A. Data Appendix

A.1. Data Sources


SH: Data on job separations and job findings available at Robert Shimer’s Homepage: http://robert.shimer.googlepages.com/.


A.2. Raw Data

Nominal GDP (GDP): nominal gross domestic product, billions of dollars, seasonally adjusted at annual rates, NIPA.

GDP Deflator (P): price index of nominal gross domestic product, index numbers, 2005=100, seasonally adjusted, NIPA.

Nominal nondurable consumption (C_{nom\ nondurables}): nominal personal consumption expenditures: nondurable goods, billions of dollars, seasonally adjusted at annual rates, NIPA.

Nominal durable consumption (C_{nom\ durables}): nominal personal consumption expenditures: durable goods, billions of dollars, seasonally adjusted at annual rates, NIPA.

Nominal consumption services (C_{nom\ services}): nominal personal consumption expenditures: services, billions of dollars, seasonally adjusted at annual rates, NIPA.

Nominal investment (I_{nom}): nominal gross private domestic investment, billions of dollars, seasonally adjusted at annual rates, NIPA.

Price index: nominal durable consumption (PC_{nom\ durables}): price index of durable goods, index numbers, 2005=100, seasonally adjusted at annual rates, NIPA.

Price index: nominal investment (PI_{nom}): price index of nominal gross private domestic investment, index numbers, 2005=100, seasonally adjusted at annual rates, NIPA.

Employment (E): civilian employment, CE16OV, seasonally adjusted, monthly, thousands, persons 16 years of age and older, FRED2.

Federal Funds Rate (FF): effective federal funds rate, H.15 selected interest rates, monthly, percent, averages of daily figures, FRED2.
Treasury bill rate ($TBill$): 3-month treasury bill: secondary market rate, H.15 selected interest rates, monthly, percent, averages of business days, discount basis, FRED2.

Population ($POP$): civilian noninstitutional population, not seasonally adjusted, monthly, thousands, FRED2.


Job separation rate ($S$): separation rate: E to U, seasonally adjusted, monthly, 1976M1-2008M12, FR. Spliced with corresponding data from Robert Shimer for the sample before 1976, quarterly. SH.

Job finding rate ($F$): Job finding rate: U to E, seasonally adjusted, monthly, 1976M1-2008M12, FR. Spliced with corresponding data from Robert Shimer for the sample before 1976, quarterly. SH.


Unemployment rate ($U$): unemployment rate labor force status: unemployment rate, LNS14000000, seasonally adjusted, percent, 16 years and over, monthly frequency, BLS.

Nominal wage ($W$): nominal hourly compensation, PRS85006103, sector: nonfarm business, seasonally adjusted, index, 1992 = 100, BLS.

Average hours ($H^{avg}$): average weekly hours, PRS85006023, sector: nonfarm business, seasonally adjusted, index, 1992 = 100, BLS.

Participation rate ($LabForce$): civilian participation rate, CIVPART, the employment situation, seasonally adjusted, monthly, percent, BLS.

A.3. Data Transformations

Raw data are transformed as follows. $POP$ is seasonally adjusted using the X12 (multiplicative) method. The indices for $W$ and $H^{avg}$ are normalized such that 2005=100. $E$, $FF$, $TBill$, $POP$, $V$, $U$ and $LabForce$ are converted to quarterly frequencies by averaging monthly observations. For the job finding rate $F$, we compute the quarterly measure from monthly data as follows:

$$F_{q1} = F_{m1} + (1 - F_{m1})F_{m2} + (1 - (1 - F_{m1})F_{m2})F_{m3},$$

where $F_{q1}$ denotes the finding rate of quarter 1 and $F_{m1}$, $F_{m2}$, $F_{m3}$ are the corresponding monthly finding rates. The case for the separation rate, $S$, follows accordingly.

Due to missing data we use $TBill$ as a proxy for the $FF$ prior 1954Q3. All data are available from 1948Q1 except for vacancies for which the first observation is 1951Q1.

We calculate the following time series which, among others, is used in the VAR:

$$\text{real } GDP = \frac{GDP}{P \times POP}$$

$$\text{hours} = \frac{H^{avg} \times E}{Pop}$$

$$\text{nominal consumption} = C_{\text{nom durable}} + C_{\text{nom service}}$$

$$\text{nominal investment} = I_{\text{nom}} + C_{\text{nom durable}}$$
The price of investment is calculated as a Torn price index using $PI_{nom}$, $PC_{nom}^{durable}$, $I_{nom}$ and $C_{nom}^{durable}$. The resulting price index $PIT$ and quantity index $QIT$ are used to calculate the relative price of investment as follows:

$$\text{relative price of investment} = \frac{PIT \cdot I_{nom}}{P \cdot QIT}.$$ 

**B. Scaling of Variables in Medium-sized Model**

We adopt the following scaling of variables. The neutral shock to technology is $z_t$ and its growth rate is $\mu_{z,t}$:

$$\frac{z_t}{z_{t-1}} = \mu_{z,t}.$$ 

The variable, $\Psi_t$, is an investment specific technology shock and it is convenient to define the following combination of our two technology shocks:

$$z_t^+ = \frac{\alpha}{\Psi_t} z_t, \quad \mu_{z+,t} = \frac{\alpha}{\Psi_t} \mu_{z,t}. \quad \tag{B.1}$$

Capital, $\bar{K}_t$, and investment, $I_t$, are scaled by $z_t^+ \Psi_t$. Consumption goods $C_t$, government consumption $G_t$, and the real wage, $\bar{W}_t/P_t$, are scaled by $z_t^+$. Also, $v_t$ is the multiplier on the nominal household budget constraint in the Lagrangian version of the household problem. That is, $v_t$ is the marginal utility of one unit of currency. The marginal utility of a unit of consumption is $v_t P_t$. The latter must be multiplied by $z_t^+$ to induce stationarity. Output, $Y_t$, is scaled by $z_t^+$. Optimal prices, $\bar{P}_t$, chosen by intermediate good firms which are subject to Calvo price setting frictions are scaled by the price, $P_t$, of the homogeneous output good. Similarly, optimal wages, $\bar{W}_t$, chosen by monopoly unions which are subject to Calvo wage setting frictions are scaled by the wage, $W_t$, of the homogenous labour input. Thus our scaled variables are:

$$k_{t+1} = \frac{K_{t+1}}{z_t^+ \Psi_t}, \quad \bar{k}_{t+1} = \frac{\bar{K}_{t+1}}{z_t^+ \Psi_t}, \quad i_t = \frac{I_t}{z_t^+ \Psi_t}, \quad c_t = \frac{C_t}{z_t^+}, \quad g_t = \frac{G_t}{z_t^+}, \quad \psi_{z+,t} = v_t P_t z_t^+, \quad \bar{w}_t = \frac{\bar{W}_t}{z_t^+ P_t}, \quad \bar{y}_t = \frac{Y_t}{z_t^+}, \quad \tilde{p}_t = \frac{\bar{P}_t}{P_t}, \quad w_t = \frac{\bar{W}_t}{\bar{W}_t}. \quad \tag{B.2}$$

We define the scaled date $t$ price of new installed physical capital for the start of period $t+1$ as $p_{k^+,t}$ and we define the scaled real rental rate of capital as $\bar{r}_t^k$:

$$p_{k^+,t} = \Psi_t P_{k^+,t}, \quad \bar{r}_t^k = \Psi_t t^k.$$ 

where $P_{k^+,t}$ is in units of the homogeneous good. The inflation rate is defined as:

$$\pi_t = \frac{P_t}{P_{t-1}}.$$
C. Equilibrium Conditions for the Medium-sized Model

C.1. Firms

We let \( s_t \) denote the firm’s marginal cost, divided by the price of the homogeneous good. The standard formula, expressing this as a function of the factor inputs, is as follows:

\[
s_t = \left( \frac{r^k P_t}{\alpha} \right)^{\alpha} \left( \frac{W_2 R_t}{1-\alpha} \right)^{1-\alpha} P^{-\alpha} z_t^{1-\alpha}.
\]

When expressed in terms of scaled variables, this reduces to:

\[
s_t = \left( \frac{\bar{r}^k}{\alpha} \right)^{\alpha} \left( \frac{\bar{w}^k R_t}{1-\alpha} \right)^{1-\alpha} P^{-\alpha} z_t^{1-\alpha}.
\] (C.1)

Productive efficiency dictates that \( s_t \) is also equal to the ratio of the real cost of labor to the marginal product of labor:

\[
s_t = \left( \frac{\mu_{\psi,t}}{1-\alpha} \right)^{\alpha} \frac{\bar{w}^k R_t}{(1-\alpha) \left( \frac{k_{t,t}}{\mu_{x,t}/H_{t,t}} \right)^{1-\alpha}}.
\] (C.2)

The only real decision taken by intermediate good firms is to optimize price when it is selected to do so under the Calvo frictions. The first order necessary conditions associated with price optimization are, after scaling:\(^1\)

\[
E_t \left[ \psi_{z+,t} y_t + \left( \frac{\bar{\pi}_{f,t+1}}{\pi_{t+1}} \right)^{\frac{1}{\lambda_f}} \beta \xi_p \pi_{t+1} - F^f_{t+1} - F^f_t \right] = 0,
\] (C.3)

\[
E_t \left[ \lambda_f \psi_{z+,t} y_t s_t + \beta \xi_p \left( \frac{\bar{\pi}_{f,t+1}}{\pi_{t+1}} \right)^{\frac{1}{\lambda_f}} K^f_{t+1} - K^f_t \right] = 0,
\] (C.4)

\[
\hat{\pi}_t = \left[ \left( 1 - \xi_p \right) \left( \frac{1 - \xi_p}{1 - \xi_p} \right)^{\frac{1}{\lambda_f}} \right]^{1-\lambda_f} \left( \frac{1 - \lambda_f}{\lambda_f} \right)
\]

\[
\hat{\pi}_{f,t} = \pi_{t+1} \left( \frac{\bar{\pi}_{f,t}}{\pi_{t+1}} \right)^{\frac{1}{\lambda_f}} \left( 1 - \lambda_f \right) \left( \frac{1 - \lambda_f}{\lambda_f} \right) \left( \frac{1 - \xi_p}{1 - \xi_p} \right)^{1-\lambda_f},
\] (C.5)

\[
\hat{\pi}_{f,t} = \pi_{t+1} \left( \frac{\bar{\pi}_{f,t}}{\pi_{t+1}} \right)^{1-\lambda_f} \left( \frac{1 - \lambda_f}{\lambda_f} \right) = K^f_t \left( F^f_t \right),
\] (C.6)

\[
\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{(1 - \beta \xi_p) (1 - \xi_p)}{\xi_p} \hat{s}_t,
\]

where a hat indicates log-deviation from steady state.

\(^1\)When we log-linearize about the steady state, we obtain,
C.2. Households

We now derive the equilibrium conditions associated with the household. We first consider the household’s consumption saving decision. We then turn to its wage decision. The Lagrangian representation of the household’s problem is:

\[ E^j_0 \sum_{i=0}^{\infty} \beta^i \left[ \ln (C_t - bC_{t-1}) - A_t \frac{h_{j,t}^{1+\phi}}{1+\phi} \right] \]

\[ v_t \left[ W_{t,j} h_{t,j} + X^k_t \bar{k}_t + R_{t-1} B_t + a_{t,j} - P_t \left( C_t + \frac{1}{\bar{\psi}_t} I_t \right) - B_{t+1} - P_t P_{k',t} \Delta_t \right] \]

\[ + \omega_t \left[ \Delta_t + (1-\delta) \bar{k}_t + \left( 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right) I_t - \bar{k}_{t+1} \right] \}

The first order condition with respect to \( C_t \) is:

\[ \frac{1}{C_t - bC_{t-1}} - E_t \frac{b \beta}{C_{t+1} - bC_t} = v_t P_t, \]

or, after expressing this in scaled terms and multiplying by \( z_t^+ \):

\[ \psi_{z,t} = \frac{1}{c_t - c_{t+1} \mu_{z,t}^{-1}} - \beta b E_t \frac{1}{c_{t+1} \mu_{z,t+1}^{-1} - b c_t}, \quad (C.8) \]

The first order condition with respect to \( \Delta_t \) is, after rearranging:

\[ P_t P_{k',t} = \frac{\omega_t}{v_t}, \quad (C.9) \]

The first order condition with respect to \( I_t \) is:

\[ \omega_t \left[ 1 - S \left( \frac{I_t}{I_{t-1}} \right) - S' \left( \frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right] + E_t \beta \omega_{t+1} S' \left( \frac{I_{t+1}}{I_t} \right) \left( \frac{I_{t+1}}{I_t} \right)^2 = P_t v_t \psi_t. \]

Making use of (C.9), multiplying by \( \Psi_t z_t^+ \), rearranging and using the scaled variables,

\[ \psi_{z+,t} P_{k',t} \left[ 1 - S \left( \frac{\mu_{z+,t+1} \mu_{k',t} i_t}{i_{t-1}} \right) - S' \left( \frac{\mu_{z+,t+1} \mu_{k',t} i_t}{i_{t-1}} \right) \frac{\mu_{z+,t+1} \mu_{k',t} i_t}{i_{t-1}} \right] \]

\[ + \beta \psi_{z+,t+1} P_{k',t+1} S' \left( \frac{\mu_{z+,t+1+1} \mu_{k',t+1} i_{t+1}}{i_t} \right) \left( \frac{i_{t+1}}{i_t} \right)^2 \mu_{z+,t+1+1} \mu_{k',t+1} = \psi_{z+,t}, \quad (C.10) \]

Optimality of the choice of \( \bar{k}_{t+1} \) implies the following first order condition:

\[ \omega_t = \beta E_t v_{t+1} X^k_{t+1} + \beta E_t \omega_{t+1} (1-\delta) = \beta E_t v_{t+1} \left[ X^k_{t+1} + P_{t+1} P_{k',t+1} (1-\delta) \right]. \]

Using (C.9) again,

\[ v_t = E_t \beta v_{t+1} \left[ \frac{X^k_{t+1} + P_{t+1} P_{k',t+1} (1-\delta)}{P_t P_{k',t}} \right] = E_t \beta v_{t+1} R^k_{t+1}, \quad (C.11) \]
where $R_{k+1}^t$ denotes the rate of return on capital:

$$R_{k+1}^t = \frac{X_{k+1}^t + P_{k+1}^t P_{k',t+1}^t (1 - \delta)}{P_t P_{k',t}}$$

Multiply (C.11) by $P_t z_t^+$ and express the results in scaled terms:

$$\psi_{z^+,t} = \beta E_t \psi_{z^+,t+1} \frac{R_{k+1}^t}{\pi_{t+1} \mu_{z^+,t+1}}. \quad (C.12)$$

Expressing the rate of return on capital, $(??)$, in terms of scaled variables:

$$R_{k+1}^t = \frac{\pi_{t+1} u_{t+1} \bar{r}_k^t - a(u_{t+1}) + (1 - \delta)p_{k',t+1}^t}{\mu_{\psi,t} \mu_{z^+,t+1}}. \quad (C.13)$$

The first order condition associated with capital utilization is:

$$\Psi_t \bar{r}_k^t = a'(u_t),$$

or, in scaled terms,

$$\bar{r}_k^t = a'(u_t). \quad (C.14)$$

The first order condition with respect to $B_{t+1}$ is:

$$v_t = \beta v_{t+1} R_t.$$  

Multiply by $z_t^+ P_t$:

$$\psi_{z^+,t} = \beta E_t \psi_{z^+,t+1} \frac{R_{k+1}^t}{\pi_{t+1} \mu_{z^+,t+1}} P_t.$$  

Finally, the law of motion for the capital stock, in terms of scaled variables is as follows:

$$\bar{k}_{t+1} = \frac{1 - \delta}{\mu_{z^+,t} \mu_{\psi,t}} \bar{k}_t + \left(1 - \tilde{S} \left( \frac{\mu_{z^+,t} \mu_{\psi,t} i_t}{i_{t-1}} \right) \right) i_t. \quad (C.16)$$

**C.3. Resource Constraint**

The resource constraint after scaling by $z_t^+$ is given by:

$$y_t = g_t + c_t + i_t + a(u_t) \frac{\bar{k}_t}{\mu_{\psi,t} \mu_{z^+,t}}. \quad (C.17)$$

In appendix D we derive a relationship between total output of the homogeneous good, $Y_t$, and aggregate factors of production which in scaled form looks as follows:

$$y_t = (\tilde{p}_t)^{-\lambda} \left[ \left( \frac{1}{\mu_{\psi,t} \mu_{z^+,t}} \right)^\alpha H_t^{1-\alpha} - \varphi \right], \quad (C.18)$$

where

$$k_t = \bar{k}_t u_t. \quad (C.19)$$

Finally, GDP is given by:

$$gdp_t = g_t + c_t + i_t. \quad (C.20)$$
C.4. Wage Setting by the Monopoly Union

We turn now to the equilibrium conditions associated with the household wage-setting decision. Consider the \( j^{th} \) household that has an opportunity to reoptimize its wage at time \( t \). We denote this wage rate by \( \tilde{W}_t \). This is not indexed by \( j \) because the situation of each household that optimizes its wage is the same. In choosing \( \tilde{W}_t \), the household considers the discounted utility (neglecting currently irrelevant terms in the household objective) of future histories when it cannot reoptimize:

\[
E^j_t \sum_{i=0}^{\infty} (\beta \xi_w)^i \left[ -A_L \left( h_{j,t+1} \right) \frac{1 + \phi}{1 + \phi} + v_{t+i} W_{j,t+i} h_{j,t+i} \right],
\]

where \( v_t \) is the multiplier on the household’s period \( t \) budget constraint. The demand for the \( j^{th} \) household’s labor services, conditional on it having optimized in period \( t \) and not again since, is:

\[
h_{j,t+i} = \left( \frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{W_{t+i}} \right)^{\frac{1}{1-\lambda_w}} \pi_{t+i} H_{t+i}.
\]

Here, it is understood that \( \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1} \equiv 1 \) when \( i = 0 \). Substituting this into the objective function and optimizing (see appendix F for details) yields the following equilibrium equations associated with wage setting:

\[
\pi_{w,t+1} = \frac{W_{t+1}}{W_t} = \frac{\bar{\omega}_{t+1} \bar{z}_{t+1}^i P_{t+1}}{\bar{\omega}_{t} \bar{z}_{t}^i P_t} = \frac{\bar{\omega}_{t+1} \mu_{z,t+1} \pi_{t+1}}{\bar{\omega}_t}, \tag{C.21}
\]

\[
h_t = \bar{\omega}_t \lambda_w H_t, \tag{C.22}
\]

\[
\hat{\omega}_t = \left( 1 - \xi_w \right) \left( \frac{1 - \xi_w \left( \frac{\tilde{\pi}_{w,t} \pi_{w,t}}{\tilde{\pi}_{w,t} \pi_{w,t}} \right)^{\frac{1}{1-\lambda_w}}}{1 - \xi_w} \right)^{\frac{1}{\lambda_w}} \hat{\omega}_t + \xi_w \left( \frac{\tilde{\pi}_{w,t} \pi_{w,t}}{\tilde{\pi}_{w,t} \pi_{w,t}} \right)^{\frac{1}{\lambda_w}}. \tag{C.23}
\]

In addition to (C.23), we have following equilibrium conditions associated with sticky wages:

\[\text{Log-linearizing these equations about the nonstochastic steady state we obtain,}\]

\[
E_t \left[ \eta_0 \hat{\omega}_{t-1} + \eta_1 \hat{\omega}_t + \eta_2 \hat{\omega}_{t+1} + \eta_3 \bar{\pi}_t + \eta_4 \bar{\pi}_{t+1} + \eta_5 \bar{\pi}_{t-1} + \eta_6 \bar{\pi}_{t+1} + \eta_7 \bar{\pi}_{t-1} + \eta_8 \bar{\pi}_{t+1} + \eta_9 \bar{\pi}_{t-1} \right] = 0,
\]

where

\[
\begin{align*}
b_w &= \frac{[\lambda_w \sigma_L - (1 - \lambda_w)]}{[(1 - \beta \xi_w)(1 - \xi_w)]}, \\
\eta_0 &= b_w \xi_w, \\
\eta_1 &= \sigma_L \lambda_w - b_w (1 + \beta \xi_w^2), \eta_2 = b_w \beta \xi_w. \\
\eta_3 &= -b_w \xi_w (1 + \beta \kappa_w), \eta_4 = b_w \beta \xi_w, \eta_5 = b_w \xi_w \kappa_w, \eta_6 = \xi_w (1 - \lambda_w), \\
\eta_7 &= -(1 - \lambda_w) \sigma_L, \eta_8 = -b_w \xi_w, \eta_9 = b_w \beta \xi_w.
\end{align*}
\]
\[
F_{w,t} = \frac{\psi_{w,t+1}}{\lambda_w} \bar{w}_t - \left( \frac{\lambda_w}{\lambda_w} \right) h_t + \beta \xi_w E_t \left( \frac{\bar{w}_{t+1}}{\bar{w}_t} \right) \left( \frac{\bar{\pi}_{w,t+1}}{\bar{\pi}_{w,t+1}} \right)^{1+\frac{\lambda_w}{\lambda_w}} F_{w,t+1}
\]  
(C.24)

\[
K_{w,t} = \left( \frac{\lambda_w}{\lambda_w} \right) h_t \left( 1+\phi \right) + \beta \xi_w E_t \left( \frac{\bar{\pi}_{w,t+1}}{\bar{\pi}_{w,t+1}} \right)^{\frac{\lambda_w}{\lambda_w}(1+\phi)} K_{w,t+1}
\]  
(C.25)

\[
\frac{1}{A_L} \left[ \frac{1 - \xi_w (\bar{\pi}_{w,t})^{1-\lambda_w}}{1 - \xi_w} \right] ^{1-\lambda_w(1+\phi)} \bar{w}_t F_{w,t} = K_{w,t}
\]  
(C.26)

\[
\bar{\pi}_{w,t+1} = \pi_t^\infty \pi_t^{(1-\kappa_w)} \mu_{z+}.
\]  
(C.27)

C.5. Equilibrium Equations

The equilibrium conditions of the model correspond to the following 28 equations,

(C.1), (C.2), (C.3), (C.4), (C.5), (C.6), (C.7), (C.16), (C.8), (C.10), (C.14),
(C.15), (C.22), (4.22), (C.17), (C.18), (4.24), (C.19), (4.25), (C.13),
(C.21), (C.24), (C.25), (C.26), (C.23), (C.27), (C.20), (C.12),

which can be used to solve for the following 28 unknowns:

\[
\tilde{r}^k, \bar{w}_t, R_t, s_t, \pi_t, p_{w',t}, k_{t+1}, \tilde{h}_{t+1}, u_t, h_t, H_t, i_t, c_t, \psi_{z+,t}, y_t,
\]

\[
K^f_t, F^f_t, \bar{\pi}_{f,t}, \bar{\pi}_t, K_{w,t}, F_{w,t}, \bar{\pi}^w_t, R^k_t, S_t, a(u_t), \bar{w}_t, \pi_{w,t}, gdp_t.
\]

D. Resource Constraint in the Medium-sized Model

We begin by deriving a relationship between total output of the homogeneous good, \(Y_t\), and aggregate factors of production. We first consider the production of the homogenous output good:

\[
Y_{t}^{\text{sum}} = \int_0^1 Y_{i,t} \, di
\]

\[
= \int_0^1 \left[ \left( z_i H_{i,t} \right)^{1-\alpha} K_{i,t}^\alpha - z_i^+ \varphi \right] \, di
\]

\[
= \int_0^1 \left[ z_i^{1-\alpha} \left( \frac{K_{i,t}}{H_{i,t}} \right)^\alpha H_{i,t} - z_i^+ \varphi \right] \, di
\]

\[
= z_i^{1-\alpha} \left( \frac{K_{i,t}}{H_{i,t}} \right)^\alpha \int_0^1 H_{i,t} \, di - z_i^+ \varphi,
\]

where \(K_t\) is the economy-wide average stock of capital services and \(H_t\) is the economy-wide average of homogeneous labor. The last expression exploits the fact that all intermediate good firms confront the same factor prices, and so they adopt the same capital services.
to homogeneous labor ratio. This follows from cost minimization, and holds for all firms, regardless whether or not they have an opportunity to reoptimize. Then,

$$Y_t^{\text{sum}} = z_t^{1-a} K_t^\alpha H_t^{1-a} - z_t^+ \varphi.$$ 

The demand for $Y_{j,t}$ is

$$\left( \frac{P_t}{P_{t,t}} \right)^{\lambda_f} = \frac{Y_{i,t}}{Y_t},$$

so that

$$\tilde{Y}_t \equiv \int_0^1 Y_{i,t} \, di = \int_0^1 Y_t \left( \frac{P_t}{P_{t,t}} \right)^{\lambda_f} \, di = Y_t \left( \frac{P_t}{P_{t,t}} \right)^{\lambda_f},$$

say, where

$$\tilde{P}_t = \left[ \int_0^1 \left( \frac{P_t}{P_{t,t}} \right)^{\lambda_f} \, di \right]^{\frac{1-\lambda_f}{\lambda_f}}.$$ 

(D.1)

Dividing by $P_t$,

$$\tilde{p}_t = \left[ \int_0^1 \left( \frac{P_t}{P_{t,t}} \right)^{\lambda_f} \, di \right]^{\frac{1-\lambda_f}{\lambda_f}},$$

or,

$$\tilde{p}_t = \left( 1 - \xi_p \right) \left( 1 - \xi_p \left( \frac{\bar{\pi}_t}{\pi_t} \right)^{1-\lambda_f} \right)^{\lambda_f} + \xi_p \left( \frac{\bar{\pi}_t}{\pi_t} \tilde{p}_{t-1} \right)^{\lambda_f} \tilde{p}_t^{1-\lambda_f}.$$ 

(D.2)

The preceding discussion implies:

$$Y_t = (\tilde{p}_t)^{\lambda_f} \tilde{Y}_t = (\tilde{p}_t)^{\lambda_f} \left[ z_t^{1-a} K_t^\alpha H_t^{1-a} - z_t^+ \varphi \right],$$

or, after scaling by $z_t^+$,

$$y_t = (\tilde{p}_t)^{\lambda_f} \left[ \left( \frac{1}{\mu_{t,t} \mu_{t,t}} \right)^{\alpha} H_t^{1-a} - \varphi \right],$$

where

$$k_t = \bar{k}_t u_t.$$ 

Finally, we adjust hours worked in the resource constraint so that it corresponds to the total number of people working, as in (F.6):

$$y_t = (\tilde{p}_t)^{\lambda_f} \left[ \left( \frac{1}{\mu_{t,t} \mu_{t,t}} \right)^{\alpha} H_t^{1-a} - \varphi \right].$$
It is convenient to also have an expression that exhibits the uses of the homogeneous output,
\[ z_t^+ y_t = G_t + C_t + I_t, \]
or, after scaling by \( z_t^+ \):
\[ y_t = g_t + c_t + \alpha_t + \alpha (u_t) \frac{\bar{k}_t}{\mu z_{z+t}}. \]

E. Optimal Price Setting in the Medium-sized Model

The profit function of the \( i \)th intermediate good firm with the substituted demand function is given by,
\[
E_t \sum_{j=0}^{\infty} \beta^j v_{t+j} P_{t+j} Y_{t+j} \left\{ \left( \frac{P_{t+j}}{P_{t+j}} \right)^{1-\frac{\lambda t}{\lambda t}} - s_{t+j} \left( \frac{P_{t+j}}{P_{t+j}} \right)^{-\lambda t} \right\},
\]
or,
\[
E_t \sum_{j=0}^{\infty} \beta^j v_{t+j} P_{t+j} Y_{t+j} \left\{ (X_{t,j} \tilde{p}_t)^{1-\frac{\lambda t}{\lambda t}} - s_{t+j} (X_{t,j} \tilde{p}_t)^{-\lambda t} \right\},
\]
where
\[
\frac{P_{t+j}}{P_{t+j}} = X_{t,j} \tilde{p}_t, \quad X_{t,j} \equiv \left\{ \begin{array}{cc}
\frac{\tilde{p}_{t+j} \cdots \tilde{p}_{t+1}}{\tilde{p}_{t+j} \cdots \tilde{p}_{t+1}^{1-\frac{\lambda t}{\lambda t}}}, & j > 0 \\
1, & j = 0
\end{array} \right.
\]
The \( i \)th firm maximizes profits by choice of \( \tilde{p}_t \). The fact that this variable does not have an index, \( i \), reflects that all firms that have the opportunity to reoptimize in period \( t \) solve the same problem, and hence have the same solution. Differentiating its profit function, multiplying the result by \( \tilde{p}_t^{\lambda t^{1-1}} \), rearranging, and scaling we obtain:
\[
E_t \sum_{j=0}^{\infty} (\beta \xi_p)^j A_{t+j} [\tilde{p}_t X_{t,j} - \lambda t s_{t+j}] = 0,
\]
where \( A_{t+j} \) is exogenous from the point of view of the firm:
\[ A_{t+j} = \psi_{z,t+j} y_{t+j} X_{t,j}. \]

After rearranging the optimizing intermediate good firm’s first order condition for prices, we obtain,
\[
\tilde{p}_t = E_t \sum_{j=0}^{\infty} (\beta \xi_p)^j A_{t+j} \lambda t s_{t+j} \frac{K_t^f}{F_t^f} = \psi_{z,t+j} y_{t+j} X_{t,j},
\]
say, where
\[
K_t^f \equiv E_t \sum_{j=0}^{\infty} (\beta \xi_p)^j A_{t+j} \lambda t s_{t+j} \]
\[
F_t^f \equiv E_t \sum_{j=0}^{\infty} (\beta \xi_p)^j A_{t+j} X_{t,j}.
\]
These objects have the following convenient recursive representations:

\[
E_t \left[ \psi_{z + t} y_t + \left( \frac{\tilde{\pi}_{t+1}}{\pi_{t+1}} \right)^{\frac{\lambda_f}{1-\lambda_f}} \beta \xi_p F_{t+1} - F_t \right] = 0
\]

\[
E_t \left[ \lambda_f \psi_{z + t} y_t s_t + \beta \xi_p \left( \frac{\tilde{\pi}_{t+1}}{\pi_{t+1}} \right)^{\frac{\lambda_f}{1-\lambda_f}} K_{t+1} - K_t \right] = 0.
\]

Turning to the aggregate price index:

\[
P_t = \left[ \int_0^1 P_{it}^{1-\lambda_f} di \right]^{(1-\lambda_f)}
\]

\[
= \left[ (1 - \xi_p) P_t^{1-\lambda_f} + \xi_p (\tilde{\pi}_t P_{t-1})^{1-\lambda_f} \right]^{(1-\lambda_f)}.
\]

After dividing by \( P_t \) and rearranging:

\[
\frac{1 - \xi_p (\tilde{\pi}_t)^{1-\lambda_f}}{1 - \xi_p} = (\tilde{P}_t)^{1-\lambda_f}.
\]

This completes the derivations of optimal decisions with respect to firms price setting.

**F. Optimal Wage Setting in the Medium-sized Model**

The objective function with the substituted labor demand function looks as follows:

\[
E_t^2 \sum_{i=0}^{\infty} (\beta \xi_w)^i \left[ -A_L \left( \frac{\tilde{W}_t \tilde{\pi}_{w,t+i+1} \cdots \tilde{\pi}_{w,t+1}}{W_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right]^{1+\phi}
\]

\[
+ v_{t+i} \tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1} \left( \frac{\tilde{W}_t \tilde{\pi}_{w,t+i+1} \cdots \tilde{\pi}_{w,t+1}}{W_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right].
\]

Recalling the scaling of variables, (B.2), we have

\[
\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1} \left( \frac{\tilde{W}_t}{W_t} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right]
\]

Recalling the scaling of variables, (B.2), we have

\[
\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1} \left( \frac{\tilde{W}_t}{W_t} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right]
\]

\[
= \tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1} \left( \frac{\tilde{W}_t}{W_t} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right]
\]

where

\[
X_{t,i} = \frac{\tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{\pi_{t+i} \pi_{t+i-1} \cdots \pi_{t+1} \mu_{z,t+i} \cdots \mu_{z,t+1}}, \quad i > 0
\]

\[
= 1, \quad i = 0.
\]
It is interesting to investigate the value of $X_{t,i}$ in steady state, as $i \to \infty$. Thus,

$$X_{t,i} = \frac{(\pi_t \cdots \pi_{t+i-1})^{\kappa_w} (\pi_i)^{(1-\kappa_w)} \mu_{z+}^i}{\pi_{t+i-1} \pi_{t+i-2} \cdots \pi_{t+1} \mu_{z,t+i-1} \cdots \mu_{z,t+i+1}}.$$ 

In steady state,

$$X_{t,i} = \frac{(\pi_i)^{\kappa_w} (\pi_i)^{(1-\kappa_w)} \mu_{z+}^i}{\pi_i \mu_{z+}^i} = 1.$$ 

Simplifying using the scaling notation,

$$E_t^i \sum_{i=0}^{\infty} (\beta \xi_w)^i \left[-A_L \frac{\left(\frac{\bar{w_t} \bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}}{1 + \phi} \right]$$

or,

$$E_t^i \sum_{i=0}^{\infty} (\beta \xi_w)^i \left[-A_L \frac{\left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}}{1 + \phi} \right] + \psi_{z^+,t+i} w_t \bar{w}_t X_{t,i} \left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i},$$

or,

$$E_t^i \sum_{i=0}^{\infty} (\beta \xi_w)^i \left[-A_L \frac{\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}}{1 + \phi} \right] w_t^{\frac{\lambda_w}{1-\lambda_w}(1+\phi)}$$

$$+ \psi_{z^+,t+i} w_t + \frac{1+\phi}{1-\lambda_w} \bar{w}_t X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} = 0.$$ 

Differentiating with respect to $w_t$,

$$E_t^i \sum_{i=0}^{\infty} (\beta \xi_w)^i \left[-A_L \frac{\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}}{1 + \phi} \right] \lambda_w (1 + \phi) w_t^{\frac{\lambda_w}{1-\lambda_w}(1+\phi)-1}$$

$$+ \psi_{z^+,t+i} \frac{\lambda_w}{1-\lambda_w} \bar{w}_t X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} = 0.$$ 

Dividing and rearranging,

$$E_t^i \sum_{i=0}^{\infty} (\beta \xi_w)^i \left[-A_L \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right]^{1+\phi}$$

$$+ \frac{\psi_{z^+,t+i}}{\lambda_w} \frac{\lambda_w}{1-\lambda_w} \bar{w}_t X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} = 0.$$ 

(F.3)
Solving for the wage rate:

\[
\frac{1}{w_t} = \frac{E_t^j \sum_{i=0}^{\infty} (\beta \xi_w)^i A_L \left( \lambda_w \frac{\bar{w}_i}{\bar{w}_{t+i}} X_{t,i} \right) H_{t+i}}{E_t^j \sum_{i=0}^{\infty} (\beta \xi_w)^i \frac{\psi_{z,t+i+1}}{\lambda_w} \bar{w}_i X_{t,i} \left( \frac{\bar{w}_i}{\bar{w}_{t+i}} X_{t,i} \right) H_{t+i}}
\]

where

\[
K_{w,t} = E_t^j \sum_{i=0}^{\infty} (\beta \xi_w)^i \left( \frac{\bar{w}_i}{\bar{w}_{t+i}} X_{t,i} \right) H_{t+i}^{1+\phi}
\]

\[
F_{w,t} = E_t^j \sum_{i=0}^{\infty} (\beta \xi_w)^i \frac{\psi_{z,t+i+1}}{\lambda_w} X_{t,i} \left( \frac{\bar{w}_i}{\bar{w}_{t+i}} X_{t,i} \right) H_{t+i}.
\]

Thus, the wage set by reoptimizing households is:

\[
w_t = \left[ \frac{A_L K_{w,t}}{w_t F_{w,t}} \right] ^{\frac{1}{1-\lambda_w(1+\phi)}}
\]

We now express \( K_{w,t} \) and \( F_{w,t} \) in recursive form:

\[
K_{w,t} = E_t^j \sum_{i=0}^{\infty} (\beta \xi_w)^i \left( \frac{\bar{w}_i}{\bar{w}_{t+i}} X_{t,i} \right) H_{t+i}^{1+\phi}
\]

\[
= H_t^{1+\phi} + \beta \xi_w \left( \frac{\bar{w}_t}{\bar{w}_{t+1}} \frac{\pi_t}{\pi_{t+1}} \frac{(1-\kappa_w) \mu_{z+}}{\lambda_w} \right) H_{t+1}^{1+\phi}
\]

\[
+ (\beta \xi_w)^2 \left( \frac{\bar{w}_t}{\bar{w}_{t+2}} \frac{\pi_t}{\pi_{t+1}} \frac{(1-\kappa_w) \mu_{z+}^2}{\lambda_w} \right) H_{t+2}^{1+\phi}
\]

or,

\[
K_{w,t} = H_t^{1+\phi} + E_t \beta \xi_w \left( \frac{\bar{w}_t}{\bar{w}_{t+1}} \frac{\pi_t}{\pi_{t+1}} \frac{(1-\kappa_w) \mu_{z+}}{\lambda_w} \right) \left( H_{t+1}^{1+\phi} + \beta \xi_w \left( \frac{\bar{w}_t}{\bar{w}_{t+1}} \frac{\pi_t}{\pi_{t+1}} \frac{(1-\kappa_w) \mu_{z+}}{\lambda_w} \right) H_{t+2}^{1+\phi} \right)
\]

\[
= H_t^{1+\phi} + \beta \xi_w E_t \left( \frac{\bar{w}_t}{\bar{w}_{t+1}} \frac{\pi_t}{\pi_{t+1}} \frac{(1-\kappa_w) \mu_{z+}}{\lambda_w} \right) K_{w,t+1}
\]

\[
= H_t^{1+\phi} + \beta \xi_w E_t \left( \frac{\bar{w}_t}{\bar{w}_{t+1}} \frac{\pi_t}{\pi_{t+1}} \right) K_{w,t+1}.
\]

(F.4)
using,
\[
\pi_{w,t+1} = \frac{W_{t+1}}{W_t} = \frac{\bar{w}_{t+1} z_{t+1}^w P_{t+1}}{\bar{w}_{t} z_{t}^w P_{t}} = \frac{\bar{w}_{t+1} \mu_{z+,t+1} \pi_{t+1}}{\bar{w}_{t}}.
\]

Also,
\[
F_{w,t} = E_t^\infty \sum_{i=0}^{\infty} (\beta \xi_w)^i \psi_{z^+,t+i} \frac{X_{t,i}}{\bar{w}_{t+i}} H_{t+i} \left( \frac{\bar{w}_{t}}{\bar{w}_{t+i}} \right) \frac{\lambda_w}{1-\lambda_w} H_{t+1}
\]
\[
+ \beta \xi_w \psi_{z^+,t+1} \frac{\bar{w}_{t}}{\bar{w}_{t+1}} \frac{\lambda_w}{1-\lambda_w} \left( \frac{\pi_{t}^{\kappa_w} \mu_{z+}^{1-\kappa_w} \mu_{z+,t+1}}{\pi_{t+1}^{\kappa_w} \mu_{z+,t+1}} \right) \frac{1+\lambda_w}{1-\lambda_w} H_{t+1}
\]
\[
+ (\beta \xi_w)^2 \psi_{z^+,t+2} \frac{\bar{w}_{t}}{\bar{w}_{t+2}} \frac{\lambda_w}{1-\lambda_w} \left( \frac{\pi_{t+1}^{\kappa_w} \mu_{z+}^{1-\kappa_w} \mu_{z+,t+2}}{\pi_{t+2}^{\kappa_w} \mu_{z+,t+2}} \right) \frac{1+\lambda_w}{1-\lambda_w} H_{t+2}
\]
\[
+ \ldots
\]

or,
\[
F_{w,t} = \frac{\psi_{z^+,t} H_{t}}{\lambda_w} + \beta \xi_w \frac{\bar{w}_{t+1}}{\bar{w}_{t}} \left( \frac{\bar{w}_{t+1}}{\bar{w}_{t}} \right) \left( \frac{1}{\pi_{w,t+1}} \pi_{w,t+1} \right) \frac{1+\lambda_w}{1-\lambda_w} F_{w,t+1},
\]

so that
\[
F_{w,t} = \frac{\psi_{z^+,t} H_{t}}{\lambda_w} + \beta \xi_w E_t \left( \frac{\bar{w}_{t+1}}{\bar{w}_{t}} \right) \left( \frac{1}{\bar{w}_{w,t+1}} \bar{w}_{w,t+1} \right) \frac{1+\lambda_w}{1-\lambda_w} F_{w,t+1}. \tag{F.5}
\]

We obtain a second restriction on \( w_t \) using the relation between the aggregate wage rate and the wage rates of individual households:
\[
W_t = \left[ (1-\xi_w) \left( \bar{W}_t \right) \frac{1}{1-\lambda_w} + \xi_w \left( \bar{w}_{w,t} W_{t-1} \right) \frac{1}{1-\lambda_w} \right]^{1-\lambda_w}.
\]

Dividing both sides by \( W_t \) and rearranging,
\[
w_t = \left[ \frac{1-\xi_w}{1-\xi_w} \left( \frac{\bar{w}_{w,t}}{W_t} \right) \frac{1}{1-\lambda_w} \right]^{1-\lambda_w}.
\]
Substituting, out for $w_t$ from the household’s first order condition for wage optimization:

$$\frac{1}{\lambda_w} \left[ \frac{1 - \xi_w (\tilde{\pi}_{w,t} / \pi_{w,t})^{1-\lambda_w}}{1 - \xi_w} \right]^{1-\lambda_w(1+\phi)} \tilde{w}_t F_{w,t} = K_{w,t}.$$  

We now derive the relationship between aggregate homogeneous hours worked, $H_t$, and aggregate household hours, $h_t \equiv \int_0^1 h_{j,t} dj$.

Substituting the demand for $h_{j,t}$ into the latter expression, we obtain,

$$h_t = \int_0^1 \left( \frac{W_{j,t}}{W_t} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_t \, dj = \frac{H_t}{(W_t)^{\frac{\lambda_w}{1-\lambda_w}}} \int_0^1 (W_{j,t})^{\frac{\lambda_w}{1-\lambda_w}} \, dj = \tilde{w}_t \frac{\lambda_w}{1-\lambda_w} H_t,$$

where

$$\tilde{w}_t \equiv \frac{\tilde{W}_t}{W_t}, \quad \tilde{W}_t = \left[ \int_0^1 (W_{j,t})^{\frac{\lambda_w}{1-\lambda_w}} \, dj \right]^{\frac{1+\lambda_w}{\lambda_w}}.$$

Also,

$$\tilde{W}_t = \left[ (1 - \xi_w) \left( \tilde{W}_t \right)^{\frac{\lambda_w}{1-\lambda_w}} + \xi_w \left( \tilde{\pi}_{w,t} \tilde{W}_t - 1 \right) \right]^{\frac{1+\lambda_w}{\lambda_w}}.$$

This completes the derivations of the optimal wage setting.

**G. The Wage-Phillips Curve in the Medium-Sized Model**

The household faces the following demand for the $j^{th}$ type of labor:

$$h_{t,j} = \left( \frac{W_t}{W_{t,j}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_t.$$  

By (G.1),

$$h_{t+i} = \left( \frac{\tilde{W}_{t+i}}{W_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} = \left( \frac{w_t \tilde{w}_t}{\tilde{w}_{t+i} X_{t,i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}.$$  

Here, we have used,

$$w_t \equiv \frac{\tilde{W}_t}{W_t}, \quad \tilde{w}_t \equiv \frac{W_t}{\tilde{z}_t F_t}, \quad \tilde{W}_{t+i} = \tilde{W}_t \tilde{\pi}_{w,t+1} \cdots \tilde{\pi}_{w,t+i},$$
and
\[ X_{t,i} = \begin{cases} \frac{\tilde{\pi}_{w,t+i-1}\ldots\tilde{\pi}_{w,t+1}}{\pi_{t+i-1}\ldots\pi_{t+1}\mu_{t+i+t},t+1} & i > 0 \\ 1 & i = 0 \end{cases} . \]

Rewriting (??),
\[
E_t \sum_{i=0}^{\infty} (\beta \xi_w)^i v_{t+i} P_{t+i} \xi_{t+i}^+ \left[ \frac{\tilde{W}_{t+i} h_{t+i}^i}{P_{t+i} \xi_{t+i}^+} - A_L \frac{(h_{t+i}^t)^{1+\phi}}{(1 + \phi) v_{t+i} P_{t+i} \xi_{t+i}^+} \right].
\]
or, using,
\[ \psi_{z+,t} = v_t P_t \xi_t^+ , \]
we have
\[
E_t \sum_{i=0}^{\infty} (\beta \xi_w)^i \psi_{z+,t+i} \left[ \frac{\tilde{W}_{t+i}^i h_{t+i}^i}{P_{t+i} \xi_{t+i}^+} - A_L \frac{(h_{t+i}^t)^{1+\phi}}{(1 + \phi) \psi_{z+,t+i}} \right].
\]
But,
\[
\frac{\tilde{W}_{t+i}^i h_{t+i}^i}{P_{t+i} \xi_{t+i}^+} = \frac{W_t (\tilde{W}_i/W_i) h_{t+i}^i}{P_t \xi_t^+} X_{t,i} = \bar{w}_i w_i h_{t+i}^i X_{t,i}
\]
\[ = \bar{w}_t w_t \left( \frac{w_i \bar{w}_t}{w_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} X_{t,i} . \]

Then,
\[
E_t \sum_{i=0}^{\infty} (\beta \xi_w)^i \psi_{z+,t+i} \left[ \bar{w}_t w_t \left( \frac{w_i \bar{w}_t}{w_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} X_{t,i} - A_L \frac{\left( \frac{w_i \bar{w}_t}{w_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}^{1+\phi}}{(1 + \phi) \psi_{z+,t+i}} \right].
\]

Differentiating with respect to \( w_t \),
\[
E_t \sum_{i=0}^{\infty} (\beta \xi_w)^i \psi_{z+,t+i} \left[ \left( \frac{\lambda_w}{1-\lambda_w} + 1 \right) \bar{w}_t \left( \frac{w_i \bar{w}_t}{w_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} X_{t,i} \right]
\]
\[ - \frac{\lambda_w}{1-\lambda_w} (1 + \phi) A_L \frac{\left( \frac{w_i \bar{w}_t}{w_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}^{1+\phi}}{w_t (1 + \phi) \psi_{z+,t+i}} \right].
\]
or
\[
E_t \sum_{i=0}^{\infty} (\beta \xi_w)^i \psi_{z+,t+i} h_{t+i}^i \left[ \bar{w}_t w_t X_{t,i} - \lambda_w MRS_{t+i}^t \right] , \tag{G.4}
\]

Here, \( MRS_{t+i}^t \) in (G.4) denotes the (scaled) cost of working for the marginal worker in period \( t+i \) whose wage was reoptimized in period \( t \) and not again reoptimized in periods \( t+1, t+2, \ldots, t+i \) : \[ MRS_{t+i}^t \equiv A_L \frac{(h_{t+i}^t)^{1+\phi}}{\psi_{z+,t+i}} . \]
According to (G.4), the union seeks to set the wage to a markup, \( \lambda_w \), over the cost of working of the marginal worker, on average.

We now expand (G.4) about a steady state in which

\[
w_t = 1, \quad x_{t,i} = 1, \quad X_{t,i} = 1, \quad \text{for all } i, \quad \bar{w}_t = \lambda_w MRS, \quad \bar{\pi}_t = \bar{\pi}, \quad \pi_{w,t} = \mu_{z+}, \quad \bar{\pi}_{w,t+1} = \pi_{\mu_{z+}}
\]

It is convenient to obtain some preliminary results. Note,

\[
X_{t,i} = \left\{
\begin{array}{ll}
-(\Delta_{\kappa_w} \hat{\pi}_{t+1} + \cdots + \Delta_{\kappa_w} \hat{\pi}_{t+i}) - (\hat{\mu}_{z+,t+i} + \cdots + \hat{\mu}_{z+,t+1}) & i > 0 \\
0 & i = 0
\end{array}
\right.
\]

\[
\hat{\pi}_{t,i} = \left\{
\begin{array}{ll}
-(\Delta_{\kappa_w} \hat{\pi}_{w,t+1} + \cdots + \Delta_{\kappa_w} \hat{\pi}_{w,t+i}) & i > 0 \\
0 & i = 0
\end{array}
\right.
\]

\[
MRS_{t+i} = -\hat{\psi}_{z+,t+i} + \phi \hat{H}_{t+i} + \phi \left( \hat{h}_{t+i} - \hat{H}_{t+i} \right)
\]

where

\[
\Delta_{\kappa_w} \hat{\pi}_{t+1} \equiv \hat{\pi}_{t+1} - \kappa_w \hat{\pi}_t, \quad \Delta_{\kappa_w} \hat{\pi}_{w,t+i} \equiv \hat{\pi}_{w,t+i} - \kappa_w \hat{\pi}_t
\]

Also, from (G.2),

\[
\hat{h}_{t+i} - \hat{H}_{t+i} = \left\{
\begin{array}{ll}
\frac{\lambda_w}{1 - \lambda_w} \left( \bar{w}_t - \Delta_{\kappa_w} \hat{\pi}_{w,t+1} - \Delta_{\kappa_w} \hat{\pi}_{w,t+i} \right) & i > 0 \\
\frac{\lambda_w}{1 - \lambda_w} \bar{w}_t & i = 0
\end{array}
\right.
\]

We have

\[
\phi \frac{\lambda_w}{1 - \lambda_w} \hat{w}_t + (\beta \xi_w) \phi \frac{\lambda_w}{1 - \lambda_w} \left( \bar{w}_t - \Delta_{\kappa_w} \hat{\pi}_{w,t+1} \right)
\]

\[
+ (\beta \xi_w)^2 \phi \frac{\lambda_w}{1 - \lambda_w} \left( \bar{w}_t - \Delta_{\kappa_w} \hat{\pi}_{w,t+1} - \Delta_{\kappa_w} \hat{\pi}_{w,t+2} \right)
\]

\[
+ (\beta \xi_w)^3 \phi \frac{\lambda_w}{1 - \lambda_w} \left( \bar{w}_t - \Delta_{\kappa_w} \hat{\pi}_{w,t+1} - \Delta_{\kappa_w} \hat{\pi}_{w,t+2} - \Delta_{\kappa_w} \hat{\pi}_{w,t+3} \right)
\]

\[
+ \ldots
\]

\[
= \frac{1}{1 - \beta \xi_w} \phi \frac{\lambda_w}{1 - \lambda_w} \hat{w}_t - \phi \frac{\lambda_w}{1 - \lambda_w} \frac{1}{1 - \beta \xi_w} (\beta \xi_w) \Delta_{\kappa_w} \hat{\pi}_{w,t+1}
\]

\[
- \phi \frac{\lambda_w}{1 - \lambda_w} \frac{1}{1 - \beta \xi_w} (\beta \xi_w)^2 \Delta_{\kappa_w} \hat{\pi}_{w,t+2}
\]

\[
\ldots
\]

Using the last expression, we can write the discounted sum of the marginal cost of working as follows:

\[
S_{MRS,t} \equiv \sum_{i=0}^{\infty} (\beta \xi_w)^i MRS_{t+i} = S_{o,t} + \phi \frac{\lambda_w}{1 - \lambda_w} \frac{1}{1 - \beta \xi_w} \left[ \hat{w}_t - S_{w,t} \right]. \quad (G.5)
\]

Here,

\[
S_{o,t} \equiv \sum_{i=0}^{\infty} (\beta \xi_w)^i \left[ -\hat{\psi}_{z+,t+i} + \phi \hat{H}_{t+i} \right] = -\hat{\psi}_{z+,t} + \phi \hat{H}_t + \beta \xi_w S_{o,t+1} \quad \quad (G.6)
\]

\[
S_{w,t} \equiv \sum_{i=1}^{\infty} (\beta \xi_w)^i \Delta_{\kappa_w} \hat{\pi}_{w,t+i} = \beta \xi_w \Delta_{\kappa_w} \hat{\pi}_{w,t+1} + \beta \xi_w S_{w,t+1}. \quad \quad (G.7)
\]
The following expression is also useful:

\[ S_{X,t} = \sum_{i=1}^{\infty} (\beta \xi_{w})^i \hat{X}_{t,i} = \frac{1}{1 - \beta \xi_{w}} \sum_{i=1}^{\infty} (\beta \xi_{w})^i \left[ -\hat{\mu}_{z+,t+i} - \Delta_{\kappa_{w}} \hat{\pi}_{t+i} \right] \quad (G.8) \]

\[ = \frac{\beta \xi_{w}}{1 - \beta \xi_{w}} \left[ -\hat{\mu}_{z+,t+1} - \Delta_{\kappa_{w}} \hat{\pi}_{t+1} \right] + \beta \xi_{w} S_{X,t+1}. \]

Because the object in square brackets in (G.4) is zero in steady state, the expansion of (G.4) does not require expanding the expression outside the square bracket. Taking this and \( \hat{w} = \lambda_{w} MRS \) into account, the expansion of (G.4) is:

\[ 0 = \frac{1}{1 - \beta \xi_{w}} (\hat{w}_t + \hat{w}_t) + S_{X,t} - S_{MRS,t}. \quad (G.9) \]

We now deduce the restriction across wages implied by the aggregate wage index. Using the wage updating equation and the fact that non-optimizing unions are selected at random, the aggregate wage index reduces to:

\[ W_t = \left[ (1 - \xi_{w}) \left( \tilde{W}_t \right)^{1/\lambda_{w}} + \xi_{w} (\tilde{\pi}_{w,t} W_{t-1})^{1/\lambda_{w}} \right]^{1 - \lambda_{w}} \]

Divide by \( W_t \) and use (G.3):

\[ 1 = \left( 1 - \xi_{w} \right) (w_t)^{1/\lambda_{w}} + \xi_{w} \left( \tilde{\pi}_{w,t} \right)^{1/\lambda_{w}}. \]

Log-linearize this expression about steady state, to obtain:

\[ \hat{w}_t = \frac{\xi_{w}}{1 - \xi_{w}} \Delta_{\kappa_{w}} \hat{\pi}_{w,t}. \]

Replace \( S_{MRS,t} \) in (G.9) using (G.5) and then substitute out for \( \hat{w}_t \) using the previous expression:

\[ \frac{1}{1 - \beta \xi_{w}} \hat{w}_t + \frac{1}{1 - \beta \xi_{w}} \xi_{w} \Delta_{\kappa_{w}} \hat{\pi}_{w,t} + S_{X,t} \]

\[ = S_{a,t} + \phi \frac{\lambda_{w}}{1 - \lambda_{w}} \frac{1}{1 - \beta \xi_{w}} \xi_{w} \Delta_{\kappa_{w}} \hat{\pi}_{w,t} - \phi \frac{\lambda_{w}}{1 - \lambda_{w}} \frac{1}{1 - \beta \xi_{w}} S_{w,t} \]

Multiply (G.10) evaluated at \( t+1 \) by \( \beta \xi_{w} \) and subtract the result from (G.10) evaluated at \( t \) to obtain:

\[ \frac{1}{1 - \beta \xi_{w}} (\hat{w}_t - \beta \xi_{w} \hat{w}_{t+1}) + \frac{1}{1 - \beta \xi_{w}} \xi_{w} \left( \Delta_{\kappa_{w}} \hat{\pi}_{w,t} - \beta \xi_{w} \Delta_{\kappa_{w}} \hat{\pi}_{w,t+1} \right) \]

\[ + (S_{X,t} - \beta \xi_{w} S_{X,t+1}) \]

\[ = (S_{a,t} - \beta \xi_{w} S_{a,t+1}) + \phi \frac{\lambda_{w}}{1 - \lambda_{w}} \frac{1}{1 - \beta \xi_{w}} \xi_{w} \left( \Delta_{\kappa_{w}} \hat{\pi}_{w,t} - \beta \xi_{w} \Delta_{\kappa_{w}} \hat{\pi}_{w,t+1} \right) \]

\[ - \phi \frac{\lambda_{w}}{1 - \lambda_{w}} \frac{1}{1 - \beta \xi_{w}} (S_{w,t} - \beta \xi_{w} S_{w,t+1}) \]

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Simplify this expression using (G.6), (G.7) and (G.8):

\[
\frac{1}{1-\beta \xi_w} (\hat{w}_t - \beta \xi_w \hat{w}_{t+1}) + \frac{1}{1-\beta \xi_w} \frac{\xi_w}{1-\xi_w} (\Delta_{\kappa_w} \hat{\pi}_{w,t} - \beta \xi_w \Delta_{\kappa_w} \hat{\pi}_{w,t+1}) \\
- \frac{\beta \xi_w}{1-\beta \xi_w} (\hat{\mu}_{z+,t+1} + \Delta_{\kappa_w} \hat{\pi}_{t+1}) \\
= -\psi_{z+,t} + \phi \hat{H}_t + \phi \frac{\lambda_w}{1-\lambda_w} \frac{1}{1-\beta \xi_w} \frac{\xi_w}{1-\xi_w} (\Delta_{\kappa_w} \hat{\pi}_{w,t} - \beta \xi_w \Delta_{\kappa_w} \hat{\pi}_{w,t+1}) \\
- \phi \frac{\lambda_w}{1-\lambda_w} \frac{1}{1-\beta \xi_w} \beta \xi_w \Delta_{\kappa_w} \hat{\pi}_{w,t+1}
\]

The relationship between wage and price inflation, the change in the real wage and technology growth is given by:

\[
\hat{w}_t = \hat{w}_{t-1} + \hat{\pi}_{w,t} - \hat{\pi}_t - \hat{\mu}_{z+,t}.
\]  

Then,

\[
\hat{\pi}_t = \hat{w}_{t-1} - \hat{w}_t + \hat{\pi}_{w,t} - \hat{\mu}_{z+,t} \\
\Delta_{\kappa_w} \hat{\pi}_t = \hat{\pi}_t - \kappa_w \hat{\pi}_{t-1} \\
= \hat{w}_{t-1} - \hat{w}_t + \hat{\pi}_{w,t} - \hat{\mu}_{z+,t} - \kappa_w \hat{\pi}_{t-1} \\
= \hat{w}_{t-1} - \hat{w}_t + \Delta_{\kappa_w} \hat{\pi}_{w,t} - \hat{\mu}_{z+,t}
\]

Use this to substitute out for \(\Delta_{\kappa_w} \hat{\pi}_{t+1}\) in (G.11):

\[
\frac{1}{1-\beta \xi_w} (\hat{w}_t - \beta \xi_w \hat{w}_{t+1}) + \frac{1}{1-\beta \xi_w} \frac{\xi_w}{1-\xi_w} (\Delta_{\kappa_w} \hat{\pi}_{w,t} - \beta \xi_w \Delta_{\kappa_w} \hat{\pi}_{w,t+1}) \\
- \frac{\beta \xi_w}{1-\beta \xi_w} (\hat{w}_t - \hat{w}_{t+1} + \Delta_{\kappa_w} \hat{\pi}_{t+1}) \\
= -\psi_{z+,t} + \phi \hat{H}_t + \phi \frac{\lambda_w}{1-\lambda_w} \frac{1}{1-\beta \xi_w} \frac{\xi_w}{1-\xi_w} (\Delta_{\kappa_w} \hat{\pi}_{w,t} - \beta \xi_w \Delta_{\kappa_w} \hat{\pi}_{w,t+1}) \\
- \phi \frac{\lambda_w}{1-\lambda_w} \frac{1}{1-\beta \xi_w} \beta \xi_w \Delta_{\kappa_w} \hat{\pi}_{w,t+1}
\]

Collecting terms in \(\hat{w}_t, \hat{w}_{t+1}, \hat{\pi}_{w,t}, \hat{\pi}_{w,t+1}\):

\[
\begin{align*}
\left[ \frac{1}{1-\beta \xi_w} - \frac{\beta \xi_w}{1-\beta \xi_w} \right] \hat{w}_t - \left[ \frac{\beta \xi_w}{1-\beta \xi_w} - \frac{\beta \xi_w}{1-\beta \xi_w} \right] \hat{w}_{t+1} \\
+ \left[ \frac{1}{1-\beta \xi_w} - \phi \frac{\lambda_w}{1-\lambda_w} \frac{1}{1-\beta \xi_w} \frac{\xi_w}{1-\xi_w} \right] \Delta_{\kappa_w} \hat{\pi}_{w,t} \\
- \left[ \frac{\beta \xi_w}{1-\beta \xi_w} - \phi \frac{\lambda_w}{1-\lambda_w} \frac{\beta \xi_w}{1-\beta \xi_w} \frac{\xi_w}{1-\xi_w} \right] \Delta_{\kappa_w} \hat{\pi}_{w,t+1} \\
- \phi \frac{\lambda_w}{1-\lambda_w} \frac{1}{1-\beta \xi_w} \beta \xi_w \Delta_{\kappa_w} \hat{\pi}_{w,t+1}
\end{align*}
\]

\[
= -\psi_{z+,t} + \phi \hat{H}_t
\]
or,

\[
\frac{1}{1 - \beta \xi_w} \frac{1 - \lambda_w}{1 - \xi_w} \left[ 1 - \phi \frac{\lambda_w}{1 - \lambda_w} \right] \Delta_{\kappa_{w,t}} \hat{\pi}_{w,t} = -\hat{\psi}_{z+t} + \phi \hat{H}_t - \hat{\omega}_t + \frac{\beta \xi_w}{1 - \beta \xi_w} \frac{1}{1 - \xi_w} \left[ 1 - \phi \frac{\lambda_w}{1 - \lambda_w} \right] \Delta_{\kappa_{w,t+1}} \hat{\pi}_{w,t+1}
\]

Let

\[
\kappa_w = \frac{(1 - \xi_w)(1 - \beta \xi_w)}{\xi_w}
\]

Multiply the previous expression by \(\kappa_w\) and rearrange:

\[
\left( 1 - \phi \frac{\lambda_w}{1 - \lambda_w} \right) \Delta_{\kappa_{w,t}} \hat{\pi}_{w,t} = \kappa_w \left( -\hat{\psi}_{z+t} + \phi \hat{H}_t - \hat{\omega}_t \right) + \left( 1 - \phi \frac{\lambda_w}{1 - \lambda_w} \right) \beta \Delta_{\kappa_{w,t+1}} \hat{\pi}_{w,t+1}.
\]