Optimal Monetary Policy in a ‘Sudden Stop’

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Abstract

During the Asian financial crises, interest rates were raised immediately, and then reduced sharply. We describe an environment in which this is the optimal monetary policy.

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1. Introduction

The Asian financial crises of 1997-98 triggered a sharp debate over the appropriate response of policy to a financial crisis. The hallmark of the crises was a “sudden stop” (Calvo, 1998): capital inflows turned into outflows and output suddenly collapsed. Some argued, appealing to the traditional monetary transmission mechanism, that a cut in the interest rate was required to slow or reverse the drop in output. Others argued that because of currency mismatches in balance sheets, the exchange rate depreciation associated with a cut in interest rates might exacerbate the crisis. They argued for an increase in interest rates. Interestingly, a look at the data indicates that both pieces of advice were followed in practice. Figure 1 shows what happened to short term interest rates in each of four Asian crisis countries. Initially they rose sharply. Within six months or so, the policy was reversed and interest rates were ultimately driven to below their pre-crisis levels. A casual observer might infer that policy was simply erratic, with policymakers trying out different advice at different times.

In this paper, we argue that the observed policy may have served a single coherent purpose. We describe a model in which the optimal response to a financial crisis is an initial sharp rise in the interest rate, followed by a fall to below pre-crisis levels.

In our model, because of the presence of real frictions, resources are slow to respond in the immediate aftermath of a shock. Over time, resource allocation becomes more flexible.\textsuperscript{1} We characterize a financial crisis as a shock in which collateral constraints unexpectedly bind and are expected to remain in place permanently. Our model has the property that when there is a binding collateral constraint and real frictions hinder resource allocation, then the monetary transmission mechanism is the reverse of what it would otherwise be. In particular, a rise in the interest rate increases economic activity and utility. Over time, as the real frictions wear off, the monetary transmission mechanism corresponds to the traditional one in which low interest rates stimulate output and raise welfare.

We now briefly explain the real and financial frictions in the model, and describe how they shape optimal policy after a financial shock. We adopt a small, tradable/non-tradable goods open economy model. The real friction is that labor in the tradeable sector is chosen prior to the realization of the current period shock.\textsuperscript{2} Thus, when the financial shock occurs, the allocation of labor to the tradeable sector cannot respond in the current period, although it can respond in subsequent periods.

We adopt two forms of financial friction.\textsuperscript{3} First, to capture the non-neutrality of money our

\textsuperscript{1}In effect, we combine into one model, the two studied in Christiano, Gust and Roldos (2004). In one model of that paper, labor in the traded good sector was fixed in each period. In another model, labor was completely flexible, thus able to shift across sectors.

\textsuperscript{2}A similar friction is used by Fernandez de Cordoba and Kehoe (2001) to study the role of capital flows following Spain’s entry to the European Community.

\textsuperscript{3}Other studies have examined the relationship between optimal interest rates and financial crises. Aghion,
model incorporates the portfolio allocation friction in the limited participation model. In the absence of collateral constraints, our model reproduces the traditional monetary transmission mechanism: when the domestic monetary authority expands the money supply, the liquidity of the banking system increases and interest rates fall, leading to an expansion in output and a depreciation of the exchange rate. Second, our model assumes firms make use of labor and a foreign intermediate input, and that these must be financed in advance. The collateral constraint that is imposed during the crisis applies to these loans.

The surprising feature of optimal policy in our model is that the nominal interest rate rises sharply in the period of the collateral shock. That this is optimal is a consequence of the interaction of the financial and real frictions. When the interest rate is increased, this acts like a tax on the employment of labor in the nontraded good sector, and raises the marginal cost of production in that sector. Because the employment of labor by firms in the traded sector is predetermined in the period of the shock, the interest rate rise does not increase the marginal cost of production in that sector. With the marginal cost of non-traded goods rising relative to the marginal cost of traded goods, the relative price of nontraded goods goes up. Other things the same, this increases the traded-good value of the physical capital stock in the non-traded sector. Because this capital is used as collateral in the import of intermediate goods, the collateral constraint is relaxed. This permits expanding imports of intermediate goods and the production of tradeable goods. Because tradeable and non-tradeable goods are complements in domestic production, this leads to an expansion in the demand for non-tradables and an expansion in overall economic activity. Welfare is increased by this policy, even though it has the effect of introducing a distortionary wedge in the labor market. The reason welfare increases is that the policy has the effect of sharply reducing another wedge, one that is associated with the collateral constraint.

The mechanism by which the higher interest rate produces higher output is novel, and so

Bacchetta and Banerjee (2000) present a model with multiple equilibria, in which a currency crisis is the bad equilibrium. The possibility of the bad equilibrium is due to the interplay between the credit constraints on private firms and the existence of nominal price rigidities. The authors show that the monetary authority should tighten monetary policy after any shock that results in the possibility of the currency crisis equilibrium. Our analysis differs from this analysis in three ways. First, equilibrium multiplicity does not play a role in our analysis. Second, our model emphasizes a different set of rigidities. Third, Aghion, Bacchetta and Banerjee focus on the prevention of crises, while we focus on their management after they occur. Similarly, Caballero and Krishnamurthy (2002) show that when the economy faces a binding international collateral constraint, a monetary expansion that would redistribute funds from consumers to distressed firms has no real effects. Given this lack of effectiveness, a monetary authority that trades-off output and an inflation target focuses on the latter and tightens monetary policy to achieve the inflation objective.


5 The collateral constraint, that incorporates a large share of nontraded sector assets on the left-hand side and a large share of external debt on the right-hand side, captures the balance sheet mismatches emphasized in the discussions and analysis of emerging markets crises. The relevance of balance sheet effects during sudden stops for emerging markets—but not for developed countries—is documented in Calvo, Izquierdo and Mejia (2004).
to further highlight its workings, we construct and analyze a simple example. The example represents a dramatic simplification of our dynamic model. There is no money, and there is only one period. In the example, a tax rate on labor plays the role of the interest rate in our dynamic, monetary model. We are able to prove that whenever the collateral constraint is binding and the equilibrium is unique, a rise in the labor tax rate must stimulate output, consumption, employment and welfare. This result may be of interest beyond the sudden stop episodes that we study here. In particular, it may be useful for shedding light on the empirical literature on the “non-Keynesian effects of fiscal policy” or “Expansionary Fiscal Consolidations”. We return to this issue in our concluding remarks.

We now briefly discuss the interaction of monetary policy and sudden stop in our model. The sudden stop is triggered by a tightening of collateral constraints. The effect of the collateral shock is to increase the shadow cost of foreign borrowing, since international debt limits - via the collateral constraint - the ability of firms to purchase foreign intermediate inputs. As a result, imports of intermediate inputs drop and, because they are crucial for domestic production, the latter falls. In addition, the sharp rise in the shadow cost of debt induces agents to pay down that debt by running a current account surplus. This process continues until the debt falls to the point where the collateral constraint is marginally non-binding, and now the economy is in a new steady state. Monetary policy has no impact on how much collateral lenders require, nor does it have an important impact on real variables in the new steady state. Monetary policy affects real variables and welfare primarily by its impact on the nature of the transition from the old to the new steady state. As discussed above, optimal monetary policy initially raises the domestic interest rate sharply and cuts it thereafter. That optimal policy cuts the interest rate eventually is no surprise. Eventually, the collateral constraint ceases to bind and resources are allocated flexibly. At this point, low interest rates are optimal for conventional Friedman-type reasons. As discussed above, in the period of the shock it is optimal to sharply raise the interest

\[ R = u'(y^H)/u'(y^L) \]

An optimal policy is for the government to issue bonds in the first period, and redistribute the proceeds to everyone (suppose the government cannot see who is constrained and who is not) in lump sum form. In the second period, the government taxes everyone in order to pay back the bonds. This policy in effect allows borrowers and lenders to exchange amongst themselves. A side effect of this policy is that the interest rate is lower. Although this example has some of the flavor of our analysis (optimal policy under binding financial constraints is associated with a high interest rate), in its details it is very different.

\[ u(c_1) + u(c_2) \]

Consider a two period economy, in which 1/2 the population (‘borrowers’) has a sequence of endowments, \( y^L \) in the first period and \( y^H \) in the second period, where \( y^L < y^H \). Suppose the other half of the population (‘lenders’) has the opposite lifetime sequence of endowments, \( y^H, y^L \). Suppose everyone has the same utility function, \( u(c_1) + u(c_2) \), where \( u \) is strictly concave and \( c_1 \) and \( c_2 \) are periods 1 and 2 consumption, respectively. Suppose also that borrowing is not permitted. Then the unique equilibrium is that everyone consumes their endowment. The borrowers are forced to do so by the non-negativity constraint on private bonds, and the lenders are prevented from lending by a very low interest rate, \( R = u'(y^H)/u'(y^L) \). An optimal policy is for the government to issue bonds in the first period, and redistribute the proceeds to everyone (suppose the government cannot see who is constrained and who is not) in lump sum form. In the second period, the government taxes everyone in order to pay back the bonds. This policy in effect allows borrowers and lenders to exchange amongst themselves. A side effect of this policy is that the interest rate is lower. Although this example has some of the flavor of our analysis (optimal policy under binding financial constraints is associated with a high interest rate), in its details it is very different.
rate. This has the effect of resisting (not reversing) the fall in nominal and real exchange rates, asset prices, output, employment and consumption, caused by the initial "sudden stop".

We compare the dynamic behavior of the variables in the model with data drawn from the Asian crisis economies. Qualitatively, the model reproduces the behavior of data for these economies reasonably well. In particular, the model reproduces the observed transitory rise in the current account, and fall of real quantities such as employment, consumption and output. The model also captures the evolution of asset prices, the real and nominal exchange rate and the behavior of the interest rate. Taken together, this evidence suggests that our model may provide a useful interpretation of the apparently erratic behavior of monetary policy exhibited in Figure 1.

The model does have quantitative empirical shortcomings. Although it captures the direction of movement in the current account, it understates the magnitude. We suspect that this reflects the absence of physical investment in the model. A reduction in investment provides agents with another margin from which to draw resources that can be used to pay off the international debt. The presence of investment may also help the model with persistence, which is another dimension on which there is some weakness. Finally, although the inflation response of the model to the financial shock matches qualitatively, it misses on magnitude.

The paper is organized as follows. First, we provide empirical evidence to support the main assumptions of the model. In particular, we show that collateral constraints were increased during the Asian financial crisis, and that it is not unreasonable to assume that at least a fraction of the assets used in the nontradable sector could be used to secure foreign borrowing by tradable sector firms. We also show that imported intermediate inputs are a large fraction of imports, and that they fell sharply during the crisis. Second, we present the simplified example discussed above. The third section presents our dynamic, monetary model. Section 4 discusses model calibration and section 5 present our simulation results. Second 6 concludes.

2. Evidence on Key Assumptions

This section discusses empirical evidence related to key features of our model. We begin by displaying evidence that collateral requirements play a role in emerging markets generally, as well as evidence that collateral constraints tightened at the onset of the Asian financial crises of 1997. Table 1 shows that up until 1996, approximately 20 percent of syndicated loans to emerging markets were secured by collateral. At the time of the financial crises of 1997, this fraction doubled to over 40 percent. Also, Edison, Luangaram and Miller (2000) show that in Thailand, banks loaned up to 70 to 80 percent of collateral before the Asian crisis, and only 50 to 60 percent after the crisis. According to Gelos and Werner (1999), survey evidence from the Bank of Thailand indicates that more than 80 percent of loans are collateralized in
Thailand. Gelos and Werner (1999) also report that around 60 percent of loans are collateralized in Mexico. Finally, a review of financial conditions of the Asian crises countries (IMF 1999) notes that lending against collateral was a widespread practice also in these countries.

There is also some indirect evidence which provides support for the notion that collateral considerations matter. Baek, Kang and Park (2004) find that the stock prices of Korean firms with higher foreign ownership suffered less during the crisis. This is consistent with our model if the foreign ownership in effect provided firms with more access to collateral for borrowing purposes. Baek, Kang and Park (2004) also report evidence that firms with better disclosure rules experienced a smaller drop in asset prices. This is consistent with our model, if we suppose that the greater transparency reduces the need for collateral. If collateral constraints are not binding on firms with better disclosure rules, then the logic in our model implies that they would have suffered less with the onset of the crisis.

In our model analysis, we assume that collateral in the non-traded good sector is available for borrowing by firms in the traded sector. Although our assumption is admittedly extreme, the evidence suggests that some sharing of collateral across sectors does occur. In several emerging markets a large share of the economy is dominated by groups of firms (‘chaebols’ in Korea) that can use internal capital markets to allocate credit among firms in the group. For example, Shin and Park (1999) report that firms in Korean chaebols guarantee bank loans taken by other firms in the same chaebol.\footnote{In Korea a large business group is often referred as a chaebol. The Korea Fair Trade Commission (KFTC) defines a business group as a “group of companies of which more than 30% of shares are owned by group’s controlling shareholder and its affiliated companies”. Chaebol firms operate in many different industries, are bound together by a nexus of explicit and implicit contracts, and maintain substantial business ties with other firms in their group. They are also characterized by an extensive arrangement of pyramidal or multi-layered share-holding arrangements and the existence of cross-debt guarantees among member firms Baek, Kang and Park (1999).} Groups typically encompass both traded and nontraded good sectors. For example, the Samsung group (one of the largest chaebols in Korea), which has member firms in the electricity, heavy machinery, chemical and financial sectors (see Shin and Park, 1999). Shin and Park (1999) also show that the sensitivity of investment to cash flow of a chaebol firm (a common measure of liquidity constraints) is significantly affected by the cash flow of other firms within the same chaebol. This is consistent with the notion that internal credit markets allow firms in chaebols to share collateral. Significantly, chaebol firms make up a large fraction of the Korean economy. For example, at the end of 1998, the top 30 chaebols in Korea accounted for 12 percent of total GNP, 48 percent of total corporate assets and 47 percent of corporate revenues (see Baek, Kang and Park, 2004). According to Claessens, Djankov, Fan, and Lang (1999), the average number of firms that belong to a group of firms in Southeast Asia was 75
percent in 1991-1996.⁸⁹

In our analysis, imports are composed of intermediate goods. Because these require finance, the ‘credit crunch’ associated with a tightening of collateral constraints inhibits the ability of firms to import intermediate goods. Because intermediate goods are assumed to be important in production, this results in a fall in production and in exports. To see that intermediate goods are an important component of imports, see Table 2. According to Table 2, intermediate good imports are 50 percent of total imports for Korea and 70 percent of total imports for Indonesia and Malaysia. Figure 2 shows real GDP and intermediate good imports and shows the close correlation between the two. To see how imports fall during a sudden stop, consider Figure 3, which displays exports and imports, measured in dollars, for four Asian countries.¹⁰ Note how imports fall more than exports (of course, this is what produces the positive swing in the current account). The fact that exports fall, despite the tremendous depreciation of the currency that occurs in a sudden stop, is consistent with the notion in our model that the fall in imports creates problems for domestic production.

In effect, the credit crunch brings on a shortage of tradeable goods according to our model. The shortage is acute, because lack of substitution in production between traded and non-traded goods causes output to slow. One expects such a shortage to manifest itself in the form of a price rise. For evidence on this, consider the data on exchange rates in Figure 4. Note that in each of the Asian crisis countries considered there is a dramatic depreciation in the aftermath of the crisis. The smallest depreciation is 143 percent (Philippines) and the largest is 169 percent (Korea). Given the relatively small movements in inflation in these countries, these movements in the nominal exchange rate correspond to movement in the real exchange rate. Assuming rough purchasing power parity in traded goods, this corresponds to a very dramatic jump in the price of traded relative to nontraded goods.

We now turn to a key assumption that causes a rise in the interest rate to be optimal in the immediate aftermath of a sudden stop. This is the assumption that labor in the tradable sector is difficult to adjust quickly. We have not found evidence that bears directly on this assumption. However, there is some indirect evidence. Botero, Djankov, La Porta, Lopez-de-Silanes and Shleifer (2003) report that there is a significant amount of labor market regulation.

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⁸ According to Claessens, Djankov, Fan, and Lang (1999, page 2), ‘A group can be described as a corporate organization where a number of firms are linked through cross-ownership or where a single individual, family or coalition of families owns a number of different firms.’

⁹ The percentages for each country break down as follows: Hong Kong, 60; Indonesia, 69; Japan, 83; South Korea, 57; Malaysia, 57; Philippines, 74; Singapore, 67; Taiwan, 53; Thailand, 42. The average over all countries is 75.

¹⁰ The data were obtained from the International Monetary Fund’s, ‘International Financial Statistics’ data base. Imports are imports of goods, services and payments associated with domestic assets issued to foreigners. Exports are defined analogously. The data for Korean, Malaysia, Phillipines ad Thailand. For all countries except the Phillipines, we used annual data.
in emerging market countries. Also, Caballero, Cowan, Engel and Miccod (2004) report that with more labor market regulation in emerging markets, employment flexibility is reduced. If the evidence found by Melitz (2003) and others for the US applies to crisis economies, then the traded sector has higher value-added, more capital per worker, higher wages, etc. All these factors are likely to be associated with greater transparency for the traded sector, which may imply that labor market regulations are applied more effectively in the traded good sector than in the non-traded good sector. If this is so, then we can suppose that labor in the traded good sector reacts less flexibly to shocks than does labor in the nontraded good sector.

3. Example

A basic result in the dynamic simulations reported in later sections is that a rise in the domestic interest rate in the period of a collateral shock places upward pressure on employment and welfare. At first glance, this result will seem puzzling since the rise in the interest rate effectively operates like a rise in the tax rate on labor. Partial equilibrium reasoning suggests such a distortion should lead to a decrease in employment and welfare, not an increase. In our model, these partial equilibrium effects are overwhelmed by a general equilibrium effect that relaxes the collateral constraint. In this section we present a drastically simplified version of our dynamic model, which allows us to show how these effects work. In the simplified example, there is no money and there is only one period.

The first subsection below displays the model. The second subsection derives the model’s qualitative properties. Here, we state our proposition and provide a heuristic proof (details are provided in Appendix B). The third subsection provides a numerical example.

3.1. Model

A final good sector produces a non-traded consumption good, \( c \), for domestic households, whose utility is as follows:

\[
    u(c, L) = c - \frac{\psi_0}{1 + \psi} \left( L^N + L^T \right)^{1 + \psi},
\]

where \( L^N \) and \( L^T \) denote labor in the nontraded and traded good sectors, respectively. The household’s budget constraint is:

\[
    pc \leq w \left( L^N + L^T \right) + \pi + T,
\]

where \( p \) is the price of consumption, \( w \) is the wage rate, \( \pi \) denotes lump-sum profits and \( T \) denotes a lump-sum transfer payment from the government. Here, we have imposed a property of the equilibrium of the model, namely that the wage rate in the non-traded and traded good sectors must be the same. All the quantities in (3.2) are measured in units of the traded good.
The consumption good is produced using intermediate goods, of which there are two types. One is a tradeable good and the other is non-traded. Each of these intermediate goods is essential in the production of the final good. The final good production function is Leontieff in terms of traded and nontraded intermediate goods:

\[ c = \min \{(1 - \gamma)c^T, \gamma c^N\} . \tag{3.3} \]

This is the same production function that we use in the dynamic model economy.

The one period in our example model is the analog of period 0 in our dynamic model. In that model, the economy is in a steady state before period 0, and then in period 0 a collateral constraint suddenly and unexpectedly becomes binding. Since employment in the traded good sector is chosen by intermediate good firms at the very beginning of the period, in period 0 employment is predetermined at the time of the collateral shock. Thus, for purposes of the analysis in this section, we treat intermediate good firms’ choice of \( L^T \) as a fixed constant, not subject to their choice. As a result, the only variable input in traded good production, from the point of view of intermediate good firms, is the imported intermediate good, \( z \). This good must be financed at the beginning of the period by foreign borrowing, which is subject to a collateral constraint. The imported intermediate good, \( z \), is essential to overall economic activity by the Leontieff assumption, (3.3). We suppose that non-traded goods are produced using a Cobb-Douglas function of labor, \( L^N \), and capital, \( K^N \). The production functions for traded and non-traded goods is given by:

\[ y^T(z) = V^\theta z^{1-\theta}, \quad y^N(L^N) = (K^N)^\alpha (L^N)^{1-\alpha} \tag{3.4} \]

respectively, where \( y^T \) and \( y^N \) denote gross output of traded and non-traded goods, respectively. Value-added in the traded good sector, \( V \), is a Cobb-Douglas function of capital and labor in that sector:

\[ V = A (K^T)^\nu (L^T)^{1-\nu}, \quad 0 < \nu < 1. \]

Production of traded and non-traded intermediate goods is carried out by a single, representative, competitive firm. This assumption allows us to sidestep potential technical complications arising from the fact that some of the economy’s collateral, the capital stock in the non-traded good sector, exists in a sector different from the sector that requires collateral for borrowing. By locating all production in a single firm, we ensure that all the economy’s collateral is available to the agents who need it for borrowing.\(^{11}\) To some extent our assumption about firms resembles the situation of actual firms in some emerging economies. See, for example, our discussion

\[^{11}\text{For an analysis of situations in which collateral is not equally distributed in the economy, see Caballero and Krishnamurthy (2001).}\]
about chaebols in section 2. An alternative interpretation of our assumption about firms is that it is a stand-in for the existence of financial institutions and markets that distribute collateral among domestic agents. We have not been able to identify empirical evidence for or against this assumption.

As indicated in the previous paragraph, the representative intermediate good firm operates the two technologies, (3.4), and seeks to maximize profits, which we denote by $\pi$:

$$\pi = p^N y^N + y^T - q^N (K^N - K^N_0) - q^T (K^T - K^T_0) - w (1 + \tau) L^N - w L^T - R^* z.$$  

Here, $p^N$ denotes the price of non-traded goods, $q^i$ denotes the price of physical capital in sector $i$, and $\tau$ denotes the labor tax rate. This tax is rebated in lump sum form to households via $T$ in their budget constraint. In addition, $K^N_0$ is the representative firm’s initial endowment of sector $i$ capital. It is convenient to express the firm’s profits in non-traded goods units:

$$\frac{\pi}{p^N} = y^N + \frac{1}{p^N} [y^T - R^* z] - \frac{q^N}{p^N} (K^N - K^N_0) - \frac{q^T}{p^N} (K^T - K^T_0) - \frac{w}{p^N} (1 + \tau) L^N - \frac{w}{p^N} L^T. \quad (3.5)$$

Foreign borrowing is subject to the constraint that a fraction of the value of the firm’s assets must be no less than the firm’s end-of-period international obligations:

$$\tau^N q^N K^N + \tau^T q^T K^T \geq R^* z $$

$$0 < \tau^N \leq 1, \quad 0 \leq \tau^T \leq 1,$$

where $\tau^N$ and $\tau^T$ are the fractions of capital in the indicated sectors that can be used for collateral.

The timing of the intermediate good firm’s decisions are as follows. First, the labor tax rate, $\tau$, becomes known. Then, a market opens in which intermediate good firms trade capital among themselves at prices, $q^N$ and $q^T$. Then $z$, $L^N$, $c^N$, $y^N$ and $y^T$ are determined and production occurs. Immediately after paying its wage bill, the intermediate good firm decides whether to default on its international loans. If it does, then the creditors can seize from the firm an amount of output equal to the firm’s obligations. It is easy to verify that the firm’s revenues, after paying the wage bill, are sufficient for this.\(^{12}\)

The resource constraints in our economy are as follows:

$$y^N = c^N, \quad y^T = c^T + z R^*.$$  

The first of these expressions states that all the output of the non-traded good sector, $y^N$, is used as inputs in the production of non-traded goods. The second says that the gross output of the traded good sector is divided between inputs into the production of final goods, $c^T$, and gross interest payments abroad for borrowing to finance the imported intermediate good, $z$.

\(^{12}\) Implicitly, we suppose that $z$ has no value to the intermediate good producer other than as an input to production. For example, the producer has no incentive to abscond with $z$ without producing anything.
3.2. Qualitative Analysis

We list 8 equations that characterize 8 equilibrium variables - \( w, p, p^N, q^N, q^T, L^N, z \) and the Lagrange multiplier on (3.6) - for our example. Consider the representative final good producer. As long as input prices are strictly positive, the final good producer always sets \( c^T = \frac{\gamma}{(1 - \gamma)} y^N \). Combining (3.3), (3.4) and the resource constraint, this implies:

\[
y^T(z) - z R^* = \frac{\gamma}{1 - \gamma} (K^N)^\alpha (L^N)^{1-\alpha}.
\]  

(3.7)

If the price of, say, \( c^T \), were zero, then the final good producer would be indifferent between purchasing an amount of \( c^T \) consistent with (3.7), or purchasing more. In such a case, we suppose that the producer resolves the indifference by imposing (3.7). Competition in final goods implies that price equals marginal cost:

\[
p = \frac{1}{1 - \gamma} + \frac{1}{\gamma} p^N, \tag{3.8}
\]

The representative intermediate good firm’s optimal choice of \( K^N \) and \( K^T \) leads to the following expressions for the price of capital in each sector:

\[
q^N = \frac{\alpha p^N (K^N)^{\alpha-1} (L^N)^{1-\alpha}}{1 - \lambda \tau^N} \tag{3.9}
\]

\[
q^T = \frac{\theta (\frac{\tau}{p})^{1-\theta} A \nu \left( \frac{L^T}{K^T} \right)^{1-\nu}}{1 - \lambda \tau^T} \tag{3.10}
\]

These are the first order necessary conditions for optimization in the Lagrangian representation of the representative intermediate good firm’s optimization problem, in which \( \lambda \geq 0 \) is the multiplier on the collateral constraint (3.6). Note that when the collateral constraint is binding, the price of capital exceeds its marginal value product. This reflects the services the capital provides in relaxing the collateral constraint.

The labor demand choice by the intermediate good firm leads it to equate the marginal cost, \((1 + \tau)w\), and value marginal product of labor in the production of non-traded goods to obtain (after making use of (3.8)),

\[
\frac{1 - \alpha}{\left( \frac{1}{1 - \gamma} p^N + \frac{1}{\gamma} \right) (1 + \tau)} (K^N)^\alpha (L^N)^{-\alpha} = \frac{w}{p}. \tag{3.11}
\]

Optimization in the choice of \( z \) leads to the following first order condition:

\[
\frac{1}{p^N} [y^T_z(z) - R^*(1 + \lambda)] = 0. \tag{3.12}
\]

Evidently, for \( p^N < \infty \), (3.12) corresponds to setting the expression in square brackets to zero. However, we will also consider the possibility \( p^N = \infty \) (this corresponds to a zero price on \( c^T \)),
in which case (3.12) does not require the expression in square brackets to be zero. Finally, the complementary slackness condition on $\lambda$ for intermediate good firm optimization is:

$$\lambda \left[ \tau^N q^N K^N + \tau^T q^T K^T - R^* z \right] = 0, \quad \lambda \geq 0, \quad \tau^N q^N K^N + \tau^T q^T K^T - R^* z \geq 0. \quad (3.13)$$

Market clearing requires that prices be strictly positive:

$$q^N, q^T, w, p^N > 0. \quad (3.14)$$

The latter, in combination with (3.9), impose an upper bound on $\lambda$, $\lambda \leq \bar{\lambda}$, where

$$\bar{\lambda} \equiv \min \left[ 1/\tau^N, 1/\tau^T \right].$$

Household optimization of employment leads to the following labor supply curve:

$$\psi_o \left( L^N + L^T \right)^\psi = \frac{w}{p}. \quad (3.15)$$

The 8 equations that characterize equilibrium are (3.7), (3.8), (3.9), (3.10), (3.11), (3.12), (3.13), (3.15), together with the non-negativity constraints, (3.14), and $0 \leq \lambda \leq \bar{\lambda}$.

In Appendix A, we establish the following proposition:

**Proposition 3.1.** Consider a parameterization of the model in which the equilibrium is unique and the collateral constraint is binding ($\lambda > 0$). Generically, a small increase in $\tau$ leads to an increase in $p^N, z, L^N$, the value of total assets and welfare.

This proposition establishes that an increase in the tax on labor raises the real exchange rate ($p^N$), asset values ($\tau^N q^N K^N + \tau^T q^T K^T$), intermediate good imports ($z$), employment ($L^N$) and welfare in the static version of our model. This is so, if the initial equilibrium is unique and the collateral constraint binds. There do exist parameter configurations for which there are multiple equilibria in our model. However, we do not discuss the impact of increases in $\tau$ in these cases, since such experiments are hard to interpret when the equilibrium is not unique.\textsuperscript{13}

In the following section, we work through a numerical example which illustrates our proposition.

We provide a sketch of the proof to this proposition here. If we drop the complementary slackness condition, (3.13), and fix the value of the multiplier, $\lambda$, we are able to compute the remaining 7 equilibrium variables in the model uniquely. We denote the asset values and level of intermediate good imports computed in this way by $q^N(\lambda; \tau), q^T(\lambda)$, and $z(\lambda)$, respectively. The variable, $\tau$, is not included in the argument of $z(\cdot)$ and $q^T(\cdot)$ because, conditional on a fixed value of $\lambda$, the equilibrium value of these variables are not a function of $\tau$. In the case of

\textsuperscript{13}The appendix contains some discussion of the nature of the equilibria when there is more than one.
z, this is obvious, since $z(\lambda)$ is defined by the requirement that the object in square brackets in (3.12) is zero. With this notation, we define the following function:

$$C(\lambda; \tau) = \tau^N q^N (\lambda; \tau) K^N + \tau^T q^T (\lambda) K^T - R^* z(\lambda).$$

Let $\lambda^*$ and $\tau^*$ denote the multiplier and labor tax rate in the type of equilibrium considered in the proposition. In addition to uniqueness, that proposition supposes $\lambda^* > 0$, so that by (3.13), $C(\lambda^*, \tau^*) = 0$. The proof requires establishing that a small increase in $\tau$ above $\tau^*$ results in a fall in the equilibrium value of the multiplier. That employment, asset values and utility are all higher in the new equilibrium then follows trivially.

We establish that the equilibrium value of $\lambda$ is decreasing in $\tau$ for $\tau \geq \tau^*$ in two steps. First, we show that $C(\lambda, \tau)$ is increasing in $\lambda$ in a neighborhood of $\lambda^*$ for given $\tau$. Second, we show that $q^N (\lambda, \tau)$ (and, hence, $C(\lambda, \tau)$) is increasing in $\tau$ for fixed $\lambda$.

To establish that $C$ is increasing in $\lambda$, the Appendix shows that for $\lambda$ approaching its upper bound, one of $q^N$ or $q^T$ diverges to $+\infty$. To see the economic motivation for this result, suppose $\tau^T < \tau^N$. The benefit of a marginal unit of $K^N$ is its collateral value, $\lambda q^N \tau^N$, plus its marginal value product. When $\lambda \to 1/\tau^N$, then $\lambda q^N \tau^N = q^N$, and the collateral value of capital equals its purchase price. In this case, $K^N$ is a ‘money-pump’: a $1$ purchase of $K^N$ generates $1$ in value as collateral plus the value marginal product of capital in production. Consequently, as $\lambda \to 1/\tau^N$ the demand for $K^N$ approaches infinity, as does its market clearing price, $q^N$. If $\tau^T > \tau^N$, then $\lambda = 1/\tau^T$. In this case, if $\lambda \to 1/\tau^T$, then $q^T \to \infty$. Because $z(\lambda)$ is bounded above, it follows that $C > 0$ for $\lambda$ sufficiently large. This implies that, generically, $C$ must be increasing in $\lambda$ at $\lambda = \lambda^*$. It may be possible to construct an example where the slope of $C$ at $\lambda = \lambda^*$ is zero, but to avoid contradicting our assumption of a unique equilibrium, that slope would have to be zero at only the point, $\lambda = \lambda^*$. Such an example is non-generic. The slope of $f$ cannot be negative at $\lambda = \lambda^*$ because in this case, $C > 0$ for sufficiently high values of $\lambda$ would require that there be a second $\lambda$ with $f = 0$, and such a scenario contradicts the hypothesis of equilibrium uniqueness. Thus, we conclude that, generically, $C$ is strictly increasing in $\lambda$ for $\lambda$ near $\lambda^*$.

That $q^N$ is increasing in $\tau$ for fixed $\lambda$ is also intuitive. The requirement that the expression in square brackets in (3.12) be zero has the effect of associating a unique $z$ with each $\lambda > 0$, independent of the value of $\tau$. By (3.7) the given value of $\lambda > 0$ also implies a unique $L^N$, independent of $\tau$. Under perfect competition, $p^N$ must be equal to the marginal cost of producing the nontraded good. For a given value of $L^N$, a higher value of $\tau$ raises that marginal cost, and so $p^N$ is increasing in $\tau$ for given $\lambda$. In view of (3.9), we conclude that $q^N$ increases in $\tau$ for given $\lambda$.

Since $C$ has a positive slope at $\lambda = \lambda^*$ and shifts up with a rise in $\tau$, it follows immediately that equilibrium $\lambda$ is falling in $\tau$ (see Figure 5). From this discussion, it is clear that what is
crucial in the result is that $\tau^N > 0$. If $\tau^N = 0$, so that capital in the non-traded good sector is useless in the collateral constraint, then an increase in $\tau$ has no impact on the equilibrium. So, although our result requires that some physical capital in the nontraded sector be available as collateral for borrowing by the traded sector, it does not require that this be the only or even the largest component of that collateral.

### 3.3. Quantitative Analysis

We illustrate the proposition in the previous subsection with a numerical example. We report the equilibrium outcomes for our model economy for a range of values of the labor tax rate. We adopt the following parameter values:

$$A = 2, \; R^* = 1.06, \; \theta = 0.8, \; \gamma = 0.43, \; \alpha = 0.25, \; \tau^N = \tau^T = 0.1, \; \psi_0 = 0.06, \; \psi = 1, \; K^N = K^T = 1, \; \nu = 0.3$$

We computed equilibrium allocations corresponding to $\tau$ in the range, 0.00 to 0.85. The upper bound on this range is just below the tax rate that would drive the price of $c^T$ to zero (see $1/p^N$ in Figure 6). The admissible set of equilibrium values of $\lambda$ belongs to the compact set, $J = [0, \bar{\lambda}]$. By considering a fine grid of $\lambda \in J$, we found that, for each value of $\tau$ considered, the equilibrium is unique. The values of utility, $1/p^N, \tau^N q^N K^N + \tau^T q^T K^T, \lambda, z, L^N$ corresponding to each $\tau$ are displayed in Figure 6. Note that for $\tau$ in the range of 0 to 0.7, $\lambda > 0$. Consistent with the proposition, utility is strictly increasing in this range. The increase in $\tau$ also raises $p^N, L^N, z$ and $\tau^N q^N K^N + \tau^T q^T K^T$. The latter has the effect of relaxing the collateral constraint, which is reflected in the fall in $\lambda$. Note that the initial value of $\lambda$ is extremely high. According to (3.12), $\lambda$ is equivalent to a tax on the purchase of the foreign intermediate input. When $\tau = 0$ this tax wedge is about 250%. By increasing the labor tax rate, the shadow tax rate on foreign borrowing is completely eliminated.

For $\tau$ beyond 0.7, utility and employment are invariant to additional increases in $\tau$. This is because in this range, $z$ is in a sense the binding constraint on domestic production. The amount of $z$, which is now pinned down by $V$ and $R^*$ in (3.12), determines $L^N$ through (3.7).

### 4. The Dynamic, Monetary Model

This section describes our dynamic, monetary model. The model builds on the structure analyzed in the previous section, and so we limit explanations and motivations to what is new here. In addition, the model is a version of the one in Christiano, Gust and Roldos (2004), and so the presentation is brief. A key difference between the two models is that here, labor in the traded

\[14\] As a result, the scarcity assumption on $z$ discussed in Appendix A is satisfied for each $\tau$ considered in the example.
good sector cannot be quickly adjusted in response to a shock, but it can be adjusted in the medium term.

4.1. Households

Household preferences over consumption and leisure are the dynamic version of the preferences in the previous section:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, L_t),$$

(4.1)

where the subscript $t$ denotes the time $t$ realization of the variable. We adopt the following specification of utility:

$$u(c, L) = \left[ c - \frac{\psi L^{1+\psi}}{1-\psi} \right]^{1-\sigma}.$$

(4.2)

The household begins the period with a stock of liquid assets, $\tilde{M}_t$. Of this, it allocates deposits, $D_t$, with the financial intermediary, and the rest, $\tilde{M}_t - D_t$, to consumption expenditures. The cash constraint that the household faces on its consumption expenditures is:

$$P_t c_t \leq W_t L_t + \tilde{M}_t - D_t,$$

(4.2)

where $W_t$ denotes the wage rate and $P_t$ denotes the price level. These nominal quantities are expressed in domestic currency units.

The household also faces a flow budget constraint governing the evolution of its assets:

$$\tilde{M}_{t+1} = R_t (D_t + X_t) + P_t^T \pi_t + [W_t L_t + \tilde{M}_t - D_t - P_t c_t].$$

(4.3)

Here, $R_t$ denotes the gross domestic nominal rate of interest, $\pi_t$ is profits which derive from household’s ownership of firms, and $X_t$ is a liquidity injection from the monetary authority. Profits, $\pi_t$, is measured in units of traded goods, and $P_t^T$ is the domestic currency price of traded goods. The term immediately to the right of the equality reflects the household’s sources of liquid assets at the beginning of period $t + 1$: interest earnings on deposits and on the liquidity injection, profits and any cash that may be left unspent in the period $t$ goods market.

The household maximizes (4.1) subject to (4.2)-(4.3), and a particular timing constraint. The household’s deposit decision is made after the realization of the collateral shock, and before the realization of the current period monetary action. As in the limited participation literature, the fact that the household makes the deposit decision before the monetary policy action is taken, is the reason monetary policy has a non-neutral effect. At the time the household makes its deposit decision, it must have a view about what the relevant price and interest rate variables are. We assume that the household expects the monetary authority to respond to the collateral shock by holding money growth constant, and that the household expects prices and interest
rates to be what they would be in a constant money growth equilibrium. All other household decisions are taken after the monetary policy action is known. Since the model is deterministic in the periods after the collateral shock, timing is irrelevant then.

4.2. Firms

The basic structure of the firm sector is the same as in the previous section, with some difference. A competitive final good firm produces the consumption good, $c_t$, and intermediate good firms produce the inputs used to produce $c_t$. We now discuss the decisions facing these firms.

4.2.1. Final Good Firms

As in the previous section, the production function of the final good firms is given by:

$$c = \min \{(1 - \gamma) c^T, \gamma c^N\},$$

(4.4)

where $c^T$ and $c^N$ denote quantities of tradeable and non-tradeable intermediate inputs, respectively. As noted above, the domestic currency price of $c$ is denoted by $P$, while $P^T$ and $P^N$ denote the money prices of the traded and nontraded inputs, respectively. The firm takes these prices parametrically.

As before, zero profits and efficiency imply the following relation between prices:

$$p = \frac{1}{1 - \gamma} + \frac{P^N}{\gamma}, \quad p = \frac{P}{P^T}.$$

(4.5)

The object, $P$, in the model corresponds to the model’s ‘consumer price index’, denominated in units of the domestic currency. The object, $p$, is the consumer price index denominated in units of the traded good.

4.2.2. Intermediate Inputs

As in the previous section, a single representative firm produces the traded and non-traded intermediate inputs. That firm manages three types of debt, two of which are short-term. The firm borrows at the beginning of the period to finance its wage bill and to purchase a foreign input, and repays these loans at the end of the period. In addition, the firm holds the outstanding stock of external (net) indebtedness, $B_t$.

The firm’s optimization problem is:

$$\max \sum_{t=0}^{\infty} \beta^t \Lambda_{t+1} \pi_t,$$

(4.6)

\[\text{--- end of excerpt ---}\]
where
\[ \pi_t = p_N^t y_N^t + y_t^T - w_t^N R_t L_t^N - w_t^T R_t L_t^T - R^* z_t - r^* B_t + (B_{t+1} - B_t), \]  
(4.7)
denotes dividends, denominated in units of traded goods. Also, \( B_t \) is the stock of external debt at the beginning of period \( t \), denominated in units of the traded good; \( R^* \) is the gross rate of interest (fixed in units of the traded good) on loans for the purpose of purchasing \( z_t \); and \( r^* \) is the net rate of interest (again, fixed in terms of the traded good) on the outstanding stock of external debt. The price, \( \Lambda_{t+1} \), is taken parametrically by firms. In equilibrium, it is the multiplier on \( \pi_t \) in the (Lagrangian representation of the) household problem:\(^{16}\)
\[ \Lambda_{t+1} = \beta \frac{u_{c,t+1} P_t^T}{P_{t+1}} \]
(4.8)
\[ = \beta \frac{u_{c,t+1} P_t^T}{P_{t+1}} \frac{1}{p_t^T p_{t+1}} (1 + x_t) \]
where
\[ p_t^T = \frac{P_t^T}{M_t} \]
Here, \( M_t \) is the aggregate stock of money at the beginning of period \( t \), which is assumed to evolve according to:
\[ \frac{M_{t+1}}{M_t} = 1 + x_t. \]
(4.9)
Note that under our notational convention, all lower case prices except one, expresses that price in units of the traded good. The exception, \( p_t^T \), is the domestic currency price of traded goods, scaled by the beginning of period stock of money. Alternatively, \( p_t^T \) is the inverse of a measure of real balances.

The firm production functions are:
\[ y_t^T = \left\{ \theta \left[ \mu_1 V_t \right]^\frac{\xi-1}{\xi} + (1 - \theta) \left[ \mu_2 z_t \right]^\frac{\xi-1}{\xi} \right\}^\frac{\xi}{\xi-1}, \]
(4.10)
\[ V = A \left( K^T \right)^{\nu} \left( L^T \right)^{1-\nu}, \]
\[ y_t^N = \left( K^N \right)^{\alpha} \left( L^N \right)^{1-\alpha}, \]
where \( \xi \) is the elasticity of substitution between value-added in the traded good sector, \( V_t \), and the imported intermediate good, \( z_t \). In the production functions, \( K^T \) and \( K^N \) denote capital in the traded and non-traded good sectors, respectively. They are owned by the representative

\(^{16}\)The intuition underlying (4.8) is straightforward. The object \( \Lambda_{t+1} \) in (4.8), is the marginal utility of one unit of dividends, denominated in traded goods, transferred by the firm to the household at the end of period \( t \). This corresponds to \( P_t^T \pi_t \) units of domestic currency. The households can use this currency in period \( t + 1 \) to purchase \( P_t^T \pi_t / p_{t+1} \) units of the consumption good. The value, in period \( t \), of these units of consumption goods is \( \beta u_{c,t+1} P_t^T \pi_t / P_{t+1} \), or \( \beta u_{c,t+1} P_t^T \pi_t / (p_{t+1} P_{t+1}) \), where \( u_{c,t} \) is the marginal utility of consumption. This is the first expression in (4.8).
intermediate input firm. We keep the stock of capital fixed throughout the analysis. It does not depreciate and there exists no technology for making it bigger.

Total employment of the firm, $L_t$, is:

$$L_t = L^T_t + L^N_t.$$  

In equilibrium, borrowing will satisfy the following restriction:

$$\frac{B_{t+1}}{(1 + r^*)^t} \to 0, \text{ as } t \to \infty. \quad (4.11)$$

We suppose that international financial markets impose that this limit cannot be positive. That it cannot be negative is an implication of firm optimality.

The firm’s problem at time $t$ is to maximize (4.6) by choice of $B_{t+j+1}, y_{t+j}', y_{t+j}^T, z_{t+j}, L^T_{t+j}; L^M_{t+j}$ and $L^N_{t+j}$, $j = 0, 1, 2, ...$ and the indicated technology. In addition, the firm takes all prices and rates of return as given and beyond its control. The firm also takes the initial stock of debt, $B_t$, as given. This completes the description of the firm problem in the pre-crisis version of the model, when collateral constraints are ignored.

The crisis brings on the imposition of the following collateral constraint:

$$\tau^N q^N_t K^N + \tau^T q^T_t K^T \geq R^* z_t + (1 + r^*)B_t + \zeta R_t \left( w^T_t L^T_t + w^N_t L^N_t \right) \quad (4.12)$$

Here, $q^i, i = N, T$ denote the value (in units of the traded good) of a unit of capital in the nontraded and traded good sectors, respectively. Also, $\tau^i$ denotes the fraction of these stocks accepted as collateral by international creditors. The left side of (4.12) is the total value of collateral, and the right side is the payout value of the firm’s external debt; $\zeta$ indicates the fraction of the wage bill that enters into the liabilities side of the collateral constraint, and represents the share of domestic loans that are collateralized and would compete with foreign creditors’ claims on the firm’s assets. Before the crisis, firms ignore (4.12), and assign a zero probability that it will be implemented. With the coming of the crisis, firms believe (correctly) that (4.12) must be satisfied in every period henceforth, and do not entertain the possibility that it will be removed.

We obtain $q^N_t$ and $q^T_t$ by differentiating the Lagrangian representation of the firm optimization problem with respect to $K^N$ and $K^T$, respectively. The equilibrium value of the asset prices, $q^i_t, i = N, T$, is the amount that a potential firm would be willing to pay in period $t$, in units of the traded good, to acquire a unit of capital and start production in period $t$. We let $\lambda_t \geq 0$ denote the multiplier on the collateral constraint (= 0 in the pre-crisis period) in firm problem. Then, $q^i_t$ solves

$$q^i_t = \frac{VMP_{k,t}^i + \beta_{N+1}^2 q^i_{t+1}}{1 - \lambda_t \tau^i}, \quad i = N, T. \quad (4.13)$$
Here, \( VMP_{k,t} \) denotes the period \( t \) value (in terms of traded goods) marginal product of capital in sector \( i \).

When \( \lambda_t \equiv 0 \), (4.13) is just the standard asset pricing equation. It is the present discounted value of the value of the marginal physical product of capital. When the collateral constraint is binding, so that \( \lambda_t \) is positive, then \( q^i_t \) is greater than this. This reflects that in this case capital is not only useful in production, but also for relieving the collateral constraint. In our model capital is never actually traded since all firms are identical. However, if there were trade, then the price of capital would be \( q^i_t \). If a firm were to default on its credit obligations, the notion is that foreign creditors could compel the sale of its physical assets in a domestic market for capital. The price, \( q^i_t \), is how much traded goods a domestic resident is willing to pay for a unit of capital. Foreign creditors would receive those goods in the event of a default. We assume that with these consequences for default, default never occurs in equilibrium.

To understand the impact of a binding collateral constraint on firm decisions, it is useful to consider the Euler equations of the firm. Differentiating Lagrangian representation of the firm problem with respect to \( B_{t+1} \):

\[
1 = \beta \frac{\Lambda_{t+2}}{\Lambda_{t+1}}(1 + r^*)(1 + \lambda_{t+1}), \quad t = 0, 1, 2, \ldots.
\]  

(4.14)

Following standard practice in the small open economy literature, we assume \( \beta(1 + r^*) = 1 \), so that\(^{17} \)

\[
\Lambda_{t+1} = \Lambda_{t+2}(1 + \lambda_{t+1}), \quad t = 0, 1, 2, \ldots.
\]  

(4.15)

A high value for \( \lambda \), which occurs when the collateral constraint is binding, raises the effective rate of interest on debt. The interpretation is that when \( \lambda \) is large, then the debt has an additional cost, beyond the direct interest cost. This cost reflects that when the firm raises \( B_{t+1} \) in period \( t \), it not only incurs an additional interest charge in period \( t + 1 \), but it is also further tightens its collateral constraint in that period. This has a cost because, via the collateral constraint, the extra debt inhibits the firm’s ability to acquire working capital in period \( t + 1 \). Thus, when \( \lambda \) is high, there is an additional incentive for firms to reduce \( \pi \) and ‘save’ by paying down the external debt. Although the firm’s actual interest rate on external debt taken on in period \( t \) is \( 1 + r^* \), it’s ‘effective’ interest rate is \( (1 + r^*)(1 + \lambda_{t+1}) \).

### 4.3. Financial Intermediary and Monetary Authority

The financial intermediary takes domestic currency deposits, \( D_t \), from the household at the beginning of period \( t \). In addition, it receives the liquidity transfer, \( X_t = x_t M_t \), from the monetary authority.\(^{18} \) It then lends all its domestic funds to firms who use it to finance their

\(^{17} \)See, for example, Obstfeld and Rogoff (1997).

\(^{18} \)In practice, injections of liquidity do not occur in the form of lump sum transfers, as they do here. It is easy to show that our formulation is equivalent to an alternative, in which the injection occurs as a result of an open
employment working capital requirements, WL. Clearing in the money market requires \( D_t + X_t = W_t L_t \), or, after scaling by the beginning-of-period \( t \) aggregate money stock,

\[
d_t + x_t = p_t^T [w_t^N L_t^N + w_t^T L_t^T],
\]

(4.16)

where \( d_t = D_t / M_t \).

The monetary authority in our model simply injects funds into the financial intermediary. Its period \( t \) decision is taken after the household has selected a value for \( D_t \), and before all other variables in the economy are determined. This is the standard assumption in the limited participation literature. It is interpreted as reflecting a sluggishness in the response of household portfolio decisions to changes in market variables. With this assumption, a value of \( x_t \) that deviates from what households expected at the time \( D_t \) was set produces an immediate reaction by firms and the financial intermediary but not, in the first instance, by households. The name, ‘limited participation’, derives from this feature, namely that not all agents react immediately to (or, ‘participate in’) a monetary shock. As a result of this timing assumption, many models exhibit the following behavior in equilibrium. An unexpectedly high value of \( x_t \) swells the supply of funds in the financial sector. To get firms to absorb the increase in funds, a fall in the equilibrium rate of interest is required. When that fall does occur, they borrow the increased funds and use them to hire more labor and produce more output.

We abstract from all other aspects of government finance. The only policy variable of the government is \( x_t \).

4.4. Equilibrium

We consider a perfect foresight, sequence-of-markets equilibrium concept. In particular, it is a sequence of prices and quantities having the properties: (i) for each date, the quantities solve the household and firm problems, given the prices, and (ii) the labor, goods and domestic money markets clear.

Clearing in the money market requires that (4.16) hold and that actual money balances, \( M_t \), equal desired money balances, \( \tilde{M}_t \). Combining this with the household’s cash constraint, (4.2), we obtain the equilibrium cash constraint:

\[
p_t^T p_t c_t = 1 + x_t.
\]

(4.17)

According to this, the total, end of period stock of money must equal the value of final output, \( c_t \). Market clearing in the traded good sector requires:

\[
y_t^T - R^* z_t - r^* B_t - c_t^T = - (B_{t+1} - B_t).
\]

(4.18)
The left side of this expression is the current account of the balance of payments, i.e., total production of traded goods, net of foreign interest payments, net of domestic consumption. The right side of (4.18) is the change in net foreign assets. Equation (4.18) reflects our assumption that external borrowing to finance the intermediate good, $z_t$, is fully paid back at the end of the period. That is, this borrowing resembles short-term trade credit. Note, however, that this is not a binding constraint on the firm, since our setup permits the firm to finance these repayments using long term debt. Market clearing in the nontraded good sector requires:

$$y_t^N = c_t^N.$$  

(4.19)

Our procedure for computing the equilibrium of the model is described in details in Appendix B and corresponds to a variation on the procedure applied in Christiano, Gust and Roldos (2004)

5. Quantitative Analysis

In this section we begin with a discussion of the parameterization of the model. We then report the results for optimal monetary policy.

5.1. Parameter Values and Steady State

The parameter values are displayed in Table 3. These were chosen to so that the model’s steady state in the absence of collateral constraints roughly matches features of Korean and Thai data during the first semester of 1997. The share of tradables in total production for Korea, assuming that tradables correspond to the non-service sectors, was approximately one third before the crisis. Combining this assumption with estimates of labor shares from Young (1995), we estimate shares of capital for the tradable and nontradable sector in Korea to be respectively 0.48 and 0.21. Based on figures for Argentina, Uribe (1997) and Rebelo and Vegh (1995) estimate the same shares to be 0.52 and 0.37. We take an intermediate point between these estimates by specifying $\nu = 0.50$ and $\alpha = 0.36$. Reinhart and Vegh (1995) estimate the elasticity of intertemporal substitution in consumption for Argentina to be equal to 0.2. We adopt a somewhat higher elasticity by setting $\sigma = 2$. We take the foreign interest rate to be equal to 6 percent and we assume a rate of money growth that implies an annual nominal domestic interest rate of 11 percent, roughly in line with the experience of Korea and Thailand in the years before the crises. We set $\psi = 1$, implying a labor supply elasticity of 1.

The parameters $\mu_1$ and $\mu_2$, in the production function were chosen to reproduce the ratio of imported intermediate inputs in manufacturing to manufacturing value-added in Korea for the year 1995. In that year the ratio is 0.4, in other words $z/V = 0.4$.

As mentioned above, the share of tradable goods in production is roughly one third, so we calibrate the remaining parameters of the model to produce a ratio of consumption of nontrad-
ables to tradables of approximately 2. In addition, we chose $\tau$ and the stock of debt in the initial steady state equilibrium so that the change in the debt from initial steady state to final steady state in the model matches the corresponding change in Korean and Thai data. Korea’s (Thailand’s) external debt started at 33% of GDP by end-1997 (60.3%) and was around 26.8% of GDP (51% of GDP) and the end of the year 2000. So, in these two countries, the external debt falls by 6 - 10% of GDP after the currency crisis. In our model, the external debt falls by 7% of GDP. Money growth in the initial steady state is set so that the nominal interest rate in the initial steady state is 11 percent, in annual terms. This is very close to the pre-crisis interest rates in Korea and Thailand. The pre-crisis steady state of the model is reported in Table 4.

5.2. Optimal Monetary Policy

We now consider the optimal monetary policy response to the unexpected imposition of the collateral constraint. The timing of the experiment can be seen in Figure 7. Up until period 0, the economy is in a nonstochastic steady state in which the collateral constraint is not binding. At the start of period 0, the firm makes its employment decision in the traded good sector. After this, the collateral constraint on borrowing is unexpectedly imposed. This constraint is binding. Then, the household makes its deposit decision. In making its deposit decision the household assumes money growth will continue at its previous constant rate. After this, the monetary action occurs. Finally, all activity occurs. The remainder of all time unfolds in a non-stochastic way. In our calculations, we focus only on equilibria that converge to a steady state and in which the collateral constraint remains in force forever.\footnote{In this respect, we follow Christiano, Gust and Roldos (2004) and Kocherlakota (2003).}

The results are reported in Figure 8. A period in the model is taken to be 6 months. As a benchmark, we include actual (semi-annual) data for Korea. Note the sharp rise in the current account. Also, the drop in GDP, relative to its pre-crisis trend, is nearly 15 percent. The drop in employment is less, though it takes longer to recover. Interestingly, this represents a substantial drop in labor productivity. The drop in consumption is a little larger and more persistent than the drop in output. Share prices fall and then recover. The interest rate rises sharply (as noted in Figure 1), and then falls substantially below its pre-crisis level. The exchange rate initially depreciates by about 50 percent, although the depreciation is ultimately smaller. Finally, inflation jumps from about 5 percent initially to about 12 percent, before stabilizing at a lower level.

Now consider the response of the model under the optimal monetary policy. Note that the current account in the model increases, though not as much as in the Korean data. We suspect that the absence of investment in our model is part of the reason for this. With domestic investment there is an additional margin that can be used to cut back domestic absorption.
and increase the current account. We expect that in a version of our model with investment, agents would exploit this margin given the very high value of the multiplier on the collateral constraint. The drops in domestic output and consumption are of a similar order of magnitude to corresponding drops in Korea, but substantially less persistent. In the case of employment, the model substantially overstates the initial drop. This is an interesting miss. In effect, the model cannot explain the substantial drop in labor productivity observed in the wake of the Korean financial crisis. The model matches the behavior of asset prices and the nominal exchange rate quite well. Although the model overstates the nominal interest rate and the rate of inflation in the wake of the Korean crisis, it matches the qualitative, non-monotonic behavior of both variables.

Overall, we believe that the model captures reasonably well the behavior of the Korean data during the currency crisis. Figure 9 provides insight into what the optimal policy is trying to accomplish, by comparing it to a particular benchmark. In the benchmark, money growth is held fixed at its pre-shock level. Note that the optimal policy raises the interest rate much more sharply than in the benchmark. This higher interest rate results in a higher (relative to the benchmark) level of output, consumption, employment, asset prices and output. In addition, it produces a stronger exchange rate. Evidently, the high interest rate policy has the effect of resisting the currency depreciation.

The economic intuition underlying these results can be found in contemplating the collateral constraint. The rise of the interest rate in period 0 slows the exchange rate depreciation and this contributes to a smaller reduction in asset prices. This relative improvement on the asset side of the collateral constraint allows for a smaller drop in imports of intermediate inputs, and a smaller reduction in real GDP, employment and consumption. Once the initial increase in interest rates and exchange rate depreciation set in motion the external adjustment process, labor is reallocated to the traded sector. From that moment onwards, the optimal monetary policy consists of reducing interest rate to values very close to the arrival steady state level of 2%. It is worth noting that during this transition period, and in consistent with the evidence on the crisis countries, interest rate cuts are associated with nominal (and real) exchange rate appreciations (Mussa, 2000).

6. Conclusion

In this paper we studied the optimal monetary policy response to a financial crisis of the kind experienced by the Asian economies in 1997-98. These crises, as many other emerging market crises, were characterized by a sudden reversal in capital inflows. Using a particular open economy model with collateral constraints, we found that the optimal monetary response to such a crisis involves an initial increase in interest rates, followed by a relatively sharp and rapid
reduction in rates in the aftermath of the crisis. Interestingly, this is the policy that was actually followed.

In our model, increasing the interest rate is very much like raising a tax. As a result, our analysis may also yield insight into the episodes of “expansionary fiscal consolidations” emphasized by a large literature initiated by Giavazzi and Pagano (1990). For example, Perotti (1999) presents some evidence that large tax increases are more likely to stimulate the economy when levels of debt are high. Based on this, he argues that a model is required in which the response of the economy to tax changes changes depends on the initial conditions, such as the level of debt. Our model is very much in this spirit.

To keep the analysis simple, our model does not include variable investment. In principle, including investment could improve the model’s empirical implications. However, whether it does so remains an important, open question. Because capital appears in the collateral constraint, investment in physical capital represents an alternative strategy - apart from paying off international debt - by which agents can reduce the burden of the collateral constraint. In effect, the imposition of the collateral constraint is equivalent to a subsidy to paying off international debt, as well as to investing in domestic capital.20 Thus, without additional assumptions, we cannot rule out the possibility that in an environment in which investment is variable, a binding collateral constraint could lead to an increase in investment, and to a fall in the current account.21 Clearly, this would deal a blow to the idea that tightening collateral constraints were the driving force behind the Asian financial crises. We suspect, however, that with reasonable investment adjustment costs and other frictions, paying off the international debt would dominate investment in physical capital as a strategy for reducing the burden of the collateral constraint. If so, then the introduction of variable investment would improve our model’s empirical implications, by magnifying the rise in the current account in the wake of a financial crisis.

At a methodological level, this paper adds to the literature that studies the impact of financial frictions on the monetary transmission mechanism. In traditional models, financial frictions have the effect of magnifying - through an ‘accelerator effect’ - the effects of monetary actions, without changing their sign. In this model we have shown that financial frictions could actually have a ‘reverse accelerator effect’, in that they reverse the sign of the effect of a monetary action.

20 For a formal statement of this, see Chari, Kehoe and McGrattan (2005).
21 Mendoza (2005) provides an example of a sudden stop similar to ours, except that he also includes investment. He finds that when collateral constraints tighten, investment drops. (Mendoza does not study the implications of sudden stop for monetary policy, which is our central focus.)
References


Table 1: Syndicated Loans to Emerging Markets  
(in billions of U.S. dollars)

<table>
<thead>
<tr>
<th>Year</th>
<th>Total</th>
<th>Secured</th>
<th>Secured as % of Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1993</td>
<td>47.5</td>
<td>7.9</td>
<td>16.5</td>
</tr>
<tr>
<td>1994</td>
<td>64.9</td>
<td>11.5</td>
<td>17.7</td>
</tr>
<tr>
<td>1995</td>
<td>93.0</td>
<td>16.1</td>
<td>17.3</td>
</tr>
<tr>
<td>1996</td>
<td>104.3</td>
<td>22.0</td>
<td>21.1</td>
</tr>
<tr>
<td>1997</td>
<td>143.7</td>
<td>61.4</td>
<td>42.7</td>
</tr>
<tr>
<td>1998</td>
<td>77.3</td>
<td>25.9</td>
<td>33.5</td>
</tr>
<tr>
<td>1999</td>
<td>73.1</td>
<td>26.3</td>
<td>35.9</td>
</tr>
</tbody>
</table>

Source: Capital Data, Loanware

Table 2: Intermediate Imports and Total Imports

Panel A: Thailand

<table>
<thead>
<tr>
<th>Year</th>
<th>Total</th>
<th>Intermediate</th>
<th>% of Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1993</td>
<td>45,995</td>
<td>17,184</td>
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</tr>
<tr>
<td>1994</td>
<td>54,338</td>
<td>19,294</td>
<td>36%</td>
</tr>
<tr>
<td>1995</td>
<td>70,718</td>
<td>25,061</td>
<td>35%</td>
</tr>
<tr>
<td>1996</td>
<td>72,248</td>
<td>24,874</td>
<td>34%</td>
</tr>
<tr>
<td>1997</td>
<td>63,286</td>
<td>21,860</td>
<td>35%</td>
</tr>
<tr>
<td>1998</td>
<td>42,403</td>
<td>14,744</td>
<td>35%</td>
</tr>
<tr>
<td>1999</td>
<td>49,919</td>
<td>18,205</td>
<td>36%</td>
</tr>
<tr>
<td>2000</td>
<td>62,181</td>
<td>23,663</td>
<td>38%</td>
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<tr>
<td>2001</td>
<td>61,847</td>
<td>22,978</td>
<td>37%</td>
</tr>
<tr>
<td>2002</td>
<td>64,317</td>
<td>24,461</td>
<td>38%</td>
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Panel B: Korea

<table>
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<th>% of Total</th>
</tr>
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<tr>
<td>83,800</td>
<td>43,987</td>
<td>52%</td>
</tr>
<tr>
<td>102,348</td>
<td>50,158</td>
<td>49%</td>
</tr>
<tr>
<td>135,119</td>
<td>64,611</td>
<td>48%</td>
</tr>
<tr>
<td>150,339</td>
<td>68,556</td>
<td>46%</td>
</tr>
<tr>
<td>144,616</td>
<td>69,361</td>
<td>48%</td>
</tr>
<tr>
<td>93,282</td>
<td>45,593</td>
<td>49%</td>
</tr>
<tr>
<td>119,752</td>
<td>57,253</td>
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</tr>
<tr>
<td>160,481</td>
<td>78,975</td>
<td>49%</td>
</tr>
<tr>
<td>141,098</td>
<td>71,929</td>
<td>51%</td>
</tr>
<tr>
<td>152,126</td>
<td>73,891</td>
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### Panel C: Malaysia

<table>
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<th>% of Total</th>
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</thead>
<tbody>
<tr>
<td>1993</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1994</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1995</td>
<td>77,601</td>
<td>50,447</td>
<td>65%</td>
</tr>
<tr>
<td>1996</td>
<td>78,426</td>
<td>52,201</td>
<td>67%</td>
</tr>
<tr>
<td>1997</td>
<td>79,036</td>
<td>51,922</td>
<td>66%</td>
</tr>
<tr>
<td>1998</td>
<td>58,293</td>
<td>40,901</td>
<td>70%</td>
</tr>
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<td>1999</td>
<td>65,389</td>
<td>48,321</td>
<td>74%</td>
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<tr>
<td>2000</td>
<td>81,963</td>
<td>61,233</td>
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<tr>
<td>2001</td>
<td>73,856</td>
<td>53,271</td>
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</tr>
<tr>
<td>2002</td>
<td>79,881</td>
<td>56,939</td>
<td>71%</td>
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### Panel D: Indonesia

<table>
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<th>% of Total</th>
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<tr>
<td>1993</td>
<td>28,376</td>
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<tr>
<td>1994</td>
<td>32,222</td>
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<td>72%</td>
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<td>1995</td>
<td>40,921</td>
<td>29,610</td>
<td>72%</td>
</tr>
<tr>
<td>1996</td>
<td>44,240</td>
<td>30,470</td>
<td>69%</td>
</tr>
<tr>
<td>1997</td>
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<td>61%</td>
</tr>
<tr>
<td>1999</td>
<td>30,600</td>
<td>18,475</td>
<td>60%</td>
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<tr>
<td>2000</td>
<td>40,367</td>
<td>26,073</td>
<td>65%</td>
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<tr>
<td>2001</td>
<td>34,669</td>
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<tr>
<td></td>
<td>24,118</td>
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### Panel E: Philippines

<table>
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<tr>
<td>1993</td>
<td>17,597</td>
<td>7,855</td>
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<td>1994</td>
<td>21,333</td>
<td>9,559</td>
<td>45%</td>
</tr>
<tr>
<td>1995</td>
<td>26,538</td>
<td>12,174</td>
<td>46%</td>
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<tr>
<td>1996</td>
<td>32,427</td>
<td>14,015</td>
<td>43%</td>
</tr>
<tr>
<td>1997</td>
<td>35,933</td>
<td>14,663</td>
<td>41%</td>
</tr>
<tr>
<td>1998</td>
<td>29,660</td>
<td>11,586</td>
<td>39%</td>
</tr>
<tr>
<td>1999</td>
<td>30,726</td>
<td>12,596</td>
<td>41%</td>
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<td>2000</td>
<td>34,491</td>
<td>16,747</td>
<td>49%</td>
</tr>
<tr>
<td>2001</td>
<td>33,058</td>
<td>15,121</td>
<td>46%</td>
</tr>
<tr>
<td>2002</td>
<td>35,427</td>
<td>14,791</td>
<td>42%</td>
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Source: CEIC Data Company Ltd
Table 3: Parameters Values of the Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>$\beta$</td>
<td>0.943</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.3</td>
</tr>
<tr>
<td>$\psi$</td>
<td>2.00</td>
</tr>
<tr>
<td>$R$</td>
<td>1.12</td>
</tr>
<tr>
<td>$R^*$</td>
<td>1.06</td>
</tr>
<tr>
<td>$r^*$</td>
<td>0.06</td>
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<tr>
<td>$\alpha$</td>
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</tr>
<tr>
<td>$K^N$</td>
<td>10</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.5</td>
</tr>
<tr>
<td>$K^T$</td>
<td>5</td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>1</td>
</tr>
<tr>
<td>$\mu_2$</td>
<td>3.5</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.08</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.7</td>
</tr>
<tr>
<td>$\psi_0$</td>
<td>0.0036</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>2</td>
</tr>
<tr>
<td>$A$</td>
<td>1.5</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.9</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Note: Here, $\beta$, $R$ and $R^*$ are expressed in annualized terms.

Table 4: Steady State Ignoring Collateral Constraint

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>$L$</td>
<td>30</td>
</tr>
<tr>
<td>$z$</td>
<td>2.73</td>
</tr>
<tr>
<td>$L^T$</td>
<td>7.75</td>
</tr>
<tr>
<td>$L^N$</td>
<td>22.25</td>
</tr>
<tr>
<td>$c^T$</td>
<td>6.17</td>
</tr>
<tr>
<td>$c^N$</td>
<td>16.68</td>
</tr>
<tr>
<td>$w$</td>
<td>0.4011</td>
</tr>
<tr>
<td>$V$</td>
<td>9.33</td>
</tr>
<tr>
<td>$p^N$</td>
<td>2.39</td>
</tr>
<tr>
<td>$y^T$</td>
<td>9.34</td>
</tr>
<tr>
<td>$p^N$</td>
<td>0.8863</td>
</tr>
<tr>
<td>$p^T$</td>
<td>0.0491</td>
</tr>
<tr>
<td>$q^T$</td>
<td>22.95</td>
</tr>
<tr>
<td>$q^N$</td>
<td>18.54</td>
</tr>
<tr>
<td>$B$</td>
<td>14.2</td>
</tr>
<tr>
<td>$\frac{B}{p^N c^N + y^T - R^* z}$</td>
<td>0.6644</td>
</tr>
</tbody>
</table>

Table 5: Arrival Steady State with Monetary Experiment

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
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<td>2.761</td>
</tr>
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<td>$L^T$</td>
<td>7.89</td>
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<td>22.74</td>
</tr>
<tr>
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<td>$c^N$</td>
<td>16.92</td>
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<tr>
<td>$w$</td>
<td>0.4177</td>
</tr>
<tr>
<td>$V$</td>
<td>9.4213</td>
</tr>
<tr>
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<tr>
<td>$y^T$</td>
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<td>$p^N$</td>
<td>0.8852</td>
</tr>
<tr>
<td>$p^T$</td>
<td>0.047</td>
</tr>
<tr>
<td>$q^T$</td>
<td>23.16</td>
</tr>
<tr>
<td>$q^N$</td>
<td>18.78</td>
</tr>
<tr>
<td>$B$</td>
<td>13.37</td>
</tr>
<tr>
<td>$\frac{B}{p^N c^N + y^T - R^* z}$</td>
<td>0.615</td>
</tr>
</tbody>
</table>
7. Appendix A: Proof of Proposition 1

Following is a proof of the proposition in section 3.2. We begin by describing the details of the mapping discussed in the text, taking the multiplier, $\lambda \geq 0$, on the collateral constraint, into candidate equilibrium prices and quantities. An equilibrium for $\lambda$ is a value for this parameter such that the complementary slackness condition on the collateral constraint is satisfied (see (3.13)). We then discuss a condition on model parameters implied by the assumption in our proposition that equilibrium is unique. The condition ensures that the traded intermediate good, $c^T$, is a ‘scarce’ factor in the production of final goods. In particular, we note that, in the absence of the collateral constraint, there is a maximum amount of $c^T$, we call this amount $c_T^0$, that can be produced, after paying for the required imported intermediate good, $z$. Given that employment, $L^T$, in the traded good sector is fixed in our static model, producing $c_T^0$ does not require the reallocation of domestic resources from other useful activities. Under these circumstances, the domestic market price of $c_T^0$ will be positive only if $c_T^0$ is ‘scarce’. That is, $c_T^0$ is scarce if with a zero price on $c^T$ and in the absence of collateral constraints, domestic demand for $c^T$ would exceed $c_T^0$. When $c_T^0$ is not scarce, then there are at least two equilibria, if there are any. Evidently, that $c_T^0$ is scarce is an implication of our assumption that equilibrium is unique.

We begin by defining a set of candidate equilibrium functions, $z(\lambda, \tau), L^N(\lambda, \tau), p^N(\lambda, \tau)$ which satisfy, for a given $\tau, \lambda \geq 0$,

\[
\frac{1}{p^N} \left[ y^T_z (z) - R^* (1 + \lambda) \right] = 0 \tag{7.1}
\]

\[
V^\theta z^{1-\theta} - R^* z = \frac{\gamma}{1-\gamma} \left( K^N \right)^\alpha \left( L^N \right)^{1-\alpha} \tag{7.2}
\]

\[
\frac{1}{p^N} = \frac{1-\gamma}{\gamma} \left[ \frac{\kappa}{1+\tau} - 1 \right] \tag{7.3}
\]

where $p^N \geq 0$ and

\[
\kappa = \frac{\gamma (1-\alpha) \left( K^N \right)^\alpha}{\psi_0 (L^N + L^T)^\psi (L^N)^\alpha}. \tag{7.4}
\]

Equations (7.1) and (7.2) are (3.12) and (3.7), respectively, reproduced here for convenience. Equation (7.3) is obtained by using (3.11) and (3.15) to substitute out for $w/p$.

Let $z_\lambda$ be the value of $z$ that sets the object in square brackets in (7.1) to zero:

\[
z_\lambda = \left( \frac{1-\theta}{R^* (1+\lambda)} \right)^\frac{1}{\theta} V, \ \lambda \geq 0. \tag{7.5}
\]

This will be our candidate equilibrium value of $z$ in case it turns out that $1/p^N > 0$. The function, $z_\lambda$, is strictly positive and strictly decreasing for each $\lambda \geq 0$, and $z_\lambda \to 0$ as $\lambda \to \infty$. Define the function, $c_T^\lambda$, by:

\[
c_T^\lambda \equiv V^\theta z_\lambda^{1-\theta} - R^* z_\lambda, \ \lambda \geq 0
\]
It is readily verified that the function, \( c_\lambda^T \), is strictly decreasing and positive for each \( \lambda \geq 0 \), and that \( c_\lambda^T \to 0 \) as \( \lambda \to \infty \). Let \( L_N^\lambda \) the value of \( L_N^\lambda \) implied by (7.2) for the given value of \( c_\lambda^T \):

\[
L_N^\lambda = \left[ \frac{(1 - \gamma)c_\lambda^T}{\gamma(K_N)} \right]^{1/\alpha}.
\]

Evidently, \( L_N^\lambda \) is strictly positive and strictly decreasing for each \( \lambda \geq 0 \), with \( L_N^\lambda \to 0 \) as \( \lambda \to \infty \).

Define the function, \( \kappa_{\lambda, \tau} \):

\[
\kappa_{\lambda, \tau} = \max \left[ \frac{\gamma (1 - \alpha)(K_N)^\alpha}{\psi_o (L_N^\lambda + L_T)^\psi (L_N^\lambda)} \right].
\]

The first object in square brackets is strictly positive and increasing in \( \lambda \geq 0 \), converging to \( \infty \) as \( \lambda \to \infty \) and converging to a positive constant as \( \lambda \to 0 \). If that constant is less than \( 1 + \tau \), there is a value of \( \lambda \), call it \( \tilde{\lambda} (\tau) \), such that the first and second terms are equal. That is, \( \tilde{\lambda} (\tau) \) is defined by

\[
\frac{\gamma (1 - \alpha)(K_N)^\alpha}{\psi_o (L_N^\lambda + L_T)^\psi (L_N^\lambda)} = 1 + \tau,
\]

if such a \( \tilde{\lambda} (\tau) \geq 0 \) exists. The function, \( \kappa_{\lambda, \tau} \), is strictly positive for \( \lambda \geq 0 \). If \( \tilde{\lambda} (\tau) \) does not exist, then \( \kappa_{\lambda, \tau} \) is strictly increasing in \( \lambda \) for all \( \lambda \geq 0 \), and otherwise \( \kappa_{\lambda, \tau} \) is strictly increasing for all \( \lambda \geq \tilde{\lambda} (\tau) \).

Let

\[
\frac{1}{p^N(\lambda, \tau)} = \frac{1 - \gamma}{\gamma} \left[ \kappa_{\lambda, \tau} \frac{1}{1 + \tau} \right] - 1.
\]

(7.6)

Note that if \( \kappa_{\lambda, \tau} > 1 + \tau \), then \( 1/p^N(\lambda) > 0 \). In this case, condition (7.1) requires the expression in square brackets to be zero, and so in this case we set \( z(\lambda, \tau) = z_\lambda \) and \( L_N^\lambda(\lambda, \tau) = L_N^\lambda \).

Suppose \( \kappa_{\lambda, \tau} = 1 + \tau \). Then condition (7.1) does not require the expression in square brackets to be zero. Let \( L_N^\lambda(\tau) \) be the unique solution to the following expression:

\[
\frac{\gamma (1 - \alpha)(K_N)^\alpha}{\psi_o (L_N^\lambda(\tau) + L_T)^\psi L_N^\lambda(\tau)} = 1 + \tau.
\]

(7.7)

Note that \( L_N^\lambda(\tau) \) is strictly decreasing in \( \tau \). If \( \kappa_{\lambda, \tau} = 1 + \tau \), we set \( L_N^\lambda(\lambda, \tau) = L_N^\lambda(\tau) \). That is:

\[
L_N^\lambda(\lambda, \tau) = \begin{cases} L_N^\lambda, & \frac{1}{p^N(\lambda, \tau)} > 0, \\ L_N^\lambda(\tau), & \frac{1}{p^N(\lambda, \tau)} = 0. \end{cases}
\]

Note,

\[
L_N^\lambda \geq L(\lambda, \tau).
\]

(7.8)

When \( \kappa_{\lambda, \tau} = 1 + \tau \), we use (7.2) to define \( z(\lambda, \tau) \):

\[
V^\theta z(\lambda, \tau)^{1 - \theta} - R^\ast z(\lambda, \tau) = \frac{\gamma}{1 - \gamma} (K_N)^\alpha (L_N^\lambda(\lambda, \tau))^{1 - \alpha}.
\]

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Condition (7.8) and the fact that \( z_\lambda \) and \( L_\lambda^N \) both satisfy (7.2) imply that the previous equation generically has two solutions. The object, \( z(\lambda, \tau) \), is taken to be the smaller of the two solutions. It is easy to verify that 

\[
    z_\lambda \geq z(\lambda, \tau).
\]

Thus, when \( 1/p^N = 0 \), then the object in square brackets in (7.1) evaluated at \( z(\lambda, \tau) \) is zero or, possibly, positive. Either way, (7.1) is satisfied.

This completes our discussion of the candidate equilibrium functions, \( z(\lambda, \tau), L^N(\lambda, \tau), p^N(\lambda, \tau) \). Note that these functions satisfy (7.1)-(7.3), as well as the condition, \( p^N \geq 0 \). Expressions (3.8) and (3.11) can then be used to compute candidate equilibrium functions for \( w \) and \( p \).

Next, we define the asset price functions, based on (3.9) and (3.10):

\[
    q^N(\lambda, \tau) = \frac{\alpha p^N(\lambda, \tau) (K^N)^{\alpha-1} L^N(\lambda, \tau)^{1-\alpha}}{1 - \lambda \tau^N}
\]
\[
    q^T(\lambda, \tau) = \frac{\theta \left( \frac{z(\lambda, \tau)}{V} \right)^{1-\theta} A^\nu \left( \frac{L^T}{K^T} \right)^{1-\nu}}{1 - \lambda \tau^T}
\]

Define:

\[
    C(\lambda, \tau) = \tau^N q^N(\lambda, \tau) K^N + \tau^T q^T(\lambda, \tau) K^T - R^* z(\lambda, \tau).
\]

An equilibrium is a value of \( \lambda \geq 0 \) such that \( C(\lambda, \tau) \geq 0 \) and \( \lambda C(\lambda, \tau) = 0 \).

It is easy to see that if \( 1/p^N = 0 \) when \( \lambda = 0 \), then it is possible to construct two equilibria. In this case, \( \tilde{\lambda}(\tau) \) exists and as \( \lambda \to \tilde{\lambda}(\tau) \) from above, \( 1/p^N \to 0 \). As a result, as \( \lambda \to \tilde{\lambda}(\tau) \) then \( q^N \to \infty \). In particular, \( C(\lambda, \tau) > 0 \) for \( \lambda \) close enough to \( \tilde{\lambda}(\tau) \). Since, for the reasons outlined in the text, \( C(\lambda, \tau) > 0 \) for \( \lambda \) close enough to \( \tilde{\lambda} \), it follows that if there is an equilibrium, there are at least two. We rule out this scenario by assuming that the traded good input, \( c^T \) is scarce. That is, we assume

\[
    \frac{\gamma}{1 - \gamma} \left( K^N \right)^{\alpha} \left( L^N(\bar{\tau}) \right)^{1-\alpha} > c^T_0,
\]

where \( \bar{\tau} \) is the largest value of the labor tax rate, \( \tau \), that we consider. The term on the left of the equality is the equilibrium demand for \( c^T \) when the collateral constraint is absent and \( 1/p^N = 0 \), and the term on the right is the maximal supply. With the above assumption, \( 1/p^N > 0 \) for \( \lambda \geq 0 \), and the argument for multiple equilibria just described does not apply.

The proof of the proposition in the text is now easy to summarize. The function, \( C(\lambda, \tau) \), is continuous and bounded for each \( 0 \leq \lambda < \bar{\lambda} \). As \( \lambda \) approaches \( \bar{\lambda} \), either \( q^N \) or \( q^T \) diverges to \( \infty \). Hence, there is some \( \lambda \) close enough to \( \bar{\lambda} \) such that \( C(\lambda, \tau) > 0 \). Generically, \( C(\lambda, \tau) \) cuts the zero line (see Figure 5) from below.
In an equilibrium with \( \lambda > 0 \), it must be that \( 1/p^N > 0 \). Suppose otherwise, that \( 1/p^N = 0 \). In this case, \( q^N = \infty \) and \( C(\lambda, \tau) > 0 \), contradicting \( C(\lambda, \tau) = 0 \). From \( 1/p^N > 0 \), it follows that \( L^N(\lambda, \tau) \) and \( q^T(\lambda, \tau) \) are not functions of \( \tau \). The only way \( \tau \) enters \( C(\lambda, \tau) \) is via \( p^N(\lambda, \tau) \) in \( q^N(\lambda, \tau) \). It is then easy to see that since \( p^N(\lambda, \tau) \) is increasing in \( \tau \), \( q^N(\lambda, \tau) \) is increasing in \( \tau \) too. Since \( C(\lambda, \tau) \) is increasing in \( \tau \) and \( C(\lambda, \tau) \) is increasing in \( \lambda \) at the equilibrium value of \( \lambda \), for given \( \tau \), it follows that equilibrium \( \lambda \) is decreasing in \( \tau \).

To see what happens to equilibrium \( p^N \) with the increase in \( \tau \), consider (7.6). According to that expression, the increase in \( \tau \) affects \( p^N \) in two ways. The direct channel via the denominator term drives \( p^N \) up. A second channel operates via \( \kappa_{\lambda,\tau} \). When \( 1/p^N > 0 \), \( \kappa_{\lambda,\tau} \) is not a function of \( \tau \), and it is an increasing function of \( \lambda \). So, the fall in \( \lambda \) drives \( p^N \) up. With both channels driving \( p^N \) up after a rise in \( \tau \), we conclude that equilibrium \( p^N \) rises with an increase in \( \tau \).

To see what happens to \( z \), note that when \( 1/p^N > 0 \), then \( z \) is determined by (7.5). The fall in \( \lambda \) induced by the rise in \( \tau \) makes \( z \) increase. Because the collateral constraint is satisfied as a strict equality, we conclude that the value of assets increases. However, it is not clear whether this is because of a rise in \( q^T \) or \( q^N \), or both.

Finally, consider utility. From (3.1):

\[
c = \frac{\psi_0}{1+\psi} (L^N + L^T)^{1+\psi} = \frac{\gamma}{(K^N)^{\alpha}} (L^N)^{1-\alpha} - \frac{\psi_0}{1+\psi} (L^N + L^T)^{1+\psi},
\]

using (3.3) and (3.4). Differentiating this function, it is easy to verify that it is strictly increasing in \( L^N \) up to the point where,

\[
\frac{\gamma (1-\alpha) K^\alpha}{\psi_0 (L^N + L^T)^\psi (L^N)^\alpha} = 1.
\]

Our assumption that \( c^T \) is scarce guarantees \( \kappa > 1 + \tau \) in (7.4). We conclude that utility is increasing in \( \tau \). Q.E.D.

It is straightforward to see what happens when the collateral function, \( C(\lambda, \tau) \), crosses the zero line twice in Figure 5, in which case there are two equilibria. When \( \tau \) is increased there exists an equilibrium in the neighborhood of the high \( \lambda \) equilibrium, which satisfies our proposition. However, there exists an equilibrium in the neighborhood of the low \( \lambda \) equilibrium, in which the results of the proposition are reversed. These observations about comparative statistics when there are multiple equilibria but no credible equilibrium selection mechanism is available are of little practical interest.

8. Appendix B: Algorithm for Solving Dynamic Model

We focus on equilibria in which the collateral constraint is satisfied as an equality for \( t \geq 0 \), and the economy converges to a steady state. In addition, we impose that the steady state is
achieved in a particular period, \( T + 1 \), and that the collateral constraint is (marginally) non-binding then. This is an approximation because, if the system did converge to a steady state, in general this would take infinite time. In practice, we set \( T = 19 \), and verify that the equilibrium conditions are satisfied within an acceptable degree of tolerance.

The computational strategy is a dynamic version of the strategy used to solve the static example in section 3. For a given sequence of (nonnegative) multipliers, \( \lambda_t \), for \( t = 0, 1, ..., T \) on the representative intermediate good firm’s collateral constraint, we use all the equilibrium conditions of the model - apart from the sequence of collateral constraints and traded goods resource constraints - to solve for equilibrium prices and quantities. We do this by solving the relevant equations backwards starting from time \( T \). We then use the traded goods sector resource constraint and the initial debt, to simulate the debt forward to period \( T \). Finally, we adjust the multipliers until the collateral constraint is satisfied as a strict equality in each period, \( t = 0, 1, ..., T \).

To describe the algorithm in greater detail, it is useful to state precisely the timing in the model. Recall that in our environment, the economy is in an initial steady state until period \( t = 0 \), when the collateral constraint unexpectedly becomes binding. At the beginning of period 0, \( L^T_0 \) is set by firms. Because this is done before the realization of the collateral shock, firms set \( L^T_0 \) to its initial steady state value. Then, the collateral shock becomes known and the household makes its deposit decision. This decision is made under the assumption that money growth will continue forever at the constant rate that occurred in the initial steady state. Next, monetary policy from period \( t = 0 \) and later, \( x_0, x_1, x_2, ... \), is revealed. Then, all other period 0 quantities and prices are realized. For periods \( t = 1, 2, ... \) our within-period timing assumptions have no effect because all period \( t \) variables are known at the start of period \( t \) for \( t = 1, 2, ... \).

Given the timing of the model, in order to compute the equilibrium response to an arbitrary sequence, \( x_0, x_1, ... \), we first have to compute the response to the sequence of constant money growths, \( x_t = x, t \geq 0 \), where \( x \) is the money growth rate in the pre-shock equilibrium. The value of \( d_0 \) in the constant money growth equilibrium, is the period 0 level of deposits that is imposed upon the equilibrium response when the arbitrary sequence, \( x_0, x_1, ... \). To find the optimal equilibrium, we search over sequences, \( x_0, x_1, ... \), to find the one that produces an equilibrium with the highest present discounted value of utility. In this search, we only need to solve the constant money growth equilibrium once.

### 8.1. Equilibrium Conditions

At time \( t \) the variables that we want to solve for are the following 16:

\[
VMP_{k,t}^N, VMP_{k,t}^T, q^N_t, q^T_t, c^N_t, c^T_t, L^N_t, L^T_t, p_t, p^N_t, p^T_{t-1}, R_t, z_t, d_t, A_t, y_t^T.
\]
The 16 equations are below. The household and firm intertemporal Euler equations imply:

$$\beta R_t = (1 + x_{t-1}) (1 + \lambda_t) (p_t^T / p_{t-1}^T),$$

for \( t = 1, 2, \ldots \). We obtain equation (8.1) by combining the following three equations. The intertemporal Euler equation associated with the household deposit decision is:

$$u_{c,t} = \frac{p_t}{p_{t+1} p_{t+1}^T} \frac{\beta R_t u_{c,t+1}}{1 + x_t}, \quad t = 1, 2, \ldots$$

(8.2)

The role of this equation in \( t = 0 \) is discussed below. The intertemporal Euler equation of the firm is:

$$\Lambda_t = (1 + \lambda_t) \Lambda_{t+1}, \quad t = 1, 2, \ldots$$

(8.3)

Here, \( \Lambda_t \) is the multiplier on the household’s period \( t-1 \) flow budget constraint in the Lagrangian representation of the household problem. This multiplier satisfies:

$$\Lambda_t = \beta \left( \frac{u_{c,t} p_t^{T-1}}{p_t} \frac{1}{1 + x_{t-1}} \right),$$

(8.4)

for \( t = 1, 2, \ldots \). Equations (8.2)-(8.4) can be combined to produce 8.1.

The nontraded good production function is:

$$c_N^t = (K_N)^{\alpha} (L_N^t)^{1-\alpha}, \quad t = 0, 1, \ldots.$$  

(8.5)

The Leontief assumption on final goods production:

$$c_T^t = \gamma \left( 1 - \gamma \right) c_N^t, \quad t = 0, 1, \ldots.$$  

(8.6)

The intermediate traded good production function:

$$y_T^t = \left\{ \theta [\mu_1 V_t]^{\xi-1} + (1 - \theta) [\mu_2 z_t]^{\xi-1} \right\}^{\frac{\xi}{\xi - 1}}, \quad t = 0, 1, \ldots.$$  

(8.7)

First order condition associated with the choice of the imported intermediate good is:

$$\left( \frac{y_T^t}{\mu_2 z_t} \right)^{\xi} \mu_2 (1 - \theta) = (1 + \lambda_t) R^*, \quad t = 0, 1, \ldots.$$  

(8.8)

Labor in the traded and nontraded sectors receives the same wage, and so the value marginal product of labor in the two sectors must be the same. This implies:

$$(1 - \alpha) p_t^N \frac{c_N^t}{L_t^N} = \left( \frac{y_T^t}{\mu_1 V_t} \right)^{\xi} \theta (1 - \nu) \frac{\mu_1 V_t}{L_t^N}, \quad t = 1, \ldots.$$  

(8.9)

Equation (8.9) does not hold for \( t = 0 \) because employment in the traded good sector is predetermined then.
The value marginal product of capital in the traded good sector, $VMP^T_{k,t}$, is:

$$VMP^T_{k,t} = \left( \frac{y^T_t}{\mu^T_1 V_t} \right)^{\frac{\xi}{\theta}} \theta U \frac{\mu_1 V_t}{K^T}, \ t = 0, 1, \ldots . \quad (8.10)$$

The value marginal product of capital in the non-traded good sector is:

$$VMP^N_{k,t} = \alpha p^N_t c^N_t, \ t = 0, 1, \ldots . \quad (8.11)$$

Combining the money market clearing condition (i.e., the wage bill equals deposits plus new money injections) together with the cash constraint for households in the goods market implies:

$$p_t p^T_t (1 - \gamma) c^T_t = (1 + x_t), \ t = 0, 1, \ldots . \quad (8.12)$$

The condition that total money spend on consumption goods is equal to the wage bill plus money allocated by households to consumption goods implies:

$$p_t (1 - \gamma) c^T_t = (1 - \alpha) \frac{p^N_t}{\xi R_t (1 + \lambda_t) L^N_t} \left( L^T_t + L^N_t \right) + \frac{1 - d_t}{p^T_t}, \ t = 0, 1, \ldots . \quad (8.13)$$

The expressions for the two asset prices are:

$$q^N_t = VMP^N_{k,t} + \lambda t^N q^N_t + \beta \frac{\Lambda_{t+2}}{\Lambda_{t+1}} q^N_{t+1}, \ t = 0, 1, \ldots . \quad (8.14)$$

$$q^T_t = VMP^T_{k,t} + \lambda t^T q^T_t + \beta \frac{\Lambda_{t+2}}{\Lambda_{t+1}} q^T_{t+1}, \ t = 0, 1, \ldots . \quad (8.15)$$

Equality of labor supply and labor demand in the non-traded good sector implies:

$$\psi_0 \left( L^N_t + L^T_t \right)^{\psi} p_t = (1 - \alpha) \frac{p^N_t}{\xi R_t (1 + \lambda_t) L^N_t} c^N_t, \ t = 0, 1, \ldots . \quad (8.16)$$

The relative price of final goods to traded goods is, given the Leontieff technology:

$$p_t = \frac{1}{1 - \gamma} + \frac{1}{\gamma} p^N_t, \ t = 0, 1, \ldots . \quad (8.17)$$

8.2. Steady State

Since the algorithm begins with the new steady state, we begin by describing how this was computed. Equation (8.1) in steady state implies:

$$R = \frac{1 + x}{\beta}, \quad (8.18)$$

where $x$ is the money growth rate in the new steady state. Equation (8.8) implies:

$$\left( \frac{y^T}{\mu^T_2 z} \right)^{\frac{\xi}{\theta}} \mu_2 (1 - \theta) = R^*. \quad (8.19)$$
Equation (8.9) implies:

\[(1 - \alpha) p^N c^N \frac{L^N}{L^N} = \left( \frac{y^T}{\mu_1 V} \right)^{\frac{1}{\theta}} (1 - \nu) \frac{\mu_1 V}{L^T}, \tag{8.20} \]

Equations (8.10), (8.11), (8.14) and (8.15) imply

\[q^N = \alpha p^N c^N \frac{K^N}{1 - \beta}, \tag{8.21} \]
\[q^T = \left( \frac{y^T}{\mu_1 V} \right)^{\frac{1}{\theta}} \mu_1 V \frac{L^T}{K^T}. \tag{8.22} \]

Equation (8.16) implies:

\[\psi_0 (L^N + L^T) \psi p = (1 - \alpha) p^N c^N \frac{\zeta R L^N}{L^N} \tag{8.23} \]

The traded goods resource constraint is:

\[y^T - R^* z - c^T = r^* B, \tag{8.24} \]

which says that net exports must equal the interest on the international debt. The collateral constraint evaluated at equality is:

\[\tau^N q^N K^N + \tau^T q^T K^T = R^* z + (1 + r^*) B + \zeta R (1 - \alpha) p^N c^N \frac{L^T + L^N}{L^N}, \tag{8.25} \]

where we have imposed

\[w = (1 - \alpha) p^N c^N \frac{L^N}{L^N}. \tag{8.26} \]

The endogenous variables here are the following nine: \(L^N, L^T, p^N, z, q^T, q^N, w, R\) and \(B\). The nine equations, (8.18)-(8.26), can be used to solve for these variables (the variables, \(c^N, y^T\) and \(V\) are solved using the relevant production functions).

### 8.3. Backward Recursion

We now discuss how prices and quantities are computed based on a given set of sequences, \(\lambda_0, \lambda_1, \ldots, \lambda_T\) and \(x_0, x_1, \ldots, x_T\). These are obtained by solving the equilibrium conditions recursively, beginning with the steady state and working backwards. We start the backward iteration in period \(T\), when \(p^T_t\) for \(t = T\) and all other variables dated \(T + 1\) and later are assumed to be in a steady state. The calculations are done in two steps. First, we proceed for \(t = T, T - 1, \ldots, 1\). After that, we consider the variables in \(t = 0\).

It is convenient to substitute out for \(p_t\) from (8.17) into (8.12), (8.13), (8.4), and (8.16). With this change, we have the following 15 unknowns:
\[ VMP_{k,t}^N, VMP_{k,t}^T, q_t^N, q_t^T, c_t^N, c_t^T, L_t^N, L_t^T, p_t^N, p_{t-1}^T, R_t, z_t, d_t, \Lambda_t, y_t^T. \]

in 15 equations. We reduce these equations to one equation in one unknown, \( L_t^N \). Thus, fix \( L_t^N \). Then, \( c_t^N \) and \( c_t^T \) are computed from (8.5) and (8.6), respectively. The variable, \( p_t^N \) is computed using (8.12) with \( p_t \) replaced with (8.17). The variables, \( z_t, L_t^T, \) and \( y_t^T \) are computed using (8.7), (8.8) and (8.9). We computed \( \Lambda_t \) using (8.3). We then computed \( p_{t-1}^{T} \) using (8.4) and the interest rate, \( R_t \), using (8.1). The variables, \( VMP_{k,t}^T \) and \( VMP_{k,t}^N \) are computed using (8.10) and (8.11). Then, (8.14) and (8.15) are solved for the asset prices, \( q_t^T \) and \( q_t^N \). We adjust \( L_t^N \) until equation (8.16) is satisfied. The variable, \( d_t \), is computed using (8.13). We proceed sequentially, for \( t = T, T-1, \ldots, 1 \).

We now consider \( t = 0 \). Relative to the previous list of unknowns, we drop 3 variables: \( p_{T-1}^T, \Lambda_0, L_0^T \). We drop \( \Lambda_0 \) because (8.3) is only satisfied for \( t = 1, 2, \ldots \). We drop \( L_0^T \) because this variable is set to its value in the initial steady state. The list of 12 unknowns for this period is:

\[ VMP_{k,0}^N, VMP_{k,0}^T, q_0^N, q_0^T, c_0^N, c_0^T, L_0^N, p_0^N, R_0, z_0, d_0, y_0^T. \]

We reduce these equations to one equation in the one unknown, \( L_0^N \). Fix the value of \( L_0^N \).

Suppose we are in the constant money growth case. This corresponds to \( x_t = x \), for \( t \geq 0 \), where \( x \) denotes the money growth rate in the pre-shock steady state equilibrium. We obtain the values of \( c_0^N \) and \( c_0^T \) from (8.5) and (8.6), as before. The variable, \( p_0^N \) is computed using (8.12) with \( p_0 \) replaced with (8.17). The variables, \( z_0, \) and \( y_0^T \) are computed using (8.7) and (8.8). We use (8.2) to compute \( R_0 \). We then obtain \( VMP_{k,0}^T \) and \( VMP_{k,0}^N \) from equations (8.10) and (8.11). Asset prices, \( q_0^N \) and \( q_0^T \), are found using (8.14) and (8.15). Equation (8.13) can be used to compute \( d_0 \). Finally, \( L_0^N \) is adjusted until (8.16) is satisfied for \( t = 0 \).

The long term debt \( B_{t+1} \) can be obtained by simulating forward the traded good market clearing conditions

\[ y_t^T - R^* z_t - r^* B_t - c_t^T = - (B_{t+1} - B_t), \quad t = 0, 1, \ldots. \quad (8.27) \]

for a fixed \( B_0 \). We adjust the \( T + 1 \) numbers, \( \lambda_t \geq 0, t = 0, \ldots, T, \) until the collateral constraints are satisfied:

\[ \tau^N q_t^N K^N + \tau^T q_t^T K^T - [R^* z_t + (1 + r^*)B_t + \zeta R_t w_t (L_t^T + L_t^N)] = 0, \quad t = 0, 1, \ldots. \quad (8.28) \]

Here, \( w_t \) denotes the wage rate, which is identified with the left side of (8.16).

Now consider the case of an arbitrary sequence, \( x_t, t \geq 0 \). In this case, \( d_0 \) is fixed at its value in the constant money growth equilibrium. Drop equation (8.2), and when \( R_0 \) is computed
in the previous algorithm, we compute it using equation (8.13). Otherwise, the algorithm with
an arbitrary sequence of \( x_t \)'s is identical to the constant money growth algorithm.

In sum, the algorithm for finding an equilibrium conditional on a sequence of \( x_t \)'s reduces to
one of solving \( T + 1 \) equations in the \( T + 1 \) unknown multipliers. In practice, we reduced the
dimension of unknowns slightly. In particular, we let \( \lambda_t, t = 0, \ldots, N - 1 \) be free parameters and
obtained \( \lambda_t \) for \( t = N, \ldots, T \) by linear interpolation between \( \lambda_{N-1} \) and \( \lambda_{T+1} = 0 \). We found that
this procedure works well for \( T = 19 \) and \( N = 17 \).

We briefly indicate the more complicated algorithm that would be required in case we did
not want to impose that the collateral constraint is satisfied as a strict equality in each period.
Let \( B \) denote the steady state value of debt which is assumed to be realized in period \( T + 1 \). The
seven equations, (8.18)-(8.24), can be used to solve for the seven variables, \( R, z, L^T, L^N, p^N, q^N \) and \( q^T \). It can then be verified that the collateral constraint is satisfied. If it is not, then a
different value of \( B \) should be tried. After this, we implement the solution algorithm described
in this section, by finding a sequence of multipliers, \( \lambda_0, \lambda_1, \ldots, \lambda_T \) with the following properties:
(i) each is non-negative, (ii) the product of the multiplier and the collateral constraint is zero in
each period, \( t = 0, 1, \ldots, T \), (iii) the collateral constraint is satisfied in each period, \( t = 0, \ldots, T \).
Once this has been accomplished, we compare \( B_{T+1} \), the debt that results from the forward
simulation step, with \( B \). If \( B \neq B_{T+1} \), then adjust \( B \) until \( B_{T+1} = B \).

8.4. Optimal Monetary Policy

To solve for the optimal monetary policy, we search over sequences of \( x_t \)'s, \( t \geq 0 \). In principle
this is a impractically high-dimensional space. We reduced the dimension of this space by making
\( x_0, x_1 \) and \( x_2 \) free parameters. We impose that the optimal monetary policy involves setting \( x_t \)
for \( t \geq 3 \) to a value slightly above the one implied by the Friedman rule, \( x = \beta - 1 + \varepsilon \), where
\( \varepsilon = 0.0087 \). We ran into numerical difficulties when we set \( x = \beta - 1 \) precisely.

To find the optimal policy in this constrained space, we searched for \( x_0, x_1, \) and \( x_2 \) on a
sequence of grids. The first grid is a coarse one, and is used to obtain a rough idea about the
location of the optimal policy. The grid of points for \( x_t \) is:

\[
\chi_t^0 = (0, .1, .2, .3, .4), \text{ for } t = 0, 1, 2.
\]

We computed an equilibrium for each of the 125 points belonging to \( \chi_0^0 \times \chi_1^0 \times \chi_2^0 \). Denote
the point on the first grid associated with the highest level of utility by \( (x_0^1, x_1^1, x_2^1) \). We then
computed a second grid of 125 points around \( (x_0^1, x_1^1, x_2^1) \). In this second grid, the grid of points
for \( x_t \) is

\[
\chi_t^1 = (x_t^1 - 0.04, x_t^1 - 0.02, x_t^1, x_t^1 + 0.02, x_t^1 + 0.04), \text{ for } t = 0, 1, 2.
\]

We then computed an equilibrium for each of the 125 points belonging to \( \chi_0^1 \times \chi_1^1 \times \chi_2^1 \). Denote
the point in this grid associated with the highest level of utility by \( (x_0^2, x_1^2, x_2^2) \). A new grid of
points was constructed around this point. Let the grid for $x_t$ be:

$$\chi_t^2 = (x_t^2 - 0.004, x_t^2 - 0.002, x_t^2, x_t^2 + 0.002, x_t^2 + 0.004), \ t = 0, 1, 2.$$  

We then computed equilibrium for each of the 125 points belonging to $\chi_0^2 \times \chi_1^2 \times \chi_2^2$. The best point on this grid is our estimate of the globally optimal monetary policy. The money growth rates associated with the optimal policy computed in this way are $x_0 = 0.24$, $x_1 = -0.01$, $x_2 = -0.01$. Also, $x_t = -0.02$, for $t > 2$.

The above calculations involve solving for an equilibrium of our model at a total of 375 sequences of money growth rates. For each of these sequences, we must solve a system of nonlinear equations (i.e., the complementary slackness conditions) for an equal number of multipliers. To solve these equations an initial guess of the multipliers is required. In all cases, we used the multipliers associated with the constant money growth rule.
SHORT-TERM INTEREST RATES 1/

Figure 1
Intermediate Goods Import vs. GDP
(Index 1995 = 100)

Sources: CEIC; and WEO.
Figure 3: Exports and Imports

Korea

Malaysia

Philippines

Thailand
EXCHANGE RATES
(national currency/US$)

Figure 4
The Effect of An Increase in the Labor Tax Rate

Figure 5: The Effect of An Increase in the Labor Tax Rate
Figure 6: Equilibrium Associated with Various Tax Rates

- $L^N$ vs. $\tau$
- $\lambda$ vs. $\tau$
- $z$ vs. $\tau$
- Utility vs. $\tau$
- $1/p^N$ vs. $\tau$
- $\tau Nq K^N + \tau^T q K^T$ vs. $\tau$
Figure 7: Timing

Collateral Shock

Monetary Action

Intermediate Good Firm Decides Employment in Traded Sector

Household Deposit Decision

Production, Consumption Occur
Figure 8: Optimal Money Growth and Korean Data
Figure 9: Optimal and Constant Money Growth

- Current account
- Employment
- Real GDP
- Consumption
- Imports
- Asset Prices
- Nominal Interest Rate
- Nominal Exchange Rate (Price of Traded)
- Inflation
- Lagrange Multiplier

Legend:
- Optimal Money Growth
- Constant Money Growth