Expectation Traps and Discretion*

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Abstract

We develop a dynamic model with optimizing private agents and a benevolent, optimizing monetary authority who cannot commit to future policies. We characterize the set of sustainable equilibria and discuss the implications for institutional reform. We show that there are equilibria in which the monetary authority pursues inflationary policies, because that is what private agents expect. We call such equilibria expectation traps. Alternative institutional arrangements for the conduct of monetary policy which impose limited forms of commitment on the policymaker can eliminate expectations traps. Journal of Economic Literature Classification Numbers: E31, E42, E50, E51, E58.

1. Introduction

A key ongoing research program in macroeconomics is the effort to design empirically plausible general equilibrium models that can be used to assess the gains and losses associated with different institutional arrangements for the conduct of monetary policy. In part this program is motivated by the experience of monetary policy in the postwar era. This experience has not been an easy one. The US lived through a dramatic rise in inflation that began in the 1960's, accelerated in the 1970's and, by most accounts, was ended only at the cost of a severe recession in the early 1980's. Various lessons can be learned from this era. From a policy perspective the critical question is: What was it about the environment that allowed this to happen? And, under current institutional arrangements for implementing monetary policy, could it happen again?

To address these types of questions, one requires a model that satisfies at least two criteria. First, the model should capture the circumstances faced by the monetary policy authority. In practice, this means taking an explicit stand on the policymaker's objectives and constraints, including whether he has access to a commitment technology. Second, the model should capture the salient features of the decision problems faced by private agents.

The effort to build models satisfying these criteria has proceeded along two tracks. On the one hand, there has been extensive work on modelling the private sector of the economy and the monetary transmission mechanism. Typically, papers in this literature abstract from explicitly modelling policymakers' decision problems. Instead, policymakers are modeled as automatons whose actions are drawn from a fixed rule, subject to occasionally random shocks.

On the other hand, there is a vast literature - stemming from the seminal papers by Kydland and Prescott [13] and Barro and Gordon [5] - devoted to understanding monetary policymakers as purposeful agents who operate subject to well - specified constraints. A particularly interesting strand of this literature investigates what happens to monetary policy when one changes the institutional framework and incentives that confront policymakers. To make progress, this literature typically abstracts from modelling the private economy in any detail. Instead the private sector is represented as a reduced form relationship connecting private sector outcomes to the actions of the monetary policy maker. For example, following Barro and Gordon [5], it is often assumed that aggregate output is a linear function of the unexpected shock to the money supply. The preferences of the policymaker are also specified in a reduced form manner. A typical assumption is that policymakers' have quadratic preferences over output and inflation. In principle it would be preferable to specify a link between the policymakers' preferences and those of private agents. But this is not possible once the decision is made to model the private sector via reduced form relationships.

Both literatures make abstractions. And both have made important contributions. But neither can successfully complete the task of quantitatively assessing the gains and losses associated with different monetary policy institutions. The first literature cannot do so because it completely abstracts from policymakers' decision problems. Consequently, it cannot ask how policymakers would respond to changes in their institutional environment. The second literature cannot do so for two reasons. First, Lucas [14] long ago pointed out the pitfalls associated with policy analyses conducted using reduced form models. Recently, Woodford [20] has repeated the relevance of Lucas' critique specifically as it applies to monetary policy. Second, granting that the principles emerging from the Barro - Gordon literature may very well remain valid in formal general equilibrium models, the enterprise of *quantitatively* evaluating the welfare properties of alternative monetary institutions must ultimately proceed with detailed, structural models of the private sector. We conclude that for real progress to be made, the two literatures must be merged.

This paper takes a modest step in this direction. Ireland [12] studies a general equilibrium monetary model in which policy is chosen by an optimizing monetary authority. We extend his analysis in three ways. First, we extend Ireland's [12] deterministic model to allow for two types of uncertainty: stochastic shocks to technology and sunspots. Second, as in Ireland [12], we use methods similar to those in Chari and Kehoe [9] to characterize the set of equilibria. These methods require that a worst equilibrium exists. Ireland [12] ensures this by imposing an exogenous upper bound on money growth. In our environment, the upper bound arises endogenously. Third, Ireland [12] focuses on the properties of the best equilibrium, while we study other equilibria as well. We use these to interpret the US inflation experience of the 1970s and to discuss the implications for institutional reform.

Our paper is related in some ways to Ball [3]. He notes the possibility of persistent inflation arising due to self fulfilling expectations in Barro - Gordon type models.[1] The basic idea is that, under discretion, policymakers can be *pushed* into pursuing inflationary policies. This can happen when the private sector, for whatever reason, expects high inflation. Under these circumstances, the central banker may find it optimal to accommodate private agents' expectations if the cost of not doing so is a recession. When the monetary authority does accommodate, private agents' expectations are self-fulfilling. We refer to such a situation as one in which the economy has fallen into an *expectation trap*.[2]

To formally articulate this argument, we consider a model economy that is populated by a representative household and three types of firms: a competitive producer of final goods, a continuum of monopolists, each of whom produces an intermediate good, and a financial intermediary. The household purchases the final consumption good, supplies labor to intermediate good firms, and lends funds to the financial intermediary. The financial intermediary receives funds from the households and makes loans to firms. Firms need loans because they must pay labor before they sell their output. There is a monetary authority which chooses monetary transfers to the household with the objective of maximizing the expected utility of the representative household. We model discretion by assuming that the policy maker acts sequentially through time and cannot commit to any future action.

The sequence of events within a period is as follows. First, all exogenous shocks, including both fundamental and non-fundamental shocks, are realized. Then, intermediate goods producers set their prices based on their expectation of the current-period money growth

[1] Ball [5] pursues a different explanation of how a one time supply shock can induce persistent effects on inflation. Using the Barro - Gordon framework, Ball [5] considers an extension of the Backus - Driffill [2] model in which there are two possible policymakers who have different preferences over inflation and unemployment. The type of policymaker in power changes stochastically. But the preferences of the policymaker in charge at any point in time is not known to the public. Both types of policymakers usually keep inflation low to enhance their reputations. In Ball's model adverse supply shocks raise the price of maintaining low inflation. Because of this, one type of policymaker will allow inflation to rise. By doing so, this policymaker reveals his type. The resulting change in reputation generates persistent effects on expected and actual inflation. It would be very interesting to embed this sort of argument within a formal general equilibrium model where the type of policymaker in power is determined endogenously.

[2] See William Fellner [11, pp.116-118] for an early discussion of the idea that expectations about monetary policy could be self-fulfilling by forcing the hands of benevolent policy makers. A reference to the possibility of expectations traps in which monetary policy responds to nonfundamental shocks also appears in Rogoff [16, pp. 245-46].

rate. Finally, the monetary authority's action is realized, and all other model variables are determined, with the output of the intermediate good being demand determined.

Absent commitment on the part of the monetary authority, two types of expectation traps can, in principle, arise in this environment. The first can arise when agents expect that monetary policy will react to shocks that don't affect preferences or technology. If these expectations are self-fulfilling, non-fundamental shocks constitute an additional set of impulses to the economy. In a second type of expectation trap monetary policy overreacts to fundamental shocks. Here, expectation traps amplify volatility by modifying the impact and propagation mechanisms from fundamental shocks to the economy. They could, for example, cause a real shock to lead to a persistent change in the inflation rate even though it would only produce a change in the level of prices under a non-accommodative monetary policy. We find these types of expectation traps interesting because they can be used to articulate the notion that during the 1960's and 70's the US fell into an expectation trap in which a benevolent, sequentially optimizing monetary authority pursued a policy which exacerbated the loss in output caused by various fundamental shocks. In the next section we argue that this view was shared by some academics and policymakers.

After our formal analysis of discretionary monetary policy, we investigate alternative institutional arrangements that can eliminate the possibility of expectation traps. One solution is full commitment on the part of the monetary authority. An obvious practical problem with this solution is that it is hard to imagine any monetary policy authority committing to a sequence of policy actions infinitely far into the future. More limited forms of commitment are practical. We consider a situation in which the monetary policy authority commits at time t to its time $t+1$ state contingent growth rate of money. We establish that expectation traps cannot occur in this regime and that the equilibrium quantities in this economy coincide with those in the Ramsey equilibrium.

The result that one period ahead limited commitment reduces the set of sustainable equilibria to a singleton - the Ramsey equilibrium - depends on particular features of our model economy. The intuition underlying the proof gives some insight into how it could be extended to accommodate different environments, say, those with multiperiod wage and price contracts. The fact that the amount of commitment required to sustain the Ramsey equilibrium depends on the details of the private sector economy, reinforces our view of the need to integrate the classic literature on time consistency with empirically plausible monetary general equilibrium models.

The remainder of this paper is organized as follows. Section 2 briefly argues that academics and policy makers believed that US monetary policy was caught in an expectation trap in the 1960s and 1970s. We describe our model in section 3, and discuss equilibrium under full commitment in section 4. Section 5 characterizes the set of sustainable outcomes under discretion, and section 6 presents examples of our two types of expectation traps. In section 7 we discuss policy implications of our analysis. Finally, we offer some concluding remarks.

2. Expectation Traps and the 1960's and 1970's

The academic literature contains numerous discussions about the possibility that a transitory real shock can lead to increased expectations of inflation, which are validated by the monetary authority.[3] This possibility is explored in the literature on the wage-price spiral. In addition, it seems to underlie policymakers' concern that an 'overheating' economy, i.e. one with high sustained rates of capacity utilization, can trigger a rise in the core inflation rate. They are also implicit in Blinder's [7] discussion of how supply shocks could have contributed to the persistent inflation of the 1970s:

> 'Inflation from special factors can "get into" the baseline rate if it causes an acceleration of wage growth. At this point policymakers face an agonizing choice—the so-called accommodation issue. To the extent that aggregate nominal demand is *not* expanded to accommodate the higher wages and prices, unemployment and slack capacity will result. There will be a recession. On the other hand, to the extent that aggregate demand *is* expanded (say, by raising the growth rate of money above previous targets), inflation from the special factor will get built into the baseline rate.' (Blinder [7, p. 264])

These ideas were not confined to academics. Our reading of Arthur Burns' speeches suggests that his views about the genesis of the inflation of the mid 1960's and 1970's

[3] See Ball [3,4] and the references therein.

have much in common with the notion that the economy was caught in an expectation trap, triggered by transitory real shocks. The principal real shock, in his view, was the expansionary fiscal policy ('the forces of excess demand') associated with the Great Society programs of the 1960s and the Vietnam war.[4] As Burns put it around the time he became Chairman of the Federal Reserve in 1970:

> 'The forces of excess demand that originally led to price inflation disappeared well over a year ago. Nevertheless, strong and stubborn inflationary forces, emanating from rising costs, linger on...' (Burns [8, p.124])

A key factor fueling rising costs, according to Burns, was the widespread expectation that the inflation which started in the mid 1960s, would continue. In a statement before the Joint Economic Committee of the US. Congress in 1971, Burns said:

> 'Consumer prices have been rising steadily since 1965 - much of the time at an accelerating rate. Continued substantial increases are now widely anticipated over the months and years ahead...in this environment, workers naturally seek wage increases sufficiently large...to get some protection against future price advances...thoughtful employers...reckon, as they now generally do, that cost increases probably can be passed on to buyers grown accustomed to inflation.' (Burns [8, p.126])

Burns clearly understood that this upward pressure on prices could not be transformed into persistent, high inflation without monetary accommodation. As he put it in a speech in 1977:

> 'Neither I nor, I believe, any of my associates would quarrel with the proposition that money creation and inflation are closely linked and that serious inflation could not long proceed without monetary nourishment. We well know–as do many others–that if the Federal Reserve stopped creating new money, or if this activity were slowed drastically, inflation would soon either come to an end or be substantially checked.' (Burns [8, p. 417])

[4]Burns believed that crop failures and oil shocks were other real disturbances that contributed to the inflation of the 1970s.

According to Burns, the Federal Reserve chose to accommodate agents' inflationary expectations for the same reason that the monetary authority in our model does so: to do otherwise would generate a costly recession. As he put in testimony before the Committee on Banking and Currency of the House of Representatives on July 30, 1974:

> '...an effort to use harsh policies of monetary restraint to offset the exceptionally powerful inflationary forces in recent years would have caused serious financial disorder and economic dislocation. That would not have been a sensible course for monetary policy.' (Burns [8, p. 171])

These quotations suggest that Burns perceived the same accommodation dilemma alluded to by Blinder, and judged that the best response was to accommodate.

3. The Model

In order to discuss the notion of an expectation trap as an equilibrium phenomenon, we consider an infinite horizon, monetary economy with uncertainty. In each period, $t = 0, 1, 2, ...$, the economy experiences an exogenously determined event, s_t, drawn from a finite set. Let $s^t = (s_0, s_1, ..., s_t)$ denote the history of exogenous events up to and including time t. The probability of s^t is given by $\mu(s^t)$. We denote the probability of s^t conditional on s^r by $\mu(s^t \mid s^r)$. The monetary authority chooses an action, A_t (a lump sum monetary transfer to the representative household) in period $t = 0, 1, 2, ...$ Let $h_t = (s^t, A_0, ..., A_t)$ denote the history of exogenous events up to date t, and of policy actions up to time t. Throughout we assume that private agents view A_t as being generated according to the policy rule

$$A_t = X_t(h_{t-1}, s_t).$$

We say that a set of histories h_{t+1}, $h_{t+2}, ...$ is induced by $X_t(\cdot, \cdot)$, $X_{t+1}(\cdot, \cdot), ...$ from $h_t = (h_{t-1}, s_t, A_t)$ if $h_{t+1} = (h_t, s_{t+1}, X_{t+1}(h_t, s_{t+1}))$, $h_{t+2} = (h_{t+1}, s_{t+2}, X_{t+2}(h_{t+1}, s_{t+2}))$, ... To conserve on notation, from here on we delete the subscript, t, on X. This should not cause confusion: that the functions $X(h_{t-1}, s_t)$ and $X(h_{r-1}, s_r)$, $r \neq t$ are different is evident from the fact that the number of elements in h_{t-1} and h_{r-1} are different. We adopt this notational convention for all functions of histories.

The commodity space in this economy consists of history contingent functions. That is, allocations, prices and policy actions are expressed as functions of history. In standard Arrow-Debreu economies, rules of this type are a function only of the history of exogenous events. The extension considered here accommodates the fact that the government in our model optimizes sequentially and that, being a 'large' agent, its actions have a non-trivial impact on private allocations and prices. At every date the government considers the consequences of all the current and future state-contingent actions that it could possibly take, and selects the set of actions that it prefers. Since its objective is a function of current and future allocations, these must be well-defined for every possible history for the government problem to be well-posed. Histories, h_t, can exclude past household actions because, though individual households optimize sequentially like the government, they are small and have no impact on aggregate allocations and prices (see Chari and Kehoe [9] for a further discussion.)

To reiterate from the introduction, the sequence of events within a period is as follows. First, the exogenous event, s_t, is realized. Then, intermediate goods producers set the prices of their goods. Finally, the monetary authority's action is realized and all other model variables are determined. We now describe the problems of the agents in our economy in detail.

3.1. Firms

Final Good Firms

The final good, $c(h_t)$, is produced by a perfectly competitive firm that combines a continuum of intermediate goods, indexed by $i \in (0, 1)$, using the following technology:

$$c(h_t) = \left[\int_0^1 (y_i(h_t))^\lambda \, di\right]^{\frac{1}{\lambda}}. \tag{3.1}$$

Here $0 < \lambda < 1$ and $y_i(h_t)$ denotes the time t input of intermediate good i. Let $P_i(h_{t-1}, s_t)$ denote the time t price of intermediate good i. This price is not a function of the time t

policy action because intermediate goods producers set their prices before A_t is realized.

The final good producer's problem is:

$$\max_{c(h_t),\{y_i(h_t)\}} P(h_t)c(h_t) - \int_0^1 P_i(h_{t-1}, s_t)y_i(h_t)di, \tag{3.2}$$

subject to (3.1). Here, $P(h_t)$ denotes the time t price of the final good. Problem (3.2) gives rise to the following input demand functions:

$$y_i(h_t) = c(h_t)\left(\frac{P(h_t)}{P_i(h_{t-1}, s_t)}\right)^{\frac{1}{1-\lambda}}. \tag{3.3}$$

In conjunction with (3.1), this implies:

$$P(h_t) = \left[\int_0^1 P_i(h_{t-1}, s_t)^{\frac{\lambda}{\lambda-1}}di\right]^{\frac{\lambda-1}{\lambda}}, \tag{3.4}$$

which expresses the time t price of the final good as a function of the time t prices of the intermediate goods. Note that this shows that in any equilibrium, the time t price of the final good is not a function of the time t policy action.

Intermediate Good Firm

Intermediate good i is produced by a monopolist using the following technology:

$$y_i(h_t) = \theta(s^t)n_i(h_t). \tag{3.5}$$

Here $n_i(h_t)$ denotes time t labor used to produce the i^{th} intermediate good and $\theta(s^t)$ denotes the stochastic economy wide level of technology at time t. Recall that $P_i(h_{t-1}, s_t)$ is chosen before the date t government policy action, A_t, is realized. We assume that the producer must supply all of the goods demanded by the final goods producers as determined by (3.3). Thus, at time t, history (h_{t-1}, s_t), producer i's problem, which we refer to as *Problem F*, is to maximize profits:

$$\max_{P_i(h_{t-1},s_t)} P_i(h_{t-1},s_t)y_i(h_t) - W(h_t)R(h_t)n_i(h_t), \tag{3.6}$$

subject to (3.3), where $h_t = (h_{t-1}, s_t, X(h_{t-1}, s_t))$. In this problem, the producer takes as given the time t wage rate, $W(h_t)$, the gross nominal interest rate, $R(h_t)$, as well as $P(h_t)$, and $c(h_t)$. The firm's unit labor costs are $W(h_t)R(h_t)$ because the firms need to pay workers before production, and must borrow these funds from the financial intermediary at interest rate $R(h_t)$.

Financial Intermediary

A perfectly competitive financial intermediary receives deposits, $I(h_t)$, from households. These funds are lent to intermediate good producers at the gross interest rate $R(h_t)$. At the end of the period, the intermediary pays $R(h_t)I(h_t)$ to households in return for deposits.

3.2. Household

At time r, history h_r, the representative household's expected present discounted utility is given by:

$$\sum_{t=r}^{\infty} \beta^{t-r} \sum_{s^t} \mu(s^t \mid s^r) U(c(h_t), n(h_t)). \tag{3.7}$$

Here $n(h_t)$ denotes total hours time t hours worked, and

$$U(c,n) = \ln(c) + \psi \ln(1-n), \tag{3.8}$$

where $\psi > 0$. The household faces the following cash constraint on its purchases:

$$P(h_t)c(h_t) \leq W(h_t)n(h_t) + M(h_{t-1}) - I(h_t) + A_t. \tag{3.9}$$

Here $M(h_{t-1})$ denotes the household's end-of-period $t-1$ holdings of cash.

The household's money holdings evolve according to:

$$M(h_t) = W(h_t)n(h_t) + M(h_{t-1}) + A_t - I(h_t) - P(h_t)c(h_t) + R(h_t)I(h_t) + D(h_t), \tag{3.10}$$

where $D(h_t)$ denotes total time t profits from the intermediate goods producers.

Let

$$Z(h_t) = [c(h_t), n(h_t), I(h_t), M(h_t)].$$

The household's problem, at date r, history h_r, is to choose a non-negative contingency plan for $Z(h_t)$ to maximize (3.7) subject to (3.9), (3.10), $n(h_t) \leq 1$ and $I(h_t) \leq M(h_{t-1}) + A_t, t \geq r$. We refer to this as *Problem H*. In solving its problem, the household observes A_r and views $A_t, t > r$ as being generated by the policy rule $X(h_{t-1}, s_t)$, so that $h_{t+1} = (h_t, s_{t+1}, X(h_t, s_{t+1}))$ for all $t > r$.

3.3. Government

At time t, given history (h_{t-1}, s_t), the government chooses a current action, A_t, and a sequence of state contingent policy rules for each future period, to maximize the representative household's utility, (3.7). It takes as given that future histories will be induced by its future policy rules from $h_t = (h_{t-1}, s_t, A_t)$. We restrict the government's choice of actions so that $A_r \geq M(h_{r-1})(\beta - 1)$. This restriction ensures that an equilibrium exists for all possible policies. (See Bewley [6] for examples of non existence of equilibrium when the growth of money is too negative.) The government also takes as given the household's contingency plan, $\{Z(h_r); r \geq t\}$. We refer to this problem as *Problem G*.

3.4. Sustainable Equilibrium

A sustainable equilibrium for our model economy is defined as follows:

Definition A *sustainable equilibrium* is a collection of history-contingent allocation rules $Z(h_t)$, pricing functions, $P_i(h_{t-1}, s_t)$, $P(h_t)$, $W(h_t)$, $R(h_t)$, and a government policy rule,

$X(h_{t-1}, s_t)$, such that:

- for all histories, h_t, the allocation rules $Z(h_t)$, solve *Problem H*,
- for all histories, (h_{t-1}, s_t), the pricing function $P_i(h_{t-1}, s_t)$ solves *Problem F*,
- for all histories, h_t, $P(h_t)$ satisfies (3.4),
- for all histories, h_t, the goods market clears, i.e., $c(h_t) = \theta(s^t)n(h_t)$, the loan market clears, i.e., $W(h_t)n(h_t) = I(h_t)$, and the money market clears, i.e., $M(h_t) = M(h_{t-1}) + A_t$.
- for all histories, (h_{t-1}, s_t), the policy rule, $X(h_{t-1}, s_t)$ solves *Problem G*.

We find it convenient to scale all time t nominal variables by the end of time $t-1$ aggregate stock of money, $M(h_{t-1})$. Let

$$\begin{aligned} p_i(h_{t-1}, s_t) &= \frac{P_i(h_{t-1}, s_t)}{M(h_{t-1})},\ p(h_t) = \frac{P(h_t)}{M(h_{t-1})},\ d(h_t) = \frac{D(h_t)}{M(h_{t-1}),} \\ w(h_t) &= \frac{W(h_t)}{M(h_{t-1})},\ i(h_t) = \frac{I(h_t)}{M(h_{t-1})},\ x(h_{t-1}, s_t) = \frac{X(h_{t-1}, s_t)}{M(h_{t-1})},\ a_t = \frac{A_t}{M(h_{t-1})}. \end{aligned}$$

In addition, we define the variable $m(h_{t-1})$ as the ratio of the household's time $t-1$ stock of money to the aggregate stock of time $t-1$ money. Of course in equilibrium $m(h_{t-1}) = 1$. Finally, let

$$z(h_t) = [c(h_t), n(h_t), i(h_t), m(h_t)]. \tag{3.11}$$

In what follows, we proceed in terms of these lower case scaled variables.

4. Private Sector Equilibrium Under Commitment

A key objective of our paper is to characterize the set of allocations that are the outcome of some sustainable equilibrium in the model economy. To do this it is useful to first analyze the behavior of the economy when the government can commit to a particular policy rule. The first subsection provides a characterization result for the set of private economy competitive equilibria corresponding to a given monetary policy rule. The second subsection characterizes the best competitive equilibrium.

4.1. Private Sector Equilibria

By commitment we mean that the government never deviates from some given policy rule, $x(h_{t-1}, s_t)$. Under these circumstances, the set of all possible date t histories can be indexed by s^t alone. This is because, under commitment, h_t can be expressed as a function of s^t using the following recursion:

$$h_{-1} = \emptyset,\ h_0 = (s^0, x(h_{-1}, s_0)),\ h_1 = (s^1, x(h_{-1}, s_0), x(x(h_{-1}, s_0), s_1)),$$

and similarly for $h_2, h_3, ..., h_t$.

Let $z^*(s^t) = [c^*(s^t),\ n^*(s^t),\ i^*(s^t),\ m^*(s^t)]$ denote the state contingent allocations under commitment. In addition, let $[w^*(s^t),\ R^*(s^t),\ p_i^*(s^t),\ p^*(s^t)]$, $x^*(s^t)$ and $d^*(s^t)$ denote the pricing functions, government policy rule, and intermediate good firm profit function under commitment, respectively.

Definition A *private sector equilibrium* is a collection of state contingent allocation rules, $z^*(s^t)$, pricing functions, $[w^*(s^t), R^*(s^t), p_i^*(s^t), p^*(s^t)]$, and a government policy rule, $x^*(s^t)$, such that (1) the allocation rules $z^*(s^t)$ solve the household's problem, (2) the pricing function $p_i^*(s^t)$ solves the intermediate good producers' problem, (3) $p^*(s^t) = \left[\int_0^1 p_i^*(s_t)^{\frac{\lambda}{\lambda-1}} di\right]^{\frac{\lambda-1}{\lambda}}$, (4) the goods market clears, i.e., $c^*(s^t) = \theta(s^t)n^*(s^t)$, the loan market clears, i.e., $w^*(s^t)n^*(s^t) = i^*(s^t)$, and the money market clears, i.e., $m^*(s^t) = 1$.

Note that a private sector equilibrium satisfies all the conditions for a sustainable equilibrium with two exceptions. First, it does not require optimality by the government. Second, private sector allocation rules and pricing functions solve the private sector optimization problems and satisfy market clearing only for histories induced by $x^*(s^t)$.

We now characterize the private sector equilibrium. The first order condition of the intermediary goods firm's problem is:

$$\lambda\theta(s^t)\left(\frac{c^*(s^t)}{y_i^*(s^t)}\right)^{1-\lambda} = \frac{w^*(s^t)}{p^*(s^t)}R^*(s^t).$$

In symmetric equilibria, $y_i^*(s^t) = c^*(s^t)$ for all i, so that,

$$\lambda\theta(s^t) = \frac{w^*(s^t)}{p^*(s^t)} R^*(s^t). \tag{4.1}$$

The Lagrangian representation of the household's problem at time 0 is:

$$\begin{aligned}
&\sum_{t=r}^{\infty} \beta^{t-r} \sum_{s^t} \mu(s^t|s^r)\{U(c^*(s^t), n^*(s^t)) \\
&+\nu(s^t)\left[m^*(s^{t-1}) - i^*(s^t) + x^*(s^t) + w^*(s^t)n^*(s^t) - p^*(s^t)c^*(s^t)\right] \\
&+\eta(s^t)[w^*(s^t)n^*(s^t) + m^*(s^{t-1}) - i^*(s^t) + x^*(s^t) - p^*(s^t)c^*(s^t) \\
&+R^*(s^t)i^*(s^t) + d^*(s^t) - (1 + x^*(s^t))m^*(s^t)]\},
\end{aligned}$$

where $\nu(s^t)$ and $\eta(s^t)$ are the non-negative Lagrange multipliers on the cash constraint and the budget constraints, respectively. In addition to the budget and cash constraints, the first order conditions for $c^*(s^t)$, $n^*(s^t)$, $m^*(s^t)$ and $i^*(s^t)$ are:

$$u_c(s^t) = (\nu(s^t) + \eta(s^t))p^*(s^t), \tag{4.2}$$

$$u_n(s^t) = -(\nu(s^t) + \eta(s^t))w^*(s^t), \tag{4.3}$$

$$(1 + x^*(s^t))\eta(s^t) = \beta \sum_{s^{t+1}} \mu(s^{t+1}|s^t)(\nu(s^{t+1}) + \eta(s^{t+1})), \tag{4.4}$$

$$\nu(s^t) + \eta(s^t) = \eta(s^t)R^*(s^t). \tag{4.5}$$

Here, $u_c(s^t)$ and $u_n(s^t)$ denote the marginal utility of consumption and labor at date t. The transversality condition for the household problem is

$$\lim_{T\to\infty} \sum_{s^T} \beta^T \mu(s^T) u_c(s^T) \frac{1 + x^*(s^T)}{p^*(s^T)} = 0. \tag{4.6}$$

These conditions can be simplified as follows. From (3.8), (4.2) and (4.3), we obtain:

$$\frac{w^*(s^t)}{p^*(s^t)} = \frac{\psi c^*(s^t)}{1 - n^*(s^t)}. \tag{4.7}$$

According to this the household equates the real time t wage rate to the marginal rate of substitution between consumption and leisure. Non-negativity of the Lagrange multipliers and (4.5) imply

$$R^*(s^t) \geq 1. \tag{4.8}$$

Relations (4.2), (4.4) and (4.5) imply

$$\frac{u_c(s^t)}{p^*(s^t)} = \beta \frac{R^*(s^t)}{1 + x^*(s^t)} \sum_{s^{t+1}} \mu(s^{t+1}|s^t) \frac{u_c(s^{t+1})}{p^*(s^{t+1})}, \tag{4.9}$$

According to (4.9), the household equates the marginal utility of spending an additional dollar on consumption to the expected marginal utility associated with investing an additional dollar with the financial intermediary.

Recall that loan market clearing requires

$$w^*(s^t) n^*(s^t) = i^*(s^t). \tag{4.10}$$

Substituting (4.10) into the scaled version of the household's cash constraint, (3.9) yields:

$$p^*(s^t) c^*(s^t) = 1 + x^*(s^t). \tag{4.11}$$

Goods market clearing implies:

$$c^*(s^t) = \theta(s^t) n^*(s^t). \tag{4.12}$$

Equations (4.9) and (4.11), together with the assumed period utility function imply

$$R^*(s^t) = \left[\beta \sum_{s^{t+1}} \frac{\mu(s^{t+1}|s^t)}{1+x^*(s^{t+1})}\right]^{-1}. \tag{4.13}$$

Equations (4.1), (4.7) and (4.12) can be used to obtain an expression relating $R^*(s^t)$ to $n^*(s^t)$. Using this expression to substitute out for $R^*(s^t)$ in (4.9) and imposing (3.8) and (4.12), we obtain:

$$\sum_{s^{t+1}} \mu(s^{t+1}|s^t) \left[\frac{n^*(s^t)}{1-n^*(s^t)} - \frac{\lambda\beta}{\psi(1+x^*(s^{t+1}))}\right] = 0. \tag{4.14}$$

Finally, substituting (4.11) into (4.6), it follows that the transversality condition is satisfied.

We are now in a position to characterize a private sector equilibrium.

Proposition 1: The allocation rule $n^*(s^t)$ is part of a private sector equilibrium if and only if $n^*(s^t)$ satisfies (4.14).

Proof. Suppose $n^*(s^t)$ satisfies (4.14). Then we construct the remaining objects in a private sector equilibrium as follows. The consumption allocation rule is $c^*(s^t) = \theta(s^t)n^*(s^t)$, the price rule is obtained from the cash constraint, (4.11). The nominal wage is obtained from (4.7), the nominal interest rate from (4.13) and, finally, $i^*(s^t)$ is obtained from (4.10). The only conditions that remain to be checked are (i) (4.8), (ii) the non-negativity constraints on the household's choices and (iii) the constraint on Problem H, $i^*(s^t) \leq 1 + x^*(s^t)$. Condition (i) holds because of the restriction on Problem G that there is a lower bound on the growth rate of money, i.e. $x^*(s^{t+1}) \geq \beta - 1$ for all s^{t+1}. We verify that condition (ii) is satisfied as follows. Non-negativity of $n^*(s^t)$ and $c^*(s^t)$ follow from (4.14) and (4.12). Non-negativity of $i^*(s^t)$ follows from (4.10). Finally, condition (iii) is verified as follows. Equation (4.14) together with the lower bound on the growth rate of money implies $\frac{\psi n^*(s^t)}{1-n^*(s^t)} \leq \lambda$. Using (4.7), we obtain $w^*(s^t) \leq \lambda p^*(s^t)\theta(s^t)$. Using relations (4.11) and (4.12), we see that $w^*(s^t)n^*(s^t) \leq \lambda(1 + x^*(s^t))$. Substituting (4.10) into this expression, the desired result follows. At the constructed set of prices, the household's Euler equations and transversality condition are satisfied. Sufficiency of the Euler equations and transversality condition for optimality of the household's problem can be established by a straightforward adaptation of the arguments in Stokey and Lucas with Prescott [17, pp. 98-99]. This completes the proof of the sufficiency portion of the proposition.

To establish the necessity portion of the proposition, suppose we have an equilibrium. Then the proof follows immediately from necessity of (4.14). ■

It is easily verified that the private sector equilibrium is unique. This would not necessarily be the case for more general utility functions. Consider for example utility functions of the form

$$U(c,n) = \left[c(1-n)^{\psi}\right]^{(1-\sigma)} /(1-\sigma), \tag{4.15}$$

where $\psi > 0$ and $\sigma > -1$. For $\sigma \neq 1$, the set of private sector equilibria under commitment in our economy may not be unique. To see this, consider the case $\theta(s^t) \equiv 1$ and $x^*(s^t) \equiv x$. In this case, the analog to (4.14) is

$$v\left(n^*(s^t), n^*(s^{t+1}), \theta(s^t), \theta(s^{t+1}), x^*(s^{t+1})\right) = 0, \tag{4.16}$$

where

$$v(n, n', \theta, \theta', x') = \frac{n}{1-n}\left[\theta n(1-n)^{\psi}\right]^{1-\sigma} - \frac{\lambda\beta}{\psi(1+x')}\left[\theta' n'(1-n')^{\psi}\right]^{1-\sigma}. \tag{4.17}$$

There exists an equilibrium in which employment is given by the deterministic analog of (4.14). But when $\sigma \neq 1$,there may be other equilibria as well, even sunspot equilibria (see Matheny [15] and Woodford [19].) To see this, note that (4.17) implicitly defines n' as a function (possibly a correspondence) from n. The implicit function theorem guarantees that there exists a differentiable function relating n' to n in a neighborhood of steady-state: $n = n' = \lambda\beta/[\lambda\beta + \psi(1+x)]$, with:

$$\frac{dn'}{dn} = \frac{2 - \sigma - (\psi(1-\sigma) - 1)\frac{\lambda\beta}{\psi(1+x)}}{(1-\sigma)\left[1 - \frac{\lambda\beta}{1+x}\right]}.$$

Here, $dn'/dn = 0.47$ when $\sigma = 10$, $\lambda = 0.8$, $\psi = 3$, $\beta = 1/1.03$, and $x = 0.05$. Because this derivative is less than one, it is possible to construct multiple deterministic equilibria and sunspot equilibria.

The expectation traps that we focus on in this paper have nothing to do with this form of multiplicity. Consequently, it is useful to proceed under assumptions which guarantee the uniqueness of a private sector equilibrium under commitment, $\sigma = 1$.

4.2. The Ramsey Equilibrium

As a benchmark, it is useful to consider the equilibrium of the model under the assumption that the government has access to a commitment technology at $t = 0$. To this end we now define a Ramsey equilibrium.

Definition A *Ramsey equilibrium* is a collection of allocation rules for private agents, $z^*(s^t)$, pricing functions $[w^*(s^t),\ R^*(s^t),\ p_i^*(s^t),\ p^*(s^t)]$, and a government policy rule, $x^*(s^t)$, that yields the highest utility for households over the set of private sector equilibria.

Proposition 2 The Ramsey equilibrium satisfies

$$n^*(s^t) = \frac{\lambda}{\psi + \lambda},\ \ R^*(s^t) = 1, \tag{4.18}$$

and

$$1 + x^*(s^{t+1}) = \beta. \tag{4.19}$$

Proof. ¿From Proposition 1, the Ramsey equilibrium maximizes discounted utility subject to (4.14). This reduces to a sequence of static problems. At each date t utility is maximized by setting $x^*(s^{t+1})$ as small as possible. Given our assumption on the lower bound on money growth, $x^*(s^{t+1}) \geq (\beta - 1)$, it is optimal to set $1 + x^*(s^{t+1}) = \beta$. Then from (4.14), it follows that $n^*(s^t) = \frac{\lambda}{\psi+\lambda}$. Finally, from (4.13), it follows that $R^*(s^t) = 1$. ■

It is important to note that even in the Ramsey equilibrium, as long as there is monopoly power ($\lambda < 1$), employment is below the social optimum, $1/(1+\psi)$. The monetary authority would like to increase employment beyond the level in a Ramsey equilibrium, but this is not achievable by monetary policy because of the constraint that $R(s^t)$ be greater than or equal to one for all s^t. This creates a temptation for a monetary authority without a commitment technology to deviate ex post from any policy, even the Ramsey policy.

5. Characterizing Sustainable Outcomes

We now turn to the task of characterizing the set of sustainable outcomes for our model economy. The characterization theorem which we provide requires the existence of a worst sustainable monetary equilibrium. In our simple model economy there is no such equilibrium absent an exogenous upper bound on the money growth rate. This reflects the fact that, in our environment, there is only a contemporaneous benefit, and no cost, from an unexpected increase in money growth. Rather than complicate the model in the text, we simply assume that there is an exogenous upper bound on the monetary growth rate, $\bar{x}$, i.e. $x(h_{t-1}, s_t) \leq \bar{x}$ for all (h_{t-1}, s^t). In the appendix, we modify the model so that there is an *endogenous* upper bound on money growth and a corresponding worst sustainable monetary equilibrium. The key assumption is that households have access to a 'backyard' technology for producing the composite consumption good outside the monetary sector of the economy. At some finitely high level of money growth, the distortions induced by inflation are sufficiently severe that the economy 'demonetizes', i.e. households abandon the monetary economy and use only the backyard technology. Since none of our conclusions are affected by this modification, for ease of exposition we relegate this version of the model to the appendix.

To characterize the set of sustainable outcomes for our model economy we must allow for allocation rules, pricing functions and government policy rules to be contingent on both the history of exogenous events and the history of policy actions. We begin by characterizing the worst sustainable equilibrium which we refer to as the 'high inflation' equilibrium. Consider the following candidate sustainable equilibrium in which the government policy rule is given by $x(h_{t-1}, s_t) = \bar{x}$ for all possible (h_{t-1}, s^t). Let x_t denote the actual time t policy action. For all histories (h_{t-1}, s^t), the candidate pricing functions are,

$$p_i(h_{t-1}, s_t) = p(h_t) = \frac{1+\bar{x}}{\theta(s^t)} \frac{\psi(1+\bar{x}) + \lambda\beta}{\lambda\beta}. \tag{5.1}$$

These candidate pricing functions reflect our assumptions that (i) firms set prices in period t prior to the realization of the period t policy action, and (ii) they do so under the expectation that the policy action will be $x(h_t, s_{t+1}) = \bar{x}$.

The candidate allocation rules, interest rate and wage functions are

$$c(h_t) = \frac{(1+x_t)}{p(h_t)},\ n(h_t) = \frac{c(h_t)}{\theta(s^t)},\ R(h_t) = \frac{\lambda}{\psi} \frac{(1-n(h_t))}{n(h_t)}, \tag{5.2}$$

$$w(h_t) = \frac{p(h_t)\psi\theta(s^t)n(h_t)}{1-n(h_t)},\ i(h_t) = (1+x_t)\frac{\psi n(h_t)}{1-n(h_t)}, \tag{5.3}$$

respectively.

By construction, these candidate rules satisfy private agent optimality and market clearing for all possible histories, h_t. So to establish that the candidate equilibrium is sustainable, we need only establish that it is consistent with optimization on the part of the government. Consider a one-shot deviation at period t, given some history, (h_{t-1}, s^t). Given our assumed upper bound on money growth, the only feasible one-shot deviation is for the government to set money growth at $x_t < \bar{x}$, and then to return to $\bar{x}$ thereafter. From (5.2), this reduces current consumption and employment. Since utility is strictly increasing in levels of equilibrium employment below $1/(1+\psi)$, this reduces current utility. Future outcomes are unaffected. Thus, the government has no incentive to pursue a one-shot deviation for any history. Since this includes histories in which there have been any number of deviations, it follows that no deviation raises welfare (see Abreu [1] and Whittle [19].) This establishes the sustainability of the candidate equilibrium.

To see that the candidate equilibrium is the worst sustainable equilibrium, note that in any other equilibrium, employment at some date is necessarily higher because of (4.14). Thus, utility must be higher too. This establishes the following proposition:

Proposition 3: The high inflation equilibrium is the worst sustainable equilibrium.

We denote by $u^d(s^t)$ the highest one period utility level associated with a deviation by the government:

$$u^d(s^t) = \max_{-1 \leq x \leq \bar{x}} U\left(\frac{1+x}{p^*(s^t)}, \frac{1+x}{\theta(s^t)p^*(s^t)}\right).$$

Let $x^d(s^t)$ denote the value of x that achieves the optimum. In addition, let $\overline{u}(s^t)$ denote the one period utility level in the high inflation equilibrium. We are now ready to state our main result. The following proposition establishes a set of restrictions on allocations and prices which are necessary and sufficient for them to be the outcomes of some sustainable equilibrium. In what follows, it is useful to define a class of policy rules characterized by a particular trigger strategy, a *grim* trigger. Such a rule specifies that if there has been a deviation to a higher money growth rate in the past, then money growth in all periods thereafter is at its upper bound (i.e., $x(h_{t-1}, s_t) = \bar{x}$). If there has been a deviation to a lower money growth rate, then future money growth is as specified in the original equilibrium. That is deviations down are forgotten. We refer to an equilibrium associated with such a policy rule as a *grim trigger equilibrium.*

Proposition 4: Let $z^*(s^t)$, $w^*(s^t)$, $R^*(s^t)$, $p_i^*(s^t)$, $p^*(s^t)$ be an arbitrary set of allocations and prices. Let $x^*(s^t)$ be an arbitrary sequence of government policies. Then, $\{z^*(s^t)$, $w^*(s^t)$, $R^*(s^t)$, $p_i^*(s^t)$, $p^*(s^t)$, $x^*(s^t)\}$ is the outcome of some sustainable equilibrium if and only if:

1. $(z^*(s^t), w^*(s^t), R^*(s^t), p_i^*(s^t), p^*(s^t), x^*(s^t))$ is a private sector equilibrium;
2. $(z^*(s^t), w^*(s^t), R^*(s^t), p_i^*(s^t), p^*(s^t), x^*(s^t))$ satisfies

$$\sum_{r=t}^{\infty} \beta^{r-t} \sum_{s^r} \mu(s^r \mid s^t) U(c^*(s^r), n^*(s^r)) \geq u^d(s^t) + \sum_{r=t+1}^{\infty} \beta^{r-t} \sum_{s^r} \mu(s^r \mid s^t) \bar{u}(s^r) \tag{5.4}$$

for all s^t, all $t = 0, 1, 2....$.

Proof. We first consider sufficiency. Suppose the allocations and prices satisfy conditions 1 and 2. We construct a particular grim trigger sustainable equilibrium which supports these outcomes. Consider the following candidate equilibrium. For all histories, h_t, in which there has been no deviation, or $x_{t-\tau} \leq x^*(s^{t-\tau})$ for some $\tau > 0$ (i.e., with possible deviations to lower money growth at some point in the past), let $x(h_{t-1}, s_t) = x^*(s^t)$, $p_i(h_{t-1}, s_t) = p_i^*(s^t)$, $p(h_t) = p^*(s^t)$, $w(h_t) = w^*(s^t)$, $R(h_t) = R^*(s^t)$ and $z(h_t) = z^*(s^t)$. For histories, h_t, in which the only deviation is down in the current period, let $p_i(h_{t-1}, s_t) = p_i^*(s^t)$, $p(h_t) = p^*(s^t)$, $R(h_t) = R^*(s^t)$, and let $c(h_t)$, $n(h_t)$, $i(h_t)$, $w(h_t)$ be given by (5.2) and (5.3), with $p(h_t) = p^*(s^t)$. For all histories with a deviation up in the past, let $x(h_{t-1}, s_t) = \bar{x}$, and let $p_i(h_{t-1}, s_t)$, $p(h_t)$ be defined by (5.1). Also, let $w(h_t)$, $R(h_t)$ and $z(h_t)$ be defined by (3.11) and (5.1)-(5.2) with x_t replaced by $\bar{x}$. For all histories with a deviation up in the current period, let $p_i(h_{t-1}, s_t) = p_i^*(s^t)$, $p(h_t) = p^*(s^t)$, and let $c(h_t)$, $n(h_t)$, $i(h_t)$, $w(h_t)$ be given by (5.2) and (5.3), with $p(h_t) = p^*(s^t)$. Finally, let $R(h_t)$ be determined by (4.13) with $x^*(s^{t+1})$ replaced by $\bar{x}$. For all histories with no deviation, these constitute a private sector equilibrium by assumption. For all other histories, they constitute a private sector equilibrium by the discussion leading up to Proposition 3. To show government optimality, note that any deviation down in money growth reduces current utility and leaves future utility unaffected. The discounted utility associated with any deviation up is at most the right side of (5.4). Thus, the candidate equilibrium is a sustainable equilibrium, which establishes sufficiency.

We now consider necessity. Suppose $(z(h_t)$, $w(h_t)$, $R(h_t)$, $p_i(h_{t-1}, s_t)$, $p(h_t)$, $x(h_{t-1}, s_t))$ is a sustainable equilibrium with outcomes $(z^*(s^t)$, $w^*(s^t)$, $R^*(s^t)$, $p_i^*(s^t)$, $p^*(s^t)$, $x^*(s^t))$. Condition 1 is satisfied by the definition of a sustainable equilibrium. We establish condition 2 by contradiction. Suppose 2 is violated for some s^t. Consider the following deviation: the government sets $x_t = x^d(s^t)$, and specifies future policies according to $x(h_{r-1}, s_r)$, $r > t$. The expected present discounted value of utility under this deviation is $u^d(s^t) + \sum_{r=t+1}^{\infty} \beta^{r-t} \sum_{s=r} \mu(s^r \mid s^t) \tilde{u}(s^r)$, where $\tilde{u}(s^r)$ denotes utility in state s^r. From the discussion preceding Proposition 4, $\bar{u}(s^r)$ is a lower bound on period utility when policy is set according to $x(h_{r-1}, s_r)$. It follows that $\tilde{u}(s^r) \geq \bar{u}(s^r)$. Therefore, the return associated with the deviation is greater than, or equal to, the right hand side of equation (5.4). Given our supposition that condition 2 is violated, this means that the proposed deviation will be implemented at s^t. Consequently, the equilibrium is not consistent with government optimization. This establishes the contradiction. ■

We conclude this section with a brief discussion of the proposition. The proposition says that a particular outcome is sustainable if, and only if, it is the outcome of the associated grim trigger equilibrium. The proposition does *not* say that a particular outcome is sustainable *only* by a grim trigger equilibrium. In general, an outcome may be sustainable by a variety of equilibria. The equilibria could involve less extreme trigger strategies, or no trigger strategy at all. The proposition is silent on the nature of the equilibria that support a given outcome. It only provides conditions under which a particular outcome is sustainable by *some* equilibrium. In the next section, we make these observations concrete through a series of examples.

6. Examples

This section reports two examples which illustrate the types of expectation traps discussed in the introduction. In both cases, we establish that the expectation trap outcome is the outcome of some sustainable equilibrium. Our first example illustrates the type of expectation trap that can arise when agents expect that monetary policy will react to non-fundamental shocks. In addition, we discuss three equilibria that can sustain this outcome. In the second example, agents expect monetary policy to react to technology shocks in a particular way. The example is constructed to articulate in a stylized way an interpretation of the US inflation experience since the mid-1960s.

Example 1: Non-Fundamental Shocks

The parameter values for the model economy are given by $\psi = 3$, $\lambda = 0.95$, $\theta_t \equiv 1$, $\sigma = 1$, $\beta = 1/1.03$, $\bar{x} = 0.30$. In each period there is a shock, $s \in \{s(1), s(2)\}$, which is drawn from a highly persistent, symmetric, two state Markov chain, where the probability of switching states is 0.10. The shock is non-fundamental, in that it has no impact on preferences or technology. Still, there is a sustainable outcome, in which money growth responds to s. When $s = s(1)$ the money growth rate is 0 percent, and when $s = s(2)$, the growth rate of money is 3 percent. The Wold representation for x_t is given by

$$x_t = 0.003 + 0.8x_{t-1} + \epsilon_t, \tag{6.1}$$

where ϵ_t is uncorrelated with past variables, and has standard deviation 0.009.

Several features of this example are worth emphasizing. First, the model is observationally equivalent to one in which policy is set according to (6.1). In this respect, the model is formally identical to a standard, monetized real business cycle model (see, for example, the 'cash-in-advance' model in Christiano and Eichenbaum [10]). The interpretation of the policy shock, however, is very different. In the literature, ϵ_t is assumed to reflect the effects of fundamental disturbances to the policy making process, e.g., shocks to preferences of policy makers. In our environment, the shocks originate outside the policy making process and reflect the non-fundamental disturbances that can generate an expectation trap.

Second, consistent with the previous observations, the dynamic response of real variables to changes in x_t resemble those in standard monetary real business cycle models. For example, Table 1 indicates that employment, consumption and beginning-of-period real balances (M/P) are low when x_t is high, while R_t and inflation are high.

Third, Table 1 contains information relevant for verifying that the outcome in this example is sustainable. The table reports, for both states of the world, the current period utility, u, of the household and the expected present value of its utility from the next period on, v, along the equilibrium path. In addition, the table reports for both states of the world the current period utility, u^d, and the expected value of utility from the next period on, v^d, associated with a one-period deviation from the equilibrium path. In the example, u^d turns out to be the level of utility associated with the socially efficient level of employment, $1/(1+\psi) = 0.25$. The deviation growth rate of money, x^d, needed to achieve this level of employment is larger in the $s = s(2)$ state of the world. To see why, note that achieving $n = 0.25$ requires setting the real value of end-of-period money balances, $(1+x)/p$, equal to 0.25. But, in the high state of the world, intermediate goods producers anticipate a high (3 percent) growth rate of money. Consequently, the price level is high. With p high, x must be high too. Since the deviation induces a higher level of consumption and employment, $u < u^d$. The outcome is nevertheless sustainable because $u + v > u^d + v^d$ in both states of the world.

Table 1: Non-Fundamental Expectation Trap

variable	$x(h_{t-1}, s_t) = 0.0$	$x(h_{t-1}, s_t) = 0.03$
n	0.2346	0.2304
M/P (beg. of period)	0.2346	0.2237
W/P	0.9196	0.8982
R	1.0330	1.0577
u	-2.2519	-2.2536
v	-75.0892	-75.0950
u^d	-2.2493	-2.2493
v^d	-76.3650	-76.3650

Fourth, we briefly consider what sort of equilibria can sustain this outcome. One such equilibrium is the grim trigger equilibrium used in the proof to Proposition 4.2. In results not reported here, we verified that a milder trigger strategy equilibrium that also sustains the outcome is one in which a deviation to a higher money growth rate triggers a shift to the maximum growth rate of money, $\bar{x}$, for one period only. Finally, we discuss a particular non-trigger equilibrium which sustains the expectations trap outcome. Loosely speaking, in this equilibrium the monetary policy rule specifies that deviations in which money growth is high in a particular state persist forever, but deviations down have no impact on future policy. Formally, for histories, (h_{t-1}, s_t), in which there has never been a deviation, $x(h_{t-1}, s_t) = \nu(s_t)$, where

$$\nu(s) = \begin{cases} 0.0, \text{ if } s = s(1) \\ 0.03, \text{ if } s = s(2) \end{cases} . \tag{6.2}$$

Next, consider histories, (h_{t-1}, s_t), in which: (i) there was one deviation in the past, when the state was $s = s(k)$, (ii) the deviation money growth rate was a, and (iii) the growth rate called for by the policy rule was $x < a$. In histories like this, the policy rule specifies that $x(h_{t-1}, s_t) = \tilde{\nu}(s_t)$, where $\tilde{\nu}(s(k)) = a$ and $\tilde{\nu}(s(j)) = \nu(s(j))$ for $j \neq k$. That is, the rule specifies that if the policy authority deviates up in a particular state, the deviation action will be followed whenever that state recurs in the future. Histories in which there has been more than one deviation are handled in the same way. Deviations down have no impact on the policy rule. Verifying that this is an equilibrium requires ensuring that there is no profitable deviation for a large number of histories. We used numerical methods to do this. The example illustrates that expectation trap outcomes need not be supportable only by grim trigger equilibria.

Example 2: Fundamental Shocks.

Apart from the stochastic process governing the state of the world, s, and the state of technology, θ_t, the parameter values used in this example coincide with those in Example 1. The state of the world can take on values numbered 1 through 7. In state one, $\theta_t = 1$, while $\theta_t = 0.90$ in the other six states. The Markov chain governing s has the following properties. If the economy is in state 1, it stays with probability 0.99, and it moves to state 2 with probability 0.01. If the economy is in states 2 through 6, the economy moves to the next higher state with probability 0.95. It stays in the current state with the complementary probability. If the economy is in state 7 it moves to state 1 with probability 0.95. In this example, there is a sustainable expectation trap outcome in which money growth is 0.0, 0.05, 0.10, 0.15, 0.20, 0.15, and 0.10 in each of states 1 through 7. This outcome has the property that money growth is typically low. Occasionally, following a decline in the state of technology, money growth and, hence, inflation rise for an extended period of time before returning to their typically low levels. Figure 1 illustrates a typical realization in which the economy is in state 1 for 5 periods, then switches to state 2 for 8 periods, and finally returns to state 1 for 2 periods. Presumably, it is easy to construct similar examples in which expectation traps like this are triggered by other fundamental shocks, like changes in government purchases.

We verified that the outcome described here is sustainable by applying Proposition 4. A question is whether there are strategies other than the grim trigger strategy that can support this outcome as a sustainable outcome. We conjecture that non-trigger strategies analogous to those described in Example 1 can support this outcome as part of a sustainable equilibrium.

We find Example 2 interesting because it illustrates another sense in which discretion makes possible volatile equilibria. Also, this type of example may form the basis for a simple explanation of the prolonged rise in inflation during the mid-1960s and 1970s. Authors such as Fellner [11] have argued that this rise in inflation was triggered by the simultaneous increase in expenditures due to the expansion of the Vietnam war and Great Society Programs. Other authors such as Blinder [7] argue that the inflation of the 1970s was triggered by the supply shocks of that decade. Our analysis raises the possibility that the long lived inflation is consistent with sequential rationality on the part of private agents and a benevolent monetary authority. It simply reflects the type of expectation trap that can arise under discretion.

7. Policy Implications

7.1. Limited Commitment

We now show that a simple institutional change in the way monetary policy is conducted eliminates the expectation traps associated with discretionary monetary policy. The change is that we impose a form of limited commitment upon the monetary authority, in which it is required to commit to a state-contingent action one period in advance. We formally model this by specifying the time t sequence of events as follows. First, the exogenous event, s_t, is realized. Second, intermediate goods producers set the time t prices of their goods. Then, the monetary authorities' date $t+1$ action, contingent upon s_{t+1}, is realized. Finally, the other date t model variables are determined. In this modified version, the history at the end of date t is $h_t = (h_{t-1}, s_t, x(s^{t+1}))$, where $x(s^{t+1})$ denotes the monetary authority's time $t+1$ state-contingent action.

Proposition 5 Under limited commitment, the set of sustainable outcomes coincides with those in the Ramsey equilibrium.

Remark Recall that in a Ramsey equilibrium, employment, n^R, is given by $n^R = \lambda/(\psi+\lambda)$ and the growth rate of the money satisfies $\sum_{s^{t+1}|s^t} \mu(s^{t+1} \mid s^t) \frac{1}{1+x(s^{t+1})} = \frac{1}{\beta}$. Thus, in a Ramsey equilibrium, allocations and prices are uniquely determined and monetary policies are determined only in an expected value sense.

Proof. In the Ramsey equilibrium, employment is independent of the state of nature. The first step in the proof is to show that in any sustainable equilibrium, the date t consumption and employment allocations depend only upon the date $t+1$ state-contingent monetary policy actions. To see this, first recall that, in a sustainable equilibrium, for all histories, the continuation outcome induced by the sustainable equilibrium must solve the consumer's optimization problem and satisfy market clearing. The analog of (4.14) is:

$$\frac{\psi}{\lambda}\frac{n(h_t)}{1-n(h_t)} = \beta \sum_{s^{t+1}} \mu(s^{t+1}|s^t)\frac{1}{1+x(s^{t+1})}. \tag{7.1}$$

It follows that, for all histories, the employment allocation at date t depends only upon the state-contingent actions of the monetary authority at date $t+1$. The second step in the proof is by contradiction. Consider a sustainable outcome such that $n(h_t) \neq n^R$ for some h_t. Consider the following sequence of deviations by the government: $x(s^{r+1}) = \frac{1}{\beta} - 1$ for $r \geq t$. Under this deviation, $n(h_r) = n^R$ for $r \geq t$. But, equations (4.14) and (7.1) imply that $n(h_t)$ is strictly less than n^R. Since utility is strictly increasing for levels of employment less than $1/(1+\psi)$, this deviation raises utility. This establishes the contradiction. ■

To understand this result, it is useful to recall the two features of our model which make expectation traps possible when there is no commitment. Private agents' actions, such as price setting decisions, depend on their expectations of future monetary policy and the monetary authority suffers a loss if it does not validate these expectations. Limited commitment eliminates expectation traps by forcing the monetary authorities to commit before private agents make their decisions. In this way, expectations are uniquely pinned down.

We conclude this section by discussing how the previous proposition would have to be modified to accommodate different environments. For example, suppose agents committed to nominal wages or prices K periods in advance. To eliminate expectation traps in this environment, we anticipate that monetary policy would have to commit $K+1$ periods in advance. In environments in which decisions depend on the entire future history of monetary policy, for example, when agents have the option to invest in physical capital, then it may not be possible to eliminate all expectation traps with limited commitment.

7.2. Fiscal Policy

In our model the monetary authority has an ever-present incentive to raise money growth because monopolistic competition leads to inefficiently low output. In principle, this inefficiency could be corrected using employment or output subsidies financed by a lump sum tax on households. If instruments like these are used appropriately, output would always be at its efficient level. In such an economy multiple equilibria associated with different paths for the price level are still possible. However, these equilibria are not interesting because real outcomes are left completely unaffected.

There are obvious problems with tax-subsidy instruments like these. For example, lump sum taxes might not be available. Under such circumstances, the government would have to use distorting taxes to finance the employment or output subsidies. We expect that in such an environment, output would still be inefficiently low and the monetary authority would still have an incentive to raise money growth. We conjecture that the socially costly fluctuations in output associated with expectation traps could still occur.

8. Concluding Remarks

This paper studied the operating characteristics of monetary policy when the monetary authority cannot commit to future policies. Our main finding is that discretion exposes the economy to expectation traps. This is true even when the monetary authority is benevolent and completely understands the structure of the economy and the effects of monetary policy. In addition, we argue that alternative institutional arrangements for the conduct of monetary policy which impose limited forms of commitment on policy makers can eliminate the possibility of expectations traps.

We began this paper by recalling that the US endured a persistent inflation episode that lasted from the mid 1960's to the end of the 1970's. We then asked two questions: What was it about the environment that allowed this to happen? And, under current institutional arrangements for implementing monetary policy, could it happen again? The analysis of this paper suggests that the institutional structure governing monetary policy during the 1960's and 1970's may have been a key factor that forced the Federal Reserve into the classic accommodation dilemma discussed by Blinder and emphasized by Arthur Burns. This in turn may have a played a major role in leading benevolent policymakers to pursue a monetary policy that allowed transitory real shocks to trigger a persistent episode of inflation. Could it happen again? We see no major difference between the institutions governing monetary policy in the 1960's and the 1990's. Certainly not of the kind that would eliminate the possibility of expectations traps. So our analysis implies that it could happen again.

FIGURE 1

9. Appendix

The characterization theorem provided in the main body of the paper makes use of the existence of a worst sustainable monetary equilibrium. In the simple model economy discussed in the text there is no worst equilibrium absent an exogenous upper bound on the money growth rate. Essentially, this is because regardless of how fast money growth is, there are only current gains associated with deviations to an even higher money growth rate. In this appendix we modify the model so that there is a high rate of money growth, such that if the monetary authority attempts to deviate to an even higher rate, the economy suffers damage. The damage is that the market economy, which allows access to an efficient production economy, collapses, forcing agents to revert to an inefficient 'backyard' production technology. This idea allows us to construct a worst, sustainable, monetary equilibrium without an exogenously imposed upper bound on money growth. This is what we do in this appendix. For simplicity, we assume that there is no uncertainty. Modifying the argument below to allow for uncertainty is straightforward.

Normalizing the level of technology to 1, we have that

$$c_m(h_t) = n_m(h_t). \tag{A1}$$

Here $n_m(h_t)$ denotes labor used in the monetary economy and $c_m(h_t)$ denotes units of consumption produced in the monetary economy. The backyard technology that households have access to transforms labor into the composite consumption good according to

$$c_b(h_t) = bn_b(h_t) \tag{A2}$$

where $0 < b < 1$, $n_b(h_t)$ denotes labor used in the backyard technology and $c_b(h_t)$ denotes units of consumption produced using the backyard technology. At each date the household must decide whether to use the backyard technology. If it does so, it cannot consume goods made in the monetary sector or supply labor to firms in the monetary sector. As we show below, these assumptions imply that there is a worst monetary equilibrium.

In the modified environment, there is the possibility of excess demand for consumption goods in the monetary sector. To describe how goods are allocated in this event, we impose a rationing rule which divides up available consumption goods equally among households. We model this rationing rule as a constraint on households. The constraint specifies that

$$c_m(h_t) \leq c^r(h_t),$$

where $c^r(h_t)$ is the maximum amount of consumption that a household can consume at history, h_t.

A sustainable equilibrium for this economy is defined analogously to the economy in the text.

Definition A *sustainable monetary equilibrium* is a sustainable equilibrium in which, for all histories induced by the government policy rule, $n_b(h_t) = 0$.

To identify the worst sustainable monetary equilibrium, it is useful to first consider a stationary, sustainable monetary equilibrium. By this we mean a sustainable monetary equilibrium in which all real variables, the nominal interest rate and the growth rate of money are constant for all histories induced by the government policy rule. We consider such an equilibrium because it is useful in defining a particular money growth rate which will be exploited in what follows. It is easy to show that, in histories induced by the government policy rule, consumption and employment are given by $c_m = n_m = \frac{\beta/(1+x)}{\psi/\lambda+\beta/(1+x)}$, where x denotes the money growth rate. The household's period utility level, $U_m(x)$, is given by $U_m(x) = U(\frac{\beta/(1+x)}{\psi/\lambda+\beta/(1+x)}, \frac{\beta/(1+x)}{\psi/\lambda+\beta/(1+x)})$. In a nonmonetary equilibrium, $c_b = bn_b$ and $n_b = \frac{1}{1+\psi}$. The household's period utility level, U_b, is given by $U(\frac{b}{1+\psi}, \frac{1}{1+\psi})$. It easy to show that if x and b are sufficiently small, then $U_m > U_b$. Since U_m is strictly decreasing in x and $\lim_{x\to\infty} U_m(x) = -\infty$, it follows that there exists some value of x, say $\overline{x}$, such that (i) $U_m(\overline{x}) = U_b$, and (ii) $U_m(x) < U_b$ for all $x > \overline{x}$.

Following is our candidate worst sustainable monetary equilibrium. We refer to it as the high inflation equilibrium, because of its similarity to the analogous equilibrium in the main text. For 'type 1' histories in which there has never been a deviation in money growth,

$$x(h_{t-1}) = \overline{x}.$$

For 'type 2' histories in which there has been at least one deviation in money growth,

$$x(h_{t-1}) = \overline{x} + \varepsilon,$$

where $\varepsilon > 0$. For type 1 histories $p_i(h_{t-1})$ and $p(h_{t-1})$ are defined as in (5.1) and the functions, $c_m(h_t)$, $n_m(h_t)$, $R(h_t)$, $w(h_t)$, $i(h_t)$, are given by (5.2) and (5.3) with $x_t = \overline{x}$. The rationing rule, $c^r(h_t)$, is set to an arbitrarily large value. For type 2 histories, the functions $p_i(h_{t-1})$, $p(h_{t-1})$, $w(h_t)$ and $R(h_t)$ are arbitrary numbers and $c_m(h_t)$, $n_m(h_t)$, $i(h_t)$ and $c^r(h_t)$ are identically equal to zero.

Proposition: The high inflation equilibrium is the worst sustainable monetary equilibrium.

Proof. We first verify that the high inflation equilibrium is a private sector equilibrium. Consider type 1 histories. By construction of (5.1), (5.2) and (5.3), the candidate rules satisfy private agent optimality and market clearing conditions under the assumption that households do not use the backyard technology. To see that this assumption is consistent with household optimality, substitute (5.1) into (5.2) to get that $c_m(h_t) = n_m(h_t) = [\beta/(1+\overline{x})] / [\psi/\lambda + \beta/(1+\overline{x})]$. So, by construction of $\overline{x}$, $U_m = U_b$. This verifies that the candidate equilibrium is a private sector equilibrium for type 1 histories. Next consider type 2 histories, h_t. Each agent takes as given that per capita employment and output will be zero for h_{t+j}, $j \geq 0$. Under these circumstances, an individual household who contemplates working and using the proceeds to purchase consumption goods expects to be rationed at zero consumption because of our rationing rule. Given the disutility of labor, such a household optimally chooses to offer no labor to the monetary sector for all h_{t+j}, $j \geq 0$. With nothing to buy in any state of the world, h_{t+j}, each agent is indifferent as to the value of $i(h_{t+j})$, and so $i(h_{t+j}) = 0$ is consistent with optimality. Since there is no market activity in h_{t+j}, rates of return and prices are not relevant and can be arbitrary. This verifies that the candidate equilibrium is a private sector equilibrium for type 2 histories.

We now establish that the candidate equilibrium satisfies government optimality. Consider type 1 histories. Deviations cause an immediate and permanent demonetization, putting period utility at U_b. At the same time, utility from not deviating is $U_m(\overline{x}) = U_b$. There is also no incentive for the government to deviate in type 2 histories, because government policy is irrelevant then.

We now establish that the previous equilibrium, which is a stationary monetary equilibrium with $x = \bar{x}$, is the worst sustainable monetary equilibrium. Suppose there were a worse sustainable monetary equilibrium with utility $\sum_{t=0}^{\infty} \beta^t U(h_t)$ for histories generated by the policy rule. Then, $\sum_{t=0}^{\infty}\beta^t\left[U_b - U(h_t)\right] < 0$, so that $U_b - U(h_t) < 0$ for at least one h_t. This is inconsistent with household optimization, establishing a contradiction. Q.E.D.

The assumption, $\varepsilon > 0$, has not been used in the proof of the above proposition. In fact, the proposition goes through for $\varepsilon = 0$ too. With the rationing rule, this economy fundamentally has multiple equilibria: At any point in time, households could expect the economy to demonetize, and it would do so. Presumably, this is actually a desirable feature for a monetary model to have. Our main point in this appendix is that the high inflation equilibrium is something suitable for use as a 'worst monetary equilibrium' in the characterization result in the text.

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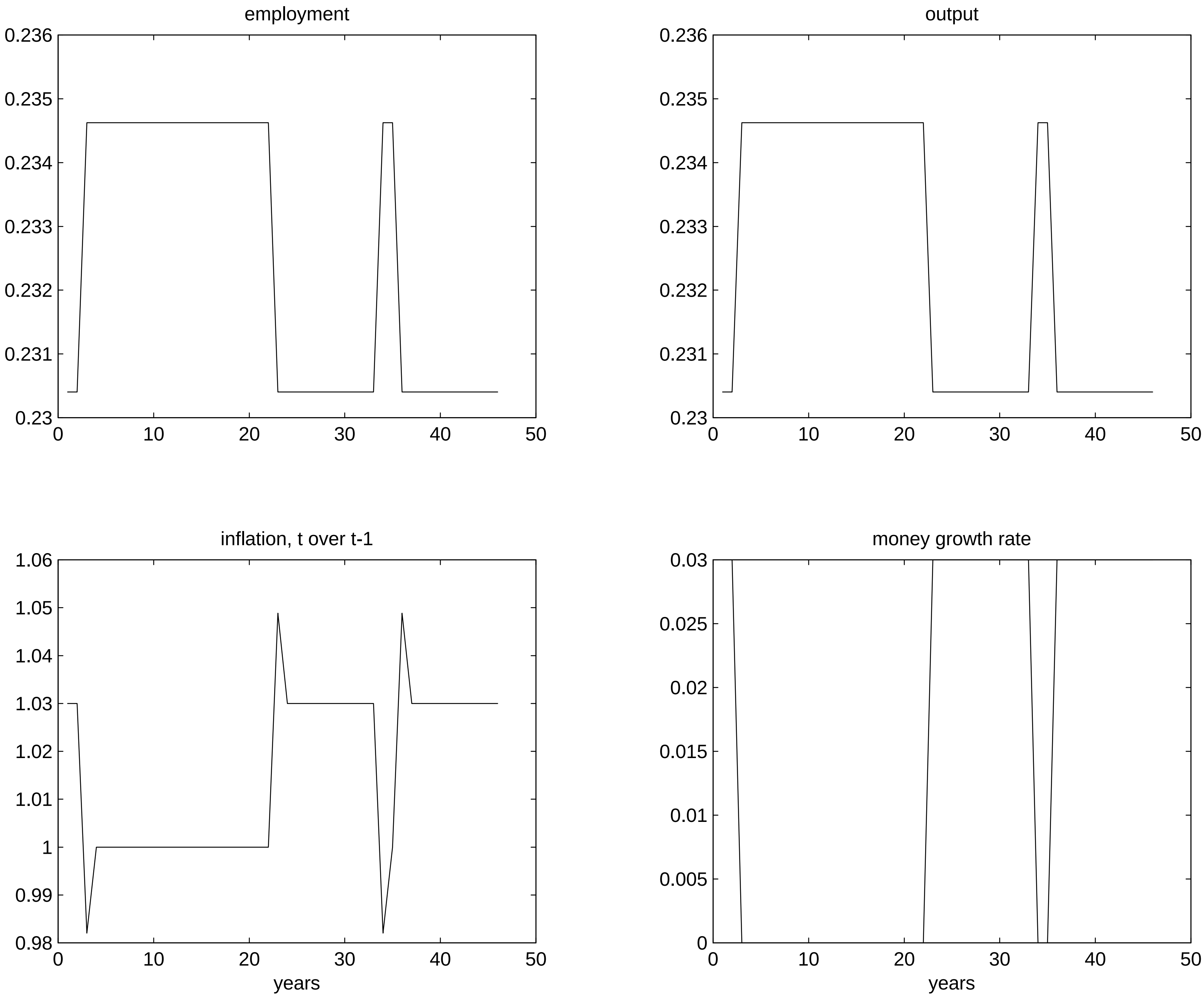

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output
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1.05
1.04
1.03
1.02
1.01
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0.99
0.98
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30
40
50
years
money growth rate
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years