

## Risk Shocks<sup>†</sup>

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*We augment a standard monetary dynamic general equilibrium model to include a Bernanke-Gertler-Gilchrist financial accelerator mechanism. We fit the model to US data, allowing the volatility of cross-sectional idiosyncratic uncertainty to fluctuate over time. We refer to this measure of volatility as risk. We find that fluctuations in risk are the most important shock driving the business cycle. (JEL D81, D82, E32, E44, L26)*

We introduce agency problems associated with financial intermediation into an otherwise standard model of business cycles. Our estimates suggest that fluctuations in the severity of these agency problems account for a substantial portion of business cycle fluctuations over the past two and a half decades.

The agency problems we introduce are those associated with asymmetric information and costly monitoring proposed by Townsend (1979). Our implementation most closely follows the work of Bernanke and Gertler (1989) and Bernanke, Gertler, and Gilchrist (1999)—henceforth, BGG.<sup>1</sup> Entrepreneurs play a central role in the model. They combine their own resources with loans to acquire raw capital. They then convert raw capital into effective capital in a process that is characterized by idiosyncratic uncertainty. We refer to the magnitude of this uncertainty as *risk*. The notion that idiosyncratic uncertainty in the allocation of capital is important in practice can be motivated informally in several ways. For example, it is well known that a large

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<sup>1</sup>Other important early contributions to the role of costly state verification in business cycles include the work of Williamson (1987); Carlstrom and Fuerst (1997); and Fisher (1999). More recent contributions include the work of Christiano, Motto, and Rostagno (2003); Arellano, Bai, and Kehoe (2012); and Jermann and Quadrini (2012).

proportion of firm start-ups end in failure.<sup>2</sup> Entrepreneurs and their suppliers of funds experience these failures as a stroke of bad luck. Even entrepreneurs whom we now think of as sure bets, such as Steve Jobs and Bill Gates, experienced failures as well as the successes for which they are famous. Another illustration of the microeconomic uncertainty associated with the allocation of capital may be found in the various wars over industry standards. In these wars, entrepreneurs commit large amounts of raw capital to one or another standard. Whether that raw capital turns into highly effective capital or becomes worthless is, to a substantial degree, up to chance.<sup>3</sup>

We model the idiosyncratic uncertainty experienced by entrepreneurs by the assumption that if an entrepreneur purchases  $K$  units of raw capital, then that capital turns into  $K\omega$  units of effective capital. Here,  $\omega \geq 0$  is a random variable drawn independently by each entrepreneur, normalized to have mean unity.<sup>4</sup> Entrepreneurs who draw a large value of  $\omega$  experience success, while entrepreneurs who draw a value of  $\omega$  close to zero experience failure. The realization of  $\omega$  is not known at the time the entrepreneur receives financing. When  $\omega$  is realized, its value is observed by the entrepreneur but can be observed by the supplier of finance only by undertaking costly monitoring. We denote the time period  $t$  cross-sectional standard deviation of  $\log \omega$  by  $\sigma_t$ . We refer to  $\sigma_t$  as *risk*. The variable  $\sigma_t$  is assumed to be the realization of a stochastic process. Thus, risk is high in periods when  $\sigma_t$  is high, and there is substantial dispersion in the outcomes across entrepreneurs. Risk is low otherwise.

Our econometric analysis assigns a large role to  $\sigma_t$  because disturbances in  $\sigma_t$  trigger responses in our model that resemble actual business cycles. The underlying intuition is simple. Following BGG, we suppose that entrepreneurs receive a standard debt contract. The interest rate on entrepreneurial loans includes a premium to cover the costs of default by the entrepreneurs who experience low realizations of  $\omega$ . The entrepreneurs and the associated financial frictions are inserted into an otherwise standard dynamic stochastic general equilibrium (DSGE) model.<sup>5</sup> According to our model, the credit spread (i.e., the premium in the entrepreneur's interest rate over the risk-free interest rate) fluctuates with changes in  $\sigma_t$ . When risk is high, the credit spread is high, and credit extended to entrepreneurs is low. With fewer financial resources, entrepreneurs acquire less raw capital. Because investment is a key input into the production of capital, it follows that investment falls. With this decline in the purchase of goods, output, consumption, and employment fall. For the reasons stressed by BGG, the net worth of entrepreneurs—an object that we identify with

<sup>2</sup>See Hall and Woodward (2010), which documents the extreme cross-sectional dispersion in payoffs to entrepreneurs backed by venture capital.

<sup>3</sup>For example, in the 1970s Sony allocated substantial resources to the construction of video equipment that used the Betamax video standard, while JVC and others used the VHS standard. After some time, VHS won the standards war, so that the capital produced by investing in video equipment that used the VHS standard was more effective than capital produced by investing in Betamax equipment. The reasons for this outcome are still hotly debated today. However, from the ex ante perspective of the companies involved and their suppliers of funds, the ex post outcome can be thought of as the realization of a random variable (for more discussion, see <http://www.mediacollege.com/video/format/compare/betamax-vhs.html>).

<sup>4</sup>The assumption about the mean of  $\omega$  is in the nature of a normalization because we allow other random variables to capture the aggregate sources of uncertainty faced by entrepreneurs.

<sup>5</sup>Our strategy for inserting the entrepreneurs into a DSGE model follows the lead of BGG in a general way. At the level of details, our model follows Christiano, Motto, and Rostagno (2003) by introducing the entrepreneurs into a version of the model proposed by Christiano, Eichenbaum, and Evans (2005) and by introducing the risk shock (and an equity shock mentioned later) studied here. To our knowledge, the first paper to appeal to variations in risk as a driver of business cycles is that of Williamson (1987).

the stock market—falls too. This occurs because the rental income of entrepreneurs falls with the decline in economic activity and because they suffer capital losses as the price of capital drops. Finally, the overall decline in economic activity results in a decline in the marginal cost of production and, thus, a decline in inflation. So, according to the model, the risk shock implies a countercyclical credit spread and procyclical investment, consumption, employment, inflation, stock market, and credit. These implications of the model correspond well to the analogous features of US business cycle data.<sup>6</sup>

We include other shocks in our model and estimate model parameters by standard Bayesian methods using 12 aggregate variables. In addition to the usual eight variables used in standard macroeconomic analyses, we also make use of four financial variables: the value of the stock market, credit to nonfinancial firms, the credit spread, and the slope of the term structure. As with any empirical analysis of this type, ours can be interpreted as a sort of accounting exercise. We in effect decompose our 12 aggregate variables into a large number of shocks. In light of the observations in the previous paragraph, it is perhaps not surprising that one of these shocks,  $\sigma_t$ , emerges as the most important by far. For example, the analysis suggests that fluctuations in  $\sigma_t$  account for 60 percent of the fluctuations in the growth rate of aggregate US output since the mid-1980s. Our conclusion that the risk shock is the most important shock depends crucially on including the four financial variables in our dataset.

Our empirical analysis treats  $\sigma_t$  as an unobserved variable. We infer its properties using our model and our 12 aggregate time series. A natural concern is that we might have relied on excessively large fluctuations in  $\sigma_t$  to drive economic fluctuations. To guard against this, we look outside the dataset used in the econometric analysis of the model for evidence on the degree of cyclical variation in  $\sigma_t$ . For this, we study a measure of uncertainty proposed in Bloom (2009). In particular, we compute the cross-sectional standard deviation of firm-level stock returns in the Center for Research in Securities Prices (CRSP) stock returns file. According to our model, the time series of this measure of uncertainty is dominated by the risk shock. We use our model to project Bloom's (2009) measure of uncertainty onto the 12 data series used in the econometric analysis of our model. We find that the degree of cyclical variation in the empirical and model-based measures of uncertainty are very similar. We interpret this as important support for the model.

Our analysis is related to a growing body of evidence which documents that the cross-sectional dispersion of a variety of variables is countercyclical.<sup>7</sup> Of course, the

<sup>6</sup>Our model complements recent papers that highlight other ways in which increased cross-sectional dispersion in an important shock could lead to aggregate fluctuations. For example, Bloom (2009) and Bloom et al. (2012) show how greater uncertainty can produce a recession by inducing businesses to adopt a wait-and-see attitude and delay investment. For another example that resembles ours, see the work of Arellano, Bai, and Kehoe (2012). For an example of how countercyclical dispersion may occur endogenously, see the work of Christiano and Ikeda (2013b).

<sup>7</sup>For example, Bloom (2009) documents that various cross-sectional dispersion measures for firms in panel datasets are countercyclical. De Veirman and Levin (2011) find similar results using the Thomas Worldscope database. Kehrig (2011) uses plant-level data to document that the dispersion of total factor productivity in US durable manufacturing is greater in recessions than in booms. Vavra (2010) presents evidence that the cross-sectional variance of price changes at the product level is countercyclical. Christiano and Ikeda (2013b) present evidence on the countercyclicality of the cross-sectional dispersion of equity returns among financial firms. Also, Alexopoulos and Cohen (2009) construct an index based on the frequency of time that words like uncertainty appear in the *New York Times* and find that this index rises in recessions. It is unclear, however, whether the Alexopoulos-Cohen evidence about uncertainty concerns variations in cross-sectional dispersion or changes in the variance of time series aggregates. Our risk shock corresponds to the former.

mere fact that cross-sectional volatility is countercyclical does not by itself prove the hypothesis in our model, that risk shocks are causal. It is in principle possible that countercyclical variation in cross-sectional dispersion is a symptom rather than a cause of business cycles.<sup>8</sup> Some support for the assumption about causal ordering in our model is provided by the work of Baker and Bloom (2011).

Our work is also related to that of Justiniano, Primiceri, and Tambalotti (2010), who stress the role of technology shocks in the production of installed capital (marginal efficiency of investment shocks). These shocks resemble our risk shock in that they primarily affect intertemporal opportunities. Our risk shock and the marginal efficiency of investment shock are hard to distinguish when we include only the eight standard macroeconomic variables in our analysis. However, the analysis strongly favors the risk shock when our four financial variables are included in the dataset. In part this is because, consistent with the data, the risk shock implies that the value of the stock market is procyclical, while the marginal efficiency of investment shock implies that it is countercyclical.

To gain intuition into our model and promote comparability with the literature, we also include a shock that we refer to as an *equity shock*. Several analyses of the recent financial crisis assign an important causal role to the equity shock (see, e.g., the work of Gertler and Kiyotaki 2010; Gertler and Karadi 2011; and Bigio 2011). This is a disturbance that directly affects the quantity of net worth in the hands of entrepreneurs.<sup>9</sup> The equity shock acts a little like our risk shock, by operating on the demand side of the market for capital. However, unlike the risk shock, the equity shock has the counterfactual implication that credit is countercyclical. Thus, the procyclical nature of credit is another reason that our econometric analysis assigns a preeminent status to risk shocks in business cycles.

The credibility of our finding about the importance of the risk shock depends on the empirical plausibility of our model. We evaluate the model's plausibility by investigating various implications of the model that were not used in constructing or estimating it. First, we evaluate the model's out-of-sample forecasting properties. We find that these are reasonable, relative to the properties of a Bayesian vector autoregression (VAR) or a simpler New Keynesian business cycle model such as the one of Christiano, Eichenbaum, and Evans (2005) (CEE) or Smets and Wouters (2007). We also examine the model's implications for data on bankruptcies, information that was not included in the dataset used to estimate the model. Finally, as discussed above, we compare the model's implications for the kind of uncertainty measures proposed by Bloom (2009). Although the match is far from perfect, overall our model performs well.

The plan of the article is as follows. The first section describes the model. Estimation results and measures of fit are reported in Section II. Section III presents the main results. We present various quantitative measures that characterize

<sup>8</sup>For example, Bachmann and Moscarini (2011) explore the idea that the cross-sectional volatility of price changes may rise in recessions as the endogenous response of the increased fraction of firms contemplating an exit decision. D'Erasmus and Boedo (2011) and Kehrig (2011) provide two additional examples of the possible endogeneity of cross-sectional volatility. Another example of endogeneity in cross-sectional volatility is provided by Christiano and Ikeda (2013b).

<sup>9</sup>In the literature, the equity shock perturbs the net worth of banks. As explained below, our entrepreneurs can be interpreted as banks.

the sense in which risk shocks are important in business cycles. We then explore the reasons the econometric results find the risk shock so important. The paper ends with a brief conclusion. Technical details, computer code and supporting analysis are provided in the online Appendix.

## I. The Model

The model incorporates the microeconomics of the debt-contracting framework of BGG into an otherwise standard monetary model of the business cycle. The first section (IA) describes the standard part of the model. Although these parts of the model can be found in many sources, we include them nevertheless so that the presentation is self-contained. In addition, the presentation fixes notation and allows us to be precise about the shocks used in the analysis. The second subsection (IB) describes the role of the entrepreneurs in the model and the agency problems that occur in supplying them with credit. The time series representations of the shocks, as well as adjustment cost functions, are reported in the third section (IC). The final section, (ID), displays the functional forms of adjustment costs and the timing assumptions that govern when agents learn about shocks.

### A. Standard Part of the Model

*Goods Production.*—A representative, competitive final goods producer combines intermediate goods,  $Y_{jt}$ ,  $j \in [0, 1]$ , to produce a homogeneous good,  $Y_t$ , using the following Dixit-Stiglitz technology:

$$(1) \quad Y_t = \left[ \int_0^1 Y_{jt}^{\frac{1}{\lambda_{f,t}}} dj \right]^{\lambda_{f,t}}, \quad 1 \leq \lambda_{f,t} < \infty,$$

where  $\lambda_{f,t}$  is a shock. The intermediate good is produced by a monopolist using the following technology:

$$(2) \quad Y_{jt} = \begin{cases} \varepsilon_t K_{jt}^\alpha (z_t l_{jt})^{1-\alpha} - \Phi z_t^* & \text{if } \varepsilon_t K_{jt}^\alpha (z_t l_{jt})^{1-\alpha} > \Phi z_t^*, \\ 0 & \text{otherwise} \end{cases}, \quad 0 < \alpha < 1.$$

Here,  $\varepsilon_t$  is a covariance stationary technology shock and  $z_t$  is a shock with a stationary growth rate. Also,  $K_{jt}$  denotes the services of effective capital, and  $l_{jt}$  denotes the quantity of homogeneous labor hired by the  $j$ th intermediate good producer. The fixed cost in the production function, (2), is proportional to  $z_t^*$ . The fixed cost is a combination of the two nonstationary stochastic processes in the model, namely,  $z_t$  and an investment-specific shock described below. The variable  $z_t^*$  has the property that  $Y_t/z_t^*$  converges to a constant in nonstochastic steady state. The monopoly supplier of  $Y_{jt}$  sets its price,  $P_{jt}$ , subject to Calvo-style frictions. Thus, in each time period  $t$  a randomly selected fraction of intermediate good firms,  $1 - \xi_p$ , can reoptimize their price. The complementary fraction set their price in this

way,  $P_{jt} = \tilde{\pi}_t P_{j,t-1}$ . The indexation term,  $\tilde{\pi}_t$ , is defined as follows:

$$(3) \quad \tilde{\pi}_t = (\pi_t^{target})^{\iota} (\pi_{t-1})^{1-\iota}.$$

Here,  $\pi_{t-1} \equiv P_{t-1}/P_{t-2}$ ,  $P_t$  is the price of  $Y_t$ , and  $\pi_t^{target}$  is the target inflation rate in the monetary authority's monetary policy rule, which is discussed below.

There exists a technology that can be used to convert homogeneous goods into consumption goods,  $C_t$ , one-for-one. Another technology converts a unit of homogeneous goods into  $\Upsilon^t \mu_{\Upsilon,t}$  investment goods, where  $\Upsilon > 1$  and  $\mu_{\Upsilon,t}$  is a shock. Because we assume these technologies are operated by competitive firms, the equilibrium prices of consumption and investment goods are  $P_t$  and  $P_t/(\Upsilon^t \mu_{\Upsilon,t})$ , respectively. The trend rise in technology for producing investment goods is the second source of growth in the model, and  $z_t^* = z_t \Upsilon^{\left(\frac{\alpha}{1-\alpha}\right)t}$ .

*Labor Market.*—The model of the labor market is taken from the work of Erceg, Henderson, and Levin (2000) and parallels the Dixit-Stiglitz structure of goods production. A representative, competitive labor contractor aggregates differentiated labor services,  $h_{i,t}$ ,  $i \in [0, 1]$ , into homogeneous labor,  $l_t$ , using the following production function:

$$(4) \quad l_t = \left[ \int_0^1 (h_{t,i})^{\frac{1}{\lambda_w}} di \right]^{\lambda_w}, \quad 1 \leq \lambda_w.$$

The labor contractor sells labor services,  $l_t$ , to intermediate good producers for nominal wage rate,  $W_t$ . The labor contractor's first-order condition for  $h_{i,t}$  represents its demand curve for that labor type. There are several ways of conceptualizing the supply of each labor type, each of which leads to the same equilibrium conditions. We find it convenient to adopt the following framework. For each labor type  $i$ , there is a monopoly union which represents all workers of that type in the economy. The union sets the wage rate,  $W_{i,t}$ , for that labor type, subject to Calvo-style frictions. In particular, a randomly selected subset of  $1 - \xi_w$  monopoly unions sets their wage optimally, while the complementary subset sets the wage according to  $W_{it} = (\mu_{z^*,t})^{\iota_w} (\mu_{z^*})^{1-\iota_w} \tilde{\pi}_{wt} W_{i,t-1}$ . Here,  $\mu_{z^*}$  denotes the growth rate of  $z_t^*$  in nonstochastic steady state. Also,

$$(5) \quad \tilde{\pi}_{w,t} \equiv (\pi_t^{target})^{\iota_w} (\pi_{t-1})^{1-\iota_w}, \quad 0 < \iota_w < 1.$$

The indexing assumptions in wage-setting ensure that wage-setting frictions are not distortionary along a nonstochastic, steady state–growth path.

*Households.*—There is a large number of identical and competitive households. We adopt the large family assumption of Andolfatto (1996) and Merz (1995) by assuming that each household contains every type of differentiated labor,  $h_{i,t}$ ,  $i \in [0, 1]$ . Each household also has a large number of entrepreneurs, but we

defer our discussion of these agents to the next subsection. Finally, households are the agents who build the raw capital in the economy.<sup>10</sup>

After goods production in period  $t$ , the representative household constructs end-of-period  $t$  raw capital,  $\bar{K}_{t+1}$ , using the following technology:

$$(6) \quad \bar{K}_{t+1} = (1 - \delta)\bar{K}_t + (1 - S(\zeta_{I,t}I_t/I_{t-1}))I_t.$$

To produce new capital, the household must purchase existing capital and investment goods,  $I_t$ . The quantity of existing capital available at the end of period  $t$  production is  $(1 - \delta)\bar{K}_t$ , where  $0 < \delta < 1$  denotes the rate of depreciation on capital. In (6),  $S$  is an increasing and convex function described below, and  $\zeta_{I,t}$  is a shock to the marginal efficiency of investment in producing capital. The household buys  $I_t$  at the price described in the previous subsection.<sup>11</sup>

In addition, the household purchases the existing stock of capital for the price  $Q_{\bar{K},t}$ . It sells new capital for the same price. The household is competitive, so it takes the price of capital and investment goods as given.

The preferences of the representative household are as follows:

$$(7) \quad E_0 \sum_{t=0}^{\infty} \beta^t \zeta_{c,t} \left\{ \log(C_t - bC_{t-1}) - \psi_L \int_0^1 \frac{h_{it}^{1+\sigma_L}}{1 + \sigma_L} di \right\}, b, \sigma_L > 0.$$

Here,  $\zeta_{c,t} > 0$  is a preference shock, and  $C_t$  denotes the per capita consumption of the members of the household. The budget constraint of the representative household is

$$(8) \quad (1 + \tau^c) P_t C_t + B_{t+1} + B_{t+40}^L + \left( \frac{P_t}{\Upsilon_t^L \mu_{\Upsilon,t}} \right) I_t + Q_{\bar{K},t} (1 - \delta) \bar{K}_t \\ \leq (1 - \tau^l) \int_0^1 W_t^i h_{i,t} di + R_t B_t + (R_t^L)^{40} B_t^L + Q_{\bar{K},t} \bar{K}_{t+1} + \Pi_t.$$

According to the left side of the budget constraint, the household allocates funds to consumption, two types of bonds, investment, and existing capital. The household's sources of funds are the earnings from differentiated labor and bonds, as well as the revenues from selling raw capital. Finally,  $\Pi_t$  represents various lump-sum payments. These include profits from intermediate goods, transfers from entrepreneurs (discussed in the next subsection), and lump-sum transfers from the government net of lump-sum taxes. Wages of differentiated labor,  $W_{i,t}$ , are set by the monopoly unions as discussed in the previous section. In addition, the household agrees to

<sup>10</sup>This task could equivalently be assigned to a competitive capital goods producer. We adopt the idea that households produce raw capital to minimize the number of agents.

<sup>11</sup>The specification of the production function for new capital in (6) is often used in DSGE models in part because it improves their fit to aggregate data (see, e.g., the work of CEE and Smets and Wouters 2007). Microeconomic evidence that also supports a specification like (6) includes the work of Matsuyama (1984); Topel and Rosen (1988); and Eberly, Rebelo, and Vincent (2012). Papers that provide interesting theoretical foundations which rationalize (6) as a reduced-form specification include those of Matsuyama (1984) and Lucca (2006).

supply whatever labor of each type that is demanded at the union-set wage rate. So the household treats labor income as exogenous.

In (8), the tax rates on consumption and wage income,  $\tau^c$  and  $\tau^l$ , are exogenous and constant. The revenues from these taxes are refunded to households in the form of lump-sum taxes via  $\Pi_t$ . The object  $B_{t+1}$  denotes one-period bonds that pay a gross nominal return,  $R_t$ , which is not contingent on the realized period  $t + 1$  state of nature. In addition, we give the household access to a long-term (ten-year) bond,  $B_{t+40}^L$ . These pay gross return,  $R_t^L$ , in period  $t + 40$ , at a quarterly rate. The nominal return on the long-term bond purchased in period  $t$ ,  $R_t^L$ , is known in period  $t$ . As discussed in the next section, the one-period bond is the source of funding for entrepreneurs and plays a critical role in the economics of the model. The long-term bond plays no direct role in resource allocation, and the market for this bond clears at  $B_{t+40}^L = 0$ . We include this bond because it allows us to diagnose the model's implications for the slope of the term structure of interest rates.

The representative household's problem in period  $t$  is to choose  $C_t, \bar{K}_{t+1}, \bar{K}_t, I_t, B_{t+1}, B_{t+40}^L$ . It makes this choice for each period with the objective of maximizing (7) subject to (8).

## B. Financial Frictions

Each of the identical households in the economy has a large number of entrepreneurs.<sup>12</sup> After production in period  $t$ , entrepreneurs receive loans from mutual funds. At this time, the state of an entrepreneur is summarized by its net worth,  $N \geq 0$ . The density of entrepreneurs with net worth,  $N$ , is denoted  $f_t(N)$ , and we denote the total net worth in the hands of all entrepreneurs at this point by

$$(9) \quad N_{t+1} = \int_0^\infty N f_t(N) dN.$$

We refer to an entrepreneur with net worth  $N$  as an  $N$ -type entrepreneur. Each  $N$ -type entrepreneur purchases raw capital using his own net worth and a loan and converts raw capital into effective capital services. In period  $t + 1$  each  $N$ -type entrepreneur earns income by supplying capital services and from capital gains; he then repays his loan and transfers funds between himself and his household. At this point, each entrepreneur's net worth in period  $t + 1$  is determined. Each entrepreneur then acquires a new loan, and the cycle continues. All markets visited by entrepreneurs are competitive.

In terms of the overall flow of funds, households are the ultimate source of funds for entrepreneurs. The most straightforward interpretation of our entrepreneurs is that they are firms in the nonfinancial business sector. However, it is also possible to interpret entrepreneurs as financial firms that are risky because they hold a nondiversified portfolio of loans to risky nonfinancial businesses.<sup>13</sup>

<sup>12</sup>In adopting the large family assumption in this financial setting, we follow Gertler and Karadi (2011) and Gertler and Kiyotaki (2010). Although we think the large-family metaphor helps to streamline the model presentation, the equations that characterize the equilibrium are the same, with one minor exception described below, as if we had adopted the slightly different presentation in BGG.

<sup>13</sup>We have in mind the banks of Gertler and Kiyotaki (2010). For a detailed discussion, see section 6 in the work of Christiano and Ikeda (2013a). To interpret our entrepreneurs as financial firms, it is necessary that there be no agency problem between the entrepreneur and the bank.

The following subsection describes the details of one period in the life of an  $N$ -type entrepreneur. The subsection after that discusses the implications for the aggregates of all entrepreneurs.

*One Period in the Life of an Entrepreneur.*—Each  $N$ -type entrepreneur obtains a loan,  $B_{t+1}^N$ , from a mutual fund, which the entrepreneur combines with  $N$  to purchase raw capital,  $\bar{K}_{t+1}^N$ , in an anonymous and competitive market at a price of  $Q_{\bar{K},t}$ . That is,  $Q_{\bar{K},t}\bar{K}_{t+1}^N = N + B_{t+1}^N$ . As explained in Section IA, entrepreneurs purchase capital from households. Entrepreneurs do not acquire capital from their own household.

After purchasing capital, each  $N$ -type entrepreneur experiences an idiosyncratic shock,  $\omega$ , which converts capital,  $\bar{K}_{t+1}^N$ , into efficiency units,  $\omega\bar{K}_{t+1}^N$ . Following BGG, we assume that  $\omega$  has a unit-mean log normal distribution that is independently drawn across time and across entrepreneurs. Denote the period  $t$  standard deviation of  $\log \omega$  by  $\sigma_t$ . The random variable,  $\omega$ , captures the idiosyncratic risk in actual business ventures. For example, in the hands of some entrepreneurs, a given amount of raw capital (e.g., metal, glass, and plastic) is a great success (e.g., the Apple iPad or the Blackberry cell phone), and in other cases, it is less successful (e.g., the NeXT computer or the Blackberry Playbook). The *risk shock*,  $\sigma_t$ , characterizes the extent of cross-sectional dispersion in  $\omega$ . We allow  $\sigma_t$  to vary stochastically over time, and we discuss its law of motion below.

After observing the period  $t + 1$  aggregate rates of return and prices, each  $N$ -type entrepreneur determines the utilization rate,  $u_{t+1}^N$ , of its effective capital and supplies an amount of capital services,  $u_{t+1}^N \omega\bar{K}_{t+1}^N$ , for a competitive market rental rate denoted by  $r_{t+1}^k$ .

At the end of period  $t + 1$  production, the  $N$ -type entrepreneur who experienced the shock,  $\omega$ , is left with  $(1 - \delta)\omega\bar{K}_{t+1}^N$  units of capital, after depreciation. This capital is sold in competitive markets to households at the price,  $Q_{\bar{K},t+1}$ . In this way, an  $N$ -type entrepreneur who draws a shock,  $\omega$ , at the end of period  $t$  enjoys rate of return  $\omega R_{t+1}^k$  at  $t + 1$ , where

$$(10) \quad R_{t+1}^k \equiv \frac{(1 - \tau^k)[u_{t+1} r_{t+1}^k - a(u_{t+1})] \Upsilon^{-(t+1)} P_{t+1} + (1 - \delta)Q_{\bar{K},t+1} + \tau^k \delta Q_{\bar{K}',t}}{Q_{\bar{K},t}}$$

Here, the increasing and convex function  $a$  captures the idea that capital utilization is costly (we describe this function below). We have deleted the superscript  $N$  from the capital utilization rate. We do so because the only way utilization affects the entrepreneur is through (10), and the choice of utilization that maximizes (10) is evidently independent of the entrepreneur's net worth. From here on, we suppose that  $u_{t+1}$  is set to its optimizing level, which is a function of  $r_{t+1}^k$  and  $\Upsilon^{-(t+1)} P_{t+1}$ . Finally,  $\tau^k$  in (10) denotes the tax rate on capital income, and we assume depreciated capital can be deducted at historical cost.

Thus, each entrepreneur in period  $t$ , regardless of net worth, has access to a stochastic, constant rate to scale technology,  $R_{t+1}^k \omega$ .<sup>14</sup> The loan obtained by an  $N$ -type

<sup>14</sup>In the case where the entrepreneur is interpreted as a financial firm, we can follow Gertler and Kiyotaki (2010) in supposing that  $R_{t+1}^k \omega$  is the return on securities purchased by the financial firm from a nonfinancial firm. The

entrepreneur in period  $t$  takes the form of a standard debt contract,  $(Z_{t+1}, L_t)$ . Here,  $L_t \equiv (N + B_{t+1}^N)/N$  denotes leverage and  $Z_{t+1}$  is the gross nominal rate of interest on debt. Let  $\bar{\omega}_{t+1}$  denote the value of  $\omega$  that divides entrepreneurs who cannot repay the interest and principal from those who can repay. In particular,

$$(11) \quad R_{t+1}^k \bar{\omega}_{t+1} Q_{\bar{K},t} \bar{K}_{t+1}^N = B_{t+1}^N Z_{t+1}.$$

Entrepreneurs with  $\omega \leq \bar{\omega}_{t+1}$  declare bankruptcy. Such an entrepreneur is monitored by a mutual fund, which then takes all the entrepreneur's assets. We have left off the superscript  $N$  on  $L_t$ ,  $\bar{\omega}_{t+1}$ , and  $Z_{t+1}$ . This is to minimize notation and is a reflection of the fact (see below) that the equilibrium value of these objects is independent of  $N$ . Note that given (11), a standard debt contract can equivalently be represented as  $(Z_{t+1}, L_t)$  or  $(\bar{\omega}_{t+1}, L_t)$ . We assume that  $N$ -type entrepreneurs value a particular debt contract according to the expected net worth in period  $t + 1$ :

$$(12) \quad E_t \left\{ \int_{\bar{\omega}_{t+1}}^{\infty} [R_{t+1}^k \omega Q_{\bar{K},t} \bar{K}_{t+1}^N - B_{t+1}^N Z_{t+1}] dF(\omega, \sigma_t) \right\} \\ = E_t [1 - \Gamma_t(\bar{\omega}_{t+1})] R_{t+1}^k L_t N.$$

Here,

$$\Gamma_t(\bar{\omega}_{t+1}) \equiv [1 - F_t(\bar{\omega}_{t+1})] \bar{\omega}_{t+1} + G_t(\bar{\omega}_{t+1}), \quad G_t(\bar{\omega}_{t+1}) = \int_0^{\bar{\omega}_{t+1}} \omega dF_t(\omega), \\ L_t = \frac{Q_{\bar{K},t} \bar{K}_{t+1}^N}{N},$$

so that  $1 - \Gamma_t(\bar{\omega}_{t+1})$  represents the share of average entrepreneurial earnings,  $R_{t+1}^k Q_{\bar{K},t} \bar{K}_{t+1}^N$ , received by entrepreneurs.<sup>15</sup> In (12) we have made use of (11) to express  $Z_{t+1}$  in terms of  $\bar{\omega}_{t+1}$ .

Before describing equilibrium in the market for loans, we discuss the mutual funds. It is convenient (though it involves no loss of generality) to imagine that mutual funds specialize in lending to entrepreneurs with specific levels of net worth,  $N$ . Each of the identical  $N$ -type mutual funds holds a large portfolio of loans that is perfectly diversified across  $N$ -type entrepreneurs. To extend loans,  $B_{t+1}^N$  per entrepreneur, the representative  $N$ -type mutual fund issues  $B_{t+1}^N$  in deposits to households at the competitively determined nominal interest rate,  $R_t$ . As discussed in Section IA, this rate is assumed not to be contingent on the realization of period  $t + 1$  uncertainty. We assume that mutual funds do not have access in period  $t$  to period  $t + 1$  state-contingent markets for funds, apart from their debt contracts with entrepreneurs. As a result, the funds received in each period  $t + 1$  state of nature must be no

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nonfinancial firm possesses a technology that generates the rate of return,  $R_{t+1}^k \omega$ , which it turns over in full to the financial firm. This interpretation requires that there be no agency costs in the financial/nonfinancial firm relationship.

<sup>15</sup>BGG show that  $\Gamma_t(\bar{\omega})$  is strictly increasing and concave,  $0 \leq \Gamma_t(\bar{\omega}) \leq 1$ ,  $\lim_{\bar{\omega} \rightarrow \infty} \Gamma_t(\bar{\omega}) = 1$ , and  $\Gamma_t(0) = 0$ .

less than the funds paid to households in that state of nature. That is, the following cash constraint

$$(13) \quad [1 - F_t(\bar{\omega}_{t+1})] Z_{t+1} B_{t+1}^N + (1 - \mu) \int_0^{\bar{\omega}_{t+1}} \omega dF_t(\omega) R_{t+1}^k Q_{\bar{K}^t, t} \bar{K}_{t+1}^N \geq B_{t+1}^N R_t$$

must be satisfied in each period  $t + 1$  state of nature. The object on the left of the inequality in (13) is the return, per entrepreneur, on revenues received by the mutual fund from its entrepreneurs. The first term on the left indicates revenues received from the fraction of entrepreneurs with  $\omega \geq \bar{\omega}_{t+1}$ , and the second term corresponds to revenues obtained from bankrupt entrepreneurs. The latter revenues are net of mutual funds' monitoring costs, which take the form of final goods and correspond to the proportion  $\mu$  of the assets of bankrupt entrepreneurs. The left term in (13) also cannot be strictly greater than the term on the right in any period  $t + 1$  state of nature because in that case mutual funds would make positive profits, and this is incompatible in equilibrium with free entry.<sup>16</sup> Thus, free entry and the cash constraint in (13) jointly imply that (13) must hold as a strict equality in every state of nature. Using this fact and rearranging (13) after substituting out for  $Z_{t+1} B_{t+1}^N$  using (11), we obtain

$$(14) \quad \Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1}) = \frac{L_t - 1}{L_t} \frac{R_t}{R_{t+1}^k},$$

in each period  $t + 1$  state of nature.

The  $(\bar{\omega}_{t+1}, L_t)$  combinations which satisfy (14) define a menu of state  $(t + 1)$ -contingent standard debt contracts offered to entrepreneurs. Entrepreneurs select the contract that maximizes their objective, (12). Since  $N$  does not appear in the constraint and appears only as a constant of proportionality in the objective, it follows that all entrepreneurs select the same  $(\bar{\omega}_{t+1}, L_t)$  regardless of their net worth.

After entrepreneurs have sold their undepreciated capital, collected capital rental receipts, and settled their obligations to their mutual fund at the end of period  $t + 1$ , a random fraction,  $1 - \gamma_{t+1}$ , of each entrepreneur's assets is transferred to their household. The complementary fraction,  $\gamma_{t+1}$ , remains in the hands of the entrepreneurs. In addition, each entrepreneur receives a lump-sum transfer,  $W_{t+1}^e$ , from the household. The objects,  $\gamma_{t+1}$  and  $W_{t+1}^e$ , are exogenous.

A more elaborate model would clarify why the transfer of funds back and forth between households and their entrepreneurs is exogenous and not responsive to economic conditions. In any case, it is clear that, given our assumptions, the larger is the net worth of a household's entrepreneurs, the greater are the resources available to the household. This is why it is in the interests of the representative household to instruct each of its entrepreneurs to maximize expected net worth. By the law of

<sup>16</sup>In an alternative market arrangement, mutual funds in period  $t$  interact with households via two types of financial instrument. One corresponds to the non-state-contingent deposits discussed in the text. Another is a financial instrument in which payments are contingent on the period  $t + 1$  state of nature. Under this complete market arrangement a mutual fund has a single zero-profit condition in period  $t$ . Using equilibrium state-contingent prices, that zero-profit condition corresponds to the requirement that the period  $t$  expectation of the left side of (13) equals the right side of (13). The market arrangement described in the text is the one implemented by BGG, and we have not explored the complete markets arrangement described in this footnote.

large numbers, this is how the household maximizes the aggregate net worth of all its entrepreneurs. Entrepreneurs comply with their household's request in exchange for perfect consumption insurance.<sup>17</sup>

*Implications for Aggregates.*—The quantity of raw capital purchased by entrepreneurs in period  $t$  must equal the quantity produced,  $\bar{K}_{t+1}$ , by households:

$$(15) \quad \bar{K}_{t+1} = \int_0^\infty \bar{K}_{t+1}^N f_t(N) dN.$$

The aggregate supply of capital services by entrepreneurs is

$$(16) \quad K_t = \int_0^\infty \int_0^\infty u_t^N \omega \bar{K}_t^N f_{t-1}(N) dF(\omega) dN = u_t \bar{K}_t,$$

where the last equality uses (15), the facts that utilization is the same for all  $N$  and that the mean of  $\omega$  is unity. Market clearing in capital services requires that the supply of capital services,  $K_t$ , equal the corresponding demand,  $\int_0^1 K_{j,t} dj$ , by the intermediate good producers in Section IA.

By the law of large numbers, the aggregate profits of all  $N$ -type entrepreneurs at the end of period  $t$  is  $[1 - \Gamma_{t-1}(\bar{\omega}_t)] R_t^k Q_{\bar{K},t-1} \bar{K}_t^N$ . Integrating this last expression over all  $N$  and using (15) evaluated at  $t - 1$ , we obtain  $[1 - \Gamma_{t-1}(\bar{\omega}_t)] R_t^k Q_{\bar{K},t-1} \bar{K}_t$ . Thus, after transfer payments, aggregate entrepreneurial net worth at the end of period  $t$  is

$$(17) \quad N_{t+1} = \gamma_t [1 - \Gamma_{t-1}(\bar{\omega}_t)] R_t^k Q_{\bar{K},t-1} \bar{K}_t + W_t^e.$$

In sum,  $N_{t+1}$ ,  $\bar{\omega}_{t+1}$ , and  $L_t$  can be determined by (14), (17), and an expression that characterizes the solution to the entrepreneur's optimization problem.<sup>18</sup> Notably, it is possible to solve for these aggregate variables without determining the distribution of net worth in the cross-section of entrepreneurs,  $f_t(N)$ , or the law of motion over time of that distribution. By the definition of leverage,  $L_t$ , these variables place a restriction on  $\bar{K}_{t+1}$ . This restriction replaces the intertemporal equation in a model such as the one in CEE, which relates the rate of return on capital,  $R_{t+1}^k$ , to the intertemporal marginal rate of substitution in consumption. The remaining two financial variables to determine are the aggregate quantity of debt extended to entrepreneurs in period  $t$ ,  $B_{t+1}$ , and their state-contingent interest rate,  $Z_{t+1}$ . Note that

$$B_{t+1} = \int_0^\infty B_{t+1}^N f_t(N) dN = \int_0^\infty [Q_{\bar{K},t} \bar{K}_{t+1}^N - N] f_t(N) dN = Q_{\bar{K},t} \bar{K}_{t+1} - N_{t+1},$$

<sup>17</sup> A variety of decentralizations of the entrepreneur side of the model is possible. An alternative is the one used by BGG, in which entrepreneurs are distinct households who maximize expected net worth as a way of maximizing utility from consumption. In this arrangement, a fraction of entrepreneurs die in each period, and the complementary fraction are born. Dying entrepreneurs consume a fraction,  $\Theta$ , of their net worth with the rest being transferred in lump-sum form to households. Entrepreneurs' motive for maximizing expected net worth is to maximize expected end-of-life consumption. The mathematical distinction between the BGG decentralization and the one pursued here is that BGG include entrepreneurial consumption in the resource constraint. Since  $\Theta$  is a very small number in practice, this distinction is very small.

<sup>18</sup> The first-order condition associated with the entrepreneur's optimization problem is

$$E_t \left\{ [1 - \Gamma_t(\bar{\omega}_{t+1})] \frac{R_{t+1}^k}{R_t} + \frac{\Gamma_t'(\bar{\omega}_{t+1})}{\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t'(\bar{\omega}_{t+1})} \left[ \frac{R_{t+1}^k}{R_t} (\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1})) - 1 \right] \right\} = 0.$$

where the last equality uses (9) and (15). Finally,  $Z_{t+1}$  can be obtained by integrating (11) relative to the density  $f_t(N)$  and solving  $Z_{t+1} = R_{t+1}^k \bar{\omega}_{t+1} L_t$ .

### C. Monetary Policy and Resource Constraint

We express the monetary authority's policy rule directly in linearized form:

$$(18) \quad R_t - R = \rho_p(R_{t-1} - R) + (1 - \rho_p) \left[ \alpha_\pi(\pi_{t+1} - \pi_t^*) + \alpha_{\Delta y} \frac{1}{4}(g_{y,t} - \mu_{z^*}) \right] + \frac{1}{400} \varepsilon_t^p,$$

where  $\varepsilon_t^p$  is a shock (in annual percentage points) to monetary policy, and  $\rho_p$  is a smoothing parameter in the policy rule. Here,  $R_t - R$  is the deviation of the net quarterly interest rate,  $R_t$ , from its steady-state value. Similarly,  $\pi_{t+1} - \pi_t^*$  is the deviation of anticipated quarterly inflation from the central bank's inflation target. The expression,  $g_{y,t} - \mu_{z^*}$  is quarterly growth in gross domestic product (GDP), in deviation from its steady state.

We complete the description of the model with a statement of the resource constraint:

$$Y_t = D_t + G_t + C_t + \frac{I_t}{\Upsilon^t \mu_{\Upsilon,t}} + a(u_t) \Upsilon^{-t} \bar{K}_t,$$

where the last term on the right represents the aggregate capital utilization costs of entrepreneurs, an expression that makes use of (15) and the fact that each entrepreneur sets the same rate of utilization on capital,  $u_t$ . Also,  $D_t$  is the aggregate resources used for monitoring by mutual funds:

$$D_t = \mu G(\bar{\omega}_t) (1 + R_t^k) \frac{Q_{\bar{K},t-1} \bar{K}_t}{P_t}.$$

Finally,  $G_t$  denotes government consumption, which we model as

$$(19) \quad G_t = z_t^* g_t,$$

where  $g_t$  is a stationary stochastic process. We adopt the usual sequence of markets equilibrium concept.

### D. Adjustment Costs, Shocks, Information, and Model Perturbations

Our specification of the adjustment cost function for investment is as follows:

$$S(x_t) = \frac{1}{2} \{ \exp[\sqrt{S''}(x_t - x)] + \exp[-\sqrt{S''}(x_t - x)] - 2 \},$$

where  $x_t \equiv \zeta_{t,t} I_t / I_{t-1}$ , and  $x$  denotes the steady-state value of  $x_t$ . Note that  $S(x) = S'(x) = 0$  and  $S''(x) = S''$ , where  $S''$  denotes a model parameter. The value

of the parameter,  $S''$ , has no impact on the model's steady state, but it does affect dynamics. Also, the utilization adjustment cost function is

$$a(u) = r^k [\exp(\sigma_a(u - 1)) - 1] \frac{1}{\sigma_a},$$

where  $\sigma_a > 0$ , and  $r^k$  is the steady-state rental rate of capital in the model. This function is designed so that utilization is unity in steady state, independent of the value of the parameter  $\sigma_a$ .

We now turn to the shocks in the model. We include a measurement error shock on the long-term interest rate,  $R_t^L$ . In particular, we interpret

$$(R_t^L)^{40} = (\tilde{R}_t^L)^{40} \eta_{t+1} \cdots \eta_{t+40},$$

where  $\eta_t$  is an exogenous measurement error shock. We refer to  $\eta_t$  as the term premium shock. The object,  $R_t^L$ , denotes the long-term interest rate in the model, while  $\tilde{R}_t^L$  denotes the long-term interest rate in the data. If in the empirical analysis we find that  $\eta_t$  accounts for only a small portion of the variance in  $\tilde{R}_t^L$ , then we infer that the model's implications for the long-term rate are good.

The model we estimate includes 12 aggregate shocks:  $\eta_t$ ,  $\varepsilon_t$ ,  $\mu_{zt}$ ,  $\lambda_{ft}$ ,  $\pi_t^*$ ,  $\zeta_{c,t}$ ,  $\mu_{\gamma,t}$ ,  $\zeta_{l,t}$ ,  $\gamma_t$ ,  $\sigma_t$ ,  $\varepsilon_t^p$ , and  $g_t$ . We model the log-deviation of each shock from its steady state as a first-order univariate autoregression. In the case of the inflation target shock, we simply fix the autoregressive parameter and innovation standard deviation to  $\rho_{\pi^*} = 0.975$  and  $\sigma_{\pi^*} = 0.0001$ , respectively. This representation is our way of accommodating the downward inflation trend in the early part of our dataset. Also, we set the first-order autocorrelation parameter on each of the monetary policy and equity shocks,  $\varepsilon_t^p$  and  $\gamma_t$ , to zero.

We now discuss the timing assumptions that govern when agents learn about shocks. A standard assumption in estimated equilibrium models is that a shock's statistical innovation (i.e., the one-step-ahead error in forecasting the shock based on the history of its past realizations) becomes known to agents only at the time that the innovation is realized. Recent research casts doubt on this assumption. For example, Alexopoulos (2011) and Ramey (2011) use US data to document that people receive information about the period  $t$  statistical innovation in technology and government spending, respectively, before the innovation is realized. These observations motivate us to consider the following shock representation:

$$(20) \quad x_t = \rho_x x_{t-1} + \overbrace{\xi_{0,t} + \xi_{1,t-1} + \cdots + \xi_{p,t-p}}^{= u_t},$$

where  $p > 0$  is a parameter. In (20),  $x_t$  is the log deviation of the shock from its nonstochastic steady state and  $u_t$  is the i.i.d. statistical innovation in  $x_t$ .<sup>19</sup> We express the variable,  $u_t$ , as a sum of i.i.d., mean zero random variables that are orthogonal to  $x_{t-j}$ ,  $j \geq 1$ . We assume that in period  $t$ , agents observe  $\xi_{j,t}$ ,  $j = 0, 1, \dots, p$ . We refer

<sup>19</sup>Expression (20) is a time series representation suggested by Davis (2008) and also used by Christiano et al. (2010).

to  $\xi_{0,t}$  as the unanticipated component of  $u_t$  and to  $\xi_{j,t}$  as the anticipated, or news, components of  $u_{t+j}$ , for  $j > 0$ . We refer to the individual terms,  $\xi_{j,t}$ ,  $j > 0$ , as news shocks. The  $\xi_{j,t}$ s are assumed to have the following correlation structure:

$$(21) \quad \rho_{x,n}^{|i-j|} = \frac{E\xi_{i,t}\xi_{j,t}}{\sqrt{(E\xi_{i,t}^2)(E\xi_{j,t}^2)}}, \quad i, j = 0, \dots, p,$$

where  $\rho_{x,n}$  is a scalar, with  $-1 \leq \rho_{x,n} \leq 1$ . The subscript  $n$  indicates news. For the sake of parameter parsimony, we place the following structure on the variances of the shocks:  $E\xi_{0,t}^2 = \sigma_x^2$ ,  $E\xi_{1,t}^2 = E\xi_{2,t}^2 = \dots = E\xi_{p,t}^2 = \sigma_{x,n}^2$ .

In sum, for a shock  $x_t$  with the information structure in (20), there are four free parameters:  $\rho_x$ ,  $\rho_{x,n}$ ,  $\sigma_{x,0}$ , and  $\sigma_{x,n}$ . For a shock with the standard information structure in which agents become aware of  $u_t$  in period  $t$ , i.e., there are no news shocks, there are two free parameters:  $\rho_x$  and  $\sigma_x$ .

We consider several perturbations of our model in which the information structure in (20) is assumed for one or more of the following set of shocks: technology, monetary policy, government spending, equity, and risk shocks. As we shall see below, the model that has the highest marginal likelihood is the one with news on the risk shock, so this is our baseline model specification. We also consider a simpler version of our model—we call it *CEE*—which does not include financial frictions. We obtain this model from our baseline model by adding an intertemporal Euler equation corresponding to household capital accumulation and dropping the three equations that characterize the financial frictions: the optimality condition characterizing the contract selected by entrepreneurs, the equation characterizing zero profits for the financial intermediaries, and the law of motion of entrepreneurial net worth. Of course, it is also necessary to delete the resources used by banks for monitoring from the resource constraint. A detailed list of the equations of our models can be found in the online Appendix and in the computer code that is also available online.

## II. Inference About Parameters and Model Fit

This section discusses the data used in the analysis, the priors and posteriors for model parameters, and measures of model fit. Finally, we report the effects on model fit of adding news to different economic shocks.

### A. Data

We use quarterly observations on 12 variables covering the period, 1985:I to 2010:II. These include 8 variables that are standard in empirical analyses of aggregate data: GDP, consumption, investment, inflation, the real wage, the relative price of investment goods, hours worked, and the federal funds rate. We interpret the relative price of investment goods as a direct observation on  $(\Upsilon^t \mu_{\Upsilon,t})^{-1}$ . The aggregate quantity variables are measured in real, per capita terms.<sup>20</sup>

<sup>20</sup> GDP is deflated by its implicit price deflator; real household consumption is the sum of household purchases of nondurable goods and services, each deflated by its own implicit price deflator; investment is the sum of gross private domestic investment plus household purchases of durable goods, each deflated by its own price deflator. The

We also use four financial variables in our analysis. For our period  $t$  measure of credit,  $B_{t+1}$ , we use data on credit to nonfinancial firms taken from the flow of funds dataset constructed by the US Federal Reserve Board.<sup>21</sup> We convert our measure of credit into real, per capita terms. Our measure of the slope of the term structure,  $R_t^L - R_t$ , is the difference between the ten-year constant maturity US government bond yield and the federal funds rate. Our period  $t$  indicator of entrepreneurial net worth,  $N_{t+1}$ , is the Dow Jones Wilshire 5000 index, converted into real, per capita terms. Finally, we measure the credit spread,  $Z_t - R_t$ , by the difference between the interest rate on BAA-rated corporate bonds and the ten-year US government bond rate.<sup>22</sup>

Prior to analysis, we transform the data as follows. In the case of consumption, investment, credit, GDP, net worth, the price of investment, and the real wage we take the logarithmic first difference and then remove the sample mean. We remove sample means separately from each variable in order to prevent counterfactual implications of the model for the low frequencies from distorting inference in the higher business cycle frequencies that interest us. For example, on average consumption grew faster than GDP in our dataset, while our model predicts that the log of the consumption to GDP ratio is stationary. We measure hours worked in log (per capita) levels, net of the sample mean. We measure inflation, the credit spread, the risk free rate and the slope of the term structure in level terms, net of their sample mean. One implication of our approach is that inference is not affected by the well-known fact that a model like ours cannot account for the fact that the slope of the term structure is on average positive.<sup>23</sup> We ensure the econometric consistency of our analysis by always applying the same transformation to the variables in the model as were applied to the actual data.

### B. Priors and Posteriors

We partition the model parameters into two sets. The first set contains parameters that we simply fix a priori. Thus, the depreciation rate  $\delta$ , capital's share  $\alpha$ , and the inverse of the Frisch elasticity of labor supply  $\sigma_L$  are fixed at 0.025, 0.4 and 1, respectively. We set the mean growth rate  $\mu_z$  of the unit root technology shock and the quarterly rate of investment-specific technological change  $\Upsilon$  to 0.41 percent and 0.42 percent, respectively. We choose these values in order to ensure that the model

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aggregate labor input is an index of nonfarm business hours of all persons. These variables are converted to per capita terms by dividing by the population over 16. (Annual population data obtained from the Organization for Economic Cooperation and Development were linearly interpolated to obtain quarterly frequency.) The real wage,  $W_t/P_t$ , is hourly compensation of all employees in nonfarm business divided by the GDP implicit price deflator,  $P_t$ . The short-term risk-free interest rate,  $R_t$ , is the three-month average of the daily effective federal funds rate. Inflation is measured as the logarithmic first difference of the GDP deflator. The relative price of investment goods,  $P_t^I/P_t = 1/(\Upsilon^t \mu_{\Upsilon,t})$ , is measured as the implicit price deflator for investment goods divided by the implicit price deflator for GDP.

<sup>21</sup> From the "flow data" tables, we take the credit market instruments components of net increase in liabilities for nonfarm, nonfinancial corporate business and nonfarm, noncorporate business. We convert our credit variable to real, per capita terms by dividing by the GDP implicit price deflator as well as by the population over 16.

<sup>22</sup> We also considered the spread measure constructed by Gilchrist and Zakrajšek (2012). They consider each loan obtained by each of a set of firms taken from the COMPUSTAT database. In each case, they compare the interest rate actually paid by the firm with what the US government would have paid on a loan with a similar maturity. When we repeated our empirical analysis using the Gilchrist-Zakrajšek spread data, we obtained similar results.

<sup>23</sup> Roughly, our model embodies the linear term structure hypothesis: the idea that the long rate is the average of future short rates.

TABLE 1—CALIBRATED PARAMETERS (*Time unit of model: quarterly*)

$\beta$	Discount rate	0.9987
$\sigma_L$	Curvature on disutility of labor	1.00
$\psi_L$	Disutility weight on labor	0.7705
$\lambda_w$	Steady-state markup, suppliers of labor	1.05
$\mu_z$	Growth rate of the economy	0.41
$\Upsilon$	Trend rate of investment-specific technological change	0.42
$\delta$	Depreciation rate on capital	0.025
$\alpha$	Power on capital in production function	0.40
$\lambda_f$	Steady-state markup, intermediate good firms	1.20
$1 - \gamma$	Fraction of entrepreneurial net worth transferred to households	$1 - 0.985$
$W^e$	Transfer received by entrepreneurs	0.005
$\eta_g$	Steady-state government spending–GDP ratio	0.20
$\pi^{target}$	Steady-state inflation rate (APR)	2.43
$\tau^c$	Tax rate on consumption	0.05
$\tau^k$	Tax rate on capital income	0.32
$\tau^l$	Tax rate on labor income	0.24

steady state is consistent with the mean growth rate of per capita, real GDP in our sample, as well as the average rate of decline in the price of investment goods. The steady-state value of  $g_t$  in (1.19) is set to ensure that the ratio of government consumption to GDP is 0.20 in steady state. Steady-state inflation is fixed at 2.4 percent on an annual basis. The household discount factor  $\beta$  is fixed at 0.9987. There are no natural units for the measurement of hours worked in the model, so we arbitrarily set  $\psi_L$  so that hours worked is unity in steady state. Following CEE, we fix the steady-state markups in the labor market  $\lambda_w$  and in the product market  $\lambda_f$  at 1.05 and 1.2, respectively. The steady-state value of the parameter controlling the rate at which the household transfers equity from entrepreneurs to itself,  $1 - \gamma$ , is set to  $1 - 0.985$ . This is fairly close to the  $1 - 0.973$  value used by BGG. Our settings of the consumption, labor, and capital income tax rates,  $\tau^c$ ,  $\tau^l$ , and  $\tau^k$ , respectively, are discussed by Christiano, Motto, and Rostagno (2010, pp. 79–80). These parameter values are reported in Table 1.

The second set of parameters to be assigned values consists of the 36 parameters listed in Table 2. We study these using the Bayesian procedures surveyed by An and Schorfheide (2007). Panel A of Table 2 considers the parameters that do not pertain to the exogenous shocks in the model. The price and wage stickiness parameters,  $\xi_p$  and  $\xi_w$ , are given relatively tight priors around values that imply prices and wages remain unchanged for, on average, one-half and one year, respectively. The posteriors for these parameters are higher. The relatively large value of the posterior mode on the parameter  $\sigma_a$  governing the capital utilization cost function implies utilization fluctuates relatively little. In most cases, there is a reasonable amount of information in the data about the parameters, indicated by the fact that the standard deviation of the posterior distribution is often less than half of the standard deviation of the prior distribution.<sup>24</sup>

<sup>24</sup>In this remark, we implicitly approximate the posterior distribution with the Laplace approximation, which is Normal.

TABLE 2—MODEL PRIORS AND POSTERiors

Parameter name	Parameter	Prior distribution			Posterior distribution	
		Prior dist	Mean	SD	Mode	SD
<i>Panel A. Economic parameters</i>						
Calvo wage stickiness	$\xi_w$	beta	0.75	0.1	0.81	0.019
Habit parameter	$b$	beta	0.5	0.1	0.74	0.050
Steady-state probability of default	$F(\bar{\omega})$	beta	0.007	0.0037	0.0056	0.0023
Monitoring cost	$\mu$	beta	0.275	0.15	0.21	0.073
Curvature, utilization cost	$\sigma_a$	normal	1	1	2.54	0.70
Curvature, investment adjust cost	$S''$	normal	5	3	10.78	1.71
Calvo price stickiness	$\xi_p$	beta	0.5	0.1	0.74	0.035
Policy weight on inflation	$\alpha_\pi$	normal	1.5	0.25	2.40	0.16
Policy smoothing parameter	$\rho_p$	beta	0.75	0.1	0.85	0.015
Price indexing weight on inflation target	$\iota$	beta	0.5	0.15	0.90	0.049
Wage indexing weight on inflation target	$\iota_w$	beta	0.5	0.15	0.49	0.11
Wage indexing weight on persistent technology growth	$\iota_\mu$	beta	0.5	0.15	0.94	0.029
Policy weight on output growth	$\alpha_{\Delta y}$	normal	0.25	0.1	0.36	0.099
<i>Panel B. Shocks</i>						
Correlation among signals	$\rho_{\sigma,n}$	normal	0	0.5	0.39	0.095
Autocorrelation, price markup shock	$\rho_{\lambda_f}$	beta	0.5	0.2	0.91	0.034
Autocorrelation, price of investment goods shock	$\rho_{\mu_\psi}$	beta	0.5	0.2	0.99	0.0085
Autocorrelation, government	$\rho_g$	beta	0.5	0.2	0.94	0.023
Autocorrelation, persistent technology growth	$\rho_{\mu_z}$	beta	0.5	0.2	0.15	0.070
Autocorrelation, transitory technology	$\rho_\epsilon$	beta	0.5	0.2	0.81	0.065
Autocorrelation, risk shock	$\rho_\sigma$	beta	0.5	0.2	0.97	0.0093
Autocorrelation, consumption preference shock	$\rho_{\zeta_c}$	beta	0.5	0.2	0.90	0.031
Autocorrelation, marginal efficiency of investment	$\rho_{\zeta_t}$	beta	0.5	0.2	0.91	0.017
Autocorrelation, term structure shock	$\rho_\eta$	beta	0.5	0.2	0.97	0.025
Standard deviation, anticipated risk shock	$\sigma_{\sigma,n}$	invg2	0.001	0.0012	0.028	0.0028
Standard deviation, unanticipated risk shock	$\sigma_{\sigma,0}$	invg2	0.002	0.0033	0.07	0.0099
SD, measurement error on net worth		Weibull	0.01	5	0.018	0.0009
<i>Standard deviations, shock innovations</i>						
Price markup	$\sigma_{\lambda_f}$	invg2	0.002	0.0033	0.011	0.0022
Investment price	$\sigma_{\mu_\psi}$	invg2	0.002	0.0033	0.004	0.0003
Government consumption	$\sigma_g$	invg2	0.002	0.0033	0.023	0.0016
Persistent technology growth	$\sigma_{\mu_z}$	invg2	0.002	0.0033	0.0071	0.0005
Equity	$\sigma_\gamma$	invg2	0.002	0.0033	0.0081	0.001
Temporary technology	$\sigma_\epsilon$	invg2	0.002	0.0033	0.0046	0.0003
Monetary policy	$\sigma_{\epsilon^p}$	invg2	0.583	0.825	0.49	0.037
Consumption preference	$\sigma_{\zeta_c}$	invg2	0.002	0.0033	0.023	0.003
Marginal efficiency of investment	$\sigma_{\zeta_t}$	invg2	0.002	0.0033	0.055	0.012
Term structure	$\sigma_\eta$	invg2	0.002	0.0033	0.0016	0.0007

Note: invg2: "inverse gamma distribution, type 2."

We treat the steady-state probability of default,  $F(\bar{\omega})$ , as a free parameter. We do this by making the variance of  $\log \omega$  a function of  $F(\bar{\omega})$  and the other parameters of the model. The mean of our prior distribution for  $F(\bar{\omega})$ , 0.007, is close to the 0.0075 value used by BGG, or the 0.0097 percent value used in Fisher (1999). The mode of the posterior distribution is not far away, 0.0056. The mean of the prior distribution for the monitoring cost,  $\mu$ , is 0.275. This is within the range of

0.20–0.36 that Carlstrom and Fuerst (1997) defend as empirically relevant. The mode of the posterior distribution for  $\mu$  is close, 0.2149. Comparing prior and posterior standard deviations, we see that there is a fair amount of information about the monitoring cost in our data and somewhat less about  $F(\bar{\omega})$ . The steady-state value of the risk shock,  $\sigma = \sqrt{\text{Var}(\log(\omega))}$ , that is implied by the mode of our model parameters is 0.26. Section IVA below discusses some independent evidence on the empirical plausibility of this result for the risk shock.

Values for the parameters of the shock processes are reported in panel B of Table 2. The posterior mode of the standard deviation of the unanticipated component of the shock to  $\log \sigma_t$ ,  $\xi_{0,t}$ , is 0.07. The corresponding number associated with the anticipated components,  $\xi_{i,t}$ ,  $i = 1, \dots, 8$ , is 0.0283. This implies that a substantial 57 percent of the variance in the statistical innovation in  $\log \sigma_t$  is anticipated.<sup>25</sup> The posterior mode on the correlation among anticipated and unanticipated shocks is 0.4. Thus, when agents receive information,  $\xi_{i,t}$ ,  $i = 0, \dots, 8$ , about current and future risk, there is a substantial correlation in news about adjacent periods, while that correlation is considerably smaller for news about horizons three periods apart and more.<sup>26</sup>

For the most part, the posterior modes of the autocorrelations of the shocks are quite large. The exception is the autocorrelation of the growth rate of the persistent component of technology growth,  $\mu_{z,t}$ . This is nearly zero, so that  $\log z_t$  is roughly a random walk. For the most part, there is substantial information in the data about the parameters of the shock processes, as measured by the small size of the posterior standard deviation relative to the prior standard deviation. The exception is the anticipated and unanticipated components of the risk shock, where the standard deviation of the posterior is larger than the standard deviation of the prior.

Table 3 reports the steady-state properties of the model when parameters are set to their mode under the prior distribution. The table also reports the analog objects in the data. Overall, the model and data match well. An exception is the model's capital output ratio, which is a little low. In part, the relatively low stock of capital reflects the effects of the financial frictions in the model. Our strategy for computing the posterior distribution of the model parameters does not make use of information in the data about the sort of ratios displayed in Table 3. It is therefore not surprising that when the model parameters are assigned their values at the posterior mode, the model's performance relative to the ratios in Table 3 deteriorates somewhat. With two exceptions that deterioration is quantitatively negligible. The exceptions are the equity-to-debt ratio and credit velocity, both of which are predicted to be 0.98.

### C. Where is the News?

In our baseline model, we place news shocks on risk and not on other variables. Much of the news literature attaches these shocks to technology and government consumption. This section reports marginal likelihood statistics which suggest that the most preferred shock to put news on is the risk shock.

<sup>25</sup>In particular,

$$0.57 = \frac{8 \times 0.0283^2}{8 \times 0.0283^2 + 0.07^2}.$$

<sup>26</sup>For example, the correlation between  $\xi_{1,t}$  and  $\xi_{4,t}$  is only  $0.4^3 = 0.06$ .

TABLE 3—STEADY-STATE PROPERTIES, MODEL AT PRIORS VERSUS DATA

Variable	Model	Sample averages
$\frac{i}{y}$	0.25	0.24 <sup>a</sup>
$\frac{c}{y}$	0.54	0.59 <sup>b</sup>
$\frac{g}{y}$	0.20	0.16
$\frac{k}{y}$	7.6	10.9 <sup>c</sup>
$\frac{N}{K-N}$ (Equity-to-debt ratio)	1.91	1.3–4.7 <sup>d</sup>
Transfer received by new entrepreneurs as percent of GDP	0.18	not known
Banks monitoring costs as percent of GDP	0.45	not known
Credit velocity	1.53	1.67 <sup>e</sup>
Inflation (APR)	2.43	2.47 <sup>f</sup>
Short-term risk free rate (APR)	4.67	4.80 <sup>g</sup>

*Notes:* All sample averages are computed over the period 1985:I–2008:II, except inflation and the short-term interest rate, which are computed over 1987:I–2008:II. Model objects are computed on the basis of the parameters evaluated at the prior mode.

<sup>a</sup>Investment includes residential, nonresidential, equipment, plants, business durables, change in inventories, and durable consumption. *Source:* BEA.

<sup>b</sup>Personal Consumption Expenditure includes nondurables and services. *Source:* BEA.

<sup>c</sup>Capital stock includes private nonresidential fixed assets, private residential, stock of consumer durables, and stock of private inventories. *Source:* BEA.

<sup>d</sup>Masulis (1983) reports an equity-to-debt ratio for US corporations in the range of 1.3–2 over the period 1937–1984.

<sup>e</sup>Credit velocity is computed as annual GDP over credit, where credit is defined as credit market instruments liabilities of nonfarm nonfinancial corporate business plus credit market instruments liabilities of nonfarm noncorporate business. *Source:* Flow of Funds Accounts of the Federal Reserve Board.

<sup>f</sup>Computed on the basis of the GDP Price Index. *Source:* BEA.

<sup>g</sup>3-month average of the daily effective Federal Funds rate. *Source:* Federal Reserve Board.

Consider Table 4. According to the first row in the table, the log marginal likelihood of our baseline model is 4,564.95. The second row shows that when we drop news from the risk shock, the fit of the model drops tremendously. In particular, the log marginal likelihood falls roughly 400 log points. Then, while keeping news off the risk shock we add news to other shocks one at a time. Results are reported in Table 4 in the order of increasing model fit. Putting news on the persistent technology shock and on government consumption adds the least to fit, compared to the scenario in which there are no news shocks at all. Putting news on the transitory technology shock or on the monetary policy shock adds a substantial amount to fit. Each of these adds roughly 300 log points to the marginal likelihood. Adding news to the equity shock adds an even larger amount to fit. The greatest improvement in fit from adding news to a single shock, apart from adding news to the risk shock, comes from adding news to the marginal efficiency of investment shock. News on the marginal efficiency of investment shock adds 40 additional log points to fit above what is achieved by adding news to the equity shock.

Because the news literature focuses relatively heavily on technology shocks, we want to give news on technology shocks the best possible chance in terms of fit. So, we also considered the case where news is added to all three technology shocks simultaneously. That adds 20 log points to fit beyond the case where there is news

TABLE 4—MARGINAL LIKELIHOOD OF PLACING NEWS ON ALTERNATIVE SHOCKS

News on:	Marginal likelihood
Risk shock, $\sigma_t$ (baseline specification)	4,564.95
No news on any shock	4184.10
Persistent technology shock, $\mu_{z,t}$	4,184.74
Government spending shock, $g_t$	4,195.93
Transitory technology shock, $\varepsilon_t$	4,423.39
Monetary policy shock, $\varepsilon_t^p$	4,486.08
Equity shock, $\gamma_t$	4,491.44
Marginal efficiency of investment shock, $\xi_{I,t}$	4,531.97
All technology shocks, $\varepsilon_t, \mu_{z,t}, \xi_{I,t}$	4,557.14

*Notes:* The marginal likelihood is computed using Geweke's (1999) modified harmonic mean method. The computations are based on a Monte Carlo Markov chain of length 200,000 for each model.

on the marginal efficiency of investment shock alone.<sup>27</sup> Table 4 shows that all these other ways of introducing news into the model adds less to model fit than does adding news to the risk shock alone (see the first row).

We infer two results from the findings in Table 4. First, news shocks have the potential to substantially improve the econometric fit of a model. Second, if one wants to place news on only one shock (as we do, for parameter parsimony reasons), then putting news on the risk shock is the best choice because it adds the most to model fit.

### III. The Risk Shock

We begin this section by discussing the various quantitative indicators which suggest that the risk shock is the most important driver of the business cycle. We then review what it is about our model and data that explains our finding. Previous studies of business cycles have stressed other shocks as the primary driving force. The last part of this section discusses which of those shocks are displaced by the risk shock.

#### A. Measuring the Importance of the Risk Shock

Consider first the results in Figure 1. The solid line in panel A displays the year-over-year growth rate in per capita, real GDP for our sample. An interpretation of this line is that it is the result of simulating our model's response to all of the estimated shocks and to the initial conditions. The dotted line shows the result of this same simulation when we feed our model only the unanticipated and anticipated components of the risk shock. The notable feature of panel A is how close the dotted and solid lines are to each other. According to the results, the decline in GDP growth associated with the 2001 recession is closely associated with the risk shock. The 2007 recession is similar. The 2007 NBER business cycle peak coincides with a peak in the component of GDP driven by the risk shock. The full magnitude of the GDP drop in the 2007–2009 recession can be accounted for by the risk shock, as well as the partial bounce back at the end of our sample. The remaining panels

<sup>27</sup> In results not reported in Table 4, we find that adding news to any two of the three technology shocks adds less to model fit than does adding news to all three of the technology shocks simultaneously.

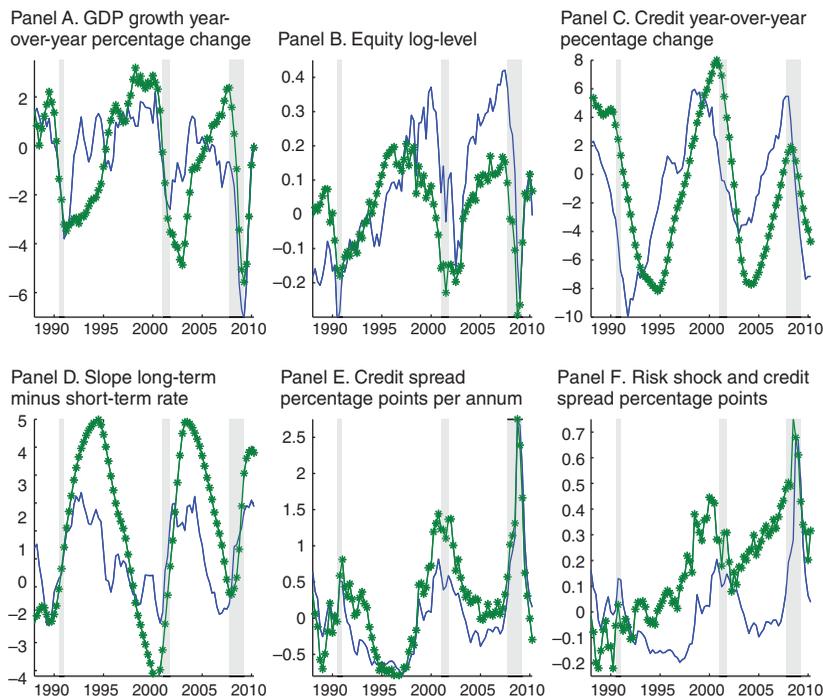


FIGURE 1. THE ROLE OF THE RISK SHOCK IN SELECTED VARIABLES

*Notes:* All data are demeaned. With the exception of panels B and F, the solid line is the data. The solid line in panel B differs from the actual data by a small, estimated measurement error. The starred line in panels A–E is the result of feeding only the estimated anticipated and unanticipated components of the risk shock to the model. Panel F displays the credit spread (solid line) and the risk shock,  $\sigma$ , (the latter expressed in percent deviation from steady state). Shaded areas indicate NBER recession dates.

in Figure 1 indicate that the risk shock is also closely associated with aggregate financial variables. Thus, panel B shows that the risk shock alone accounts for a large portion of the fluctuations in the log level of per capita, real equity. Panel C shows that a large part of the movements in the year-over-year growth rate in real per capita credit are accounted for by the risk shock. Panel D indicates that the risk shock accounts for a substantial component of the fluctuations in the slope of the term structure of interest rates. Panel E shows that the risk shock accounts for a very large part of the movements in the credit spread. In sum, the risk shock accounts for a large part of the movements of the key variables in our dataset.

To gain additional insight into the results in panel E, panel F displays the estimated risk shock and our measure of the credit spread.<sup>28</sup> Note that although the risk shock,  $\sigma_t$ , and the credit spread are positively related, they are by no means perfectly correlated. This is so, despite the result in panel E which shows that when we feed only the estimated anticipated and unanticipated components of  $\sigma_t$  to the baseline

<sup>28</sup> The estimated risk shock was obtained in the same way used to compute the starred line in panels A–E in Figure 1. We fed the estimated anticipated and unanticipated components of the risk shock to the time series representation for risk. The risk variable reported in the figure is  $100 \times (\sigma_t - \sigma) / \sigma$ .

model, the resulting simulated credit spread tracks the corresponding empirical measure closely. We infer that the credit spread is a complicated dynamic function of the news about the risk shock,  $\sigma_t$ , and not just a simple function of the  $\sigma_t$  itself.

Our final indicator of the importance of risk shocks appears in Table 5. That table reports the percentage of the variance in the level of several variables at business cycle frequencies which is contributed by our various shocks.<sup>29</sup> This is done for several specifications of our model. The entries in the first column of data, labeled *Risk*, have a format,  $x|y|z$ , where  $x$ ,  $y$ , and  $z$  each denote the percentage of business cycle variance due to various components of the innovations to risk. The variable  $x$  pertains to both anticipated and unanticipated components,  $\xi_{0,t}, \dots, \xi_{8,t}$ ;  $y$  pertains to the unanticipated component,  $\xi_t^0$ ; and  $z$  pertains to the anticipated component,  $\xi_{1,t}, \dots, \xi_{8,t}$ . The sum,  $x + y + z$ , does not always add to unity because there is a small amount of correlation between the shocks (see (21)). For now, we consider only the first row of each panel. The results in those rows are computed using our baseline model, evaluated at the posterior mode of the parameters.

Consistent with the evidence in panel A of Figure 1, over 60 percent of the business cycle variance in output is accounted for by the risk shock. Indeed, the risk shock is by far more important for GDP than are any of the other shocks. Interestingly, with one exception the risk shock affects the economy primarily via its unanticipated component. The unanticipated component of risk is more than twice as important as the anticipated component, for GDP. It is four times as important in the case of consumption. In the exceptional case, the credit spread, the anticipated and unanticipated components of risk are of roughly equal importance. This evidence complements the findings in Table 4, that news is important in the modeling of business cycles.

The risk shock is particularly important for the financial variables. Interestingly, the risk shock makes the linear term structure model of interest rates look good, because our term premium shock accounts for only 7 percent of the fluctuations in the slope of the term structure.<sup>30</sup> More than half the business cycle variance in the slope of the term structure is attributed to the risk shock.

### B. Why is the Risk Shock So Important?

The answer to the question in the title of this subsection is that, when fed to our model, the risk shock generates responses that resemble the business cycle. One way that we show this is by studying our model's impulse responses to disturbances in risk. In principle, model impulse responses point to another way to evaluate a model, namely, by comparing them to analogous objects estimated using minimally restricted vector autoregressions (VAR). However, the model developed here implies that standard methods for identifying VARs do not work.<sup>31</sup> These considerations

<sup>29</sup>We compute the variance of the (log) levels of the variables in the frequency domain, leaving off frequencies lower than the business cycle.

<sup>30</sup>To save space, we do not display this result in Table 5. With the exception of the slope of the term structure, the term premium shock accounts for essentially 0 percent of the variance of the variables in the model.

<sup>31</sup>The results in Figure 1, panel E and in Table 5 suggest that the risk shock and the credit spread are very similar. This might tempt one to interpret one-step-ahead forecast errors in the credit spread computed using a limited list of aggregate variables as shocks to  $\sigma_t$  that are unexpected by economic agents. Under this interpretation, the estimated dynamic responses in economic variables to the one-step-ahead forecast error in the credit spread would constitute an empirical estimate of the economy's response to risk shocks. The impulse responses obtained by this

TABLE 5—VARIANCE DECOMPOSITION AT BUSINESS CYCLE FREQUENCY (Percent)

Shock variable	Risk $\sigma_t$	Equity $\gamma_t$	M.E.I. $\zeta_{l,t}$	Technol. $\varepsilon_t, \mu_{z,t}$	Markup $\lambda_{f,t}$	M.P. $\epsilon_t$	Demand $\zeta_{c,t}$	Exog.Spend. $g_t$
GDP	62 16 38	0	13	2	12	2	4	3
<i>drop all fin. var</i>	1 0 1	0	44	12	22	3	11	8
<i>CEE</i>	[-]	[-]	[39]	[18]	[31]	[4]	[3]	[5]
Consumption	16 3 12	0	11	3	19	2	46	3
<i>drop all fin. var</i>	0 0 0	0	2	15	26	3	51	2
<i>CEE</i>	[-]	[-]	[6]	[12]	[9]	[1]	[67]	[5]
Investment	73 18 46	0	21	0	4	1	1	0
<i>drop all fin. var</i>	2 0 2	0	85	2	7	2	2	0
<i>CEE</i>	[-]	[-]	[57]	[10]	[24]	[3]	[5]	[0]
Equity	69 23 35	2	23	0	1	2	0	0
Credit spread	95 39 42	1	3	0	0	0	0	0
Credit	64 12 46	10	17	2	4	1	1	0
Slope	56 12 38	0	17	3	8	6	2	0

*Notes:* For each variable indicated in the first column, variance decompositions are generated by the baseline model evaluated at the mode of the posterior distribution. Results in the row marked *drop all fin. var* are generated by the baseline model evaluated at the mode of the posterior distribution when our four financial variables are dropped. Results in the rows marked *CEE* are generated by the *CEE* model (i.e., the model without financial frictions), evaluated at the mode of the posterior distribution computed based on our eight standard macroeconomic variables. Numbers in each row may not add up to 100 due to rounding. The table does not display results for shocks (such as  $\pi_t^*$  and  $\mu_{\tau,t}$ ) whose contribution is less than 1/2 of 1 percent. To save space, we also dropped results for the term premium shock. With one exception it contributes roughly zero to the variance of all variables. In the exceptional case, the term premium shock accounts for 7 percent of the variance of *Slope*, the slope of the term structure. Data on equity is also explained by measurement error, which is estimated to contribute 3 percent in the baseline model. The contribution of the risk shock,  $\sigma_t$ , is presented in the following way: the first entry is the contribution of the entire shock, the second entry is the contribution of  $\xi_0$ , and the third entry is the contribution of  $\xi_1, \dots, \xi_8$ . The latter two contributions do not sum up to the first entry as they ignore the correlation between the  $\xi$ s. Business cycle frequency is measured as a periodic component with cycles of 8–32 quarters, obtained using the model spectrum.

motivate us to also consider a second type of evidence, one based on the implications of risk shocks for the dynamic cross-correlations of aggregate output with various macroeconomic variables. Finally, we ask which variables in our dataset account for the preeminence of the risk shock over other variables.

*Impulse Response Functions.*—As stressed in the introduction, the economic intuition underlying the response of the model to a jump in the risk shock is simple. With a rise in risk, the probability of a low  $\omega$  increases, and banks raise the interest rate charged on loans to entrepreneurs to cover the resulting costs. Entrepreneurs respond by borrowing less, so credit drops. With fewer financial resources, entrepreneurs purchase less capital, which has the consequence that investment is lower.

VAR strategy could be compared with the impulses implied by the model. But this VAR strategy is not justified in our framework, for several reasons. One is our finding that agents anticipate a substantial portion of the one-step-ahead forecast error in risk by as much as two years in advance. Ramey (2011) in particular has emphasized the specification error consequences of a VAR strategy which ignores that agents have advance information about statistical innovations in shocks. (See also the work of Blanchard, L'Huillier, and Lorenzoni 2013.) An interesting application of the model of this article would be to quantify the specification error consequences of the VAR identification strategy described above.

The drop in investment leads to a fall in output and consumption. The fall in investment produces a fall in the price of capital, which reduces the net worth of entrepreneurs, and this magnifies the impact of the jump in risk through standard accelerator effects. The decline in economic output leads to a fall in costs, and, thus, inflation is reduced. The decline in credit is smaller in percentage terms than the decline in net worth, because in these dynamic responses there is a partially offsetting effect on credit. In particular, when the price of capital drops, there is an expectation that it will return to steady state. Other things the same, the resulting higher prospective return on capital raises credit. The net impact of all these effects on credit is negative. But, the decline is muted, and this is why credit falls less than net worth, in percentage terms. In what follows, we display the impulse response functions which support the intuition just described. A more detailed exploration of the economics of these impulse responses can be found in our online Appendix, sections D and I.

Figure 2 displays the dynamic response of various variables to an unanticipated shock in risk (i.e.,  $\xi_{0,t}$ , the solid line) and to a two-year-ahead anticipated shock (i.e.,  $\xi_{8,t}$ , the line with circles). (The other lines will be discussed later.) Both shocks occur in period 0. To simplify the interpretation of the impulse responses, each of  $\xi_{0,0}$  and  $\xi_{8,0}$  are disturbed in isolation, ignoring the fact that according to our empirical analysis, the anticipated and unanticipated shocks are correlated. In addition, we restrict both shocks to be the same magnitude, with  $\xi_{0,0} = \xi_{8,0} = 0.10$ .

Panel H of Figure 2 displays the dynamic response of  $\sigma_t$  to the two shocks. The response of  $\sigma_t$  to  $\xi_{8,0}$  mirrors the response to  $\xi_{0,0}$ , except that it is displaced by eight periods. According to panel A, the dynamic response of the credit spread to  $\xi_{0,0}$  and to  $\xi_{8,0}$  differs in roughly the same way that the response of  $\sigma_t$  to  $\xi_{0,0}$  and  $\xi_{8,0}$  differs.<sup>32</sup> Still, the response of the credit spread is countercyclical in each case. The dynamic responses of the other variables to  $\xi_{0,0}$  and to  $\xi_{8,0}$  are much more similar. In particular, credit, investment, output, and inflation all drop immediately and persistently in response to both  $\xi_{0,0}$  and  $\xi_{8,0}$ . Interestingly, in all these cases the eventual response to  $\xi_{8,0}$  exceeds the eventual response to  $\xi_{0,0}$ . The slope of the term structure of interest rates,  $R_t^L - R_t$ , is countercyclical in response to each shock to risk. The peak response of  $R_t^L - R_t$  to  $\xi_{8,0}$  is bigger than the peak response of  $R_t^L - R_t$  to  $\xi_{0,0}$ .

Consider panel F, which displays the response of consumption to a jump in risk. There is perhaps a small qualitative difference in the response of consumption to the  $\xi_{0,0}$  and  $\xi_{8,0}$  shocks. Consumption drops immediately in response to  $\xi_{0,0}$ , while it exhibits almost no response in the immediate aftermath of a disturbance in  $\xi_{8,0}$ . Still, in both cases consumption eventually drops sharply. This negative response of consumption to a jump in risk may at first glance seem surprising. The rise in risk in effect corresponds to an increased tax on investment, and this is why investment falls. With flexible prices one expects this decrease in the demand for current goods to drive down the price of current goods relative to future goods, i.e., the real interest rate. This drop in the real interest rate would then be expected to stimulate

<sup>32</sup>Note that  $\xi_{0,t}$  has a smaller impact on the period  $t$  interest rate spread than on subsequent values of the spread. This is because the period  $t$  spread corresponds to loans extended in period  $t - 1$ . Disturbances in  $\xi_{0,t}$  affect  $\sigma_t$ , which has a direct impact on loans extended in period  $t$  and, therefore, on the period  $t + 1$  spread. The fact that  $\xi_{0,t}$  has some effect on the period  $t$  spread reflects the state contingency in the interest rate paid by entrepreneurs.

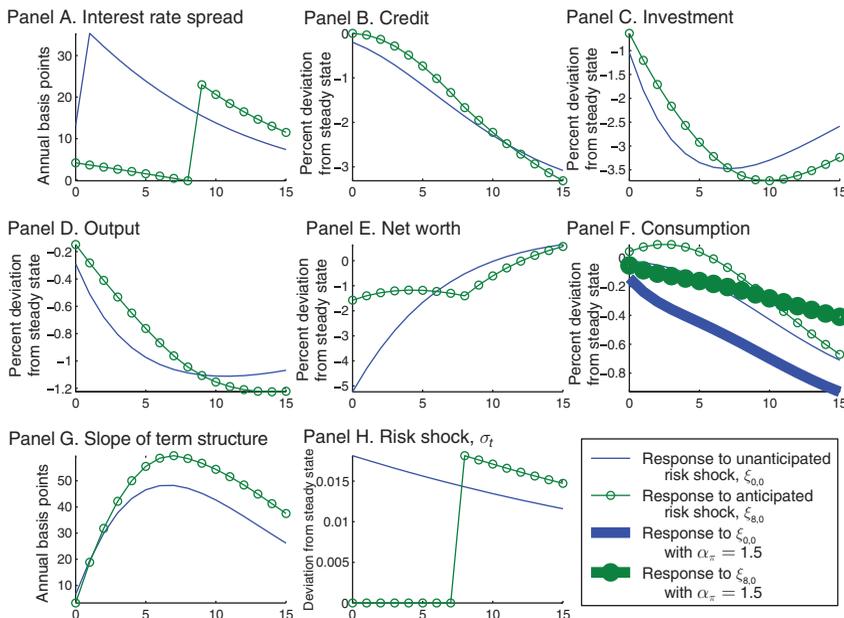


FIGURE 2. DYNAMIC RESPONSES TO UNANTICIPATED AND ANTICIPATED COMPONENTS OF THE RISK SHOCK

consumption. In fact, consumption does rise in response to a jump in risk in the flexible wage and price version of our model. But consumption and investment move up and down together over the business cycle in the data. So, any econometric estimator working with the flexible price and wage version of our model would assign a very small role to risk shocks in business cycles. Price and wage frictions are essential to our finding that the risk shock is important.

The reason that consumption falls after a rise in risk in our model is that the real interest rate is not exclusively determined by market forces when wages and prices are not flexible. In our baseline model, monetary policy also plays a key role in determining the real rate of interest. Moreover, our standard representation of monetary policy is known to imply that the real interest rate responds less to shocks than it does when wages and prices are flexible (see, for example, the work of Christiano, Trabandt, and Walentin 2010). We conclude that consumption falls after a rise in risk because the real interest rate falls by less than it would if wages and prices were flexible. The results in Figure 2 lend support to this intuition. Panel F in that figure displays the drop in consumption when the weight on inflation in the Taylor rule,  $\alpha_\pi$ , is reduced to 1.5. Because inflation falls in the wake of a positive shock to risk, the reduced value of  $\alpha_\pi$  implies that the interest rate falls by less after a positive shock to risk. Consistent with the intuition outlined above, the smaller value of  $\alpha_\pi$  results in a larger drop in consumption after a positive shock to risk. The impact is particularly noticeable for the anticipated shock,  $\xi_{8,0}$ . The cut in the value of  $\alpha_\pi$  does not have an interesting impact on any of the other responses in Figure 2, and so we do not display those in the figure. A more extended discussion of these observations about consumption appears in the online technical Appendix, Section I.

*Dynamic Cross Correlations.*—Here, we define the business cycle as the dynamic cross-correlations between output and the variables in Figure 3. Before we computed the correlations displayed in Figure 3, our data on output, credit, investment, equity, and consumption were logged and converted to year-over-year growth rates. The gray area in each graph is a 95 percent confidence interval centered about the empirical correlations, which are not themselves displayed. In the figure, *slope* indicates the slope of the term premium,  $R^L - R$ , and *credit spread* indicates  $Z - R$ , the premium of the interest rate paid by (nondefaulting) entrepreneurs over the risk-free rate. The lines with circles in Figure 3 display the model-implied correlations when only the anticipated and unanticipated shocks to risk are activated. We emphasize two results in Figure 3. First, the dynamic correlations implied by the model with only risk shocks resemble the correlations when all the shocks are activated. This illustrates how risk shocks are a dominant shock in the model. Second, the dynamic correlations with only the risk shock resemble broadly the corresponding objects in the data, and in this sense, they generate what looks like a business cycle.

Taken together, the impulse response functions and cross-correlation analysis quantify the sense in which risk shocks in the model generate dynamics that resemble the business cycle. This is the principal reason our econometric analysis assigns such an important role to risk shocks in its account of business cycles.

*The Risk Shock and Financial Data.*—Our conclusion that the risk shock is the most important shock driving the business cycle depends sensitively on the fact that we include financial variables in the analysis. We can see this by examining the rows beyond the first one in the panels of Table 5. The rows marked *drop all fin. var* report variance decompositions at the posterior mode of our baseline model when our four financial variables are dropped from the analysis.<sup>33</sup> The rows marked *CEE* allow one to see what happens to inference about the importance of shocks when a model without financial frictions is used. The results in the *CEE* rows are computed using the *CEE* model discussed in Section ID, evaluated at the mode of the posterior distribution of its parameters. The dataset underlying that posterior distribution is the same as the dataset underlying the calculations reported in the rows labeled *drop all fin. var*. The entries for *CEE* corresponding to risk and equity shocks are empty, since these shocks do not appear in the *CEE* model. In addition, we do not include the term premium shock in the *CEE* model, so the entry corresponding to this shock is also empty.

The key result in Table 5 is that when all financial variables are dropped, the risk shock vanishes in importance, and the marginal efficiency of investment shock appears to be the most important driver of the business cycle. Moreover, when our model is not permitted to see the financial variables, it reaches a similar conclusion as does *CEE* regarding the historical importance of different shocks. In particular, the major shock driving GDP fluctuations is the marginal efficiency of investment shock,  $\zeta_{I,t}$ .

To some extent, the degree to which the risk shock is pushed out when the financial variables are dropped is overstated in Table 5. The log of the posterior density

<sup>33</sup>The four variables dropped are credit, the credit spread, equity and the slope of the term structure. The number of model parameters is reduced somewhat in the *drop all fin. var* case. Dropping equity implies that the measurement error variance for equity drops from the set of model parameters. Similarly, dropping the slope of the term structure implies that the parameters governing the term premium shock,  $\eta_t$ , also drop from the set of model parameters.

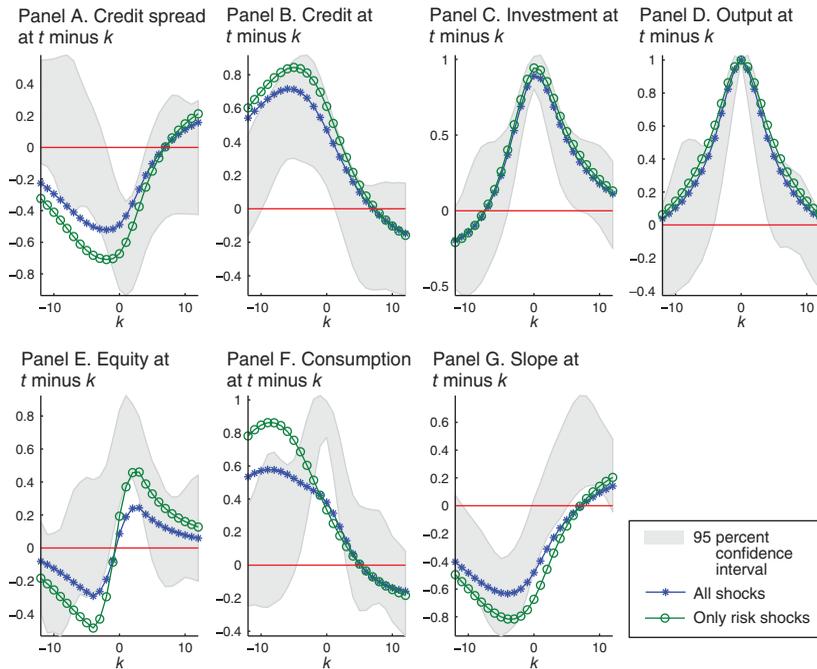


FIGURE 3. SELECTED CROSS-CORRELATIONS WITH CONTEMPORANEOUS OUTPUT, MODEL AND DATA

at the mode on which the results in the *drop all fin. var* row of Table 5 are based is, apart from an additive constant, equal to 3221.3. But we found another local maximum for the posterior distribution where the log of the posterior is, apart from the same additive constant, 3,218.4. The difference in the criterion at these two points is a trivial 2.9 log points. However, the properties of the model at the alternative parameterization resemble those of our baseline model in that the marginal efficiency of investment plays only a modest role, and the risk shock is the most important shock. For this reason, we conclude that in the absence of financial variables, it is hard to distinguish a parameterization of the model in which the risk shock is important and the marginal efficiency of investment is not important from another in which the reverse is true.<sup>34</sup> When the financial data are introduced, it is no longer the case that these two parameterizations are hard to distinguish.<sup>35</sup>

### C. Why Do Risk Shocks Drive Out Other Intertemporal Shocks?

Our model includes three shocks that affect intertemporal decisions: risk,  $\sigma_t$ ; the marginal efficiency of investment,  $\zeta_{t,i}$ ; and shocks to equity,  $\gamma_t$ . We find that the risk shock is far more important than the other two shocks. For example, according to Table 5, disturbances in  $\sigma_t$  account for 62 percent of the fluctuations in output while

<sup>34</sup>A related observation is made in Justiniano, Primiceri, and Tambalotti (2010).

<sup>35</sup>Our results show that the posterior distribution, when none of the four financial variables are included, has a second local maximum near the mode. When we included some or all the financial data, we never encountered another local maximum near the mode.

shocks to  $\zeta_{i,t}$  and  $\gamma_t$  account for only 13 and 0 percent of the business cycle component of output, respectively. We discuss the reasons for these findings below.

*Marginal Efficiency of Investment Shock.*—Our finding for  $\zeta_{i,t}$  differs sharply from results in the literature, which assign a very substantial role in business cycles to  $\zeta_{i,t}$ .<sup>36</sup> We reproduced the finding in the literature for  $\zeta_{i,t}$  by computing the variance decompositions implied by the *CEE* model.<sup>37</sup> According to the results in Table 5, the *CEE* model implies that  $\zeta_{i,t}$  is the most important shock driving output and that it accounts for 39 percent of the business cycle fluctuations in that variable. Here, we seek to understand at an intuitive level why the risk shock reduces the importance of the marginal efficiency of investment. We focus in particular on the role played by equity.

Consider Figure 4, which displays the dynamic response of the variables in our model to several shocks. To facilitate comparison, we repeat the impulse responses to the unanticipated component in risk,  $\xi_{0,0}$ , from Figure 2 (the solid lines). The lines with circles display the dynamic responses to an innovation in  $\zeta_{i,t}$  in our model. For ease of comparison, we have scaled this innovation so that the maximal decline in output coincides with the maximal decline in the output response to  $\xi_{0,0}$ . Consider panel E of Figure 4, which displays the dynamic responses in equity. Note in particular that equity is countercyclical in response to the innovation in  $\zeta_{i,t}$ . Evidently, the marginal efficiency of investment shock has the strongly counterfactual implication that the value of equity is countercyclical. This stands in sharp contrast to the risk shock, which, consistent with the data, implies that the value of equity is procyclical.

Another way to see the contrasting implications of risk versus the marginal efficiency of investment for the cyclical properties of equity appears in Figure 5. The solid lines indicate historical observations on year-over-year output growth and on the real value of the stock market. The starred lines indicate the results of simulating the indicated model responses to the indicated shocks. The left column of graphs reproduces the relevant portions of Figure 1. It shows what output and equity would have been according to the baseline model at its posterior mode if only the estimated risk shocks had been active in our sample. The right column of graphs shows what output and equity would have been according to the *CEE* model at its posterior mode if only the marginal efficiency of investment shocks had been active.<sup>38</sup> Note that each type of shock accounts well for the dynamics of output growth. However, when equity is brought into the picture, the implications of the two perspectives on the sources of economic fluctuations differ sharply. The risk shock accounts well for the fluctuations in equity. In contrast, the marginal efficiency of investment shock predicts stock market booms when there are busts and busts when there are booms.

The intuition for these results is very simple. Consider a Marshallian cross representation of the market for capital with the price of capital,  $Q_{\bar{K},t+1}$ , on the vertical axis and the quantity of capital,  $\bar{K}_{t+1}$ , on the horizontal (see Figure 6). The supply curve corresponds to the marginal cost of building capital, derived from the household's technology for constructing capital discussed just after (8). The marginal efficiency of investment shock perturbs this supply curve. Entrepreneurs are the source

<sup>36</sup> See, for example, Justiniano, Primiceri, and Tambalotti (2010, 2011).

<sup>37</sup> See Sections ID and IIIB for a discussion of the *CEE* model and its parameters.

<sup>38</sup> In the *CEE* model, we proxy equity by the real price of capital,  $Q_{\bar{K},t+1}/P_t$ .

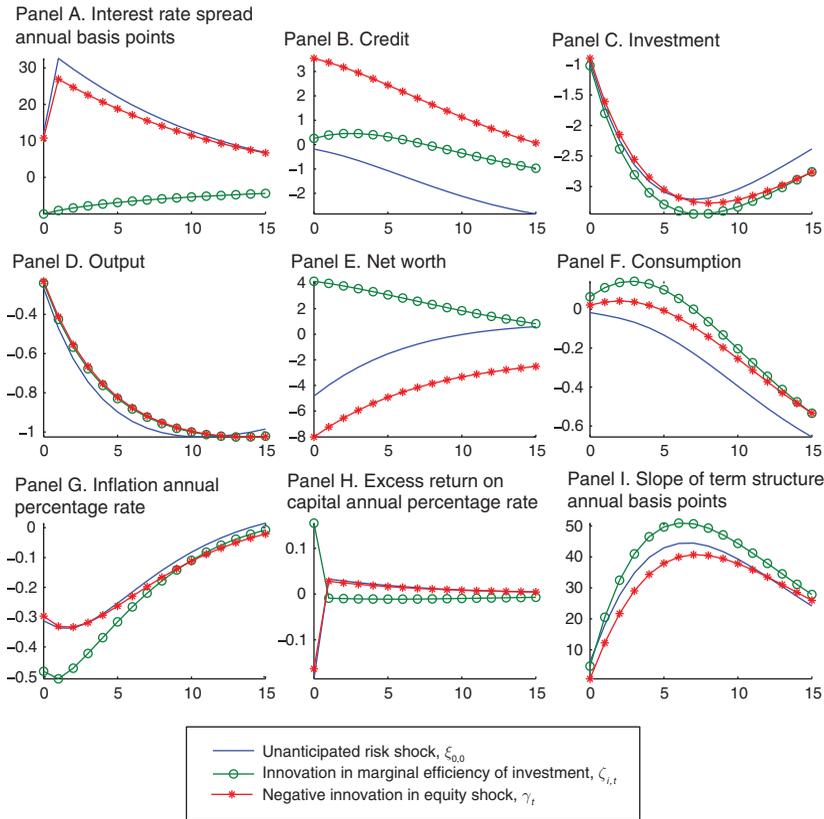


FIGURE 4. DYNAMIC RESPONSES TO THREE SHOCKS

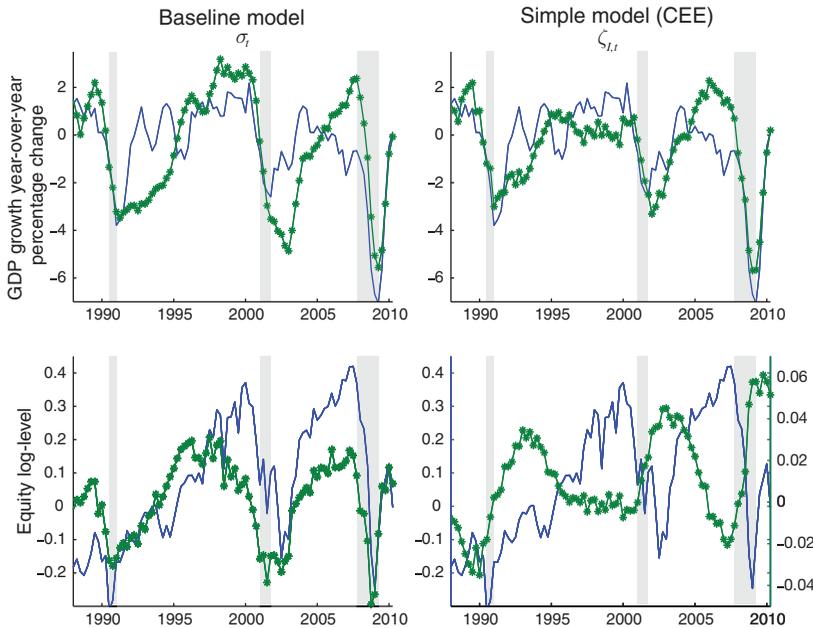


FIGURE 5. HISTORICAL DECOMPOSITIONS IN TWO MODELS

Notes: First row of graphs—actual GDP growth (solid line) and model simulated growth (starred line). Second row of graphs—same as first row, except data pertains to log level of real, per capita equity. Columns—simulation of indicated model in response to smoothed estimate of indicated shock.

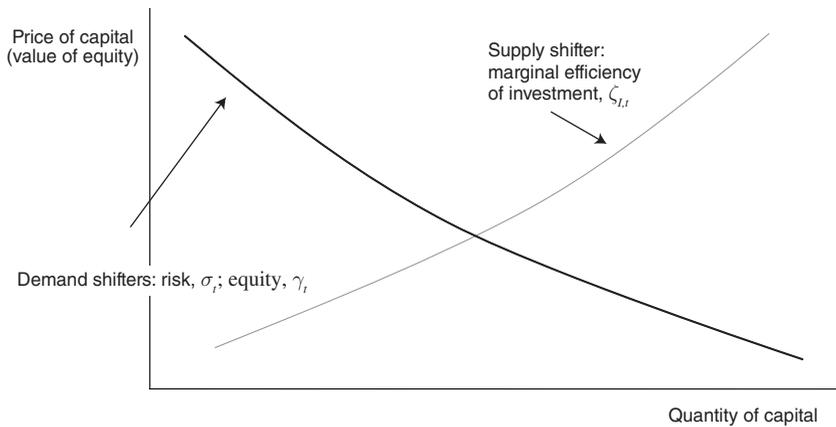


FIGURE 6. THE RISK AND EQUITY SHOCKS, VERSUS THE MARGINAL EFFICIENCY OF INVESTMENT

of demand for capital. The demand curve is perturbed by the equity and risk shocks,  $\gamma_t$  and  $\sigma_t$ , that affect the terms of entrepreneurial loan contracts with banks. The price of capital is a major input determining entrepreneurs' net worth,  $N_{t+1}$ , which we identify with the value of equity in the data.<sup>39</sup> For purposes of gaining intuition, we can think of the price of capital and the value of equity as being the same thing.

Now, suppose that there is a shock to the marginal efficiency of investment which shifts the supply curve to the left. The figure indicates that the equilibrium quantity of capital decreases. This in turn implies that fewer investment goods are purchased by the producers of capital goods, so that there is a decline in production and employment. This explains why the  $\zeta_{I,t}$  shock implies that investment is procyclical. A similar logic reaches the conclusion that the  $\sigma_t$  and  $\gamma_t$  shocks also imply procyclical investment. This intuition is consistent with the results in Figure 4, panel C.<sup>40</sup> Although the demand and supply shocks have the same implications for the cyclical properties of investment, they have opposite implications for the price of capital and, hence, the value of equity. This explains the results in panel E of Figure 4.

Inspecting Figure 4, it is also clear that the credit spread plays a role in differentiating between the risk shocks and  $\zeta_{I,t}$  shocks. According to panel A of Figure 4, the marginal efficiency of investment predicts, counterfactually, that the credit spread is procyclical. The risk shock predicts, correctly, that the credit spread is countercyclical.<sup>41</sup>

*Equity Shock.*—The risk shock,  $\sigma_t$ , also drives out equity shocks,  $\gamma_t$  (recall Table 5). To understand why this is so, consider the dynamic response of our baseline model to a negative innovation in  $\gamma_t$  (see Figure 4). According to panel B, equity and risk shocks have opposite implications for the cyclicity of credit.

<sup>39</sup>The equation that characterizes net worth is given in (17). The price of capital enters that expression via the rate of return on capital, (10).

<sup>40</sup>The dynamic responses to an innovation in  $\gamma_t$  are displayed with the curve indicated by \*s, and the equity innovation has been scaled so that the maximal decline in output coincides with the maximal decline in output in response to a risk shock.

<sup>41</sup>Note from panel F that consumption is countercyclical in the first two years after a  $\zeta_{I,t}$  shock. However, this failure of the model is not robust to alternative parameterizations. For example, when we reduce the coefficient on inflation in the interest rate rule to 1.5, then consumption falls after a  $\zeta_{I,t}$  shock, for the reasons discussed in Section IIIB above.

The reason equity shocks counterfactually imply countercyclical credit is explored in detail in Appendix D of the online Appendix. The idea is that an equity shock has two effects on credit. The first is a partial equilibrium effect. A drop in  $\gamma_t$  directly reduces the net worth of entrepreneurs, and partial equilibrium analysis of the debt contract implies that this reduces the amount that entrepreneurs borrow in period  $t$  (panel E of Figure 4 shows the response of net worth to a decline in  $\gamma_t$ ). The second, general equilibrium, effect follows from the fact that entrepreneurs with fewer resources buy less capital, and this drives down the price of capital. Because the price of capital is expected to return back up to steady state over time, the period  $t$  drop in the price of capital triggers a jump in the expected return to capital. This can be seen in panel H, which shows the immediate drop in the excess return to capital,  $(1 + R_t^k)/(1 + R_{t-1})$ , in period  $t$  as the price of period  $t$  capital drops, followed by a persistently high expected excess return. The jump in the expected return on capital causes entrepreneurs to receive more credit in period  $t$ . Thus, the partial equilibrium effect causes a fall in credit in the wake of a drop in  $\gamma_t$  and the general equilibrium effect causes a rise. In our model, the general equilibrium effect dominates, and this is why credit rises. Although credit expands, it does not expand by enough to offset the initial decline in net worth that causes the contraction in spending by entrepreneurs in the first place.

The risk shock also triggers the two effects described in the previous paragraph (the general equilibrium effect may be seen in panel H). However, Figures 3 and 4 indicate that the partial equilibrium effect dominates, so that the risk shock correctly implies procyclical credit. We suspect that this numerical result is robust because a contractionary risk shock does not have the direct, negative effect on net worth that a contractionary equity shock has. To see this, suppose credit did increase in the wake of a contractionary risk shock. Because there is no direct negative shock to net worth, we expect the overall resources available to entrepreneurs to expand. This would cause them to buy more capital, driving its price up and, hence, its anticipated rate of return down. But this drop in the anticipated rate of return is inconsistent with the assumed initial rise in credit. This is why we expect a rise in risk to robustly lead to a fall in credit.

We conclude that the credit data favor the risk shock over the equity shock because the former correctly predicts credit is procyclical, while the latter incorrectly predicts credit is countercyclical.

#### IV. Various Measures of Model Out-of-Sample Performance

The key finding of this article is that variations in risk,  $\sigma_t$ , are the most important impulse to business cycles. Whether this finding should be taken seriously depends on how seriously we take the underlying model. In this section, we offer a defense of the model based on various out-of-sample measures of fit.

We begin by examining two variables not used in our formal econometric analysis. The first of these is a measure of uncertainty recently proposed by Bloom (2009). The second is an indicator of bankruptcy rates. We use our model to project these two variables onto the sample data used in model estimation. If our analysis overstates the importance of risk shocks in the business cycle, then we expect the model to overstate the degree of cyclical variation in Bloom's measure of uncertainty and

in the bankruptcy rate. We show that, in fact, the predicted and actual degrees of cyclical variation in these two variables are very similar.

We then turn to the Federal Reserve's survey of senior loan officers to test another aspect of our analysis. Our model stresses that the origins of business cycle fluctuations lie in agency problems in the nonfinancial sector.<sup>42</sup> Other research, particularly work that focusses on the events since 2007, explores the idea that agency problems lie specifically inside the financial sector.<sup>43</sup> We display evidence in the survey of senior loan officers that lends support to the approach taken in this article.

We also examine a more conventional measure of model fit, the model's pseudo-real-time out-of-sample root mean square forecast errors (RMSE). By pseudo-real-time we mean that forecasts are computed using model parameters estimated only on revised data available at the date of the forecast. We compare the RMSEs of our baseline model with those implied by CEE as well as RMSEs implied by a Bayesian Vector Autoregression. We find that our model compares well against all these alternatives. These results are reported in the online Appendix, Section J.

### A. Implications for Uncertainty

In an influential paper, Bloom (2009) points to cyclical variation in the cross-sectional standard deviation of firm-level stock returns as evidence of the importance in business cycles of what he calls uncertainty. These data, for nonfinancial business firms, are depicted by the solid line marked with '+' in Figure 7.<sup>44</sup> As Bloom (2009) emphasizes, this measure of uncertainty is relatively high during recessions. In the 1990 and 2007 recessions, it is highest near to the business cycle trough, while in the 2001 recession it rose sharply somewhat before the recession started (vertical gray areas indicate NBER recession periods).

We computed the analog of Bloom's measure of uncertainty in our model. Conditional on the period  $t$  aggregate shocks, an entrepreneur with idiosyncratic shock  $\omega$  earns the following, as a ratio to the entrepreneur's net worth:  $R_t^e(\omega) \equiv \max\{0, [\omega - \bar{\omega}_t]\} \times R_t^k L_{t-1}$ . Here,  $L_{t-1}$  denotes leverage, and  $R_t^k$  is the cross-sectional average return on capital. According to the model,  $R_t^e(\omega)$  is not a function of the entrepreneur's level of net worth,  $N$ . The standard deviation, *std*, of the entrepreneurial return on equity in a cross-section which includes only nonbankrupt entrepreneurs (i.e., those with  $\omega > \bar{\omega}_t$ ) is  $std(R_t^e(\omega) | \omega > \bar{\omega}_t) = R_t^k L_{t-1} \sqrt{\text{Var}(\omega - \bar{\omega}_t | \omega > \bar{\omega}_t)}$ . Here,  $\text{Var}(x|D)$  denotes the variance of  $x$

<sup>42</sup>In Section IB we indicated that in principle some of our entrepreneurs could be interpreted as financial firms. However, our measure of credit in the data corresponds to borrowing by nonfinancial firms. So in the empirical analysis, we in effect take the position that our entrepreneurs are nonfinancial firms.

<sup>43</sup>See the work of Christiano and Ikeda (2013a) as well as the studies that they cite. Related research develops the idea that problems in the financial sector are a source of business cycle disturbances, without developing a detailed structural model of those disturbances. See, for example, Ajello (2012) and the references that he cites.

<sup>44</sup>There are two differences between the data studied by Bloom (2009) (see row 2 of his Table I) and our data. First, the time period in our model is quarterly, while the Center for Research in Securities Prices (CRSP) data used by Bloom (2009) are monthly. To ensure comparability, we use the data constructed by Ferreira (2012) which aggregates the monthly CRSP returns to quarterly returns. Second, we work specifically with data on nonfinancial firms rather than all firms, as does Bloom (2009). This choice of data is more consistent with our analysis, given the way we map from entrepreneurial credit and interest rate spreads into the data in our econometric analysis. However, there would have been virtually no change to Figure 7 if we had instead reported results based on CRSP data for nonfinancial and financial firms.

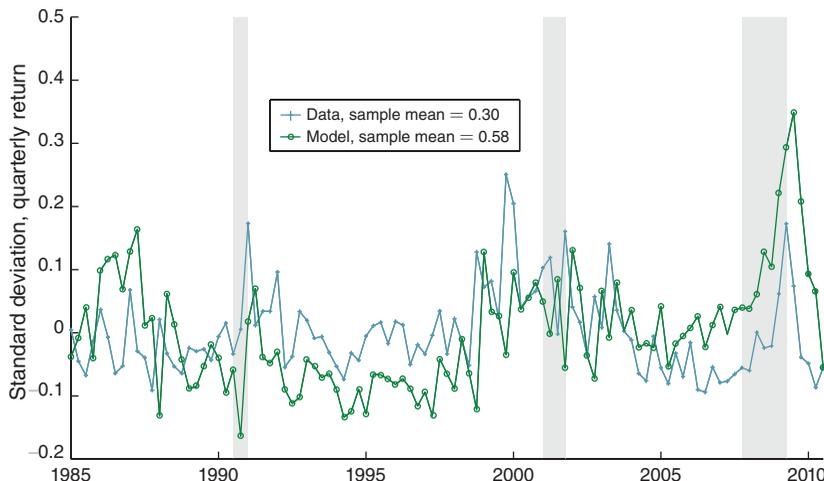


FIGURE 7. TIME SERIES, CROSS SECTION STANDARD DEVIATIONS OF QUARTERLY RATES OF RETURN MODEL AND DATA (Nonfinancial firm equity, CRSP)

Note: Shaded areas correspond to NBER dates, sample means removed from data and reported in legend.

conditional on the event,  $D$ .<sup>45</sup> We use our estimated model and the Kalman smoother to compute the projection of  $std(R_t^e(\omega)|\omega > \bar{\omega}_t)$  onto the dataset used in our formal Bayesian analysis. The results are depicted by the solid line marked with “o” in Figure 7. The empirical and model-implied data differ somewhat in terms of levels. The mean of the model and data variables is 0.58 and 0.30, respectively. Presumably, the mean of the model variable could be reduced by small adjustments in parameter values, without substantially altering the dynamic properties of the model. The real test of the model lies in comparing the magnitude of variation in the two volatility measures. To focus on this degree of variation, the two volatility measures in Figure 7 are expressed as a deviation from their respective sample means. Note that the magnitude and timing of the variation in the two volatility measures is similar. For example, both series indicate that volatility is relatively high toward the end of the 1990 and 2007 recessions. Also, the model implies that volatility is relatively high before the onset of the 2001 recession, as in the data. Because the volatility data played no role in the estimation of the model, this similarity between model and data provides evidence in support of the model.

Our model analysis also has the effect supporting Bloom’s inference from the volatility data that uncertainty is an important force in business cycles. Such support is helpful because, in addition to the usual problem of inferring causality from correlations, the degree of cyclical in Bloom’s volatility measure may at first glance not seem very big. According to our model, a key driving force of the business cycle is variations in risk and the model predicts roughly the degree of variation in volatility

<sup>45</sup>Ferreira (2012) shows that

$$\text{Var}(R_t^e(\omega)|\omega \geq \bar{\omega}_t) = \frac{1}{1 - F(\bar{\omega}_t)} e^{\sigma^2} \left[ 1 - \Phi\left(\frac{\log \bar{\omega}_t}{\sigma} - \frac{3}{2}\sigma\right) \right] - \left( \frac{1 - G(\bar{\omega}_t)}{1 - F(\bar{\omega}_t)} \right)^2.$$

For completeness, Ferreira’s derivation is reproduced in Appendix G of the online technical Appendix to this paper.

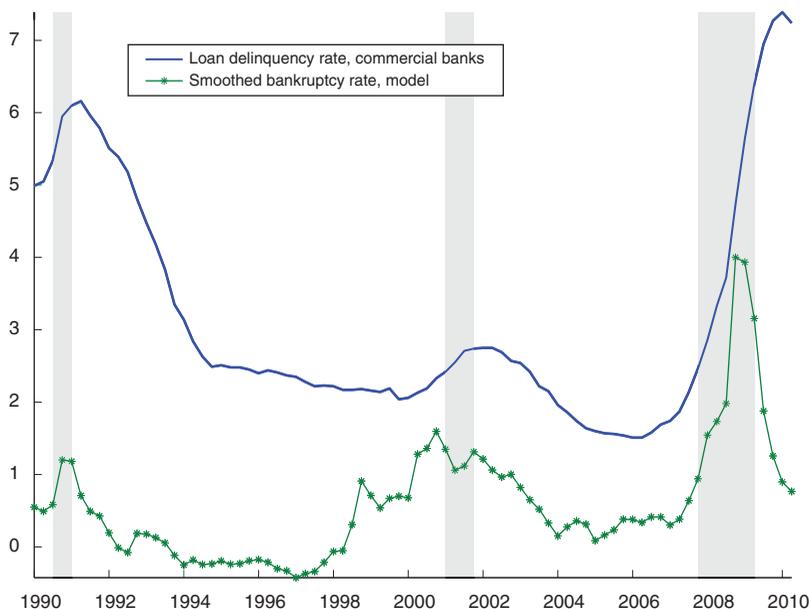


FIGURE 8. MODEL BANKRUPTCY RATE VERSUS LOAN DELINQUENCY RATE

Note: Shaded areas indicate NBER recession dates.

that is observed. This represents support for Bloom's inference because our concept of risk is similar to his concept of uncertainty. We use risk to refer exclusively to variations in idiosyncratic, microeconomic uncertainty. Bloom uses uncertainty to refer both to risk and to changes in aggregate volatility.

### B. Implications for Bankruptcy Rates

For our second out-of-sample test of the model, we use the two-sided Kalman smoother to estimate the period  $t$  default rate,  $F_{t-1}(\bar{\omega}_t)$ , implied by our model and compare it with the delinquency rate on all loans extended by commercial banks.<sup>46</sup> The results are reported in Figure 8. Note that the default rate implied by our model rises and falls with each of the three recessions in our sample, just as the loan delinquency rate does. However, the match between our model's default rate and the delinquency rate is not perfect since the latter lags recessions somewhat. Still, the two variables are reasonably similar, bearing in mind that empirical measures of default played no role in model estimation.

### C. Senior Loan Officer Opinion Survey

Each quarter, the Federal Reserve surveys the opinions of senior loan officers at commercial banks. We focus on a key question in this survey: "If you have tightened

<sup>46</sup>The data were obtained from the St. Louis Federal Reserve Bank's online data base, FRED. The FRED mnemonic for the delinquency rate on commercial bank loans is DRALACBS.

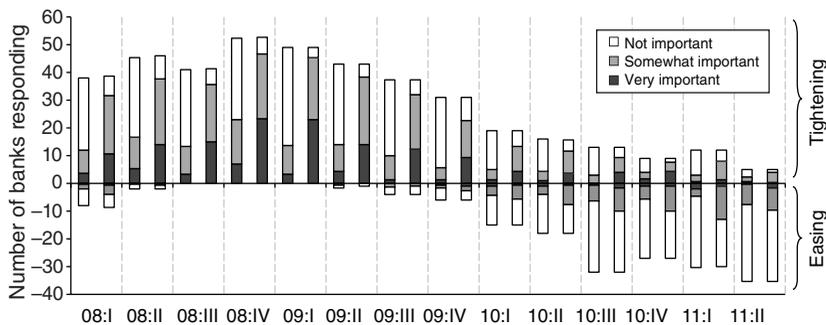


FIGURE 9. CONTRIBUTION OF BANK BALANCE SHEET FACTORS AND NONFINANCIAL FIRM MACROECONOMIC FACTORS TO TIGHTENING/EASING

*Note:* For each quarter, the first bar refers to the contribution of bank balance sheet factors, and the second bar refers to the contribution of nonfinancial firm macroeconomic factors.

or eased over the last three months, what are the reasons?” Loan officers are referred to the following seven potential considerations for tightening or easing bank credit: (i) bank capital position; (ii) liquidity conditions in secondary markets for loans; (iii) current and expected liquidity position; (iv) less favorable or more uncertain macroeconomic outlook; (v) tolerance to risk; (vi) industry-specific developments; (vii) bank competition. For each of these seven considerations, the respondent is asked to report whether it was very important, somewhat important, or not important in the decision to tighten or ease bank credit. We collected the reasons into two categories: factors having to do with banks’ own balance sheets (considerations (i), (ii), (iii)) and factors associated with macroeconomic conditions not related to banks’ balance sheets (considerations (iv), (v), (vi)).<sup>47</sup>

We summarize respondents’ answers in Figure 9, which covers the period from the first quarter of 2008 to the second quarter of 2011.<sup>48</sup> There are potentially four bars associated with each quarter in Figure 9. The length of the two bars above the zero line in a particular quarter indicate how many banks reported that they were tightening credit in that quarter. The length of the two bars extending below the zero line indicates how many banks reported that they were easing credit in that quarter. Evidently, in late 2008 and early 2009, no bank was easing credit. In each quarter, the left bar summarizes the importance assigned to factors having to do with the banks’ balance sheets, and the right bar summarizes the importance assigned to macro factors originating outside the banks. Each bar has a black part, a gray part, and a white part. The length of the black part indicates the average number of very important responses across the three considerations in the associated category. Similarly, the length of the gray part indicates the average number of somewhat important responses, and the length of the white part indicates the number of not important responses. The sum of the average responses is equal to the number of banks tightening or easing. This is why the length of the bars on the right and the left is always equal.

<sup>47</sup> Consideration (vii) was not included in either of the two categories.

<sup>48</sup> The survey of loan officers begins before 2008. However, the Fed did not publish how many banks responded to each question prior to 2008.

The key result is that the black and gray areas extend further for the bars on the right than for the bars on the left. That is, changing conditions outside banks' balance sheets are relatively more important than changes in banks' own balance sheets in determining whether banks tighten or ease credit conditions.

We view the evidence in Figure 9 as providing some support for our choice to leave out considerations strictly related to banks' balance sheets from the model. It is important, however, to stress the limitations of the evidence in Figure 9. First, the evidence applies to a relatively short subperiod of our dataset. At the same time, this evidence is perhaps notable because it covers a period when many think problems in banks' balance sheets were a principal reason for the business cycle contraction.<sup>49</sup> Second, the loan officer survey covers only a portion of the financial system, namely, the commercial banks. What is true about the commercial banks need not necessarily be true for financial firms as a whole. Still, we regard the evidence in Figure 9 as supportive of our model.

## V. Conclusion

We started with a model that combines CEE with BGG and added the assumption that the cross-sectional standard deviation of an idiosyncratic productivity shock varies over time, as in Christiano, Motto, and Rostagno (2003). We call this cross-sectional standard deviation a *risk shock*. When we study US macroeconomic data over the period 1985–2010, we conclude that the risk shock accounts for a large share of the fluctuations in GDP and other macroeconomic variables. It is the fact that we include financial variables in an otherwise standard macroeconomic dataset that allows us to differentiate the risk shock from more standard macroeconomic shocks. To evaluate the credibility of our result, we study the implications of our model for variables not included in the dataset used to assign values to the model parameters. In particular, we examine the implications of the model for loan delinquency rates, for out-of-sample forecasts, and for features of the cross-sectional dispersion of firm-level stock returns recently stressed by Bloom (2009) and others. We find that the model does well on these dimensions and infer that its implications for the risk shock deserve to be taken seriously.

Our analysis assumes that variations in risk are exogenous. Presumably, in reality there is a large endogenous component to risk shocks. Understanding these endogenous components is an important task for future research. Examples of how cyclical variations in risk may arise endogenously are explored in Bachmann and Moscarini (2011) and Christiano and Ikeda (2013b).

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<sup>49</sup> See Christiano and Ikeda (2013a).

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