

Discussion of:  
'Networks and the Macroeconomy: An Empirical  
Exploration'  
by Daron Acemoglu, Ufuk Akcigit and William Kerr\*

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# 1. Introduction

A typical firm sells about one-half of its output to other firms and materials purchases from other firms account for roughly half of the firm's input costs. The fact that an individual firm's production occurs within a network of firms is typically ignored by macroeconomists.<sup>1</sup> Acemoglu et al show that the patterns of amplification and propagation of industry shocks predicted by standard network models are supported by the data. Moreover, they conjecture that incorporating networks into macroeconomic models has first order implications for macroeconomics. In this comment I describe three examples that support the authors' conjecture. Using a simple model with price setting frictions, I show that incorporating the network structure of production

- implies quantitatively large costs of inflation,
- cuts the slope of the Phillips curve in half,
- raises questions about the efficacy of the Taylor principle for stabilizing inflation.

According to conventional wisdom, the social cost of inflation is large.<sup>2</sup> Yet, standard economic models typically imply that the cost of inflation is modest at best.<sup>3</sup> In the words of Paul Krugman, 'one of the dirty little secrets of economic analysis is that even though inflation is universally regarded as a terrible scourge, efforts to measure its costs come up with embarrassingly small numbers.'<sup>4</sup> I show that mixing networks and price setting frictions can produce quantitatively large costs of inflation, consistent with the conventional view. The

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<sup>1</sup>For a recent exception, see Ascari, Phaneuf and Sims (2015). This work (done independently from the work described in this comment) makes observations about the cost of inflation that are similar to mine. Early work on the significance of networks can be found in Basu (1995) and Rotemberg and Woodford (1993), among others. As Rotemberg and Woodford point out, when there are no market frictions such as monopoly power or rigidities in pricing, then the standard assumption that output is a function only of capital and labor, and not materials, involves no loss of generality. Thus, the practice in the real business cycle literature of implicitly ignoring materials is defensible. Much of the New Keynesian literature - which stresses the importance of frictions - has imported the real business cycle approach to materials. However, as stressed by Basu and Rotemberg and Woodford, when there are frictions then the real business cycle approach can be highly misleading.

<sup>2</sup>A simple revealed-preference argument suggests that the American public thinks the economic costs of inflation are high. It is widely believed that the severe recession of the early 1980s was brought about by Federal Reserve Chairman Paul Volcker as a side-effect of his strategy for ending the high inflation of the 1970s. Despite the perceived high cost of Volcker's anti-inflation policy, his public reputation is very high. I infer that the public believes that the benefit of Volcker's strategy - ridding the economy of high inflation - is preferred to the cost - the punishing recession of the early 1980s. That is, the public assigns a high cost to inflation.

<sup>3</sup>By 'standard economic models', I also mean the standard practice of linearizing models about a steady state in which prices are not distorted. See Ascari (2004) for an early discussion of the dangers of this practice.

<sup>4</sup>See Krugman (1997).

basic idea is that the allocative and other distortions generated by price frictions are amplified by network effects in a way that resembles the familiar mechanism by which networks magnify the effects of firm-level technology shocks (see, e.g., Jones (2013)).

I work with a standard New Keynesian (NK) model with Calvo-style price frictions, modified to incorporate production networks in the manner suggested in Basu (1995).<sup>5</sup> The model has the property, well-known in the input-output literature, that network effects magnify a one percent technology shock at the level of the gross output of firms into a two percent shock to aggregate value-added. The distortions to the allocation of resources produced by price-setting frictions have effects on gross output that resemble those of a negative technology shock (see Yun (1996)). Given the results for technology shocks, it is then not surprising that price distortions large enough to cause a one percent loss to gross output are magnified into a two percent or greater loss to aggregate value-added, i.e., gross domestic product (GDP).

In the model that I work with, the severity of the allocative distortions in gross output is an increasing function of aggregate inflation.<sup>6</sup> By magnifying these allocative distortions, networks in effect magnify the cost of inflation. I show that these costs can be quite large even relative to the rather modest - by world standards - levels of inflation observed in the United States. For example, the model implies that the high inflation of the 1970s may have had a cost equivalent to a loss of 10 percent of GDP or more in each year that the inflation was high. Also, there have been proposals to raise the inflation target to perhaps 4 percent, at an annual rate, as a way to reduce the likelihood of the zero lower bound on the nominal interest becoming binding again.<sup>7</sup> According to the calculations below, the cost of such a level of inflation might be a loss of output as high as 1-2 percent per year. This would be a steep price to pay if, say, zero lower bound episodes are likely to only occur once every 50 to 75 years.

These cost of inflation numbers are big, perhaps too big. My key point is that introducing networks into macroeconomics - the part with price frictions - matters. The NK model, which is ordinarily thought of as a model that assigns little or no cost to anticipated inflation, actually predicts an abundance of such costs when networks are taken into account.

My calculations involve several assumptions, all presented in detail below. Apart from the assumption that production occurs in networks, the assumptions I make are standard. Still, greater scrutiny no doubt will imply that some of these assumptions deserve adjustment. But,

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<sup>5</sup>At the level of detail, the model corresponds to the one in Christiano, Trabandt and Walentin (2011). For a model that uses a similar production structure, but a different specification of price adjustment frictions, see Nakamura and Steinsson (2010). For other work that emphasizes the importance of materials in production, see Huang and Liu (2001,2005).

<sup>6</sup>For an extensive discussion of the implications of the allocative distortions resulting from inflation - in a setting that ignores networks - see Ascari and Sbordone (2014).

<sup>7</sup>For a review, see Ball (2013).

it is quite possible that adjustments would actually imply an even higher cost of inflation. For example, the modeling shortcut that I take to capture networks leaves out much of the richness and detail that Acemoglu et al describe. A consequence of the simplification is that the number of prices in my framework is substantially less than what would appear in a more empirically realistic analysis. With additional prices there could come additional possibilities for distortions and more reasons for inflation to be costly when there are price setting frictions.<sup>8</sup>

In my analysis, I assume that the frequency of price adjustment is constant. One might suppose that such an assumption implies an overstatement of the cost of a 4 percent inflation, or of the higher inflation in the 1970s. The idea is that firms would increase the frequency of price adjustment with higher inflation. The empirical results in Golosov and Lucas (2007, Figure 3) suggest that this is in fact not the case for levels of inflation observed in the United States. They present evidence that the average frequency of price adjustment changes very little for inflation rates in the range of zero to 16 percent per year.<sup>9</sup>

Another reason that networks are important has been recognized for a long time, but has not played a significant role in the New Keynesian literature. In particular, networks imply a strategic complementarity among price setters.<sup>10</sup> Consider an individual firm in an environment where the prices of all other firms are inertial (i.e., they respond weakly to shocks). This implies that the firm's price of materials is also inertial, contributing to inertia in its own marginal costs. With inertial marginal costs, the firm has an incentive to be inertial in the setting of its own price.<sup>11</sup> I derive the Phillips curve for this economy and show that the slope of the Phillips curve in terms of marginal cost is cut in half in the presence of networks, as the intuition suggests.

There is a third reason that networks in combination with price-setting frictions may be important. There is a widespread consensus that inflation targeting has valuable macroeconomic stabilizing powers and that the Taylor principle - an interest rate rule with a large coefficient on inflation - represents a good strategy for implementing inflation targeting. Under this principle, the monetary authority raises the interest rate by more than 1 percent when inflation rises by 1 percent. Through a *demand channel*, such an increase in the interest rate induces a fall in GDP and thereby brings inflation back down to target. Suppose

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<sup>8</sup>Consistent with these observations, Ascari, Phaneuf and Sims (2015) find bigger costs of inflation than I do. They work with a standard medium-sized NK model, one that includes, among other things, frictions in the setting of wages.

<sup>9</sup>See also Alvarez, Gonzalez-Rozada, Neumeyer and Beraja, (2011, Figure 10). They report empirical evidence which suggests that the frequency of price adjustment is insensitive to inflation rates in the range of 0 to 10 percent, at an annual rate.

<sup>10</sup>Recent papers that have recognized this fact include Basu (1995), Christiano, Trabandt and Walentin (2011), Nakamura and Steinsson (2010), and Huang and Liu (2001, 2005).

<sup>11</sup>This strategic complementarity argument goes back at least to Blanchard (1987) and Gordon (1981).

now that the funds paid by firms for their variable inputs need to be acquired in advance. These funds will be assigned an interest cost, either because the funds have to be raised in credit markets or because the funds are obtained by suppressing dividends which have an opportunity cost. That there is an interest component of the cost of variable inputs opens up a second, *working capital channel*. By raising marginal costs, higher interest rates help to push inflation up. In principle, the working capital channel could be stronger than the demand channel, with the possibility that the Taylor principle becomes what might better be called the Taylor curse. A sharp rise in the interest rate, rather than being an antidote to inflation, could actually be a trigger for more inflation.

In standard macroeconomic models with price setting frictions, but no network effects, the working capital channel is not strong enough to overwhelm the demand channel by enough to destabilize the economy.<sup>12</sup> So, standard models provide support to the Taylor principle. However, I show below that when network effects are taken into account, the possibility that the working capital channel is stronger than the demand channel is much greater. In that case, an interest rate rule with a big coefficient on inflation may not work to stabilize inflation and the broader economy.<sup>13</sup> This discussion is relatively brief because it summarizes the findings reported in Christiano, Trabandt and Walentin (2011).

These examples are why I think Acemoglu et al are right in their conjecture that network effects have first order implications for macroeconomics.

The following section provides a rough sketch of the model used in both parts of my analysis. Details about the model and its solution are provided in the appendix. The subsequent two sections focus on the first and second reasons that network effects may be important for macroeconomics, respectively.

## 2. A Business Cycle Model with Networks and Price Frictions

I adopt the usual Dixit-Stiglitz framework, in which there is a homogeneous good,  $Y_t$ , that is produced by a representative competitive firm using the following production function:

$$Y_t = \left[ \int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad \varepsilon > 1.$$

The representative firm takes output and input prices,  $P_t$  and  $P_{i,t}$ ,  $i \in (0, 1)$ , as given and chooses  $Y_{i,t}$ ,  $i \in (0, 1)$  to maximize profits. The  $i^{th}$  input,  $Y_{i,t}$ , is produced by a monopolist

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<sup>12</sup>Christiano, Eichenbaum and Evans (2005) show that the working capital channel is large enough that a monetary policy-induced rise in the interest rate drives inflation up. But, this effect is only transitory and not enough to wipe out the basic stabilizing effects of the Taylor principle.

<sup>13</sup>For another example in which the Taylor principle does not stabilize inflation or the broader economy, see Christiano, Ilut, Motto and Rostagno (2010).

using labor,  $N_{i,t}$ , and materials,  $I_{i,t}$ , using the following technology:

$$Y_{i,t} = A_t N_{i,t}^\gamma I_{i,t}^{1-\gamma},$$

$0 < \gamma \leq 1$ . The monopolist sets its price,  $P_{i,t}$ , subject to Calvo price-setting frictions. The monopolist can set  $P_{i,t}$  optimally with  $1 - \theta$  and with probability  $\theta$  the  $i^{th}$  monopolist must set  $P_{i,t} = P_{i,t-1}$  without price indexation. The no-indexation assumption is suggested by the same microeconomic observations that motivate price setting frictions in the first place. Those observations show that many prices remain unchanged for extended periods of time (see Eichenbaum, Jaimovich, and Rebelo 2011 and Klenow and Malin 2011). The no-indexation assumption has the implication that the degree of distortion in relative prices is increasing in steady state inflation.

The  $i^{th}$  monopolist is competitive in the market for materials and labor. It acquires materials by purchasing the homogeneous good and converting it one-for-one into  $I_{i,t}$ . Through an analogous mechanism, the  $i^{th}$  monopolist sells some of its output to other firms as materials for their use in production. In effect, the  $i^{th}$  firm is embedded in a network, in which some of its output is sold to other firms for their use as materials and some of its inputs are materials acquired from other firms.

The effective price of labor and materials for intermediate good firms is denoted by  $\bar{W}_t$  and  $\bar{P}_t$ , respectively, where

$$\bar{W}_t = (1 - \nu) [1 - \psi + \psi R_t] W_t, \quad \bar{P}_t = (1 - \nu) [1 - \psi + \psi R_t] P_t.$$

Here,  $W_t$  denotes the competitively determined price of labor;  $\psi$  is the fraction of input costs that must be paid in advance;  $R_t$  is the gross nominal rate of interest; and  $\nu$  is a lump sum tax-financed subsidy to costs.

Regarding  $\nu$ , let

$$L_t = \frac{\text{marginal utility cost of labor}_t}{\text{marginal product of labor}_t} = \frac{C_t N_t^\varphi}{F_{N,t}}. \quad (2.1)$$

The functional form for the marginal utility cost of labor reflects that I work with a standard representation of representative household utility. The denominator will be explained below. In NK literature,  $\nu$  is typically set to ensure that, in the steady state, the cost and benefit of labor are equated,  $L = 1$ . To achieve this,  $\nu$  must be set to  $\nu^*$ , where

$$1 - \nu^* = \frac{\varepsilon - 1}{(1 - \psi + \psi R) \varepsilon} \frac{1 - \beta \theta \bar{\pi}^\varepsilon}{1 - \theta \bar{\pi}^\varepsilon} \frac{1 - \theta \bar{\pi}^{(\varepsilon-1)}}{1 - \beta \theta \bar{\pi}^{(\varepsilon-1)}}, \quad R = \frac{\bar{\pi}}{\beta}. \quad (2.2)$$

Here,  $R$  and  $\bar{\pi}$  denote the steady state gross nominal rate of interest and gross inflation, respectively. The subsidy is increasing in  $\psi$ , to minimize working capital distortions; it

is decreasing in  $\varepsilon$  to mitigate the effects of monopoly distortions. Inflation impacts on  $\nu^*$  through conflicting channels. The marginal firm that adjusts its price sets it high if inflation is high, and other things the same this implies a bigger subsidy to reduce the resulting markup. At the same time, firm that do not adjust their price in effect drag the aggregate price level down when inflation is high and this suggests a reduction in the subsidy. Not surprisingly, the effects of  $\bar{\pi}$  on  $\nu^*$  turn out to be quantitatively small.

In my analysis, I set  $\nu = \nu^*$ . This allows me to focus on the specific impact of  $\bar{\pi}$ . The presence of monopoly power does in fact amplify the distortionary effects of inflation when  $\nu = 0$ . But, I found that this effect is quantitatively small.<sup>14</sup>

In equilibrium, aggregate gross output,  $Y_t$ , is related to aggregate employment,  $N_t$ , and the aggregate use of materials,  $I_t$ , by the following gross output production function:

$$Y_t = p_t^* A_t N_t^\gamma I_t^{1-\gamma}, \quad (2.3)$$

where  $0 \leq p_t^* \leq 1$  is a function of degree of price dispersion. I refer to  $p_t^*$  as the Tack Yun distortion because the expression was derived in Yun (1996). The law of motion of the Tack Yun distortion is given by:

$$p_t^* = \left[ (1 - \theta) \left( \frac{1 - \theta \bar{\pi}_t^{(\varepsilon-1)}}{1 - \theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta \bar{\pi}_t^\varepsilon}{p_{t-1}^*} \right]^{-1}. \quad (2.4)$$

To understand this expression, note that when prices are flexible (i.e.,  $\theta = 0$ ), then there is no price dispersion and  $p_t^* = 1$ . The upper bound on  $p_t^*$  is attained when  $\theta = 0$  because all firms face the same marginal cost and demand elasticity, so that if they could all flexibly set their prices, then they would all set the same price. Producers of  $Y_t$  would in this case demand an equal amount,  $Y_{i,t}$ , from each of the  $i \in (0, 1)$  intermediate good producers. For a given level of aggregate resources,  $N_t$  and  $I_t$ , the greatest amount of  $Y_t$  is produced when labor and materials are distributed in equal amounts across sectors. This is why  $p_t^* = 1$  when  $\theta = 0$ . When there are price setting frictions,  $\theta > 0$ , then there is price dispersion and homogeneous output is lost because of the resulting unequal allocation of resources across  $i \in (0, 1)$ . Price dispersion is greater at higher rates of inflation and for this reason the loss of homogeneous output is greater then too.

Cost minimization by firms leads to the following relationship between aggregate mate-

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<sup>14</sup>Consistent with the observations in Rotemberg and Woodford (1993), networks also amplify the distortionary effects of monopoly power. This can be seen in the expression for the steady state value of  $F_{N,t}$  that appears in (A.31) in the Appendix. For example, when  $\gamma = 1$ , so that there are no networks, then monopoly power has no impact on  $F_N$ . But, when  $\gamma = 1/2$  then the degree of monopoly power potentially has a very large impact on  $F_N$ . The impact of monopoly power on  $F_N$  is set to zero with my assumption,  $\nu = \nu^*$ . This assumption does not distract from my central point, which has to do with the effect of inflation on  $F_N$ .

rials and gross output:

$$I_t = \frac{\mu_t}{p_t^*} Y_t, \quad (2.5)$$

where  $\mu_t$  is the cost share of materials at the individual firm level. When prices are flexible ( $\theta = 0$ ) and there is no working capital channel ( $\psi = 0$ ), then  $\mu_t = 1 - \gamma$ . In the version of the model with price frictions and a working capital channel,  $\mu_t$  fluctuates with changes in the markup of the aggregate price over marginal cost and with changes in the interest rate,  $R_t$ .

Using (2.5) to substitute out for  $I_t$  in (2.3), and solving the latter for  $Y_t$  we obtain:

$$Y_t = \left[ p_t^* A_t \left( \frac{\mu_t}{p_t^*} \right)^{1-\gamma} \right]^{\frac{1}{\gamma}} N_t. \quad (2.6)$$

Gross Domestic Product (*GDP*) in this economy is just consumption:

$$C_t = Y_t - I_t = \left( 1 - \frac{\mu_t}{p_t^*} \right) Y_t.$$

Making use of (2.6), we obtain the *value-added production function*:

$$C_t = F_{N,t} N_t,$$

where  $F_{N,t}$  is the marginal value-added produced by labor (i.e., the object in the denominator of (2.1).) In particular,

$$F_{N,t} = \chi_t \left[ A_t \gamma^\gamma (1 - \gamma)^{1-\gamma} \right]^{\frac{1}{\gamma}}, \quad (2.7)$$

where

$$\chi_t \equiv (p_t^* \omega_t)^{\frac{1}{\gamma}}, \quad \omega_t \equiv \left( \frac{1 - \frac{\mu_t}{p_t^*}}{\gamma} \right)^\gamma \left( \frac{\frac{\mu_t}{p_t^*}}{1 - \gamma} \right)^{1-\gamma}. \quad (2.8)$$

The object,  $F_{N,t}$ , is Total Factor Productivity (TFP) in this model.<sup>15</sup> In a version of the model with capital the expression for TFP would also be given by (2.7) and (2.8). Numerical experiments suggest that when  $\nu = \nu^*$ , then  $\omega_t \simeq 1$ . The reason is simple. In the steady state

$$\frac{\mu}{p^*} = 1 - \gamma,$$

in which case  $\omega_t$  attains its maximal value of unity. It follows that variations in  $\mu_t/p_t^*$  have no first-order impact on  $\omega_t$ . That is, to a first order approximation  $\omega_t = 1$ .

The percent loss in value added due to inflation, what I call the *allocative cost of inflation*, is:

$$100 (1 - \chi_t).$$

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<sup>15</sup>The expression in (2.8) is consistent with a central theme in Basu (1995), that networks in conjunction with price setting frictions and monopoly power in effect provide a theory of endogenous TFP.



This measure of loss takes the aggregate quantity of inputs,  $N_t$ , as given. A complete analysis of the cost of inflation would also consider its impact on  $N_t$ . An advantage of the more restricted analysis done here is that the allocative cost of inflation is a function of a small subset of model equations. A good approximation for the allocative cost of inflation is:

$$100(1 - \chi_t) \simeq 100 \left( 1 - (p_t^*)^{\frac{1}{\gamma}} \right) \simeq \frac{1}{\gamma} 100(1 - p_t^*). \quad (2.9)$$

The first approximate equality in (2.9) reflects previous discussion and the second is a first order Taylor series expansion about  $p_t^* = 1$ . The last expression in (2.9), and the fact,  $\gamma = 1/2$ , are the basis for the claim in the introduction that introducing networks doubles the model's implication for the cost of inflation.

The intuition for why networks double the cost of inflation is simple. First, we see from (2.7) that in the presence of networks, the exogenous disturbance in the value-added production function is a magnified version of the shock,  $A_t$ , in the gross output production function. This is the well-known ‘multiplier effect’ in the network literature (see, e.g., Jones (2013)). When  $A_t$  is treated as an unobserved variable, then this multiplier effect is of limited substantive interest. Given observations on  $C_t$  and  $N_t$ , all we observe is  $TFP$  and whether we think of the whole of  $TFP$  as a shock, or of  $TFP$  as a smaller shock that has been magnified is immaterial. Second, we can see that price dispersion,  $p_t^*$ , is also magnified in the value-added production function. This is potentially of substantial significance because, according to (2.4),  $p_t^*$  is a function of inflation, which is observable.

In the version of the model with flexible prices and no working capital channel, we have enough equilibrium conditions to determine all the equilibrium variables of interest, including  $C_t$ ,  $N_t$ ,  $I_t$ ,  $p_t^*$  and  $\chi_t$ . In the presence of the Calvo price setting frictions, we are short one equation for determining these variables. I fill this gap in the standard way, with a Taylor rule for setting the nominal rate of interest:

$$R_t/R = (R_{t-1}/R)^\rho \exp[(1 - \rho) 1.5(\bar{\pi}_t - \bar{\pi})], \quad (2.10)$$

where  $\bar{\pi}$  represents the monetary authority's inflation target. This is the value of  $\bar{\pi}_t$  in steady state.

### 3. Networks, Price Frictions and the Cost of Inflation

According to the NK model described in the previous section, inflation gives rise to a misallocation of resources, which results in a  $100(1 - \chi_t)$  percent loss in GDP. This section shows that the loss can be quantitatively large. Networks play an essential role in this result. I will focus in particular on three levels of inflation: the 8 per cent average annual rate during

the high inflation of the 1970s in the United States;<sup>16</sup> the level of 4 percent that has been proposed by some as a way to reduce the risk of hitting the zero lower bound on the nominal rate of interest; and the 2 percent level which corresponds roughly to the normal inflation rate in recent years before the crisis.

The first subsection examines the distortions using steady state calculations. The steady state has the advantage that it is characterized by relatively simple, transparent expressions. The second subsection reports distortions implied by simulating the dynamic formulas, (2.4) and (2.8) using actual US inflation data.

### 3.1. Cost of Steady State Inflation

According to (2.8), the Tack Yun distortion,  $p_t^*$ , is a key input into the cost of inflation,  $\chi_t$ . Its steady state value, according to (2.4), is:

$$p^* = \frac{1 - \theta \bar{\pi}^\varepsilon}{1 - \theta} \left( \frac{1 - \theta}{1 - \theta \bar{\pi}^{(\varepsilon-1)}} \right)^{\frac{\varepsilon}{\varepsilon-1}}. \quad (3.1)$$

Another input to  $\chi$  is the cost share of materials,  $\mu$ , scaled by  $p^*$ . In the Appendix, I show that

$$\frac{\mu}{p^*} = (1 - \gamma) \frac{1 - \nu^*}{1 - \nu}, \quad (3.2)$$

where  $\nu^*$  is defined in (2.2). Substituting the last expression into (2.8), taking into account,  $\nu = \nu^*$ , we obtain

$$\chi = (p^*)^{\frac{1}{\gamma}}. \quad (3.3)$$

In my analysis of (3.3), I consider only  $\theta = 3/4$ , the case in which prices are held unchanged for 1 year, on average. I consider three values of  $\varepsilon$ . I use the value of  $\varepsilon = 6$  used by Christiano, Eichenbaum and Evans (2005), which implies a steady state price markup of 20 percent for the intermediate good producers. I also allow for greater amounts of competition by considering price markups of 15 and 10 percent. The underlying economics suggests that whether  $\chi$  is increasing or decreasing in  $\varepsilon$  is ambiguous and depends on other parameters. An increase in  $\varepsilon$  drives  $\chi$  up because larger values increase the response of resource allocations to a given distortion in relative prices. At the same time, there is another effect of  $\varepsilon$  which goes the other way: an increase in  $\varepsilon$  raises the elasticity of substitution between goods, making the consequences of a given degree of misallocation less severe.

My quantitative findings are reported in Table 1. Panels a-c report (3.3) for the three inflation rates of interest. The first and second columns report results for when there are

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<sup>16</sup>Here and below, I use data on the consumer price index, CPIAUCSL, taken from the FRED database, maintained by the Federal Reserve Bank of St. Louis. The 8 percent annual average reported in the text is based on data for the period, 1972Q1-1983Q4.

no networks ( $\gamma = 1$ ) and when there are networks ( $\gamma = 1/2$ ), respectively.<sup>17</sup> The results are presented in sets of three numbers. Numbers in parentheses and in braces correspond to markups of 15 and 10 percent, respectively. The first number in each set of three corresponds to a markup of 20 percent.

Consider the results in Panel a first. Even in the absence of networks, the cost of a permanent inflation is high enough to warrant serious concern. In the 10 percent markup case, for example, 10 percent of GDP is lost in each period. Still, when networks are introduced then costs are doubled, making the cost of the kind of inflation observed in the 1970s truly alarming.

Consider Panel b, which examines the effects of a permanent 4 percent annual inflation rate. As discussed in the introduction, this level of inflation has been proposed as a device to reduce the probability of a binding zero lower bound event. When there are no networks, Table 1 suggests that the price of raising the inflation target is already significant. However, the presence of networks doubles the cost to a range of 1 to 2 percent of GDP lost. Costs in such a range may warrant placing greater emphasis on other ways to avoid zero lower bound problems.<sup>18</sup> Finally, panel c is consistent with the conventional view that the cost of inflation in the 2 percent range are small. Evidently, the table illustrates the general convexity property of the cost of distortions. When inflation doubles, the cost more than doubles.

| <b>Table 1: Percent of GDP Lost<sup>1</sup> Due to Inflation, <math>100(1 - \chi_t)</math></b>   |                                  |
|--|----------------------------------|
| Without networks ( $\gamma = 1$ )  | With networks ( $\gamma = 1/2$ ) |
| <b>a: Steady state inflation: 8 percent per year</b>   |                                  |
| 2.41 <sup>2</sup> (3.92) [10.85]   | 4.76 (7.68) [20.53]              |
| <b>b: Steady state inflation: 4 percent per year</b>   |                                  |
| 0.46 (0.64) [1.13]   | 0.91 (1.27) [2.25]               |
| <b>c: Steady state inflation: 2 percent per year</b>   |                                  |
| 0.10 (0.13) [0.21]   | 0.20 (0.27) [0.42]               |
| Notes: (1) table reports (3.3)   |                                  |
| (2) number not in parentheses assumes a markup of 20 percent; number in parentheses: 15 percent; number in square brackets: 10 percent |                                  |
|  |                                  |

### 3.2. Dynamic Distortions

Next, we turn to the time series evidence on  $100(1 - \chi_t)$  displayed in Figure 1. The objects that determine  $\chi_t$  are not all observable. In the case of the Tack Yun distortion,  $p_t^*$ , I require

<sup>17</sup>The numerical results in the first column of Table 1 are consistent with the findings reported in Ascari and Sbordone (2014), especially their Figure 7.

<sup>18</sup>See, for example, Rogoff (2014).

an initial condition (see (2.4)). However, I found that the initial condition has a negligible dynamic effect on  $p_t^*$ . Figure 1 reports the time series on  $p_t^*$  when the initial value of  $p_t^*$  (in 1947Q3) is set to unity. The results after 1960 are essentially unchanged if I instead set the initial condition to the much lower value of 0.80.

To compute  $\chi_t$  I also need time series data on  $\mu_t/p_t^*$  in order to construct  $\omega_t$  (see (2.8)). For the reasons explained in section 2, I adopt the approximation,  $\omega_t = 1$ .<sup>19</sup>

Figure 1 has two panels. Figures 1a and 1b display results for the case where the markup is 20 percent and 15 percent, respectively. Each panel displays three time series: quarterly observations on quarterly gross inflation in the consumer price index; quarterly observations on  $100(1 - p_t^*)$  and quarterly observations on  $100(1 - \chi_t) = 100(1 - (p_t^*)^{1/\gamma})$ .

Consider panel a first. There are several notable results. First, the percent of GDP lost when it is assumed there are networks is roughly twice as large when the network structure of production is ignored (I set  $\gamma = 1/2$ ). Second, the losses due to inflation can be quantitatively large. In normal, low inflation, times, the average cost is well below 1 percent but when inflation rises a bit, the costs rise sharply.

In panel b the costs are higher. On average, 2.65 percent of *GDP* is lost. Also, more than 4 percent of GDP is lost in 18 percent of the 273 quarters considered. And, in those quarters the average loss of GDP was 10 percent. Finally, the losses during the high inflation of the 1970s are quantitatively large. Indeed, they reach a maximum value of over 20 percent during the high inflation period (see Figure 1b). The average loss in the high inflation period was 9.73 percent of GDP per period, according to the results in Figure 1b. This exceeds the 7.68 percent loss reported in Table 1a because of the convexity of the loss as a function of inflation. These numbers certainly vindicate the conventional view that the cost of the high inflation in the 1970s was high.

Clearly, the NK model has no trouble rationalizing the view that high inflation imposes a big cost on society. If anything, one is suspicious that the cost suggested by the model is implausibly high.

## 4. Networks and the Slope of the Phillips Curve

The standard Phillips curve reported in the literature only holds when there are no price distortions in the steady state. When there are price distortions - as most of the discussion here assumes - then the Phillips curve is more complicated than and includes additional

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<sup>19</sup>See the first approximate equality in (2.9). The approximation depends crucially on  $\nu = \nu^*$ . In practice, this greatly exaggerates the role of the tax system in undoing monopoly distortions. (Indeed, actual the tax systems provide their own distortions.) However, some preliminary calculations suggest that allowing monopoly power to also reduce  $\chi_t$  by setting  $\nu = 0$  has only a small impact on the contribution of inflation to  $\chi_t$ .

variables. Still, to preserve comparability I display the usual Phillips curve for the special case when there is no steady state inflation. For the same reason, I also drop working capital, so that  $\psi = 0$ . The appendix shows that the Phillips curve for this model is:

$$\pi_t = \frac{(1 - \theta)(1 - \beta\theta)}{\theta} \gamma (1 + \varphi) x_t + \beta E_t \pi_{t+1}, \quad (4.1)$$

where  $\pi_t \equiv \bar{\pi}_t - 1$  denotes the net inflation rate. Also,  $x_t$  represents the output gap, the log of the ratio of GDP (i.e., consumption) to potential GDP. As explained in the Appendix, potential GDP can be interpreted either as the level of output that would occur if prices were flexible, or the level of output in the Ramsey equilibrium. The key thing to note in (4.1) is that the slope of the Phillips curve in terms of the output gap is cut in half by the presence of networks.<sup>20</sup> The underlying intuition, based on strategic complementarities, is explained in the introduction.

## 5. Networks and the Taylor Principle

There is a consensus that inflation targeting is a monetary policy with excellent operating characteristics, at least in ‘normal’ times when the zero lower bound on the interest rate is not binding. Inflation targeting can be operationalized by applying a rule like (2.10) with a coefficient on inflation that is substantially greater than unity, i.e., that satisfies the ‘Taylor principle’. The idea is that the Taylor principle serves two important objectives. One is the achievement of low average inflation. By raising the interest rate when inflation is above target, the central bank reigns in the demand for goods and services. Working through the demand channel, this policy generates a slowdown in economic activity, which brings inflation back down to target by reducing marginal costs.

The second objective of the Taylor principle is to anchor inflation expectations. Unanchored inflation expectations can be a source of instability in inflation as well as in aggregate output and employment. To see this, suppose that for some reason there is a jump in inflation expectations. The resulting lower real rate of interest stimulates spending and output. By producing a rise in marginal cost, the increase in output contributes to a rise in inflation. In this way, the initial jump in inflation expectations is self-fulfilling and so inflation is without an anchor.

The Taylor principle helps to short-circuit this loop from higher inflation expectations to higher actual inflation. This is also accomplished by working through the demand channel. When the monetary authority raises the interest rate vigorously in response to inflation, the demand channel produces a fall in spending which reduces output and hence inflation. As people become aware of the lower actual inflation, the inflation expectations that initiated

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<sup>20</sup>Here, and throughout, I assume that the empirically plausible value of  $\gamma$  is 1/2.

the loop would evaporate before it could have much of an effect on the macroeconomy. Under rational expectations, the initial rise in inflation expectations would not occur in the first place.

The stabilizing effects of the Taylor principle depend on the demand channel being the primary avenue through which monetary policy operates. When firms have to borrow to pay for their variable inputs, then the interest rate is a part of marginal cost and monetary policy also operates through a working capital channel. If the working capital channel is sufficiently important then, instead of curbing inflation, a jump in the nominal rate of interest could actually ignite inflation.

Whether the working capital or demand channel dominate has been studied extensively in the type of model described in the previous section.<sup>21</sup> The general finding is that when  $\gamma = 1$  the demand channel dominates the working capital channel and the Taylor principle achieves the two objectives described above. This is so, even when the working capital channel is strongest, with  $\psi = 1$ . When gross output and value added coincide, there is not enough borrowing for the working capital channel to overwhelm the demand channel. However, when we take into account the network nature of production (i.e.,  $\gamma = 2$ ), then the amount of borrowing for working capital purposes is potentially much greater. As a result, when  $\psi = 1$  and there is no interest rate smoothing in the Taylor rule, i.e.,  $\rho = 0$ , the non-stochastic steady state equilibrium is indeterminate. Even though monetary policy satisfies the Taylor principle, there are many equilibria. These equilibria can be characterized in terms of the loop from higher expected inflation to higher actual inflation discussed above. When there is no working capital channel, then the Taylor principle short-circuits this loop by raising the interest rate and preventing the rise in expected inflation from occurring. But, when the working capital channel is sufficiently strong, then the rise in the interest rate simply reinforces the loop from higher expected inflation to higher actual inflation. In this way the Taylor principle could become the ‘Taylor curse’ referred to in the introduction.

It is interesting that the Taylor principle works as hoped for when there is substantial interest rate smoothing in monetary policy, i.e.,  $\rho$  is large. Presumably, the intuition for this is that demand responds most strongly to long term interest rates rather than to short term interest rates. As a result, the strength of the demand channel is increasing in  $\rho$  while the working capital channel, which is only a function of the short term interest rate, remains unaffected.

The example highlights how the integration of network effects could be important for the design of monetary policy. This is a topic that deserves further study. It is important to know, for example, how pervasive working capital is in the data.<sup>22</sup> It is also important to

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<sup>21</sup>See Christiano, Trabandt and Walentin (2011).

<sup>22</sup>For one relevant study, see Barth and Ramey (2002).

understand better how the working capital channel works in network environments that are closer to the more realistic one advocated in the Acemoglu et al paper, in which firms buy materials directly from other firms.

## 6. Conclusion

Acemoglu et al suggest that the introduction of networks into macroeconomic models could have first-order consequences for the kinds of questions that interest macroeconomists. I have described three examples in which this is the case. The examples convince me that following the lead of Acemoglu et al and others by integrating networks into macroeconomics represents an important priority.

### A. Appendix: Model Used in the Analysis

This comment makes use of a standard NK model, extended to include network effects following the suggestion of Basu (1995). At the level of detail, the model corresponds to the one analyzed in Christiano, Trabandt and Walentin (2011) (CTW). I include a description of the model in this appendix for three reasons. First, CTW do not describe all the connections between gross output and value-added that I use in my discussion. Second, I explain the properties of the first-best allocations discussed in the text. Third, the text loosely describes some subtleties associated with the Phillips curve in the present context, and the appendix explains these carefully. Fourth, conclusions from stochastic simulations of the model are summarized in section 3 and the discussions in sections 4 and 5 are also based on analyses that make use of all parts of the model.

#### A.1. Households

There are many identical, competitive households who maximize utility,

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left( u(C_t) - \frac{N_t^{1+\varphi}}{1+\varphi} \right), \quad u(C_t) \equiv \log C_t,$$

subject to the following budget constraint:

$$\text{s.t. } P_t C_t + B_{t+1} \leq W_t N_t + R_{t-1} B_t + \text{Profits net of taxes}_t.$$

Here,  $C_t$  denotes consumption;  $W_t$  denotes the nominal wage rate;  $N_t$  denotes employment;  $P_t$  denotes the nominal price of consumption;  $B_{t+1}$  denotes a nominal one period bond purchased in period  $t$  which pays off a gross, nominal non-state contingent return,  $R_t$ , in

period  $t + 1$ . Also, the household earns lump sum profits and pays lump sum taxes to the government. Optimization by households implies:

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}} \quad (\text{A.1})$$

$$C_t N_t^\varphi = \frac{W_t}{P_t}. \quad (\text{A.2})$$

## A.2. Goods Production

The structure of production has the Dixit-Stiglitz structure that is standard in the NK literature, extended to consider network effects.

### A.2.1. Homogeneous Goods

A representative, homogenous good firm produces output,  $Y_t$ , using the following technology:

$$Y_t = \left[ \int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad \varepsilon > 1. \quad (\text{A.3})$$

The firm takes the price of homogeneous goods,  $P_t$ , and the prices of intermediate goods,  $P_{i,t}$ , as given and maximizes profits,

$$P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} dj,$$

subject to (A.3). Optimization leads to the following first order condition:

$$Y_{i,t} = Y_t \left( \frac{P_t}{P_{i,t}} \right)^\varepsilon, \quad (\text{A.4})$$

for all  $i \in (0, 1)$ . Combining the first order condition with the production function, we obtain the following equilibrium condition:

$$P_t = \left( \int_0^1 P_{i,t}^{(1-\varepsilon)} di \right)^{\frac{1}{1-\varepsilon}}. \quad (\text{A.5})$$

### A.2.2. Intermediate Goods

The intermediate good,  $i \in (0, 1)$ , is produced by a monopolist using the following technology:

$$Y_{i,t} = A_t N_{i,t}^\gamma I_{i,t}^{1-\gamma}, \quad 0 < \gamma \leq 1.$$

Here,  $N_{i,t}$  and  $I_{i,t}$  denotes the quantity of labor and materials, respectively, used by the  $i^{\text{th}}$  producer. The producer obtains  $I_{i,t}$  by purchasing the homogenous good,  $Y_t$ , and converting it one-for-one into materials. Both  $N_{i,t}$  and  $I_{i,t}$  are acquired in competitive markets.



Firms experience Calvo-style frictions in setting their price. That is, the  $i^{\text{th}}$  firm sets its period  $t$  price,  $P_{i,t}$ , as follows:

$$P_{i,t} = \begin{cases} \tilde{P}_t & \text{with probability } 1 - \theta \\ P_{i,t-1} & \text{with probability } \theta \end{cases}, \quad 0 \leq \theta < 1. \quad (\text{A.6})$$

Here,  $\tilde{P}_t$  denotes the price selected in the probability  $1 - \theta$  event that it can choose its price. Firms that cannot optimize their price must simply set it to whatever value it took on in the previous period.

Given its current price (however arrived at), the firm must satisfy the  $Y_{it}$  that is implied by (A.4). Linear homogeneity of its technology and our assumption that the  $i^{\text{th}}$  firm acquires materials and labor in competitive markets implies that marginal cost is independent of  $Y_{i,t}$ . By studying its cost minimization problem we find that  $s_t$ , the  $i^{\text{th}}$  firm's marginal cost (scaled by  $P_t$ ) is

$$s_t = \left( \frac{\bar{P}_t/P_t}{1 - \gamma} \right)^{1-\gamma} \left( \frac{\bar{W}_t/P_t}{\gamma} \right)^\gamma \frac{1}{A_t}. \quad (\text{A.7})$$

Here,  $\bar{P}_t$  and  $\bar{W}_t$  denote the net price, after taxes and interest rate costs, of materials and labor, respectively. In particular,

$$\begin{aligned} \bar{W}_t &= (1 - \nu)(1 - \psi + \psi R_t) W_t \\ \bar{P}_t &= (1 - \nu)(1 - \psi + \psi R_t) P_t, \end{aligned}$$

where  $\psi$  represents the fraction of input costs that must be financed in advance so that, for example, one unit of labor used during period  $t$  costs  $\psi W_t R_t$  units of currency at the end of the period.<sup>23</sup> Also,  $\nu$  is the subsidy discussed in the text.

Another implication of the  $i^{\text{th}}$  firm's cost minimization problem is that cost of materials,  $\bar{P}_t I_{i,t}$ , as a fraction of total cost, is equal to  $1 - \gamma$ . This implies,

$$I_{i,t} = \mu_t Y_{i,t}, \quad (\text{A.8})$$

where  $\mu_t$  is the share of materials in gross output, and:

$$\mu_t = \frac{(1 - \gamma) s_t}{(1 - \nu)(1 - \psi + \psi R_t)}. \quad (\text{A.9})$$

I now turn to the problem of one of the  $1 - \theta$  randomly selected firms that has an opportunity to select its price,  $\tilde{P}_t$ , in period  $t$ . Such a firm is concerned about the value of its cash flow (i.e., revenues net of costs) in period  $t$  and in later periods:

$$E_t \sum_{j=0}^{\infty} (\beta\theta)^j v_{t+j} \left[ \tilde{P}_t Y_{i,t+j} - P_{t+j} s_{t+j} Y_{i,t+j} \right] + \Phi_t. \quad (\text{A.10})$$

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<sup>23</sup>We assume that banks create credits which they provide in the amount,  $(\psi W_t N_t + \psi P_t I_t)$ , to firms at the beginning of the period. At the end of the period they receive  $R_t (\psi W_t N_t + \psi P_t I_t)$  back from firms and the profits,  $(R_t - 1)(\psi W_t N_t + \psi P_t I_t)$ , are transferred to households in lump sum form.

The objects in square brackets are the cash flows in current and future states in which the firm does not have an opportunity to reset its price. The expectation operator in (A.10) integrates over aggregate uncertainty, while the firm-level idiosyncratic uncertainty is manifest in the presence of  $\theta$  in the discounting. In (A.10), cash flows in each period are weighed by the associated date and state-contingent value that the household assigns to cash. The firm takes these weights as given and

$$v_{t+j} = \frac{u'(C_{t+j})}{P_{t+j}}. \quad (\text{A.11})$$

The second term in (A.10),  $\Phi_t$ , represents the value of cash flow in future states in which the firm is able to reset its price. Given the structure of our environment,  $\Phi_t$  is not affected by the choice of  $\tilde{P}_t$ . The problem of a firm that is able to choose its price is to select a value of  $\tilde{P}_t$  that maximizes (A.10) subject to (A.4), taking  $\bar{P}_t, \bar{W}_t, P_t$  and  $W_t$  as given.

To solve the firm problem I first substitute out for  $Y_{i,t}$  and  $v_t$  using (A.4) and (A.11), respectively. I then differentiate (A.10) taking into account that  $\Phi_t$  is independent of  $\tilde{P}_t$ . The solution to this problem is obtained by a standard set of manipulations. In particular, let

$$\tilde{p}_t \equiv \frac{\tilde{P}_t}{P_t}, \quad \bar{\pi}_t \equiv \frac{P_t}{P_{t-1}}, \quad X_{t,j} = \begin{cases} \frac{1}{\bar{\pi}_{t+j}\bar{\pi}_{t+j-1}\dots\bar{\pi}_{t+1}}, & j \geq 1 \\ 1, & j = 0. \end{cases},$$

$$X_{t,j} = X_{t+1,j-1} \frac{1}{\bar{\pi}_{t+1}}, \quad j > 0$$

Then, the (scaled by  $P_t$ ) solution to the firm problem is:

$$\tilde{p}_t = \frac{E_t \sum_{j=0}^{\infty} (\beta\theta)^j (X_{t,j})^{-\varepsilon} \frac{Y_{t+j}}{C_{t+j}} \frac{\varepsilon}{\varepsilon-1} s_{t+j}}{E_t \sum_{j=0}^{\infty} (\beta\theta)^j (X_{t,j})^{1-\varepsilon} \frac{Y_{t+j}}{C_{t+j}}} = \frac{K_t}{F_t}, \quad (\text{A.12})$$

where

$$K_t = \frac{\varepsilon}{\varepsilon-1} \frac{Y_t}{C_t} s_t + \beta\theta E_t \left( \frac{1}{\bar{\pi}_{t+1}} \right)^{-\varepsilon} K_{t+1} \quad (\text{A.13})$$

$$F_t = \frac{Y_t}{C_t} + \beta\theta E_t \left( \frac{1}{\bar{\pi}_{t+1}} \right)^{1-\varepsilon} F_{t+1}. \quad (\text{A.14})$$

### A.3. Economy-wide Variables and Equilibrium

By the usual result associated with Calvo-sticky prices, we have the following cross-price restriction:

$$P_t = \left( \int_0^1 P_{i,t}^{(1-\varepsilon)} di \right)^{\frac{1}{1-\varepsilon}} = \left[ (1-\theta) \tilde{P}_t^{(1-\varepsilon)} + \theta P_{t-1}^{(1-\varepsilon)} \right]^{\frac{1}{1-\varepsilon}}. \quad (\text{A.15})$$

Dividing by  $P_t$  and rearranging,

$$\tilde{p}_t = \left[ \frac{1 - \theta \bar{\pi}_t^{(\varepsilon-1)}}{1 - \theta} \right]^{\frac{1}{1-\varepsilon}}. \quad (\text{A.16})$$

Combining this expression with (A.12), we obtain a useful equilibrium condition:

$$\frac{K_t}{F_t} = \left[ \frac{1 - \theta \bar{\pi}_t^{(\varepsilon-1)}}{1 - \theta} \right]^{\frac{1}{1-\varepsilon}}. \quad (\text{A.17})$$

It is convenient to express real marginal cost, (A.7), in terms that do not involve prices:

$$s_t = (1 - \nu) (1 - \psi + \psi R_t) \left( \frac{1}{1 - \gamma} \right)^{1-\gamma} \left( \frac{1}{\gamma} C_t N_t^\varphi \right)^\gamma \frac{1}{A_t}, \quad (\text{A.18})$$

where (A.2) has been used to substitute out for  $W_t/P_t$ .

I now derive the value-added production function. To this end, we first compute the equilibrium relationship between aggregate inputs,  $I_t$  and  $N_t$ , and gross output,  $Y_t$ . I do this by adapting the argument in Yun (1996). I then adapt a version of the argument in Jones (2013) to obtain the mapping from aggregate employment to  $GDP$ .

Let  $Y_t^*$  denote the unweighted sum of  $Y_{i,t}$  and then substitute out for  $Y_{i,t}$  in terms of prices using (A.4):

$$Y_t^* \equiv \int_0^1 Y_{i,t} di = Y_t \int_0^1 \left( \frac{P_{i,t}}{P_t} \right)^{-\varepsilon} di = Y_t \left( \frac{P_t}{P_t^*} \right)^\varepsilon,$$

where

$$P_t^* \equiv \left[ \int_0^1 P_{i,t}^{-\varepsilon} di \right]^{\frac{-1}{\varepsilon}} = \left[ (1 - \theta) \tilde{P}_t^{-\varepsilon} + \theta (P_{t-1}^*)^{-\varepsilon} \right]^{\frac{-1}{\varepsilon}}, \quad (\text{A.19})$$

using the analog of the result in (A.15). In this way, we obtain the following expression for  $Y_t$ :

$$\begin{aligned} Y_t &= p_t^* Y_t^*, \quad p_t^* \equiv \left( \frac{P_t^*}{P_t} \right)^\varepsilon, \\ p_t^* &= \begin{cases} \leq 1 & \text{not } P_{i,t} = P_{j,t}, \text{ all } i, j \\ = 1 & P_{i,t} = P_{j,t}, \text{ all } i, j \end{cases}, \end{aligned}$$

where  $p_t^*$  is the Tack Yun distortion discussed in the text.

By a standard calculation, I obtain the law of motion for  $p_t^*$  by dividing (A.19) by  $P_t^*$ , using (A.16) and rearranging,

$$p_t^* = \left[ (1 - \theta) \left( \frac{1 - \theta \bar{\pi}_t^{(\varepsilon-1)}}{1 - \theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta \bar{\pi}_t^\varepsilon}{p_{t-1}^*} \right]^{-1}. \quad (\text{A.20})$$

Then,

$$\begin{aligned} Y_t &= p_t^* Y_t^* = p_t^* \int_0^1 Y_{i,t} di = p_t^* A_t \int_0^1 N_{i,t}^\gamma I_{i,t}^{1-\gamma} di \\ &= p_t^* A_t \left( \frac{N_t}{I_t} \right)^\gamma I_t, \end{aligned}$$

where I have used the fact that all firms have the same materials to labor ratio. In this way, we obtain

$$Y_t = p_t^* A_t N_t^\gamma I_t^{1-\gamma}. \quad (\text{A.21})$$

I substitute out for  $I_t$  in (A.21) by noting from (A.8) that

$$I_t \equiv \int_0^1 I_{i,t} di = \mu_t \int_0^1 Y_{i,t} di = \mu_t Y_t^* = \frac{\mu_t}{p_t^*} Y_t. \quad (\text{A.22})$$

Use this to solve out for  $I_t$  in (A.21):

$$Y_t = \left( p_t^* A_t \left( \frac{\mu_t}{p_t^*} \right)^{1-\gamma} \right)^{\frac{1}{\gamma}} N_t. \quad (\text{A.23})$$

*GDP* for this economy is simply

$$C_t = Y_t - I_t. \quad (\text{A.24})$$

I conclude,

$$GDP_t = Y_t - I_t = \left( 1 - \frac{\mu_t}{p_t^*} \right) Y_t = F_{N,t} \times N_t, \quad (\text{A.25})$$

after some rearranging. Here, *TFP* denotes total factor productivity and is given by:

$$F_{N,t} = \left( p_t^* A_t \left( 1 - \frac{\mu_t}{p_t^*} \right)^\gamma \left( \frac{\mu_t}{p_t^*} \right)^{1-\gamma} \right)^{\frac{1}{\gamma}}, \quad (\text{A.26})$$

or,

$$F_{N,t} = \chi_t (A_t \gamma^\gamma (1-\gamma)^{1-\gamma})^{\frac{1}{\gamma}},$$

where

$$\chi_t \equiv \left( p_t^* \left( \frac{1 - \frac{\mu_t}{p_t^*}}{\gamma} \right)^\gamma \left( \frac{\frac{\mu_t}{p_t^*}}{1 - \gamma} \right)^{1-\gamma} \right)^{\frac{1}{\gamma}}. \quad (\text{A.27})$$

There are 12 variables to be determined for the model:

$$K_t, F_t, \bar{\pi}_t, p_t^*, s_t, C_t, Y_t, N_t, I_t, \mu_t, R_t, \chi_t.$$

There are 11 equilibrium conditions implied by private sector decisions : (A.13), (A.14), (A.17), (A.20), (A.1), (A.21), (A.24), (A.23), (A.18), (A.9), (A.27). In the special case of

flexible prices ( $\theta = 0$ ) and no working capital ( $\psi = 0$ ) then the classical dichotomy obtains. That is, equilibrium consumption and employment can be solved and, given the subsidy in (2.2), we obtain:

$$N_t = 1, C_t = [A_t (\gamma)^\gamma (1 - \gamma)^{1-\gamma}]^{\frac{1}{\gamma}}.$$

In the case that is of interest in this discussion, we need an additional equation to solve the model variables. To this end, I adopt the specification of monetary policy, (2.10). It can be verified that the steady state of the model is determinate when the smoothing parameter is 0.8 but indeterminate when the smoothing parameter is zero. The economic reasons for this are discussed in section 5.

To compute the dynamic properties of the model I solve the model using second order perturbation (with pruning), using Dynare.<sup>24</sup> For this, I require the model steady state in which the shocks,  $a_t$ , are held at their steady state values of zero. The steady state is discussed in the next section.

The parameter values I assign to the model are as follows:

$$\begin{aligned} \bar{\pi} &= 1.025^{\frac{1}{4}}, \psi = 1, \gamma = \frac{1}{2}, \beta = 1.03^{-0.25}, \\ \theta &= 0.75, \varepsilon = 6, \varphi = 1, \nu = \nu^*. \end{aligned}$$

The time series representation I use for  $a_t$  is that it is roughly a first order autoregression in its first difference. In particular,

$$a_t = (\rho_1 + \rho_2) a_{t-1} - \rho_1 \rho_2 a_{t-2} + \varepsilon_t, E\varepsilon_t^2 = 0.01^2,$$

where  $\rho_1 = 0.99$  and  $\rho_2 = 0.3$ .

#### A.4. Model Steady State

This section displays the steady state of the model. I derive the steady state expressions, (2.2) and (3.2), as well as the result that  $L$  in (2.1) is equal to unity when  $\nu = \nu^*$ . In addition, the steady state is required in the next subsection to derive the linearized Phillips curve which is discussed in the text.

Removing the time subscripts from time series variables in the model equilibrium conditions, (A.1), (A.17), (A.13), (A.14), (A.20), we find:

$$\begin{aligned} R &= \frac{\bar{\pi}}{\beta}, K_f \equiv \frac{K}{F} = \left[ \frac{1 - \theta}{1 - \theta \bar{\pi}^{(\varepsilon-1)}} \right]^{\frac{1}{\varepsilon-1}}, \\ s &= K_f \frac{\varepsilon - 1}{\varepsilon} \frac{1 - \beta \theta \bar{\pi}^\varepsilon}{1 - \beta \theta \bar{\pi}^{\varepsilon-1}}, p^* = \frac{1 - \theta \bar{\pi}^\varepsilon}{1 - \theta} \left( \frac{1 - \theta}{1 - \theta \bar{\pi}^{(\varepsilon-1)}} \right)^{\frac{\varepsilon}{\varepsilon-1}} \end{aligned} \quad (\text{A.28})$$

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<sup>24</sup>The code is available on my website.

The steady state share of materials in costs, scaled by  $p^*$ , is given by

$$\frac{\mu}{p^*} = \frac{(1-\gamma)s/p^*}{(1-\nu)(1-\psi+\psi R)}, \quad (\text{A.29})$$

according to (A.9). Let  $\nu^*$  be defined by,

$$\frac{\mu}{p^*} = (1-\gamma) \frac{1-\nu^*}{1-\nu}. \quad (\text{A.30})$$

Equating (A.29) and (A.30), and using (A.28), we obtain (2.2). This establishes (3.2).

In steady state the marginal product of labor is

$$\begin{aligned} F_N &= \left[ p^* \left( 1 - (1-\gamma) \frac{1-\nu^*}{1-\nu} \right)^\gamma \left( (1-\gamma) \frac{1-\nu^*}{1-\nu} \right)^{1-\gamma} \right]^{\frac{1}{\gamma}} \\ &= \left( p^* \left( \frac{1 - (1-\gamma) \frac{1-\nu^*}{1-\nu}}{\gamma} \right)^\gamma \left( \frac{1-\nu^*}{1-\nu} \right)^{1-\gamma} \right)^{\frac{1}{\gamma}} (\gamma^\gamma (1-\gamma)^{1-\gamma})^{\frac{1}{\gamma}}, \end{aligned} \quad (\text{A.31})$$

using (A.26) and (A.30).

According to (A.29) and (A.30),

$$s = (1-\psi+\psi R)(1-\nu^*)p^*.$$

Using this expression to substitute out for  $s$  in the steady state version of (A.18),

$$\frac{1-\nu^*}{1-\nu} p^* (1-\gamma)^{1-\gamma} (\gamma)^\gamma = (CN^\varphi)^\gamma,$$

using the fact,  $A_t = 1$ , in steady state. Combining the last expression with (A.31), we obtain

$$\frac{1-\nu^*}{1-\nu} \frac{F_N^\gamma}{\left( \frac{1-(1-\gamma)\frac{1-\nu^*}{1-\nu}}{\gamma} \right)^\gamma \left( \frac{1-\nu^*}{1-\nu} \right)^{1-\gamma}} = (CN^\varphi)^\gamma.$$

After rearranging, the latter expression reduces to

$$\frac{\gamma}{\gamma + \frac{\nu^*-\nu}{1-\nu^*}} = L. \quad (\text{A.32})$$

From this we can see the result cited in the text, that  $L = 1$  when  $\nu = \nu^*$ .

We can solve (A.32) for  $N$  by using  $C = F_N N$  :

$$N = \left[ \frac{\gamma}{\gamma + \frac{\nu^*-\nu}{1-\nu^*}} \right]^{\frac{1}{1+\varphi}}.$$

Finally,

$$\begin{aligned} C &= F_N N, \quad Y = \frac{C}{\gamma}, \\ I &= (1-\gamma)Y, \quad F = \frac{1/\gamma}{1-\beta\theta\bar{\pi}^{\varepsilon-1}}, \quad K = K_f \times F, \end{aligned}$$

so that all steady state variables are now available.

## A.5. Output Gap

The output gap is a key variable in the Phillips curve and I discuss that variable here. The output gap is the log deviation of equilibrium output from a benchmark level of output. Three possible benchmarks include: (i) output in the Ramsey equilibrium, (ii) the equilibrium when prices are flexible and (iii) the first best equilibrium, when output is chosen by a benevolent planner. In the latter case this corresponds to maximizing<sup>25</sup>

$$u(C_t) = \frac{N_t^{1+\varphi}}{1+\varphi}$$

subject to the maximal level of consumption that can be produced by allocating resources efficiently across sectors:

$$C_t = (A_t \gamma^\gamma (1-\gamma)^{1-\gamma})^{\frac{1}{\gamma}} N_t.$$

Optimization implies:

$$C_t^* = (A_t \gamma^\gamma (1-\gamma)^{1-\gamma})^{\frac{1}{\gamma}}, \quad N_t^* = 1. \quad (\text{A.33})$$

It is easily verified that when there is no working capital channel,  $\psi = 0$ , then (i), (ii) and (iii) coincide (that the equilibrium allocations under flexible prices are given by (A.33) was discuss in section A.3). With  $\psi > 0$  the three benchmarks differ. In this case expressions (i) and (ii) are relatively complicated, while (iii) stands out for its analytic simplicity. This is why I work with (iii) as my benchmark. This leads me to the following definition of the output gap,  $X_t$ :

$$X_t = \frac{C_t}{C_t^*}.$$

The log deviation from the steady state is:

$$\begin{aligned} x_t &\equiv \hat{X}_t = \hat{C}_t - \hat{C}_t^* \\ &= \hat{C}_t - \frac{1}{\gamma} \hat{A}_t, \end{aligned} \quad (\text{A.34})$$

where  $x_t$  is also  $\log(X_t/X)$  for  $X_t$  sufficiently close to its steady state value,  $X$ .

## A.6. Phillips Curve

Linearizing (A.13), (A.14) and (A.17), about steady state,

$$\hat{K}_t = (1 - \beta\theta\bar{\pi}^\varepsilon) \left[ \hat{Y}_t + \hat{s}_t - \hat{C}_t \right] + \beta\theta\bar{\pi}^\varepsilon E_t \left( \varepsilon \hat{\pi}_{t+1} + \hat{K}_{t+1} \right) \quad (\text{A.35})$$

$$\hat{F}_t = (1 - \beta\theta\bar{\pi}^{\varepsilon-1}) \left( \hat{Y}_t - \hat{C}_t \right) + \beta\theta\bar{\pi}^{\varepsilon-1} E_t \left( (\varepsilon - 1) \hat{\pi}_{t+1} + \hat{F}_{t+1} \right) \quad (\text{A.36})$$

$$\hat{K}_t = \hat{F}_t + \frac{\theta\bar{\pi}^{(\varepsilon-1)}}{1 - \theta\bar{\pi}^{(\varepsilon-1)}} \hat{\pi}_t. \quad (\text{A.37})$$

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<sup>25</sup>I assume the planner has the capacity to avoid the frictions associated with working capital, when  $\psi > 0$ .

Here,  $\hat{x}_t = (x_t - x)/x$  and  $x$  denotes the steady state value of  $x_t$ . Substitute out for  $\hat{K}_t$  in (A.35) using (A.37) and then substituting out for  $\hat{F}_t$  using (A.36), we obtain,

$$\begin{aligned} & (1 - \beta\theta\bar{\pi}^{\varepsilon-1}) \left( \hat{Y}_t - \hat{C}_t \right) + \beta\theta\bar{\pi}^{\varepsilon-1} E_t \left( (\varepsilon - 1) \hat{\pi}_{t+1} + \hat{F}_{t+1} \right) \\ + \frac{\theta\bar{\pi}^{(\varepsilon-1)}}{1 - \theta\bar{\pi}^{(\varepsilon-1)}} \hat{\pi}_t &= (1 - \beta\theta\bar{\pi}^\varepsilon) \left[ \hat{Y}_t + \hat{s}_t - \hat{C}_t \right] + \beta\theta\bar{\pi}^\varepsilon E_t \left( \varepsilon \hat{\pi}_{t+1} + \hat{F}_{t+1} + \frac{\theta\bar{\pi}^{(\varepsilon-1)}}{1 - \theta\bar{\pi}^{(\varepsilon-1)}} \hat{\pi}_{t+1} \right). \end{aligned}$$

Collecting terms,

$$\begin{aligned} \hat{\pi}_t &= \frac{(1 - \theta\bar{\pi}^{(\varepsilon-1)}) (1 - \beta\theta\bar{\pi}^\varepsilon)}{\theta\bar{\pi}^{(\varepsilon-1)}} \hat{s}_t + \beta E_t \hat{\pi}_{t+1} \\ &+ (1 - \bar{\pi}) (1 - \theta\bar{\pi}^{(\varepsilon-1)}) \beta \left[ \hat{Y}_t - \hat{C}_t + E_t \left( \hat{F}_{t+1} + \left( \varepsilon + \frac{\theta\bar{\pi}^{(\varepsilon-1)}}{1 - \theta\bar{\pi}^{(\varepsilon-1)}} \right) \hat{\pi}_{t+1} \right) \right]. \end{aligned} \quad (\text{A.38})$$

This expression reduces to the usual Phillips curve in the special case,  $\bar{\pi} = 1$ .

We require  $\hat{s}_t$ . Combining (A.2), (A.25) and (A.7),

$$s_t = (1 - \nu) (1 - \psi + \psi R_t) \left( \frac{1}{1 - \gamma} \right)^{1-\gamma} \left( \frac{1}{\gamma} C_t^{1+\varphi} \right)^\gamma \frac{TFP_t^{-\varphi\gamma}}{A_t}.$$

Totally differentiating,

$$\hat{s}_t = \frac{\psi R}{(1 - \psi + \psi R)} \hat{R}_t + (1 + \varphi) \gamma \hat{C}_t - \varphi \gamma \widehat{TFP}_t - \hat{A}_t.$$

We obtain  $\widehat{TFP}_t$  by totally differentiating (A.26):

$$\widehat{TFP}_t = \frac{1}{\gamma} \hat{A}_t,$$

where terms in  $\hat{p}_t^*$  and  $\hat{\mu}_t$  disappear because we set  $\bar{\pi} = 1$ . Then,

$$\hat{s}_t = \frac{\psi R}{(1 - \psi + \psi R)} \hat{R}_t + \gamma (1 + \varphi) \left[ \hat{C}_t - \frac{1}{\gamma} \hat{A}_t \right].$$

Using (A.34), this establishes:

$$\hat{s}_t = \frac{\psi R}{(1 - \psi + \psi R)} \hat{R}_t + \gamma (1 + \varphi) x_t.$$

Substituting this expression for marginal cost into (A.38) with  $\bar{\pi} = 1$ , we obtain the representation of the Phillips curve displayed in the text. The object,  $x_t$ , is a conventional measure of the output gap when  $\psi = 0$ . When  $\psi > 0$  it must be interpreted as the percent deviation between actual and first best output (see section A.5).



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