Optimal Fiscal and Monetary Policy with Sticky Prices*

Henry E. Siu†

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Abstract

In this paper, I study the properties of the Ramsey equilibrium in a model with distortionary taxation, nominal non-state-contingent debt, and costs of surprise inflation. To do this, I modify the standard cash-credit good economy studied in the optimal policy literature to include sticky prices. With this modification, the Ramsey planner must balance the shock absorbing benefits of surprise inflation against the associated resource misallocation costs.

The results of this modification are striking, as introducing price rigidity generates large departures from the case with fully flexible prices. For even small amounts of price stickiness, optimal monetary policy displays very little volatility in inflation. Optimal tax rates display much greater volatility than with fully flexible prices. The Friedman Rule is no longer optimal, as the nominal interest rate fluctuates across states of nature in the Ramsey equilibrium. Finally, optimal tax rates and real debt holdings no longer inherit the serial correlation properties of the underlying shocks; with sticky prices, these variables exhibit behavior similar to a random walk.

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†Department of Economics, Northwestern University, Evanston, IL 60208, USA; tel: 847.491.8239; fax: 847.491.7001; email: hankman@northwestern.edu

1. INTRODUCTION

A well known result in the optimal policy literature is that in stochastic environments, tax distortions should be smoothed across time and states of nature. For instance, in worlds where governments finance stochastic government spending through taxing labor income and issuing one-period debt, state-contingent returns on that debt allow tax rates to be roughly constant across states of nature (see Lucas and Stokey, 1983; and Chari et al., 1991). In monetary models, similar tax smoothing can be achieved even when nominal returns on debt are not state-contingent. In these models, varying the price level in response to shocks allows the government to achieve appropriate ex post real returns on debt across states (see Chari et al., 1991, 1995; and Chari and Kehoe, 1998). Generating a surprise inflation in the period of a positive government spending shock allows the government to decrease its real liabilities by reducing the real value of its outstanding debt; in this way, the government is able to attenuate the otherwise large increase in distortionary tax rates required to maintain present value budget balance. Clearly, in environments in which nominal returns to debt are not state-contingent, surprise inflation is an important policy tool, since it can be used to generate real returns which are. Sims (2000) extends this analysis to address the welfare costs of dollarization; in essence, replacing debt denominated in domestic currency with debt denominated in foreign currency eliminates the feasibility of state-contingent returns generated in this manner.[1]

A quantitative property of these models is that given plausible parameter values, optimal monetary policy displays an extremely high degree of volatility in the inflation rate. For instance, in a model calibrated to post-war US data, Chari et al. (1991) find a two standard deviation interval on the annual inflation rate to have bounds of approximately $+40\%$ and -40%. This extreme volatility is due to the fact that surprise inflation in these models is costless. The aim of this paper is to determine the optimal degree of volatility when

[1] For further discussion, as well as discussion of these results in relation to the literature on the Fiscal Theory of the Price Level, see Woodford (1998) and Christiano and Fitzgerald (2000). Interestingly, these papers, as well as Sims (2000), leave as an open question the optimal degree of inflation volatility when surprise inflation is costly.

surprise inflation is no longer costless, but still has shock absorbing benefits due to its effect on the government's inherited debt burden. This is an important consideration given that models which consider optimal monetary policy alone typically prescribe stable and near zero inflation when nominal rigidities are present (see King and Wolman, 1999; and Khan et al., 2000).[2]

To study this question I introduce sticky prices into the standard cash-credit good model. When some prices in the economy are set before the realization of government spending, surprise inflation causes the relative price of sticky price and flexible price firms to deviate from unity. This relative price distortion generates costly misallocation of real resources. Optimal policy on the part of the government must balance the shock absorbing benefits of surprise movements in the price level against these misallocation costs.

In quantitative examples, I show that this modification has a striking impact on the optimal degree of inflation volatility. For parameterizations in which the flexible price model prescribes extreme volatility, the analogous sticky price model prescribes essentially stable deflation at the rate of time preference. This is true even when the proportion of sticky price setters is small. For instance, when 2% of price setting firms post prices before the realization of shocks, the standard deviation of inflation falls by a factor of 5 (relative to the case of fully flexible prices); when 5% of firms have sticky prices, the standard deviation falls by a factor of 17. When the model is parameterized so that the marginal product of labor is diminishing, 5% sticky prices causes the standard deviation of inflation to fall by a factor of 70. Faced with sticky prices, a benevolent government finances increased spending largely through increased tax revenues. In the case of persistent spending shocks, tax revenues are gradually increased, and real debt is accumulated as high spending regimes progress. During spells of low spending, tax revenues fall and accumulated debt is paid off.

The optimal nominal interest rate is no longer zero in the sticky price model, as prescribed by the Friedman Rule. Instead, the interest rate is small but positive when government spending is low, and is zero when spending is high. Finally, the serial correlation properties

[2] Correia et al. (2001) find similar results in a model of optimal fiscal and monetary policy with nominal rigidites in which the government is able to issue state-contingent debt.

of Ramsey tax rates and real government debt differ markedly in the two environments. In contrast to Barro's (1979) random walk result, Chari et al. (1991) show that with flexible prices, optimal tax rates and bond holdings inherit the serial correlation properties of the model's underlying shocks.[3] With sticky prices, the autocorrelations of these objects are near unity regardless of the persistence in the shock process, partially reviving Barro's result. This finding is similar to that of Marcet et al. (2000) who consider the Ramsey problem in a model with incomplete markets; in fact, I show that the sequence of restrictions imposed on the set of feasible equilibria by sticky prices and market incompleteness turn out to be analytically similar.

The remainder of the paper is structured as follows. Section 2 presents a cash-credit good model with price setting on the part of intermediate good producers; a subset of these firms post prices before the realization of the state of nature. Section 3 characterizes competitive equilibrium, and develops the primal representation of equilibrium. I show that the primal representation requires consideration of two sequences of cross-state restrictions not present in the flexible price version of the economy. Section 4 presents the Ramsey allocation problem. The existence of the cross-state restrictions makes solving this problem difficult. Section 5 discusses the characteristics of the Ramsey equilibrium that are key to developing a solution method. In particular, I show that the solution to the problem builds upon the recursive contracts approach developed in Marcet and Marimon (1999). Section 6 presents quantitative results. Section 7 provides additional analysis of the cross-state constraint introduced by sticky prices and its implications. Section 8 concludes.

2. THE MODEL

2.1 Households

Let $s^t = (s_0, \dots, s_t)$ denote the history of events up to date t. The date 0 probability of observing history s^t is given by $\pi\left(s^t\right)$. The initial state, s^0, is given so that $\pi\left(s^0\right) = 1$. There is a large number of identical, infinitely-lived households in the economy. The representative

[3] For the initial exposition of this result in a real barter economy, see Lucas and Stokey (1983).

household's objective function is given by:

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi\left(s^t\right) U\left(c_1\left(s^t\right), c_2\left(s^t\right), l\left(s^t\right)\right),$$

where the period utility function takes the form:

$$U\left(c_1, c_2, l\right)=\left\{\left[(1-\gamma) c_1^{\phi}+\gamma c_2^{\phi}\right]^{\frac{1}{\phi}}(1-l)^{\psi}\right\}^{1-\sigma} /(1-\sigma).$$

Here, l denotes the share of the household's unit time endowment devoted to labor services, c_1 denotes units of consumption good purchased in cash, and c_2 denotes consumption purchased on credit.

The household maximizes its expected lifetime utility subject to several constraints. The first is the flow budget constraint. This constraint is relevant during securities trading in each period, which occurs after observation of the current realization, s_t:

$$M\left(s^t\right)+B\left(s^t\right) \leq R\left(s^{t-1}\right) B\left(s^{t-1}\right)+M\left(s^{t-1}\right)-P\left(s^{t-1}\right) c_1\left(s^{t-1}\right)$$

$$-P\left(s^{t-1}\right) c_2\left(s^{t-1}\right)+\left(1-\tau\left(s^{t-1}\right)\right)\left[\int_0^1 \Pi_i\left(s^{t-1}\right) d i+w\left(s^{t-1}\right) l\left(s^{t-1}\right)\right].$$

This must hold for all s^t. Holdings of cash chosen at s^t are denoted $M\left(s^t\right)$. Holdings of one-period government bonds, which mature at the beginning of period $t+1$, are denoted $B\left(s^t\right)$. The non-state-contingent nominal return on debt is denoted by $R\left(s^t\right)$. I adopt the timing structure used in Lucas and Stokey (1983), so that nominal wealth carried into state s^t can be transformed linearly between s^t cash and bond holdings.

The household's asset holdings derive from bond income, excess cash holdings from the previous period, and after-tax production income earned during the previous period, less consumption purchases made on credit. The household's state s^t production income (payable at the beginning of period $t+1$) derives from wage payments, $w\left(s^t\right) l\left(s^t\right)$, as well as dividends / profits earned from intermediate goods producers. Here, $\Pi_i\left(s^t\right)$ denotes intermediate good firm i's profit, where $i \in[0,1]$. Finally, $\tau\left(s^t\right)$ is a uniform, distortionary income tax rate levied on both dividend and labor income.

After securities trading, the household supplies labor, $l\left(s^t\right)$, at the wage rate $w\left(s^t\right)$, and purchases consumption, $c_1\left(s^t\right)$ and $c_2\left(s^t\right)$, at the price $P\left(s^t\right)$. Purchases of the cash good are subject to a cash-in-advance constraint, so that:

$$M\left(s^t\right) \geq P\left(s^t\right) c_1\left(s^t\right), \quad \forall s^t.$$

State s^t purchases made in cash are settled at s^t, while purchases made on credit are settled at the beginning of period $t+1$.[4]

This setup generates the standard first-order necessary conditions (FONCs):

$$\begin{aligned}
-U_l\left(s^t\right) &= U_2\left(s^t\right)\left(1-\tau\left(s^t\right)\right)\frac{w\left(s^t\right)}{P\left(s^t\right)}, \\
U_1\left(s^t\right) &= U_2\left(s^t\right) R\left(s^t\right), \\
\frac{U_1\left(s^t\right)}{P\left(s^t\right)} &= \beta R\left(s^t\right) \sum_{s^{t+1}|s^t} \pi\left(s^{t+1}|s^t\right)\frac{U_1\left(s^{t+1}\right)}{P\left(s^{t+1}\right)},
\end{aligned}$$

where the conditional probability $\pi\left(s^{t+1}|s^t\right) = \pi\left(s^{t+1}\right)/\pi\left(s^t\right)$. Here and in the rest of the paper, $U_1 = \partial U/\partial c_1$ (and similarly for c_2), and $U_l = \partial U/\partial l$.

2.2 Final Good Firms

Firms in the final good sector transform intermediate goods into output according to the production function:

$$Y = \left[\int_0^1 Y_i^{\frac{1}{\mu}} di\right]^{\mu}, \ \mu > 1$$

where Y is the final good firm's output, and Y_i is the input purchased from intermediate good firm i. Final goods are transformed linearly into household and government consumption, so that the following condition holds:

$$c_1\left(s^t\right) + c_2\left(s^t\right) + g\left(s^t\right) \leq Y\left(s^t\right), \quad \forall s^t.$$

[4] Though seemingly artificial, the cash-credit good distinction portrays in a simple manner the idea that only a fraction of final goods is purchased with cash balances, and that these purchases require foregoing interest. Moreover, if money demand was introduced in the more familiar single-good, cash-in-advance context, the optimal tax rate and inflation rate would be indeterminate; see Chari and Kehoe (1998) for discussion.

Final good firms receive payment both in the form of cash at period t (on sales of c_1), and cash at the beginning of period $t+1$ (on sales of c_2 and g). Firms must hold cash received on c_1 sales overnight (earning no interest); in equilibrium, the law of one price dictates that all final goods are sold at the uniform, state s^t price, $P\left(s^t\right)$.

The market for final goods is perfectly competitive and since technology displays constant returns to scale, equilibrium profits in this sector equal zero. The inputs used in this sector are produced by firms with monopoly power over their particular good i.

There are two types of intermediate good firms: sticky price and flexible price firms. At the end of each period, before the realization of next period's shock, a fraction $(1-v) \in [0,1]$ of the firms must post their prices for next period; these are the sticky price firms. Aside from this restriction, the two types of firms are identical. Therefore, in a symmetric equilibrium:

$$Y\left(s^t\right) = \left[vY_f\left(s^t\right)^{\frac{1}{\mu}} + (1-v)\,Y_s\left(s^t\right)^{\frac{1}{\mu}}\right]^{\mu},$$

where s stands for sticky and f stands for flexible.

The representative final good firm's problem is to choose the values of intermediate good inputs to maximize:

$$P\left(s^t\right)Y\left(s^t\right) - \int_{i\in f} P_i\left(s^t\right)Y_i\left(s^t\right)di - \int_{i\in s} P_i\left(s^{t-1}\right)Y_i\left(s^t\right)di.$$

This produces the following FONCs:

$$\frac{P_f\left(s^t\right)}{P\left(s^t\right)} = \left(\frac{Y\left(s^t\right)}{Y_f\left(s^t\right)}\right)^{\frac{\mu-1}{\mu}},$$

$$\frac{P_s\left(s^{t-1}\right)}{P\left(s^t\right)} = \left(\frac{Y\left(s^t\right)}{Y_s\left(s^t\right)}\right)^{\frac{\mu-1}{\mu}},$$

when type s and f firms act symmetrically. Note that given the timing structure, the representative sticky price firm's price at state s^t, $P_s\left(s^{t-1}\right)$, is a function of the history s^{t-1} only, and is identical across realizations of s_t. Of course, the value of $Y_s\left(s^t\right)$ will differ across date t realizations since demand depends on the relative price $P_s\left(s^{t-1}\right)/P\left(s^t\right)$.

2.3 Intermediate Good Firms

Each intermediate good firm i belonging to the continuum $[0,1]$ produces a differentiated product according to:

$$Y_i = L_i^{\alpha},\ \alpha \leq 1.$$

Labor services are hired from a perfectly competitive labor market at the state s^t wage rate, $w\left(s^t\right)$. If $\alpha < 1$, I interpret production as requiring an additional non-reproducible factor that is specific to the firm such as land, non-reproducible capital or entrepreneurial talent. Returns to this factor are paid to the household in the form of profit. This allows for an additional degree of curvature in investigating the distortions due to asymmetry in prices, and consequently, asymmetry in labor allocations across flexible and sticky price firms.

2.3.1 Flexible Price Firms After observing the current realization, s_t, the representative flexible price firm sets its price in order to maximize profit:

$$\Pi_f\left(s^t\right) = P_f\left(s^t\right) Y_f\left(s^t\right) - w\left(s^t\right) L_f\left(s^t\right),$$

taking the final good firm's demand as given. Again, $Y_f = L_f^{\alpha}$. The FONC for this problem is:

$$P_f\left(s^t\right) = \frac{\mu}{\alpha} w\left(s^t\right) L_f\left(s^t\right)^{1-\alpha},$$

which states the familiar condition that labor is hired up to the point where the wage is equal to a fraction, $\frac{1}{\mu}$, of the marginal revenue product of labor.

2.3.2 Sticky Price Firms At the end of date $t-1$, before observing s_t, the representative sticky price firm's problem is to choose a price $P_s\left(s^{t-1}\right)$, identical across states $\left(s^t \mid s^{t-1}\right)$, in order to maximize:

$$\sum_{s^t|s^{t-1}} \pi\left(s^t|s^{t-1}\right) \Xi\left(s^t\right) \left[P_s\left(s^{t-1}\right) Y_s\left(s^t\right) - w\left(s^t\right) L_s\left(s^t\right)\right].$$

The term in square brackets is the firm's s^t profit, $\Pi_s\left(s^t\right)$. The marginal value of dividends at s^t, $\Xi\left(s^t\right) = \left(1-\tau\left(s^t\right)\right) U_2\left(s^t\right)/P\left(s^t\right)$, and the final good firm's demand are taken as given. After some algebra, the FONC from this problem is:

$$\sum_{s^t|s^{t-1}} \pi\left(s^t|s^{t-1}\right)\Xi\left(s^t\right)\left[P_s\left(s^{t-1}\right)-\frac{\mu}{\alpha}w\left(s^t\right)L_s\left(s^t\right)^{1-\alpha}\right]P\left(s^t\right)^{\frac{\mu}{\mu-1}}Y\left(s^t\right)=0.$$

This expression is similar to the FONC for the flexible price firm, except that the term in square brackets is weighted across the possible states, s^t.

2.4 Government

The government faces a standard flow budget constraint:

$$M\left(s^t\right)+B\left(s^t\right)+\tau\left(s^{t-1}\right)\left[\int_0^1 \Pi_i\left(s^{t-1}\right)di+w\left(s^{t-1}\right)l\left(s^{t-1}\right)\right]$$

$$=M\left(s^{t-1}\right)+R\left(s^{t-1}\right)B\left(s^{t-1}\right)+P\left(s^{t-1}\right)g\left(s^{t-1}\right),$$

which must hold $\forall s^t$. As in previous studies, government consumption, $g\left(s^t\right)$, is determined exogenously and is assumed to transit between $\left\{\underline{g},\bar{g}\right\}$ with symmetric transition probability $0<\rho<1$. The government purchases final goods on credit.

3. CHARACTERIZING EQUILIBRIUM

An imperfectly competitive equilibrium, in which intermediate good firms of each type behave symmetrically, is defined in the usual way. In particular, consider the following definition.

Definition 1 Given the household's initial real claims on the government, a_0, and the stochastic process $\left\{g\left(s^t\right)\right\}$, a symmetric, imperfectly competitive equilibrium is an allocation, $\{c_1\left(s^t\right), c_2\left(s^t\right), l\left(s^t\right), L_f\left(s^t\right), L_s\left(s^t\right), Y\left(s^t\right), Y_f\left(s^t\right), Y_s\left(s^t\right), B\left(s^t\right)\}$, price system, $\{R\left(s^t\right), P\left(s^t\right), P_f\left(s^t\right), P_s\left(s^{t-1}\right), w\left(s^t\right)\}$, and government policy, $\left\{M\left(s^t\right), \tau\left(s^t\right)\right\}$, such that:

- $\{c_1(s^t),\ c_2(s^t),\ l(s^t),\ M(s^t),\ B(s^t)\}$ solves the representative household's optimization problem subject to the sequence of household flow budget constraints and cash-in-advance constraints;

- $\{Y(s^t), Y_f(s^t), Y_s(s^t)\}$ solves the representative final good firm's optimization problem;

- $\{P_f(s^t), L_f(s^t), Y_f(s^t)\}$ solves the representative flexible price firm's optimization problem;

- $P_s(s^{t-1})$ solves the representative sticky price firm's optimization problem (with $L_s(s^t)$ and $Y_s(s^t)$ being demand determined);

- the sequence of government flow budget constraints is satisfied;

- the labor market clears:

$$l(s^t) = vL_f(s^t) + (1-v)L_s(s^t), \ \ \forall s^t;$$

- and $R(s^t) \geq 1, \ \ \forall s^t$.

The final condition ensures that the consumer does not find it profitable to buy money and sell bonds, so that the cash-in-advance constraint holds with equality. Bond market clearing at each state has been implicitly assumed, as both issues and holdings are denoted by the single variable, $B(s^t)$; the same is true of money, $M(s^t)$. Clearing in the market for each intermediate good i has also been assumed, with purchases and production of each denoted by $Y_i(s^t)$. Clearing in the final goods market is satisfied by Walras' Law; indeed, this condition can be obtained by combining the two flow budget constraints (holding with equality), the labor market clearing condition and the final good firm's FONCs. Finally, note that it is initial real, as opposed to nominal, claims that are taken as given; this ensures that the initial price level, $P(s^0)$, cannot be used by the government to generate zero real indebtedness or arbitrarily large revenues in period 0. As a result, the value of $P(s^0)$ is simply a normalization and inconsequential for the Ramsey analysis to follow.

3.1 The Primal Approach

To simplify the analysis of optimal policy, I adopt the standard approach of characterizing equilibrium in primal form. This involves restating the equilibrium conditions in terms of real allocations alone. I show that the primal representation for the sticky-price economy requires consideration of five constraints.

The first two constraints:

$$U_1\left(s^t\right) \geq U_2\left(s^t\right), \tag{1}$$

$$c_1\left(s^t\right) + c_2\left(s^t\right) + g\left(s^t\right) = \left[vL_f\left(s^t\right)^{\frac{\alpha}{\mu}} + (1-v)\,L_s\left(s^t\right)^{\frac{\alpha}{\mu}}\right]^{\mu}, \tag{2}$$

guarantee that $R\left(s^t\right) \geq 1$ and that the final goods market clears; these must hold $\forall s^t$. Call these the no arbitrage and aggregate resource constraints.

The third constraint is:

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi\left(s^t\right)C\left(s^t\right) = U_1\left(s^0\right)a_0, \tag{3}$$

where

$$C\left(s^t\right) = U_1\left(s^t\right)c_1\left(s^t\right) + U_2\left(s^t\right)c_2\left(s^t\right) + U_l\left(s^t\right)\Lambda\left(s^t\right),$$
$$\Lambda\left(s^t\right) = \frac{\mu}{\alpha}\left[vL_f\left(s^t\right) + (1-v)\,L_f\left(s^t\right)^{1-\frac{\alpha}{\mu}}L_s\left(s^t\right)^{\frac{\alpha}{\mu}}\right].$$

This is the standard implementability constraint modified to account for: (1) the presence of monopolistic competition in intermediate goods, and (2) the asymmetric behavior between the two 'types' of intermediate good firms, s and f. The fourth constraint is simply a rewriting of the sticky price firm's FONC in terms of real allocations:

$$\sum_{s^{t+1}|s^t}\pi\left(s^{t+1}|s^t\right)U_l\left(s^{t+1}\right)\left[L_f\left(s^{t+1}\right)^{1-\frac{\alpha}{\mu}}L_s\left(s^{t+1}\right)^{\frac{\alpha}{\mu}} - L_s\left(s^{t+1}\right)\right] = 0, \ \ \forall s^t. \tag{4}$$

The final constraint is the sticky price constraint. This condition ensures that $P_s\left(s^t\right)$ is identical across realizations of s_{t+1}. Given that government spending can take on two possible values ($\bar{g}$ and $\underline{g}$), let $\bar{s}^{t+1}$ and $\underline{s}^{t+1}$ denote the two possible date $t+1$ histories following s^t; the sticky price constraint is:

$$A\left(\bar{s}^{t+1}\right)\sum_{r=t+1}^{\infty}\sum_{s^r|\bar{s}^{t+1}}\beta^r\pi\left(s^r|\bar{s}^{t+1}\right)C\left(s^r\right)=A\left(\underline{s}^{t+1}\right)\sum_{r=t+1}^{\infty}\sum_{s^r|\underline{s}^{t+1}}\beta^r\pi\left(s^r|\underline{s}^{t+1}\right)C\left(s^r\right), \quad (5)$$

where

$$A\left(s^{t+1}\right)=\frac{1}{U_1\left(s^{t+1}\right)}\left[v\left(\frac{L_f\left(s^{t+1}\right)}{L_s\left(s^{t+1}\right)}\right)^{\frac{\alpha}{\mu}}+1-v\right]^{1-\mu}.$$

Proposition 2 In any competitive equilibrium, the allocation for consumption and labor, $\{c_1\left(s^t\right), c_2\left(s^t\right), L_f\left(s^t\right), L_s\left(s^t\right)\}$, must satisfy the five constraints, (1) through (5). Furthermore, given sequences $\{c_1\left(s^t\right), c_2\left(s^t\right), L_f\left(s^t\right), L_s\left(s^t\right)\}$ that satisfy these constraints, it is possible to construct all of the remaining competitive equilibrium (real) allocation, price and policy variables.

Proof. To derive the implementability constraint, (3), take the household's date t flow budget constraint, multiply it by $\beta^t\pi\left(s^t\right)U_1\left(s^t\right)/P\left(s^t\right)$, and sum over all s^t and t. This can be simplified to read:

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi\left(s^t\right)\left\{U_1\left(s^t\right)c_1\left(s^t\right)+U_2\left(s^t\right)\left[c_2\left(s^t\right)-\left(1-\tau\left(s^t\right)\right)I\left(s^t\right)\right]\right\}=U_1\left(s^0\right)a_0,$$

where $I\left(s^t\right)$ is:

$$I\left(s^t\right)=v\frac{\Pi_f\left(s^t\right)}{P\left(s^t\right)}+(1-v)\frac{\Pi_s\left(s^t\right)}{P\left(s^t\right)}+\frac{w\left(s^t\right)}{P\left(s^t\right)}l\left(s^t\right).$$

This is done by using the second and third household FONC, the cash-in-advance constraint, and the transversality condition on real bond holdings:

$$\lim_{r\to\infty}\beta^r\pi\left(s^r\right)U_1\left(s^r\right)b\left(s^r\right)=0, \quad \forall s^r,$$

where $b\left(s^r\right)=B\left(s^r\right)/P\left(s^r\right)$. Using the final good firm's FONCs, the flexible price firm's FONC, and labor market clearing, $I\left(s^t\right)$ can be simplified to read:

$$I\left(s^t\right)=\frac{w\left(s^t\right)}{P\left(s^t\right)}\Lambda\left(s^t\right).$$

Substitute this in above and use the household's FONC with respect to labor to obtain the implementability constraint.

To get the sticky price firm's FONC in terms of real allocations, substitute in the household's FONC with respect to labor supply:

$$\sum_{s^t|s^{t-1}} \pi\left(s^t|s^{t-1}\right) U_l\left(s^t\right) \frac{P\left(s^t\right)}{w\left(s^t\right)} \left[P_s\left(s^{t-1}\right) - \frac{\mu}{\alpha} w\left(s^t\right) L_s\left(s^t\right)^{1-\alpha}\right] P\left(s^t\right)^{\frac{1}{\mu-1}} Y\left(s^t\right) = 0.$$

Again, using the remaining production-side FONCs, this can be simplified to read:

$$\sum_{s^t|s^{t-1}} \pi\left(s^t|s^{t-1}\right) U_l\left(s^t\right) \frac{\mu}{\alpha} \left[L_f\left(s^t\right)^{1-\frac{\alpha}{\mu}} L_s\left(s^t\right)^{\frac{\alpha}{\mu}-\alpha} - L_s\left(s^t\right)^{1-\alpha}\right] P\left(s^t\right)^{\frac{\mu}{\mu-1}} Y\left(s^t\right) = 0.$$

Multiplying the constraint by $\frac{\alpha}{\mu} P_s\left(s^{t-1}\right)^{\frac{\mu}{1-\mu}}$, a constant across states $\left(s^t \mid s^{t-1}\right)$, results in the expression (4).

For the sticky price constraint, first rewrite the household's date $t+1$ flow budget constraint as:

$$\frac{P\left(s^{t+1}\right)}{P\left(s^t\right)} = \frac{R\left(s^t\right) b\left(s^t\right) + \left(1-\tau\left(s^t\right)\right) Y\left(s^t\right) - c_2\left(s^t\right)}{c_1\left(s^{t+1}\right) + b\left(s^{t+1}\right)}.$$

The price $P_s\left(s^t\right)$, chosen at the end of state s^t, must be identical across states $\left(s^{t+1} \mid s^t\right)$. Using the final good firm's FONC, this can be stated as:

$$\frac{P\left(\bar{s}^{t+1}\right)}{P\left(s^t\right)} \left(\frac{Y\left(\bar{s}^{t+1}\right)}{L_s\left(\bar{s}^{t+1}\right)^{\alpha}}\right)^{\frac{\mu-1}{\mu}} = \frac{P\left(\underline{s}^{t+1}\right)}{P\left(s^t\right)} \left(\frac{Y\left(\underline{s}^{t+1}\right)}{L_s\left(\underline{s}^{t+1}\right)^{\alpha}}\right)^{\frac{\mu-1}{\mu}}$$

for states $\bar{s}^{t+1}$ and $\underline{s}^{t+1}$ following s^t. Substitute the household's flow budget constraint into this equation to obtain:

$$\left[c_1\left(\bar{s}^{t+1}\right) + b\left(\bar{s}^{t+1}\right)\right] \left(\frac{L_s\left(\bar{s}^{t+1}\right)^{\alpha}}{Y\left(\bar{s}^{t+1}\right)}\right)^{\frac{\mu-1}{\mu}} = \left[c_1\left(\underline{s}^{t+1}\right) + b\left(\underline{s}^{t+1}\right)\right] \left(\frac{L_s\left(\underline{s}^{t+1}\right)^{\alpha}}{Y\left(\underline{s}^{t+1}\right)}\right)^{\frac{\mu-1}{\mu}}.$$

Finally, use the household's date r flow budget constraint, multiply it by $\beta^r \pi\left(s^r\right) \frac{U_1(s^r)}{P(s^r)}$, and sum over states $\left(s^r \mid s^{t+1}\right)$ and dates $r \geq t+2$ to get:

$$b\left(s^{t+1}\right) = \sum_{r=t+2}^{\infty} \sum_{s^r|s^{t+1}} \beta^{r-t-1} \pi\left(s^r|s^{t+1}\right) \frac{C\left(s^r\right)}{U_1\left(s^{t+1}\right)} + \frac{U_2\left(s^{t+1}\right)}{U_1\left(s^{t+1}\right)} c_2\left(s^{t+1}\right) + \frac{U_l\left(s^{t+1}\right)}{U_1\left(s^{t+1}\right)} \Lambda\left(s^{t+1}\right).$$

To get the sticky price constraint, (5), substitute this into the equation above for $b\left(\bar{s}^{t+1}\right)$ and $b\left(\underline{s}^{t+1}\right)$.

Given sequences $\left\{c_1\left(s^t\right), c_2\left(s^t\right), L_f\left(s^t\right), L_s\left(s^t\right)\right\}$ that satisfy the above constraints, it is easy to construct all of the remaining competitive equilibrium (real) allocation, price and policy variables. At state s^t:

$$\frac{M\left(s^t\right)}{P\left(s^t\right)} = c_1\left(s^t\right),$$

$$R\left(s^t\right) = \frac{U_1\left(s^t\right)}{U_2\left(s^t\right)},$$

$$Y\left(s^t\right) = \left[vL_f\left(s^t\right)^{\frac{\alpha}{\mu}} + (1-v)\,L_s\left(s^t\right)^{\frac{\alpha}{\mu}}\right]^{\mu},$$

$$\frac{P_f\left(s^t\right)}{P\left(s^t\right)} = \left(\frac{Y\left(s^t\right)}{L_f\left(s^t\right)^{\alpha}}\right)^{\frac{\mu-1}{\mu}},$$

$$\frac{P_s\left(s^{t-1}\right)}{P\left(s^t\right)} = \left(\frac{Y\left(s^t\right)}{L_s\left(s^t\right)^{\alpha}}\right)^{\frac{\mu-1}{\mu}},$$

$$\frac{w\left(s^t\right)}{P\left(s^t\right)} = \frac{\alpha}{\mu}\frac{P_f\left(s^t\right)}{P\left(s^t\right)}L_f\left(s^t\right)^{\alpha-1},$$

$$\tau\left(s^t\right) = 1 + \frac{U_l\left(s^t\right)}{U_2\left(s^t\right)}\frac{P\left(s^t\right)}{w\left(s^t\right)}.$$

Real government debt at state s^t satisfies:

$$b\left(s^t\right) = \sum_{r=t+1}^{\infty}\sum_{s^r|s^t}\beta^{r-t}\pi\left(s^r|s^t\right)\frac{C\left(s^r\right)}{U_1\left(s^t\right)} + \frac{U_2\left(s^t\right)}{U_1\left(s^t\right)}c_2\left(s^t\right) + \frac{U_l\left(s^t\right)}{U_1\left(s^t\right)}\Lambda\left(s^t\right).$$

Finally, the condition:

$$\frac{P\left(s^{t+1}\right)}{P\left(s^t\right)} = \frac{R\left(s^t\right)b\left(s^t\right) + \left(1-\tau\left(s^t\right)\right)Y\left(s^t\right) - c_2\left(s^t\right)}{\left[c_1\left(s^{t+1}\right) + b\left(s^{t+1}\right)\right]}$$

defines the inflation rate between states s^{t+1} and s^t. ■

4. THE RAMSEY PROBLEM

The Ramsey allocation problem is the following: find the fiscal and monetary policy that induces competitive equilibrium associated with the highest value of the household's expected lifetime utility. Specifically, I assume that the government is able to commit to its policy plan at the beginning of time, and that in all periods, maximizing agents in the economy behave taking this policy plan as given.[5] Given the results of Proposition 2, solving the Ramsey problem is equivalent to finding the allocation $\{c_1(s^t), c_2(s^t), L_f(s^t), L_s(s^t)\}$ that maximizes the household's utility subject to the constraints (1) through (5).

Note that the two sequences of cross-state restrictions, (4) and (5), complicate the analysis relative to environments in which the implementability constraint provides the sole 'intertemporal link.' Note also that the appearance of the expected values of future choice variables in (5) makes solving the Ramsey problem particularly difficult; because of the sticky price constraint, period t allocations are not simply functions of the realizations of s_{t-1} and s_t since the entire infinite history of shocks, s^∞, matter for optimal period t decisions. Further details on the issues involved in solving this problem are presented in the next section. In the rest of this section, I formally state the Ramsey allocation problem, and show that the Friedman Rule is not optimal in this environment.

The Ramsey problem is to choose consumption and labor, $\{c_1(s^t), c_2(s^t), L_f(s^t), L_s(s^t)\}$, and multipliers, λ and $\{\theta(s^t), \delta(s^t), \eta(s^t), \xi(s^t)\}$ to solve the Lagrangian:

$$\max \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) U(c_1(s^t), c_2(s^t), l(s^t)) + \tag{6}$$
$$\lambda \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) C(s^t) + \lambda [C(s^0) - U_1(s^0) a_0] +$$
$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \theta(s^t) [Y(s^t) - c_1(s^t) - c_2(s^t) - g(s^t)] +$$
$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \delta(s^t) [U_1(s^t) - U_2(s^t)] +$$

[5] Discussion of time consistency issues and the relationship to Stackelberg equilibrium are contained in Lucas and Stokey (1983), Chari et al. (1995), and Woodford (1998).

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi\left(s^t\right)\eta\left(s^t\right)\sum_{s^{t+1}|s^t}\beta\pi\left(s^{t+1}|s^t\right)U_l\left(s^{t+1}\right)h\left(s^{t+1}\right)+$$

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi\left(s^t\right)\xi\left(s^t\right)\sum_{s^{t+1}|s^t}\tilde{\imath}\left(s^{t+1}\right)A\left(s^{t+1}\right)\sum_{r=1}^{\infty}\sum_{s^{t+r}|s^{t+1}}\beta^r\pi\left(s^{t+r}|s^{t+1}\right)C\left(s^{t+r}\right).$$

where,

$$\begin{aligned}
C\left(s^t\right) &= U_1\left(s^t\right)c_1\left(s^t\right)+U_2\left(s^t\right)c_2\left(s^t\right)+U_l\left(s^t\right)\Lambda\left(s^t\right),\\
\Lambda\left(s^t\right) &= \frac{\mu}{\alpha}\left[vL_f\left(s^t\right)+(1-v)L_f\left(s^t\right)^{1-\frac{\alpha}{\mu}}L_s\left(s^t\right)^{\frac{\alpha}{\mu}}\right],\\
h\left(s^{t+1}\right) &= L_f\left(s^{t+1}\right)^{1-\frac{\alpha}{\mu}}L_s\left(s^{t+1}\right)^{\frac{\alpha}{\mu}}-L_s\left(s^{t+1}\right),\\
\tilde{\imath}\left(s^{t+1}\right) &= \begin{cases} -1 & , if\ g\left(s_{t+1}\right)=\underline{g} \\ +1 & , if\ g\left(s_{t+1}\right)=\bar{g} \end{cases},\\
A\left(s^{t+1}\right) &= \frac{1}{U_1\left(s^{t+1}\right)}\left[v\left(\frac{L_f\left(s^{t+1}\right)}{L_s\left(s^{t+1}\right)}\right)^{\frac{\alpha}{\mu}}+1-v\right]^{1-\mu}.
\end{aligned}$$

The second line of the maximization states the implementability constraint, (3); the third and fourth lines give the aggregate resource constraint and the no arbitrage constraint, (1) and (2); the fifth line states the sticky price firm's FONC, (4); the sixth line states the sticky price constraint, (5).

With the definition of the Ramsey problem, (6), it is possible to show that the Friedman Rule is not optimal. That is, once sticky prices are introduced into the imperfectly competitive, cash-credit good model, the condition $R\left(s^t\right)=1$ (or equivalently, $U_1\left(s^t\right)=U_2\left(s^t\right)$) does not hold $\forall s^t$, $t\geq 1$. To see this, equate the first order conditions with respect to $c_1\left(s^t\right)$ and $c_2\left(s^t\right)$, set $\delta\left(s^t\right)=0$ (so that we ignore the no arbitrage constraint), and simplify to get:

$$\frac{\xi\left(s^{t-1}\right)\tilde{\imath}\left(s^t\right)A\left(s^t\right)}{\pi\left(s^t|s^{t-1}\right)}\left[\Theta\left(s^t\right)\left(\frac{U_1\left(s^t\right)}{U_2\left(s^t\right)}-1\right)+\frac{\phi-1}{c_1\left(s^t\right)}\right]\sum_{r=0}^{\infty}\sum_{s^{t+r}|s^t}\beta^r\pi\left(s^{t+r}|s^t\right)C\left(s^{t+r}\right)$$

$$=\left[U_1\left(s^t\right)-U_2\left(s^t\right)\right]\Upsilon\left(s^t\right),$$

where

$$\Theta\left(s^t\right)=\frac{(1-\phi-\sigma)\gamma c_2\left(s^t\right)^{\phi-1}}{(1-\gamma)c_1\left(s^t\right)^{\phi}+\gamma c_2\left(s^t\right)^{\phi}},$$

$$\Upsilon\left(s^{t}\right)=1+(1-\sigma)\left[\left(\lambda+\sum_{r=0}^{t-1}\frac{\xi\left(s^{r}\right)\tilde{\imath}\left(s^{r+1}\right)A\left(s^{r+1}\right)}{\pi\left(s^{r+1}|s^{r}\right)}\right)\ell\left(s^{t}\right)-\eta\left(s^{t-1}\right)\frac{\psi h\left(s^{t}\right)}{1-l\left(s^{t}\right)}\right],$$

$$\ell\left(s^{t}\right)=\left(1-\frac{\psi\Lambda\left(s^{t}\right)}{1-l\left(s^{t}\right)}\right).$$

Hence, $U_1\left(s^t\right)=U_2\left(s^t\right)$ if and only if:

$$\xi\left(s^{t-1}\right)\sum_{r=0}^{\infty}\sum_{s^{t+r}|s^{t}}\beta^{r}\pi\left(s^{t+r}|s^{t}\right)C\left(s^{t+r}\right)=0.$$

Note that $\sum_{r=0}^{\infty}\sum_{s^{t+r}|s^{t}}\beta^{r}\pi\left(s^{t+r}|s^{t}\right)C\left(s^{t+r}\right)$ is the present (utility) value of the real government surplus from state s^t onward. Since this value is generically different from zero, $U_1\left(s^t\right)=U_2\left(s^t\right)$ if and only if $\xi\left(s^{t-1}\right)=0$. That is, since $\left\{\xi\left(s^t\right)\right\}$ is the sequence of multipliers associated with the sticky price constraint, the Friedman Rule is optimal if and only if sticky prices are not present in the model. This is summarized in Proposition 3. In appendix A, I derive the primal representation for the model with monopolistic competition in which all firms set prices flexibly, and provide a proof of the optimality of the Friedman Rule for that environment.

Proposition 3 For the imperfectly competitive, cash-credit good model with sticky prices, the Friedman Rule is not optimal; that is, $R\left(s^t\right)=1$ does not hold $\forall s^t$. However, if the model is modified so that all prices are flexible (all prices are set after observation of the current period realization of government spending), optimality of the Friedman Rule is restored.

Given this result, it is not surprising that non-optimality of the Friedman Rule for this economy stems from the sticky price constraint, (5), which restricts the set of feasible equilibria relative to the case with flexible prices. Further discussion of this result is deferred to section 7, where I consider the implications of the sticky price constraint on the Ramsey equilibrium. Also, note that this result differs from that emphasized in Schmitt-Grohé and Uribe (2001a). In particular, they derive an imperfectly competitive model where the use of cash is motivated by a desire to minimize transaction costs. In their model, the

Friedman Rule is not optimal even **without** sticky prices. This is due to an assumption that the profits of intermediate good firms are untaxed. Indeed, if I modify the flexible price, cash-credit good economy so that profit income goes untaxed, the Friedman Rule breaks down as well. Further discussion of this result, as well as its relationship to the 'uniform commodity taxation rule' is contained in appendix A.[6]

5. A RECURSIVE SOLUTION METHOD

Solving the Ramsey problem, (6), is made difficult by the fact that 'future' decision variables (variables of states s^{t+r}, $r \geq 1$) appear in the 'current' sticky price constraint (at state s^t). One of the consequences of this is that **current** period decision variables depend upon the whole history of **past** shocks, s^t. As described in Marcet and Marimon (1999), this can be remedied through the introduction of a costate variable which summarizes the evolution of the ξ-multipliers on the sequence of sticky price constraints.

However, the multiplicative form of the sticky price constraint further complicates the analysis, because all **future** shocks following s^t also enter into **current** period decisions! The easiest way to see this is to consider one of the FONCs of the Ramsey problem; for instance, consider the FONC with respect to $c_1\left(s^t\right)$:

$$U_1\left(s^t\right) + \left[\lambda + \xi\left(s^0\right) i\left(s^1\right) A\left(s^1\right) + \ldots + \xi\left(s^{t-2}\right) i\left(s^{t-1}\right) A\left(s^{t-1}\right)\right] C_1\left(s^t\right) +$$

$$\xi\left(s^{t-1}\right) i\left(s^t\right) \left\{ A\left(s^t\right) C_1\left(s^t\right) + A_1\left(s^t\right) \left[C\left(s^t\right) + \sum_{r=1}^{\infty} \sum_{s^{t+r}|s^t} \beta^r \pi\left(s^{t+r}|s^t\right) C\left(s^{t+r}\right) \right] \right\} +$$

$$\eta\left(s^{t-1}\right) U_{1l}\left(s^t\right) h\left(s^t\right) + \delta\left(s^t\right)\left[U_{11}\left(s^t\right) - U_{12}\left(s^t\right)\right] = \theta\left(s^t\right),$$

where

$$i\left(s^t\right) = \begin{cases} -1/\pi\left(s^t|s^{t-1}\right) & , \text{ } if\ g\left(s^t\right) = \underline{g} \\ 1/\pi\left(s^t|s^{t-1}\right) & , \text{ } if\ g\left(s^t\right) = \bar{g} \end{cases}.$$

[6] See also Schmitt-Grohé and Uribe (2001a) for further discussion on the relationship between the Friedman Rule and taxation of profit income.

Here, and in the rest of the paper, partial derivatives of functions such as $A\left(s^t\right)$ are denoted $A_1 = \partial A/\partial c_1$ (and similarly for c_2), and $A_{L_f} = \partial A/\partial L_f$ (and similarly for L_s). Clearly, both the history of events up to s^t as well as events following s^t impact upon state s^t decisions.

To make analysis of the problem tractable, first define the variable:

$$\kappa\left(s^{t-1}\right) = \sum_{r=0}^{t-2} \xi\left(s^r\right) i\left(s^{r+1}\right) A\left(s^{r+1}\right), \quad t \geq 2,$$

which acts to summarize the history of sticky price constraints up to s^t. Since the initial state s^0 is given, $\kappa\left(s^{t-1}\right) = 0$ for $t = 0, 1$. This costate variable evolves according to the law of motion:

$$\kappa\left(s^t\right) = \kappa\left(s^{t-1}\right) + \xi\left(s^{t-1}\right) i\left(s^t\right) A\left(s^t\right), \quad t \geq 1.$$

Next, define the recursive function $q\left(s^t\right)$ as:

$$q\left(s^t\right) = C\left(s^t\right) + \beta \sum_{s^{t+1}|s^t} \pi\left(s^{t+1}|s^t\right) q\left(s^{t+1}\right), \quad t \geq 1.$$

This acts to summarize the impact of future decision variables upon decisions made at the current state.

With these two definitions, the FONC for $c_1\left(s^t\right)$ can be rewritten as:

$$U_1\left(s^t\right) + \left[\lambda + \kappa\left(s^{t-1}\right)\right] C_1\left(s^t\right) + \xi\left(s^{t-1}\right) i\left(s^t\right)\left[A\left(s^t\right) C_1\left(s^t\right) + A_1\left(s^t\right) q\left(s^t\right)\right] +$$

$$\eta\left(s^{t-1}\right) U_{1l}\left(s^t\right) h\left(s^t\right) + \delta\left(s^t\right)\left[U_{11}\left(s^t\right) - U_{12}\left(s^t\right)\right] = \theta\left(s^t\right).$$

The FONCs for $c_2\left(s^t\right)$, $L_f\left(s^t\right)$, and $L_s\left(s^t\right)$ possess a similar form and are displayed in appendix B. Inspection of these FONCs reveal that, for $t \geq 1$, Ramsey allocations are stationary in the state $\left(\kappa\left(s^{t-1}\right), \ \left(g\left(s_t\right) \mid g\left(s_{t-1}\right)\right)\right)$. Accordingly, denote:

$$\left(g\left(s_t\right) \mid g\left(s_{t-1}\right)\right) \quad \text{by} \quad \Gamma \in \left\{\left(\underline{g}|\underline{g}\right), \ \left(\bar{g}|\underline{g}\right), \ \left(\underline{g}|\bar{g}\right), \ \left(\bar{g}|\bar{g}\right)\right\},$$

and

$$\kappa\left(s^{t-1}\right) \quad \text{by} \quad \kappa \in \mathcal{R}.$$

For the sake of exposition, I continue to use the '$\mid$' relation to denote the timing of shocks; therefore, $(\bar{g}|\underline{g})$ represents '$\bar{g}$ at date t following $\underline{g}$ at date $t-1$.'

Hence, allocations such as $c_1\left(s^t\right)$ are stationary functions, denoted $c_1\left(\kappa,\Gamma\right)$, and similarly for $c_2\left(s^t\right)$, $L_f\left(s^t\right)$, and $L_s\left(s^t\right)$. In addition, the multipliers on the cross-state restrictions, (4) and (5), are stationary functions of $\left(\kappa\left(s^{t-1}\right),\ g\left(s_{t-1}\right)\right)$; that is, $\eta\left(s^{t-1}\right)=\eta\left(\kappa,g_{-1}\right)$ and similarly for $\xi\left(s^{t-1}\right)$, where $g_{-1}\in\left\{\underline{g},\,\bar{g}\right\}$ denotes the realization of government spending at date $t-1$ and κ is as defined above.

The costate evolves according to $\kappa'=\kappa+\xi\left(\kappa,g_{-1}\right)i\left(\Gamma\right)A\left(\kappa,\Gamma\right)$. Notice that dependance of real variables (such as real government debt) and policy variables (such as tax rates) on κ imparts a persistent component to these objects. Finally, note that the recursive function $q\left(s^t\right)$ is stationary as well, and satisfies:

$$q\left(\kappa,\Gamma\right)=C\left(\kappa,\Gamma\right)+\beta\sum_{\Gamma'\,|\,\Gamma}\pi\left(\Gamma'\right)q\left(\kappa';\Gamma'\right).$$

Chari et al. (1995) show that inference on the quantitative properties of Ramsey policies and prices are sensitive to the choice of solution method. In particular, they find that non-linear approximation methods provide important improvements in accuracy relative to linearization methods.[7] As a result, a non-linear method is used to derive the results presented in this paper. Appendix B contains a detailed description of the solution algorithm. The algorithm builds upon the projection techniques presented in Judd (1992) in order to approximate the recursive, stationary function $q\left(\kappa,\Gamma\right)$; this function is used to characterize the solution to the Ramsey problem.

6. QUANTITATIVE RESULTS

In this section I present results which illustrate the quantitative properties of the Ramsey equilibrium with sticky prices. As a benchmark, I also present results for the analogous model where all intermediate good firms set prices flexibly (that is, all firms $i\in[0,1]$ set prices, $P_i\left(s^t\right)$, after observing s_t). In the flexible price model, it is possible to show

[7] See Marcet et al. (2000) for a discussion on inaccuracy of second-order approximations in environments of the type considered here.

Parameter	β	σ	γ	ϕ	μ	α	ρ	$\underline{g}$	$\bar{g}$
Value	0.97	1.25	0.57	0.83	1.05	1.00	0.95	0.054	0.066

Table 1. Baseline Parameter Values.

that optimal decisions at any date are constrained only to satisfy an aggregate resource constraint and implementability constraint. As a result, the Ramsey allocation, policy and price system depend only on the current value of government spending. This makes the flexible price model particularly tractable, and exact solutions can be found.[8] Further details regarding this benchmark model are contained in appendix A.

6.1 Parameterization

Table 1 contains the baseline parameter values common to both versions of the model. The first two parameters are typical values used in models calibrated to annual data (see for instance, Chari et al., 1991; and Jones et al., 2000). The values of γ and ϕ are the estimated values found in Chari et al. (1991).

The value of μ corresponds to a 5% steady-state markup in the intermediate good sector, which is somewhat smaller than that considered in other studies with sticky prices.[9] This was chosen as a compromise between the existing literature with sticky prices, and the literature on optimal policy with flexible prices and no market power. As in previous studies of optimal policy, the value of labor's share of national income, α, is initially set to unity (see Chari et al., 1991).

The persistence parameter ρ is initially set so that the first-order autocorrelation on government spending is 0.90. The values of $\bar{g}$ and $\underline{g}$ deliver a coefficient of variation on government spending of 0.10; the mean value of government spending is 20% of GDP. The value of ψ is set so that, in the non-stochastic steady state, 30% of the household's time

[8] See, for instance, Schmitt-Grohé and Uribe (2001a). For discussion of this result in a model with perfect competition, see Chari and Kehoe (1998).

[9] Chari et al. (2000) and Khan et al. (2000) consider $\mu = 1.11$, while others estimate this value to be higher still.

Rate (in %)	Flexible	5% Sticky	10% Sticky
Income Tax			
mean	21.77	26.66	26.56
std. deviation	0.098	1.532	1.593
autocorrelation	0.895	0.968	0.966
Inflation			
mean	−1.949	−2.869	−2.862
std. deviation	10.06	0.595	0.439
autocorrelation	−0.010	0.375	0.349
Money Growth			
mean	−1.981	−2.832	−2.814
std. deviation	9.696	2.858	3.196
autocorrelation	−0.007	−0.499	−0.499
Nominal Interest			
mean	0	0.132	0.146
std. deviation	N/A	0.387	0.457

Table 2. Taxes, Prices and Money at various degrees of Price Rigidity.

is spent working. Initial real claims on the government are set so that in the stationary equilibrium of the flexible price model, the government's real debt to GDP ratio is 0.45. Finally, for the sticky price model, results are presented with the fraction of sticky price firms set at 5% and 10% (v equal to 0.95 and 0.90, respectively).

6.2 Sticky Prices and Volatility

Simulation results for the baseline model are reported in tables 2 and 3. For a given value of ρ, the same realization of the exogenous shock sequence was used for all versions of the model. In table 2, all rates are expressed as (annual) percentages.

The introduction of sticky prices has a large impact on the volatilities of the income tax rate and the inflation rate. When the fraction of sticky price firms increases from 0% to 5%, the standard deviation of the tax rate increases by a factor of 16 (and its coefficient of variation increases by a factor of 13); the standard deviation of the inflation rate falls by a factor of 17. When the fraction of sticky price firms increases from 0% to 10%, the standard deviation of the tax rate increases by a factor of 17, and the standard deviation of inflation falls by a factor of 23. At 5% sticky prices, the volatility of inflation is remarkably small; if the inflation rate was normally distributed, ninety percent of observations would lie between -3.85% and -1.89%. The analogous interval for the flexible price model is bounded by -18.50% and 14.60%.[10]

In fact, optimal inflation volatility is small even when the degree of nominal rigidity is less than that displayed in table 2. Figure 1 plots the standard deviation of the Ramsey inflation rate at various values of v. Notice that the volatility falls quickly as the fraction of sticky price firms moves from 0% to 1%. With 2% sticky prices, the standard deviation is only 1.84%, five times smaller than that of the flexible price case.

Despite the breakdown of the Friedman Rule, table 2 indicates that nominal interest rates are still close to zero with sticky prices. In particular, the value of the interest rate in the Ramsey equilibrium depends largely on the realization of government spending, Γ. When current spending is high, irrespective of the previous spending shock (or the value of the costate), the interest rate is constrained by the 0% lower bound. When current spending is low, the interest rate is positive. In transition states, $\left(\underline{g}|\bar{g}\right)$, the interest rate attains its largest values; for the 5% sticky price simulation, the maximum value obtained was 2.71%. In continuation states, $\left(\underline{g}|\underline{g}\right)$, the values are much smaller; in the same simulation, the nominal interest rate has a mean of 0.16% in these states. This behavior accounts for the small magnitude of mean interest rates and large standard deviations presented in table 2.

Table 3 presents simulation results for real variables. The 5% sticky price model displays approximately 32% greater volatility in output and aggregate labor relative to the flexible

[10]During the completion of this paper, I have learned of independent work by Schmitt-Grohé and Uribe (2001b) who present similar results in a model where costs of inflation are imposed as a quadratic cost of price adjustment.

	Flexible	5% Sticky	10% Sticky
Std. Deviations			
Y	0.0043	0.0055	0.0056
l	0.0043	0.0055	0.0056
c_1	0.0002	0.0009	0.0010
c_2	0.0015	0.0035	0.0036
L_s	N/A	0.0285	0.0160
L_f	N/A	0.0050	0.0051
p_s	N/A	0.494%	0.215%
p_f	N/A	0.022%	0.023%

Table 3. Allocations and Relative Prices at various degrees of Price Rigidity.

price model. More striking, the volatilities of c_1 and c_2 are respectively, 224% and 135% greater. In absolute terms, however, the volatility in these variables is still small for the sticky price models. The coefficients of variation for c_1 and c_2 are approximately 0.025 and 0.018 when 5% of firms have sticky prices. The final two rows present the volatility of p_s and p_f; these refer to the ratio of sticky and flexible intermediate good prices to the aggregate price level, in percentage terms. These have very small standard deviations, again indicating the government's strong incentive to minimize resource allocation distortions.

The differences in Ramsey outcomes between the sticky and flexible price models are more dramatic when the value of labor's share, α, is less than unity. This can be seen in table 4, where α is set at 0.64. Recall that the production of intermediate goods is given by $Y_i = L_i^{\alpha}$, for all firms $i \in [0, 1]$. When $\alpha < 1$, the marginal physical product of labor is diminishing and no longer constant. Evidently, with a greater degree of curvature in production, the Ramsey planner's incentive to reduce misallocation costs is strengthened. With 5% sticky prices, the standard deviation of the tax rate increases by a factor of 19 (and its coefficient of variation increases by a factor of 16) relative to the case with flexible prices; the standard deviation of the inflation rate falls by a factor of 70. These results indicate that a benevolent government is faced with a strong incentive to eliminate the

Rate (in %)	Flexible	5% Sticky
Income Tax		
mean	14.73	17.27
std. deviation	0.059	1.091
autocorrelation	0.895	0.993
Inflation		
mean	−2.406	−2.968
std. deviation	7.281	0.104
autocorrelation	−0.011	0.807
Money Growth		
mean	−2.431	−2.966
std. deviation	6.904	0.770
autocorrelation	−0.007	−0.361
Nominal Interest		
mean	0	0.037
std. deviation	N/A	0.125

Table 4. Taxes, Prices and Money when Labor's Share is 0.64.

resource allocation distortions that arise from surprise inflation, relative to the distortions due to variability in tax rates across states of nature.

With an exogenous increase in spending, the present value of the government's real liabilities increase. When all prices are flexible, the government finances this principally through a large decrease in the real value of its outstanding debt by generating a surprise inflation. This is not true when there are sticky prices. This can be seen in figures 2 and 3, where I display 21-period time series of simulated data. The data in figure 2 are generated from the baseline parameterization of the flexible price model, and figure 3 from the baseline 5% sticky price model. The scale of the vertical axis in each panel is preserved across figures 2 and 3; this is done to reflect differences in orders of magnitude across models.

In period 5, government spending transits from its low state to its high state. Spending stays high until period 16, when it changes again. When all prices are flexible, the government responds contemporaneously to the increase in real spending by generating a large surprise inflation; in panel B, the inflation rate jumps from -4.7% at date 4 to 48.6% at date 5. This has the effect of sharply reducing the real value of inherited debt, as seen in panel C. Real inherited debt falls by 36% in the period of the shock; this value falls a further 15% in the period after the shock (when payment on date 5 spending is made), due principally to a reduction in real bond issues in period 5.

This allows the government to keep the tax rate and real tax revenues (panels D and E) essentially constant during the transition to the high spending regime. The tax rate increases 0.2%, and tax revenues increase by 3.7%, in period 5. When real government spending falls in period 16, there is a surprise deflation and the value of real debt rises. Again the tax rate and tax revenues move very little.

In the sticky price model, there is very little surprise inflation in response to the increase in spending. The inflation rate increases from -2.9% in period 4 to -1.7% in period 5, and again to -1.4% in period 6 when payment on the increased spending is due. As a result, there is a much smaller fall in the real value of inherited debt; inherited debt falls by only 2.1% in period 5 (due to surprise inflation), and 2.7% in period 6 (due to inflation and reduced bond issues in period 5).

Instead, the government finances its increased spending largely through increased tax revenue and, as the high spending regime persists, through bond issue. In period 5, the tax rate falls 0.4%. This tax cut, coupled with the wealth effect of lower future earnings, stimulates labor supply, so that real tax revenues increase by 1.6%. In period 6, there is a sharp 1.5% increase in the tax rate which generates a 4.8% increase in real tax revenues. Tax revenues increase by an additional 1.2% between periods 6 and 15 as the high spending regime continues. During this time, real debt issue increases by 7.0%. When government spending falls, tax revenues are lowered, and the government gradually pays down the accumulated debt (with a one period lag).

	persistent shocks		i.i.d. shocks	
	Flexible	5% Sticky	Flexible	5% Sticky
Gov't. Spending				
autocorrelation	0.895		−0.013	
Tax Rate (%)				
std. deviation	0.098	1.532	0.098	0.795
autocorrelation	0.895	0.968	−0.013	0.987
Real Debt				
autocorrelation	0.895	0.999	−0.013	0.994
Inflation Rate (%)				
std. deviation	10.06	0.595	4.277	1.121
autocorrelation	−0.010	0.375	−0.293	−0.077
Nom. Int. Rate (%)				
mean	0	0.132	0	0.053
std. deviation	N/A	0.387	N/A	0.064

Table 5. Effects of Varying the Persistence in Government Spending.

6.3 Sticky Prices and Persistence

With sticky prices, the serial correlation of the Ramsey tax rate exhibits a noticeable deviation from the case with flexible prices. Tables 2 and 4 show that for the flexible price model, the tax rate inherits the autocorrelation of government spending (see also Lucas and Stokey, 1983; and Chari et al., 1991). However, with sticky prices, the autocorrelation is much closer to unity.

Table 5 shows that this is also true of the persistence in real government debt. For the baseline parameterization, the autocorrelation of simulated government spending is equal to 0.895; columns 2 and 3 show that the autocorrelation of real debt is 0.895 with flexible prices, and 0.999 with sticky prices. Columns 4 and 5 present the same statistics for the case of i.i.d. spending shocks (and all other parameters as in the baseline case). Again, tax rates and real debt inherit the serial correlation properties of the shock process when all prices are flexible. With sticky prices, the autocorrelation of these variables is near unity. In this sense, introducing price rigidity moves optimal policy towards Barro's (1979) random walk result. As in Marcet et al. (2000), this behavior is driven by the introduction of a costate variable summarizing the history of binding constraints. In the present model, this costate summarizes the history of binding sticky price constraints.

7. IMPLICATIONS OF THE STICKY PRICE CONSTRAINT

The introduction of sticky prices causes the optimality of the Friedman Rule to break down. As well, optimal tax rates and real bond holdings display a high degree of persistence, regardless of the persistence in the underlying shocks. Both of these features of the Ramsey equilibrium are better understood upon closer inspection of the sticky price constraint.

Note that this constraint requires that the present value of real government surpluses be approximately equal across states $\underline{s}^t$ and $\bar{s}^t$ following s^{t-1}, $t \geq 1$. That is, condition (5) can be rewritten as:

$$\tilde{A}\left(\underline{s}^t\right)\mathcal{PV}\left(\underline{s}^t\right) = \tilde{A}\left(\bar{s}^t\right)\mathcal{PV}\left(\bar{s}^t\right),$$

where

$$\tilde{A}\left(s^t\right) = \left[v\left(\frac{L_f\left(s^t\right)}{L_s\left(s^t\right)}\right)^{\frac{\alpha}{\mu}} + 1 - v\right]^{1-\mu},$$

$$\mathcal{PV}\left(s^t\right) = \sum_{r=t}^{\infty}\sum_{s^r|s^t}\beta^{r-t}\pi\left(s^r|s^t\right)\frac{U_1\left(s^r\right)}{U_1\left(s^t\right)}\left[\frac{\tau\left(s^r\right)Y\left(s^r\right) - g\left(s^r\right)}{R\left(s^r\right)} + \frac{M\left(s^r\right)}{P\left(s^r\right)}\left(\frac{R\left(s^r\right)-1}{R\left(s^r\right)}\right)\right].$$

Appendix C contains a more detailed derivation of this result. In the expression for $\mathcal{PV}\left(s^t\right)$, the first term in square brackets represents the government's real primary budget surplus at s^r, $r \geq t$, adjusted for the timing structure on spending and tax income in the government's budget constraint. The second term represents the real interest savings the government earns from issuing money relative to debt. Evidently, $\mathcal{PV}\left(s^t\right)$ is the present value of real government surpluses (from all sources) from s^t onward.

Hence the sticky price constraint can be interpreted as follows: for states following s^{t-1}, the $\mathcal{PV}\left(s^t\right)$'s must be equalized up to the factor $\tilde{A}\left(\bar{s}^t\right)/\tilde{A}\left(\underline{s}^t\right)$. This cross-state restriction on present values is obviously absent from the flexible price model. In fact, Chari and Kehoe (1998) show that with flexible prices, $\mathcal{PV}\left(\underline{s}^t\right) > \mathcal{PV}\left(\bar{s}^t\right)$; that is, the present value of government surpluses is larger when current spending is low, relative to when current spending is high. Note that this is true even in the case of i.i.d. spending shocks, since $\mathcal{PV}\left(s^t\right)$ includes the current period surplus.

With sticky prices, the results of section 6 indicate that the Ramsey equilibrium displays essentially constant deflation; hence, labor allocations across sticky and flexible price firms are approximately symmetric and $\tilde{A}\left(s^t\right) \simeq 1$, $\forall s^t$.[11] Given the Ramsey planner's strong incentive to minimize resource allocation distortions, the sticky price constraint requires that $\mathcal{PV}\left(\underline{s}^t\right) \simeq \mathcal{PV}\left(\bar{s}^t\right)$. This allows for some insight into the behavior of the nominal interest rate, tax rate and real bond holdings in the Ramsey equilibrium.

First, consider a deviation from the Friedman Rule. Raising the nominal interest rate from zero has two first order effects on $\mathcal{PV}\left(s^t\right)$: it decreases the real value of the time-adjusted primary surplus and increases the real value of interest savings. Since the primary surplus is orders of magnitudes greater than interest savings,[12] an increase in the nominal rate decreases the present value of government surpluses. Hence with sticky prices, the Ramsey planner uses a positive nominal interest rate during periods of low spending to decrease $\mathcal{PV}\left(\underline{s}^t\right)$, and help satisfy the sticky price constraint. This alleviates the need to use surprise inflation which causes deviations of $\tilde{A}\left(\bar{s}^t\right)/\tilde{A}\left(\underline{s}^t\right)$ from unity.

Finally, the sticky price constraint can be interpreted as an approximation to the constraint found in the following model: one without money or sticky prices, but in which the real return on debt is non-state-contingent. This environment is exactly the one considered by Marcet et al. (2000). When their model is simplified so that government spending takes

[11] For instance, in the baseline parameterization with 5% sticky prices, the average simulated value of $\tilde{\mathsf{A}}\left(\mathsf{s}^t\right)$ is 0.9996 with standard deviation 0.0049.

[12] For the baseline model with 5% sticky prices, the average simulated value of the primary surplus is 1800 times greater than that of interest savings.

on only two values, the sequence of constraints imposed by market incompleteness is:

$$\mathcal{PV}\left(\underline{s}^t\right) = \mathcal{PV}\left(\bar{s}^t\right),$$

for both states $\underline{s}^t$ and $\bar{s}^t$ following s^{t-1} (for a derivation of this, see appendix C). In the sticky price model, with $\tilde{A}\left(s^t\right) \simeq 1$ in all states, $\mathcal{PV}\left(\underline{s}^t\right) \simeq \mathcal{PV}\left(\bar{s}^t\right)$. Given the Ramsey planner's aversion to volatile inflation, the restrictions on the set of feasible equilibria imposed by sticky prices and incomplete markets are approximately equivalent. It is not surprising then, that the quantitative properties of real variables (namely the serial correlation in tax rates and debt holdings) are similar in the two economies.

8. CONCLUSION

This paper characterizes optimal fiscal and monetary policy with sticky price setting in intermediate goods markets. With sticky prices, a benevolent government must balance the benefits of surprise inflation (which alters the real value of outstanding debt liabilities) with its resource misallocation costs. The results of this study show that the introduction of small amounts of price rigidity generates large departures from the case with fully flexible prices studied previously in the literature.

With a small fraction of firms setting prices before the realization of government spending, the Ramsey solution prescribes essentially constant deflation. Hence, responses in the real value of the government's inherited debt are largely attenuated. Instead, periods of high and low government spending are financed by the collection of larger and smaller amounts of tax revenue. Persistent spells of high spending are accompanied by increasing tax collection and the accumulation of debt; spells of low spending are accompanied by falling tax collection and the reduction of accumulated debt. In summary, the extreme volatility in optimal inflation rates described in the previous literature is sensitive to small departures from the assumption of flexible price setting.

The behavior of nominal interest rates in the Ramsey equilibrium ceases to be characterized by the Friedman Rule. However, the quantitative departure from zero percent nominal interest is small. Finally, tax rates and real government debt display a high degree of persistence with the introduction of sticky prices; this is true regardless of the persistence properties of the underlying shock process.

APPENDICES

A. THE FLEXIBLE PRICE MODEL AND OPTIMALITY OF THE FRIEDMAN RULE

In this appendix, I first present the FONCs and budget constraints which must hold in an imperfectly competitive equilibrium for the cash-credit good model with flexible prices. This is followed by a proof of the second statement in Proposition 3, namely that the Friedman Rule is optimal in this model. Finally, I discuss why the Friedman Rule breaks down when profit income goes untaxed.

A.1 The Primal Representation

The first set of equilibrium conditions are the household FONCs, the household's flow budget constraint, and the cash-in-advance constraint; these are identical to those presented in section 2. The government's flow budget constraint is identical as well. The final good and intermediate good production functions are also identical to those above, and are restated here:

$$Y\left(s^t\right) = \left[\int_0^1 Y_i\left(s^t\right)^{\frac{1}{\mu}}\,di\right]^{\mu},$$

$$Y_i\left(s^t\right) = L_i\left(s^t\right)^{\alpha},\quad i \in [0,1]\,.$$

However, because there is no sticky price / flexible price distinction, $Y_i\left(s^t\right) = \bar{Y}\left(s^t\right)$ for all i in a symmetric equilibrium. From the final good firm's production function and FONC, $Y\left(s^t\right) = \bar{Y}\left(s^t\right)$ and $P_i\left(s^t\right) = P\left(s^t\right)$ for all i. Imposing labor market clearing, $\bar{Y}\left(s^t\right) = l\left(s^t\right)^{\alpha}$, and from the intermediate good firm's FONC, $w\left(s^t\right)/P\left(s^t\right) = \frac{\alpha}{\mu} l\left(s^t\right)^{\alpha-1}$. Clearing in the final goods market can again be obtained by combining the household and government budget constraints.

Proposition 4 Equilibrium can be characterized in primal form as an allocation $\{c_1(s^t),$ $c_2(s^t),\ l(s^t)\}$ that satisfies the following three constraints:

$$U_1(s^t) \geq U_2(s^t),$$

$$c_1(s^t) + c_2(s^t) + g(s^t) = l(s^t)^{\alpha},$$

which must hold for all s^t, and:

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) D(s^t) = U_1(s^0) a_0,$$

where

$$D(s^t) = U_1(s^t) c_1(s^t) + U_2(s^t) c_2(s^t) + U_l(s^t) \frac{\mu}{\alpha} l(s^t).$$

Furthermore, given allocations which satisfy these constraints, it is possible to construct all of the remaining equilibrium allocation, price and policy variables.

Proof. Again, the constraint $U_1(s^t) \geq U_2(s^t)$ is required so that the household does not find it profitable to buy money and sell bonds. The second constraint is the aggregate resource constraint. To obtain the implementability constraint, take the household's date t budget constraint, multiply it by $\beta^t \pi(s^t) U_1(s^t)/P(s^t)$, and sum over all s^t and t. Using the household FONCs, the cash-in-advance constraint and the transversality condition on real bonds, this can be simplified to read:

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \left\{ U_1(s^t) c_1(s^t) + U_2(s^t) c_2(s^t) + U_l(s^t) \frac{P(s^t)}{w(s^t)} l(s^t)^{\alpha} \right\} = U_1(s^0) a_0.$$

Finally, use the intermediate good firm's FONC to get the expression above.

With sequences $\{c_1(s^t),\ c_2(s^t),\ l(s^t)\}$ that satisfy these three constraints, construct the remaining equilibrium objects at s^t as:

$$\frac{M(s^t)}{P(s^t)} = c_1(s^t),$$

$$R(s^t) = \frac{U_1(s^t)}{U_2(s^t)},$$

$$\bar{Y}\left(s^t\right) = l\left(s^t\right)^{\alpha},$$

$$\frac{w\left(s^t\right)}{P\left(s^t\right)} = \frac{\alpha}{\mu} l\left(s^t\right)^{\alpha-1},$$

$$\tau\left(s^t\right) = 1 + \frac{U_l\left(s^t\right)}{U_2\left(s^t\right)} \frac{P\left(s^t\right)}{w\left(s^t\right)}.$$

Real bond holdings at state s^t satisfy:

$$b\left(s^t\right) = \sum_{r=t+1}^{\infty} \sum_{s^r|s^t} \beta^{r-t} \pi\left(s^r|s^t\right) \frac{D\left(s^r\right)}{U_1\left(s^t\right)} + \frac{U_2\left(s^t\right)}{U_1\left(s^t\right)} c_2\left(s^t\right) + \frac{U_l\left(s^t\right)}{U_1\left(s^t\right)} \frac{\mu}{\alpha} l\left(s^t\right).$$

Finally, the condition:

$$\frac{P\left(s^{t+1}\right)}{P\left(s^t\right)} = \frac{R\left(s^t\right) b\left(s^t\right) + \left(1 - \tau\left(s^t\right)\right) Y\left(s^t\right) - c_2\left(s^t\right)}{\left[c_1\left(s^{t+1}\right) + b\left(s^{t+1}\right)\right]}$$

defines the inflation rate between states s^{t+1} and s^t. ■

Notice that this is the natural simplification of the primal representation for the sticky price economy presented in section 3. In particular, without sticky prices, the constraint (5) is irrelevant so that $\xi\left(s^t\right) \equiv 0$. Symmetry requires $L_i\left(s^t\right) = l\left(s^t\right)$ for all i; this implies that $\Lambda\left(s^t\right) \equiv \frac{\mu}{\alpha} l\left(s^t\right)$ in (3). In addition, since the term in square brackets of (4) is equal to zero, this constraint holds trivially. In fact, it is possible to show that:

$$\eta\left(s^t\right) \equiv \eta = \frac{\lambda\left(1-v\right)\left(1-\frac{\mu}{\alpha}\right)}{1-\frac{\alpha}{\mu}}.$$

A.2 When is the Friedman Rule Optimal?

With these observations it is easy to show that the Friedman Rule is optimal for this economy. Consider the Ramsey problem, (6), where $C\left(s^t\right)$ is replaced by $D\left(s^t\right)$, and the sticky price firm's FONC and sticky price constraint are no longer relevant. Omit the no arbitrage condition, and consider the maximization problem with the interest rate unconstrained. Equate the FONCs with respect to $c_1\left(s^t\right)$ and $c_2\left(s^t\right)$ and simplify to get:

$$\left[U_1\left(s^t\right) - U_2\left(s^t\right)\right] \Upsilon\left(s^t\right) = 0,$$

where

$$\Upsilon\left(s^t\right) = 1 + (1-\sigma)\lambda\left(1 - \frac{\mu}{\alpha}\frac{\psi l\left(s^t\right)}{1-l\left(s^t\right)}\right).$$

Since $\Upsilon\left(s^t\right)$ is generically different from zero, $\left[U_1\left(s^t\right) - U_2\left(s^t\right)\right]\Upsilon\left(s^t\right) = 0$ is satisfied if and only if $U_1\left(s^t\right) = U_2\left(s^t\right)$.

It is worth noting that this result depends crucially upon the assumption that both labor and profit income are taxed at the uniform rate, $\tau\left(s^t\right)$. In particular, if the model is modified so that the tax rate on profits is zero, the Friedman Rule is no longer optimal. To see this, modify the flexible price model in this manner and derive the primal representation. It is easy to show that equilibrium is characterized by the same aggregate resource constraint and no arbitrage constraint, but the implementability constraint becomes:

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi\left(s^t\right)\tilde{D}\left(s^t\right) = U_1\left(s^0\right)a_0,$$

where

$$\tilde{D}\left(s^t\right) = U_1\left(s^t\right)c_1\left(s^t\right) + U_2\left(s^t\right)\left[c_2\left(s^t\right) - \left(1 - \frac{\alpha}{\mu}\right)l^{\alpha}\right] + U_l\left(s^t\right)l\left(s^t\right).$$

Inspection of the Ramsey FONCs with this set of constraints reveals that the Friedman Rule is not optimal; in particular, it is possible to show that in the Ramsey equilibrium, $U_1\left(s^t\right) > U_2\left(s^t\right)$ for all s^t, $t \geq 1$. Therefore, the nominal interest rate should be strictly positive.

To gain intuition for this, note that the implementability constraint and aggregate resource constraint of this economy are equivalent to those derived from a repeated sequence of static, real barter economies. This economy has: a production technology with a linear transformation frontier between c_1 and c_2; distinct consumption tax rates; no taxes on income; and an un-taxed endowment of the good c_2 which, in equilibrium, is equal to $(1-\alpha/\mu)\,l^{\alpha}$. Because of the endowment, optimal consumption plans require $U_1 > U_2$. Since the law of one price dictates that the untaxed prices of c_1 and c_2 are equal, the government must levy a higher tax rate on c_1 in order to induce the optimal allocation. In the context of the cash-credit good model, this is achieved through a positive nominal interest rate which acts as an effective tax on the cash good.

Hence, the fact that $R\left(s^t\right) > 1$ is optimal can be understood as an exception to the uniform commodity taxation rule. Specifically, despite the fact that preferences are assumed to be homothetic in c_1 and c_2, and weakly separable in leisure, optimal tax rates on the two consumption goods are not equal when there is an endowment of the good c_2 (see Chari and Kehoe, 1998). The presence of the untaxed profit income acts as a wealth endowment denominated in the credit good. This is a readily identifiable interpretation, since profit income is transformed into credit at a 1-to-1 rate in the consumer's budget constraint.

B. THE SOLUTION ALGORITHM

The algorithm I develop is based on the projection methods described in Judd (1992). Here, I describe the approximation to the function, $q(\cdot)$, characterizing the solution to the Ramsey problem. As well, I present the functional equation, $R(\cdot) = 0$, that summarizes the conditions $q(\cdot)$ must satisfy.

First, define the variable $k = \lambda + \kappa$. To see why this is helpful, notice that in the FONC for $c_1\left(s^t\right)$ displayed in section 5, the costate $\kappa\left(s^t\right)$ appears only in conjunction (in an additive manner) with the multiplier on the implementability constraint, λ; this is true for all of the relevant FONCs of the Lagrangian, (6). In order to solve the Ramsey problem, I express the function $q(k, \Gamma)$ as a linear function of Chebychev polynomials:

$$q(k, \Gamma) = \sum_{j=0}^{N-1} \omega_j(\Gamma) T_j(\varphi(k)),$$

where $T_j(\cdot)$ is the j-th order Chebychev polynomial, and $\varphi(\cdot)$ maps the domain of k into the interval $[-1, +1]$. Notice that since the state variable Γ takes on four possible values, the problem is to solve for four q-functions, each a function of the continuous variable k and indexed by a particular value of Γ; more precisely, I solve for four $(N \times 1)$ coefficients vectors, $\vec{\omega}(\Gamma)$, one for each element of Γ.

The solution finds coefficient vectors to satisfy the functional equation:

$$R\left(k,\Gamma;\vec{\omega}\right)=q\left(k,\Gamma\right)-\left[C\left(k,\Gamma\right)+\beta\sum_{\Gamma'\,|\,\Gamma}\pi\left(\Gamma'\right)q\left(k',\Gamma'\right)\right]=0,$$

for all k and Γ. Here, $k'=\lambda+\kappa'$, and the value of κ' is determined according to the law of motion:

$$\kappa'=\kappa+\xi\left(k,g_{-1}\right)i\left(\Gamma\right)A\left(k,\Gamma\right).$$

Clearly, it is not feasible to set $R\left(k,\Gamma;\vec{\omega}\right)=0$ for all possible values of k. Instead, I consider a Galerkin / collocation method that sets $R\left(k,\Gamma;\vec{\omega}\right)=0$ for M prespecified values of k, where $M\geq N$. Given the linear structure of the approximation function, the collocation procedure can be implemented in an iterative algorithm. This algorithm is described in the steps below.

1. Choose an initial guess for the vector $\vec{\omega}\left(\Gamma\right)$ for the four values of Γ; call the initial guess $\vec{\omega}^0$, and the resulting guess of the q-function, $q^0\left(k;\Gamma\right)$.

2. Consider the first collocation value of the costate variable; call it k_1. Recall that there are M collocation points.

3. Consider one of the values of g_{-1}. Recall that $g_{-1}\in\left\{\underline{g},\,\bar{g}\right\}$.

4. Choose initial guesses for the values of $\eta\left(k_1,g_{-1}\right)$ and $\xi\left(k_1,g_{-1}\right)$; call these guesses η and ξ.

5. Given the values of k_1, η, and ξ, solve the following set of FONCs for c_2, L_f, L_s, and δ; the fact that these latter four objects are functions of $\left(k_1,\Gamma\right)$ is suppressed for the sake of exposition:

$$\begin{aligned}&U_2+k_1C_2+\eta U_{2l}h+\delta\left[U_{12}-U_{22}\right]+\\ \xi i&\left\{AC_2+A_2\left[C+\beta\sum_{\Gamma'\,|\,\Gamma}\pi\left(\Gamma'\right)q^0\left(k',\Gamma'\right)\right]\right\}=\theta,\end{aligned}\tag{a}$$

$$vU_l + k_1 C_{L_f} + \eta\left(vU_{ll}h + U_l h_{L_f}\right) + \delta v\left(U_{1l} - U_{2l}\right) +$$

$$\xi i\left\{AC_{L_f} + A_{L_f}\left[C + \beta\sum_{\Gamma'\,|\,\Gamma}\pi\left(\Gamma'\right)q^0\left(k',\Gamma'\right)\right]\right\} = -\theta Y_{L_f}, \qquad \text{(b)}$$

$$(1-v)\,U_l + k_1 C_{L_s} + \eta\left((1-v)\,U_{ll}h + U_l h_{L_s}\right) + \delta\left(1-v\right)\left(U_{1l} - U_{2l}\right) +$$

$$\xi i\left\{AC_{L_s} + A_{L_s}\left[C + \beta\sum_{\Gamma'\,|\,\Gamma}\pi\left(\Gamma'\right)q^0\left(k',\Gamma'\right)\right]\right\} = -\theta Y_{L_s}, \qquad \text{(c)}$$

$$U_1 > U_2, \;\; if\ \delta = 0$$

$$U_1 = U_2, \;\; if\ \delta > 0. \qquad \text{(d)}$$

Here, $\theta = \theta\left(k_1, \Gamma\right)$ is defined by the FONC with respect to c_1:

$$\theta = U_1 + k_1 C_1 + \eta U_{1l}h + \delta\left(U_{11} - U_{12}\right) +$$

$$\xi i\left\{AC_1 + A_1\left[C + \beta\sum_{\Gamma'\,|\,\Gamma}\pi\left(\Gamma'\right)q^0\left(k',\Gamma'\right)\right]\right\},$$

i is the indicator function:

$$i\left(\Gamma\right) = \begin{cases} -1/\rho & ,\ if\ g = \underline{g}\ and\ g_{-1} = \underline{g} \\ 1/\left(1-\rho\right) & ,\ if\ g = \bar{g}\ and\ g_{-1} = \underline{g} \\ -1/\left(1-\rho\right) & ,\ if\ g = \underline{g}\ and\ g_{-1} = \bar{g} \\ 1/\rho & ,\ if\ g = \bar{g}\ and\ g_{-1} = \bar{g} \end{cases},$$

$c_1 = Y - c_2 - g$, and:

$$k' = k_1 + \xi\left(k_1, g_{-1}\right)\, i\, A\left(k_1, \Gamma\right).$$

Solve this system of equations for both realizations of current period government spending, $\underline{\Gamma} = \left(\underline{g}\,|\,g_{-1}\right)$ and $\bar{\Gamma} = \left(\bar{g}\,|\,g_{-1}\right)$, following g_{-1}.

6. Evaluate the sticky price constraint:

$$A\left(k_1,\underline{\Gamma}\right)\left[C\left(k_1,\underline{\Gamma}\right)+\beta\sum_{\Gamma'\,|\underline{\Gamma}}\pi\left(\Gamma'\right)q^0\left(k',\Gamma'\right)\right]=$$

$$A\left(k_1,\bar{\Gamma}\right)\left[C\left(k_1,\bar{\Gamma}\right)+\beta\sum_{\Gamma'\,|\bar{\Gamma}}\pi\left(\Gamma'\right)q^0\left(k',\Gamma'\right)\right].$$

   Adjust the value of $\xi\left(k_1,g_{-1}\right)$ and repeat step (5) until the constraint holds with equality.

7. Evaluate the sticky price firm's FONC:

$$\pi\left(\underline{\Gamma}\right)U_l\left(k_1,\underline{\Gamma}\right)h\left(k_1,\underline{\Gamma}\right)+\pi\left(\bar{\Gamma}\right)U_l\left(k_1,\bar{\Gamma}\right)h\left(k_1,\bar{\Gamma}\right)=0.$$

   Adjust the value of $\eta\left(k_1,g_{-1}\right)$ and repeat steps (5) and (6) until the constraint holds with equality.

8. Repeat steps (4) through (7) for the other value of g_{-1}.

9. Repeat steps (3) through (8) for the other collocation values, k_m, for $m=2,\ldots,M$.

10. Define the $(M\times N)$ matrix X as:

$$X=\begin{bmatrix} T_0\left(\varphi\left(k_1\right)\right) & T_1\left(\varphi\left(k_1\right)\right) & \cdots & T_{N-1}\left(\varphi\left(k_1\right)\right)\\ \vdots & \vdots & \ddots & \vdots\\ T_0\left(\varphi\left(k_M\right)\right) & T_1\left(\varphi\left(k_M\right)\right) & \cdots & T_{N-1}\left(\varphi\left(k_M\right)\right)\end{bmatrix}.$$

    Also, define the $(M\times 1)$ vector Z^0 as:

$$Z^0=\begin{bmatrix} C\left(k_1;\Gamma\right)+\beta\sum_{\Gamma'\,|\,\Gamma}\pi\left(\Gamma'\right)q^0\left(k';\Gamma'\right)\\ \vdots\\ C\left(k_M;\Gamma\right)+\beta\sum_{\Gamma'\,|\,\Gamma}\pi\left(\Gamma'\right)q^0\left(k';\Gamma'\right)\end{bmatrix}.$$

    The vector Z is indexed by '0' to reinforce the fact that its value depends on the current guess of the coefficient vector, $\vec{\omega}^0$. Since the approximation function $q\left(k,\Gamma\right)=X\vec{\omega}$, the functional equation characterizing $q\left(\cdot\right)$ can be written as:

$$R\left(k,\Gamma;\vec{\omega}\right)=X\vec{\omega}-Z=0.$$

Consequently, the new iterate of the coefficient vector, $\vec{\omega}^1$, is obtained as:

$$\vec{\omega}^1 = \left(X'X\right)^{-1}\left(X'Z^0\right).$$

With this new guess, repeat steps (2) through (9) until the coefficient vector converges to $\vec{\omega}^*$, which produces the solution $q^*(k,\Gamma)$.

11. Finally, choose a candidate value of λ and solve for the date 0 values. Using these values and the solution to the recursive problem for dates $t \geq 1$, evaluate the implementability constraint, (3):

$$C(g_0,\lambda) + \beta\left[\pi\left(\underline{g}|g_0\right)q^*\left(\lambda,\left(\underline{g}|g_0\right)\right) + \pi\left(\bar{g}|g_0\right)q^*\left(\lambda,\left(\bar{g}|g_0\right)\right)\right] = U_1(g_0,\lambda)\,a_0.$$

Adjust the value of λ until this expression holds with equality.

For the quantitative results reported in section 6, I use $M = 33$ and $N = 28$. These values produce highly accurate numerical results. To see this, note that real bond holdings can be derived as:

$$b(k,\Gamma) = \frac{q^*(k,\Gamma)}{U_1(k,\Gamma)} - c_1(k,\Gamma);$$

as a result, the values of the residual function, $R(k,\Gamma;\vec{\omega}^*)$, are easily expressed as approximation errors in real bond holdings. For the relevant range of k values, the maximum absolute percentage error in real bond holdings for the 5% sticky price model is 0.004% for $\Gamma = \left(\underline{g}|\underline{g}\right)$, 0.072% for $\Gamma = \left(\bar{g}|\underline{g}\right)$, 0.492% for $\Gamma = \left(\underline{g}|\bar{g}\right)$, and 0.025% for $\Gamma = (\bar{g}|\bar{g})$.

C. DERIVATIONS FROM SECTION 7

C.1 Rewriting the Sticky Price Constraint

Here I derive in more detail the version of the sticky price constraint displayed in section 7. For convenience, I reproduce condition (5) here in a slightly modified form:

$$\tilde{A}\left(\bar{s}^t\right)\sum_{r=t}^{\infty}\sum_{s^r|\bar{s}^t}\beta^{r-t}\pi\left(s^r|\bar{s}^t\right)\frac{C(s^r)}{U_1\left(\bar{s}^t\right)} = \tilde{A}\left(\underline{s}^t\right)\sum_{r=t}^{\infty}\sum_{s^r|\underline{s}^t}\beta^{r-t}\pi\left(s^r|\underline{s}^t\right)\frac{C(s^r)}{U_1\left(\underline{s}^t\right)},$$

where

$$C\left(s^{r}\right)=U_{1}\left(s^{r}\right)c_{1}\left(s^{r}\right)+U_{2}\left(s^{r}\right)c_{2}\left(s^{r}\right)+U_{l}\left(s^{r}\right)\Lambda\left(s^{r}\right),$$

$$\tilde{A}\left(s^{t}\right)=\left[v\left(\frac{L_{f}\left(s^{t}\right)}{L_{s}\left(s^{t}\right)}\right)^{\frac{\alpha}{\mu}}+1-v\right]^{1-\mu},$$

for $\bar{s}^{t}$ and $\underline{s}^{t}$ following s^{t-1}.

Note, however, that the term:

$$\sum_{r=t}^{\infty}\sum_{s^{r}|s^{t}}\beta^{r-t}\pi\left(s^{r}|s^{t}\right)\frac{C\left(s^{r}\right)}{U_{1}\left(s^{t}\right)} \tag{7}$$

represents the present value of real government surpluses from s^{t} onward. To see this, consider the expression:

$$\frac{U_{1}\left(s^{t}\right)}{P\left(s^{t}\right)}\left[M\left(s^{t}\right)+B\left(s^{t}\right)\right]. \tag{8}$$

Manipulating this as in the proof to Proposition 2 and dividing by $U_{1}\left(s^{t}\right)$ produces (7). However, the term (7) can be written in an alternative form. Again, start with (8); use the government's date r flow budget constraint, multiply it by $\beta^{r}\pi\left(s^{r}\mid s^{t}\right)\frac{U_{1}(s^{r})}{P(s^{r})}$, and sum over states $\left(s^{r}\mid s^{t}\right)$ and dates $r\geq t+1$ to obtain:

$$\sum_{r=t}^{\infty}\sum_{s^{r}|s^{t}}\beta^{r-t}\pi\left(s^{r}|s^{t}\right)U_{1}\left(s^{r}\right)\left[\frac{\tau\left(s^{r}\right)Y\left(s^{r}\right)-g\left(s^{r}\right)}{R\left(s^{r}\right)}+\frac{M\left(s^{r}\right)}{P\left(s^{r}\right)}\left(\frac{R\left(s^{r}\right)-1}{R\left(s^{r}\right)}\right)\right].$$

Dividing this by $U_{1}\left(s^{t}\right)$ obtains the expression for $\mathcal{PV}\left(s^{t}\right)$ presented in section 7. Hence, (7) is equivalent to $\mathcal{PV}\left(s^{t}\right)$, and the sticky price constraint can be rewritten as:

$$\tilde{A}\left(\bar{s}^{t}\right)\mathcal{PV}\left(\bar{s}^{t}\right)=\tilde{A}\left(\underline{s}^{t}\right)\mathcal{PV}\left(\underline{s}^{t}\right),$$

for $\bar{s}^{t}$ and $\underline{s}^{t}$ following s^{t-1}.

C.2 The Cross State Restriction with Incomplete Markets

Here I describe a real economy with non-state contingent debt, and derive the cross-state restriction that this imposes on the Ramsey problem. This restriction is simply a rewriting of the measurability constraint analyzed in Marcet et al. (2000) for the simple case when government spending takes on two possible values, $\{\underline{g}, \bar{g}\}$. This presentation follows closely that of Chari et al. (1991), Chari and Kehoe (1998), and Marcet et al. (2000).

The representative household maximizes utility derived from consumption, $c\left(s^t\right)$, and leisure, $1-l\left(s^t\right)$. In each period, the household is subject to the following budget constraint:

$$c\left(s^t\right)+\mathsf{b}\left(s^t\right)=\left(1-\tau\left(s^t\right)\right)l\left(s^t\right)+\mathsf{R}\left(s^{t-1}\right)\mathsf{b}\left(s^{t-1}\right),$$

where $\tau\left(s^t\right)$ is the labor income tax rate, and $\mathsf{b}\left(s^t\right)$ are holdings of one-period real government bonds. These mature at the beginning of period $t+1$, and earn a non-state-contingent real return of $\mathsf{R}\left(s^t\right)$. Production is constant returns to labor, generating the following aggregate resource constraint:

$$c\left(s^t\right)+g\left(s^t\right)=l\left(s^t\right).$$

The government sets the tax rate and issues real bonds in order to satisfy its budget constraint:

$$\mathsf{b}\left(s^t\right)+\tau\left(s^t\right)l\left(s^t\right)=\mathsf{R}\left(s^{t-1}\right)\mathsf{b}\left(s^{t-1}\right)+g\left(s^t\right).$$

All constraints given above must hold $\forall s^t$.

To derive the cross-state restriction, take the government's state s^t budget constraint and multiply by the marginal utility of consumption, $U_c\left(s^t\right)$, to get:

$$U_c\left(s^t\right)\left[\tau\left(s^t\right)l\left(s^t\right)-g\left(s^t\right)+\mathsf{b}\left(s^t\right)\right]=U_c\left(s^t\right)\mathsf{R}\left(s^{t-1}\right)\mathsf{b}\left(s^{t-1}\right).$$

Add to this the government's date r budget constraint, multiplied by $\beta^{r-t}\pi\left(s^r|s^t\right)U_c\left(s^r\right)$, for all $\left(s^r \mid s^t\right)$ and $r\geq t+1$. Using the household's FONC:

$$U_c\left(s^t\right)=\beta\mathsf{R}\left(s^t\right)\sum_{s^{t+1}|s^t}\pi\left(s^{t+1}|s^t\right)U_c\left(s^{t+1}\right),$$

produces:

$$\sum_{r=t}^{\infty}\sum_{s^r|s^t}\beta^{r-t}\pi\left(s^r|s^t\right)U_c\left(s^r\right)\left[\tau\left(s^r\right)l\left(s^r\right)-g\left(s^r\right)\right]=U_c\left(s^t\right)\mathsf{R}\left(s^{t-1}\right)\mathsf{b}\left(s^{t-1}\right).$$

Clearly, the summation term in this expression is the present (utility) value of real government surpluses. Defining:

$$\mathcal{PV}\left(s^t\right) = \sum_{r=t}^{\infty} \sum_{s^r|s^t} \beta^{r-t} \pi\left(s^r|s^t\right) \frac{U_c\left(s^r\right)}{U_c\left(s^t\right)} \left[\tau\left(s^r\right) l\left(s^r\right) - g\left(s^r\right)\right],$$

the expression becomes $\mathcal{PV}\left(s^t\right) = \mathsf{R}\left(s^{t-1}\right) \mathsf{b}\left(s^{t-1}\right)$. Note that $\mathsf{R}\left(s^{t-1}\right) \mathsf{b}\left(s^{t-1}\right)$ is known at state s^{t-1} and must be the same across realizations of government spending at date t. Denote these states as $\bar{s}^t$ and $\underline{s}^t$. Hence, the cross-state restriction imposed by non-state-contingent real returns can be expressed as:

$$\mathcal{PV}\left(\bar{s}^t\right) = \mathcal{PV}\left(\underline{s}^t\right).$$

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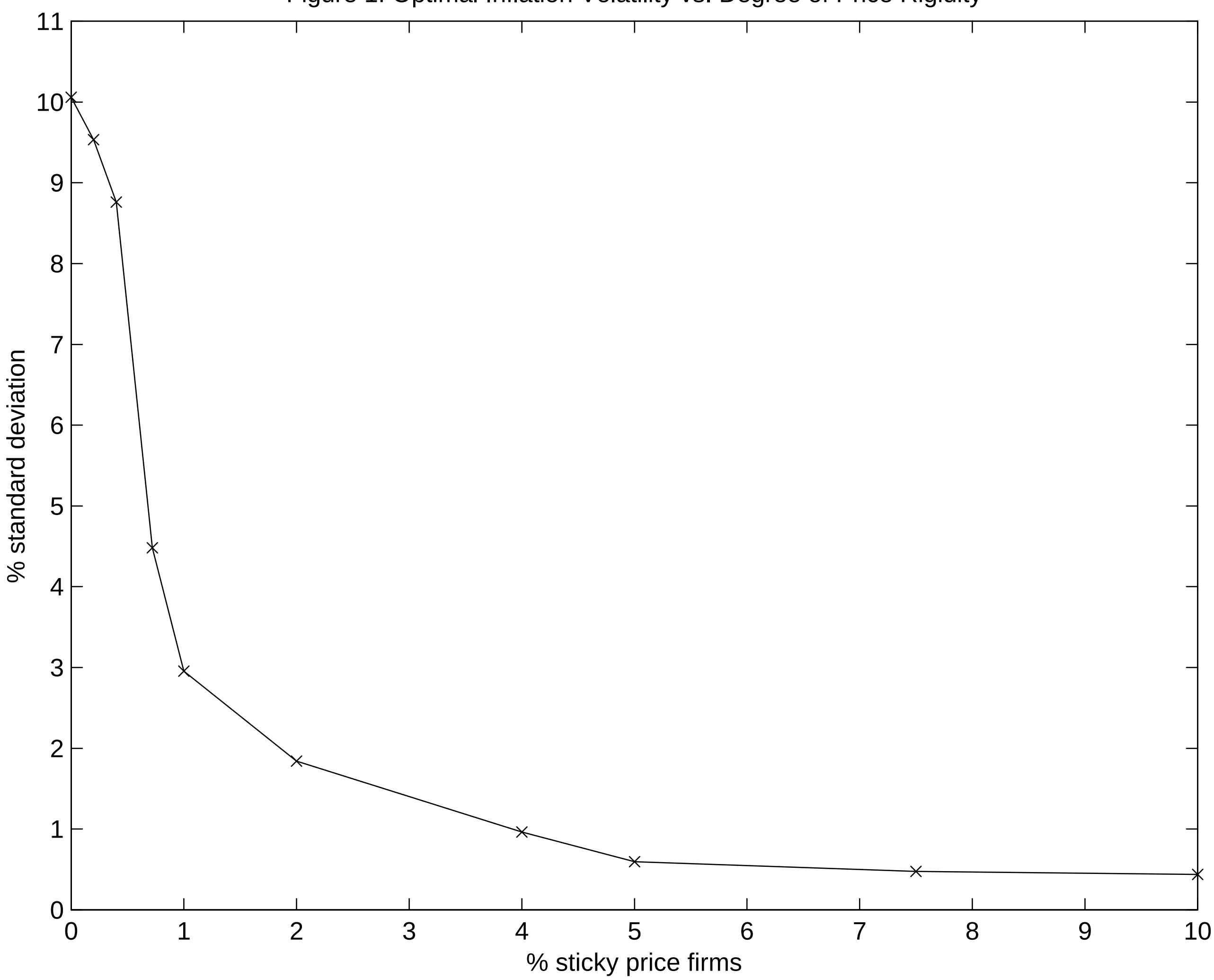
Figure 1: Optimal Inflation Volatility vs. Degree of Price Rigidity



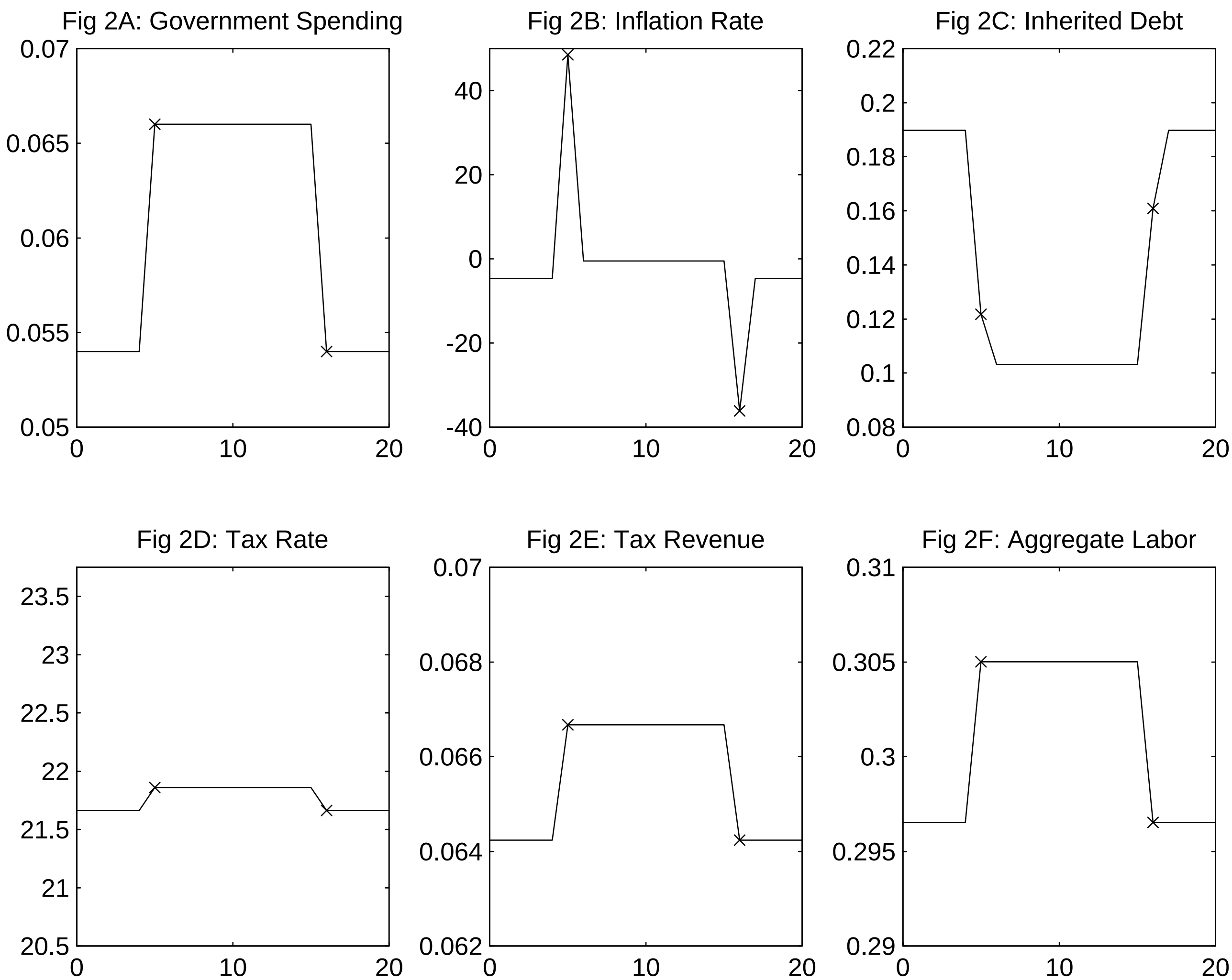


Fig 2A: Government Spending
0.07
0.065
0.06
0.055
0.05
0
10
20
Fig 2B: Inflation Rate
40
20
0
-20
-40
0
10
20
Fig 2C: Inherited Debt
0.22
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0
10
20
Fig 2D: Tax Rate
23.5
23
22.5
22
21.5
21
20.5
0
10
20
Fig 2E: Tax Revenue
0.07
0.068
0.066
0.064
0.062
0
10
20
Fig 2F: Aggregate Labor
0.31
0.305
0.3
0.295
0.29
0
10
20

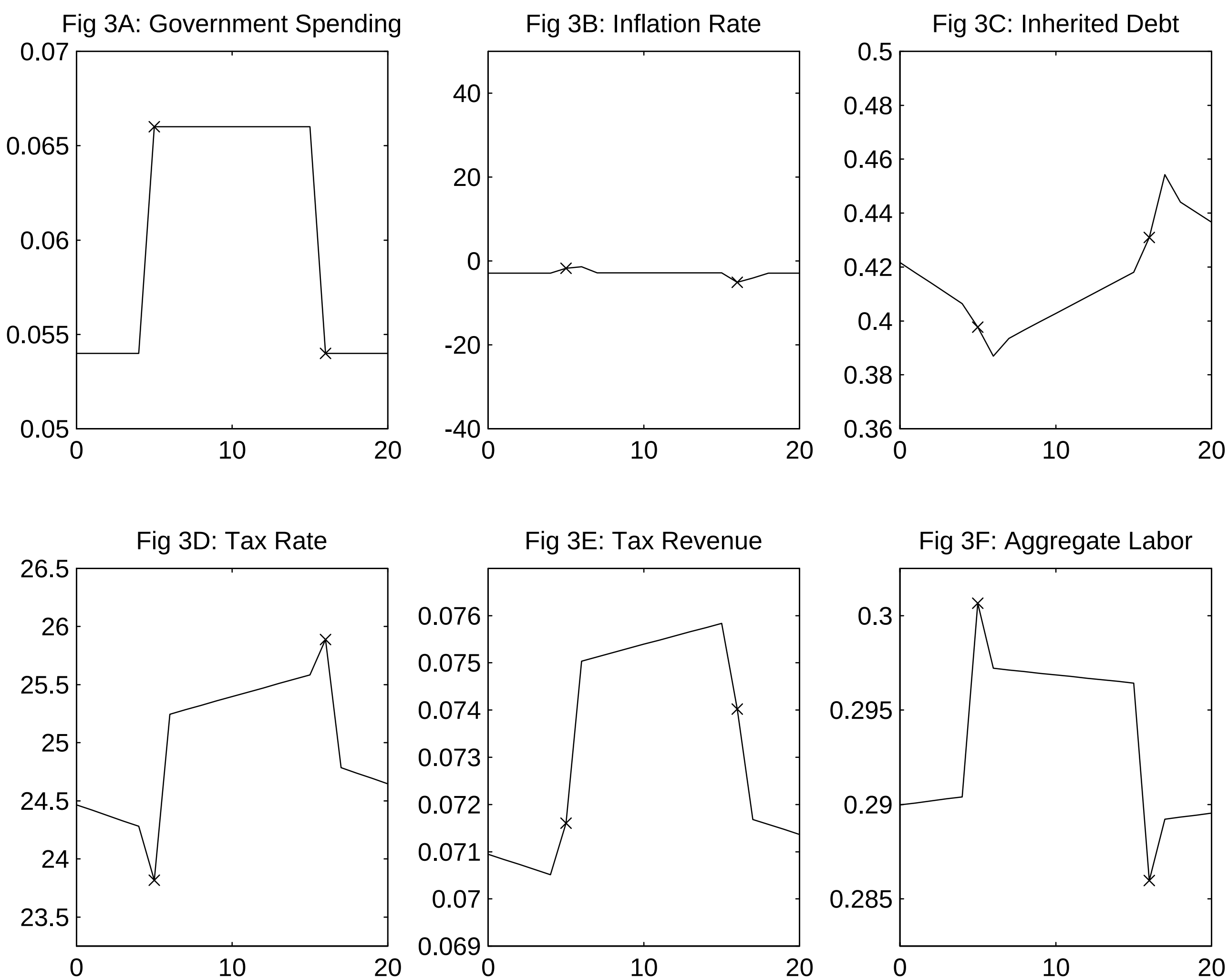


Fig 3A: Government Spending
0.07
0.065
0.06
0.055
0.05
0
10
20
Fig 3B: Inflation Rate
40
20
0
-20
-40
0
10
20
Fig 3C: Inherited Debt
0.5
0.48
0.46
0.44
0.42
0.4
0.38
0.36
0
10
20
Fig 3D: Tax Rate
26.5
26
25.5
25
24.5
24
23.5
0
10
20
Fig 3E: Tax Revenue
0.076
0.075
0.074
0.073
0.072
0.071
0.07
0.069
0
10
20
Fig 3F: Aggregate Labor
0.3
0.295
0.29
0.285
0
10
20