Watson} = 1.43.$$

These regressions show that an increase in military spending is followed by further increases, and only later does military spending return to its normal value. Increases in military employment also tend to predict subsequent increases in military purchases.

To compute the responses of the other variables to changes in military spending, we also estimate regressions explaining private value added, hours, and real wages with the logarithms of military spending and employment. These are analogous to the relationships reported in (5) and (6). We let the level of each variable be explained by three of its own lags, the current and three lags of our two military variables, and a constant and a trend. Instead of reporting the resulting coefficients, we report only the implied impulse response functions to a unit change in η_t^m . These impulse response functions are computed by combining the coefficients from the regressions explaining our three variables with (43) and (44).

This is only one of many possible procedures for computing impulse responses. We also experimented with specifications in which the variables of interest are included as first differences instead of as levels.²³ The result is that, because fewer parameters are needed to capture the effect of military spending, the statistical significance of the military spending regressors is enhanced. The implied short-run responses from estimation in first differences are essentially identical to those from estimation in levels, though the long-run responses differ. Since we are more confident of the validity of the short-term responses, our discussion of impulse responses is limited to what hap-

²³ This would be more efficient if the series were integrated and there was no cointegrating vector. On the other hand, it is less robust, in that the estimates cease to be consistent if these integration assumptions are false.

pens within 10 quarters of a given shock and places most emphasis on the first few quarters.

VI. Estimated and Simulated Impulse Responses

In this section we compare the predictions of the oligopolistic and competitive models to the U.S. experience since World War II. As discussed above, our version of the competitive model differs from the oligopolistic one not only in its markup but also in its preference parameters. The theoretical impulse response functions are derived from the solution to (39). This solution depends on current and expected future values of \hat{G}_t and \hat{H}_t^g , so that we must first convert \hat{m}_t and \hat{h}_t^m into these variables.

To make this conversion, we note first that total government spending on goods and services, S^g , consists of government purchases of privately produced goods and services, G , and of government employees, H^g . Thus the percentage deviation of total goods and services from their steady-state value can be written as

$$\hat{S}_t^g = \frac{G}{S^g} \hat{G}_t + \left(1 - \frac{G}{S^g}\right) \hat{H}_t^g, \quad (45)$$

where $1 - (G/S^g)$ represents the average ratio of government value added to total government purchases of goods and services, which equals .55 in our sample. When we ignore changes in nonmilitary purchases, we get

$$\hat{S}_t^g = \frac{S^m}{S^g} \hat{m}_t, \quad (46)$$

where S^m/S^g is the average ratio of military purchases of goods and services to government purchases and equals .39. When we continue to ignore changes in nonmilitary purchases, we also get

$$\hat{H}_t^g = \frac{H^m}{H^g} \hat{H}_t^m, \quad (47)$$

where H^m/H^g is the average ratio of military to government employees or of military compensation of employees to government compensation of employees. Both of these ratios equal about .25. Equations (45)–(47) can be solved for \hat{G}_t and \hat{H}_t^g as functions of \hat{m}_t and \hat{H}_t^m . Given this correspondence between the two sets of variables, we compute the expectations $E_t \hat{G}_{t+k}$ and $E_t \hat{H}_{t+k}^g$ using the stochastic processes for \hat{m}_t and \hat{H}_t^m , (43) and (44).

We now compare the models' predictions of output, hours, and real wages with those generated by the regressions discussed in Section V.

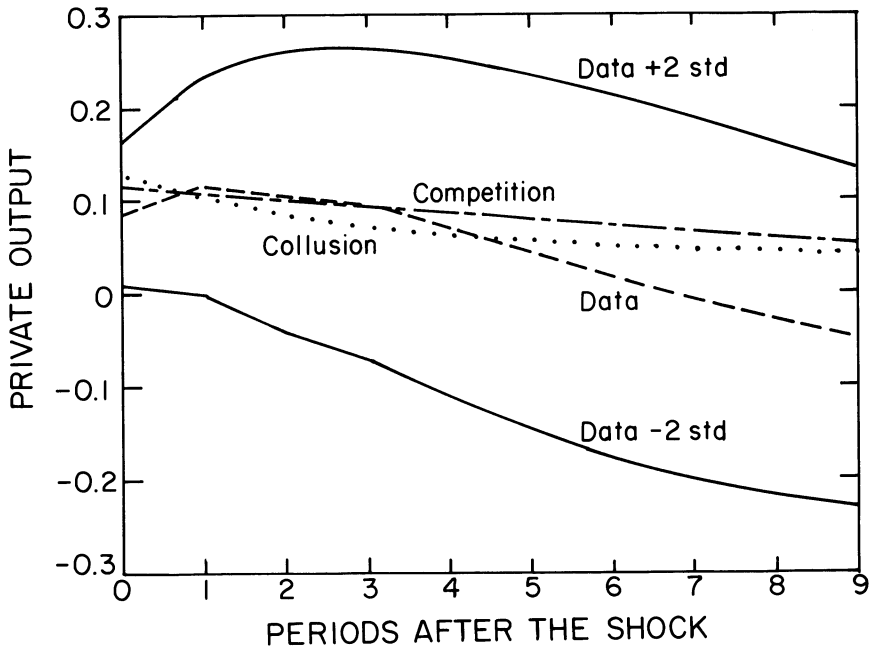


FIG. 1.—Response of private value added to a military spending shock

To get a sense of the performance of the two models, we have also plotted the two-standard-error confidence intervals around our empirical impulse responses.²⁴

Figure 1 presents the response of private value added to a unit shock in military spending. The positive association between military purchases and output has been noted before. Both Hall (1986) and García-Milá (1987) show that, in postwar U.S. data, increases in military purchases raise GNP, though slightly less than one for one. Since government value added rises much less than one for one with military purchases, private output must rise as well.

Our choice of parameters ensures that our noncompetitive model tracks this response reasonably well. The competitive version predicts the output response about as well. This is a certain measure of success for the preference parameters proposed by Aiyagari et al. (1989), since they did not choose these parameters to make this particular simulation fit these particular facts. In particular, their simulations assume rather different values for the capital/output ratio, for the

²⁴ These intervals are computed using the asymptotic method applied by Poterba, Rotemberg, and Summers (1986).

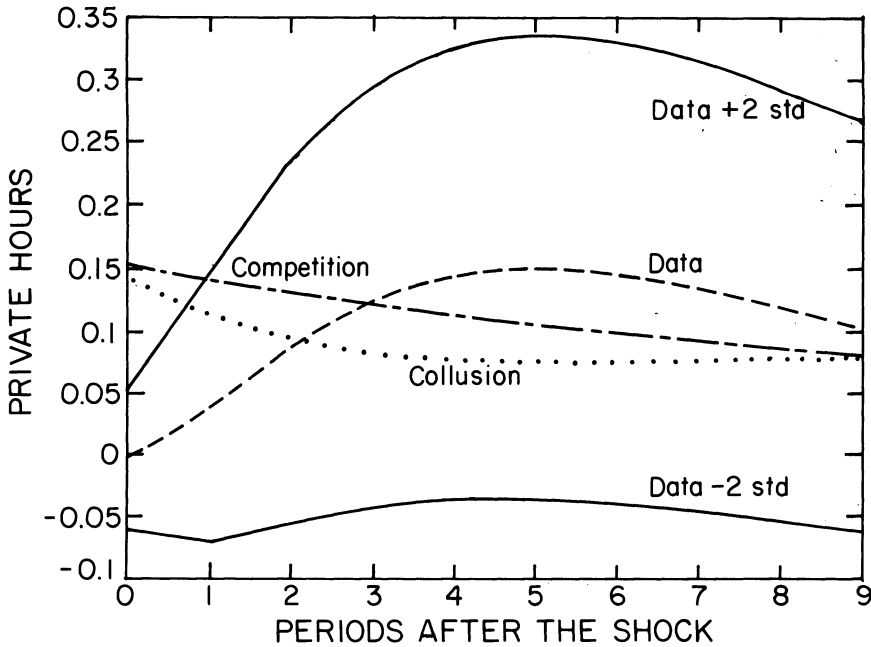


FIG. 2.—Response of private hours to a military spending shock

parameters of the stochastic process describing government purchases, and so forth.

Figure 2 presents the response of hours worked. Here the competitive model encounters well-known difficulties. The actual percentage increase in hours worked is smaller than the percentage increase in output. Given diminishing returns, the competitive model predicts that the percentage increase in hours worked should be larger than the percentage increase in output.²⁵ Models with increasing returns such as ours can in principle account for this discrepancy. Indeed, Hall (1987) uses the extent to which productivity increases in response to demand variations such as changes in military purchases to measure the degree of increasing returns. However, to explain the observed change in productivity, the ratio of steady-state average cost

²⁵ Christiano and Eichenbaum (1992) point out that actual productivity is less procyclical than is implied by models in which technology is the only random influence on output. They seek to remedy this by letting government purchases be random as well. This solution is not altogether convincing given that productivity and output rise together in response to unexpected changes in military spending. What is more, productivity actually rises more for a given percentage increase in output when that increase in output is correlated with current and lagged values of military spending than when it is not.

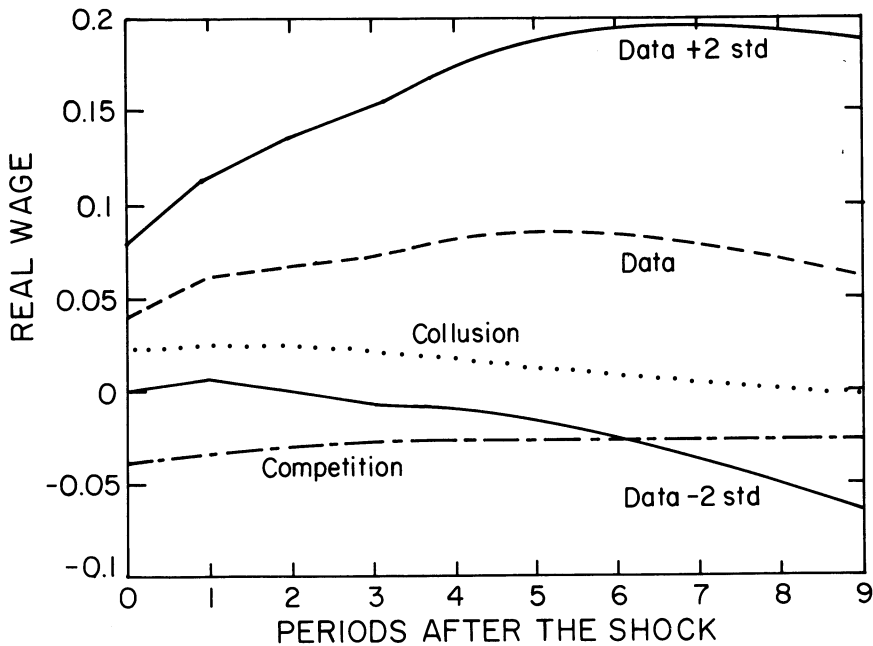


FIG. 3.—Response of real wages to a military spending shock

to marginal costs (and thus the steady-state markup) must be in excess of two. Our conservative μ of 1.2 is not sufficient to explain the initial burst in productivity.²⁶

Figure 3 presents the response of real hourly earnings in manufacturing. The observed positive effect on real wages is the principal difference between the predictions of our oligopolistic and competitive models.²⁷ Our competitive model's predicted response falls outside the two-standard-error confidence band, whereas that for the oligopolistic model is contained within it. In fact, if anything, the point estimate for the response is actually larger than that predicted by our model. This is, once again, the result of choosing a relatively conservative value for μ , which requires ϵ_μ to be modest.

²⁶ One reason for choosing a low value of μ even though we could fit this fact better with a larger μ is that the size of μ that is needed would imply a level of increasing returns that might be regarded as implausible. In any event, there exist possible explanations for procyclical productivity in response to demand shocks that do not rely on increasing returns (see, e.g., Lucas 1970; Eden 1990; Rotemberg and Summers 1990).

²⁷ It also differentiates the predictions of our oligopolistic model from textbook Keynesian models with rigid nominal wages. In those models, output expands only when real wages fall. It is also worth pointing out that the increase in real wages is even stronger when earnings in manufacturing are deflated by the overall GNP deflator instead of deflated by the deflator for private value added.

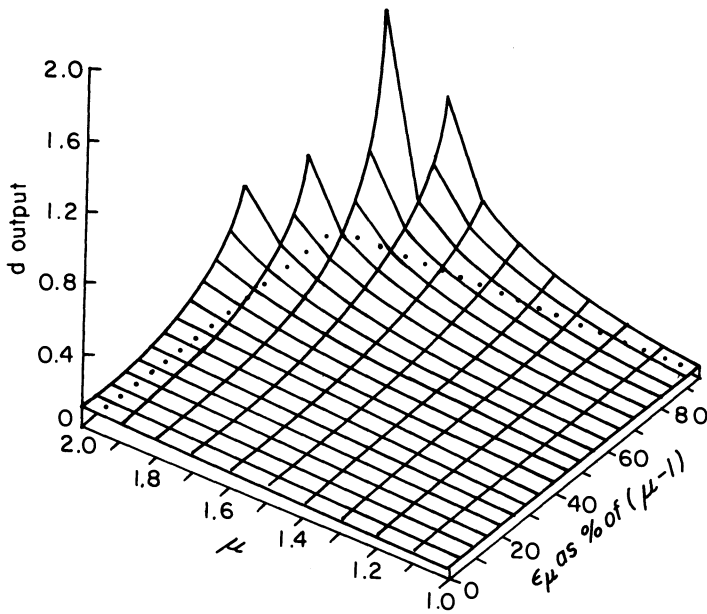


FIG. 4.—Sensitivity of the first-period output response to μ and ϵ_μ

To see this and to gauge more generally how our results depend on our assumed parameter values, consider figures 4, 5, 6, and 7. They display how the initial responses of output and of the real wage vary as we vary the parameters. Figures 4 and 5 show how these responses vary as we vary μ and ϵ_μ holding the preference parameters fixed at the values we use when simulating the oligopolistic model. Since ϵ_μ can take values only between zero and $\mu - 1$, ϵ_μ is scaled as a percentage of $\mu - 1$. These figures show that both the increase in output and the increase in the real wage get more pronounced as we increase either μ or ϵ_μ . A higher value of μ implies that output rises more for any increase in the labor input because μ also indexes the degree of increasing returns. A higher value of ϵ_μ implies a larger increase in labor demand for a given increase in military purchases.

Figures 6 and 7 show how the initial responses depend on the preference parameters σ and ϵ_{Hw} . These are plotted using the competitive benchmark values for μ and ϵ_μ , to show that the preference parameters are not responsible for the failures of the competitive model that we discuss. Figure 6 shows that a large value of ϵ_{Hw} is needed to obtain substantial increases in output following increases in military purchases. The reason is that output increases in the competitive model only if labor supply goes up, and for a given wealth decrease and increase in interest rates, labor supply goes up more

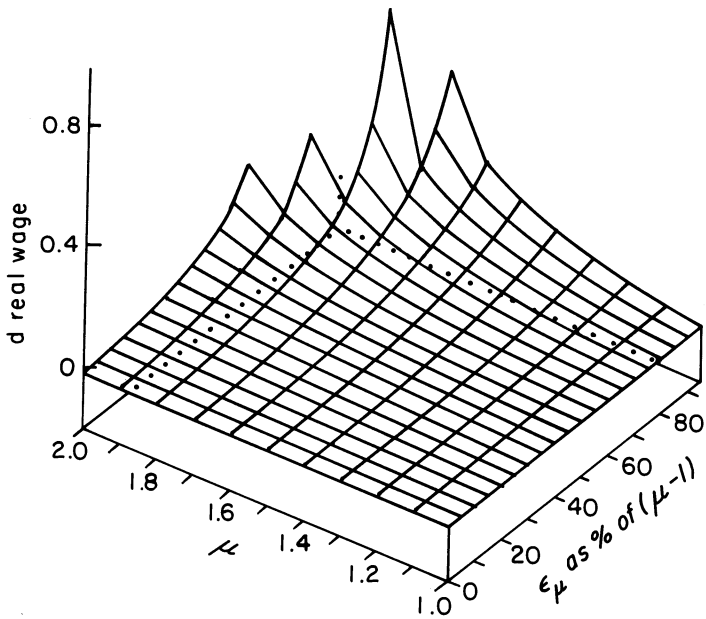


FIG. 5.—Sensitivity of the first-period real-wage response to μ and ϵ_μ

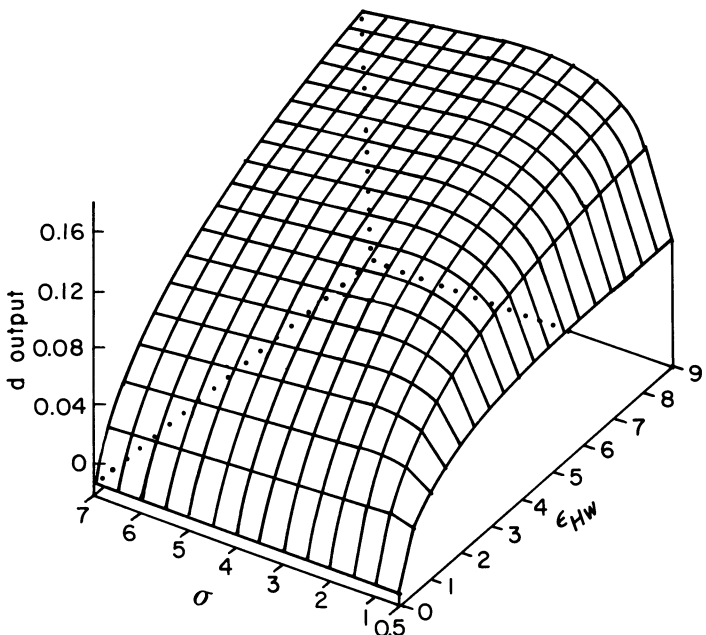


FIG. 6.—Sensitivity of the first-period output response to σ and ϵ_{Hw}

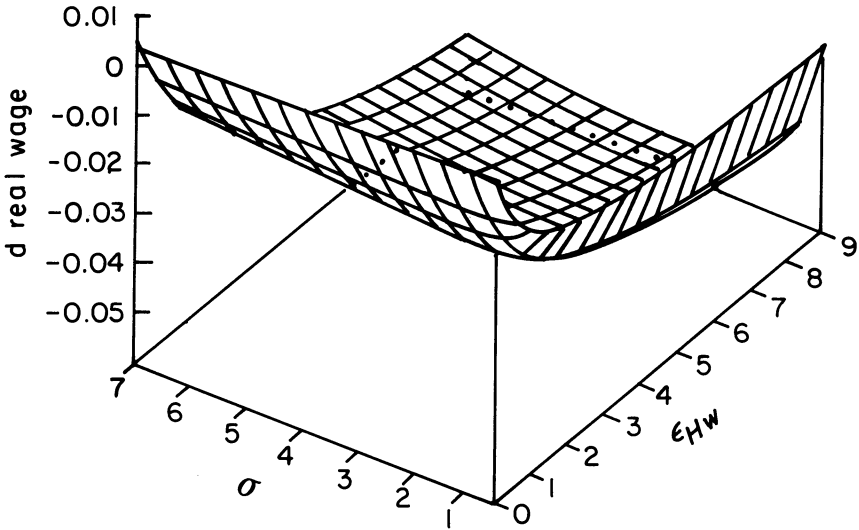


FIG. 7.—Sensitivity of the first-period real-wage response to σ and ϵ_{Hw}

the higher ϵ_{Hw} is. But for firms to actually increase their labor input, the real wage must fall. This is clear from figure 7, which shows that high values of ϵ_{Hw} imply large reductions in real wages.

While figures 6 and 7 deal only with the cases in which μ and ϵ_{μ} equal one and zero, respectively, the sensitivity with respect to ϵ_{Hw} is similar when they equal 1.2 and .19. Even in this oligopolistic case, higher values of ϵ_{Hw} imply that output is higher and real wages are lower than with smaller values of ϵ_{Hw} . It is for this reason that our oligopolistic model, the output response of which is increased by the higher value for μ , performs better when ϵ_{Hw} is relatively small. In the oligopolistic case, higher values of σ also increase the output and real-wage responses. The reason is that these higher values of σ lead to larger interest rate increases (as people are less willing to substitute their consumption over time). These higher interest rates lower X and lead to lower markups.

We have dealt with estimated and simulated responses to changes in η_t^m . We do not conduct as exhaustive an analysis of the estimated and simulated responses to the orthogonal shock η_t^p . This shock leads to changes in the composition of military spending rather than changes in their overall level. We are less interested in the estimated effect of such changes in composition, despite the fact that they matter according to both our models, because we are less confident in our ability to measure such innovations well. Errors in measuring either \hat{m}_t or \hat{h}_t^m contribute an error term to η_t^p , whereas errors in

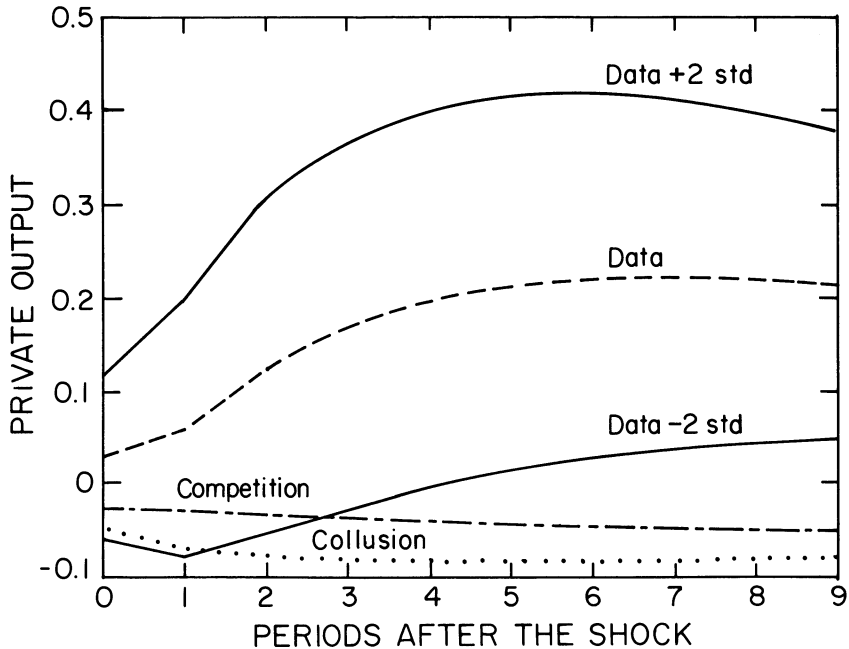


FIG. 8.—Response of private value added to η_t^p

measuring \hat{h}_t^m do not contribute to our constructed \hat{m}_t . Furthermore, the competitive and collusive models do not differ as much in their predictions about the response to an innovation in η_t^p . Both models predict that an increase in η_t^p , which raises \hat{H}_t^g and lowers \hat{G}_t , should lower private value added (as more labor is diverted to the government sector), lower private hours, and raise real wages.

Unfortunately, as figures 8, 9, and 10 demonstrate, this shock actually raises output and hours while lowering real wages. The resolution of this puzzle appears to require that a positive η_t^p be associated with an increase in labor supply. One possible explanation, which we are exploring in ongoing research, is that the exogenous events that lead to increases in military purchases are associated with varying amounts of increased patriotism. When patriotism rises more for a given increase in military purchases, the government increases H_t^m relative to G_t . The reason might be that recruiting is easier when patriotism happens to be higher or that efforts to stimulate patriotism are greater in the case of foreign threats that particularly require the commitment of American personnel. At the same time, increases in patriotism cause increases in labor supply. Thus increases in η_t^p are associated with increases in patriotism and labor supply, whereas increases in η_t^m are not.

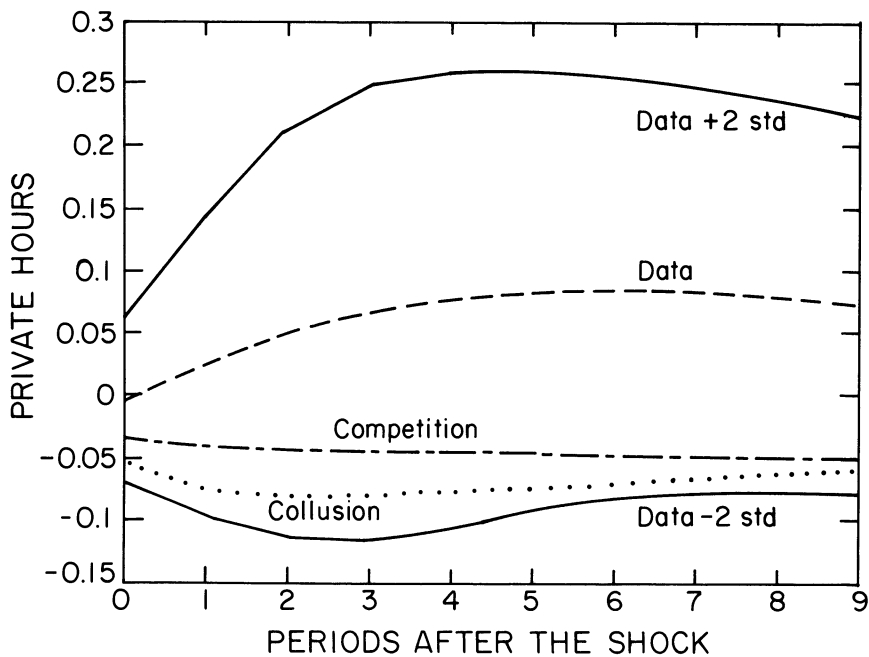


FIG. 9.—Response of private hours to η_t^p

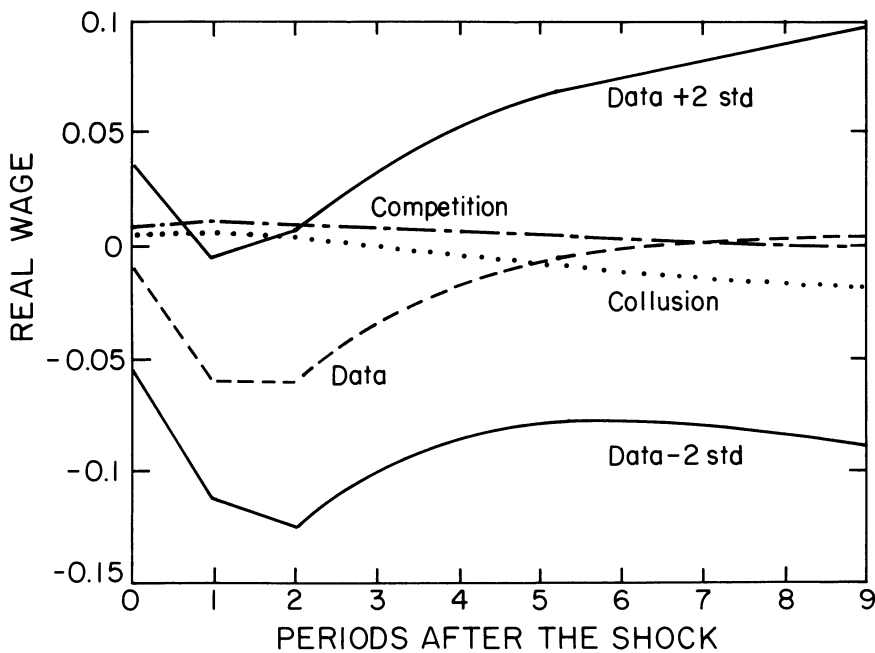


FIG. 10.—Response of real wages to η_t^p

While the anomalous estimated responses to an innovation in η_t^p cannot be explained by the models in this paper, they certainly provide no support for the view that the competitive model can explain the effects of military purchases. In particular, the postwar data suggest that increases in military employment are associated with lower, and not higher, real wages.

VII. Robustness of the Empirical Findings

Up to this point, our empirical work has used only postwar U.S. data to show that real wages rise together with output in response to increases in military purchases. In this section, we show that this finding is robust in that similar conclusions can be obtained from a longer sample of data that includes the two world wars. This is of interest because military purchases rose much more sharply in these episodes than they did at any time in the postwar era. A separate reason for analyzing the world wars is that popular discussions attribute the end of the Great Depression in large part to the rise in military spending after 1939.

Figure 11 displays the annual changes in the logarithm of real military purchases, in the logarithm of real GNP, and in the logarithm of real wages from 1891 to 1988. The real GNP series comes from

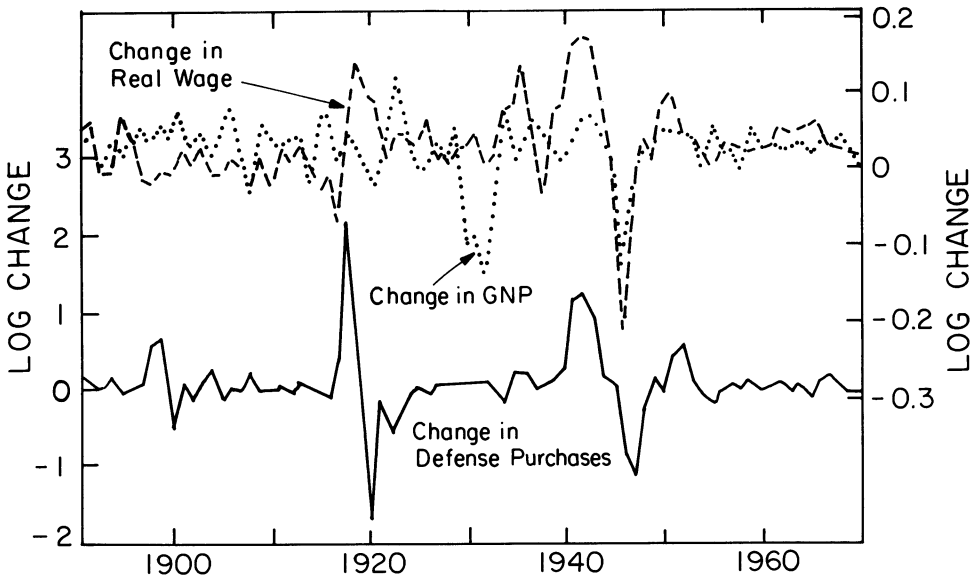


FIG. 11.—Changes in defense purchases, output, and real wages, 1891–1970

Romer (1989). Nominal military purchases are obtained from the *Historical Statistics of the United States*. They equal the sum of expenditures on the Departments of the Air Force, Navy, and Army (formerly War Department) from 1890 to 1953, and they equal the expenditures on the Department of Defense from 1954 on. We divide these nominal expenditures by Romer's GNP deflator to obtain a series on real military purchases. Real wages are equal to nominal wages divided by Romer's deflator. The nominal wage series is obtained by linking hourly earnings in manufacturing from 1890 to 1926, hourly earnings of production workers in manufacturing from 1926 to 1947, and hourly compensation of employees from 1947 on. The picture shows that increases in military purchases, particularly around the two world wars and around the Korean War, raise both GNP and real wages. This visual impression is confirmed by regressions. In particular, using annual data from 1891 to 1970, we obtain

$$\begin{aligned} \Delta \log \text{GNP}_t &= .019 + .333 \Delta \log \text{GNP}_{t-1} + .333 \Delta \log \text{mil}_t, \\ & \quad (.007) \quad (.11) \quad \quad (.014) \\ R^2 &= .237; \text{ Durbin-Watson} = 1.87; \end{aligned} \tag{48}$$

$$\begin{aligned} \Delta \log w_t &= .011 + .371 \Delta \log w_{t-1} + .020 \Delta \log \text{mil}_t, \\ & \quad (.004) \quad (.10) \quad \quad (.008) \\ R^2 &= .176; \text{ Durbin-Watson} = 2.02. \end{aligned} \tag{49}$$

This figure and associated regressions suggest that our postwar results are robust: GNP and real wages rise in response to increases in military spending.

However, it is important to stress that the limitations of the prewar data mean that they are not as informative about the weaknesses of the competitive model and the strengths of the oligopolistic one. Since the regressions for 1890–1970 use GNP rather than private value added, they could be consistent with a fall in private value added following increases in military purchases. In fact, separate analysis of the World War II episode shows that privately produced output did in fact rise in this period; we have no comparable data for World War I. Another problem with this longer sample is that we do not have separate data on military employment and that the existence of price controls may distort our estimates of real wages. To some degree, our postwar observations that increases in employment lower rather than raise real wages suggest that the first of these weaknesses is not very serious. Price controls are more problematic, though one can at least observe that in 1941, before price controls were instituted, military purchases, private value added, and real wages all rose together.

VIII. Concluding Remarks

In this conclusion we briefly discuss some alternative interpretations for the facts put forth in this paper. The first possibility is that the competitive model is accurate and that we have misspecified the model in important respects.

One possibility along these lines is that the one-sector structure of our model is misleading. Increases in government purchases may actually lower product wages in the sectors that are led to produce more while raising them only in those sectors in which output falls. Our measurement of increased real wages may be due only to inappropriate aggregation of the two sectors. While this possibility cannot be dismissed, we note that it implies that the prices of the sectors that expand ought to rise relative to other goods' prices. In particular, the prices of military goods ought to rise relative to other prices when military spending rises. Yet regressions using postwar data show that the deflator for military purchases falls relative to the GNP deflator when military spending goes up. Similarly, the disaggregated post-1972 data show that both the ammunition deflator and the military equipment deflator fall more relative to that for private value added the more real military purchases rise.

A second competitive explanation of our findings is that increases in the demand for goods lead firms to use their capital more intensely. A simple model of variations in the "workweek of capital" such as that of Lucas (1970) will not suffice, however. In that model, capital can be used more intensely by employing it for additional shifts. But one must ask why capital should not be fully employed. In Lucas's explanation, overtime hours are more costly than regular hours, and hours of both kinds are employed until the marginal product of each kind of hour equals its wage. But, then, it is again impossible for firms to be induced to hire more hours of either kind in response to an increase in demand, unless one or both real wages fall because of a labor supply shift. The variable workweek of capital can explain our real wage puzzle only if the real wage increase is an artifact of aggregation; both regular and overtime wages might fall while the composition of hours shifts toward overtime hours to such an extent that average hourly earnings increase. But increases in military purchases raise real wages even when we use as our measure of the nominal wage the time series on hourly earnings in manufacturing that are corrected for overtime and industry shifts.²⁸

²⁸ For further discussion of this type of explanation and other models of variable utilization of the capital stock, see Rotemberg and Woodford (1991, app. 1).

The second class of possibilities is that aggregate demand does indeed affect the economy by changing the markup but that implicit collusion plays no role. We therefore briefly survey alternative reasons for countercyclical markups. The oldest proposal is due to Robinson (1933).²⁹ Robinson's view, which has recently been revived by Lindbeck and Snower (1987) and Bils (1989), is that decreases in aggregate demand also reduce the elasticity of demand faced by the typical monopolistically competitive firm. This leads the firms (which are regarded as maximizing profits independently in each period) to increase the markup. One difficulty with this view is a lack of persuasive reasons for the elasticity of demand to vary with the strength of demand. What is more, it is not enough that the elasticity depend on the level of sales (or output) alone. Suppose that the markup μ is only a function of Y so that (1) can be replaced by

$$F_H(K_t, H_t, z_t) = \mu(K_t, H_t, z_t) w_t.$$

This still describes a relationship between H_t and w_t that depends only on K_t and z_t so that it cannot be affected by aggregate demand. Aggregate demand can still affect employment only by shifting labor supply. Increases in real wages following increases in aggregate demand could still be consistent with this story if the derivative of μ with respect to Y were so large that the labor demand curve sloped upward.

For labor demand to shift with aggregate demand in this type of model, the shifts in aggregate demand must be accompanied by changes in the elasticity of demand even at unchanged output. This is easiest to rationalize if the shift in aggregate demand also changes the composition of demand and if the components differ in their demand elasticity. Indeed Lindbeck and Snower propose that government demand is more elastic than private-sector demand, and Bils proposes that demand of the young is more elastic than that of high-income consumers.

In such a model it would be sheer coincidence if all the shocks that increase aggregate demand (changes in the perceived profitability of investment, changes in the tastes of foreigners, etc.) also shifted demand toward more elastic customers. One would expect instead that some increases in demand would increase but others would decrease

²⁹ Similar ideas are expressed by Kalecki (1938) and Keynes (1939), with Kalecki developing more fully the role of markup variation in the generation of business cycles. Another early hypothesis is that of Hall and Hitch (1939), who explained countercyclical markups in terms of strategic interactions among oligopolists and decreasing average costs. In these respects their view is a precursor of our own, although their model is entirely static.

the markup. This story thus tends to be inconsistent with the long tradition of macroeconomics that treats all changes in aggregate demand (i.e., all shifts from future to current spending at given interest rates) as having roughly similar effects.

Stiglitz (1984) proposes several other models of countercyclical markups. Some of these are also static. Others are dynamic but have implications rather different from ours. In one model an incumbent monopolist faces the threat of future entry and responds by limit pricing. As real interest rates go up, the incumbent is less worried about future entry, and so he charges a higher price (which is closer to the price he would charge in the absence of potential entry). This theory predicts that markups should rise when the real interest rate rises, whereas ours predicts that markups should fall in this case.

Increased interest rates have similar effects if prices are set as in the model of Phelps and Winter (1970). Suppose that customers are divided into two groups. There are some who have already experienced and liked the good. They are willing to pay a great deal. There are also customers who have yet to try the good and so are willing to pay less. Low prices are then an investment in new customers. When firms are discounting the future highly, they will invest little and exploit their existing customer base by charging high prices. So, insofar as increases in military purchases raise real interest rates, they ought to raise markups. For a more detailed comparison of the Phelps and Winter model and our implicit collusion model, see Rotemberg and Woodford (1991). There we present further econometric evidence for interest rate effects of the sign predicted by our model.

Gottfries (1986) and Greenwald and Stiglitz (1988) propose a more complicated version of this theory in which informational barriers impede financial flows between firms and outside investors. Thus the observed real interest rate is not the rate used by firms when they select investment projects. In particular, falls in demand exacerbate the firm's liquidity problems and thus raise the rate at which firms discount the future. This reduces investment in physical assets as well as investment and customers so that prices rise. According to this version of the theory, increases in demand ought to raise output only if they also raise physical investment. We have thus run regressions explaining investment with military purchases. In these regressions, increases in military spending tend to reduce physical investment. Given that output rises and markups fall, this theory appears to be contradicted. Still, the general relationship between physical investment and markups deserves to be explored further.

An alternative to all these approaches is the view that countercyclical markups are a result of some kind of sluggishness on the part of firms in changing the prices at which they sell their output, even if

marginal costs have changed.³⁰ According to this view, it is posted money prices that are slow to adjust. So a prediction that differentiates this view from all the “real” theories just discussed would be the claim that markups should decline most when costs are increasing most rapidly in *money* terms. However, a detailed comparison between this kind of model and the others is difficult because choice-based dynamic models of this type, where real interest rates and changes in the expectation of future demand play a role, remain to be developed. Still, the role of nominal rigidities in explaining markup variation is an important problem for future research.

Appendix

In this Appendix we make somewhat more specific assumptions about the aggregator functions f and g and use them to characterize the optimal symmetric equilibrium of each oligopoly.

Assumptions Regarding the Aggregator Functions

Our assumptions concern the behavior of f and g near the boundary of the positive orthant. In the case of the aggregator function f for the composite products of the I industries, we assume that *isoquants of f never intersect the boundary of the positive orthant*. This means that each industry's output is essential; purchases from each industry will be positive in the case of any finite relative prices. This implies that

$$\lim_{\rho \rightarrow 0} D^j(\rho, \dots, \rho) = \infty$$

and hence that

$$\lim_{\gamma^i \rightarrow 0} \pi(\gamma^i; \gamma) = -\infty \quad (\text{A1})$$

for all γ .

In the case of function g , which aggregates the products of the m firms in each industry, we assume that, instead of being essential, the individual products are good substitutes. In particular, we assume that if one firm's output (firm j 's) approaches zero but all other firms (one of which has the index k) produce the same positive level of output, *the marginal rate of substitution g_k/g_j approaches a limiting value of $\tilde{\gamma}$* , which lies between zero and one. The existence of this positive limit implies that there is a finite price differential that results in zero purchases from one of the firms. Below, we further assume that the required differential is relatively small. On the other hand, $\tilde{\gamma} < 1$ implies that the products are not perfect substitutes. As we discuss below, this is important for the existence of a steady state around which we can linearize the equilibrium dynamics.

³⁰ See, e.g., Woodford (1991). For a survey of evidence for and theoretical models of nominal price rigidity, see Rotemberg (1987). Rotemberg and Summers (1990) show that price rigidity can also rationalize the increase in measured productivity that accompanies increases in military spending.

Our second assumption implies that

$$D^j(\rho, \dots, \rho^j, \dots, \rho) = 0$$

for all $\rho^j \geq \rho/\bar{\gamma}$ and hence that

$$\pi^d(\underline{\gamma}^i; \underline{\gamma}) = 0 \quad (\text{A2})$$

for all $\underline{\gamma}^i \leq \bar{\gamma}$, and for any $\underline{\gamma}$.

Characterization of the Optimal Symmetric Subgame Perfect Equilibrium for an Industry

Following Abreu (1986), it suffices to restrict attention to equilibria of the following two-phase form. Play both on and off the equilibrium path is described by two stochastic processes, $\{\bar{\gamma}_t^i, \underline{\gamma}_t^i\}$, each random variable being measurable with respect to the history of aggregate state variables up through period t . The collusive agreement specifies that all firms in industry i should charge a markup $\bar{\gamma}_t^i$ in period t if there has been no deviation from the agreement by any firm in the industry in period $t - 1$, but that all should charge $\underline{\gamma}_t^i$ if there has been any deviation. In the initial period ($t = 0$), all firms are directed to charge $\bar{\gamma}_0^i$. In equilibrium, $\gamma_t^i = \bar{\gamma}_t^i$ in all periods and under all realizations of the aggregate shocks.

A pair of markup processes $\{\bar{\gamma}_t^i, \underline{\gamma}_t^i\}$ describes a symmetric subgame perfect equilibrium (SSPE) if and only if the following set of incentive constraints is satisfied. First of all, there must be no incentive to deviate in period t in the "collusive phase" (i.e., if there has been no deviation in period $t - 1$) by a firm that intends to comply with the collusive agreement thereafter. This requires that

$$\begin{aligned} \pi^d(\bar{\gamma}_t^i; \gamma_t) Q_t &\leq \pi(\bar{\gamma}_t^i; \gamma_t) Q_t \\ &+ E_t \left\{ \left(\frac{\rho_{t+1}}{\rho_t} \right) [\pi(\bar{\gamma}_{t+1}^i; \gamma_{t+1}) - \pi(\underline{\gamma}_{t+1}^i; \gamma_{t+1})] Q_{t+1} \right\}. \end{aligned} \quad (\text{A3})$$

There must similarly be no incentive for a one-period deviation in the "punishment phase" (i.e., if there has been a deviation in period $t - 1$), which requires that

$$\begin{aligned} \pi^d(\underline{\gamma}_t^i; \gamma_t) Q_t &\leq \pi(\underline{\gamma}_t^i; \gamma_t) Q_t \\ &+ E_t \left\{ \left(\frac{\rho_{t+1}}{\rho_t} \right) [\pi(\bar{\gamma}_{t+1}^i; \gamma_{t+1}) - \pi(\underline{\gamma}_{t+1}^i; \gamma_{t+1})] Q_{t+1} \right\}. \end{aligned} \quad (\text{A4})$$

If, in addition

$$\lim_{T \rightarrow \infty} E_t \left\{ \left(\frac{\rho_T}{\rho_t} \right) [\pi^d(\underline{\gamma}_T^i; \gamma_T) - \pi(\underline{\gamma}_T^i; \gamma_T)] Q_T \right\} = 0 \quad (\text{A5})$$

at all times, one can show that (A3) and (A4) are both necessary and sufficient for the processes $\{\bar{\gamma}_t^i, \underline{\gamma}_t^i\}$ to describe an SSPE. This is the familiar "one-stage deviation principle" for multistage games with observed actions (Fudenberg and Tirole 1991, sec. 4.2). The proof, to which we now turn, is similar to theorem 4.2 in Fudenberg and Tirole, although (A5) is weaker than their assumption of "continuity at infinity."

Let σ denote any contemplated deviation beginning at date t , and for any $T > t$, let σ_T denote the strategy that agrees with σ at all dates prior to T but

involves compliance with the collusive strategy from date T onward. Without loss of generality, we may consider only plans σ such that if the deviator ever returns to compliance, there is never a subsequent deviation. (If deviations of this kind are never profitable, then neither are plans involving multiple alternations between deviation and compliance.) For any such plan σ , let $V_t(\sigma)$ denote the present value of profits (discounted back to time t) that the firm deviating at t expects to obtain. Then $\{\bar{\gamma}_t^i, \underline{\gamma}_t^i\}$ describe an SSPE if and only if, for any deviation σ and any date t , $V_t(\sigma) \leq 0$.

Conditions (A3) and (A4) imply that, for any deviation σ , $V_t(\sigma_{t+1}) \leq 0$ so that deviations that last only one period are unprofitable. Furthermore, (A4) implies that $V_t(\sigma_{t+k+1}) \leq V_t(\sigma_{t+k})$ for all $k \geq 1$; hence, by induction on k , $V_t(\sigma_{t+k}) \leq 0$ for all $k \geq 1$. It is also evident that, for any $k \geq 1$,

$$V_t(\sigma) - V_t(\sigma_{t+k}) \leq \limsup_{t \rightarrow \infty} \Delta_{t,k}^T, \tag{A6}$$

where

$$\begin{aligned} \Delta_{t,k}^T \equiv & E_t \left\{ \left(\frac{\rho_{t+k}}{\rho_t} \right) s_{t+k} [\pi^d(\underline{\gamma}_{t+k}^i) - \pi(\underline{\gamma}_{t+k}^i)] Q_{t+k} \right\} \\ & + \sum_{j=k+1}^{T-t} E_t \left\{ \left(\frac{\rho_{t+j}}{\rho_t} \right) s_{t+j-1} [s_{t+j} \pi^d(\underline{\gamma}_{t+j}^i) + (1 - s_{t+j}) \pi(\underline{\gamma}_{t+j}^i) - \pi(\bar{\gamma}_{t+j}^i)] Q_{t+j} \right\} \end{aligned}$$

for all $T \geq t + k$. We have suppressed the second argument (γ_{t+j}) from each of the π and π^d functions for simplicity. In this equation, $\{s_{t+j}\}$ is a random variable taking the value one if the plan σ implies deviations at that date and in that contingency, and the value zero otherwise. The term $\Delta_{t,k}^T$ represents an upper bound for the present value of profits between $t + k$ and T from following σ instead of σ_{t+k} . Condition (A4) then implies that $\Delta_{t,k}^T \leq \Delta_{t,k+1}^T$ for any $k \geq 1$, $T \geq t + k - 1$. Iterating, one finds that

$$\begin{aligned} \Delta_{t,k}^T & \leq \Delta_{t,T-t}^T = E_t \left\{ \left(\frac{\rho_T}{\rho_t} \right) s_T [\pi^d(\underline{\gamma}_T^i) - \pi(\underline{\gamma}_T^i)] Q_T \right\} \\ & \leq E_t \left\{ \left(\frac{\rho_T}{\rho_t} \right) [\pi^d(\underline{\gamma}_T^i) - \pi(\underline{\gamma}_T^i)] Q_T \right\}. \end{aligned}$$

Consequently, (A5) implies that

$$\limsup_{T \rightarrow \infty} \Delta_{t,k}^T \leq 0,$$

which, together with (A6), implies that $V_t(\sigma) \leq V_t(\sigma_{t+k}) \leq 0$. Hence, given (A5), (A3) and (A4) are necessary and sufficient conditions for $\{\bar{\gamma}_t^i, \underline{\gamma}_t^i\}$ to be an SSPE.

In the *optimal* SSPE, $\bar{\gamma}_t^i$ represents the play in period t associated with the best possible SSPE for the subgame beginning in period t , and $\underline{\gamma}_t^i$ represents the play associated with the *worst* possible SSPE for that same subgame. (In the pessimal equilibrium, all firms are directed to charge $\underline{\gamma}_t^i$ in period t and then charge $\bar{\gamma}_{t+j}^i$ in all subsequent periods if no deviation from equilibrium play has occurred.) Hence both $\bar{\gamma}_t^i$ and $\underline{\gamma}_t^i$ are independent of past pricing in industry i . The reason is that one wishes to make the disincentive to deviation from the collusive agreement as strong as possible in each period.

As noted above, in any SSPE the present discounted value of profits must at all times be nonnegative. Hence if an equilibrium exists in which this is

zero, it must be the worst one. Sufficient conditions for the existence of an equilibrium of this kind are as follows. Suppose that there exists a stochastic process $\{\bar{\gamma}_t^i\}$, measurable with respect to the history of aggregate state variables through period t , such that

$$\pi^d(\bar{\gamma}_t^i)Q_t \leq \sum_{j=0}^{\infty} E_t \left\{ \left(\frac{\rho_{t+j}}{\rho_t} \right) \pi(\bar{\gamma}_{t+j}^i) Q_{t+j} \right\} < \infty \quad (\text{A7})$$

and

$$-\pi(\bar{\gamma})Q_t \leq \sum_{j=1}^{\infty} E_t \left\{ \left(\frac{\rho_{t+j}}{\rho_t} \right) \pi(\bar{\gamma}_{t+j}^i) Q_{t+j} \right\} < \infty, \quad (\text{A8})$$

where $\bar{\gamma}$ is the relative price referred to in (A2) above. (We continue to suppress the second argument in π and π^d in these expressions.) Then it is possible to construct a process $\{\underline{\gamma}_t^i\}$ such that $\{\bar{\gamma}_t^i, \underline{\gamma}_t^i\}$ describe an SSPE, in the punishment phase of which the present discounted value of profits is zero. One simply chooses $\underline{\gamma}_t^i$ to solve

$$\pi(\underline{\gamma}_t^i)Q_t + \sum_{j=1}^{\infty} E_t \left\{ \left(\frac{\rho_{t+j}}{\rho_t} \right) \pi(\bar{\gamma}_{t+j}^i) Q_{t+j} \right\} = 0. \quad (\text{A9})$$

By (A7), the second term in (A9) is a finite, nonnegative quantity. (Recall that $\pi^d \geq 0$.) Given (A1), there must accordingly be a $0 < \gamma_t^i \leq 1$ that solves (A10) below, and indeed it must be unique since $\pi(\gamma^i)$ is monotonically increasing for $\gamma^i \leq 1$. Then (A9) says that in the punishment phase the present discounted value of profits is zero.

It remains to be shown that the processes $\{\bar{\gamma}_t^i, \underline{\gamma}_t^i\}$ so obtained describe a subgame perfect equilibrium. We proceed by showing that (A3), (A4), and (A5) are satisfied. Substitution of (A9) into the right-hand side of (A3) makes the latter expression equal to the right-hand side of (A7). Hence (A7) and (A9) imply (A3). A similar substitution of (A9) into the right-hand side of (A4) makes that expression equal zero. Hence (A4) is satisfied if and only if

$$\pi^d(\underline{\gamma}_t^i) = 0. \quad (\text{A10})$$

But comparison of (A8) and (A9) indicates that $\pi(\underline{\gamma}_t^i) \leq \pi(\bar{\gamma})$ so that $\underline{\gamma}_t^i \leq \bar{\gamma}$. Then (A2) implies (A10), and (A4) is satisfied. Finally, substitution of (A9) and (A10) into (A5) makes the latter equivalent to

$$\lim_{T \rightarrow \infty} \sum_{j=T+1-t}^{\infty} E_t \left\{ \left(\frac{\rho_{t+j}}{\rho_t} \right) \pi(\bar{\gamma}_{t+j}^i) Q_{t+j} \right\} = 0. \quad (\text{A11})$$

But (A11) must hold, given the well-defined and finite sums on the right-hand sides of (A7) and (A9). Hence (A7)–(A9) imply (A3), (A4), and (A5), and so describe an SSPE.

It is furthermore evident that if *any* process $\{\bar{\gamma}_t^i\}$ exists that satisfies (A7) and (A8), the process describing the “collusive phase” of the *best* SSPE must satisfy them. For we have shown that if any such process exists, it can be supported as the collusive phase of an SSPE. Hence at all times, the present discounted value of profits in the *optimal* SSPE must be at least as high as in the collusive phase of that equilibrium, and so (A9) holds a fortiori for the process describing the collusive phase at the optimal SSPE. (The present

discounted value of profits must still be finite in the optimal equilibrium in order for firms' objective function to be well defined.) Furthermore, we have shown that an equilibrium exists in which the present value of profits is zero, and so this must be true in the punishment phase of the optimal equilibrium. Then (A9) holds for the optimal equilibrium, and this plus (A3) implies (A7).

Hence the process describing the collusive phase of the optimal SSPE is that process $\{\bar{\gamma}_t^i\}$ satisfying (A7) and (A8) that achieves the highest value for the right-hand side of (A7). This can be broken into the two problems of first finding the optimal process for periods $t + 1$ and later (since a higher present value of profits for periods $t + 1$ onward only makes the constraints [A7] and [A8] bind less tightly in period t) and then finding the value of $\bar{\gamma}_t^i$ that maximizes $\pi(\bar{\gamma}_t^i)$ subject to (A7) and (A8). Furthermore, constraint (A8) does not bind, so $\bar{\gamma}_t^i$ maximizes $\pi(\bar{\gamma}_t^i)$ subject to (A7). This is just the characterization of the equilibrium markup process in the text at (13), where X_t^i equals the right-hand side of (A8) because $V_t^i = 0$.

This characterization, of course, depends on the assumption that there exists *some* process $\{\bar{\gamma}_t^i\}$ satisfying (A7) and (A8). It is easily shown that there exist processes satisfying (A7); $\bar{\gamma}_t^i = \gamma^{ib}(\gamma_t)$ would be one such. It is less obvious that (A8) can be satisfied at the same time. This is why the condition (14) is required in the text as an additional condition for an equilibrium. We may note, however, that processes will exist satisfying (A8) as well if $\bar{\gamma}$ is close enough to one. For we can make $\bar{\gamma}$ arbitrarily close to one by suitable choice of the aggregator function g , without affecting the rate at which $\pi(\gamma^i; \gamma)$ decreases with decreasing γ^i , which depends only on the function f . Hence if goods within the same industry are very close substitutes but goods in different industries are not particularly substitutable, $\pi(\bar{\gamma}; \gamma)$ will be a very small negative quantity. In any event, if (14) holds, then (A8) holds for each industry i .

Existence of a Steady-State Growth Path

We wish to show the existence of a steady state in the absence of stochastic variation in government purchases (and for a suitable choice of the initial capital stock). This is an equilibrium in which the detrended state variables $\bar{Y}_t, \bar{X}_t, \bar{K}_t, \bar{H}_t, \bar{W}_t, \bar{\lambda}_t$, and μ_t are constant for all time, given constant values for \bar{G}_t and \bar{H}_t^g .

It follows from (20) and (29) that in such an equilibrium, one must have

$$\frac{\bar{X}}{\bar{Y}} = \kappa \left(\frac{\mu - 1}{\mu} \right), \tag{A12}$$

where

$$\kappa \equiv \frac{\alpha \beta \gamma_N (\gamma_z)^{1-\sigma}}{1 - \alpha \beta \gamma_N (\gamma_z)^{1-\sigma}}.$$

The denominator of this expression must be positive for κ itself to be positive. Since $\alpha \leq 1$, the denominator is positive as long as the real rate of return is higher than the growth rate, which requires that

$$\beta \gamma_N (\gamma_z)^{1-\sigma} < 1. \tag{A13}$$

Without (A13) the budget constraint of the representative household is not well defined in the standard real business cycle (RBC) model.

It also follows from (22) that

$$\mu = \mu\left(\frac{\bar{X}}{\bar{Y}}\right). \quad (\text{A14})$$

With parameter values that satisfy (A13), conditions (A12) and (A14) determine the steady-state values of μ and \bar{X}/\bar{Y} . Because of (13), these values must also satisfy

$$-\pi\left(\bar{\gamma}, \frac{\mu}{1 + s_M(\mu - 1)}\right) \leq (1 - s_M) \frac{\bar{X}}{\bar{Y}}. \quad (\text{A15})$$

Given a steady-state markup μ satisfying (A12), (A14), and (A15), the steady-state values \bar{Y} , \bar{K} , \bar{H}^p , \bar{W} , and $\bar{\lambda}$ are determined by the equations

$$\bar{Y} = F(\bar{K}, \bar{H}^p) - \Phi, \quad (\text{A16})$$

$$F_H(\bar{K}, \bar{H}^p) = \mu\bar{w}, \quad (\text{A17})$$

$$\mu[\beta^{-1}\gamma_z^\sigma - (1 - \delta)] = F_K(\bar{K}, \bar{H}^p), \quad (\text{A18})$$

$$C(\bar{w}, \bar{\lambda}) + [\gamma_z\gamma_N - (1 - \delta)]\bar{K} + \bar{G} = \bar{Y}, \quad (\text{A19})$$

$$\bar{H}^p + \bar{H}^g = H(\bar{w}, \bar{\lambda}). \quad (\text{A20})$$

For the level of investment to be positive in the steady state, (A19) requires (as standard RBC models do) that

$$\gamma_z\gamma_N > 1 - \delta. \quad (\text{A21})$$

Together with (A13), (A21) also ensures that the marginal product of capital in (A18) is positive. Given parameter values satisfying (A13) and (A21), existence of a unique solution to (A16)–(A20) follows under standard assumptions on the form of preferences and technology.

To ensure that (40) is satisfied in the steady state, we treat Φ as an endogenous instead of an exogenous variable. This simplifies the analysis of the steady state given that we determined the steady-state value of μ using (A12) and (A14). Substituting (40) into (A9) yields

$$\bar{Y} = \frac{F(\bar{K}, \bar{H}^p)}{\mu}. \quad (\text{A22})$$

With Φ given by (40), the steady-state values \bar{Y} , \bar{K} , \bar{H}^p , \bar{w} , and $\bar{\lambda}$ are determined by (A17)–(A20) and (A22). But conditions (A17)–(A20) and (A22) are exactly the conditions that describe the steady state of a standard one-sector neoclassical growth model (RBC model), in the case of a production function $\bar{F}(K, H) = F(K, H)/\mu$. Hence, unique steady-state values exist, given μ , under quite weak and familiar assumptions.

We return, then, to the question of existence of a steady-state markup μ satisfying (A12), (A14), and (A15). The right-hand side of (A12) is a continuous function of μ , monotonically increasing from a value of zero when $\mu = 1$ to a value of $\kappa > 0$ as $\mu \rightarrow \infty$. The right-hand side of (A14) is a continuous nondecreasing function of \bar{X}/\bar{Y} . It has a value of $\mu^B > 1$ (the value of μ corresponding to $\gamma = \gamma^B$) when $\bar{X}/\bar{Y} = 0$ and a value of μ^M , the value of μ when $\gamma = \gamma^M$, for all \bar{X}/\bar{Y} large enough. It follows that the two equations (A12) and (A14) must have a unique solution, with $\mu^B < \mu \leq \mu^M$ and $\bar{X}/\bar{Y} > 0$. If this solution also satisfies (A15), a steady state exists (and is unique). From (10) and (A12), condition (A15) is equivalent to

$$(1 - \tilde{\gamma})D^j\left(\frac{\tilde{\gamma}}{\gamma}, \dots, \frac{\tilde{\gamma}}{\gamma}\right) \leq \kappa(\gamma^B - 1). \tag{A23}$$

Hence, in addition to the standard assumptions on preferences and technology made in RBC models, if the preferences over differentiated goods imply that (A23) holds, a unique steady state exists.

It remains to consider whether in this steady state the incentive compatibility constraint (14) always binds strictly, as is assumed in Section III. This occurs if and only if the solution to (A12) and (A14) satisfies $\mu < \mu^M$, which is true if and only if

$$\mu\left(\kappa\left(\frac{\mu^M - 1}{\mu^M}\right)\right) < \mu^M.$$

With (17), this is equivalent to

$$\kappa < \frac{\gamma^M \phi(\gamma^M)}{\gamma^M - 1}, \tag{A24}$$

where $\phi(\gamma)$ is defined in (15). Hence, conditions (A23) and (A24) ensure that the incentive compatibility constraint binds at the steady state so that collusion is imperfect and sensitive to variations in demand conditions.

How realistic are conditions (A23) and (A24)? In the case we are most interested in, goods within the same industry are close (though not perfect) substitutes. This means that $\tilde{\gamma}$ is only slightly less than one and that γ^B is only slightly greater than one. Such considerations alone, however, do not indicate whether (A23) holds. They require, however, that the function $D^j(1, \dots, \rho, \dots, 1)$ decrease rapidly from a value of one when $\rho = 1$ to a value of zero when $\rho = 1/\tilde{\gamma}$ (a quantity only slightly above one). Suppose that $D^j(1, \dots, \rho, \dots, 1)$ is close to being a linear function of ρ over this range. Then $D^j_i(1, \dots, 1) = -\tilde{\gamma}/(1 - \tilde{\gamma})$, from which it follows that $\gamma^B \approx \tilde{\gamma}/(2\tilde{\gamma} - 1)$. In the case in which $\tilde{\gamma} \approx 1$, it follows that $\gamma^B - 1 \approx 1 - \tilde{\gamma}$. Hence (A23) is plausible as long as

$$\kappa > D^j\left(\frac{1}{\gamma^M}, \dots, \frac{1}{\gamma^M}\right). \tag{A25}$$

The right-hand side of (A25) is necessarily greater than one, but if goods of different industries are not particularly good substitutes, it need not be much larger. On the other hand, κ is likely to be much larger than one. Note that

$$\kappa = \frac{\alpha(1 + g)}{1 + r - \alpha(1 + g)},$$

where r is the steady-state real rate of return, and g is the steady-state growth rate of real output. If a “period” (the length of time before which a deviator cannot be punished) is short, then κ is large. For example, if a period is a quarter, as assumed in our calibration in the text, then $r = .015$ and $g = .008$. If $\alpha = 1$ (collusive agreements last forever), κ is greater than 100. But if $\alpha = .9$, collusive agreements last 10 quarters on average, and κ is close to nine. This suggests that (A25) and (A23) are not unreasonable assumptions.

Condition (A24) is more problematic since it requires that κ not be too large. With $\alpha < 1$, however, this inequality may well be satisfied. In the case of near-perfect substitutes within the same industry, a deviating firm can

expect to capture essentially all the industry's sales with only a small reduction in prices below those of the other firms. Hence, $\pi^d(\gamma; \gamma) \approx m\pi(\gamma; \gamma)$, where m is the number of firms per industry. It follows that $\phi(\gamma) \approx (m - 1)[(\gamma - 1)/\gamma]$, so that the right-hand side of (A24) approximately equals $m - 1$. Thus (A24) requires that $\kappa < m - 1$. But if $\alpha = .9$, as assumed above, κ is less than nine, so that 10 firms per industry are sufficient for (A24) to hold, in the case of close substitutes.

We can now comment on our assumption of less than perfect substitutes within a single industry, which contrasts with Rotemberg and Saloner (1986) and most of the literature on oligopolistic collusion. Were we to assume perfect substitutability, then, as just explained, we would have $\phi(\gamma) = (m - 1)[(\gamma - 1)/\gamma]$ so that

$$\frac{\mu - 1}{\mu} = \min \left[\frac{1}{m - 1} \frac{\bar{X}}{\bar{Y}}, \frac{\mu^M - 1}{\mu^M} \right]. \quad (\text{A26})$$

Comparison of (A12) and (A26) indicates that one solution is always $\mu = 1$, $\bar{X}/\bar{Y} = 0$. If $\kappa < m - 1$, this is the only kind of possible steady state, but this steady state with no collusion is not of interest to us. If $\kappa > m - 1$, there is also a solution $\mu = \mu^M$, $\bar{X}/\bar{Y} = \kappa(\mu^M - 1)/\mu^M$, and this is the optimal collusive agreement. But we are also not interested in these steady states with perfect collusion since the equilibrium markup would not be affected by small demand shocks.

Finally, if $\kappa = m - 1$, solutions with $1 < \mu < \mu^M$ are possible. But in this case, every μ in that interval is a solution, as is $\mu = \mu^M$, so the optimal collusive agreement involves this latter markup. In this special case, the incentive compatibility constraint is satisfied with equality in the steady state, so that small perturbations in government purchases can lower markups by making it impossible to sustain $\mu = \mu^M$. But $\mu(X/Y)$ is not differentiable at the steady-state value of \bar{X}/\bar{Y} , so that our linearization technique (which requires differentiable equilibrium conditions) cannot be applied to characterize the effects of small shocks. Hence, we cannot use this case for our purposes either. We thus assume that goods within the same industry are close, but not perfect, substitutes. This allows a steady state to exist in which $\mu^B < \mu < \mu^M$ and in which the equilibrium conditions are differentiable.

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