Two Reasons Why Money and Credit May be Useful in Monetary Policy

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Abstract

We describe two examples which illustrate in different ways how money and credit may be useful in the conduct of monetary policy. Our first example shows how monitoring money and credit can help anchor private sector expectations about inflation. Our second example shows that a monetary policy that focuses too narrowly on inflation may inadvertently contribute to welfare-reducing boom-bust cycles in real and financial variables. The example is of some interest because it is based on a monetary policy rule fit to aggregate data. We show that a policy of monetary tightening when credit growth is strong can mitigate the problems identified in our second example.

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1. Introduction

The current consensus is that money and credit have essentially no constructive role to play in monetary policy. When Michael Woodford first suggested this possibility before a large gathering of prominent economists in Mexico City in 1996, the audience was mystified (Woodford, 1998). The consensus at the time was the one forged by Milton Friedman, according to which inflation is ‘always and everywhere a monetary phenomenon’. The pendulum has now swung to the other extreme, in the form of a new consensus which de-emphasizes money completely. We briefly review the reasons for this shift, before presenting two examples which suggest the pendulum may have swung too far.

The experience of the 1970s showed that the inflation expectations of the public can lose their anchor and that when this happens the social costs are high. To stabilize inflation expectations, monetary authorities have evolved versions of the following policy: When the evidence suggests that inflation will rise above some numerical objective, the monetary authority responds proactively by tightening monetary policy. Monetary policy is loosened in response to signs that inflation will fall below the numerical objective.¹ A rough characterization of such a policy expresses the interest rate, $R_t$, as a function of expected inflation, $\pi_{t+1}^{e}$, and other variables, $x_t$:

$$R_t = \rho R_{t-1} + (1 - \rho) \left[ \alpha_x \left( \pi_{t+1}^{e} - \pi_t^* \right) + \alpha_x x_t \right].$$  \hspace{1cm} (1.1)

¹In practice, monetary policy strategies differ according to how vigorously the central bank responds to changing signals about future inflation, and how much weight it assigns to other factors, such as the state of the real economy. Strategies also differ according to how heavily they make use of formal econometric models of the economy. In recent years, there has been much progress towards integrating formal models into the design of monetary policy. For example, Giannoni and Woodford (2005), Svensson and Tetlow (2005), and Svensson and Woodford (2005) propose replacing (1.1) by the optimal policy relative to a specified objective function.
Here, $\pi_t^*$ denotes the monetary authority’s inflation target. When $\rho > 0$, policy acts to minimize large movements in the interest rate from one period to the next.\footnote{For a rationale, see Woodford (2003b).} Although we have included only the one-period-ahead forecast of inflation in this rule, what we have to say goes through in the more plausible case where central bank policy is driven by the longer-term outlook for inflation. We will refer to the rule as a Taylor rule, though that is not strictly speaking accurate since the rule John Taylor discusses reacts only to current inflation and output. We chose to include expected inflation in (1.1) to account for the fact that in practice monetary authorities must anticipate economic developments in advance, since policy actions may have very little immediate impact and thus may take time to exert their influence on the economy.\footnote{This point was stressed by Svensson (1997).} We do not mean to suggest that any central bank’s policy is governed by a rigid rule like (1.1). We think of (1.1) only as a rough characterization, one that allows us to make our points about the role of money and credit in monetary policy.

There are two reasons for the current consensus that money and credit have essentially no role to play in monetary policy. First, these variables are not included in (1.1). Second, monetary theory lends some support to the notion that money demand and supply are virtually irrelevant in determining the operating characteristics of (1.1). For intuition, recall the undergraduate textbook IS-LM model with an aggregate supply side. In this model, money balances do not enter the spending decisions underlying the IS curve, and they do not enter the considerations determining the supply curve. If monetary policy is characterized by an interest rate rule like (1.1), then the equilibrium of the model is determined independently of the LM curve.\footnote{The notion that money balances literally do not interact with consumption and investment} That is, the operating characteristics of (1.1) can be studied without taking
a stand on the nature of money demand or money supply.\(^5\)

In what follows we present two examples in which a strategy such as (1.1) is not successful at stabilizing the economy. In each example outcomes are improved if: (a) the central bank carefully monitors monetary indicators and (b) it reacts or threatens to react to such indicators in case inflation expectations or asset price formation get out of control. By "monetary indicators" we mean aggregates defined both on the liability (i.e. money proper) and the asset side (i.e. credit) of monetary institutions. After presenting the examples, we provide some concluding remarks. An appendix discusses our first example in greater detail.

2. First Example: Anchoring Inflation Expectations

Our first example illustrates points emphasized by Benhabib, Schmitt-Grohe and Uribe (2001,2002a,b), Carlstrom and Fuerst (2002,2005) and Christiano and Ros-decisions is implausible. Most theories of money demand rest on the premise that money balances play a role in facilitating transactions and that money balances therefore do interact with other decisions. However, experience has shown that those theories also imply that the role of money in consumption, investment and employment decisions is quantitatively negligible (See, for example, McCallum, 2001.) That is, the insight based on the textbook macro model that one can ignore money demand and money supply when monetary policy is governed by (1.1) is a very good approximation in a broad class of models.

\(^5\)A third reason that is sometimes given for ignoring money demand is that money demand is unstable. This overstates the instability of money demand and understates the stability of non-financial variables. A simple graph of the money velocity based on the St. Louis Fed’s measure of transactions balances, MZM, against the interest rate shows a reasonably stable relation. At the same time, the US consumption to output ratio suddenly began to trend up since the early 1980s, and is now about 6 percentage points of GDP higher than it used to be. This change in trend almost fully explains a similar change in trend in the US current account. No one would suggest not looking at the current account, consumption or GDP because of this evidence of instability.
tagno (2001) (BSU-CF-CR). Although (1.1) may be effective at anchoring inflation expectations in some models, the finding is not robust to small, empirically plausible, changes in model specification. This is of concern because there is considerable uncertainty about the correct model specification.

We begin by discussing why (1.1) is effective in anchoring inflation expectations in the simple New-Keynesian model. We then introduce a slight modification to the environment which captures in spirit of many of the examples in BSU-CF-CR. The modification is motivated by the evidence that firms need to borrow substantial amounts of working capital to finance variable inputs like labor and intermediate goods. This modification introduces a supply-side channel for monetary policy and creates the possibility for inflation expectations to lose their anchor. This is so, even if monetary policy acts aggressively against inflation by assigning a high value to $\alpha \pi$ in (1.1). The resulting instability affects all the variables in the model, including money and credit. A commitment by the monetary authority to monitor these variables and to react when they exhibit instability that is not clearly linked.

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6 As Benhabit, Schmitt-Grohe and Uribe (2001) note, even in this model there is a global problem of multiple equilibria. In addition to the ‘normal’ equilibrium, there is another equilibrium in which the interest rate drops to zero. However, an escape clause strategy in which the monetary authority commits to deviating from (1.1) to a policy of controlling the money supply in the event that the interest rate drops to zero eliminates this equilibrium in many models. This observation reinforces our basic point that money may have a constructive role to play in monetary policy. For further discussion, see Christiano and Rostagno (2001).

7 See, for example, Christiano, Eichenbaum and Evans (1996) and Barth and Ramey (2002). The existence of a supply-side channel for monetary policy is potentially an explanation for the ‘price puzzle’, the finding in structural vector autoregressions that inflation tends to rise for a while after a monetary tightening (see Christiano, Eichenbaum, and Evans, 1999). Additional evidence on the importance of the supply-side channel is provided in the appendix.
to fundamental economic shocks (including money demand shocks) keeps inflation expectations anchored within a narrow range. In effect, the strategy corresponds to operating monetary policy according to (1.1) with a particular ‘escape clause’: a commitment to control money and credit aggregates directly in case these variables misbehave. The strategy works like the textbook analysis of a bank run. The government’s commitment to supply liquidity in the event of a bank run eliminates the occurrence of a bank run in the first place, so that government never has to act on its commitment. Similarly, the monetary authority’s commitment to monitor money growth and reign it in if necessary implies that inflation expectations and thus money growth never get out of line in the first place.

Here, we provide an intuitive discussion. A formal, numerical analysis is presented in the appendix. Suppose that the economy is described by the IS-LM model augmented by a supply curve, as in Figure 1. On the vertical axis of Figure 1a, we display the nominal rate of interest and on the horizontal axis appears aggregate real output, $y$. Note that the IS curve is a function of expected inflation because the spending decisions summarized in that curve are a function of the real interest rate. The LM curve summarizes money market equilibrium in the usual way. Figure 1b displays the supply side of the economy, in which higher output is associated with higher inflation. The curve captures the idea that higher output raises pressure on scarce resources, driving up production costs and leading businesses to post higher prices.

Suppose that monetary policy responds to a one percentage point rise in expected inflation, $\pi^e$, by raising the nominal rate of interest by more than one percentage point (the ‘Taylor principle’). It is easy to see that a monetary authority which follows the Taylor principle in the simple model of Figure 1 succeeds in anchoring the public’s expectations about inflation. In particular, suppose a belief begins to circulate that
inflation will rise, so that $\pi^e$ jumps. The monetary authority reacts by reducing the money supply so that the nominal rate of interest rises by more than the rise in $\pi^e$ (see Figure 1a). The resulting shift up in the LM curve causes output to fall from $y_1$ to $y_2$. The fall in output, by reducing costs, leads to a fall in inflation (Figure 1b). Thus, in the given model and under the given monetary policy, a spontaneous jump in expected inflation produces a chain of events that ultimately places downward pressure on actual inflation. Under these circumstances, a general fear that inflation will rise could not persist for long. Thus, inflation expectations are anchored under the Taylor principle in the given model.

To see how crucial the Taylor principle is for anchoring inflation expectations in the model of Figure 1, suppose the monetary authority did not apply the Taylor principle. That is, the monetary authority responds to a one percent rise in expected inflation by raising the nominal rate of interest by less than one percent. In terms of Figure 1, this means that the monetary authority shifts the LM curve up by less than the rise in $\pi^e$. The resulting fall in the real rate of interest implies an increase in spending. The rise in spending leads to a rise in output and, hence, costs. The rise in costs in turn places upward pressure on inflation. In this way a rise in expected inflation initiates a chain of events that ultimately produces a rise in actual inflation. The outcome is that inflation expectations are self-fulfilling and have no anchor.

Now consider the modification to the economy that we mentioned in the introduction. In particular, suppose that when the nominal interest rate is increased, the output-inflation trade-off shifts up (see Figure 2b). This could occur because an increase in the interest rate directly increases the cost of production by raising expenses associated with financing inventories, the wage bill and other variable costs.\(^8\)

\(^8\)We cite evidence in the appendix which indicates borrowing for variable inputs may be substantial in practice.
Prices might also rise as a by-product of the tightening in balance sheets that occurs as higher interest rates drive asset values down. Suppose, as before, that inflation expectations rise and the monetary authority follows the Taylor principle. The monetary authority shifts the LM curve up by more than the amount of the increase in $\pi^e$, so that the real rate of interest rises. Spending falls. If the supply curve did not shift, then our previous analysis indicates that actual inflation would fall and the higher $\pi^e$ would not be confirmed. But, under the modified scenario tightening monetary conditions produce such a substantial rise in costs that actual inflation rises. In this scenario, a rise in inflation expectations produces a chain of events that ultimately results in higher inflation. The outcome is that, despite the application of the Taylor principle, inflation expectations have no anchor.

It is asking too much of our simple diagrams to use them to think through what happens over time when inflation expectations have lost their anchor. For this, an explicit dynamic equilibrium model is required. The analyses reported in BSU-CF-CR do this, and there we see that when things go wrong all economic variables fluctuate over time in response to non-fundamental economic shocks. Among these variables is the money supply. It is shown that a monetary policy which commits to deviating from the Taylor rule as soon as money is observed to respond to non-fundamental shocks in effect anchors inflation expectations. To implement this policy requires a public commitment to monitor the money supply carefully and to expend resources analyzing the reasons for its fluctuations. Paradoxically, in practice it will seem like the monitoring policy is pointless.

A concluding remark about this example deserves emphasis. According to the theoretical analyses that support the idea of an escape clause strategy, *all* variables

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*It is possible to construct examples in which even this policy will not anchor expectations, though these examples seem unlikely. For further discussion, see Christiano and Rostagno (2001).*

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in the economy exhibit instability when inflation expectations lose their anchor. The models imply that an escape clause strategy which abandons (1.1) in favor of stabilizing any economic variable - not necessarily money per se - works equally well. When we conclude that the right variable to control in the event that the escape clause is activated is money, we introduce considerations that lie outside the models. Economic models assume that the monetary authority has perfect control over any one variable in the economy with its one policy instrument. It can control money as easily as the current account or Gross National Product. In reality, there is only one variable that the monetary authority controls directly and credibly, and that is money. All other variables that it may attempt to control - the interest rate, the current account, etc. - can only be controlled indirectly, by virtue of the monetary authority’s control of the money supply.\footnote{The monetary aggregate directly controlled by the US Federal Reserve is the nonborrowed reserves of banks.} What is crucial for the escape clause strategy to work is that the central bank be able to credibly control whatever variable it commits to control in the event that the escape clause is activated. In practice, there really is only one such variable: money.

3. Second Example: Asset Market Volatility

Our second example summarizes the analysis of Christiano, Ilut, Motto and Rostagno (2007). This example builds on the analysis of Beaudry and Portier (2004, 2006), which suggests that a substantial fraction of economic fluctuations may be triggered by the arrival of signals about future improvements in productivity. We find that when such a signal shock is fed to a standard model used in the analysis of business cycles, it produces patterns that in many ways resembles the boom-bust
cycles that economies experience periodically.\textsuperscript{11} In the model, the response of investment, consumption, output and stock prices greatly exceed what is socially efficient. The excess volatility reflects two features of the model: (i) there are frictions in the setting of wages and (ii) monetary policy focuses too narrowly on inflation. The finding should be cause for concern, because there is substantial evidence that wage frictions are important and because the monetary policy rule used in our analysis is a version of (1.1) in which the parameters have been estimated using aggregate data. Because the nominal wage rate is relatively sticky in the model, an overly narrow focus on inflation stabilization in effect reduces to real wage stabilization. Such a policy produces bad outcomes because it interferes with the efficient allocation of resources. Although this is well known as a matter of principle (see, e.g., Erceg, Henderson and Levin, 2000), what is less well known is that is that a policy rule like (1.1) can make the monetary authorities unwitting participants in boom-bust episodes. Although additional empirical research is necessary, there is indirect evidence consistent with the view that a narrow focus on inflation stabilization may produce instability. For example, Cecchetti and Ehrmann (2002) present evidence that suggests that adopting an inflation targeting monetary regime may increase output volatility.\textsuperscript{12}

\textsuperscript{11}We use a variant of the model proposed in Christiano, Eichenbaum and Evans (2005) and further analyzed in Smets and Wouters (2003, 2007).

\textsuperscript{12}Several countries refocused their monetary policy more narrowly on inflation by formally adopting inflation targeting in the 1980s and 1990s. To isolate the impact of this policy change on output volatility, one has to disentangle the effects of the change from the effects of all the other factors that produced a moderation in volatility in this period (the ‘Great Moderation’). Cecchetti and Ehrmann (2002) do this by computing the standard deviation of output growth in countries that adopted inflation targeting and in countries that are non-targeters. They report that the average volatility across non-targeters in the 1985-1989 period and the 1993-1997 period is 10.12 and 7.41,
There is a long tradition which locates the cause of boom-bust cycles in excessive credit creation. A recent review of this tradition, and some evidence to support it, is provided in Eichengreen and Mitchener (2004). Motivated by this strand of literature, we introduce credit into our model. We do this by introducing frictions into the financing of capital, following the lead of Bernanke, Gertler and Gilchrist (1999). We find that when (1.1) is amended to include credit growth, then the response of the economy to the signal shock much more closely resembles the efficient response. We find that adding credit to (1.1) also brings the model response to other shocks more closely in line with the efficient response.

We now summarize the analysis with a little more detail. We begin by briefly describing the baseline model used in the analysis. We then turn to the results.

3.1. Model

To accommodate frictions in price-setting, we adopt the usual Dixit-Stiglitz specification of final good production:

$$Y_t = \left[ \int_0^1 Y_{jt} \lambda^d d\lambda \right]^{1/\lambda_f}, \quad 1 \leq \lambda_f < \infty,$$

where $Y_t$ denotes aggregate output and $Y_{jt}$ denotes the $j$th intermediate good. Intermediate good $j$ is produced by a price-setting monopolist according to the following respectively. The analogous results for targeters is 7.47 and 6.92 (see their Table 1). Thus, targeters experienced a change of -.55 and non-targeters achieved a change of -2.71. Assuming the policy regime is the only difference between targeters and the targeters, one infers that inflation targeting per se increased volatility by 2.16 percentage points. Presumably, this is an over estimate. But, if the correct number was only half as large, it would still be cause for concern.

13See also Borio and Low (2002).

14We considered shocks to the cost of investment goods, a cost-push shock, a shock to actual technology, a shock to the discount rate and a shock to the production function for converting investment goods into installed capital.
technology:

\[
Y_{jt} = \begin{cases} 
\varepsilon_t K_{jt}^{\alpha} (z_t l_{jt})^{1-\alpha} - \Phi z_t & \text{if } \varepsilon_t K_{jt}^{\alpha} (z_t l_{jt})^{1-\alpha} > \Phi z_t, \\
0, & \text{otherwise}
\end{cases}, \quad 0 < \alpha < 1, 
\]

where \( \Phi z_t \) is a fixed cost and \( K_{jt} \) and \( l_{jt} \) denote the services of capital and homogeneous labor. Capital and labor services are hired in competitive markets at nominal prices, \( P_t r^k_t \), and \( W_t \), respectively. The object, \( z_t \), is the deterministic source of growth in the economy, with \( z_t = \mu_z z_{t-1} \) and \( \mu_z > 1 \). The other technology factor, \( \varepsilon_t \), is stochastic. The time series representation of \( \varepsilon_t \) is specified as follows:

\[
\log \varepsilon_t = \rho \log \varepsilon_{t-1} + \varepsilon_{t-p} + \xi_t, 
\]

where \( \varepsilon_t \) and \( \xi_t \) are uncorrelated over time and with each other. Here, \( \varepsilon_t \) is a ‘news’ shock, which signals a move in \( \log \varepsilon_{t+p} \). The other shock, \( \xi_t \), reflects that although there is some advance information on \( \varepsilon_t \), that information is not perfect. In the simulation experiment, we consider the following impulse. Up until period 1, the economy is in a steady state. In period \( t = 1 \), a signal occurs which suggests \( \varepsilon_{1+p} \) will be high. But, when period \( 1 + p \) occurs, the expected rise in technology in fact does not happen because of a contrary move in \( \xi_{1+p} \). We refer to a disturbance in \( \varepsilon_t \) as a ‘signal shock’.

The firm sets prices according to a variant of Calvo sticky prices.\(^{15}\) In each period

\(^{15}\)Price-setting frictions only play a small role in our analysis. Because prices are set in a forward-looking way, they do help the model produce a fall in inflation throughout the boom period. We conjecture that our basic results about the relationship between monetary policy and boom-bust episodes are robust to the use of any other form of price setting frictions that entail forward-looking behavior. The reason we specifically use Calvo-sticky prices is that they are computationally convenient to work with and they are consistent with some key features of the data: the fact that there are many small price changes (Midrigan (2005)) and the fact that the hazard rate of price changes for many individual goods is roughly constant (Nakamura and Steinsson (2006)).

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an intermediate good firm can reoptimize its price with probability, \( 1 - \xi_p \). With the complementary probability, a firm cannot reoptimize. The \( i^{th} \) firm that cannot reoptimize sets its price according to:

\[
P_{it} = \tilde{\pi}_t P_{i,t-1},
\]

where

\[
\tilde{\pi}_t = \pi_{t-1} \bar{\pi}^{1-t}.
\] (3.4)

Here, \( \pi_t \) denotes the gross rate of inflation, \( \pi_t = P_t/P_{t-1} \), and \( \bar{\pi} \) denotes steady state inflation. If the \( i^{th} \) firm is able to optimize its price at time \( t \), it chooses \( P_{i,t} = \tilde{P}_t \) to optimize discounted profits:

\[
E_t \sum_{j=0}^{\infty} (\beta \xi_p)^j \lambda_{t+j} [P_{i,t+j} Y_{i,t+j} - P_{t+j} s_{t+j} (Y_{i,t+j} + \Phi z_{t+j})],
\] (3.5)

where \( \lambda_{t+j} \) is the multiplier on firm profits in the household’s budget constraint. Also, \( P_{i,t+j}, j > 0 \) denotes the price of a firm that sets \( P_{i,t} = \tilde{P}_t \) and does not reoptimize between \( t+1, \ldots, t+j \). The equilibrium conditions associated with firms are standard.

We model the labor market as in Erceg, Henderson and Levin (2000).\(^\text{16}\) The homogeneous labor employed by firms in (3.2) is ‘produced’ from specialized labor inputs according to the following linear homogeneous technology:

\[
l_t = \left[ \int_0^1 (h_{t,i})^{\lambda_w} \lambda_w \right]^{\lambda_w}, \quad 1 \leq \lambda_w.
\] (3.6)

We suppose that this technology is operated by perfectly competitive labor contractors, who hire specialized labor from households at wage, \( W_{jt} \), and sell homogeneous

\(^{16}\text{This particular representation of wage frictions is thought to be vulnerable to the critique of Barro (1977). Christiano, Ilut, Motto and Rostagno (2007) report that when a variant of the labor market model suggested by Gertler, Sala and Trigari (2007) is used instead, the basic qualitative results described here go through. The latter model is not vulnerable to the Barro critique.}\)
labor services to the intermediate good firms at wage, $W_t$. Optimization by labor contractors leads to the following demand for $h_{t,i}$:

$$h_{t,i} = \left( \frac{W_{t,i}}{W_t} \right)^{\frac{\lambda_w}{1-\lambda_w}} l_t, \ 1 \leq \lambda_w. \tag{3.7}$$

The $j^{th}$ household maximizes utility

$$E_{s}^{t} \sum_{t=0}^{\infty} \beta^{t} \left\{ u(C_{t+1} - bC_{t+1} - \psi L_{t,j} \frac{h_{t,j}}{1+\sigma_L} - v \left( \frac{P_{t+1}C_{t+1}}{M_{t+1}^j} \right)^{1-\sigma_q} \right\} \tag{3.8}$$

subject to the constraint

$$P_t (C_t + I_t) + M_{t+1}^d - M_t^d + T_{t+1} \leq W_{t,j}l_{t,j} + P_t r_t K_t + (1 + R_{t-1}) T_t + A_{j,t}, \tag{3.9}$$

where $M_t^d$ denotes the household’s beginning-of-period stock of money and $T_t$ denotes nominal bonds issued in period $t-1$, which earn interest, $R_{t-1}$, in period $t$. This nominal interest rate is known at $t-1$. The magnitude of $\nu$ controls how much money balances households hold on balance. We found that when $\nu$ is set to reproduce the velocity of money in actual data, the properties of the model are virtually identical to what they are when $\nu$ is set essentially to zero. Thus, we simplify the analysis without losing anything if we work with the ‘cashless limit’, the version of the model in which $\nu$ is zero.

The $j^{th}$ household is the monopoly supplier of differentiated labor, $h_{j,t}$. With probability $1 - \xi_w$ it has the opportunity to choose its wage rate. With probability $\xi_w$ the household’s wage rate evolves as follows:

$$W_{j,t} = \tilde{\pi}_{w,t} \mu W_{j,t-1},$$

where

$$\tilde{\pi}_{w,t} \equiv (\pi_{t-1})^{t_w} \tilde{\pi}_{1-t_w}. \tag{3.10}$$
In (3.9), the variable, $A_{j,t}$ denotes the net payoff from insurance contracts on the risk that a household cannot reoptimize its wage rate, $W^j_t$. The existence of these insurance contracts have the consequence that in equilibrium all households have the same level of consumption, capital and money holdings. We have imposed this equilibrium outcome on the notation by dropping the $j$ subscript.

The household chooses investment in order to achieve the desired level of its capital, according to the following technology:

$$K_{t+1} = (1 - \delta)K_t + (1 - S \left( \frac{I_t}{I_{t-1}} \right))I_t,$$  \hspace{1cm} (3.11)

where

$$S(x) = \frac{a}{2} (x - \exp(\mu_z))^2,$$

with $a > 0$. For interesting economic environments in which (3.11) is the reduced form, see Lucca (2006) and Matsuyama (1984).17

The household’s problem is to maximize (3.8) subject to the demand for labor, (3.7), the Calvo wage-setting frictions, the technology for building capital, (3.11), and its budget constraint, (3.9).

The monetary authority’s policy rule is a version of (1.1). Let the target interest rate be denoted by $R^*_t$:

$$R^*_t = \alpha_\pi [E_t (\pi_{t+1}) - \pi] + \alpha_y \log \left( \frac{Y_t}{Y_t^+} \right),$$

where $Y_t^+$ is aggregate output on a nonstochastic steady state growth path (we ignore a constant term here). The monetary authority manipulates the money supply to

17We also explored a specification in which capital adjustment costs are a function of the level of investment. This, however, led investment to fall in response a positive signal about future technology.
ensure that the equilibrium nominal rate of interest, $R_t$, satisfies:

$$R_t = \rho_t R_{t-1} + (1 - \rho_t) R_t^*.$$  \hfill (3.12)

### 3.2. Results

We assign the following values to the model parameters:

$$\beta = 1.01358^{-0.25}, \mu_z = 1.0136^{0.25}, b = 0.63, a = 15.1,$$

$$\alpha = 0.40, \delta = 0.025, \psi_L = 109.82, \sigma_L = 1, \rho = 0.83, p = 12,$$

$$\lambda_f = 1.20, \lambda_w = 1.05, \xi_p = 0.63, \xi_w = 0.81, \iota = 0.84,$$

$$\iota_w = 0.13, \rho_i = 0.81, \alpha_{\pi} = 1.95, \alpha_y = 0.18, \upsilon = 0.$$

For further discussion, see Christiano, Ilut, Motto and Rostagno (2007).

The solid line in Figure 3 displays the response of the economy to a signal in period 1 that technology will improve 1 percent in period 13. The starred lines represent the response of the efficient allocations. The efficient allocations are obtained by dropping the monetary policy rule, (3.12), and computing the best allocations that are consistent with the remaining model equations.\footnote{All calculations were done using the model-solution and simulation package, DYNARE. Although the computation of the Ramsey-efficient allocations is conceptually straightforward, the algebra required to derive the equations that characterize those allocations is laborious. In doing the calculations we benefited greatly from the code prepared for Levin, Lopez-Salido (2004) and Levin, Onatski, Williams, and Williams (2005), which automatically does the required algebra in a format that can be input into DYNARE. It is perhaps worth stressing that monetary authority in the efficient allocations does not have any information advantage over private agents.} Note first how the equilibrium responses overshoot the efficient responses to a signal shock. The percent responses in output, consumption, investment and hours worked are roughly three times greater

in the equilibrium than they would be under an ideal monetary policy. Also, the price of capital rises and then falls in the monetary equilibrium. The price of capital corresponds to the price of equity in our model.\textsuperscript{19} The pattern of responses of the solid line corresponds, qualitatively, to the pattern in a typical boom-bust episode.\textsuperscript{20} Since the starred line represents the best that is feasible with monetary policy, according to the analysis here most of the boom-bust episode reflects bad (in a welfare sense) monetary policy.

We verified that the problem with monetary policy is that it focuses too narrowly on inflation. If the coefficient on inflation in the monetary policy rule, (3.12), is reduced to nearly unity, then the solid line and the starred lines essentially coincide. Also, if the frictions in wage setting are removed by setting $\xi_w = 0$, then the starred and solid lines also nearly coincide.

We found that if price frictions are eliminated, then the results are for the most part unchanged. One difference is that inflation falls less quickly in the early phase of the boom-bust.\textsuperscript{21} Regardless of sticky prices, however, everyone - the monetary authority included - expects prices to fall when the positive technology shock actually occurs. The anticipated monetary easing generated by this is enough to produce the boom in the immediate aftermath of the signal. We conclude that it is the interaction of sticky wages and a monetary policy too narrowly focused on inflation that accounts

\begin{itemize}
\item \textsuperscript{19}Note how the price of capital falls in the efficient allocations. For an extensive discussion of this, see Christiano, Ilut, Motto and Rostagno (2007).
\item \textsuperscript{20}For what happens in boom-bust episodes, see, for example, Adalid and Detken (2006) and Bordo and Wheelock (2007).
\item \textsuperscript{21}Presumably, in a more complete boom-bust scenario an actual rise in technology would accompany the signal shock, and this would help place downward pressure on the price level in the wake of the signal shock. In our analysis we do not allow for a rise in actual technology during the boom in order to isolate the very large role played by expectations.
\end{itemize}
for the excessive volatility in allocations.

To understand the economics of the analysis, consider the dynamic behavior of the real wage. In the equilibrium with the Taylor rule, the real wage falls, while efficiency dictates that it rise. In effect, in the Taylor rule equilibrium the markets receive a signal that the cost of labor is low, and this is part of the reason that the economy expands so strongly. The ‘correct’ signal would be sent by a high real wage, and this could be accomplished by allowing the price level to fall. However, in the monetary policy regime governed by our Taylor rule, (3.12), this fall in the price level is not permitted to occur: any threatened fall in the price level is met by a proactive expansion in monetary policy. Not surprisingly, when we redo our analysis with a Taylor rule in which longer-term inflation expectations appear, the problem is made even worse.

As noted above, when Christiano, Ilut, Motto and Rostagno (2007) introduce credit into the model they find that the solid line in the figure essentially drops to the starred line. That is, allowing monetary policy to react to credit growth causes the model response to a signal shock to virtually coincide with the efficient response.

4. Conclusion

We described two examples that illustrate in different ways how money and credit may be useful in monetary policy. The first example shows how a commitment to monitor money and control it directly in the event that it behaves erratically can help anchor inflation expectations. The second example shows how a policy that focuses too narrowly on inflation may inadvertently contribute to welfare-reducing boom-bust cycles. According to the example, a policy of monetary tightening when credit growth is strong can attenuate this unintended effect of too-narrow inflation
stabilization.

We emphasize that in our examples, the problem is not the stabilization of inflation expectations or inflation per se. The examples show that there can be trade-offs between overly rigid inflation stabilization and the stabilization of asset prices and output. The design of an efficient inflation stabilization program must balance trade-offs, and to get these right one must get the structure of the economy right. Fortunately, there has been great progress in recent years as increasingly sophisticated macroeconomic models are developed and fit to data. In addition, there have been substantial strides in the conceptual aspects of designing policies to stabilize inflation.\footnote{Each of our two examples perturbs the standard sticky price model in directions that appear to be empirically plausible. The first example integrates financial frictions in the supply side of the economy. The second example introduces frictions in the setting of wages. These examples suggest to us that as the models used for monetary policy analysis become more realistic, money and credit will come to play a direct role in monetary policy.}

\footnote{The recent work in ‘flexible inflation targeting’ (see Bernanke, 2003, for an informal discussion) focuses on replacing (1.1) by the optimal policy. This requires taking a stand on the model of the economy. When the economy is our benchmark economy, the monetary authority observes the shocks striking the economy as they occur and the monetary authority’s objective is social welfare, then optimal policy corresponds to the policy captured by the Ramsey policies exhibited in Figure 1. Discussions of this approach appear in, among other places, Benigno and Woodford (2007), Gianonni and Woodford (2005) and Svensson and Woodford (2005). This approach obviously requires that the model economy be correctly specified and that the appropriate commitment technology exist to resist the time inconsistency associated with optimal plans.}
5. Appendix

In this appendix, we describe a perfect foresight version of the standard New Keynesian model.\textsuperscript{23} We show that when a working capital channel is added, then the model displays the kind of multiplicity of equilibria discussed in the text. Although the analysis is similar in spirit to the ones in BSU-CF-CR, the detailed example is new. For this reason, we develop the example carefully.

We need only consider the perfect foresight version of the model because the argument is based only on the properties of the model in a neighborhood of the perfect foresight steady state. The first section below describes the agents in the economy. Because the Taylor rule assumed in the equilibrium of our model involves deviations from the optimal equilibrium, the second subsection presents a careful discussion of optimality. We describe the best equilibrium that is supportable by some feasible monetary and fiscal policy (i.e., the ‘Ramsey’ equilibrium). We also describe a different concept, the best allocations that are feasible given only the preferences and technology in the economy and ignoring the price-setting frictions. Although the two concepts are different along a transition path, they coincide in steady state. The third subsection below describes our Taylor rule, and presents our basic result.

5.1. The Agents in the Economy

Households are assumed to have the following preferences:

\[
\sum_{t=0}^{\infty} \beta^t \left( \log C_t - \frac{N_t^{1+\phi}}{1+\phi} \right),
\]

\textsuperscript{23}See, for example, Woodford (2003a) and the references he cites.
where $N_t$ denotes employment and $C_t$ denotes consumption. We suppose that households participate in a labor market and in a bond market, leading to the following efficiency conditions:

$$-c_t = -rr + r_t - c_{t+1} - \pi_{t+1},$$

$$\varphi n_t + c_t = w_t - p_t,$$

where

$$rr \equiv -\log \beta, \ c_t \equiv \log C_t, \ r_t \equiv \log R_t, \ w_t = \log W_t, \ p_t = \log P_t, \ \pi_t \equiv p_t - p_{t-1}.$$

Here, $P_t$ denotes the price of consumption goods, $W_t$ denotes the nominal wage rate, and $R_t$ denotes the gross nominal rate of interest from $t$ to $t+1$.

Final output is produced by a representative competitive producer with technology:

$$Y_t = \left( \int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{-\frac{1}{\epsilon}}, \ \infty > \epsilon \geq 1, \quad (5.1)$$

where $Y_t(i)$ is an intermediate good purchased at price $P_t(i), i \in [0, 1]$. Final output, $Y_t$, is sold at price $P_t$ and the representative final good firm takes $P_t(i)$ and $P_t$ as given. Optimization by the representative final good producer implies:

$$Y_t(i) = Y_t \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} \quad (5.2)$$

Substituting (5.2) into (5.1) and rearranging, we obtain:

$$P_t = \left( \int_0^1 P_t(i)^{(1-\epsilon)} di \right)^{\frac{1}{1-\epsilon}}. \quad (5.3)$$

The $i^{th}$ intermediate good producer is a monopolist in the market for $Y_t(i)$, but interacts competitively in the labor market. The $i^{th}$ intermediate good producer’s technology is given by:

$$Y_t(i) = N_t(i),$$

21
and the producer’s marginal cost is:

\[(1 - \nu_t) \frac{W_t}{P_t} (1 + \psi r_t),\]

where \(\nu_t\) is a potential subsidy received by the intermediate good firm from the government. Any subsidy is assumed to be financed by a lump-sum tax to households.

The intermediate good producer faces Calvo-style frictions in the setting of prices. A fraction, \(\theta\), of intermediate good firms cannot change price:

\[P_t(i) = P_{t-1}(i - 1)\]

and the complementary fraction, \(1 - \theta\), sets price optimally:

\[P_t(i) = \tilde{P}_t.\]

5.2. Ramsey (‘Natural’) Equilibrium

Given the nature of technology, the ideal allocation of labor occurs when it is spread equally across the different intermediate good producers:

\[N_t(i) = N_t\ \text{all } i,\]

so that

\[Y_t = N_t, \ y_t = n_t, \quad (5.4)\]

where \(y_t = \log Y_t\). Efficiency in the total level of employment requires equating the marginal cost of work in consumption units (the marginal rate of substitution between work and leisure - \(MRS_t\)) to the marginal benefit (the marginal product of labor, \(MP_{L,t}\)). In logs:

\[\log MRS_t = \log MP_{L,t} = 0 \quad (5.5)\]
Combine (5.4) and (5.5) and \( c_t = y_t \) to obtain:

\[
y_t (1 + \varphi) = 0
\]

so that natural level of output and employment are:

\[
y_t^* = n_t^* = 0. \tag{5.6}
\]

Under the ideal allocations, employment, consumption and output are all equal to unity in each period.

The ideal allocations do not in general coincide with the Ramsey allocations: the ones attainable by some feasible choice of monetary and fiscal policy. However, the Ramsey allocations do converge to the ideal allocations in steady state.

We express the Ramsey equilibrium as the solution to a particular constrained optimization problem. The pricing frictions and the technology imply, as shown in Yun (1996, 2005):

\[
C_t = Y_t = p_t^* N_t, \tag{5.7}
\]

where \( p_t^* \) is the following measure of price dispersion:

\[
p_t^* = \left[ \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} di \right]^{-1},
\]

where \( P_t \) is defined in (5.3). Note that \( p_t^* = 1 \) when all intermediate goods prices are the same. The law of motion for \( p_t^* \) is obtained by combining the last equation with the first order condition for firms that optimize their price. The resulting expression is:

\[
p_t^* = \left[ (1 - \theta) \left( \frac{1 - \theta (\pi_t)^{\frac{\varepsilon-1}{\varepsilon}}} {1 - \theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta \pi_t^\varepsilon}{p_t^{*1-1}} \right]^{-1}. \tag{5.8}
\]
Note that $p^*_{t-1} = \pi_t = 1$ implies $p^*_t = 1$. That is, if there was no price dispersion in the previous period (i.e., $p^*_{t-1} = 1$) and there is no aggregate inflation in the current period (i.e., $\pi_t = 1$), then there is also no price dispersion in the current period. To understand the intuition behind this result, recall that non-optimizers leave their price unchanged. If in addition there is no aggregate inflation, then it must be that optimizers also leave their price unchanged. If everyone leaves their price unchanged and all intermediate firm prices were identical in the previous period, then it follows that all intermediate good firm prices must be the same in the current period.

The first order necessary conditions associated with firms that optimize their prices can be shown to be\(^\text{24}\):\(^\dagger\)

\[
1 + \left( \frac{1}{\pi_{t+1}} \right)^{1-\varepsilon} \beta \theta F_{t+1} = F_t \quad (5.9)
\]

\[
\frac{\varepsilon}{\varepsilon - 1} \frac{(1 - \nu_t) W_t (1 + \psi r_t)}{P_t} + \beta \theta \left( \frac{1}{\pi_{t+1}} \right)^{-\varepsilon} K_{t+1} = K_t \quad (5.10)
\]

\[
F_t \left[ \frac{1 - \theta \left( \frac{1}{\pi_t} \right)^{1-\varepsilon}}{1 - \theta} \right]^{1/\varepsilon} = K_t, \quad (5.11)
\]

where $F_t$ and $K_t$ represent auxiliary variables. Finally, we restate the household’s inter- and intra-temporal Euler equations for convenience:

\[
\frac{1}{C_t} = \frac{\beta}{C_{t+1}} (1 + r_t) / \pi_{t+1}, \quad \frac{W_t}{P_t} = N_t^\phi C_t \quad (5.12)
\]

The allocations in a Ramsey equilibrium solve the Ramsey problem:

\[
\max_{N_t, K_t, \nu_t, C_t, \phi_t} \sum_{t=0}^{\infty} \beta^t \left( \log C_t - \frac{N_t^{1+\phi}}{1+\phi} \right), \quad (5.13)
\]

subject to (5.7)-(5.12) and the given value of $p^*_{t-1}$. We now establish the following proposition:\(^\text{24}\)

\[^\dagger\text{See, for example, Benigno and Woodford, (2004).}\]
Proposition 5.1. The allocations in a Ramsey equilibrium are uniquely defined by the following expressions:

\[
\pi_t = \arg \max_{\pi_t} p_t^* = \left[ \frac{(p_{t-1}^*)^{(e-1)}}{1 - \theta + \theta (p_{t-1}^*)^{(e-1)}} \right]^{\frac{1}{e-1}},
\]

(5.14)

\[
N_t = 1
\]

(5.15)

\[
p_t^* = \left[ 1 - \theta + \theta (p_{t-1}^*)^{e-1} \right]^{\frac{1}{e-1}},
\]

(5.16)

\[
\lim_{T \to \infty} P_T = p_{-1}^* P_{-1},
\]

(5.17)

\[
1 - \nu_t = \frac{\varepsilon - 1}{\varepsilon (1 + \psi r_t)}
\]

(5.18)

\[
1 + r_t = \frac{1}{\beta}
\]

(5.19)

for \( t = 0, 1, 2, .. \)

According to (5.14) and (5.15), the only restrictions that bind on the solution are the resource constraint, (5.7), and the law of motion for \( p_t^* \), (5.8). In particular, \( \pi_t, N_t \) and \( C_t \) may be chosen to solve (5.13) subject to (5.7) and (5.8), ignoring the other restrictions on the Ramsey problem. The other restrictions may then be solved for the remaining choice variables in the optimization problem, (5.13). The details are reviewed in what follows.

Substitute out for \( C_t \) in (5.13) using (5.7) and then maximize the result with respect to \( N_t \) and \( \pi_t \). The solution to this problem is characterized by (5.14) and (5.15). Then, substitute out for \( \pi_t \) from (5.14) into (5.8) to obtain (5.16). The latter is a stable linear difference equation in \( (p_t^*)^{e-1} \), with slope equal to \( \theta \). Because \( 0 < \theta < 1 \), the difference equation is globally stable and has a unique fixed point at \( p_t^* = 1 \). Combining (5.16) and (5.14), we obtain:

\[
\pi_t = \frac{p_{t-1}^*}{p_t^*}.
\]

(5.20)
Note that although the resource allocation distortion, \( p_t^* \), is minimized according to (5.14), it is not necessarily eliminated in each period. If \( p_{t-1}^* \neq 1 \) the resource allocation distortion is eliminated gradually over time. Combining (5.20) with the global stability of (5.16) we obtain (see Yun, 2005):

\[
\lim_{T \to \infty} \frac{P_T}{P_{-1}} = \lim_{T \to \infty} \frac{p_{t-1}^*}{p_T^*} = p_{-1}^*.
\]

This establishes (5.17).

Impose (5.7), (5.12) and (5.18) on (5.10) and divide by \( p_t^* \), to obtain:

\[
1 + \beta \theta \left( \frac{1}{\pi_t+1} \right)^{1-\varepsilon} K_{t+1} \frac{p_t^*}{p_t^{*+1}} = \frac{K_t}{p_t^*},
\]

(5.21)

where (5.20) has been used. We use (5.21) to define \( K_t \), so that (5.10) is satisfied. Define

\[
F_t = \frac{K_t}{p_t^*}
\]

Note

\[
\left[ \frac{1 - \theta \left( \pi_t \right)^{\varepsilon-1}}{1 - \theta} \right]^{\frac{1}{1-\varepsilon}} = \left[ \frac{1 - \theta \left( \frac{p_{t-1}^*}{p_t^*} \right)^{\varepsilon-1}}{1 - \theta} \right]^{\frac{1}{1-\varepsilon}} = \left[ \frac{1 - \theta \left( \frac{p_{t-1}^*}{p_t^*} \right)^{\varepsilon-1}}{1 - \theta + \theta \left( p_{t-1}^* \right)^{\varepsilon-1}} \right]^{\frac{1}{1-\varepsilon}} = \frac{1}{p_t^*},
\]

so that (5.11) is satisfied. Note that given (5.21) and our definition of \( F_t \), it follows that (5.9) is satisfied. Finally, (5.19) is obtained by substituting (5.15), (5.20) and (5.7) into (5.12). Since all the constraints on the problem, (5.13), are satisfied, the proposition is established.
Four observations on the proposition are worth emphasizing. First, if \( p_{t-1}^* \neq 1 \) then \( \pi_t = 1 \) for \( t = 0, 1, \ldots \) is not optimal. However, \( \pi_t \) does optimally converge to unity so that the allocations converge to what we call the ‘ideal’ allocations asymptotically. To get a sense of how long the transition path is, consider the case \( p_{t-1}^* = 0.91 \). With this initial condition, output is 9 percent below potential, for any given aggregate level of employment. The transition of inflation and the distortion (or, equivalently, consumption) to the steady state is indicated in Figure A (we use the following quarterly parameter values, \( \theta = 0.75, \varepsilon = 5, \varphi = 1, \beta = 0.99 \)). Note how very far the inflation rate is from its steady state. The Ramsey level of consumption also remains substantially below its steady state value. The example assumes that the average duration of prices is one year (i.e., \( 1/(1 - \theta) = 4 \)). With our parameterization, it takes about one year for the inflation rate and consumption to converge to the Ramsey steady state.

A second observation worth emphasizing is that equation (5.18) implies the labor subsidy on firms is chosen to completely eliminate the monopoly power distortion and the distortion due to the financial friction. It is interesting that this distortion is eliminated entirely, because the cross-sectoral distortion is not eliminated during the transition (note that \( p_t^* \) takes about three years to converge). The theory of the second best might have led us to expect that if one distortion could not be eliminated, then the other would not either.

Third, we note that the environment does not rationalize price level targeting. Equation (5.17) implies that after a shock which drives the price level up in period \(-1\), average inflation is below steady state. This is because a shock to the aggregate price level creates price dispersion, and this drives \( p_{t-1}^* \) down. However, a shock in \(-1\) which drives the aggregate price level down also drives \( p_{t-1}^* \) down and so such a shock is followed by a period of below steady state inflation too. Since price level targeting
requires that inflation be high after a shock that drives the price level down and low after a shock that drives the price level up, it follows that price level targeting is not a property of a Ramsey equilibrium. The environment does rationalize inflation targeting, though it does not rationalize returning inflation to target immediately. It instead rationalizes driving inflation to its target over time. Fourth, the fact that (5.7) and (5.8) are the only restrictions that bind on the Ramsey problem indicates that the Ramsey policy is time consistent. This is because neither (5.7) or (5.8) incorporates expectations about the future. Thus, the Ramsey policy would be implemented by a policymaker that has no ability to commit to future inflation.

The benchmark economy that we use is the steady state of the Ramsey equilibrium. This corresponds to the ideal allocations defined at the beginning of this section.

5.3. Equilibrium with Taylor Rule Monetary Policy

In our economy with a Taylor rule, we suppose that fiscal policy, \( v_t \), is set to ensure that the steady state corresponds to the steady state of the Ramsey equilibrium:

\[
1 - v_t = \frac{\varepsilon - 1}{\varepsilon (1 + \psi r_r)}.
\]  

(5.22)

Under the Taylor rule, the interest rate deviates from the Ramsey or natural rate according to whether expected inflation is higher or lower than steady state inflation:

\[
\hat{r}_t = \tau \pi_{t+1}, \quad \hat{r}_t \equiv r_t - rr.
\]

We repeat here the household intertemporal equilibrium condition in the Taylor rule equilibrium:

\[
y_t = - [\hat{r}_t - \pi_{t+1}] + y_{t+1}.
\]
The intertemporal equilibrium condition in steady state (either Ramsey or in the economy with the Taylor rule) is:

\[ y_t^* = -[rr_t^* - rr] + y_{t+1}^*, \]

where \( y_t^* \) is defined in (5.6) and \( rr_t^* \) is the Ramsey (or, ‘natural’) rate of interest. We deduce that \( rr_t^* = rr \). Subtracting the steady state intertemporal condition from the one in the economy with the Taylor rule, we obtain the ‘New Keynesian IS equation’:

\[ x_t = -[\hat{r}_t - \pi_{t+1}] + x_{t+1}. \]

The log of (5.7) implies:

\[ y_t = \log p_t^* + n_t. \]

In a sufficiently small neighborhood of steady state, \( \log p_t^* \approx 0 \) (see Yun, 1996, 2005) and we impose this from here on. This is appropriate because we are concerned with the properties of equilibrium in a small neighborhood of steady state. The Calvo reduced form inflation equation implies

\[ \pi_t = \beta \pi_{t+1} + \kappa \times \widehat{mc}_t, \quad \kappa = \frac{(1 - \theta)(1 - \beta \theta)}{\theta}, \]

where \( \widehat{mc}_t \) denotes the log deviation of real marginal cost in the Taylor rule equilibrium from its log steady state value, \( mc^* \):

\[ mc^* = \log (1 - \nu) + \log \frac{W_t}{P_t} + \log (1 + \psi rr) \]

\[ = \log \frac{\varepsilon - 1}{\varepsilon}, \]

using (5.16), the household’s static Euler equation and the fact that output and employment are unity in steady state. Then,

\[ \widehat{mc}_t = \log (1 - \nu_t) + \log \frac{W_t}{P_t} + \log (1 + \psi r_t) - mc_t^* \]

\[ = \log (1 - \nu) + \varphi n_t + c_t + \psi r_t - \log (1 - \nu) - \psi rr \]

\[ = (\varphi + 1) x_t + \psi \hat{r}_t, \]
using the household’s static Euler equation and (5.7). Also, we have used the approximation,
\[ \log(1 + \psi r_t) \simeq \psi r_t. \]
Substituting this into the Phillips curve, we obtain:
\[ \pi_t = \beta \pi_{t+1} + \kappa (\varphi + 1) x_t + \kappa \psi \hat{r}_t. \]
Collecting the equilibrium conditions, we obtain:
\[ \begin{align*}
\pi_t &= \beta \pi_{t+1} + \kappa (\varphi + 1) x_t + \kappa \psi \hat{r}_t \text{ ‘Phillips curve’} \\
x_t &= -[\hat{r}_t - \pi_{t+1}] + x_{t+1} \text{ ‘IS curve’} \\
\hat{r}_t &= \tau \pi_{t+1} \text{ ‘Taylor rule’}. 
\end{align*} \]

### 5.4. Determinacy Properties of the Nonstochastic Steady State

Using the monetary policy rule to substitute out for \( \hat{r}_t \):
\[ \begin{align*}
x_t + \sigma (\tau - 1) \pi_{t+1} - x_{t+1} &= 0 \\
\pi_t - \lambda x_t - (\beta + \gamma \tau) \pi_{t+1} &= 0,
\end{align*} \]
where \( \sigma \) is the intertemporal elasticity of substitution (assumed unity in the previous derivation). Also,
\[ \begin{align*}
\lambda &= \kappa (\varphi + 1) \\
\gamma &= \kappa \psi.
\end{align*} \]
Expressed in matrix form, the system is:
\[ \begin{bmatrix} \sigma (\tau - 1) & -1 \\ - (\beta + \gamma \tau) & 0 \end{bmatrix} \begin{bmatrix} \pi_{t+1} \\ x_{t+1} \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & -\lambda \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \]
30
for $t = 0, 1, 2, \ldots$, or, after inverting:

$$
\begin{pmatrix}
\pi_{t+1} \\
x_{t+1}
\end{pmatrix} = A
\begin{pmatrix}
\pi_t \\
x_t
\end{pmatrix},
$$

(5.23)

where

$$
A = \frac{1}{\beta + \tau \gamma}
\begin{bmatrix}
1 & -\lambda \\
a & b
\end{bmatrix},
$$

$a = \sigma (\tau - 1)$, $b = \beta + \tau \gamma - \lambda a$.

Note that the eigenvalues of $A$ solve:

$$
\mu = \frac{1}{2} (b + 1) \pm \frac{1}{2} \sqrt{b^2 - 4a\lambda - 2b + 1}.
$$

Local uniqueness of the steady state equilibrium, $x_t = 0 = \pi_t$, requires that both eigenvalues of $A$ exceed unity in absolute value. To see why, note first that both $x_t$ and $\pi_t$ are endogenous variables whose values are determined at time $t$ (they are period $t$ ‘jump’ variables). If one or both eigenvalues of $A$ were less than unity in absolute value, one could set some combination of $x_0$ and $\pi_0$ different from zero, and the solution to (5.23) describes a path that eventually takes the system back to steady state (i.e., $(x_t, \pi_t) \to 0$, as $t \to \infty$). Because there is an uncountable number of such combinations, $(x_0, \pi_0)$, each of which follows a path back to steady state and each such path satisfies the equilibrium conditions, it follows that there is a multiplicity of equilibria. Consider, for example, the parameter values:

$$
\theta = 0.75, \ k = 0.085, \ \tau = 1.5, \ \sigma = 1, \ \varphi = 1, \ \psi = 1, \ \beta = 0.99, \ \gamma = k.
$$

These parameter values are standard. They imply that the average time between price changes is one year (see $\theta$); the coefficient on expected inflation, $\tau$, represents an aggressive reaction to inflation; households have log utility (see $\sigma$); the (Frisch) labor supply elasticity is unity (see $\varphi$); and the discount rate is 4 percent per year (see $\beta$). We set $\psi = 1$, so that intermediate good firms are assumed to have to
finance 100% of their variable input costs (i.e., labor) in advance. We found that in this example, the smallest (in absolute value) root of $A$ is 0.94. So, we have a multiplicity of equilibria, as in the discussion of the text. We found that the smallest root of $A$ is less than unity for all $\psi \geq 0.08$. For $\psi$ smaller than this, we reproduce the standard result that both eigenvalues of $A$ are greater than unity. This is consistent with the intuition described in the body of the paper.

To assess the empirical plausibility of the range of values of $\psi$ that produce multiplicity of equilibrium, we examined the Quarterly Financial Report for Manufacturing, Mining, and Trade Corporations: 2006, Quarter 1, issued June 2006 (US Bureau of the Census, available online at http://www.census.gov/prod/2006pubs/qfr06q1.pdf). According to Table 1.0, page 2, sales in 2006Q1 were $S = $1.4 trillion. According to Table 1.1, page 4, short term liabilities (bank loans and commercial paper with maturity less than one year, plus trade credit, plus other current liabilities) totaled $L = $1.3 trillion. If we take $S/L$ as a (very) crude estimate of $\psi$, we conclude that the range of values of $\psi$ that generate multiple equilibria is empirically reasonable.
References


Annual Conference of the Central Bank of Chile, Santiago, Chile: Central Bank of Chile, pg. 247-274.


Figure 1a

Figure 1b
Figure 2a

Figure 2b

Phillips curve

Higher π e confirmed and likely to persist
Figure 3: Benchmark Monetary Model and Associated Ramsey-efficient Allocations

- **Output**: Ramsey Allocations of Simple Monetary Model vs. Simple Monetary Model
- **Investment**: Ramsey Allocations of Simple Monetary Model vs. Simple Monetary Model
- **Consumption**: Ramsey Allocations of Simple Monetary Model vs. Simple Monetary Model
- **Hours Worked**: Ramsey Allocations of Simple Monetary Model vs. Simple Monetary Model
- **Net Real Rate of Interest**: Ramsey Allocations of Simple Monetary Model vs. Simple Monetary Model
- **Price of Capital**: Ramsey Allocations of Simple Monetary Model vs. Simple Monetary Model
- **Net Nominal Rate of Interest**: Ramsey Allocations of Simple Monetary Model vs. Simple Monetary Model
- **Real Wage**: Ramsey Allocations of Simple Monetary Model vs. Simple Monetary Model
Figure A: Ramsey Equilibrium Starting with 9 Percent Distortion

$C_t$, distortion, $p^*$

inflation (APR)