Bank Leverage and Social Welfare

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We describe a general equilibrium model in which there is a particular agency problem in banks. The agency problem arises because bankers must exert an unobserved and costly effort to increase the likelihood of good outcomes on the asset side of their balance sheet. We focus on a scenario in which aggregate banker net worth is too low to insulate creditors from bad outcomes on the bankers' balance sheet. In this case, the banking system is distorted because there is a pecuniary externality associated with bank borrowing. The result is that banks borrow too much in equilibrium. Social welfare is increased by imposing a leverage restriction on banks. We formalize this argument and provide a numerical example. We suspect that in a dynamic version of the model, it is desirable to restrict bank leverage in good times so that banks have a cushion in the event of a negative shock to net worth.

I. The Model

There are two periods, 1 and 2. The representative household consists of a representative worker and a mass of bankers that are ex ante identical. The bankers and the worker are endowed with N and Y units of the good in period 1, respectively.

The problem of the household in period 1 is to divide Y into consumption, c, and deposits, d, with a mutual fund, subject to $c + d \leq Y$. In period 2 the household's budget constraint is $C \leq Rd + \pi$, where R denotes the return on deposits, π denotes the lump sum profits brought home by the bankers, and C denotes period 2 consumption. Bankers and workers in a household share equally in period 1 and period 2 consumption. The household chooses d to optimize lifetime utility, $u(c) + \beta C$, where $0 < \beta < 1$ and u is a strictly increasing, strictly concave and twice differentiable function. Linearity of utility in the second period is assumed for analytic simplicity. An interior optimum for the household choice of d implies:

$$\beta R = u'(Y - d). \tag{1}$$

We refer to this expression as the *supply of deposits*.

Each banker leaves the household in period 1 and sets up a bank. The banker has an option to take a deposit, d, from a mutual fund. Under this option, the banker combines its net worth with the deposit, and invests $N + d$ in a productive asset. The return on the asset is technologically determined and can be either good, g, or bad, b. If the return is i, for $i = g, b$, then the banker receives $R^i(N + d)$ at the start of period 2, for $i = g, b$, where $R^g > R^b$. By making an effort, e, the banker increases the probability, $p(e)$, that it receives a good return on its asset, where $p(e) = \min\left\{\bar{a} + \bar{b}e, 1\right\}$, $\bar{a}, \bar{b} > 0$. The banker obtains d from a mutual fund as part of a deposit contract. The contract also specifies the return, R_d^i, paid by the banker to the mutual fund in period 2 contingent on the realized return on the banker's investment, $i = g, b$, which the mutual fund observes. The banker's level of effort is not observed by the mutual fund.

The mutual fund is perfectly diversified across the risky bankers and makes profits, $\left[p(e) R_d^g + (1 - p(e)) R_d^b\right] d - Rd$. The value of the mutual fund's outside option is zero and we assume the banker makes a take-it-or-leave it offer. So, the mutual fund's profits from its

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dealings with a banker are driven to zero:

$$p(e)\, R_d^g + (1 - p(e))\, R_d^b = R. \tag{2}$$

The the contract offered by the bank satisfies two additional conditions. The first corresponds to the cash constraints:

$$R^g (N + d) \geq R_d^g d, \;\; R^b (N + d) \geq R_d^b d. \tag{3}$$

We refer to the latter constraint as the *bad-state cash constraint*. These constraints simply say that the bank cannot offer a contract in which it promises to do something that is infeasible. The cash constraints reflect our assumption that the bank has no other access to finance apart from the deposit contract. The final condition satisfied by the contract is implied by our assumption that the mutual fund cannot observe the banker's level of effort, e. As a result, the contract (correctly) takes it for granted that after the banker receives a loan contract, d, R_d^g, R_d^b, it chooses whatever value of e it deems to be privately optimal. The equation which characterizes the banker's choice of e is the banker's *incentive constraint*.

The banker's objective is to maximize expected return, with a deduction for the banker's utility cost of exerting effort, $e^2 (N + d)/2$. Note that the marginal cost of e is increasing in the size of the bank's balance sheet. In adopting this specification, we follow the suggestion of Ferrante and Prestipino (private communication).[1] By instructing all its bankers to behave in this way, the household maximizes the total return received from all its bankers, after deducting for the banker's utility cost of effort.[2] In exchange, the household offers each banker perfect consumption insurance against the uncertain return on its investment. In considering which deposit contract to offer the mutual fund, the banker's objective is $V\left(e, d, R_d^g, R_d^b\right)$, where

$$\begin{aligned} V&\left(e, d, R_d^g, R_d^b\right) \\ &= p(e)\, \beta \left[R^g (N + d) - R_d^g d\right] \\ &+ (1 - p(e))\, \beta \left[R^b (N + d) - R_d^b d\right] \\ &- \frac{1}{2} e^2 (N + d). \end{aligned} \tag{4}$$

The banker's *deposit contract problem* is to select the parameters of the loan contract, R_d^g, R_d^b, d, to optimize (4) subject to (2), (3) and the banker's incentive constraint[3] :

$$\begin{aligned} &e(N + d) \\ &= \beta \bar{b} \left[\left(R^g - R^b\right)(N + d) - \left(R_d^g - R_d^b\right) d\right]. \end{aligned} \tag{5}$$

Let V denote the optimized value of (4). The banker must decide whether or not to take a deposit contract by comparing V with the value of its outside option. For its outside option the banker can do one of two things. It can choose to deposit N with a mutual fund, set $e = 0$ and earn RN at the start of period 2. The value of this option from the point of view of period 1 is βRN. Alternatively, the banker can invest N, choose the optimal level of effort, $e^* = \beta \bar{b} \left(R^g - R^b\right)$[4] and earn a return,

$$V^* = \beta \left[p^* \beta R^g + \left(1 - p^*\right) \beta R^b\right] N - \frac{(e^*)^2}{2} N,$$

where p^* is the probability of success implied by e^*. The value in period 1 of the banker's outside option is $V^o = \max\left(V^*, \beta RN\right)$. The *banker's problem* is to select $\max\left(V, V^o\right)$.

We study a *deposit contract equilibrium*, in which bankers choose the deposit contract. We define such an equilibrium as follows:

DEFINITION 1: *A deposit contract equilibrium is a set of numbers, R, p, e, d, R_d^g, R_d^b such that $d > 0$ and*

[1] In the online technical appendix, we illustrate the Ferrante and Prestipino observations that the model is uninteresting if the cost of effort is simply $e^2/2$.

[2] Banker effort is observable to its own household.

[3] This is the incentive constraint for interior e. When e is at its upper bound, $(1 - \bar{a})/\bar{b}$, the equality is replaced by $\leq$. When e is at its lower bound, the equality is replaced by $\geq$. In the calculations done for the numerical example, we do not restrict ourselves to interior solutions, though the solutions do end up being interior. See the online technical appendix for details.

[4] We assume, $\beta \bar{b} \left(R^g - R^b\right) < (1 - \bar{a})/\bar{b}$.

(i) conditional on R, the four numbers, e, d, R_d^g, R_d^b solve the banker's deposit contract problem, with $V \geq V^o$,

(ii) R satisfies household optimality, (1).

II. The Desirability of Leverage Restrictions

If N or R^b are small enough, then the bad-state cash constraint binds in the banker's deposit contract problem. In this case, it is necessary that R_d^b be low in order to satisfy (3). Because of the zero profit condition, a mutual fund must be compensated with $R_d^g > R$. The resulting distortion to banker effort (see (5)) has the implication that a leverage restriction improves social welfare.

A. Equilibrium with a Binding Leverage Constraint

Suppose there is a regulation that places an upper bound, $\bar{d}$, on the amount of deposits that a banker may take:

$$\bar{d} - d \geq 0. \tag{6}$$

We assume that $\bar{d}$ is less than what the banker would choose in the absence of the regulation, so that the constraint is binding. Although the deposit choice itself is now off the table, the banker must still choose the terms of the deposit contract, R_d^g and R_d^b. We assume that the bad-state cash constraint and the incentive constraint are binding. The former implies

$$R_d^b = (N + d)\, R^b / d. \tag{7}$$

The zero profit condition, (2), implies

$$R_d^g - R_d^b = \left(R - R_d^b\right) / p\,(e)\,. \tag{8}$$

Using these two expressions to substitute $R_d^g - R_d^b$ out of (5), the incentive constraint becomes:

$$\beta \bar{b} \left[R^g - R^b - \frac{R \frac{d}{N+d} - R^b}{p\,(e)} \right] - e = 0. \tag{9}$$

It is convenient to study the Lagrangian representation of the banker problem, in which the banker chooses e and d, to maximize (4) subject to (6) and (9). Interior optimality of d implies:

$$\begin{aligned} & p\,(e)\, R^g + (1 - p\,(e))\, R^b \\ = \; & R + \frac{e^2}{2\beta} + \eta \bar{b} \frac{\frac{RN}{(N+d)^2}}{p\,(e)} + \frac{1}{\beta}\xi, \end{aligned} \tag{10}$$

where $\xi \geq 0$ and $\eta \geq 0$ denote the multipliers on (6) and (9), respectively.[5] The left side of this expression captures the benefit of a marginal increase in d arising from the increase in expected earnings on assets. The right side is the sum of four costs associated with a marginal increase in bank borrowing: (i) the rise in expected interest costs[6], (ii) the additional utility cost of effort; (iii) the extra distortion to the incentive constraint; (iv) the cost of tightening the bank's leverage constraint.

The equilibrium for the regulated economy is computed as follows. The value of R can be read directly from the deposit supply relation, (1), with $d = \bar{d}$. Then, e is computed using (9).[7] The multipliers, η and ξ, can be obtained from the banker first order conditions for d and e (the latter is not displayed). Finally, R_d^g and R_d^b are obtained from (7) and (8).

B. Decision Problem for Benevolent Regulator

The regulator's choice of $\bar{d}$ affects all the equilibrium variables. So, the benevolent regulator in effect chooses d, e, R, R_d^g and R_d^b to optimize social welfare subject to the equilibrium conditions of the private sector. We use the deposit supply equation, the bad-state incentive constraint and the zero profit condition to eliminate R, R_d^g and R_d^b. The regulator's problem is

[5] The non-negativity of ξ is the Kuhn-Tucker result for the sign of the multiplier on an inequality constraint. The sign of the multiplier on the incentive constraint, which is an equality constraint, can be motivated intuitively as follows. Suppose the Lagrangian representation of the banker's contract problem is solved with $\eta = 0$. The solution is $e = \beta \bar{b} \left(R^g - R^b\right)$ which, when substituted into (9), makes the object on the left of the equality negative. The Lagrangian problem penalizes this violation of the incentive constraint by making η positive.

[6] Recall, via the zero profit condition, that $R = p\,(e)\, R_b^g + (1 - p\,(e))\, R_d^b$, where R_d^i are the interest rate costs on deposits, $i = g, b$. This is why R is referred to as the *expected* interest cost of an additional deposit.

[7] In case there is more than one e that solves (9) for the given $\bar{d}$, pick the one associated with the higher value of the banker objective.

to maximize social welfare,

$$\max_{0\leq d\leq Y,\ 0\leq e\leq(1-\bar{a})/\bar{b}} u\left(Y-d\right)-\frac{e^2}{2}\left(N+d\right) +\beta\left[p\left(e\right)R^g+\left(1-p\left(e\right)\right)R^b\right]\left(N+d\right) \tag{11}$$

subject to the banker's incentive constraint, (9), in which R is replaced by (1). Interior optimality of d implies:

$$p\left(e\right)R^g+\left(1-p\left(e\right)\right)R^b=R +\frac{e^2}{2\beta}+\eta\bar{b}\frac{\frac{RN}{(N+d)^2}}{p\left(e\right)}+\eta\bar{b}\frac{\frac{-u''(Y-d)}{\beta}\frac{d}{N+d}}{p\left(e\right)}. \tag{12}$$

Here, $\eta\geq 0$ denotes the multiplier on the banker's incentive constraint and $-u''\left(Y-d\right)/\beta > 0$ denotes the derivative of R with respect to d, taking into account (1). The left side of (12) captures the benefit of a marginal increase in d from the point of view of the regulator. By comparing (10) with (12), we see that the regulator and the banker have the same assessment of these benefits. The four terms on the right side of (12) represent the four costs the regulator associates with a marginal increase in bank borrowing. The first three costs coincide with the first three costs perceived by the banker (see (i), (ii) and (iii) after (10)). The fourth term in (12) captures the tightening of the banker's incentive constraint that occurs as a marginal increase in bank borrowing raises R (see the $-u''$ term in (12)). The banker does not internalize this general equilibrium effect on the market interest rate, R, of increasing its borrowing. The effect constitutes an externality, a *pecuniary externality,* because extra borrowing by one banker tightens the incentive constraints of all the other bankers via the price system. This externality exists only when the bad-state cash and incentive constraints are binding. In this case, the unregulated banking system is characterized by over borrowing.

The regulator can induce the banker to internalize the externality by implementing a binding leverage constraint. This is because, as we saw above, the leverage constraint introduces a fourth cost into the banker's first order condition for e (see (iv) after (10)). That cost precisely matches the cost assigned by the regulator if $\xi=\eta\bar{b}\left(-u''\left(Y-d\right)d\right)/((N+d)\,p(e))$. It is easily verified that if the value of d that solves the regulator's problem is imposed as a leverage restriction on the private economy, then the private economy equilibrium coincides with the equilibrium selected by the regulator. An ingredient in this result is the fact that optimality of the e choice by the banker and by the regulator lead to the same first order condition.

III. Numerical Example

We assigned the following parameter values:

$$\begin{aligned} R^g &= 1.20,\ R^b=0.6,\ \beta=0.99, \\ \bar{a} &= 0.1,\ \bar{b}=1,\ N=0.1,\ Y=2. \end{aligned}$$

In addition, we assume that $u\left(c\right)=\log\left(c\right)$. Let $L\equiv\left(N+d\right)/N$ denote the level of leverage in the unregulated economy. In the numerical example, $L=8.10$. Let $\bar{L}$ denote the maximal amount of bank leverage permitted under the regulation and let $f\equiv\bar{L}/L$. We considered a range of values of f, $0.9\leq f\leq 1$. The results are displayed in Figure 1. Note that social welfare, (11), is maximized by restricting leverage by 6 percent relative to its value in the unregulated equilibrium (see Panel a). Panel e indicates that as the leverage restriction gets tighter, deposits move in the opposite direction relative to their first best level (i.e., the value of d that solves (11) ignoring the incentive constraint). This illustrates the distinction between constrained and unconstrained efficiency. By tightening the leverage constraint, the regulator reduces the distortions to incentives: the interest rate spread (R_d^g-R in Panel b), $R_d^g-R_d^b$ and η (Panel c) all fall. The reduced distortions lead to a rise in effort, e, (Panel d) and the efficiency of the banking system is improved. The latter effect more than compensates for the negative effect of moving d in the opposite direction from its first best level. Panel f confirms that the value to a banker of issuing deposits is higher than the value of its outside option, as is required for a deposit contract equilibrium. Interestingly, the value of the deposit contract, V, increases with a tightening in the leverage constraint. This reflects that leverage restrictions have an effect that resembles collusion among banks, allowing them to earn monopsony profits. This has important implications for the dynamic version of this model (see, Christiano and Ikeda (2014)).

[Figure 1: six figures should be placed here. See attached pdf file.]

Bigio, Saki. 2014. "Leverage Restrictions in a Business Cycle Model: A Comment." In Bauducco, Christiano and Raddatz, editors, *Macroeconomic and Financial Stability: Challenges for Monetary Policy*. Banco Central de Chile.

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Figure 1: Impact on Model of Reducing Leverage from L in Unregulated Economy to f*L