Understanding the Great Recession†

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We argue that the vast bulk of movements in aggregate real economic activity during the Great Recession were due to financial frictions. We reach this conclusion by looking through the lens of an estimated New Keynesian model in which firms face moderate degrees of price rigidities, no nominal rigidities in wages, and a binding zero lower bound constraint on the nominal interest rate. Our model does a good job of accounting for the joint behavior of labor and goods markets, as well as inflation, during the Great Recession. According to the model the observed fall in total factor productivity and the rise in the cost of working capital played critical roles in accounting for the small drop in inflation that occurred during the Great Recession. (JEL E12, E23, E24, E31, E32, E52)

The Great Recession has been marked by extraordinary contractions in output, investment, and consumption. Mirroring these developments, per capita employment and the labor force participation rate have dropped substantially and show little sign of improving. The unemployment rate has declined from its Great Recession peak. But this decline primarily reflects a sharp drop in the labor force participation rate, not an improvement in the labor market. Indeed, while vacancies have risen to their prerecession levels, this rise has not translated into an improvement in employment. Despite all this economic weakness, the decline in inflation has been relatively modest.

We seek to understand the key forces driving the US economy in the Great Recession. To do so, we require a model that provides an empirically plausible account of key macroeconomic aggregates, including labor market outcomes like employment, vacancies, the labor force participation rate, and the unemployment rate. To this end, we extend the medium-sized dynamic, stochastic general equilibrium (DSGE) model in Christiano, Eichenbaum, and Trabandt (2013)—henceforth, CET—to endogenize the labor force participation rate. To establish the empirical

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credibility of our model, we estimate its parameters using pre-2008 data. We argue that the model does reasonably well at accounting for the dynamics of 12 key macroeconomic variables over this period.

We show that four shocks can account for the key features of the Great Recession. Two of these shocks capture—in a reduced form way—frictions which are widely viewed as having played an important role in the Great Recession. The first shock is motivated by the sharp increase in credit spreads observed in the post-2008 period. To capture this phenomenon, we introduce a perturbation into households’ first order condition for optimal capital accumulation. We refer to this perturbation as the financial wedge. One interpretation of this wedge is that it reflects variations in bankruptcy costs and other costs of financial intermediation. An alternative interpretation is that the wedge reflects a change in the desirability of bonds issued by nonfinancial firms to finance their acquisition of capital. This change could arise due to variations in the risk or liquidity premium associated with nonfinancial firms. Motivated by models like Bigio (2013), we allow the financial wedge to impact on the cost of working capital.

The second shock is motivated by the notion that in the crisis there was a flight to safe and/or liquid assets. For convenience, we capture this idea as in Smets and Wouters (2007) and Fisher (2014), by introducing a perturbation to agents’ inter-temporal Euler equation governing the accumulation of the risk-free asset. We refer to this perturbation as the consumption wedge. An alternative interpretation of this shock comes from the literature stressing a reduction in consumption as a trigger for a zero lower bound (ZLB) episode (see Eggertsson and Woodford 2003; Eggertsson and Krugman 2012; and Lorenzoni and Guerrieri 2012).

The third shock in our analysis is a neutral technology shock that captures the observed decline, relative to trend, in total factor productivity (TFP). The final shock in our analysis corresponds to the changes in government consumption that occurred during the Great Recession.

Our main findings can be summarized as follows. First, our model can account, quantitatively, for the key features of the Great Recession, including the ongoing decline in the labor force participation rate. According to our model, if the labor force participation rate had not fallen, then the decline in employment, consumption, and output that occurred during the Great Recession would have been substantially smaller. Second, our model implies that the vast bulk of the decline in economic activity is due to the financial wedge and, to a smaller extent, the consumption wedge. Third, the rise in government consumption associated with the American Recovery and Reinvestment Act of 2009 did have a peak multiplier effect of about 1.6, but the rise in government spending was too small to have a substantial effect on aggregate economic activity. In addition, for reasons discussed in the main text, we cannot attribute the long duration of the Great Recession to the substantial decline in government consumption that began around the start of 2011. The peak multiplier associated with the decline in government spending is roughly equal to 0.9. Fourth, consistent with the basic findings in CET, we are able to account for the

1 For a formalization of this interpretation, see Christiano and Davis (2006).
2 The findings with respect to the financial wedge are consistent with Del Negro, Giannoni, and Schorfheide (2014), who reach their conclusion using a different methodology than the one that we use.
general behavior of real wages during the Great Recession, even though we do not allow for sticky wages. Fifth, our model can account for the relatively small decline in inflation with only a moderate amount of price stickiness.

Our last finding is perhaps surprising in light of arguments by Hall (2011) and others that New Keynesian (NK) models imply inflation should have been much lower than it was during the Great Recession. Del Negro et al. (2014) argue that Hall’s conclusions do not hold if the Phillips curve is sufficiently flat. In contrast, our model accounts for the behavior of inflation after 2008 by incorporating two key features of the data into our analysis: (i) the prolonged slowdown in TFP growth during the Great Recession and (ii) the rise in the cost of firms’ working capital as measured by the spread between the corporate-borrowing rate and the risk-free interest rate. In our model, these forces drive up firms’ marginal costs, exerting countervailing pressures on the deflationary forces operative during the post-2008 period.

Our paper may be of independent interest from a methodological perspective for three reasons. First, our analysis of the Great Recession requires that we do stochastic simulations of a model that is highly nonlinear in several respects: (i) we work with the actual nonlinear equilibrium conditions; (ii) we confront the fact that the ZLB on the nominal interest rate is binding in parts of the sample and not in others; and (iii) our characterization of monetary policy allows for forward guidance, a policy rule that is characterized by regime switches in response to the values taken on by endogenous variables. The one approximation that we use in our solution method is certainty equivalence. Second, as we explain below, our analysis of the Great Recession requires that we adopt an unobserved components representation for the growth rate of neutral technology. This leads to a series of challenges in solving the model and deriving its implications for the data. Third, we note that traditional analyses of vacancies and unemployment based on the Beveridge curve would infer that there was a deterioration in the efficiency of labor markets during the Great Recession. We argue that this conclusion is based on a technical assumption which is highly misleading when applied to data from the Great Recession.

The remainder of this paper is organized as follows. Section I describes our model. Sections II and III describe the data, methodology, and results for estimating our model on pre-2008 data. In Sections IV and V, we use our model to study the Great Recession. We close with a brief conclusion. Many technical details of our analysis are relegated to a separate technical Appendix that is available on request.

I. The Model

In this section, we describe a medium-sized DSGE model whose structure is, with one important exception, the same as the one in CET. The exception is that we modify the framework to endogenize the labor force participation rate. We suppose that an individual can be in one of three states: out of the labor force, unemployed or employed in the market. In our model, the household faces the following
tradeoff: It can keep people at home producing a nonmarket-produced consumption good, or it can send people to the market to seek employment. The wages earned in the labor market can be used to acquire a market-produced consumption good or an investment good. When wages are low and/or the job finding rate is low, then households choose a lower labor force participation rate.

A. Households and Labor Force Dynamics

The economy is populated by a large number of identical households. Each household has a unit measure of members. Members of the household can be engaged in three types of activities: (i) \((1 - L_t)\) members specialize in home production in which case we say they are not in the labor force and that they are in the nonparticipation state; (ii) \(l_t\) members of the household are in the labor force and are employed in the production of a market good, and (iii) \((L_t - l_t)\) members of the household are unemployed, i.e., they are in the labor force but do not have a job.

We now describe aggregate flows in the labor market. We derive an expression for the total number of people searching for a job at the end of a period. This allows us to define the job finding rate, \(f_t\), and the rate at which workers transit from nonparticipation into labor force participation, \(e_t\).

At the end of each period a fraction \((1 - \rho)\) of randomly selected employed workers is separated from the firm with which they had been matched. Thus, at the end of period \(t - 1\) a total of \((1 - \rho) l_{t-1}\) workers separate from firms and \(\rho l_{t-1}\) workers remain attached to their firm. Let \(u_{t-1}\) denote the unemployment rate at time \(t - 1\), so that the number of unemployed workers at time \(t - 1\) is \(u_{t-1} L_{t-1}\). The sum of separated and unemployed workers is given by:

\[
(1 - \rho) l_{t-1} + u_{t-1} L_{t-1} = (1 - \rho) l_{t-1} + \frac{L_{t-1} - l_{t-1}}{L_{t-1}} L_{t-1}
\]

\[
= L_{t-1} - \rho l_{t-1}.
\]

We assume that a separated worker and an unemployed worker have an equal probability, \(1 - s\), of exiting the labor force. It follows that \(s\) times the number of separated and unemployed workers, \(s(L_{t-1} - \rho l_{t-1})\), remain in the labor force and search for work. We refer to \(s\) as the “staying rate.”

The household chooses \(r_t\), the number of workers that it transfers from nonparticipation into the labor force. Thus, the labor force in period \(t\) is:

\[
L_t = s(L_{t-1} - \rho l_{t-1}) + \rho l_{t-1} + r_t.
\]

The total number of workers searching for a job at the start of \(t\) is \(s(L_{t-1} - \rho l_{t-1}) + r_t\) which, according to the previous expression, can be expressed as follows:

\[
(1.1) \quad s(L_{t-1} - \rho l_{t-1}) + r_t = L_t - \rho l_{t-1}.
\]
By its choice of \( r \), the household in effect chooses \( L_t \). We require \( r_t \geq 0 \), so that the restriction on the household’s choice of \( L_t \) is

\[
1 \geq L_t \geq s(L_{t-1} - \rho l_{t-1}) + \rho l_{t-1}.
\]  

(1.2)

It is of interest to calculate the probability, \( e_t \), that a nonparticipating worker is selected to be in the labor force. We assume that the \( (1 - s) (L_{t-1} - \rho l_{t-1}) \) workers who separate exogenously into the nonparticipation state do not return home in time to be included in the pool of workers relevant to the household’s choice of \( r \). As a result, the universe of workers from which the household selects \( r_t \) is \( 1 - L_{t-1} \). It follows that \( e_t \) is given by\(^5\)

\[
e_t = \frac{r_t}{1 - L_{t-1}} = \frac{L_t - s(L_{t-1} - \rho l_{t-1}) - \rho l_{t-1}}{1 - L_{t-1}},
\]  

(1.3)

which is nonnegative by (1.2).

The law of motion for employment is:

\[
l_t = (\rho + x_t) l_{t-1} = \rho l_{t-1} + x_t l_{t-1},
\]  

(1.4)

where \( x_t \) denotes the hiring rate.

The job finding rate is the ratio of the number of new hires divided by the number of people searching for work, given by (1.1):

\[
f_t = \frac{x_t l_{t-1}}{L_t - \rho l_{t-1}}.
\]  

(1.5)

B. Household Maximization

Members of the household derive utility from a market consumption good and a good produced at home.\(^6\) The home good, \( C_t^H \), is produced using the labor of individuals that are not in the labor force, \( 1 - L_t \):

\[
C_t^H = \eta_t^H (1 - L_t) - \mathcal{F}(L_t, L_{t-1}; \eta_t^L).
\]  

(1.6)

\(^5\)We include the staying rate, \( s \), in our analysis for a substantive as well as a technical reason. The substantive reason is that, in the data, workers move in all directions between unemployment, nonparticipation, and employment. The gross flows are much bigger than the net flows. Setting \( s < 1 \) helps the model account for these patterns. The technical reason for allowing \( s < 1 \) can be seen by setting \( s = 1 \) in (1.3). In that case, if the household wishes to make \( L_t - L_{t-1} < 0 \), it must set \( e_t < 0 \). That would require withdrawing from the labor force some workers who were unemployed in \( t - 1 \) and stayed in the labor force as well as some workers who were separated from their firm and stayed in the labor force. But, if some of these workers are withdrawn from the labor force then their actual staying rate would be lower than the fixed number, \( s \). So, the actual staying rate would be a nonlinear function of \( L_t - L_{t-1} \) with the staying rate below \( s \) for \( L_t - L_{t-1} < 0 \) and equal to \( s \) for \( L_t - L_{t-1} \geq 0 \). This kink point is a nonlinearity that would be hard to avoid because it occurs precisely at the model’s steady state. Even with \( s < 1 \) there is a kink point, but it is far from steady state and so it can be ignored when we solve the model.

\(^6\)Erceg and Levin (2013) also exploit this type of tradeoff in their model of labor force participation. However, their households find themselves in a very different labor market than ours do. In our analysis the labor market is a version of the Diamond-Mortensen-Pissarides model, while in their analysis, the labor market is a competitive spot market.
The term $F(L_t, L_{t-1}; \eta^L_t)$ captures the idea that it is costly to change the number of people in the labor force, $L_t$. We include the adjustment costs in $L_t$ so that the model can account for the gradual and hump-shaped response of the labor force to a monetary policy shock (see subsection IIIC). $\eta^H_t$ and $\eta^L_t$ are processes, discussed below, that ensure balanced growth.

An employed worker gives his wage to his household. An unemployed worker receives government-provided unemployment compensation which it gives to its household. Unemployment benefits are financed by lump-sum taxes paid by households. The details of how workers find employment and receive wages are explained below. All household members have the same concave preferences over consumption, so each is allocated the same level of consumption.

The period utility function of the representative household is:

$$\mathcal{U}_t = \ln \tilde{C}_t + v\left(\frac{M_{t+1}}{P_t}\right),$$

where $M_{t+1}$ denotes beginning-of-period $t + 1$ money holdings and $v$ is an increasing and concave function. To accommodate the scenario in which the market rate of interest is zero, we require that $v'(m) = 0$ for some finite $m \geq 0$. Our analysis does not require any other restrictions on $v$. In equation (1.7),

$$\tilde{C}_t = \left[ (1 - \omega)(C_t - b\bar{C}_{t-1})^\chi + \omega(C^H_t - b\bar{C}^H_{t-1}) \right]^{\frac{1}{\chi}}, \quad 0 < \chi < 1.$$

Here, $C_t$ denotes purchases of the market consumption good. The parameter $b$ controls the degree of habit formation in household preferences. We assume $0 \leq b < 1$. A bar over a variable indicates its economy-wide average value.

According to (1.7) the household does not suffer disutility from the activities of people in the three states of the labor market. Given the ordinal nature of utility in our setting, this assumption can be thought of simply as a normalization. We think that work in the labor market does generate disutility. But, so does work in the home and the experience of being unemployed. The omission of labor disutility from (1.7) corresponds to the assumption that this disutility is (i) additively separable from the utility of consumption and (ii) the same in the three labor market states. Given complete consumption insurance, (i) and (ii) imply that the only effect of moving from one labor market state on household utility operates through its impact on the budget constraint. Formally, we could subtract a term, $\zeta_t > 0$, in (1.7) that is invariant to the distribution of workers across states. We do not include such a term because it has no impact on equilibrium allocations and prices.

We see no obvious reason to think that the disutility from working in the home and the market sector are different. It is also not obvious to us that the disutility from the activities of being unemployed and from being employed are very different. On the one hand, time use surveys suggest that the unemployed have more leisure (see, e.g., Aguiar, Hurst, and Karabarbounis 2013). On the other hand, Hornstein, Krusell, and Violante (2011) argue that the value of unemployment is quite low. In addition, there are a number of studies that report that unemployed people
experience adverse physical and mental health consequences (see, e.g., Brenner 1979; Schimmack, Schupp, and Wagner 2008; Sullivan and von Wachter 2009). In addition, our assumption about $\zeta_t$ improves the business cycle performance of the model. See CET for an extended discussion.\footnote{Our assumption that $\zeta_t$ is the same across all labor market states implies that the analysis of Chodorow-Reich and Karabarbounis (2014) does not apply to our environment.}

The flow budget constraint of the household is as follows:

\begin{equation}
P_t C_t + P_{L_t} I_t + A_{t+1} \leq (R_{K_t} u^K_t - a(u^K_t) P_{L_t}) K_t + (L_t - l_t) P_{P}^D D + l_t W_t - T_t + B_t + M_t. \tag{1.9}
\end{equation}

Here,

\begin{equation}
A_{t+1} \geq \frac{B_{t+1}}{R_t} + M_{t+1} \tag{1.10}
\end{equation}

denotes the household’s end of period $t$ financial assets. According to (1.10), financial assets are composed of interest-bearing discount bonds, $B_{t+1}/R_t$, and cash, $M_{t+1}$. In principle there are three types of bonds that households can purchase: government bonds, bonds that are used to finance working capital, and bonds that can be used to purchase physical capital. In our benchmark model these three bonds are perfect substitutes from the perspective of the household. The variable, $T_t$, denotes lump-sum taxes net of transfers and firm profits and $R_{K_t}$ denotes the nominal rental rate of capital services. The variable, $u^K_t$, denotes the utilization rate of capital. We assume that the household sells capital services in a perfectly competitive market, so that $R_{K_t} u^K_t K_t$ represents the household’s earnings from supplying capital services. The increasing convex function $a(u^K_t)$ denotes the cost, in units of investment goods, of setting the utilization rate to $u^K_t$. The variable, $P_{L_t}$, denotes the nominal price of an investment good and $I_t$ denotes household purchases of investment goods. In addition, the nominal wage rate earned by an employed worker is denoted by $W_t$ and $\eta_t^D$ denotes exogenous unemployment benefits received by unemployed workers from the government. The term $\eta_t^D$ is a process that ensures balanced growth and will be discussed below.

When the household chooses $L_t$, it takes the aggregate job finding rate, $f_t$, and the law of motion linking $L_t$ and $l_t$ as given:

\begin{equation}
l_t = \rho l_{t-1} + f_t (L_t - \rho l_{t-1}). \tag{1.11}
\end{equation}

Relation (1.11) is consistent with the actual law of motion of employment because of the definition of $f_t$ (see equation (1.5)).

The household owns the stock of capital which evolves according to:

\begin{equation}
K_{t+1} = (1 - \delta_K) K_t + \left[ 1 - S(I_t/I_{t-1}) \right] I_t. \tag{1.12}
\end{equation}
The function $S(\cdot)$ is an increasing and convex function capturing adjustment costs in investment. We assume that $S(\cdot)$ and its first derivative are both zero along a steady state growth path.

The sources of uncertainty in this economy are a monetary policy shock and two technology shocks. We now define the household problem which is broken into two stages. The first and second stages occur before and after the period $t$ monetary policy shock is realized, respectively. In the first stage the household decides its quantity variables and the total size of its financial portfolio. In the second stage it chooses the composition of that portfolio between cash and interest bearing bonds.

Let the vector, $s_t$, denote current and lagged values of the two technology shocks. Let the vector, $\mathbf{X}_t$, denote the household’s own state variables at the start of time $t$:

$$\mathbf{X}_t \equiv (K_t, L_{t-1}, I_{t-1}, \bar{C}_{t-1}, \bar{C}^H_{t-1}, I_{t-1}, B_t, M_t).$$

The corresponding aggregate quantities are denoted by $\mathbf{X}^a_t$. The aggregate variables, $\eta^T_t, f_t, P_t, P_t W,P_t T$, which enter the household budget constraint, are functions of $\mathbf{X}^a_t$ and $s_t$. The variable, $R_t$, is a function of $\mathbf{X}^a_t$, $s_t$ and also the monetary policy shock, $\epsilon_{R,t}$.

The sequence of events in period $t$ is as follows. The household observes $\mathbf{X}_t, \mathbf{X}^a_t$, $s_t$ and chooses:

$$\mathbf{Y}_t \equiv \left(u^K_t, I_t, C_t, C^H_t, L_t, I_t, A_t, K_{t+1}\right).$$

Then $\epsilon_{R,t}$ is realized, $R_t$ is determined, and the household chooses $B_{t+1}$ and $M_{t+1}$ to solve

$$\tilde{W}(\mathbf{X}_t, \mathbf{X}^a_t, \mathbf{Y}_t, s_t, \epsilon_{R,t}) = \max_{B_{t+1}, M_{t+1}} \left\{ v\left(\frac{M_{t+1}}{P_t}\right) + \beta \mathbb{E} W(\mathbf{X}^a_{t+1}, \mathbf{X}^a_{t+1}, s_{t+1}) \right\},$$

subject to (1.10), the given values of $\mathbf{Y}_t$ and the laws of motion of $\mathbf{X}^a_t$, and $s_{t}$. Here, the expectation is over the distribution of $\mathbf{X}^a_{t+1}, s_{t+1}$ conditional on $\mathbf{X}_t, \mathbf{X}^a_t$, $\mathbf{Y}_t, s_t$, and $\epsilon_{R,t}$. The vector, $\mathbf{Y}_t$, is chosen at the start of time $t$ to solve the following problem

$$W(\mathbf{X}_t, \mathbf{X}^a_t, s_t) = \max_{\mathbf{Y}_t} \left\{ \ln C_t + \beta \mathbb{E}_{\epsilon_{R,t}} \tilde{W}(\mathbf{X}_t, \mathbf{X}^a_t, \mathbf{Y}_t, s_t, \epsilon_{R,t}) \right\},$$

subject to (1.2), (1.6), (1.8), (1.9), (1.11), (1.12), and the laws of motion of $\mathbf{X}^a_t, s_t$. Here, the expectation operator is over values of $\epsilon_{R,t}$.

**C. Final Good Producers**

A final homogeneous market good, $Y_t$, is produced by competitive and identical firms using the following technology

$$Y_t = \left[\int_0^1 (Y_{j,t})^{\frac{1}{\lambda}} dj\right]^{\lambda},$$

(1.13)
where $\lambda > 1$. The representative firm chooses specialized inputs, $Y_{j,t}$, to maximize profits:

$$P_t Y_t - \int_0^1 P_{j,t} Y_{j,t} \, dj,$$

subject to the production function (1.13). The firm’s first order condition for the $j$th input is:

$$Y_{j,t} = \left( \frac{P_t}{P_{j,t}} \right)^{\lambda - 1} Y_t. \tag{1.14}$$

Finally, we note that the homogeneous output, $Y_t$, can be used to produce either consumption goods or investment goods. The production of the latter uses a linear technology in which one unit of $Y_t$ is transformed into $\Psi_t$ units of $I_t$.

### D. Retailers

As in Ravenna and Walsh (2008), the $j$th input good is produced by a monopolist retailer, with production function:

$$Y_{j,t} = k_j(z_t h_{j,t})^{1-\alpha} - \eta_t \phi. \tag{1.15}$$

The retailer is a monopolist in the product market and is competitive in the factor markets. Here $k_{j,t}$ denotes the total amount of capital services purchased by firm $j$. Also, $\eta_t \phi$ represents an exogenous fixed cost of production, where $\phi$ is a positive scalar and $\eta_t \phi$ is a process, discussed below, that ensures balanced growth. We calibrate the fixed cost so that retailer profits are zero along the balanced growth path. In (1.15), $z_t$ is a technology shock whose properties are discussed below. Finally, $h_{j,t}$ is the quantity of an intermediate good purchased by the $j$th retailer. This good is purchased in competitive markets at the price $P_{j,t}$ from a wholesaler. As in Christiano, Eichenbaum, and Evans (2005)—henceforth, CEE—we assume that to produce in period $t$, the retailer must borrow a share $\zeta$ of $P_{j,t} h_{j,t}$ at the interest rate, $R_t$, that he expects to prevail in the current period. In this way, the marginal cost of a unit of $h_{j,t}$ is

$$P_{j,t} \left[ \zeta R_t + (1 - \zeta) \right], \tag{1.16}$$

where $\zeta$ is the fraction of the intermediate input that must be financed. The retailer repays the loan at the end of period $t$ after receiving sales revenues. The $j^{th}$ retailer sets its price, $P_{j,t}$, subject to the demand curve, (1.14), and the Calvo sticky price friction (1.17). In particular,

$$P_{j,t} = \begin{cases} P_{j,t-1} & \text{with probability } \xi \\ \bar{P}_t & \text{with probability } 1 - \xi \end{cases}. \tag{1.17}$$
Here, \( \tilde{P}_t \) denotes the price set by the fraction \( 1 - \xi \) of producers who can re-optimize. We assume these producers make their price decision before observing the current period realization of the monetary policy shock, but after the other time \( t \) shocks. Note that, unlike CEE, we do not allow the nonoptimizing firms to index their prices to some measure of inflation. In this way, the model is consistent with the observation that many prices remain unchanged for extended periods of time (see Eichenbaum, Jaimovich, and Rebelo 2011; and Klenow and Malin 2011).

E. Wholesalers and the Labor Market

A perfectly competitive representative wholesaler firm produces the intermediate good using labor only. Let \( l_{t-1} \) denote employment of the wholesaler at the end of period \( t - 1 \). Consistent with our discussion above, a fraction \( 1 - \rho \) of workers separates exogenously from the wholesaler at the end of the period. A total of \( \rho l_{t-1} \) workers are attached to the wholesaler at the start of period \( t \). To meet a worker at the beginning of the period, the wholesaler must pay a fixed cost, \( \eta_t^c \kappa \), and post a suitable number of vacancies. Here, \( \kappa \) is a positive scalar and \( \eta_t^c \) is a process, discussed below, that ensures balanced growth. To hire \( x_t l_{t-1} \) workers, the wholesaler must post \( x_t l_{t-1} / Q_t \) vacancies where \( Q_t \) denotes the aggregate vacancy filling rate, which the representative firm takes as given. Posting vacancies is costless. We assume that the representative firm is large, so that if it posts \( x_t l_{t-1} / Q_t \) vacancies, then it meets exactly \( x_t l_{t-1} \) workers.

Free entry ensures that firms make zero profits in equilibrium. That is, the cost of meeting a worker must equal the value of a match:

\[
(1.18) \quad \eta_t^c \kappa = J_t,
\]

where the objects in (1.18) are expressed in units of the final good.

At the beginning of the period, the representative wholesaler is in contact with a total of \( l_t \) workers (see equation (1.4)). This pool of workers includes workers with whom the firm was matched in the previous period, plus the new workers that the firm has just met. Each worker in \( l_t \) engages in bilateral bargaining with a representative of the wholesaler, taking the outcome of all other negotiations as given. We assume that bargaining occurs after the realization of the technology shocks, but before the realization of the monetary policy shock. Denote the equilibrium real wage rate by:

\[
w_t \equiv W_t / P_t.
\]

In equilibrium all bargaining sessions conclude successfully, so the representative wholesaler employs \( l_t \) workers. Production begins immediately after wage negotiations are concluded and the wholesaler sells the intermediate good at the real price, \( \vartheta_t \equiv P_t^b / P_t \).

Consistent with Hall and Milgrom (2008)—henceforth, HM—and CET, we assume that wages are determined according to the alternating offer bargaining protocol proposed in Rubinstein (1982) and Binmore, Rubinstein, and Wolinsky.
(1986). Let \( w^p_t \) denote the expected present discounted value of the wage payments by a firm to a worker that it is matched with:

\[
w^p_t = w_t + \rho E_t m_{t+1} w^p_{t+1}.
\]

Here \( m_t \) is the time \( t \) household discount factor which firms and workers view as an exogenous stochastic process beyond their control. This discount factor is defined as follows. Let \( \lambda_{C,t} \) denote the multiplier on the household budget constraint, (1.9), in the Lagrangian representation of the household problem. Then, \( m_{t+1} \equiv \beta \lambda_{C,t+1} P_{t+1}/(\lambda_{C,t} P_t) \).

The value of a worker to the firm, \( J_t \), can be expressed as follows:

\[
J_t = \vartheta^p t - w^p_t.
\]

Here \( \vartheta^p_t \) denotes the expected present discounted value of the marginal revenue product associated with a worker to the firm:

\[(1.19) \quad \vartheta^p_t = \vartheta_t + \rho E_t m_{t+1} \vartheta^p_{t+1}.
\]

We now define the value of the typical household member in each of the three labor market states. In each case, the flow value experienced by the labor market member is the marginal contribution of that member to household utility, (1.7), measured in units of the market consumption good. In each case we evaluate a time \( t \) value function at a point in time when the time \( t \) labor force adjustment costs are sunk. In the case of an employed worker, the contribution to household welfare is simply the real wage, \( w_t \). Let \( V_t \) denote the value to a worker of being matched with a firm that pays \( w_t \) in period \( t \). Then,

\[(1.20) \quad V_t = w_t + E_t m_{t+1} [\rho V_{t+1} + (1 - \rho) s \{ f_{t+1} \bar{V}_{t+1} + (1 - f_{t+1}) U_{t+1} \} + (1 - \rho)(1 - s)(\mathcal{L}_{t+1} + N_{t+1})].
\]

The first of the period \( t + 1 \) terms reflect that with probability, \( \rho \), today’s match persists in period \( t + 1 \) with the household member enjoying utility, \( V_{t+1} \). With probability \( 1 - \rho \) the match breaks up in which case there are three possibilities. First, with probability \( s \) the household member remains in the labor force, in which case the household member meets another firm with probability \( f_{t+1} \) and goes into unemployment with probability \( 1 - f_{t+1} \). Here, \( \bar{V}_{t+1} \) denotes the value of working for another firm in period \( t + 1 \). In equilibrium, \( \bar{V}_{t+1} = V_{t+1} \). Also, \( U_{t+1} \) in (1.20) is the value of being an unemployed worker in period \( t + 1 \). The third possibility for matches that break up is that the household member goes out of the labor force. This happens with probability \( 1 - s \). In this case, the worker gives rise to a labor adjustment cost, which we denote by \( \mathcal{L}_{t+1} \). The value of being out of the labor force, after possible adjustment costs have been accounted for, is denoted by \( N_{t+1} \). The
adjustment costs incurred by a household member that moves from the labor force to nonparticipation contributes the following to household welfare (see (1.6)):

\[
L_t = \lambda_t F_1(L_t, L_{t-1}; \eta_t^H) + E_t m_{t+1} \lambda_{t+1} F_2(L_{t+1}, L_t; \eta_{t+1}^L).
\]

Here, \( \lambda_t \) denotes the contribution to household utility of the nonmarket produced good, \( C_t^H \), expressed in units of the market consumption good, \( C_t \). Given our functional forms,

\[
\lambda_t = \frac{\omega}{1 - \omega} \left( \frac{C_t - b C_{t-1}}{C_t^H - b C_{t-1}^H} \right)^{1-\chi}.
\]

It is convenient to rewrite (1.20) as follows:

(1.21) \( V_t = w_t^p + A_t \),

where

(1.22) \( A_t = (1 - \rho) E_t m_{t+1} \left[ s f_{t+1} \bar{V}_{t+1} + s (1 - f_{t+1}) U_{t+1} \right] \)

\[ + (1 - s) \left( L_{t+1} + N_{t+1} \right) \] \( + \rho E_t m_{t+1} A_{t+1} \).

According to (1.21), \( V_t \) consists of two components. The first is the expected present value of wages received by the workers from the firm with which he is currently matched. The second corresponds to the expected present value of the payments that a worker receives in all dates and states when he is separated from that firm.

We assume that the only contribution of unemployed workers to household resources is unemployment compensation, \( \eta_t^D D \). Thus, the value of unemployment, \( U_t \), is given by:

(1.23) \( U_t = \eta_t^D D + \bar{U}_t \).

The variable, \( \bar{U}_t \), denotes the continuation value of unemployment:

(1.24) \( \bar{U}_t \equiv E_t m_{t+1} \left[ s f_{t+1} V_{t+1} + s (1 - f_{t+1}) U_{t+1} + (1 - s) \left( L_{t+1} + N_{t+1} \right) \right] \).

Expression (1.24) reflects our assumption that an unemployed worker finds a job in the next period with probability \( sf_{t+1} \), remains unemployed with probability

---

8The derivative takes into account that \( L_t = 1 - N_t \) so that

\[
\frac{d}{d\eta_t^L} F(L_t, L_{t-1}; \eta_t^L) = - F_1(L_t, L_{t-1}; \eta_t^L).
\]

Also, \( \lambda_t \equiv \lambda_{H,t}/(\lambda_C P_t) \) where \( \lambda_{H,t} \geq 0 \) denotes the multiplier on (1.6) in the Lagrangian representation of the household problem, while \( \lambda_C \geq 0 \) denotes the multiplier on the household budget constraint.
s(1 − f_{t+1}) and exits the labor force with probability 1 − s. In the case when the unemployed worker exits the labor market, then he contributes to labor force adjustment costs by L_{t+1}.

The value of nonparticipation is:

\[ N_t = \lambda_t \eta_t^H + E_t m_t \left[ e_t (f_t V_t + (1 - f_t) U_t + L_t) \right] + (1 - e_t) N_{t+1} \]

Expression (1.25) reflects our assumption that a nonparticipating worker is selected to join the labor force with probability $e_{t+1}$, defined in (1.3). In addition, (1.25) indicates that a household member who does not participate in the labor force in period $t$ and in period $t + 1$ does not contribute to labor force adjustment costs in period $t + 1$. However, the household member that does not participate in the labor market in period $t$, but does participate in period $t + 1$ gives rise to labor adjustment costs in $t + 1$. This is captured by $-L_{t+1}$ in (1.25). Finally, the time $t$ flow term in (1.25) is the marginal product of labor, $\eta_t^H$, in producing $C_t^H$, times the corresponding value, $\lambda_t$.

The basic structure of alternating offer bargaining is the same as it is in CET. Each matched worker-firm pair (both those who just matched for the first time and those who were matched in the past) bargain over the current wage rate, $w_t$. Each time period (a quarter) is subdivided into $M$ periods of equal length, where $M$ is even. The firm makes a wage offer at the start of the first subperiod. It also makes an offer at the start of every subsequent odd subperiod in the event that all previous offers have been rejected. Similarly, workers make a wage offer at the start of all even subperiods in the case when all previous offers have been rejected. Because $M$ is even, the last offer is made, on a take-it-or-leave-it basis, by the worker. When the firm rejects an offer it pays a cost, $\eta_t^\gamma$, of making a counteroffer. Here $\gamma$ is a positive scalar and $\eta_t^\gamma$ is a process that ensures balanced growth.

In subperiods $j = 1, \ldots, M - 1$, the recipient of an offer can either accept or reject it. If the offer is rejected the recipient may declare an end to the negotiations or he may plan to make a counteroffer at the start of the next subperiod. In the latter case there is a probability, $\delta$, that bargaining breaks down and the wholesaler and worker revert to their outside option. For the firm, the value of the outside option is zero and for the worker the outside option is unemployment.\(^9\) Given our assumptions, workers and firms never choose to terminate bargaining and go to their outside options.

It is always optimal for the firm to offer the lowest wage rate subject to the condition that the worker does not reject it. To know what that wage rate is, the wholesaler must know what the worker would counteroffer in the event that the firm’s offer was rejected. But, the worker’s counteroffer depends on the firm’s counteroffer in case the worker’s counteroffer is rejected. We solve for the firm’s initial offer beginning

\(^9\)We could allow for the possibility that when negotiations break down the worker has a chance of leaving the labor force. To keep our analysis relatively simple, we do not allow for that possibility here.
with the worker’s final offer and working backwards. Since workers and firms know everything about each other, the firm’s opening wage offer is always accepted.

Our environment is sufficiently simple that the solution to the bargaining problem has the following straightforward characterization

\[ \alpha_1 J_t = \alpha_2 (V_t - U_t) - \alpha_3 \eta_t \gamma + \alpha_4 (\theta_t - \eta_t^D D), \]

where

\[ \begin{align*}
\alpha_1 &= 1 - \delta + (1 - \delta)^M \\
\alpha_2 &= 1 - (1 - \delta)^M \\
\alpha_3 &= \alpha_2 \frac{1 - \delta}{\delta} - \alpha_1 \\
\alpha_4 &= \frac{1 - \delta}{2 - \delta} \frac{\alpha_2}{M} + 1 - \alpha_2.
\end{align*} \]

The technical Appendix contains a detailed derivation of (1.26) and describes the procedure that we use for solving the bargaining problem.

To summarize, in period \( t \) the problem of wholesalers is to choose the hiring rate, \( x_t \), and to bargain with the workers that they meet. These activities occur before the monetary policy shock is realized and after the other shocks are realized.

F. Innovations to Technology

In this section we describe the laws of motion of technology. Turning to the investment-specific shock, we assume that \( \ln \mu_{\Psi,t} \equiv \ln(\Psi_t / \Psi_{t-1}) \) follows an AR(1) process:

\[ \ln \mu_{\Psi,t} = (1 - \rho \Psi) \ln \mu_{\Psi} + \rho \Psi \ln \mu_{\Psi,t-1} + \sigma \Psi \varepsilon_{\Psi,t}. \]

Here, \( \varepsilon_{\Psi,t} \) is the innovation in \( \ln \mu_{\Psi,t} \), i.e., the error in the one-step-ahead forecast of \( \ln \mu_{\Psi,t} \) based on the history of past observations of \( \ln \mu_{\Psi,t} \).

For reasons explained later, it is convenient for our post-2008 analysis to adopt a components representation for neutral technology. In particular, we assume that the growth rate of neutral technology is the sum of a permanent \( (\delta^P_t) \) and a transitory \( (\delta^T_t) \) component:

\[ \ln(\delta^P_t) = \ln(z_t / z_{t-1}) = \ln(\gamma) + \mu_{P,t} + \mu_{T,t}, \]

where

\[ \mu_{P,t} = \rho_P \mu_{P,t-1} + \sigma_P \varepsilon_{P,t}, \quad \left| \rho_P \right| < 1, \]

\[ \text{Unobserved components representations have played an important role in macroeconomic analysis. See, for example, Erceg and Levin (2003) and Edge, Laubach, and Williams (2007).} \]
and

\begin{equation}
\mu_{T,t} = \rho_T \mu_{T,t-1} + \sigma_T (\varepsilon_{T,t} - \varepsilon_{T,t-1}), \quad |\rho_T| < 1 .
\end{equation}

In (1.28) and (1.29), \( \varepsilon_{p,t} \) and \( \varepsilon_{T,t} \) are mean zero, unit variance, i.i.d shocks. To see why (1.29) is the transitory component of \( \ln(z_t) \), suppose \( \mu_{p,t} \equiv 0 \) so that \( \mu_{T,t} \) is the only component of technology and (ignoring the constant term) \( \ln(\mu_{z,t}) = \mu_{T,t} \), or

\[ \ln(\mu_{z,t}) = \ln(z_t) - \ln(z_{t-1}) = \rho_T (\ln(z_{t-1}) - \ln(z_{t-2})) + \sigma_T (\varepsilon_{T,t} - \varepsilon_{T,t-1}) . \]

Dividing by \( 1 - L \), where \( L \) denotes the lag operator, we have

\[ \ln(z_t) = \rho_T \ln(z_{t-1}) + \sigma_T \varepsilon_{T,t} . \]

Thus, a shock to \( \varepsilon_{T,t} \) has only a transient effect on the forecast of \( \ln(z_t) \). By contrast a shock, say \( \Delta \varepsilon_{p,t} \), to \( \varepsilon_{p,t} \) shifts \( E_t \ln(z_{t+j}) \), \( j \to \infty \) by the amount, \( \Delta \varepsilon_{p,t}/(1 - \rho_p) \).

We assume that when there is a shock to \( \ln(z_t) \), agents do not know whether it reflects the permanent or the temporary component. As a result, they must solve a signal extraction problem when they adjust their forecast of future values of \( \ln(z_t) \) in response to an unanticipated move in \( \ln(z_t) \). Suppose, for example, there is a shock to \( \varepsilon_{p,t} \), but that agents believe most fluctuations in \( \ln(z_t) \) reflect shocks to \( \varepsilon_{T,t} \). In this case they will adjust their near term forecast of \( \ln(z_t) \), leaving their longer-term forecast of \( \ln(z_t) \) unaffected. As time goes by and agents see that the change in \( \ln(z_t) \) is too persistent to be due to the transitory component, the long-run component of their forecast of \( \ln(z_t) \) begins to adjust. Thus, a disturbance in \( \varepsilon_{p,t} \) triggers a sequence of forecast errors for agents who cannot observe whether a shock to \( \ln(z_t) \) originates in the temporary or permanent component of \( \ln(\mu_{z,t}) \).

Because agents do not observe the components of technology directly, they do not use the components representation to forecast technology growth. For forecasting, they use the univariate Wold representation that is implied by the components representation. The shocks to the permanent and transitory components of technology enter the system by perturbing the error in the Wold representation. To clarify these observations we first construct the Wold representation.

Multiply \( \ln(\mu_{z,t}) \) in (1.27) by \( (1 - \rho_p L)(1 - \rho_T L) \), where \( L \) denotes the lag operator:

\begin{equation}
(1 - \rho_p L)(1 - \rho_T L) \ln(\mu_{z,t}) = (1 - \rho_T L) \sigma_p \varepsilon_{p,t} + (1 - \rho_p L) \\
\times (\sigma_T \varepsilon_{T,t} - \sigma_T \varepsilon_{T,t-1}) .
\end{equation}

Let the stochastic process on the right of the equality be denoted by \( \mathcal{W}_t \). Evidently, \( \mathcal{W}_t \) has a second order moving average representation, which we express in the following form:

\begin{equation}
\mathcal{W}_t = (1 - \theta_1 L - \theta_2 L^2) \sigma \eta_t, \quad E\eta_t = 1 .
\end{equation}
We obtain a mapping from $\rho_P, \rho_T, \sigma_P, \sigma_T$ to $\theta_1, \theta_2, \sigma_\eta$ by computing the variance and two lagged covariances of the object to the right of the equality in (1.30). We then find the values of $\theta_1, \theta_2,$ and $\sigma_\eta$ for which the variance and two lagged covariances of $W_t$ and the object on the right of the equality in (1.30) are the same. In addition, we require that the eigenvalues in the moving average representation of $\eta_t$, (1.31), lie inside the unit circle. The latter condition is what guarantees that the shock in the Wold representation is the innovation in technology. In sum, the Wold representation for $\ln(\mu_{z,i})$ is:

\begin{equation}
(1.32) \quad (1 - \rho_P L)(1 - \rho_T L)\ln(\mu_{z,i}) = (1 - \theta_1 L - \theta_2 L^2) \sigma_\eta \eta_t.
\end{equation}

The mapping from the structural shocks, $\varepsilon_{P,t}$ and $\varepsilon_{T,t}$, to $\eta_t$ is obtained by equating the objects on the right of the equalities in (1.30) and (1.31):

\begin{equation}
(1.33) \quad \eta_t = \theta_1 \eta_{t-1} + \theta_2 \eta_{t-1} + \frac{\sigma_P}{\sigma_\eta} (\varepsilon_{P,t} - \rho_T \varepsilon_{P,t-1})
\end{equation}

\begin{equation}
\quad + (1 - \rho_P L) \frac{\sigma_T}{\sigma_\eta} (\varepsilon_{T,t} - \varepsilon_{T,t-1}).
\end{equation}

According to this expression, if there is a positive disturbance to $\varepsilon_{P,t}$, this triggers a sequence of one-step-ahead forecast errors for agents, consistent with the intuition described above.\footnote{An alternative approach to agents' forecasting problem is to set it up as a Kalman filtering problem. One can show that the solution to that problem and the one obtained with our Wold representation coincide.}

When we estimate our model, we treat the innovation in technology, $\eta_t$, as a primitive and are not concerned with the decomposition of $\eta_t$ into the $\varepsilon_{P,t}$'s and $\varepsilon_{T,t}$'s. In effect, we replace the unobserved components representation of the technology shock with its representation in (1.32). That representation is an autoregressive, moving average representation with two autoregressive parameters, two moving average parameters and a standard deviation parameter. Thus, in principle it has five free parameters. But, since the Wold representation is derived from the unobserved components model, it has only four free parameters. Specifically, we estimate the following parameters: $\rho_P, \rho_T, \sigma_P$ and the ratio $\sigma_T/\sigma_P$.

Although we do not make use of the decomposition of the innovation, $\eta_t$, into the structural shocks when we estimate our model, it turns out that the decomposition is very useful for interpreting the post-2008 data.

G. Market Clearing, Monetary Policy, and Functional Forms

The total supply of the intermediate good is given by $l_t$ which equals the total quantity of labor used by the wholesalers. So, clearing in the market for intermediate goods requires

\begin{equation}
(1.34) \quad h_t = l_t,
\end{equation}
where

$$h_t \equiv \int_0^1 h_{j,t} dj.$$  

The capital services market clearing condition is:

$$u_t K_t = \int_0^1 k_{j,t} dj.$$  

Market clearing for final goods requires:

$$C_t + (I_t + a(u^K K_t) / \Psi_t + \eta_t \kappa x_t I_{t-1} + G_t = Y_t.$$  

The right-hand side of the previous expression denotes the quantity of final goods. The left-hand side represents the various ways that final goods are used. Homogeneous output, $Y_t$, can be converted one-for-one into either consumption goods, goods used to hire workers, or government purchases, $G_t$. In addition, some of $Y_t$ is absorbed by capital utilization costs. Homogeneous output, $Y_t$, can also be used to produce investment goods using a linear technology in which one unit of the final good is transformed into $\Psi_t$ units of $I_t$. Perfect competition in the production of investment goods implies,

$$P_{I,t} = \frac{P_t}{\Psi_t}.$$  

Clearing in the loan market requires that the demand for loans by retailers, $\kappa h_t P^h_t$, equals the supply, $B_{t+1}/R_t$:

$$\kappa h_t P^h_t = \frac{B_{t+1}}{R_t}.$$  

We adopt the following specification of monetary policy:

$$\ln(R_t/R) = \rho_R \ln(R_{t-1}/R)$$

$$+ (1 - \rho_R) \left[ 0.25 r \pi \ln \left( \frac{\pi_t}{\pi^A} \right) + 0.25 r \Delta \pi \ln \left( \frac{\pi_t^A}{\pi_{t-4}^{A^4}} \right) \right]$$

$$+ \sigma_R \varepsilon_{R,t},$$

where $\pi_t^A \equiv P_t / P_{t-4}$ and $\pi^A$ is the monetary authority’s inflation target. The object, $\pi^A$ is also the value of $\pi_t^A$ in nonstochastic steady state. The shock, $\varepsilon_{R,t}$, is a

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12 We also estimated a version of the model in which the output gap, i.e., the level of output relative to its balanced growth path, enters the monetary policy rule. We always found the estimated coefficient on the output gap to be very small.
unit variance, zero mean, and serially uncorrelated disturbance to monetary policy. The variable, $O_t$, denotes Gross Domestic Product:

$$O_t = C_t + I_t/\Psi_t + G_t,$$

where $G_t$ denotes government consumption, which is assumed to have the following representation:

$$(1.37) \quad G_t = \eta^g_t g_t,$$

Here, $\eta^g_t$ is a process that guarantees balanced growth and $g_t$ is an exogenous stochastic process. The constant, $\mu^g_0$, is the value of $O_t/O_{t-4}$ in nonstochastic steady state. Also, $R$ denotes the steady state value of $R_t$. Finally, we require that money demand equals money supply.

The sources of long-term growth in our model are the neutral and investment-specific technological progress shocks discussed in the previous subsection. The growth rate in steady state for the model variables is a composite, $\Phi_t$, of these two technology shocks:

$$\Phi_t = \Psi_t^{\alpha} z_t.$$

The variables $Y_t/\Phi_t, C_t/\Phi_t, w_t/\Phi_t$ and $I_t/(\Psi_t\Phi_t)$ converge to constants in nonstochastic steady state.

If objects like the fixed cost of production, the cost of hiring, etc., were constant, they would become irrelevant over time. To avoid this implication, it is standard in the literature to suppose that such objects are proportional to the underlying source of growth, which is $\Phi_t$ in our setting. However, this assumption has the unfortunate implication that technology shocks of both types have an immediate effect on the vector of objects

$$(1.38) \quad \Omega_t = [\eta^g_t, \eta^D_t, \eta^\gamma_t, \eta^\kappa_t, \eta^\phi_t, \eta^L_t, \eta^H_t]^\prime.$$

Such a specification seems implausible and so we instead proceed as in Christiano, Trabandt, and Walentin (2012) and Schmitt-Grohé and Uribe (2012). In particular, we suppose that the objects in $\Omega_t$ are proportional to a long moving average of composite technology, $\Phi_t$:

$$(1.39) \quad \Omega_{i,t} = \Phi_t^\theta (\Omega_{i,t-1})^{1-\theta},$$

where $\Omega_{i,t}$ denotes the $i$th element of $\Omega_t$, $i = 1, \ldots, 7$. Also, $0 < \theta \leq 1$ is a parameter to be estimated. Note that $\Omega_{i,t}$ has the same growth rate in steady state as GDP. When $\theta$ is very close to zero, $\Omega_{i,t}$ is virtually unresponsive in the short-run to an innovation in either of the two technology shocks, a feature that we find very attractive on a priori grounds.
We adopt the investment adjustment cost specification proposed in CEE. In particular, we assume that the cost of adjusting investment takes the form:

\[ S(I_t/I_{t-1}) = 0.5 \exp \left[ \sqrt{S''(I_t/I_{t-1} - \mu_\Phi \times \mu_\Psi)} \right] + 0.5 \exp \left[ - \sqrt{S''(I_t/I_{t-1} - \mu_\Phi \times \mu_\Psi)} \right] - 1. \]

Here, \( \mu_\Phi \) and \( \mu_\Psi \) denote the steady state growth rates of \( \Phi_t \) and \( \Psi_t \). The value of \( I_t/I_{t-1} \) in nonstochastic steady state is \( (\mu_\Phi \times \mu_\Psi) \). It is straightforward to verify that \( S(\mu_\Phi \times \mu_\Psi) = S'(\mu_\Phi \times \mu_\Psi) = 0 \). Our specification of the adjustment costs has the convenient feature that the steady state of the model is independent of the value of \( S'' \).

The adjustment cost function for the labor force is specified as follows:

\[ \mathcal{F}(L_t, L_{t-1}; \eta_t) = 0.5 \eta_t \phi_L(L_t/L_{t-1} - 1)^2. \]

We assume that the cost associated with setting capacity utilization is given by

\[ a(u^K_t) = 0.5\sigma_a \sigma_b (u^K_t)^2 + \sigma_b (1 - \sigma_a) u^K_t + \sigma_b (\sigma_a/2 - 1), \]

where \( \sigma_a \) and \( \sigma_b \) are positive scalars. We normalize the steady state value of \( u^K_t \) to unity, so that the adjustment costs are zero in steady state, and \( \sigma_b \) is equated to the steady state of the appropriately scaled rental rate on capital. Our specification of the cost of capacity utilization and our normalization of \( u^K_t \) in steady state has the convenient implication that the model steady state is independent of \( \sigma_a \).

Finally, we discuss the determination of the equilibrium vacancy filling rate, \( Q_t \). We posit a standard matching function:

\[ x_t l_{t-1} = \sigma_m (L_t - \rho l_{t-1})^\sigma (l_{t-1} v_t)^{1-\sigma}, \]

where \( l_{t-1} v_t \) denotes the economy-wide average number of vacancies and \( v_t \) denotes the aggregate vacancy rate. Then,

\[ Q_t = \frac{x_t}{v_t}. \]

II. Data and Econometric Methodology for Pre-2008 Sample

We estimate our model using a Bayesian variant of the strategy in CEE that minimizes the distance between the dynamic response to three shocks in the model and the analog objects in the data. The latter are obtained using an identified VAR for postwar quarterly US times series that include key labor market variables. The particular Bayesian strategy that we use is the one developed in Christiano, Trabandt, and Walentin (2010) — henceforth, CTW.
CTW estimate a 14 variable VAR using quarterly data that are seasonally adjusted and cover the period 1951:1 to 2008:IV. To facilitate comparisons, our analysis is based on the same VAR that CTW use. As in CTW, we identify the dynamic responses to a monetary policy shock by assuming that the monetary authority observes the current and lagged values of all the variables in the VAR, and that a monetary policy shock affects only the Federal Funds Rate contemporaneously. As in Altig et al. (2011), Fisher (2006) and CTW, we make two assumptions to identify the dynamic responses to the technology shocks: (i) the only shocks that affect labor productivity in the long-run are the innovations to the neutral technology shock, $\eta_t$, and the innovations to the investment-specific technology shock $\epsilon_{\Psi_t}$, and (ii) the only shocks that affect the price of investment relative to consumption in the long-run are the innovations to the investment-specific technology shock $\epsilon_{\Psi_t}$. These assumptions are satisfied in our model. Standard lag-length selection criteria lead CTW to work with a VAR with two lags.\textsuperscript{13} The assumptions used to identify the effects of monetary policy and technology shocks are satisfied in our model.

We include the following variables in the VAR:

\begin{equation}
\begin{bmatrix}
\Delta \ln(\text{relative price of investment}) \\
\Delta \ln(\text{real GDP/hours}) \\
\Delta \ln(\text{GDP deflator}) \\
\ln(\text{capacity utilization}) \\
\ln(\text{hours}) \\
\ln(\text{real GDP/hours}) - \ln(\text{real wage}) \\
\ln(\text{nominal } C/\text{nominal GDP}) \\
\ln(\text{nominal } I/\text{nominal GDP}) \\
\ln(\text{job vacancies}) \\
\text{job separation rate} \\
\text{job finding rate} \\
\ln(\text{hours/labor force}) \\
\text{federal funds rate}
\end{bmatrix}
\end{equation}

See Section A of the technical Appendix in CTW for details about the data. Here, we briefly discuss the job vacancy data. Our time series on vacancies splices together a help-wanted index produced by the Conference Board with a job openings measure produced by the Bureau of Labor Statistics in their Job Openings and Labor Turnover Survey (JOLTS). According to JOLTS, a “job opening” is a position that the firm would fill in the event that a suitable candidate appears. A job vacancy in our model corresponds to this definition of a “job opening.” To see this, recall that

\textsuperscript{13} See CTW for a sensitivity analysis with respect to the lag length of the VAR.
in our model the representative firm is large. We can think of our firm as consisting of a large number of plants. Suppose that the firm wants to hire \( z \) people per plant when the vacancy filling rate is \( Q \). The firm instructs each plant to post \( z/Q \) vacancies with the understanding that each vacancy which generates a job application will be turned into a match.\(^{14}\) This is the sense in which vacancies in our model meet the JOLTS definition of a job opening. Of course, it is possible that the people responding to the JOLTS survey report job opening numbers that correspond more closely to \( z \). To the extent that this is true, the JOLTS data should be thought of as a noisy indicator of vacancies in our model. This measurement issue is not unique to our model. It arises in the standard search and matching model (see, for example, Shimer 2005).

Given an estimate of the VAR we compute the implied impulse response functions to the three structural shocks. We stack the contemporaneous and 14 lagged values of each of these impulse response functions for 13 of the variables listed above in a vector, \( \hat{\psi} \). We do not include the job separation rate because that variable is constant in our model. We include the job separation rate in the VAR to ensure the VAR results are not driven by an omitted variable bias.

The logic underlying our model estimation procedure is as follows. Suppose that our structural model is true. Denote the true values of the model parameters by \( \theta_0 \). Let \( \psi(\theta) \) denote the model-implied mapping from a set of values for the model parameters to the analog impulse responses in \( \hat{\psi} \). Thus, \( \psi(\theta_0) \) denotes the true value of the impulse responses whose estimates appear in \( \hat{\psi} \). According to standard classical asymptotic sampling theory, when the number of observations, \( T \), is large, we have

\[
\sqrt{T}(\hat{\psi} - \psi(\theta_0)) \overset{d}{\sim} N(0, W(\theta_0, \zeta_0)).
\]

Here, \( \zeta_0 \) denotes the true values of the parameters of the shocks in the model that we do not formally include in the analysis. When we estimate the model we work with a log-linearized solution. Consequently, \( \psi(\theta_0) \) is not a function of \( \zeta_0 \). However, the sampling distribution of \( \hat{\psi} \) is a function of \( \zeta_0 \). We find it convenient to express the asymptotic distribution of \( \hat{\psi} \) in the following form:

\[
(2.2) \quad \hat{\psi} \overset{d}{\sim} N(\psi(\theta_0), V),
\]

where

\[
V = \frac{W(\theta_0, \zeta_0)}{T}.
\]

For simplicity our notation does not make the dependence of \( V \) on \( \theta_0, \zeta_0 \) and \( T \) explicit. We use a consistent estimator of \( V \). Motivated by small sample considerations, that estimator has only diagonal elements (see CTW). The elements in \( \hat{\psi} \) are

\(^{14}\) Some plants will hire more than \( z \) people and others will hire fewer. By the law of large numbers, there is no uncertainty at the firm level about how many people will be hired.
graphed in Figures 1–3 (see the solid lines). The gray areas are centered, 95 percent probability intervals computed using our estimate of $V$.

In our analysis, we treat $\hat{\psi}$ as the observed data. We specify priors for $\theta$ and then compute the posterior distribution for $\theta$ given $\hat{\psi}$ using Bayes’ rule. This computation requires the likelihood of $\hat{\psi}$ given $\theta$. Our asymptotically valid approximation of this likelihood is motivated by (2.2):

$$f(\hat{\psi}|\theta, V) = \left(\frac{1}{2\pi}\right)^{\frac{N}{2}} |V|^{-\frac{1}{2}} \exp\left[-\frac{1}{2}(\hat{\psi} - \psi(\theta))' V^{-1} (\hat{\psi} - \psi(\theta))\right].$$

The value of $\theta$ that maximizes the above function represents an approximate maximum likelihood estimator of $\theta$. It is approximate for three reasons: (i) the central limit theorem underlying (2.2) only holds exactly as $T \to \infty$, (ii) our proxy for $V$ is guaranteed to be correct only for $T \to \infty$, and (iii) $\psi(\theta)$ is calculated using a linear approximation.

Treating the function, $f$, as the likelihood of $\hat{\psi}$, it follows that the Bayesian posterior of $\theta$ conditional on $\hat{\psi}$ and $V$ is

$$f(\theta|\hat{\psi}, V) = \frac{f(\hat{\psi}|\theta, V)p(\theta)}{f(\hat{\psi}|V)}. $$

Here, $p(\theta)$ denotes the priors on $\theta$ and $f(\hat{\psi}|V)$ denotes the marginal density of $\hat{\psi}$:

$$f(\hat{\psi}|V) = \int f(\hat{\psi}|\theta, V)p(\theta) d\theta.$$

The mode of the posterior distribution of $\theta$ is computed by maximizing the value of the numerator in (2.4), since the denominator is not a function of $\theta$. The posterior distribution of $\theta$ is computed using a standard Markov Chain Monte Carlo (MCMC) algorithm.

III. Empirical Results, Pre-2008 Sample

This section presents results for the estimated model. First, we discuss the priors and posteriors of structural parameters. Second, we discuss the ability of the model to account for the dynamic response of the economy to a monetary policy shock, a neutral technology shock and an investment-specific technology shock.

A. Calibration and Parameter Values Set a priori

We set the values for a subset of the model parameters a priori. These values are reported in panel A of Table 1. We also set the steady state values of five endogenous model variables, listed in panel B of Table 1. We specify $\beta$ so that the steady state annual real rate of interest is 3 percent. The depreciation rate on capital, $\delta_k$, is set to imply an annual depreciation rate of 10 percent. The growth rate of composite technology, $\mu \Phi$, is equated to the sample average of real per capita
GDP growth. The growth rate of investment-specific technology, $\mu_q$, is set so that $(\mu_q \times \mu_{\phi})$ is equal to the sample average of real, per capita investment growth. We assume the monetary authority’s inflation target is 2 percent per year and that the profits of intermediate good producers are 0 in steady state. We set the steady state value of the vacancy filling rate, $Q$, to 0.7, as in den Haan, Ramey, and Watson (2000) and Ravenna and Walsh (2008). The steady state unemployment rate, $u$, is set to the average unemployment rate in our sample, 0.055. We assume the parameter $M$ to be equal to 60 which roughly corresponds to the number of business days in a quarter. We set $\rho = 0.9$, which implies a match survival rate that is consistent with both Hall and Milgrom (2008) and Shimer (2012). Finally, we assume that the steady state value of the ratio of government consumption to gross output is 0.2.

Two additional parameters pertain to the household sector. We set the elasticity of substitution in household utility between home and market produced goods, $(1 - b)/(1 - \chi)$, to 3. Specifically, we estimate the consumption habit parameter $b$ and always set $\chi$ so that $(1 - b)/(1 - \chi) = 3$. This magnitude of the elasticity of substitution is similar to the estimate in Aguiar, Hurst and Karabarbounis (2013).\(^{15}\) We set the steady state labor force to population ratio, $L$, to 0.67.

To make the model consistent with the five calibrated values for $L, Q, G/Y, u$, and profits, we select values for five parameters: the weight of market consumption in the utility function, $\omega$; the constant in front of the matching function, $\sigma_m$; the fixed cost of production, $\phi$; the cost for the firm to make a counteroffer, $\gamma$; and the scale

---

\(^{15}\)We take our elasticity of substitution parameter from the literature to maintain comparability. However, there is a caveat. To understand this, recall the definition of the elasticity of substitution. It is the percent change in $C^H$ in response to a 1 percent change in the corresponding relative price, say $\lambda$. From an empirical standpoint, it is difficult to obtain a direct measure of this elasticity because we do not have data on $C^H$ or $\lambda$. As a result, structural relations must be assumed, which map from observables to $C^H$ and $\lambda$. Since estimates of the elasticity are presumably dependent on the details of the structural assumptions, it is not clear how to compare values of this parameter across different studies, which make different structural assumptions.
Table 2—Priors and Posteriors of Model Parameters

<table>
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<tr>
<th>Price setting parameters</th>
<th>Prior</th>
<th>Posterior</th>
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<tbody>
<tr>
<td>Price stickiness</td>
<td>$\xi$</td>
<td>Beta 0.66, 0.15</td>
</tr>
<tr>
<td>Price markup parameter</td>
<td>$\lambda$</td>
<td>Gamma 1.20, 0.05</td>
</tr>
<tr>
<td>Working capital share</td>
<td>$\sigma_w$</td>
<td>Beta 0.50, 0.20</td>
</tr>
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<tr>
<th>Monetary authority parameters</th>
<th>Prior</th>
<th>Posterior</th>
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</thead>
<tbody>
<tr>
<td>Taylor rule: interest rate smoothing</td>
<td>$\rho_p$</td>
<td>Beta 0.75, 0.15</td>
</tr>
<tr>
<td>Taylor rule: inflation coefficient</td>
<td>$r_n$</td>
<td>Gamma 1.70, 0.10</td>
</tr>
<tr>
<td>Taylor rule: GDP growth coefficient</td>
<td>$r_{\Delta y}$</td>
<td>Gamma 0.20, 0.05</td>
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<tr>
<th>Preferences and technology</th>
<th>Prior</th>
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<tbody>
<tr>
<td>Market and home consumption habit</td>
<td>$b$</td>
<td>Beta 0.66, 0.10</td>
</tr>
<tr>
<td>Capacity utilization adjustment cost</td>
<td>$\sigma_a$</td>
<td>Gamma 0.50, 0.40</td>
</tr>
<tr>
<td>Investment adjustment cost</td>
<td>$S^*$</td>
<td>Gamma 2.00, 0.50</td>
</tr>
<tr>
<td>Capital share</td>
<td>$\alpha$</td>
<td>Beta 0.33, 0.03</td>
</tr>
<tr>
<td>Technology diffusion</td>
<td>$\theta$</td>
<td>Beta 0.50, 0.20</td>
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<thead>
<tr>
<th>Labor market parameters</th>
<th>Prior</th>
<th>Posterior</th>
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<tbody>
<tr>
<td>Probability of bargaining breakup</td>
<td>$100\delta$</td>
<td>Gamma 0.25, 0.10</td>
</tr>
<tr>
<td>Replacement ratio</td>
<td>$D_{lw}$</td>
<td>Beta 0.25, 0.10</td>
</tr>
<tr>
<td>Hiring cost to output ratio</td>
<td>$s_l$</td>
<td>Gamma 1.00, 0.30</td>
</tr>
<tr>
<td>Labor force adjustment cost</td>
<td>$\phi_L$</td>
<td>Gamma 100.0, 50.0</td>
</tr>
<tr>
<td>Probability of staying in labor force</td>
<td>$s$</td>
<td>Beta 0.85, 0.05</td>
</tr>
<tr>
<td>Matching function parameter</td>
<td>$\sigma$</td>
<td>Beta 0.50, 0.10</td>
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<tr>
<th>Shocks</th>
<th>Prior</th>
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<tr>
<td>Std. monetary policy shock</td>
<td>$400\sigma_g$</td>
<td>Gamma 0.65, 0.05</td>
</tr>
<tr>
<td>AR(1) persistent comp. neutral tech.</td>
<td>$\rho_p$</td>
<td>Gamma 0.50, 0.07</td>
</tr>
<tr>
<td>AR(1) transitory comp. neutral tech.</td>
<td>$\rho_T$</td>
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</tr>
<tr>
<td>Std. ratio neutral tech. shocks</td>
<td>$\sigma_T/\sigma_p$</td>
<td>Gamma 6.00, 0.45</td>
</tr>
<tr>
<td>AR(1) investment spec. tech. shock</td>
<td>$\rho_q$</td>
<td>Beta 0.75, 0.10</td>
</tr>
<tr>
<td>Std. investment spec. tech. shock</td>
<td>$100\sigma_q$</td>
<td>Gamma 0.10, 0.05</td>
</tr>
</tbody>
</table>

Notes: $s_l$ denotes the steady state hiring to gross output ratio (in percent). Posterior mean and distributions of parameters are based on a standard MCMC algorithm with 500,000 draws (100,000 draws used for burn-in, and the draw acceptance rate is 0.24).

The priors and posteriors for the model parameters about which we do Bayesian inference are summarized in Table 2. A number of features of the posterior mean of the estimated parameters of our model are worth noting.

First, the posterior mean of $\xi$ implies a moderate degree of price stickiness, with prices changing on average once every four quarters. This value lies within the range reported in the literature.

Second, the posterior mean of $\delta$ implies that there is a 0.13 percent chance of an exogenous breakup in negotiations when a wage offer is rejected.

The posterior means and distributions of parameters are based on a standard MCMC algorithm with 500,000 draws (100,000 draws used for burn-in, and the draw acceptance rate is 0.24).
Third, the posterior means of our model parameters, along with the assumption that the steady state unemployment rate equals 5.5 percent, implies that it costs firms about 0.6 days of marginal revenue to prepare a counteroffer during wage negotiations (see Table 3).

Fourth, the posterior mean of steady state hiring costs as a percent of gross output is equal to 0.5 percent. This result implies that steady state hiring costs as a percent of total wages of newly-hired workers is equal to 7 percent. Silva and Toledo (2009) report that, depending on the exact costs included, the value of this statistic is between 4 and 14 percent, a range that encompasses the corresponding statistic in our model.

Fifth, the posterior mean of the replacement ratio is 0.32. HM summarize the literature and report a range of estimates from 0.12 to 0.36 for the replacement ratio. So our estimate falls within their range. It is well known that Diamond (1982), Mortensen (1982), and Pissarides (1985) —henceforth, DMP—style models with Nash wage bargaining require a replacement ratio in excess of 0.9 to account for fluctuations in labor markets (see e.g., CET for an extended discussion). For the reasons stressed in CET, alternating offer bargaining between workers and firms mutes the sensitivity of real wages to aggregate shocks. This property underlies our model’s ability to account for the estimated response of the economy to monetary policy shocks and shocks to neutral and investment-specific technology with a relatively low replacement ratio.

Sixth, the posterior mean of $s$ implies that a separated or unemployed worker leaves the labor force with probability $1 - s = 0.16$. As a practical matter, we found that there was only little information in the data about $s$. 

| Table 3—Model steady states and implied parameters |
|-----------------|-----------------|-----------------|
| Variable        | At estimated posterior mean | Description |
| $K/Y$           | 6.45             | Capital to gross output ratio (quarterly) |
| $C/Y$           | 0.59             | Market consumption to gross output ratio |
| $I/Y$           | 0.21             | Investment to gross output ratio |
| $l$             | 0.63             | Employment to population ratio |
| $R$             | 1.0125           | Gross nominal interest rate (quarterly) |
| $R^{real}$      | 1.0075           | Gross real interest rate (quarterly) |
| $\sigma_{\lambda}$ | 0.73          | Marginal cost (inverse markup) |
| $\gamma$        | 0.036            | Capacity utilization cost parameter |
| $Y$             | 0.73             | Gross output |
| $\phi/Y$        | 0.36             | Fixed cost to gross output ratio |
| $\sigma_{\mu}$  | 0.66             | Level parameter in matching function |
| $f$             | 0.63             | Job finding rate |
| $\vartheta$     | 0.885            | Marginal revenue of wholesaler |
| $x$             | 0.1              | Hiring rate |
| $J$             | 0.06             | Value of firm |
| $V$             | 263.3            | Value of work |
| $U$             | 262.4            | Value of unemployment |
| $N$             | 263.0            | Value of not being in the labor force |
| $\nu$           | 0.14             | Vacancy rate |
| $e$             | 0.05             | Probability of leaving nonparticipation |
| $\omega$        | 0.47             | Home consumption weight in utility |
| $C^H$           | 0.32             | Home consumption |
| $w$             | 0.88             | Real wage |
| $\gamma/(\vartheta/M)$ | 0.63        | Counteroffer costs as share of daily revenue |
Seventh, the posterior mean of $\theta$ which governs the responsiveness of the elements of $\Omega_t$ to technology shocks, is small (0.11). So, variables like government purchases and unemployment benefits are quite unresponsive in the short-run to technology shocks.

Eighth, the posterior means of the parameters governing monetary policy are similar to those reported in the literature (see for example Justiniano, Primiceri, and Tambalotti 2010).

Ninth, we turn to the parameters of the unobserved components representation of the neutral technology shock. According to the posterior mean, the standard deviation of the shock to the transient component is roughly five times the standard deviation of the permanent component. So, according to the posterior mean, most of the fluctuations (at least, at a short horizon) are due to the transitory component of neutral technology. The permanent component of neutral technology has an autocorrelation of roughly 0.75, so that a minus one percent shock to the permanent component eventually drives the level of technology down by about 4 percent. The temporary component is also fairly highly autocorrelated.

Many authors conclude that the growth rate of neutral technology follows roughly a random walk (see, for example, Prescott 1986). Our model is consistent with this view. We find that the first order autocorrelation of $\ln(z_t/z_{t-1})$ in our model is 0.01, which is very close to zero. For discussions of how a components representation, in which the components are both highly autocorrelated, can nevertheless generate a process that looks like a random walk, see Christiano and Eichenbaum (1990) and Quah (1990).

Tenth, the posterior mean of the fraction of retailers’ intermediate input bill, $\kappa$, that must be financed by working capital, is equal to 0.56.

Table 4 reports the frequency with which workers transit between the three states that they can be in. The table reports the steady state frequencies implied by the model and the analog statistics calculated from data from the Current Population Survey. Note that we did not use these data statistics when we estimated or calibrated the model.\(^{17}\) Nevertheless, with two minor exceptions, the model does very

\(^{17}\) Our data does include the job-finding rate. However, our impulse response matching procedure only uses the dynamics of that variable and not its level.
well at accounting for those statistics of the data. The exceptions are that the model somewhat understates the frequency of transition from unemployment into unemployment and slightly overstates the frequency of transition from unemployment to out-of-the labor force. Finally, we note that in the data over half of newly employed people are hired from other jobs (see Diamond 2010, 316). Our model is consistent with this fact: in the steady state of the model, roughly 53 percent of newly employed workers in a given quarter come from other jobs.\footnote{We reached this conclusion as follows. Workers starting a new job at the start of period \( t \) come from three states: employment, unemployment and not-in-the labor force. The quantities of these people are \( (1 - \rho) l_{t-1} s_f, f_s u_{t-1} L_{t-1} \) and \( f_s e_t (1 - L_t) \), respectively. We computed these three objects in steady state using the information in Tables 1, 2 and 3. The fraction reported in the text is the ratio of the first number to the sum of all three.}

Overall, we view these findings as additional evidence in support of the notion that our model of the labor market is empirically plausible.

C. Impulse Response Functions

The thin solid lines in Figures 1–3 present the impulse response functions to a monetary policy shock, a neutral technology shock, and an investment-specific technology shock implied by the estimated VAR. The gray areas represent 95 percent probability intervals. The thick solid lines correspond to the impulse response functions of our model evaluated at the posterior mean of the structural parameters. The thin dashed lines correspond to the 95 percent highest probability density interval for the model impulse response functions. Figure 1 shows that the model does a reasonable job at reproducing the estimated effects of an expansionary monetary policy shock, including the hump-shaped rises in real GDP and hours worked, the rise in the labor force participation rate, and the muted response of inflation. Notice that real wages respond by much less than hours worked to a monetary policy shock. Even though the maximal rise in hours worked is roughly 0.15 percent, the maximal rise in real wages is only 0.04 percent. Significantly, the model accounts for the hump-shaped fall in the unemployment rate as well as the rise in the job-finding rate and vacancies that occur after an expansionary monetary policy shock. The model does understate the rise in the capacity utilization rate. The sharp rise of capacity utilization in the estimated VAR may reflect that our data on the capacity utilization rate pertains to the manufacturing sector, which probably overstates the average response across all sectors in the economy.

From Figure 2 we see that the model does a good job of accounting for the estimated effects of a negative innovation, \( \eta_t \), to neutral technology (see equation (1.32)). Note that the model is able to account for the initial fall and subsequent persistent rise in the unemployment rate. The model also accounts for the initial rise and subsequent fall in vacancies and the job finding rate after a negative shock to neutral technology. The model is consistent with the relatively small response of the labor force participation rate to a neutral technology shock.

Turning to the response of inflation after a negative neutral technology shock, note that our VAR implies that the maximal response occurs in the period of the
shock. Our model has no problem reproducing this observation. See CTW for intuition.

Figure 3 reports the VAR-based estimates of the responses to an investment-specific technology shock. The figure also displays the responses to $\varepsilon_\Psi$, implied by our model evaluated at the posterior mean of the parameters. Note that in all cases the model impulses lie in the 95 percent probability interval of the VAR-based impulse responses.

Viewed as a whole, the results of this section provide evidence that our model does well at accounting for the cyclical properties of key labor market and other macro variables in the pre-2008 period.

IV. Modeling the Great Recession

In this section we provide a quantitative characterization of the Great Recession. We suppose that the economy was buffeted by a sequence of shocks that began in...
2008:III. Using simple projection methods, we estimate how the economy would have evolved in the absence of those shocks. The difference between how the economy would have evolved and how it did evolve is what we define as the Great Recession. We then extend our modeling framework to incorporate four candidate shocks that in principle could have caused or impacted economic activity in the Great Recession. In addition, we provide an interpretation of monetary policy during the Great Recession, allowing for a binding ZLB and forward guidance. Finally, we discuss our strategy for stochastically simulating our model.

A. Characterizing the Great Recession

The solid lines in Figure 4 display the behavior of key macroeconomic variables since 2001 which corresponds to the start of the recession prior to the Great Recession. Our sample ends in 2013:II. Note that we include two measures of the spread between the corporate-borrowing rate and the interest rate paid by the US government. Given the importance of these spreads we display their behavior from 1985:I to provide a better perspective on their evolution. Our benchmark spread measure corresponds to the one constructed in Gilchrist and Zakrajbek (2012)—henceforth, GZ. These data
are displayed in panel M of Figure 4. Panel N of Figure 4 displays the Moody’s seasoned Baa corporate bond yield relative to the yield on the ten-year Treasury bond.

A number of features in Figure 4 are worth noting. First, there was a large drop in per capita GDP. While some growth began in late 2009, per capita GDP has still not returned to its pre-crisis level as of the end of our sample. Second, there was a very substantial decline in consumption and investment. While the latter showed strong growth since late 2009, it has not yet surpassed its pre-crisis peak in per capita terms. Strikingly, although per capita consumption initially grew starting in late 2009, it stopped growing around the middle of 2012. The stop of consumption growth is mirrored by a slowdown in the growth rate of GDP and investment at around the same time. Interestingly, this time period coincides with the events surrounding the debt ceiling crisis and the sequester. For example, in Spring 2012, Federal Reserve Chairman Bernanke warned lawmakers of a “massive fiscal cliff” involving year-end tax increases and spending cuts.\textsuperscript{20}

\textsuperscript{20} According to the Huffington Post (http://www.huffingtonpost.com/2012/12/27/fiscal-cliff-2013_n_2372034.html) in Autumn of 2012, many economists warned that if left unaddressed, concerns about the “fiscal cliff,” could trigger a recession.
Third, vacancies dropped sharply in late 2008 and then rebounded almost to their prerecession levels. At the same time, unemployment rose sharply, but then only fell modestly. Kocherlakota (2010) interprets these observations as implying that firms had positions to fill, but the unemployed workers were simply not suitable. This explanation is often referred to as the mismatch hypothesis. Davis, Faberman, and Haltiwanger (2012) provide a different interpretation of these observations. In their view, what matters for filling jobs is the intensity of firms’ recruiting efforts, not vacancies per se. They argue that the intensity with which firms recruited workers after 2009 did not rebound in the same way that vacancies did. Perhaps surprisingly, our model can account for the joint behavior of unemployment and vacancies, even though the forces stressed by Kocherlakota (2010) and Davis, Faberman, and Haltiwanger (2012) are absent from our framework.
Fourth, we note that despite the steep drop in GDP, inflation dropped by only about 1 to 1.5 percentage points. Authors like Hall (2011) argue that this joint observation is particularly challenging for NK models.

Finally, note that both measures of the corporate bond spread rose sharply in the middle of 2008. While the spreads have declined from their peak levels, both are still elevated relative to their historical averages. Also, both spreads are higher than their values at the end of 2007 and substantially above their values in the ten years prior to the Great Recession. Interestingly, the Moody’s spread measure has not come down quite as much as the GZ spread. The persistence in the rise of the corporate spread plays an important role in our analysis below.

**Target Gap Ranges.**—To assess how the economy would have evolved absent the large shocks associated with the Great Recession, we adopt a simple and transparent procedure. For each variable, we fit a linear trend from date $x$ to 2008:II, where $x \in \{1985:1, 2003:1\}$. To characterize what the data would have looked like absent the shocks that caused the financial crisis and Great Recession, we extrapolate the trend line for each variable. According to our model, all the nonstationary variables in the analysis are difference-stationary. Our linear extrapolation procedure implicitly assumes that the shocks in the estimation period were small relative to the drift terms in the time series. Given this assumption, our extrapolation procedure approximately identifies how the data would have evolved, absent shocks after 2008:II.

For each value of $x$, we calculate, at various horizons, the difference between the projected value of each variable and its actual value. We refer to this difference as the target gaps. If we knew the “correct” value of $x$, these target gaps would represent our estimates of the economic effects of the shocks that hit the economy in 2008:III and later. In effect, these target gaps would be the objects that we are trying to explain. But we do not know the correct value of $x$. So, we construct the min-max range for the target gaps, using all the values of $x \in \{1985:1, 2003:1\}$. The min-max ranges of the target gaps for all the variables correspond to the gray intervals displayed in Figures 7 and 8. The objective is to assess whether, given plausible shocks, the model-implied values of the endogenous variables in the post 2008:II period are within the target gap ranges depicted in Figures 7 and 8.

Some features of these target gap ranges are worth emphasizing. First, any projection of the values for the labor force and employment after 2008:II are controversial because of ongoing demographic changes in the US population. Our procedure attributes a mean target gap decline by 2013:II of roughly 2.0 percentage points due to cyclical factors. Projections for the labor force to population ratio published by the Bureau of Labor Statistics in November 2007 suggest that the cyclical component in the decline in this ratio was roughly 2 percentage points. In contrast,

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21 In a previous draft of this paper we only considered $x$ equal to 2001:I. Hall (forthcoming) uses a procedure identical to ours corresponding to a value of $x$ equal to 1990:I.

22 There are of course many alternative procedures for projecting the behavior of the economy. For example, we could use separate ARMA time series models for each of the variables or we could use multivariate methods including the VAR estimated with pre-2008:I data. A challenge for a multivariate approach is the nonlinearity associated with the ZLB. Still, it would be interesting to pursue alternative projection approaches in the future.

23 See Erceg and Levin (2013), figure 1.
Reifschneider, Wascher, and Wilcox (2013) and Sullivan (2013) estimate that the cyclical component of the decline in the labor force to population ratio is equal to 1 percentage point and 0.75 percentage points, respectively. The range of the target gaps for the labor force participation rate displayed in Figure 8 indicates considerable uncertainty about the precise value to be targeted.

Second, according to Figure 8, the mean fall of the employment to population ratio is about 4 percent. According to Figure 8, actual employment to population fell by about 5 percent. So, our procedure ascribes 20 percent of the actual fall of employment to noncyclical factors. Krugman (2014) and Shimer (2014) argue that one-third of the fall in the employment to population ratio is due to noncyclical factors. So, like us, they ascribe a relatively small portion of the fall to noncyclical factors. In contrast, Kapon and Tracy (2014) argue that the cyclical component of the decline was smaller.

B. The Shocks Driving the Great Recession

We suppose that the Great Recession was triggered by four shocks. Two of these shocks are wedges which capture in a reduced form way frictions which are widely viewed as having been important during the Great Recession. The other sources of shocks that we allow for are government consumption and technology.

The Consumption Wedge.—The first shock that we consider is a shock to households’ preferences for safe and/or liquid assets. We capture this shock by introducing a perturbation, $\Delta^b_t$, to agents’ intertemporal Euler equation associated with saving via risk-free bonds. The object, $\Delta^b_t$, is the consumption wedge we discussed in the introduction. The Euler equation associated with the nominal risk-free bond is given by:

\[(4.1) \quad 1 = (1 + \Delta^b_t) E_t m_{t+1} R_t / \pi_{t+1}. \]

See Fisher (2014) for a discussion of how a positive realization of $\Delta^b_t$ can, to a first-order approximation, be interpreted as reflecting an increase in the demand for risk-free bonds.\(^{25}\)

We obtain an empirical measure of $\Delta^b_t$ as follows. As it turns out ex post measures of the time series on $m_{t+1}$ implied by the estimated version of our model do not display substantial variation over time. Hence, we set $m_{t+1} = \beta$. Ignoring covariance terms, equation (4.1) can then be written as,

\[(4.2) \quad (1 + \Delta^b_t) = E_t \pi_{t+1} / (\beta R_t).\]


\(^{25}\)The shock is also similar to the “flight-to-quality” shock found to play a substantial role in the start of the Great Depression in Christiano, Motto, and Rostagno (2003).
In conjunction with Federal Funds rate data on \( R_t \) and measuring \( E_t \pi_{t+1} \) using the one-quarter ahead core CPI-inflation forecasts from the Survey of Professional Forecasters, (4.2) yields a time series on \( \Delta_t^b \). We then apply the procedure discussed above to compute target gaps to obtain measures of \( \Delta_t^b \). The results are displayed in panel B of Figure 7. It is clear that there is substantial uncertainty about the target gaps for \( \Delta_t^b \) stemming from uncertainty about the estimation period for the linear trend.

We assume that agents forecast \( \Delta_t^b \) using the following AR(2) process:

\[
\Delta_t^b = 1.5 \Delta_{t-1}^b - 0.56 \Delta_{t-2}^b + \varepsilon_t^b,
\]

where \( \varepsilon_t^b \) is a mean zero, unit variance i.i.d shock. The steady state value of \( \Delta_t^b \) is zero. This process implies that at each point in time, agents forecast a persistent rise in \( \Delta_t^b \) consistent with the mean target gap.

We assume that the actual sequence of values of \( \Delta_t^b \) that occurred after 2008:I is depicted by the solid line with circles in panel B of Figure 7. This sequence is identical to the mean target gap until 2010:IV and is constant at the 2010:IV value thereafter. Notice that the actual mean of the target gap rises in 2011:I. If we assume a target gap consistent with that rise, then the model would generate a pronounced and counterfactual contraction beginning in 2011:I. Our assumption that \( \Delta_t^b \) is constant amounts to an informal estimate of that shock using the structure of the model.

The Financial Wedge.—The second shock that we consider is a shock to agents’ intertemporal Euler equation for capital accumulation:

\[
1 = \left( 1 - \Delta_t^k \right) E_t m_{t+1} R_{t+1}^k / \pi_{t+1},
\]

where \( \Delta_t^k \) is the financial wedge. We use the GZ spread data to construct an empirical measure of \( \Delta_t^k \). Taking the ratio of (4.1) and (4.3), ignoring the expectation operator, and rearranging we obtain:

\[
1 - \Delta_t^k = \frac{(1 + \Delta_t^b) R_t}{R_{t+1}^k} \simeq (1 + \Delta_t^b) (1 - \Delta_t^k),
\]

where \( \Delta_t^k \) denotes \( R_t^k - R_t \). The time period in our model is quarterly, but the average duration of the bonds in GZ’s data is about seven years. We suppose that the \( \Delta_t^k \)’s are related to the GZ spread as follows:

\[
\Gamma_t = E \left[ \frac{\Delta_t^k + \Delta_{t+1}^k + \cdots + \Delta_{t+27}^k}{7} \big| \Omega_t \right],
\]

\(^{26}\)Nonlinear versions of standard Kalman smoothing methods could be used instead to estimate the sequences of all the exogenous shocks in the post-2008:II data. In practice, this approach is computationally challenging and we defer it to future work. See e.g., Gust, Lopez-Salido, and Smith (2012) who estimate a nonlinear DSGE model subject to an occasionally binding ZLB constraint.
where $\Gamma_t$ denotes the GZ spread minus the projection of that spread as of 2008:II. Also, $\Omega_t$ denotes the information available to agents at time $t$. In (4.4) we sum over $\Delta^k_{t+j}$ for $j = 0, ..., 27$ because $\Delta^k_t$ is a tax on the one quarter return to capital while $\Gamma_t$ applies to $t + j$, $j = 0, 1, ..., 27$ (i.e., seven years). Also, we divide the sum in (4.4) by 7 to take into account that $\Delta^k_t$ is measured in quarterly decimal terms while our empirical measure of $\Gamma_t$ is measured in annual decimal terms.

We assume that agents forecast $\Gamma_t$ using a mean zero, first order autoregressive representation (AR(1)), with autoregressive coefficient, $\rho_\Gamma = 0.5$. This low value of $\rho_\Gamma$ captures the idea that agents thought the sharp increase in the financial wedge was transitory in nature. To solve their problem, agents actually work with the $\Delta^k_t$ s. But, for any sequence, $\Gamma_t$, $E_t \Gamma_{t+j}$, $j = 1, 2, ..., 7$, they can compute a sequence, $\Delta^k_{t+j}$, $E_t \Delta^k_{t+j}$, $j = 1, 2, 3, ..., 7$ that satisfies (4.4). We assume that the actual sequence of values of $\Gamma_t$ that occurred after 2008:I is depicted by the solid line with circles in panel A of Figure 7. If we assume that $\Gamma_t$ is equal to the mean value of the target gap depicted in Figure 7, then the model counterfactually implies that the recovery from the Great Recession would have began around mid-2011. Our assumptions about $\Gamma_t$ amount to an informal estimate of that shock using the structure of the model. It is important to recall that the Moody’s spread displays somewhat more persistence than the GZ spread. In the Appendix we derive the target gaps for the financial wedge using the Moody’s spread. We show that those target gaps are quantitatively more important than the bankruptcy interpretation, especially in the recent crisis. While we are sympathetic to their view, our analysis does not require us to take a stand on the relative plausibility of the two interpretations of the financial wedge. The only distinction between the two interpretations lies in their impact on the resource constraint. The quantitative magnitude of this impact is likely to be very small.

Recall that firms finance a fraction, $\kappa$, of the intermediate input in advance of revenues (see (1.16)). In contrast to the existing DSGE literature, we allow for a

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27 In performing this computation, we impose that $E_t \Gamma_{t+j} \rightarrow 0$ and $E_t \Delta^k_{t+j} \rightarrow 0$ as $j \rightarrow \infty$.

28 For a formalization of this perspective, see Christiano and Davis (2006).

29 See Ilut and Schneider (2014) for a third interpretation of $\Delta^k_t$, which relies on ambiguity aversion.
risk working capital channel in the sense that the financial wedge also applies to working capital loans. Specifically, we replace (1.16) with

$$P_t = \varepsilon R_t \left( 1 + \hat{\Delta}_t^k \right) + (1 - \varepsilon).$$

where $\varepsilon = 0.56$, as estimated (see Table 2). The risky working capital channel captures in a reduced form way the frictions modeled in e.g., Bigio (2013). As a practical matter we think of our measure of $\hat{\Delta}_t^k$ as a noisy signal of the actual wedge in the market for working capital. In practice the latter wedge is given by

$$\hat{\Delta}_t^k = 0.33\Delta_t^k.$$

We found that if the financial and working capital wedges were equal the model generated a counterfactually high level of inflation during the Great Recession. Our assumption that the working capital wedge is about one-third as large as the financial wedge amounts to an informal estimate of that shock using the structure of the model.

Total Factor Productivity (TFP) Shocks.—We now turn to a discussion of TFP. Various measures produced by the Bureau of Labor Statistics (BLS) are reported in panel A of Figure 5. Each measure is the log of value-added minus the log of capital
and labor services weighted by their shares in the income generated from producing the measure of value-added. In each case, we report a linear trend line fitted to the data from 2001:I through 2008:II. The start date corresponds to the start of the recession prior to the Great Recession. We then project the numbers forward after 2008:II. We do the same for three additional measures of TFP in panel B of Figure 5. Two are taken from Fernald (2014) and the third is taken from the Penn World Tables. The bottom panel of Figure 5 displays log TFP minus the post-2008:II projection for log TFP. Note that, with one exception, (i) TFP is below its pre-2008 trend during the Great Recession, and (ii) it remains well below its pre-2008 trend all the way up to the end of our dataset. The exception is Fernald’s (2014) utilization-adjusted TFP measure, which briefly rises above trend in 2009. Features (i) and (ii) of TFP play an important role in our empirical results.

To assess the robustness of (i) and (ii), we redid our calculations using an alternative way of computing the trend lines. Figure 6 reproduces the basic calculations for three of our TFP measures using a linear trend that is constructed using data starting in 1985. While there are some interesting differences across the figures, they all share the two key features, (i) and (ii), discussed above. Specifically, it appears that TFP was persistently low during the Great Recession.

We now explain why we adopt an unobserved components time series representation of . If we assume that agents knew in 2008:III that the fall in TFP would turn out to be so persistent, then our model generates a counterfactual surge in inflation. We infer that agents only gradually became aware of the persistence in the decline of TFP. The notion that it took agents time to realize that the drop in TFP was highly persistent is consistent with other evidence. For example, Figure 4 in Swanson and Williams (forthcoming) shows that professional forecasts consistently underestimated how long it would take the economy to emerge from the ZLB.

The previous considerations are the reason that we work with the unobserved components representation for in (1.27). In addition, these considerations underlie our prior that the standard deviation of the transitory shock is substantially larger than the standard deviation of the permanent shock. We imposed this prior in estimating the model on pre-2008 data.

At this point, it is worth repeating the observation made in Section IIIB that we have not assumed anything particularly exotic about technology growth. As noted above, our model implies that the growth rate of technology is roughly a random walk, in accordance with a long tradition in business cycle theory. What our analysis in effect exploits is that a process that is as simple as a random walk can have components that are very different from a random walk.

Our analysis involves simulating the response of the model to shocks. So, we must compute a sequence of realized values of . Unlike government spending

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30 The BLS measure is only available at an annual frequency. We interpolate the annual data to a quarterly frequency using a standard interpolation routine described in Boot, Feibes, and Lisman (1967).

31 To assess the robustness of the inference for TFP we adopted the following exercise. First, we computed the target gap for TFP using the procedure discussed in the main text. Second, we measured TFP using the BLS time series for the private business sector. Then the solid line with dots in Figure 5 corresponding to the model implied measure of TFP lies well within the min-max range for TFP target gap.
and interest rate spreads, we do not directly observe $\ln(z_t)$. In our model log TFP does not coincide with $\ln(z_t)$. The principle reason for this is the presence of the fixed cost in production in our model. But, the behavior of model-implied TFP is sensitive to $\ln(z_t)$.

To our initial surprise, the behavior of inflation is also very sensitive to $\ln(z_t)$ So, from this perspective both inflation and TFP contain substantial information about $\ln(z_t)$. These observations led us to choose a sequence of realized values for $\ln(z_t)$ that, conditional on the other shocks, allows the model to account reasonably well for inflation and log TFP.
The bottom panel of Figure 5 reports the measure of TFP for our model, computed using a close variant of the Bureau of Labor Statistics’ procedure. The black line with dots displays the model’s simulated value of TFP relative to trend (how we detrend and solve the model is discussed below). Note that model TFP lies within the range of empirical measures reported in Figure 5. The bottom panel of Figure 6 shows that we obtain the same result when we detrend our three empirical measures of TFP using a trend that begins in 1985.

Nonlinear versions of the standard Kalman smoothing methods could be used instead to estimate the sequence of $\ln(z_t)$ in the post-2008:II data. In practice, this approach is computationally challenging and we defer it to future work. For convenience, we assume there was a one-time shock to $\ln(z_t)$ in 2008:III. For the reasons given above, we assume that the shock was to the permanent component of $\ln(z_t)$, i.e., $\varepsilon^P_t$. We selected a value of $-0.4$ percent for that shock so that, in conjunction with our other assumptions, the model does a reasonable job of accounting for post-2008:II inflation and log TFP. This one-time shock leads to a persistent move in $\ln(z_t)$, which eventually puts $z_t$ roughly $1.6$ percent below the level it would have been in the absence of the shock. The shock to $\varepsilon^P_t$ also leads to a sequence of one-step-ahead forecast errors for agents, via (1.33). Our specification of $\ln(z_t)$ captures features (i) and (ii) of the TFP data that were discussed above.

**Government Consumption Shocks.**—Next we consider the shock to government consumption. The variable $\eta^g_t$ defined in (1.37) is computed using the simulated path of neutral technology, $\ln(z_t)$ (see (1.38) and (1.39)). Then, $g_t$ is measured by dividing actual government consumption in Figure 4, by $\eta^g_t$. Agents forecast the period $t$ value of $\eta^g_t$ using current and past realizations of the technology shocks. We assume that agents forecast $g_t$ by using the following AR(2) process:

$$\ln(g_t/g) = 1.6 \ln(g_{t-1}/g) - 0.64 \ln(g_{t-2}/g) + \varepsilon^G_t,$$

where $\varepsilon^G_t$ is a mean zero, unit variance i.i.d shock. We choose the roots for the AR(2) process such that the first and second order autocorrelations of $\Delta \ln G_t$ in our estimated model are close to the data for the sample 1951:I to 2008:II. We assume that the realized values of the government consumption target gap, i.e., the shocks to government consumption, equal the mean data values depicted in panel D of Figure 7.

**C. Monetary Policy**

We make two key assumptions about monetary policy during the post 2008:III period. We assume that the Fed initially followed a version of the Taylor rule that respects the ZLB on the nominal interest rate. We assume that there was an

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32 Our measure of TFP is the ratio of GDP (i.e., $C + I + G$) to capital and labor services, each raised to a power that corresponds to their steady state share of total income.

33 For simplicity, in our calculations we assume that the investment-specific technology shock remains on its steady state growth path after 2008.
unanticipated regime change in 2011:III, when the Fed switched to a policy of forward guidance.

Taylor Rule.—We now define our version of the Taylor rule that takes the non-negativity constraint on the nominal interest rate into account. Let $Z_t$ denote a gross “shadow” nominal rate of interest, which satisfies the following Taylor-style monetary policy rule:

\[(4.6) \quad \ln(Z_t) = \ln(R) + r_\pi \ln(\pi_t^A/\pi^A) + 0.25r_{\Delta y} \ln(O_t/(O_{t-4}^{\Delta y})).\]

The actual policy rate, $R_t$, is determined as follows:

\[(4.7) \quad \ln(R_t) = \max\{\ln(R/a), \rho_R \ln(Z_{t-1}) + (1 - \rho_R) \ln(Z_t)\}.\]

In 2008:II, the federal funds rate was close to two percent (see Figure 4). Consequently, because of the ZLB, the federal funds rate could only fall by at most two percentage points. To capture this in our model, we set the scalar $a$ to 1.004825.

Absent the ZLB constraint, the policy rule given by (4.6)–(4.7) coincides with (1.36), the policy rule that we estimated using pre-2008 data.
Forward Guidance.—We interpret forward guidance as a monetary policy that commits to keeping the nominal interest rate at zero until there is substantial improvement in the state of the economy. Initially, in 2011:III the Fed did not quantify what they meant by “substantial improvement.” Instead, they reported how long they thought it would take until economic conditions would have improved substantially. In December 2012 the Fed became more explicit about what the state of the economy would have to be for them to consider leaving the ZLB. In particular, the Fed said that it would keep the interest rate at zero as long as inflation remains below 2.5 percent and unemployment remains above 6.5 percent. They did not commit to any particular action in case one or both of the thresholds are breached.

In modeling forward guidance we begin with the period, 2011:III–2012:IV. We do not know what the Fed’s thresholds were during this period. But, we do know that in 2011:III, the Fed announced that it expected the interest rate to remain at zero until mid-2013 (see Campbell et al., 2012). According to Swanson and Williams (forthcoming), when the Fed made its announcement, the number of quarters that professional forecasters expected the interest rate to remain at zero jumped from four quarters to seven or more quarters. We assume that forecasters believed the Fed’s announcement and thought that the nominal interest rate would be zero for about eight quarters. Interestingly, Swanson and Williams (forthcoming) also report that forecasters continued to expect the interest rate to remain at zero for seven or more quarters in each month through January 2013. Clearly, forecasters were repeatedly revising upwards their expectation of how long the ZLB episode would last. To capture this scenario in a parsimonious way we assume that in each quarter, beginning in 2011:III and ending in 2012:IV, agents believed the ZLB would remain in force for another eight quarters. Thereafter, we suppose that they expected the Fed to revert back to the Taylor rule, (4.6) and (4.7).34

Beginning in 2013:I, we suppose that agents believed the Fed switched to an explicit threshold rule. Specifically, we assume that agents thought the Fed would keep the Federal Funds rate close to zero until either the unemployment rate fell below 6.5 percent or inflation rose above 2.5 percent. We assume that as soon as these thresholds are met, the Fed switches back to our estimated Taylor rule, (4.6) and (4.7). The latter feature of our rule is an approximation because, as noted above, the Fed did not announce what it would do when the thresholds were met.

D. Solving the Model

Our specification of monetary policy includes a nonnegativity constraint on the nominal interest rate, as well as regime switching. A subset of the latter depends on realizations of endogenous variables. We search for a solution to our model in the space of sequences.35 The solution satisfies the equilibrium conditions which take the form of a set of stochastic difference equations that are restricted by initial

34 Our model of monetary policy is clearly an approximation. For example, it is possible that in our stochastic simulations the Fed’s actual thresholds are breached before eight quarters. Since we do not know what those thresholds were, we do not see a way to substantially improve our approach. Later, in December 2013, the Fed did announce thresholds, but there is no reason to believe that those were their thresholds in the earlier period.

35 Our procedure is related to the one proposed in Fair and Taylor (1983).
and end-point conditions. Our solution strategy makes one approximation: certainty equivalence. That is, wherever an expression like $E_x f(x_{t+j})$ is encountered, we replace it by $f(E_x x_{t+j})$, for $j > 0$.

Let $y_t$ denote the vector of shocks operating in the post-2008:II period:

$$y_t = (\Delta_t^b \ \Delta_t^k \ \varrho_t)'.$$

The law of motion and agents’ information sets about $y_t$ have been discussed above.

Let $\varrho_t$ denote the $N \times 1$ vector of period $t$ endogenous variables, appropriately scaled to account for steady growth. We express the equilibrium conditions of the model as follows:

$$E[f(\varrho_{t+1}, \varrho_t, \varrho_{t-1}, y_t, y_{t+1}) | \Omega_t] = 0. \tag{4.8}$$

Here, the information set is given by

$$\Omega_t = \{ \varrho_{t-1-j}, y_{t-j}, j \geq 0 \}.$$

Our solution strategy proceeds as follows. As discussed above, we fix a sequence of values for $y_t$ for the periods after 2008:II. We suppose that at date $t$ agents observe $y_{t-s}, s \geq 0$ for each $t$ after 2008:II. At each such date $t$, they compute forecasts, $y_{t+1}^f, y_{t+2}^f, y_{t+2}^f, \ldots$, of the future values of $y_t$. It is convenient to use the notation $y_t^f \equiv y_t$.

We adopt an analogous notation for $\varrho_t$. In particular, denote the expected value of $\varrho_{t+j}$ formed at time $t$ by $\varrho_{t+j}^f$, where $\varrho_t^f \equiv \varrho_t$. The equilibrium value of $\varrho_t$ is the first element in the sequence, $\varrho_{t+j}^f, j \geq 0$. To compute this sequence we require $y_t^f, j \geq 0$, and $\varrho_{t-1}$. For $t$ greater than 2008:III we set $\varrho_{t-1} = \varrho_{t-1}^f$. For $t$ corresponding to 2008:III, we set $\varrho_{t-1}$ to its nonstochastic steady state value. We now discuss how we computed $\varrho_{t+j}^f, j \geq 0$. We do so by solving the equilibrium conditions and imposing certainty equivalence. In particular, $\varrho_t^f$ must satisfy:

$$E[f(\varrho_{t+1}, \varrho_t, \varrho_{t-1}, y_t, y_{t+1}) | \Omega_t]$$

$$\simeq f(\varrho_{t+1}^f, \varrho_{t-1}^f, y_t^f, y_{t+1}^f) = 0.$$

Evidently, to solve for $\varrho_{t+j}^f$ requires $\varrho_{t+j-1}^f$. Relation (4.8) implies:

$$E[f(\varrho_{t+2}, \varrho_{t+1}, \varrho_t, y_t, y_{t+2}) | \Omega_t]$$

$$\simeq f(\varrho_{t+2}^f, \varrho_{t+1}^f, \varrho_t^f, y_t^f, y_{t+2}^f) = 0.$$

Proceeding in this way, we obtain a sequence of equilibrium conditions involving $\varrho_{t+j}, j \geq 0$. Solving for this sequence requires a terminal condition. We obtain this condition by imposing that $\varrho_{t+j}$ converges to the nonstochastic steady state value of
With this procedure it is straightforward to implement our assumptions about monetary policy.

V. The Great Recession: Empirical Results

In this section we analyze the behavior of the economy from 2008:III to the end of our sample, 2013:II. First, we investigate how well our model accounts for the data. Second, we use our model to assess which shocks account for the Great Recession. In addition, we also investigate the role of the risky working capital channel, the ZLB, forward guidance, and the labor force participation rate.

A. The Model’s Implications for the Great Recession

Figure 8 displays our empirical characterization of the Great Recession, i.e., the difference between how the economy would have evolved absent the post-2008:II shocks and how it did evolve. In addition, we display the relevant model analogs. For this, we assume that the economy would have been on its steady state growth path in the absence of the post-2008:II shocks. This is an approximation that simplifies...
the analysis and is arguably justified by the fact that the volatility of the economy is much greater after 2008 than it was before. The model analog to our empirical characterization of the Great Recession is the log difference between the variables on the steady state growth path and their response to the post-2008:II shocks.

Figure 8 indicates that the model does quite well at accounting for the behavior of our 11 endogenous variables during the Great Recession. Notice in particular that the model is roughly consistent with the modest decline in real wages despite the absence of nominal rigidities in wage setting. Also, notice that the model accounts reasonably well for the average level of inflation despite the fact that our model incorporates only a moderate degree of price stickiness: firms change prices on average once a year. In addition, the model also accounts well for the key labor market variables: labor force participation, employment, unemployment, vacancies and the job finding rate.

Figure 9 provides another way to assess the model’s implications for vacancies and unemployment. There, we report a scatter plot with vacancies on the vertical axis and unemployment on the horizontal axis. The model variables in Figure 9 are taken from panels D and I of Figure 8. The data correspond to the target gap in the gray area of Figure 8 generated by using a linear trend starting in 2001, the start of the recession prior to the Great Recession. Although the variables are expressed in deviations from trend, the resulting Beveridge curve has the same key features as those in the raw data (see, for example, Diamond 2010, Figure 4). In particular, notice how actual vacancies fall and unemployment rises from late 2008 to late 2009. This downward relationship is referred to as the Beveridge curve. After 2009,
vacancies rise but unemployment falls by less than one would have predicted based on the Beveridge curve that existed before 2009. That is, it appears that after 2009 there was a shift up in the Beveridge curve. This shift is often interpreted as reflecting a deterioration in match efficiency, captured in a simple environment like ours by a fall in the parameter governing productivity in the matching function (see $\sigma_m$ in (1.41)). This interpretation reflects a view that models like ours imply a stable downward relationship between vacancies and unemployment, which can only be perturbed by a change in match efficiency. However, this downward relationship is in practice derived as a steady state property of models and is in fact not appropriate for interpreting quarterly data. To explain this, we consider a simple example.\footnote{We include this example for completeness. It can be found in other places, for example, Yashiv (2008).}

Suppose that the matching function is given by:

$$h_t = \sigma_{m,t} V_t^\alpha U_t^{1-\alpha}, \quad 0 < \alpha < 1,$$

where $h_t$, $V_t$ and $U_t$ denote hires, vacancies and unemployment, respectively. Also, $\sigma_{m,t}$ denotes a productivity parameter that can potentially capture variations in match efficiency. Dividing the matching function by the number of unemployed, we obtain the job finding rate $f_t \equiv h_t/U_t$, so that:

$$f_t = \sigma_{m,t} (V_t/U_t)^\alpha.$$

The simplest search and matching model assumes that the labor force is constant so that:

$$1 = l_t + U_t,$$

where $l_t$ denotes employment and the labor force is assumed to be of size unity. The change in the number of people unemployed is given by:

$$U_{t+1} - U_t = (1 - \rho) l_t - f_t U_t,$$

where $(1 - \rho) l_t$ denotes the employed workers that separate into unemployment in period $t$ and $f_t U_t$ is the number of unemployed workers who find jobs. In steady state, $U_{t+1} = U_t$, so that:

$$U_t = (1 - \rho)/(f_t + 1 - \rho).$$

Combining this expression with the definition of the finding rate and solving for $V_t$, we obtain:

$$V_t = \left[ \frac{(1 - \rho)(1 - U_t)}{\sigma_{m,t} U_t^{1-\alpha}} \right]^{1/\alpha}.$$

\footnotesize{(5.1)}
This equation clearly implies (i) a negative relationship between $U_t$ and $V_t$ and (ii) the only way that relationship can shift is with a change in the value of $\sigma_{m,t}$ or in the value of the other matching function parameter, $\alpha$. Results (i) and (ii) are apparently very robust, as they do not require taking a stand on many key relations in the overall economy. In the technical Appendix, we derive a similar result for our model, which also does not depend on most of our model details, such as the costs for arranging meetings between workers and firms, the determination of the value of a job, etc.

While the steady state Beveridge curve described in the previous paragraph may be useful for many purposes, it is misleading for interpreting data from the Great Recession, when the steady state condition, $U_{t+1} - U_t = 0$, is far from being satisfied.

If we don’t impose the steady state condition $U_{t+1} = U_t$, we obtain the following relationship between $V_t$ and $U_t$:

$$V_t = \left(1 - \rho \right) \frac{1 - U_t}{\sigma_{m,t}} \frac{U_{t+1} - U_t}{\sigma_{m,t}}^{1/\alpha}.$$

During severe recessions, the steady state condition, $U_{t+1} = U_t$, will not be satisfied. The variable $U_{t+1} - U_t$ is a large positive number in the downturn of a severe recession, and then becomes negative as the economy recovers. This effect can easily generate what looks like a shift in the “standard” Beveridge curve.

Figure 9 shows that our model accounts for the so-called shift in the Beveridge curve, even though the productivity parameter in our matching function is constant. The only difference between the analysis in Figure 9 and our model’s steady state Beveridge curve is that we do not impose the $U_{t+1} - U_t = 0$ condition. Thus, according to our analysis the data on vacancies and unemployment present no reason to suppose that there has been a deterioration in match efficiency. There may have been such a deterioration to some extent, but it does not seem to be a first order feature of the Great Recession.

Below we show that the most important shock driving the real side of the economy into the Great Recession was the financial wedge. It follows that shocks to that wedge were the key drivers of variations in $U_{t+1} - U_t$ and the apparent shift in the Beveridge curve.

B. The Causes of the Great Recession

Figures 10 through 14 decompose the impact of the different shocks and the risky working capital channel on the economy in the post-2008:III period. We determine

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37 In principle, a change in the separation rate, $1 - \rho$, could also have shifted the Beveridge curve during the Great Recession. This explanation is not consistent with the data because the separation rate fell from an average level of 3.7 percent before the Great Recession to an average of 3.1 percent after 2009. These numbers were calculated using JOLTS data available at the BLS Website.

38 Eichenbaum (forthcoming) shows that the simple model above as summarized by (5.2) also captures the key features of the Beveridge curve over the Great Recession.
the role of a shock by setting that shock to its steady state value and redoing the simulations underlying Figure 8. The resulting decomposition is not additive because of the nonlinearities in the model.

Effects of Neutral Technology.—Figure 10 displays the effect of the neutral technology shock on the post-2008 simulation. For convenience, the solid line reproduces the corresponding solid line in Figure 8. The dashed line displays the behavior of the economy when neutral technology shock is shut down (i.e., $\epsilon_t^p = 0$ in 2008:III). Comparing the solid and dashed lines, we see that the neutral technology slowdown had a significant impact on inflation that arises from its effect on marginal costs. Had it not been for the decline in neutral technology, there would have been substantial deflation, as predicted by very simple NK models that do not allow for a drop in technology during the ZLB period. Also note that according to the model the negative technology shock pushed output, investment, and consumption down. At the same time that shock led to an increase in employment and the labor force. The latter effect reflects that according to our estimated model agents...
perceive technology shocks to be transitory. See CTW for the intuition underlying the result that employment and output move in opposite directions in response to transitory neutral technology shocks.

Effects of Risky Working Capital Channel.—Medium-sized DSGE models typically abstract from the risky working capital channel. A natural question is: how important is that channel in allowing our model to account for the moderate degree of inflation during the Great Recession? To answer that question, we redo the simulation underlying Figure 8, replacing (4.5) with (1.16). The results are displayed in Figure 11. We find that the risky working capital channel plays an important role in allowing the model to account for the moderate decline in inflation that occurred during the Great Recession. In the presence of a risky working capital requirement, a higher interest rate due to a positive financial wedge shock directly raises firms’ marginal cost. Other things equal, this rise leads to inflation. Gilchrist, Schoenle, Sim, and Zakrajišek (2012) provide firm-level evidence consistent with the importance of our risky working capital channel. They find that firms with bad
balance sheets raise prices relative to firms with good balance sheets. From our perspective, firms with bad balance sheets face a very high cost of working capital and therefore, high marginal costs.

While the risky working capital channel has a significant impact on inflation, it has essentially no effect on real quantities. The reason is as follows. Recall that agents forecast the corporate bond spread using an AR(1) process with an AR coefficient of 0.5. So they think that the rise in spreads is very transitory as well as any inflation stemming from this channel. It follows that agents think that change in the real interest rate associated with the change in the corporate spread is not long lasting. But changes in consumption demand depend on the change in the long-term real interest rate. That rate is not much affected by transitory changes in the short-term real interest rate. The previous argument makes clear that our analysis of the effect of the risky working capital channel depends on our assumption that agents expected the ZLB episode to be relatively short-lived (see Swanson and Williams forthcoming for evidence on this point).

Taken together, the negative technology shocks and the risky working capital channel explain the relatively modest disinflation that occurred during the Great Recession. Essentially they exerted countervailing pressure on the disinflationary forces that were operative during the Great Recession.

Effects of Financial and Consumption Wedges.—Figures 12 and 13 report the effects of the financial and consumption wedges, respectively. The financial wedge is clearly the most important shock in terms of driving the economy into the ZLB and in terms of accounting for the drop in economic activity and inflation after 2008. The fact that the nominal interest rate remains at zero after 2011 when there is no financial wedge reflects our specification of monetary policy. Notice that the model attributes the substantial drop in the labor force participation rate mostly due to the financial wedge. The consumption wedge drives down the labor force participation rate to a lesser extent than the financial wedge. The rationale for the effects of the wedges is straightforward. Both wedges lead to deteriorations in labor market conditions: drops in the job vacancy and finding rates and in the real wage. We do not think these wedge shocks were important in the pre-2008 period. In this way, the model is consistent with the fact that labor force participation rates are not very cyclical during normal recessions, while being very cyclical during the Great Recession.40

A natural question is: what features of our model account for the persistence of the Great Recession? The answer is: the persistence of the financial and consumption wedges. Suppose we set the wedges to zero starting in 2010:I, then the unemployment rate would have been 5.5 percent by early 2012. Most of this effect stems from setting the financial wedge to zero.

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39 If we increase the AR coefficient that agents use to forecast the corporate bond spread, then we obtain larger effects of the risky working capital channel on real quantities in Figure 11.

40 See Erceg and Levin (2013), for an analysis which reaches a qualitatively similar conclusion using a small scale, calibrated model.
**Effects of Government Consumption and the ZLB.**—We now turn to Figure 14, which analyzes the role of government consumption in the Great Recession. Government consumption passes through two phases (see Figure 7). The first phase corresponds to the expansion associated with the American Recovery and Reinvestment Act of 2009. The second phase involves a contraction that began at the start of 2011. The first phase involves a maximum rise of 2.75 percent in government consumption (i.e., 0.55 percent relative to steady state GDP) and a maximum rise of about 0.9 percent in GDP. This implies a maximum government consumption multiplier of 0.9/0.55 or 1.6.\(^1\) In the second phase the decline in government spending is much more substantial, falling by a maximum of about 10 percent, or 2 percent relative to steady state GDP. At the same time, the resulting drop in GDP is about 1.8 percent (see Figure 14). So, in the second phase, the government spending

\(^1\) Christiano, Eichenbaum, and Trabandt (2014) provide some evidence that our conclusions for the multiplier are robust to allowing for distortionary labor income taxes and no lump-sum taxes.
multiplier is only 1.8/2 or 0.9. In light of this result, it is difficult to attribute the long duration of the Great Recession to the recent decline in government consumption.

The second phase findings may at first seem inconsistent with existing analyses, which suggest that the government consumption multiplier may be very large in the ZLB. Indeed, Christiano, Eichenbaum, and Rebelo (2011) show that a rise in government consumption that is expected to not extend beyond the ZLB has a large multiplier effect. But, they also show that a rise in government consumption that is expected to extend beyond the ZLB has a relatively small multiplier effect. The intuition for this is straightforward. An increase in spending after the ZLB ceases to bind has no direct impact on spending in the ZLB. But, it has a negative impact on household consumption in the ZLB because of the negative wealth effects associated with the (lump-sum) taxes required to finance the increase in government spending. A feature of our simulations is that the increase in government consumption in the first phase is never expected by agents to persist beyond the ZLB. In the second phase the decrease in government consumption is expected to persist beyond the end of the ZLB.
Christiano, Eichenbaum, and Rebelo (2011) also argue that the size of the multiplier depends on how binding the ZLB is. By binding we mean how low the nominal interest rate would be if we ignored the ZLB. Figure 15 displays the effects of ignoring the ZLB in our model’s predictions for the Great Recession.\footnote{Since we ignore the ZLB here, we also shut down forward guidance in this decomposition.} Two things are worth noting. First, the ZLB was much more binding at the stage when government consumption was rising. This fact helps explain why the multiplier associated with the increase in government consumption is higher than the one associated with the decrease. Second, the output effects stemming from the ZLB are generally modest. Put differently, the model attributes most of the magnitude and persistence of the Great Recession to the wedges and the slowdown in the growth rate of TFP.

Of course absent nominal rigidities, the effects of the shocks would be very different. For example, it is well known that it is very hard to obtain comovement of consumption and investment in a real business cycle model that does not allow for nominal rigidities.
**Effects of Forward Guidance.**—Figure 16 displays the impact of the monetary policy regime switch to forward guidance in 2011. The dashed line represents the model simulation with all shocks, when the Taylor rule is in place throughout the period. The figure indicates that without forward guidance the Fed would have started raising the interest rate in early 2014. By keeping the interest rate at zero, the monetary authority caused output to be about two percent higher and the unemployment rate to be about one percentage point lower. Interestingly, this relationship is consistent with Okun’s law.

**Effects of Labor Force Participation.**—One of the key contributions of our paper is to endogenize the labor force participation rate. A natural question is to what extent the prolonged fall in the labor force participation rate contributed to the decline in overall economic activity. Figure 17 displays the model’s predictions for how the economy would have evolved post-2008 if we hold the labor force participation rate constant. Clearly, the decline in employment, consumption and output that occurred during the Great Recession would have been substantially smaller.
VI. Conclusion

This paper argues that the bulk of movements in aggregate real economic activity during the Great Recession were due to financial frictions. We reach this conclusion by looking at the data through the lens of an estimated New Keynesian model in which firms face moderate degrees of price rigidities, no nominal rigidities in wages and a ZLB constraint that started to bind in early 2009. Our model does a good job of accounting for the joint behavior of labor and goods markets, as well as inflation, during the Great Recession. According to the model, the observed fall in TFP relative to trend and the rise in the cost of working capital played key roles in accounting for the small drop in inflation that occurred during the Great Recession.
REFERENCES


