Leverage Restrictions in a Business Cycle Model *

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Abstract

We modify an otherwise standard medium-sized DSGE model, in order to study the macroeconomic effects of placing leverage restrictions on financial intermediaries. The financial intermediaries ('bankers') in the model must exert effort in order to earn high returns for their creditors. An agency problem arises because banker effort is not observable to creditors. The consequence of this agency problem is that leverage restrictions on banks generate a very substantial welfare gain in steady state. We discuss the economics of this gain. As a way of testing the model, we explore its implications for the dynamic effects of shocks.

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1 Introduction

We seek to develop a business cycle model with a financial sector, which can be used to study the consequences of policies to restrict the leverage of financial institutions ('banks').[1] Because we wish the model to be consistent with basic features of business cycle data, we introduce our banking system into a standard medium sized DSGE model such as Christiano, Eichenbaum and Evans (2005) (CEE) or Smets and Wouters (2007). Banks in our model operate in perfectly competitive markets. Our model implies that social welfare is increased by restricting bank leverage relative to what leverage would be if financial markets were unregulated. With less leverage, banks are better able to use their net worth to insulate creditors in case there are losses on bank balance sheets. Our model implies that by reducing risk to creditors, agency problems are mitigated and the efficiency of the banking system is improved. We explore the economics of our result by studying the model's steady state. We also display various dynamic features of the model to assess its empirical plausibility.

There are two types of motivations for restrictions on banking leverage. One motivates leverage restrictions as a device to correct an agency problem in the private economy. Another motivates leverage restrictions as a device to correct a commitment problem in the government.[2] In this paper we focus on the former type of rationale for leverage restrictions.

We posit the existence of an agency problem between banks and their creditors. By bank creditors we have in mind real-world depositors, holders of debt securities like bonds and commercial paper, and also holders of bank preferred stock.[3] As a result, bank credit in our

[1] By 'banks' we mean all financial institutions, not just commercial banks.

[2] For example, Chari and Kehoe (2012) show that a case for leverage restrictions can be built on the assumptions that (i) bankruptices are ex post inefficient and (ii) governments are unable to commit ex ante to not bailout failed banks. See also Gertler, Kiyotaki and Queralto (2011) for a discussion. In the general discussion of Adrian, Colla and Shin (forthcoming), Robert Hall draws attention to the implications for bank leverage decisions of the expectation of government intervention in a crisis episode.

[3] Our logic for including bank preferred stock in bank 'credit' is as follows. In our model, the liability side of bank balance sheets has only 'bank debt' and 'bank net worth'. For the vast majority of banks in our model, their asset portfolio performs well enough that debt holders receive a high return and bank net worth generally earns a positive return. In the case of banks in our model whose portfolio of assets performs poorly, net worth is wiped out and debt holders earn a low return. The reason we think of preferred stock as part of bank debt in the model is: (i) dividend payments on preferred stock are generally not contingent on the overall performance of the bank's assets, unless the performance of the assets is so bad that common stock holders are wiped out; and (ii) like ordinary debt, holders of preferred stock do not enjoy voting rights. Our model abstracts from the differences that do exist between the different components of what we call bank debt. For example, dividends on preferred stock are paid after interest and principal payments on a bank's bonds, commercial paper and deposits. In addition, the tax treatment of preferred stock is different from the tax treatment of a bank's bond and commercial paper. The reason we identify the common stock portion of bank liabilities with bank net worth in our model is that holders of common stock are residual claimants. As a result, they are the recipients of increases in bank earnings (magnified by leverage) and they suffer losses when earnings are low (and, these losses are magnified by leverage). Financial firms are very important in the market for preferred stock. For example, Standard and Poor's computes an overall index of the price and yield on preferred stock. In their index for December 30, 2011, 82 percent of the firms belong to the financial sector (see https://www.sp-indexdata.com/idpfiles/strategy/prc/active/factsheets/fs-sp-us-preferred-stock-index-ltr.pdf).

model is risky. To quantify this risk, we calibrate the model to the premium paid by banks for funds in the interbank market. This premium is on average about 50 basis points at an annual rate.[4] To simplify the analysis, we assume there is no agency problem on the asset side of banks' balance sheets. The role of banks in our model is to exert costly effort to identify good investment projects. The source of the agency problem in our model is our assumption that bank effort is not observed. Under these circumstances it is well known that competitive markets do not necessarily generate the efficient allocations. In our analysis, the fact that banker effort is unobserved has the consequence that restricting the amount of liabilities a bank may issue raises welfare.

As in any model with hidden effort, the resulting agency problem is mitigated if the market provides the agent (i.e., the banker) with the appropriate incentives to exert effort. For this, it is useful if the interest rate that the banker pays to its creditors is not sensitive to the performance of the asset side of its balance sheet. In this case, the banker reaps the full reward of its effort. But, this requires that the banker have sufficient net worth on hand to cover the losses that will occasionally occur even if a high level of effort is expended. The creditors in low net worth banks which experience bad outcomes on their portfolio necessarily must share in bank losses. Understanding this in advance, creditors require that low net worth bankers with well-performing porfolios pay a high interest rate. Under these circumstances, the banker does not enjoy the full fruits of its effort and so its incentive to exert effort is correspondingly reduced.

We analyze the steady state properties of the model and show that a leverage restriction moves equilibrium consumption and employment in the direction of the efficient allocations that would occur if effort were observable. In particular, when banks are restricted in how many liabilities they can issue, then they are more likely to be able to insulate their creditors from losses on the asset side of their balance sheet. In this way leverage restrictions reduce the interest rate spread faced by banks and promote their incentive to exert effort. We calibrate our model's parameters so that leverage is 20 in the absence of regulation. When a regulation is imposed that limits leverage to 17, steady state welfare jumps an amount that is equivalent to a permanent 1.19 percent jump in consumption.[5]

After obtaining these results for the steady state of the model, we turn to its dynamic

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[4] We measure the interest rate on the interbank market by the 3 month London interbank offer rate (LIBOR). The interest rate premium is the excess of LIBOR over the 3 month rate on US government Treasury bills.

[5] In our analysis, we do not factor in the bureaucratic and other reporting costs of leverage restrictions. If we do so, presumably the steady state welfare benefit of leverage would be smaller. However, because the benefits reported in this paper are so large, we expect our finding that welfare increases to be robust.

properties. We display the dynamic response of various variables to four shocks. Of these, one is a monetary policy shock, two are shocks to bank net worth and fourth is a shock to the cross-sectional dispersion of technology.[6] In each case, a contractionary shock drives down consumption, investment, output, employment, inflation and bank net worth, just as in actual recessions. In addition, all four shocks raise the cross-sectional dispersion of bank equity returns. We use Center for Research on Security Prices (CRSP) data to show that this implication is consistent with the data. The countercyclical nature of various measures of dispersion has been a subject of great interest since Bloom (2009) drew attention to the phenomenon. A factor that may be of independent interest is that our paper provides examples of how this increase in dispersion can occur endogenously. Finally, we show that the shocks in our model imply that bank leverage is countercyclical. We show that broad-based empirical measures of leverage are consistent with this implication.

The paper is organized as follows. The next section describes the circumstances of the bankers. We then describe the general macroeconomic environment into which we insert the bank. After that we report our findings for leverage and for the dynamic properties of our model. A last section includes concluding remarks.

2 Banks, Mutual Funds and Entrepreneurs

We begin the discussion in period t, after goods production for that period has occurred. There is a mass of identical bankers with net worth, N_t. The bankers enter into competitive and anonymous markets, acquire deposits from mutual funds and lend their net worth and deposits to entrepreneurs. Mutual funds take deposits from households and make loans to a diversified set of banks. The assumption that mutual funds stand between households and banks is made for convenience. Our bankers are risky and if households placed deposits directly with banks they would choose to diversify across banks. The idea that households diversify across a large set of banks seemed awkward to us. Instead, we posit that households place their deposits with mutual funds, and then mutual funds diversify across banks. Another advantage of our assumption that mutual funds stand between households and banks is that this allows us to define a risk free rate of interest. However, nothing of substance hinges on the presence of the mutual funds.

Each entrepreneur has access to a constant returns to scale investment technology. The technology requires as input an investment at the end of goods production in period t and

[6] For the latter we consider a risk shock, as in Christiano, Motto and Rostagno (2012).

produces output during production in $t+1$. Entrepreneurs are competitive, earn no rent and there is no agency problem between entrepreneurs and banks. The bank from which an entrepreneur receives its loan receives the full rate of return earned by entrepreneurs on their projects.

There are 'good' and 'bad' entrepreneurs. We denote the gross rate of return on their period t investment by R^g_{t+1} and R^b_{t+1}, respectively, where $R^g_{t+1} > R^b_{t+1}$ in all period $t+1$ states of nature. These represent exogenous stochastic processes from the point of view of entrepreneurs. We discuss the factors that determine these rates of return in the next section. There, we situate entrepreneurs and bankers in the broader macro economy.

A key function of banks is to identify good entrepreneurs. To do this, bankers exert a costly effort. In our baseline model this effort is not observable to the mutual funds that supply the banks with funds, and this creates an agency problem on the liability side of a bank's balance sheet. As a convenient benchmark, we also consider the version of the model in which banker effort is observable to the mutual fund which supplies the bank with deposits, d_t.

At the end of production in period t each banker takes deposits, d_t and make loans in the amount, $N_t + d_t$, to entrepreneurs. We capture the idea that banks are risky with the assumption that a bank can only invest in one entrepreneur.[7] The quantities, N_t and d_t are expressed in per capita terms.

We denote the effort exerted by a banker to find a good entrepreneur by e_t. The banker identifies a good entrepreneur with probability $p_t(e_t)$ and a bad entrepreneur with the complementary probability. For computational simplicity, we adopt the following simple representation of the probability function:

$$p(e) = \min\left\{1, \bar{a} + \bar{b}e\right\}, \ \bar{a}, \bar{b} \geq 0$$

Because we work with equilibria in which $p(e) > 1/2$, our model implies that when bankers exert greater effort, the mean return on their asset increases and its variance decreases.

Mutual funds are competitive and perfectly diversified across good and bad banks. As a result of free entry, they enjoy zero profits:

$$p(e_t) R^d_{g,t+1} + (1 - p(e_t)) R^d_{b,t+1} = R_t, \tag{1}$$

[7] We can describe the relationship between a bank and an entrepreneur in search theoretic terms. Thus, the bank exerts an effort, e_t, to find an entrepreneur. Upon exerting this effort a bank meets exactly one entrepreneur in a period. We imagine that the outside option for both the banker and the entrepreneur at this point is zero. We suppose that upon meeting, the bank has the option to make a take-it-or-leave-it offer to the entrepreneur. Under these circumstances, the bank will make an offer that puts the entrepreneur on its outside option of zero. In this way, the banker captures all the rent in their relationship.

in each period $t+1$ state of nature. Here, $R^d_{g,t+1}$ and $R^d_{b,t+1}$ denote the gross return received from good and bad banks, respectively. In (1), $p(e_t)$ is the fraction of banks with good returns and $1-p(e_t)$ is the fraction of banks with bad returns.[8] The following two subsections discuss the deposit contracts between banks and mutual funds that emerge in equilibrium. The first discussion reviews the case when mutual funds observe e_t. The case that we consider empirically relevant is the one in which the e_t selected by a bank is not observed by the mutual fund that provides the bank with deposits. The latter case is considered in the subsequent section. After that we describe the aggregate law of motion of banker net worth. Finally, we describe the changes to the environment when there are binding leverage restrictions.

2.1 Deposit Contracts When Banker Effort is Observable

A loan contract between a banker and a mutual fund is characterized by four objects,

$$\left(d_t, e_t, R^d_{g,t+1}, R^d_{b,t+1}\right). \tag{2}$$

In this section, all four elements of the contract are assumed to be directly verifiable by the mutual fund. Throughout this paper, we assume that sufficient sanctions exist so that verifiable deviations from a contract never occur.

The representative mutual fund takes R_t as given. We assume the banker's only source of funds for repaying the mutual fund is the earnings on its investment. Regardless of the return on its asset, the banker must earn enough to pay its obligation to the mutual fund:

$$R^g_{t+1}\left(N_t+d_t\right)-R^d_{g,t+1}d_t\geq 0,\ \ R^b_{t+1}\left(N_t+d_t\right)-R^d_{b,t+1}d_t\geq 0.$$

Mutual funds are obviously only interested in contracts that are feasible, so the above inequalities represent restrictions on the set of contracts that mutual funds are willing to consider. In practice, only the second inequality represents a restriction.

In equilibrium, each bank has access to a menu of contracts, defined by the objects in (2)

[8] We obtain (1) as follows. The period t measure of profits for mutual funds is

$$E_t\lambda_{t+1}\left[p(e_t)R^d_{g,t+1}+(1-p(e_t))R^d_{b,t+1}-R_t\right],$$

where the product of λ_{t+1} and the associated conditional probability is proportional to the state contingent price of cash. In addition, we assume the only source of funds for mutual funds in period $t+1$ is the revenues from banks, so that mutual funds have the following state-by-state non-negativity constrain:

$$p(e_t)R^d_{g,t+1}+(1-p(e_t))R^d_{b,t+1}-R_t\geq 0.$$

Equation (1) is implied by the zero profit condition and the above non-negativity constraint.

which satisfy (1) and

$$R^b_{t+1}\left(N_t+d_t\right)-R^d_{b,t+1}d_t\geq 0, \tag{3}$$

as well as non-negativity of e_t and d_t. The problem of the banker is to select a contract from this menu.

A banker's ex ante reward from a loan contract is:

$$E_t\lambda_{t+1}\left\{p\left(e_t\right)\left[R^g_{t+1}\left(N_t+d_t\right)-R^d_{g,t+1}d_t\right]+\left(1-p\left(e_t\right)\right)\left[R^b_{t+1}\left(N_t+d_t\right)-R^d_{b,t+1}d_t\right]\right\}-\frac{1}{2}e_t^2, \tag{4}$$

where $e_t^2/2$ is the banker's utility cost of expending effort and λ_{t+1} denotes the marginal value of profits to the household. As part of the terms of the banker's arrangement with its own household, the banker is required to seek a contract that maximizes (4).[9] Formally, the banker maximizes (4) by choice of e_t, d_t, $R^d_{g,t+1}$, an $R^d_{b,t+1}$ subject to (1) and (3). In Appendix A, we show that (3) is non-binding and that the following are the optimization conditions:

$$\begin{aligned}
e &: \; e_t=E_t\lambda_{t+1}p_t'\left(e_{t+1}\right)\left(R^g_{t+1}-R^b_{t+1}\right)\left(N_t+d_t\right) && (5)\\
d &: \; E_t\lambda_{t+1}\left[p_t\left(e_t\right)R^g_{t+1}+\left(1-p_t\left(e_t\right)\right)R^b_{t+1}-R_t\right]=0 && (6)\\
\mu &: \; R_t=p_t\left(e_t\right)R^d_{g,t+1}+\left(1-p_t\left(e_t\right)\right)R^d_{b,t+1}. && (7)
\end{aligned}$$

Here, the letter before the colon indicates the variable being differentiated in the Lagrangian version of the bank's optimization problem. The object, μ denotes the multiplier on (1). Note from (5) how the size of the base, N_t+d_t, on which banks make profits affects effort, e_t. Also, note from (5) that in setting effort, e, the banker looks only at the sum, N_t+d_t, and not at how this sum breaks down into the component reflecting banker's own resources, N_t, and the component reflecting the resources, d_t, supplied by the mutual fund. By committing to care for d_t as if these were the banker's own funds, the banker is able to obtain better contract terms from the mutual fund. The banker is able to commit to the level of effort in (5) because e_t is observable to the mutual fund.

The values of the state contingent return on the deposits of banks with good and bad investments, $R^d_{g,t+1}$, $R^d_{b,t+1}$ are not uniquely pinned down. These returns are restricted only by (7) and (3). For example, the following scenario is compatible with the equations, $R^d_{g,t+1}=R^g_{t+1}$, $R^d_{b,t+1}=R^b_{t+1}$. It may also be possible for the equations to be satisfied by a non-state

[9] Throughout the analysis we assume the banker's household observes all the variables in (4) and that the household has the means (say, because the household could threaten to withhold the perfect consumption insurance that it provides) to compel the banker to do what the household requires of it.

contingent pattern of returns, $R_{g,t+1}^d = R_{b,t+1}^d = R_t$. However, (3) indicates that the latter case requires N_t to be sufficiently large.

2.2 Deposit Contracts When Banker Effort is Not Observable

We now suppose that the banker's effort, e_t, is not observed by the mutual fund. Thus, whatever d_t, $R_{g,t+1}^d$, $R_{b,t+1}^d$ and e_t is specified in the contract, a banker always chooses e_t ex post to maximize (4). The first order condition necessary for optimality is:

$$e : e_t = E_t \lambda_{t+1} p_t' (e_t) \left[\left(R_{t+1}^g - R_{t+1}^b \right) (N_t + d_t) - \left(R_{g,t+1}^d - R_{b,t+1}^d \right) d_t \right]. \tag{8}$$

Note that $R_{g,t+1}^d > R_{b,t+1}^d$ reduces the banker's incentive to exert effort. This is because in this case the banker receives a smaller portion of the marginal increase in expected profits caused by a marginal increase in effort. The representative mutual fund understands that e_t will always be selected according to (8). Since the mutual fund is only interested in contracts that will actually be implemented, it will only offer contracts that satisfy not just (3), but also (8). Thus, we assume that the menu of contracts that exists in equilibrium is the set of $\left(d_t, e_t, R_{g,t+1}^d, R_{b,t+1}^d\right)$'s that satisfy (1), (3) and (8). The banker's problem now is to maximize (4) subject to these three conditions. In the appendix, we show that the conditions for optimization are:

$$\begin{aligned}
e &: \; E_t \left(\lambda_{t+1} + \nu_{t+1} \right) p_t' (e_t) \left(R_{g,t+1}^d - R_{b,t+1}^d \right) d_t + \eta_t = 0 \\
d &: \; 0 = E_t \left(\lambda_{t+1} + \nu_{t+1} \right) \left[p_t (e_t) \left(R_{t+1}^g - R_{g,t+1}^d \right) + (1 - p_t (e_t)) \left(R_{t+1}^b - R_{b,t+1}^d \right) \right] \\
R_g^d &: \; \nu_{t+1} p_t (e_t) + \eta_t \lambda_{t+1} p_t' (e_t) = 0 \\
\mu &: \; R_t = p_t (e_t) R_{g,t+1}^d + (1 - p_t (e_t)) R_{b,t+1}^d \\
\eta &: \; e_t = E_t \lambda_{t+1} p_t' (e_t) \left[\left(R_{t+1}^g - R_{t+1}^b \right) (N_t + d_t) - \left(R_{g,t+1}^d - R_{b,t+1}^d \right) d_t \right] \\
\nu &: \; \nu_{t+1} \left[R_{t+1}^b (N_t + d_t) - R_{b,t+1}^d d_t \right] = 0, \;\; \nu_{t+1} \geq 0, \;\; \left[R_{t+1}^b (N_t + d_t) - R_{b,t+1}^d d_t \right] \geq 0.
\end{aligned} \tag{9}$$

Here, η_t is the multiplier on (8), ν_{t+1} is the multiplier on (3). The date on a multiplier indicates the information on which it is contingent. Thus, η_t, ν_t and μ_t are each contingent on the period t realization of aggregate shocks. For computational simplicity, we only consider parameter values such that the cash constraint, (3), is always binding. The first three equations in (9) correspond to first order conditions associated with the Lagrangian representation of the banker problem, with the names corresponding to the variable being differentiated.

The magnitude of the multiplier, $\nu_{t+1} \geq 0$, is a measure of the inefficiency of the banking system. If ν_{t+1} is zero, then $\eta_t = 0$ is zero by the R_g^d condition in (9). Then, combining the e

equation with the η equation, we see that e_t is set efficiently, in the sense that it is set according to (5). When $\nu_{t+1} > 0$ then $\eta_t < 0$ and e_t is below the level indicated by (5).[10]

A notable feature of the model concerns its implication for the cross-sectional variance on the rate of return on bank equity. In period $t+1$, the realized rate of return on bank equity for the $p(e_t)$ successful banks and for the $1-p(e_t)$ unsuccessful banks is, respectively,

$$\frac{R^g_{t+1}(N_t+d_t)-R^d_{g,t+1}d_t}{N_t}, \frac{R^b_{t+1}(N_t+d_t)-R^d_{b,t+1}d_t}{N_t}.$$

Given our assumption that the cash constraint is binding for unsuccessful banks, the second of the above two returns is zero. So, the period t cross-sectional standard deviation, s^b_{t+1}, and mean, E^b_{t+1}, of bank equity returns is:[11]

$$\begin{aligned} s^b_{t+1} &= [p(e_t)(1-p(e_t))]^{1/2}\frac{R^g_{t+1}(N_t+d_t)-R^d_{g,t+1}d_t}{N_t}, \\ E^b_{t+1} &= p(e_t)\frac{R^g_{t+1}(N_t+d_t)-R^d_{g,t+1}d_t}{N_t}. \end{aligned} \tag{10}$$

When e_t increases, banks become safer in the sense that their Sharpe ratio, E^b_{t+1}/s^b_{t+1}, increases.

2.3 Law of Motion of Aggregate Bank Net Worth

In the next section, we assume that each banker is a member of one of a large number of identical households. Each household has sufficiently many bankers that the law of large numbers applies. We assume that the bankers in period t all have the same level of net worth, N_t. We assume in $t+1$ they pool their net worth after their period $t+1$ returns are realized. In this way, we avoid the potentially distracting problem of having to model the evolution of the distribution of banker net worth. After bankers have pooled their net worth in period $t+1$, an exogenous fraction, $1-\gamma_{t+1}$, of this net worth is transferred to their household. At this point, the representative household makes an exogenous lump sum transfer, T_{t+1}, to the net worth of its banker. After pooling and transfers, the worth of a banker in the representative household in period $t+1$ is given by:

$$N_{t+1} = \gamma_{t+1}\left\{p(e_t)\left[R^g_{t+1}(N_t+d_t)-R^d_{g,t+1}d_t\right]+(1-p(e_t))\left[R^b_{t+1}(N_t+d_t)-R^d_{b,t+1}d_t\right]\right\}+T_{t+1}. \tag{11}$$

[10] In Appendix A we show that ν_{t+1} is positive in any period $t+1$ state of nature if, and only if, it is positive in all period $t+1$ states of nature.

[11] Recall that if a random variable has a binomial distribution and takes on the value x^h with probability p and x^l with probability $1-p$, then the variance of that random variable is $p(1-p)\left(x^h-x^l\right)^2$.

We assume that γ_{t+1} and T_{t+1} are exogenous shocks, realized in $t+1$. A rise in T_{t+1} is equivalent to an influx of new equity into the banks. Similarly, a rise in γ_{t+1} also represents a rise in equity. Thus, we assume that the inflow or outflow of equity into the banks is exogenous and is not subject to the control of the banker. The only control bankers have over their net worth operates through their contol over deposits and the resulting impact on their earnings.

In the unobserved effort model, where we assume the cash constraint is always binding in the bad state, we have:

$$N_{t+1} = \gamma_{t+1} p\left(e_t\right)\left[R_{t+1}^g\left(N_t + d_t\right) - R_{g,t+1}^d d_t\right] + T_{t+1}. \tag{12}$$

The object in square brackets is the realized profits of good banks. It is possible for those to make losses on their deposits (i.e., $R_{t+1}^g < R_{g,t+1}^d$), however we assume that those profits are never so negative that they cannot be covered by earnings on net worth.

When there is no aggregate uncertainty, the d and μ equations (9) imply that the expected earnings of a bank on deposits is zero. Then,

$$p_t\left(e_t\right) R_{t+1}^g + \left(1 - p_t\left(e_t\right)\right) R_{t+1}^b = R_t. \tag{13}$$

Equation (13) and the μ equation in (9) together imply that the law of motion has the following form:

$$N_{t+1} = \gamma_{t+1} R_t N_t + T_{t+1}. \tag{14}$$

When there is aggregate uncertainty, equation (13) holds only in expectation. It does not hold in terms of realized values.

2.4 Restrictions on Bank Leverage

We now impose an additional constraint on banks, that they must satisfy:

$$\frac{N_t + d_t}{N_t} \leq L_t, \tag{15}$$

where L_t denotes the period t restriction on leverage. The banker problem now is (A.3) with the additional constraint, $N_t L_t - (N_t + d_t) \geq 0$. Let $\Lambda_t \geq 0$ denote the multiplier on that constraint. It is easy to verify that the equilibrium conditions now are (9) with the zero in the d equation replaced by Λ_t, plus the following complementary slackness condition:

$$\Lambda_t\left[N_t L_t - \left(N_t + d_t\right)\right] = 0,\ \Lambda_t \geq 0,\ N_t L_t - \left(N_t + d_t\right) \geq 0.$$

Thus, when the leverage constraint is binding, we use the d equation to define Λ_t and add the equation

$$N_t L_t = (N_t + d_t).$$

Interestingly, since the d equation does not hold any longer with $\Lambda_t = 0$, the expected profits of banks in steady state are positive. As a result, (14) does not hold in steady state. Of course, (11) and (12) both hold. Using the μ equation to simplify (11):

$$N_{t+1} = \gamma_{t+1} \left\{ \left[p_t(e_t) R^g_{t+1} + (1 - p_t(e_t)) R^b_{t+1} \right] (N_t + d_t) - R_t d_t \right\} + T_{t+1}. \tag{16}$$

The modified d equation in the version of the model without aggregate uncertainty is:

$$\Lambda_t = (\lambda_{t+1} + \nu_{t+1}) \left[p_t(e_t) \left(R^g_{t+1} - R^d_{g,t+1} \right) + (1 - p_t(e_t)) \left(R^b_{t+1} - R^d_{b,t+1} \right) \right]. \tag{17}$$

Substituting this into (16):

$$N_{t+1} = \gamma_{t+1} \left\{ \left[\frac{\Lambda_t}{\lambda_{t+1} + \nu_{t+1}} + R_t \right] (N_t + d_t) - R_t d_t \right\} + T_{t+1},$$

or

$$N_{t+1} = \gamma_{t+1} \left\{ R_t N_t + \left[\frac{\Lambda_t}{\lambda_{t+1} + \nu_{t+1}} \right] (N_t + d_t) \right\} + T_{t+1}.$$

From here we see that banks make profits on deposits when the leverage constraint is binding, so that $\Lambda_t > 0$.

3 The General Macroeconomic Environment

In this section, we place the financial markets of the previous section into an otherwise standard macro model, along the lines of Christiano, Eichenbaum and Evans (2005) or Smets and Wouters (2007). The financial market has two points of contact with the broader macroeconomic environment. First, the rates of return on entrepreneurial projects are a function of the rate of return on capital. Second, there is a market clearing condition in which the total purchases of raw capital by entrepreneurs, $N_t + d_t$, is equal to the total supply of raw capital by capital producers. In the following two subsections, we first describe goods production and the problem of households. The second subsection describes the production of capital and its links to the entrepreneur. Later subsections describe monetary policy and other aspects of the macro model.

3.1 Goods Production

Goods are produced according to a Dixit-Stiglitz structure. A representative, competitive final goods producer combines intermediate goods, Y_{jt}, $j \in [0,1]$, to produce a homogeneous good, Y_t, using the following technology:

$$Y_t = \left[\int_0^1 Y_{j,t}{}^{\frac{1}{\lambda_f}} dj \right]^{\lambda_f}, \; 1 \leq \lambda_f < \infty. \tag{18}$$

The intermediate good is produced by a monopolist using the following technology:

$$Y_{j,t} = \begin{cases} \bar{K}_{j,t}^{\alpha} \left(z_t l_{jt} \right)^{1-\alpha} - \Phi z_t^* & \text{if } \bar{K}_{j,t}^{\alpha} \left(z_t l_{j,t} \right)^{1-\alpha} > \Phi z_t^* \\ 0 & \text{otherwise} \end{cases}, \; 0 < \alpha < 1. \tag{19}$$

Here, z_t follows a determinist time trend. Also, $\bar{K}_{j,t}$ denotes the services of capital and $l_{j,t}$ denotes the quantity of homogeneous labor, respectively, hired by the j^{th} intermediate good producer. The fixed cost in the production function, (19), is proportional to z_t^*, which is discussed below. The variable, z_t^*, has the property that Y_t/z_t^* converges to a constant in non-stochastic steady state. The monopoly supplier of $Y_{j,t}$ sets its price, $P_{j,t}$, subject to Calvo-style frictions. Thus, in each period t a randomly-selected fraction of intermediate-goods firms, $1 - \xi_p$, can reoptimize their price. The complementary fraction sets its price as follows:

$$P_{j,t} = \pi P_{j,t-1}.$$

Let π_t denote the gross rate of inflation, P_t/P_{t-1}, where P_t is the price of Y_t. Then, π denotes the steady state value of inflation.

There exists a technology that can be used to convert homogeneous goods into consumption goods, C_t, one-for-one. Another technology converts a unit of homogenous goods into Υ^t investment goods, where $\Upsilon > 1$. This parameter allows the model to capture the observed trend fall in the relative price of investment goods. Because we assume these technologies are operated by competitive firms, the equilibrium prices of consumption and investment goods are P_t and

$$P_{I,t} = \frac{P_t}{\Upsilon^t},$$

respectively. The trend rise in technology for producing investment goods is the second source of growth in the model, and

$$z_t^* = z_t \Upsilon^{\left(\frac{\alpha}{1-\alpha}\right)t}.$$

Our treatment of the labor market follows Erceg, Henderson and Levin (2000), and parallels the Dixit-Stiglitz structure of goods production. A representative, competitive labor contractor aggregates the differentiated labor services, $h_{i,t},\ i \in [0,1]$, into homogeneous labor, l_t, using the following production function:

$$l_t = \left[\int_0^1 (h_{i,t})^{\frac{1}{\lambda_w}} \, di \right]^{\lambda_w}, \ 1 \leq \lambda_w. \tag{20}$$

The labor contractor sells labor services, l_t, to intermediate good producers for a given nominal wage rate, W_t. The labor contractor also takes as given the wages of the individual labor types, $W_{i,t}$.

A representative, identical household supplies each of the differentiated labor types, $h_{i,t},\ i \in [0,1]$, used by the labor contractors. By assuming that all varieties of labor are contained within the same household (this is the 'large family' assumption introduced by Andolfatto (1996) and Merz (1995)) we avoid confronting difficult - and potentially distracting - distributional issues. For each labor type, $i \in [0,1]$, there is a monopoly union that represents workers of that type belonging to all households. The i^{th} monopoly union sets the wage rate, W_{it}, for its members, subject to Calvo-style frictions. In particular, a randomly selected subset of $1 - \xi_w$ monopoly unions set their wage to optimize household utility (see below), while the complementary subset sets the wage according to:

$$W_{i,t} = \mu_{z^*} \pi W_{i,t-1}.$$

Here, μ_{z^*} denotes the growth rate of z_t^*. The wage rate determines the quantity of labor demanded by the competitive labor aggregators. Households passively supply the quantity of labor demanded.

3.2 Households

The representative household is composed of a unit measure of agents. Of these, a fraction ϱ are workers and the complementary fraction are bankers. Per capita household consumption is C_t, which is distributed equally to all household members. Average period utility across all workers is given by:

$$\log(C_t - b_u C_{t-1}) - \tilde{\psi}_L \int_0^1 \frac{h_{i,t}^{1+\sigma_L}}{1+\sigma_L} di, \ \tilde{\psi}_L, \sigma_L \geq 0.$$

The object, $b_u \geq 0$, denotes the parameter controlling the degree of habit persistence. The period utility function of a banker is:

$$\log(C_t - b_u C_{t-1}) - \tilde{\varrho} e_t^2, \ \tilde{\varrho} \equiv \frac{1}{2(1-\varrho)}. \tag{21}$$

The representative household's utility function is the equally-weighted average across the utility of all the workers and bankers:

$$\log(C_t - b_u C_{t-1}) - \psi_L \int_0^1 \frac{h_{i,t}^{1+\sigma_L}}{1+\sigma_L} di - \frac{1}{2} e_t^2, \ \psi_L \equiv \varrho \tilde{\psi}_L.$$

The representative household's discount value of a stream of consumption, employment and effort is valued as follows:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log(C_t - b_u C_{t-1}) - \psi_L \int_0^1 \frac{h_{i,t}^{1+\sigma_L}}{1+\sigma_L} di - \frac{1}{2} e_t^2 \right\}, \ \psi_L, b_u, \sigma_L > 0. \tag{22}$$

Bankers behave as described in section 2. They are assumed to do so in exchange for the perfect consumption insurance received from households. Although the mutual funds from which bankers obtain deposits do not observe banker effort, e_t, we assume that a banker's own household observes everything that it does. By instructing the bankers to maximize expected net worth (taking into account their own costs of exerting effort), the household maximizes total end-of-period banker net worth.[12]

The representative household takes e_t and labor earnings as given. It chooses C_t and the quantity of a nominal bond, B_{t+1}, to maximize (22) subject to the budget constraint:

$$P_t C_t + B_{t+1} \leq \int_0^1 W_{i,t} h_{i,t} di + R_t B_t + \Pi_t.$$

Here, Π_t denotes lump sum transfers of profits from intermediate good firms and bankers and taxes. In addition, the household has access to a nominally non-state contingent one-period

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A brief observation about units of measure. We measure the financial objects that the banker works with, N_t and d_t in per capita terms. Bankers are a fraction, $1-\varrho$, of the population, so that in per banker terms, bankers work with $N_t/(1-\varrho)$ and $d_t/(1-\varrho)$. We assume the banker values profits net of the utility cost of its effort as follows:

$$E_t \lambda_{t+1} \left\{ p(e_t) \left[R_{t+1}^g \left(\frac{N_t + d_t}{1-\varrho} \right) - R_{g,t+1}^d \frac{d_t}{1-\varrho} \right] + (1 - p(e_t)) \left[R_{t+1}^b \left(\frac{N_t + d_t}{1-\varrho} \right) - R_{b,t+1}^d \frac{d_t}{1-\varrho} \right] \right\} - \tilde{\varrho} e_t^2.$$

Multiplying this expression by $1-\varrho$ and using (21), we obtain (4).

bond with gross payoff R_t in period $t+1$. Loan market clearing requires that, in equilibrium:

$$B_t = d_t. \tag{23}$$

3.3 Monetary Policy

We express the monetary authority's policy rule directly in linearized form:

$$R_t - R = \rho_p (R_{t-1} - R) + \left(1 - \rho_p\right) \left[\alpha_\pi (\pi_{t+1} - \pi) + \alpha_{\Delta y} \frac{1}{4} (g_{y,t} - \mu_{z^*})\right] + \frac{1}{400} \varepsilon_t^p, \tag{24}$$

where ε_t^p is a shock to monetary policy and ρ_p is a smoothing parameter in the policy rule. Here, $R_t - R$ is the deviation of the period t net quarterly interest rate, R_t, from its steady state. Similarly, $\pi_{t+1} - \pi$ is the deviation of anticipated quarterly inflation from the central bank's inflation target. The expression, $g_{y,t} - \mu_{z^*}$ is quarterly GDP growth, in deviation from its steady state. Finally, ε_t^p is an *iid* shock to monetary policy with standard deviation, σ_p. Note that the shock is in units of annual percentage points.

3.4 Capital Producers, Entrepreneurial Returns and Market Clearing Conditions

In this section we explain how entrepreneurial returns are linked to the underlying return on physical capital. In addition, we discuss the agents that produce capital, the capital producers. Finally, we present the final goods market clearing condition and the market clearing for capital.

The sole source of funds available to an entrepreneur is the funds, $N_t + d_t$, received from its bank after production in period t. An entrepreneur uses these funds to acquire raw capital, $\tilde{K}_{t+1}$, and convert it into effective capital units,

$$P_{k',t} \tilde{K}_{t+1} = N_t + d_t,$$

where $P_{k't}$ is the nominal price of a unit of new, raw capital. This is the market clearing condition for capital. Good and bad entrepreneurs convert one unit of raw capital into

$$e^{g_t}, e^{b_t},$$

units of effective capital, respectively, where $g_t > b_t$. Once this conversion is accomplished, entrepreneurs rent their homogeneous effective capital into the $t+1$ capital market. Thus, in period $t+1$ the quantity of effective capital is $\bar{K}_{t+1}$, where

$$\bar{K}_{t+1} = \left[p\left(e_t\right)e^{g_t} + \left(1 - p\left(e_t\right)\right)e^{b_t}\right]\tilde{K}_{t+1}. \tag{25}$$

Here, e_t is the level of effort expended by the representative banker in period t. Note that if e_t is low in some period, then the effective stock of capital is low in period $t+1$. This reduction has a persistent effect, because - as we shall see below - effective capital is the input into the production of new raw capital in later periods. This effect of banker effort into the quantity of effective capital reflects their role in allocating capital between good and bad entrepreneurs. The object in square brackets in (25) resembles the 'capital destruction shock' adopted in the literature, though here it is an endogenous variable. We refer to it as a measure of the allocative efficiency of the banking system.

Entrepreneurs rent the services of effective capital in a competitive, period $t+1$ capital market. The equilibrium nominal rental rate in this market is denoted by $P_{t+1}r_{t+1}^k$.[13] Entrepreneurs' effective capital, $\bar{K}_{t+1}$, depreciates at the rate δ while it is being used by firms to produce output. The nominal price at which entrepreneurs sell used effective capital to capital producers is denoted $P_{k,t+1}$. The rates of return enjoyed by good and bad entrepreneurs are given by:

$$R_{t+1}^g = e^{g_t}R_{t+1}^k,\ R_{t+1}^b = e^{b_t}R_{t+1}^k, \tag{26}$$

where

$$R_{t+1}^k \equiv \frac{r_{t+1}^k \Upsilon^{-t-1}P_{t+1} + \left(1-\delta\right)P_{k,t+1}}{P_{k't}}.$$

Here, R_{t+1}^k is a benchmark return on capital. The actual return enjoyed by entrepreneurs scales the benchmark according to whether the entrepreneur is good or bad.

We assume there is a large number of identical capital producers. The representative capital producer purchases the time t stock of effective capital and time t investment goods, I_t, and produces new, raw capital using the following production function:

$$\tilde{K}_{t+1} = \left(1-\delta\right)\bar{K}_t + \left(1 - S\left(I_t/I_{t-1}\right)\right)I_t, \tag{27}$$

where S is an increasing and convex function defined below. The number of capital producers

[13]Here, the real rental rate on capital has been scaled. That actual real rental rate of capital is $r_{t+1}^k\Upsilon^{-t-1}$. The latter is a stationary object, according to the model. In the model, the rental rate of capital falls in steady state because the capital stock grows at a rate faster than z_t due to the trend growth in the productivity of making investment goods.

is large enough that they behave competitively. However, there is no entry or exit by entrepreneurs in order to avoid complications that would otherwise arise due to the presence of lagged investment in the production function for new capital. The representative capital producer takes the price of 'old' effective capital, $P_{k,t}$, as given, as well as the price of new, raw capital, $P_{k't}$. If we denote the amount of effective capital that the capital producer purchases in period t by x_t and the amount of raw capital that it sells in period t by y_t, then its objective is to maximize:

$$\sum_{j=0}^{\infty} \lambda_{t+j} \left\{ P_{k',t+j} y_{t+j} - P_{k,t+j} x_{t+j} - P_{I,t+j} I_{t+j} \right\},$$

where λ_t denotes the multiplier on the household budget constraint and $P_{I,t}$ denotes the price of investment goods. The multiplier and the prices are denominated in money terms. Substituting out for y_t using the production function, we obtain:

$$\max_{\{x_{t+j}, I_{t+j}\}_{j=0}^{\infty}} \sum_{j=0}^{\infty} \lambda_{t+j} \left\{ P_{k',t+j} \left[x_{t+j} + \left(1 - S\left(I_{t+j}/I_{t+j-1}\right)\right) I_{t+j} \right] - P_{k,t+j} x_{t+j} - P_{I,t+j} I_{t+j} \right\}$$

From this expression, we see that the capital producer will set $x_t = \infty$ if $P_{k',t} > P_{k,t}$ or set $x_t = 0$ if $P_{k',t} < P_{k,t}$. Since neither of these conditions can hold in equilibrium, we conclude that

$$P_{k',t} = P_{k,t} \text{ for all } t.$$

Thus, the problem is simply to choose I_{t+j} to maximize:

$$\begin{aligned} &\lambda_t \left\{ P_{k',t} \left[\left(1 - S\left(I_t/I_{t-1}\right)\right) I_t \right] - P_{I,t} I_t \right\} \\ &+ E_t \lambda_{t+1} \left\{ P_{k',t+1} \left(1 - S\left(I_{t+1}/I_t\right)\right) I_{t+1} - P_{I,t+1} I_{t+1} \right\} + ... \end{aligned}$$

The first order necessary condition for a maximum is:

$$\lambda_t \left[P_{k',t} \left(1 - S\left(I_t/I_{t-1}\right) - S'\left(I_t/I_{t-1}\right) \frac{I_t}{I_{t-1}} \right) - P_{I,t} \right] + E_t \lambda_{t+1} P_{k',t+1} S'\left(I_{t+1}/I_t\right) \left(\frac{I_{t+1}}{I_t} \right)^2 = 0. \tag{28}$$

Market clearing in the market for old capital requires:

$$x_t = (1 - \delta) \bar{K}_t.$$

Combining (27) with (25), we have the equilibrium law of motion for capital:

$$\bar{K}_{t+1} = \left[p_t(e_t) e^{g_t} + (1 - p_t(e_t)) e^{b_t} \right] \left[(1 - \delta) \bar{K}_t + \left(1 - S\left(I_t/I_{t-1}\right)\right) I_t \right].$$

Finally, we have the market clearing condition for final goods, Y_t, which is:

$$Y_t = G_t + C_t + \frac{I_t}{\Upsilon^t} + a(u_t)\Upsilon^{-t}\bar{K}_t,$$

3.5 Shocks, Adjustment Costs, Resource Constraint

The adjustment cost function on investment is specified as follows:

$$S\left(\frac{I_t}{I_{t-1}}\right) = \left(\exp\left[\frac{1}{2}\sqrt{S''}\left(\varsigma_{I,t}\frac{I_t}{I_{t-1}} - \mu_{z^*}\Upsilon\right)\right] + \exp\left[-\frac{1}{2}\sqrt{S''}\left(\varsigma_{I,t}\frac{I_t}{I_{t-1}} - \mu_{z^*}\Upsilon\right)\right] - 2\right),$$

where the parameter, S'', controls the curvature of the adjustment cost function. Also, we specify that T_t and G_t evolve as follows:

$$T_t = z_t^* \tilde{T}_t, \; G_t = z_t^* \tilde{g},$$

where $\tilde{g}$ is a parameter and the additive equity shock, $\tilde{T}_t$, obeys the following law of motion:

$$\log\left(\tilde{T}_t/\tilde{T}\right) = \rho_T \log\left(\tilde{T}_{t-1}/\tilde{T}\right) - \varepsilon_t^T.$$

The multiplicative equity shock, γ_t, obeys the following law of motion:

$$\log(\gamma_t/\gamma) = \rho_\gamma \log\left(\gamma_{t-1}/\gamma\right) - \varepsilon_t^\gamma.$$

Our third financial shock is a risk shock, Δ_t, like the one considered in Christiano, Motto and Rostagno (2012). In particular, let

$$\begin{aligned} b_t &= b - \Delta_t \\ g_t &= g + \Delta_t. \end{aligned}$$

Thus, Δ_t is a shock to the spread between the return to good banks and the return to bad banks. We assume

$$\Delta_t = \rho_\Delta \Delta_{t-1} + \varepsilon_t^\Delta.$$

The innovations to our three financial shocks are iid and

$$E\left(\varepsilon_t^T\right)^2 = (\sigma_T)^2, \; E\left(\varepsilon_t^\gamma\right)^2 = (\sigma_\gamma)^2, \; E\left(\varepsilon_t^\Delta\right)^2 = (\sigma_\Delta)^2.$$

4 Results

We first consider the steady state implications of our model for leverage. We then turn to the dynamic implications.

4.1 Model Parameterization

Our baseline model is the one in which banker effort is not observable and there are no leverage restrictions on banks. There are four shock processes, and these are characterized by 7 parameters

$$\begin{aligned}\sigma_p &= 0.25,\ \sigma_T = \sigma_\gamma = 0.01,\ \sigma_\Delta = 0.001 \\ \rho_T &= \rho_\gamma = \rho_\Delta = 0.95.\end{aligned}$$

The monetary policy shock is in annualized percentage points. Thus, its standard deviation is 25 basis points. The two other three shocks are in percent terms. Thus, the innovation to the equity shocks are 1 percent each and the innovation to risk is 0.1 percent. The autocorrelations are 0.95 in each.

Apart from the parameters of the shock processes, that model has the 25 parameters displayed in Table 1. Among these parameters, values for the following eight:

$$b, g, \bar{a}, \tilde{T}, \tilde{g}, \Phi, \mu_{z*}, \Upsilon,$$

where chosen to hit the eight calibration targets listed in Table 2.

The first calibration target in Table 2 is based on the evidence in Figure 1. That figure reproduces data constructed in Ferreira (2012). Each quarterly observation in the figure is the cross-sectional standard deviation of the quarterly rate of return on equity for financial firms in the CRSP data base. The sample mean of those observations is 0.2, after rounding. The analog in our model of the volatility measure in Figure 1 is s^b in (10). We calibrate the model so that in steady state, $s^b = 0.20$. The cyclical properties of the volatility data, as well as HP-filtered GDP data in Figure 1 are discussed in a later section.

Our second calibration target in Table 2 is the interest rate spread paid by financial firms. We associate the interest rate spread in the data with $R_g^d - R$ in our model. Loosely, we have in mind that R_g^d is the interest rate on the face of the loan contract.[14] The 60 annual

[14] The return, $R_{g,t}^d$ fluctuates with aggregate uncertainty in our model. In this respect, it does not look like the rate of return on the face of a loan document.

basis point interest rate spread in Table 2 is the sample average of the data on spreads in Figure 2. That figure displays quarterly data on the spread on 3 month loans, measured by the London Interbank Offer Rate, over the rate on 3-month US government securities. The data are reported in annual percent terms.

The third calibration target is leverage, L, which we set to 20. We based this on sample leverage data reported in CGFS (2009, Graph 3). According to the results reported there, the leverage of large US investment banks averaged around 25 since 1995 and the leverage of US commercial banks averaged around 14 over the period.[15] Our value, $L = 20$, is a rough average of the two.

For the remaining calibration targets we use the average growth of US per capita GDP and the average decline in US durable good prices. We set the allocative efficiency of the financial system in steady state to unity. We suspect that this is in the nature of a normalization. Finally, we set the fixed cost in the production function so that profits of the intermediate good firms in steady state are zero. We do not allow entry or exit of these firms, and the implausibility of this assumption is perhaps minimized with the zero steady state profit assumption.

The parameters pertaining to the financial sector that remain to be determined are $\bar{b}$ and γ. The parameter, $\bar{b}$, is important in our analysis. If $\bar{b}$ is sufficiently low, then the unobserved and observed equilibria are similar and the essential mechanism emphasized in this paper is absent. With low $\bar{b}$, our baseline model inherits the property of the observable effort equilibrium, that binding leverage reduces social welfare. If $\bar{b}$ is too high, then the incentive to exert effort is substantial and there ceases to exist an interior equilibrium with $p(e) < 1$ in the baseline model. We balance these two extremes by setting $\bar{b} = 0.3$. With $\bar{b} = 0.2$, social welfare falls when leverage is restricted by a very modest amount, to 19.999. The parameter, γ, resembles a similar object in Bernanke, Gertler and Gilchrist (1999), who assign a value of 0.98 to it. We found that with such a large value of γ, the dynamic response of variables to a monetary policy shock is very different from the results based on vector autoregressions (VARs) reported in CEE. In particular, a jump in the monetary policy shock in (24) drives inflation and output up, rather than down. We are still exploring the economic reasons for this result. However, we noticed that with $\gamma = 0.85$, the impulse responses to a monetary policy shock appear more nearly in line with the results reported in CEE. This is why we chose the value, $\gamma = 0.85$. We are investigating what the implications of micro data may be for the value of this parameter.

The parameters in Panel B were assigned values that are standard in the literature. The steady state inflation rate corresponds roughly to the actual US experience in recent decades.

[15] The data are base on information about Bear Stearns, Goldman Sachs, Lehman Brothers, Merrill Lynch and Morgan Stanley.

The Calvo sticky price and wage parameters imply that prices and wages on average remain unchanged for about a year. Similarly, the parameter values in Panel C are also fairly standard.

4.2 The Steady State Effects of Leverage

We consider the impact on welfare and other variables of imposing a binding leverage restriction. The results are reported in Table 3. The first column of numbers displays the steady state properties of our baseline model, the unobservable effort model without any leverage restrictions. In that model, the assets of the financial system are 20 times its net worth. The second column of numbers shows what happens to the steady state of the model when all parameter are held at their values in Table 1, but a binding leverage restriction of 17 is imposed. The last two columns of numbers report the same results as in the first two columns, but they apply to the version of our model in which effort is observable. We first consider the results for the unobserved effort version of the model.

When leverage restrictions are imposed, Table 3 indicates that bank borrowing, d, declines. A consequence of this is that the interest rate spread on banks falls. To gain intuition into this result, we can see from the fact that the multiplier, ν, on the cash constraint, (3), is positive, that the cash constraint is binding (for ν, see (9)). This means that the creditors of banks with poorly performing assets must share in the losses, i.e., R_b^d is low. However, given the zero profit condition of mutual funds, (13), it follows that R_g^d must be high. That is, $R_b^d < R$ and $R_g^d > R$. We can see from (3) that, for given R^b and bank net worth, creditors of ex post bad banks suffer fewer losses the smaller are their deposits. This is why the value of R_b^d that solves (3) with equality increases with lower deposits. This in turn implies, via the mutual funds' zero profit condition that R_g^d falls towards R as d falls. Thus, deposit rates fluctuate less with the performance of bank portfolios with smaller d. This explains why the interest rate spread falls from 60 basis points in the baseline model to 21 basis points with the imposition of the leverage restriction. A closely related result is that ν falls with the introduction of the binding leverage constraint.

The reduction in the interest rate spread faced by banks helps to improve the efficiency of the economy by giving banks an incentive to increase e (see (8)). But these effects alone only go part way in explaining the full impact of imposing a leverage restriction on this economy. There is also an important general equilibrium, dynamic effect of the leverage restriction that operates via its impact on banker net worth.

To understand this general equilibrium effect, we observe that a leverage restriction in effect allows banks to collude and behave like monopsonists. Deposits are a key input for banks and unregulated competition drives the profits that banks earn on deposits to zero. We can see this from the d equation in (9). That equation shows that in an unregulated banking system, the profits earned by issuing deposits are zero in expectation. This zero profit condition crucially depends on banks being able to expand deposits in case they earn positive profits on them. When a binding leverage restriction is imposed, this competitive mechanism is short-circuited. The d equation in (9) is replaced by (16), where $\Lambda \geq 0$ is the multiplier on the leverage constraint in the banker problem. When this multiplier is positive the bankers make positive profits on deposits. To explain this further, it is useful to focus on a particular decomposition of the rate of return on equity for banks. This rate of return is:

$$\frac{\left[p\left(e_t\right)R^g_{t+1}+\left(1-p\left(e_t\right)\right)R^b_{t+1}\right]N_t+\left[p\left(e_t\right)\left(R^g_{t+1}-R^d_{g,t+1}\right)+\left(1-p\left(e_t\right)\right)\left(R^b_{t+1}-R^d_{b,t+1}\right)\right]d_t}{N_t}-1$$

$$\begin{aligned} = \; & \overbrace{p\left(e_t\right)R^g_{t+1}+\left(1-p\left(e_t\right)\right)R^b_{t+1}-1}^{\text{equity portion of bank rate of return on bank equity}} \\ & \overbrace{+\left[p\left(e_t\right)\left(R^g_{t+1}-R^d_{g,t+1}\right)+\left(1-p\left(e_t\right)\right)\left(R^b_{t+1}-R^d_{b,t+1}\right)\right]\frac{d_t}{N_t}}^{\text{deposit contribution to rate of return on bank equity}} \end{aligned}$$

These three objects are displayed in Table 3, after substituting out for $R^d_{g,t+1}$ an $R^d_{b,t+1}$ using the mutual fund zero profit condition. The d equation in (9) implies that, in steady state, the object in brackets in the deposit contribution to banks' return on equity is zero.[16] So, the fact that d_t/N_t is very large when leverage is 20 has no implication for bank profits. However, with the imposition of the leverage restriction, the object in square brackets becomes positive and then the large size of d_t/N_t is very important. Indeed, it jumps from 0 to 9.76 (APR) when the leverage restriction is imposed. This is the primary reason why banks' rate of return on equity jumps from only 4.59 percent per year in the absence of regulations to a very large 14.96 percent per year when the leverage restriction is imposed. A small additional factor behind this jump is that the equity portion of bankers' rate of return on equity jumps a little too. That reflects the improvement in the efficiency of the banking system as e rises with the imposition of the leverage regulation. To see this, recall from (26) that the gross return on bank assets is given by:

$$\begin{aligned} & p\left(e\right)R^g+\left(1-p\left(e\right)\right)R^b \\ = \; & \left[p\left(e\right)e^g+\left(1-p\left(e\right)\right)e^b\right]R^k. \end{aligned} \tag{29}$$

From this we see that the gross return on bank assets can rise even if R^k falls a little, if the

[16] Here, we also use the mutual fund zero profit condition.

allocative efficiency of the banking system improves enough.[17]

With the high rate of profit it is not surprising that in the new steady state associated with a leverage restriction, bank net worth is higher. Indeed, it is a substantial 17 percent higher. This effect on bank net worth mitigates one of the negative consequence of the leverage restriction. We can see this from (8), which shows that banker effort is not just decreasing with an increased spread between R_b^d an R_g^d, but it is also a function of the total quantity of assets under management. Thus, the bank profits occasioned by the imposition of leverage restrictions raise banker net worth and mitigate the negative impact on banker efficiency of a fall in deposits.

As a way of summarizing the results in Table 3 for the unobserved effort model of this section, we examine the impact of leverage on welfare. We suppose that the social welfare function is given by:

$$u = \log\left(c - \frac{b}{\mu_{z^*}} c\right) - \frac{\psi_L}{1+\sigma_L} h^{1+\sigma_L} - \frac{1}{2} e^2,$$

where c represents C_t / z_t^* in steady state. Let u^l and u^{nl} denote the value of this function in the equilibrium with leverage imposed and not imposed, respectively. Let $u^{nl}(\chi)$ denote utility in the equilibrium without leverage in which consumption, c^{nl}, is replaced by $(1+\chi)\,c^{nl}$. We measure the utility improvement from imposing leverage by the value of χ that solves $u^{nl}(\chi) = u^l$. That is,

$$\chi = e^{u^l - u^{nl}} - 1.$$

In the table we report 100χ. Note that the welfare improvement from imposing leverage is a very substantial 1.19 percent. We suspect that, if anything, this understates the welfare improvement somewhat. According to the table, the quantity of capital falls a small amount with the imposition of the leverage restriction while the efficiency of the banking system improves. This suggests that during the transition between steady states (which is ignored in our welfare calculations), investment must be relatively low and consumption correspondingly high.

We now discuss the last two columns in Table 3. The column headed 'non-binding' describes properties of the equilibrium of our model when effort is observable and the model parameters take on the values in Table 1. The column headed 'binding' indicates the equilibrium when leverage is restricted to 17. We do not report interest rate spreads for the observable effort model because, as indicated above, spreads are not uniquely determined in that model. Comparing the results in the last two columns with the results in the first two columns allows us to

[17] The rate of return, R^k, on capital falls somewhat because the capital labor ratio rises, and this reduces the rental rate of capital. This is the only input into R^k that changes with the imposition of leverage.

highlight the central role in our analysis played by the assumption that effort is not observable. The welfare results in the table provide two ways to summarize the results.

First, note that imposing a leverage restriction on the model when effort is observed implies a very substantial 2.70 percent drop in welfare.[18] Evidently, leverage restrictions are counter-productive when effort is observable. Second, the results indicate that the lack of observability of effort implies a substantial reduction in welfare. In the absence of a leverage restriction, the welfare gain from making effort observable is 6.11 percent.[19] When a binding leverage limit of 17 is in place, then the welfare gain from making effort observable is also a substantial 2.03 percent.[20]

We now discuss why it is that the observable effort equilibrium is so much better than the equilibrium in which effort is not observable. We then sum up by pointing out that the benefits of the leverage restriction on the unobserved effort economy explaining what it is about the leverage restriction that improves welfare.

Making effort observable results in higher consumption and output, and lower employment. These additions to utility are partially offset by the utility cost of extra effort by bankers. This extra effort by bankers in the observable effort equilibrium is the key to understanding why consumption and capital are higher and labor lower, in that equilibrium. To see this, note that the steady state version of (6), combined with (29), imply:

$$R = \left[p\left(e\right) e^{g} + \left(1 - p\left(e\right)\right) e^{b} \right] R^{k}.$$

When e rises with observability of effort, the object in square brackets (the allocative efficiency of the banking system) increases and, absent a change in R^k, would cause a rise in R. Imagine that that rise in R did occur, stimulating more deposits. That would lead to more capital, thus driving R^k down. In the new steady state, R is the same as it was before effort was made observable. Thus, across steady states R^k must fall by the same amount that the efficiency of the banking system rises. The fall in R^k implies a rise in the capital to labor ratio, k/h. According to Table 3, this rise is accomplished in part by an increase in k and in part by a decrease in h. The higher steady state capital is sustained by higher intermediation, $N + d$,

[18] The simultaneous drop in the capital stock and the absence of any change in the efficiency of the banking system suggests that when the transition is taken into account, the drop in welfare may be smaller.

[19] It is not clear how taking into account the transition between steady states would affect this welfare calculation. In the steady state with observable effort, the quantity of capital is higher but the efficiency of the banking system is also greater. The impact of the transition on welfare depends on the extent to which the higher amount of capital reflects increased efficience and/or a reduction in consumption during the transition.

[20] The observations about the impact of the transition on welfare calculations made in the previous footnote apply here as well.

and this primarily reflects a higher level of deposits.[21] Imposing the leverage restriction on the unobserved effort economy moves consumption, employment and effort in the same direction that making effort observable does. This is why imposing the leverage restriction raises welfare.

4.3 Dynamic Properties of the Model

In this section we consider the dynamic effects of a monetary policy shock and four financial shocks.

4.3.1 Monetary Policy Shock

Figure 3 displays the responses in our baseline model to a 25 basis point shock to monetary policy. First, consider the standard macroeconomic variables. The shock has a persistent, hump-shaped and long-lived effect on output, consumption and investment. The maximal decline of 0.3 and 0.5 percentage points, respectively, in GDP and investment occur after about two years. In the case of consumption the maximal decline occurs three years after the shock and the maximal decline is a little over 0.3 percent. Inflation drops a modest 8 annualized basis points. Unlike the pattern reported in CEE, the response in inflation does not display a hump-shape. However, direct comparison between the results in Figure 3 and VAR-based estimates of the effects of monetary policy shocks reported in CEE and other places is not possible. The latter estimates often assume that aggregate measures of economic activity and prices and wages are predetermined within the quarter to a monetary policy shock. In our model, this identifying assumption is not satisfied. One way to see this is to note that the actual rise in the interest rate is only 15 basis points in the period of the shock. The fact that the interest rate does not rise the full 25 basis points of the policy shock reflects the immediate negative impact on the interest rate of the fall in output and inflation. Still, it seems like a generally positive feature of the model that the implied impulse responses correspond, in a rough qualitative sense, to the implications of VAR studies for aggregate variables and inflation.

Now, consider the impact on financial variables. The reduction in output and investment reduces R^k by two channels: it reduces the rental rate of capital and the value of capital, $P_{k'}$. Both of these have the effect of reducing bank net worth. The reduction in bank net worth leads to a tightening of the cash constraint, (3). The result is that the interest rate spread on banks increases and banker effort declines. That is, $p(e)$ falls 70 basis points. This in turn is manifest in a rise in the cross-sectional dispersion of bank equity returns. Interestingly,

[21] In the case with no leverage restriction, the rise in $N + d$ is entirely due to a rise in d.

cross-sectional dispersion in the rate of return on financial firm equity is countercyclical in the data (see Figure 1). Finally, bank assets, $N + d$, and bank liabilities, d, both decline.

The relative size of the decline in $N + d$ and in d is of some interest. To pursue this, it is useful to focus on a particular decomposition of the percent change in bank leverage. Let Δx denote $(x - x^s)/x^s$, where x^s is a reference value (perhaps its lagged value) of a variable, x. Then, letting L denote bank leverage, $(N + d)/N$, we have[22]

$$\Delta L = (L - 1)\left[\Delta d - \Delta (N + d)\right].$$

Using this expression we can infer from Figure 3 that our model implies a rise in leverage in the wake of a monetary-policy induced contraction. Recent literature suggests this implication is counterfactual (see Adrian, Colla and Shin (2012)). We suspect that a version of the model could be constructed in which credit responds more and net worth less, so that leverate is procyclical.

4.3.2 Financial Shocks

The dynamic responses of the model variables to our three financial shocks are displayed in Figures 4, 5 and 6. A notable feature of these figures is how similar they are, at least qualitatively. In each case, consumption, investment, output, inflation and the risk free rate all fall in response to the shock. The interest rate spread rises and the cross-sectional dispersion in bank equity returns jumps as $p(e)$ falls. Finally, bank assets and liabilities both fall. However, the former fall by a greater percent, so that leverage is countercyclical in each case. It is perhaps not surprising that the risk shock has the greatest quantitative impact on $p(e)$.

5 Conclusion

Bank leverage has received considerable attention in recent years. Several questions have been raised about leverage:

[22] Note that

$$\Delta (N + d) = \frac{N}{N + d}\Delta N + \frac{d}{N + d}\Delta d,$$

so that

$$\Delta N = \frac{N + d}{N}\Delta (N + d) - \frac{d}{N}\Delta d.$$

Also,

$$\Delta L = \Delta (N + d) - \Delta N.$$

The formula in the text follows by substituting out for ΔN from the first expression.

- Should bank leverage be restricted, and how should those restrictions be varied over the business cycle?
- How should monetary policy react to bank leverage, if at all?

This paper describes an environment that can in principle be used to shed light on these questions. We have presented some preliminary results by studying the implications for leverage in steady state. We showed that steady state welfare improves substantially with a binding welfare restriction. There are several ways to understand the economics of this result. We pursue one way in this paper. Bigio (2012a) takes an alternative approach, in which he relates the improvement in welfare to the operation of a pecuniary externality. Either way, leverage restrictions help to correct a problem in the private economy. For this reason, we think the model environment is an interesting one for studying the questions listed above.

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Table 1: Baseline Model Parameter Values		
Meaning	Name	Value
Panel A: financial parameters		
return parameter, bad entrepreneur	b	-0.09
return parameter, good entrepreneur	g	0.00
constant, effort function	$\bar{a}$	0.83
slope, effort function	$\bar{b}$	0.30
lump-sum transfer from households to bankers	$\tilde{T}$	0.38
fraction of banker net worth that stays with bankers	γ	0.85
Panel B: Parameters that do not affect steady state		
steady state inflation (APR)	$400(\pi - 1)$	2.40
Taylor rule weight on inflation	α_π	1.50
Taylor rule weight on output growth	$\alpha_{\Delta y}$	0.50
smoothing parameter in Taylor rule	ρ_p	0.80
curvature on investment adjustment costs	S''	5.00
Calvo sticky price parameter	ξ_p	0.75
Calvo sticky wage parameter	ξ_w	0.75
Panel C: Nonfinancial parameters		
steady state gdp growth (APR)	μ_{z^*}	1.65
steady state rate of decline in investment good price (APR)	Υ	1.69
capital depreciation rate	δ	0.03
production fixed cost	Φ	0.89
capital share	α	0.40
steady state markup, intermediate good producers	λ_f	1.20
habit parameter	b_u	0.74
household discount rate	$100(\beta^{-4} - 1)$	0.52
steady state markup, workers	λ_w	1.05
Frisch labor supply elasticity	$1/\sigma_L$	1.00
weight on labor disutility	ψ_L	1.00
steady state scaled government spending	$\tilde{g}$	0.89

Table 2: Steady state calibration targets for baseline model		
Variable meaning	variable name	magnitude
Cross-sectional standard deviation of quarterly non-financial firm equity returns	s^b	0.20
Fnancial firm interest rate spreads (APR)	$400(R_g^d - R)$	0.60
Financial firm leverage	L	20.00
Allocative efficiency of the banking system	$p(e)e^g + (1 - p(e))e^b$	1
Profits of intermediate good producers (controled by fixed cost, Φ)		0
Government consumption relative to GDP (controlled by $\tilde{g}$)		0.20
Growth rate of per capita GDP (APR)	$400(\mu_z^* - 1)$	1.65
Rate of decline in real price of capital (APR)	$400(\Upsilon - 1)$	1.69

Table 3: Steady State Properties of the Model					
Variable meaning	Variable name	Unobserved Effort		Observed Effort	
		Leverage Restriction		Leverage Restriction	
		non-binding	binding	non-binding	binding
Spread	$400(R_g^d - R)$	0.600	0.211	NA	NA
Multiplier on cash constraint	v	0.060	0.040	0	0
scaled consumption	c	1.84	1.88	2.01	1.95
scaled GDP	y	4.43	4.37	4.68	4.43
labor	h	1.18	1.16	1.15	1.14
scaled capital stock	k	51.52	51.40	59.75	53.86
capital output ratio	$k/(c+i+g)$	11.63	11.75	12.78	12.15
bank assets	$N+d$	51.52	51.31	59.55	53.68
bank net worth	N	2.58	3.02	2.58	3.16
bank deposits	d	48.94	48.29	56.98	50.52
bank leverage	$(N+d)/N$	20.00	17.00	23.12	17.00
bank return on equity (APR)	$400\left(\frac{\left[p(e_t)R_{t+1}^g+(1-p(e_t))R_{t+1}^b\right](N_t+d_t)-R_t d_t}{N_t} - 1\right)$	4.59	14.96	4.59	17.63
equity portion of bank return (APR)	$400(p(e_t)R_{t+1}^g + (1-p(e_t))R_{t+1}^b - 1)$	4.59	5.20	4.59	5.36
deposit portion of bank return (APR)	$400[p(e_t)R_{t+1}^g + (1-p(e_t))R_{t+1}^b - R_t]\frac{d_t}{N_t}$	0.00	9.76	0.00	12.27
benchmark return on capital (APR)	$400(R^k - 1)$	4.59	4.47	3.23	4.00
bank efficiency	$p(e)e^g + (1-p(e))e^b$	1.000	1.002	1.003	1.003
fraction of firms with good balance sheets	$p(e)$	0.962	0.982	1.000	1.000
Benefit of leverage (in c units)	100χ	NA	1.19	NA	-2.70
Benefit of making effort observable (in c units)	100χ	NA	NA	6.11	2.03
Note: (i) NA, not applicable, indicates that the number is not defined. (ii) All calculations based on a single set of parameter values, reported in Table 1.					

Figure 1: Cross-section standard deviation financial firm quarterly return on equity, HP-filtered US real GDP

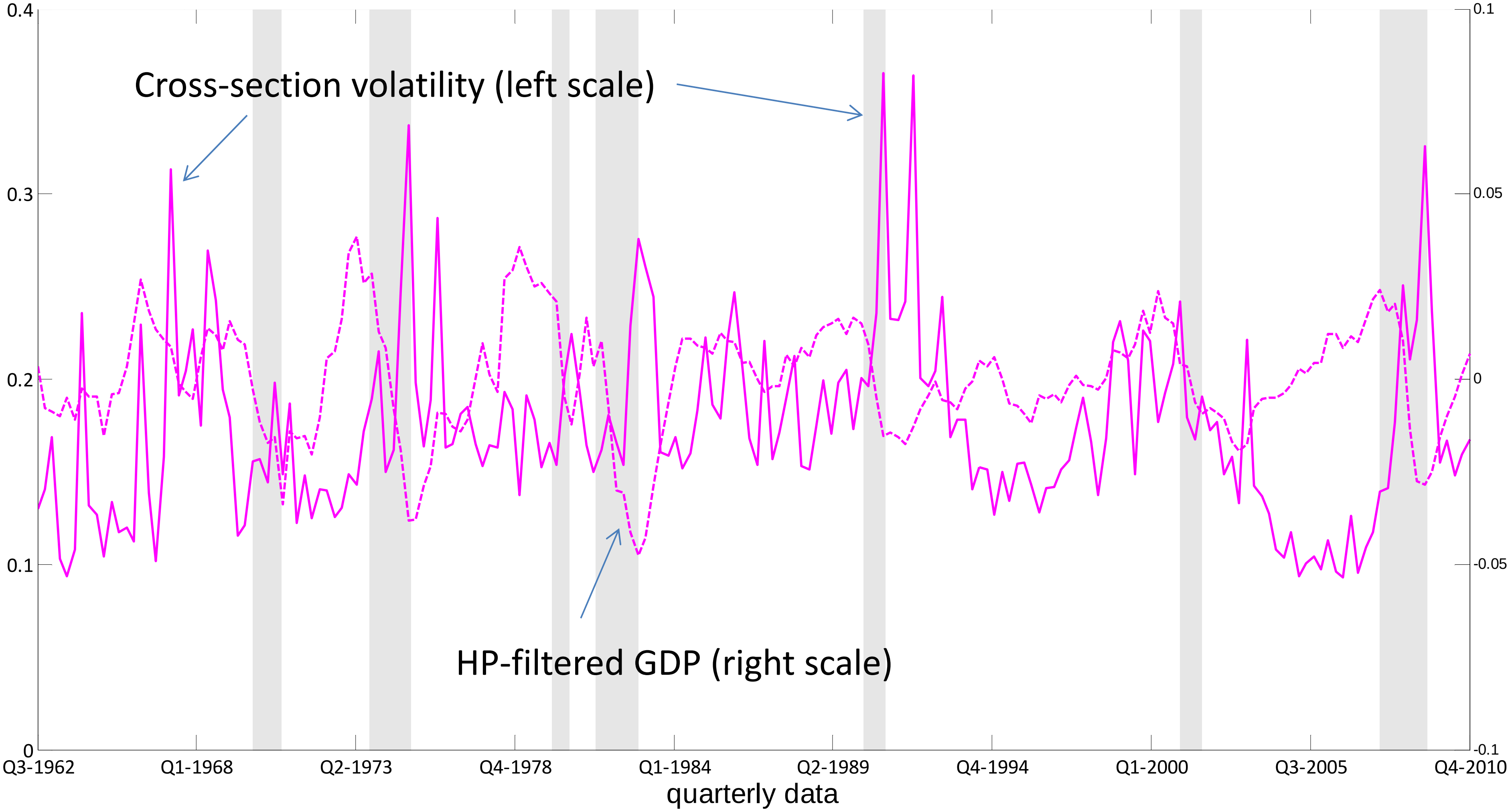


Figure 2: 3 month US Libor versus 3 month T-bill (APR)

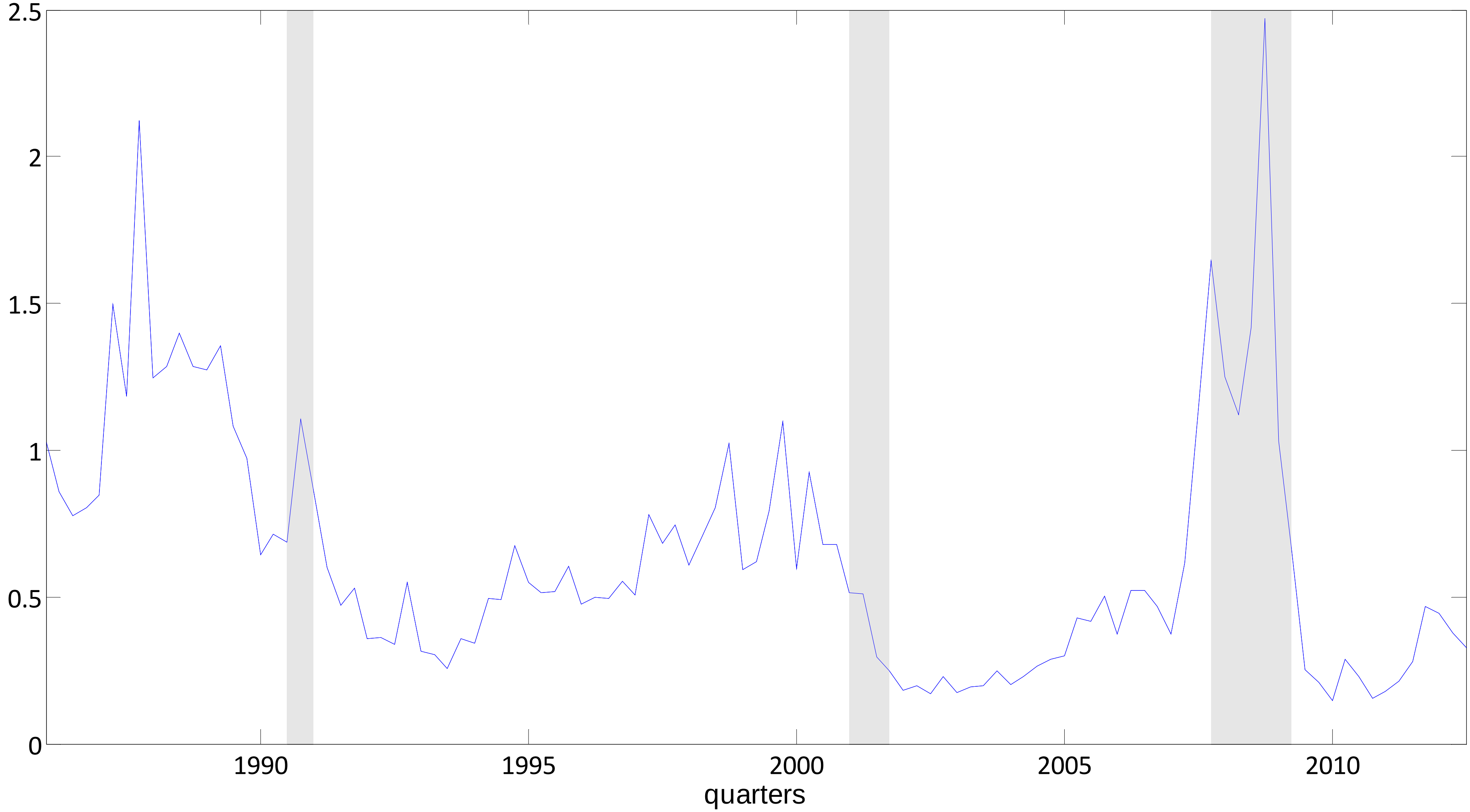


Figure 3: Dynamic Response of Baseline Model to Monetary Policy Shock

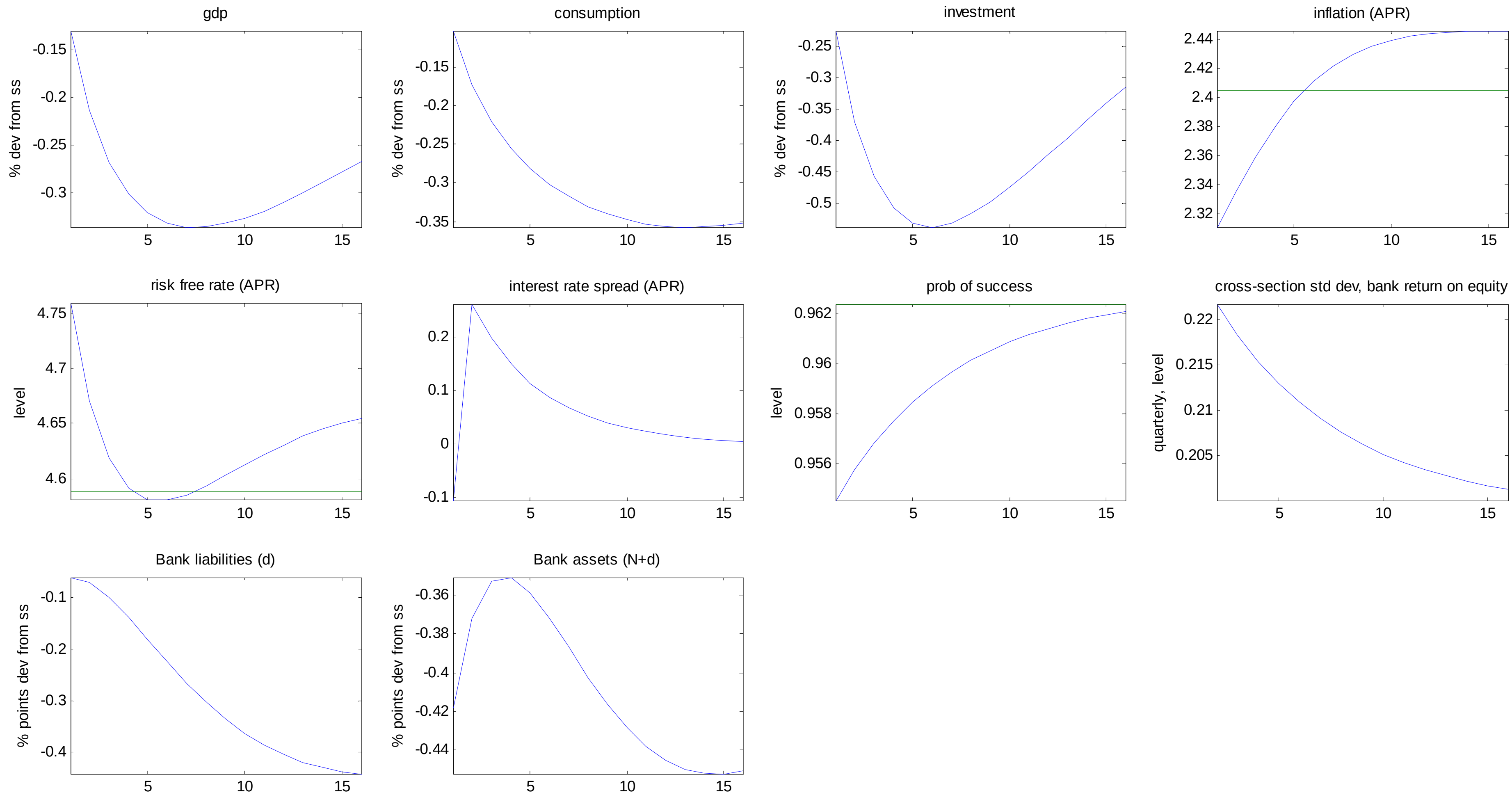


Figure 4: Dynamic Response of Baseline Model to Υ Shock

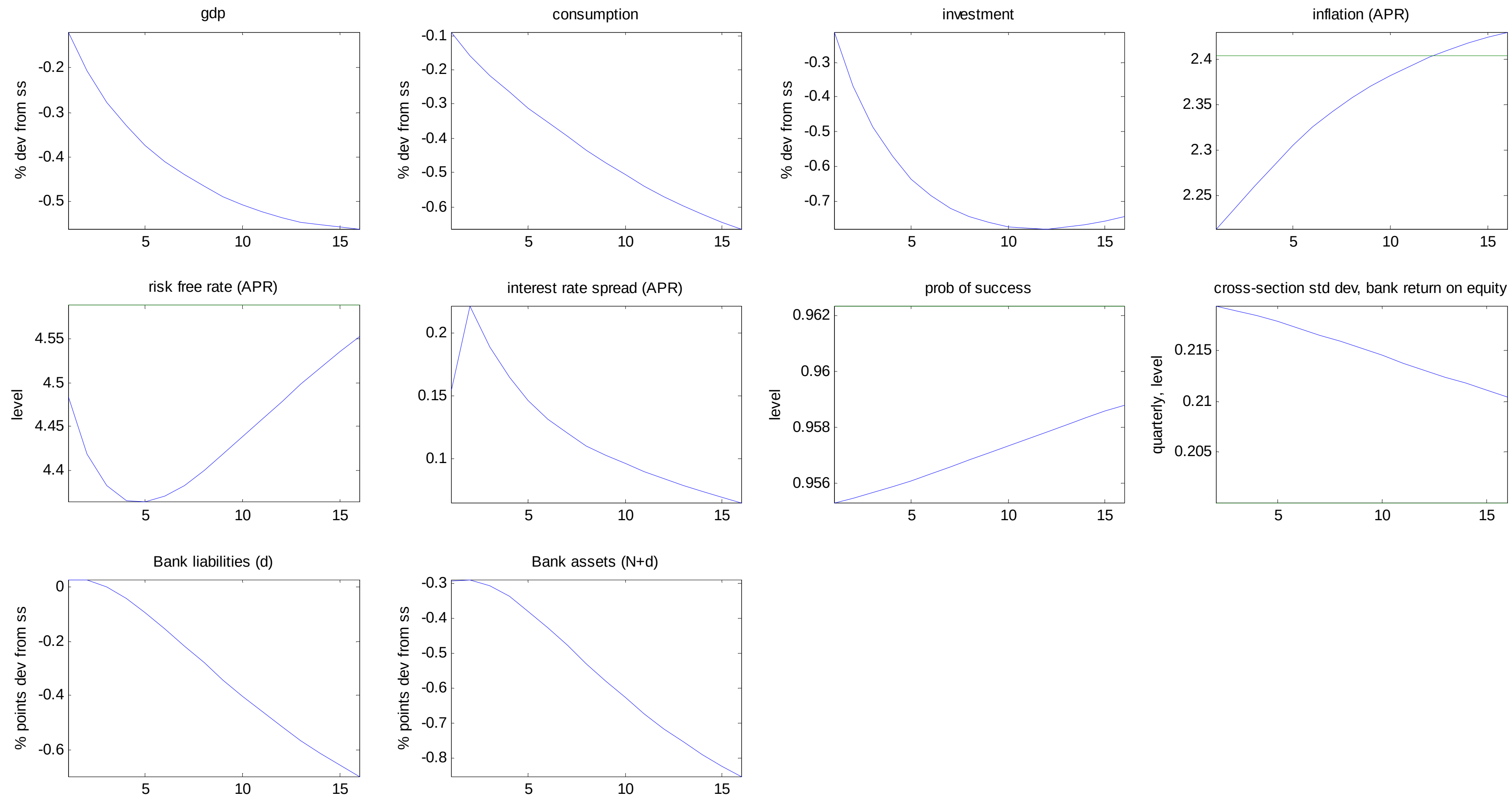


Figure 5: Dynamic Response of Baseline Model to Risk Shock

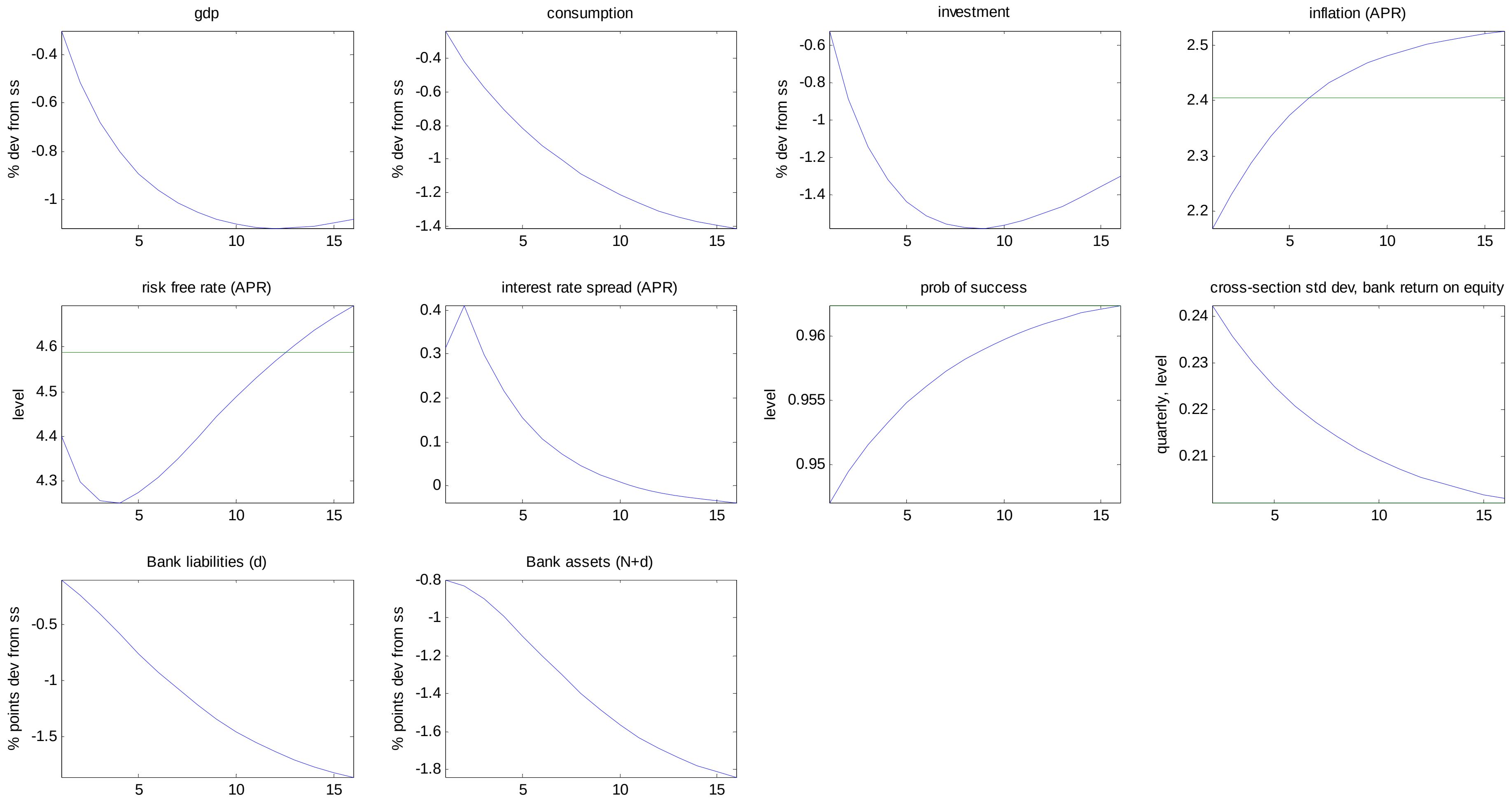
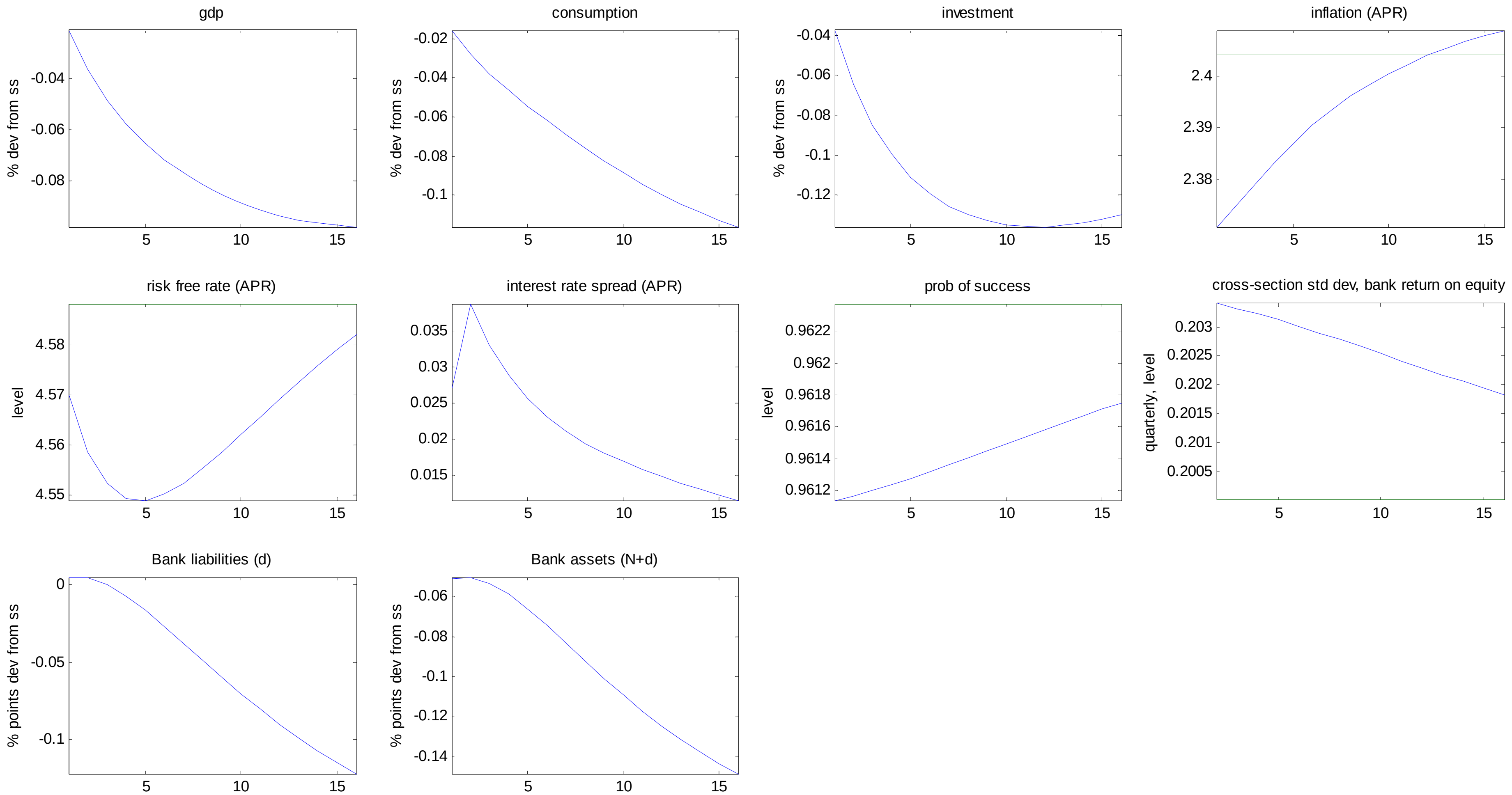


Figure 6: Dynamic Response of Baseline Model to *T* Shock



Technical Appendix to "Leverage Restrictions in a Business Cycle Model",
Lawrence Christiano and Daisuke Ikeda

A Appendix A: Derivation of Financial Sector Equilibrium Conditions

This appendix derives the equilibrium conditions associated with the financial sector. The first subsection considers the conditions associated with the case where banker effort is observable. We then consider the unobservable effort case.

A.1 Observable Effort

The Lagrangian representation of the banker's problem in the observable effort rerpresentation of the problem is:

$$\begin{aligned}&\max_{e,d,R_g^d,R_b^d} E_t\lambda_{t+1}\left\{p_t\left(e_t\right)\left[R_{t+1}^g\left(N_t+d_t\right)-R_{g,t+1}^d d_t\right]+\left(1-p_t\left(e_t\right)\right)\left[R_{t+1}^b\left(N_t+d_t\right)-R_{b,t+1}^d d_t\right]\right\}\\&-\frac{1}{2}e_t^2 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{(A.1)}\\&+E_t\left\{\mu_{t+1}\left[p_t\left(e_t\right)R_{g,t+1}^d d_t+\left(1-p_t\left(e_t\right)\right)R_{b,t+1}^d d_t-R_t d_t\right]+\nu_{t+1}\left[R_{t+1}^b\left(N_t+d_t\right)-R_{b,t+1}^d d_t\right]\right\}\end{aligned}$$

where μ_{t+1} is the Lagrange multiplier on (1) and $\nu_{t+1}\geq 0$ is the Lagrange multiplier on (3). Note that the constraints must be satisfied in each period $t+1$ state of nature, which is indicated by the fact that the multipliers, μ_{t+1} an ν_{t+1}, are contingent upon the realization of period $t+1$ uncertainty. The first order conditions associated with the banker problem are:

$$\begin{aligned}&e: 0=E_t\{\lambda_{t+1}p_t'\left(e_t\right)\left[\left(R_{t+1}^g-R_{t+1}^b\right)\left(N_t+d_t\right)-\left(R_{g,t+1}^d-R_{b,t+1}^d\right)d_t\right]-e_t\\&+\mu_{t+1}p_t'\left(e_t\right)\left(R_{g,t+1}^d-R_{b,t+1}^d\right)d_t\}\\&d: 0=E_t\{\lambda_{t+1}\left[p_t\left(e_t\right)\left(R_{t+1}^g-R_{g,t+1}^d\right)+\left(1-p_t\left(e_t\right)\right)\left(R_{t+1}^b-R_{b,t+1}^d\right)\right]\\&+\mu_{t+1}\left[p_t\left(e_t\right)R_{g,t+1}^d+\left(1-p_t\left(e_t\right)\right)R_{b,t+1}^d-R_t\right]+\nu_{t+1}\left(R_{t+1}^b-R_{b,t+1}^d\right)\}\\&R_g^d: 0=-\lambda_{t+1}p_t\left(e_t\right)d_t+\mu_{t+1}p_t\left(e_t\right)d_t\\&R_b^d: 0=-\lambda_{t+1}\left(1-p_t\left(e_t\right)\right)d_t+\mu_{t+1}\left(1-p_t\left(e_t\right)\right)d_t-\nu_{t+1}d_t\\&\mu: p_t\left(e_t\right)R_{g,t+1}^d d_t+\left(1-p_t\left(e_t\right)\right)R_{b,t+1}^d d_t=R_t d_t\\&\nu: \nu_{t+1}\left[R_{t+1}^b\left(N_t+d_t\right)-R_{b,t+1}^d d_t\right]=0,\ \nu_{t+1}\geq 0,\ R_{t+1}^b\left(N_t+d_t\right)-R_{b,t+1}^d d_t\geq 0,\end{aligned}$$

where "$x:$" in the first column indicates the first order condition with respect to the variable, x. In the R_g^d and R_b^d equations, we differentiate state by state. In the results reported above the density of the state does not appear. This reflects our assumption that the density is strictly positive over all states, so that we can divide through by that density. We make this assumption throughout. Adding the R_g^d and R_b^d equations, we obtain:

$$\mu_{t+1}=\lambda_{t+1}+\nu_{t+1}. \tag{A.2}$$

Substituting (A.2) back into the R_g^d equation, we find

$$\nu_{t+1}=0,$$

so that the cash constraint is non-binding. Substituting the latter two results back into the system of equations, they reduce to (5), (6) and (7) in the text. To see this, note that

$\mu_{t+1} = \lambda_{t+1}$ in the e equation results in a simple cancellation that implies (5). Equation (6) is derived in a similarly simple way. Finally, equation (7) is simply the μ equation repeated.

Now suppose we impose a leverage restriction, (15). This only affects the d equation above, since d_t is the only choice variable in the leverage restriction. As a result, our findings, $\nu_{t+1} = 0$ and $\mu_{t+1} = \lambda_{t+1}$ are unaffected. That is, the cash constraint remains non-binding and the effort equation remains as in (5). The only change implied by a binding leverage constraint is that the 0 in the d equation is replaced by the multiplier on the leverage constraint.

A.2 Unobservable Effort

Given the indicated set of contracts, the Lagrangian representation of the banker's problem now is:

$$\begin{aligned}
&\max_{\left(e_t,d_t,R^d_{g,t+1},R^d_{b,t+1}\right)} E_t\lambda_{t+1}\left\{p_t\left(e_t\right)\left[R^g_{t+1}\left(N_t+d_t\right)-R^d_{g,t+1}d_t\right]+\left(1-p_t\left(e_t\right)\right)\left[R^b_{t+1}\left(N_t+d_t\right)-R^d_{b,t+1}d_t\right]\right\}\\
&-\frac{1}{2}e_t^2 \qquad (A.3)\\
&+E_t\mu_{t+1}\left[p_t\left(e_t\right)R^d_{g,t+1}d_t+\left(1-p_t\left(e_t\right)\right)R^d_{b,t+1}d_t-R_td_t\right]\\
&+\eta_t\left(e_t-E_t\lambda_{t+1}p_t'\left(e_t\right)\left[\left(R^g_{t+1}-R^b_{t+1}\right)\left(N_t+d_t\right)-\left(R^d_{g,t+1}-R^d_{b,t+1}\right)d_t\right]\right)\\
&+E_t\nu_{t+1}\left[R^b_{t+1}\left(N_t+d_t\right)-R^d_{b,t+1}d_t\right].
\end{aligned}$$

where η_t is the Lagrange multiplier on (8). Note that this multiplier is not contingent on the realization of the period $t+1$ state of nature since the constraint is on the effort level exerted by the banker in t. To understand the solution to this problem, consider the first order necessary conditions associated with the banker problem, (A.3):

$$\begin{aligned}
e &: E_t\lambda_{t+1}p_t'\left(e_t\right)\left[\left(R^g_{t+1}-R^b_{t+1}\right)\left(N_t+d_t\right)-\left(R^d_{g,t+1}-R^d_{b,t+1}\right)d_t\right]\\
&\quad -e_t+E_t\mu_{t+1}p_t'\left(e_t\right)\left(R^d_{g,t+1}-R^d_{b,t+1}\right)d_t\\
&\quad +\eta_t\left(1-E_t\lambda_{t+1}p_t''\left(e_t\right)\left[\left(R^g_{t+1}-R^b_{t+1}\right)\left(N_t+d_t\right)-\left(R^d_{g,t+1}-R^d_{b,t+1}\right)d_t\right]\right)=0\\
d &: 0=E_t\lambda_{t+1}p_t\left(e_t\right)\left(R^g_{t+1}-R^d_{g,t+1}\right)+E_t\lambda_{t+1}\left(1-p_t\left(e_t\right)\right)\left(R^b_{t+1}-R^d_{b,t+1}\right)\\
&\quad +E_t\mu_{t+1}\left[p_t\left(e_t\right)R^d_{g,t+1}+\left(1-p_t\left(e_t\right)\right)R^d_{b,t+1}-R_t\right]\\
&\quad -\eta_tE_t\lambda_{t+1}p_t'\left(e_t\right)\left[\left(R^g_{t+1}-R^b_{t+1}\right)-\left(R^d_{g,t+1}-R^d_{b,t+1}\right)\right]+E_t\nu_{t+1}\left(R^b_{t+1}-R^d_{b,t+1}\right)\\
R^d_g &: -\lambda_{t+1}p_t\left(e_t\right)+\mu_{t+1}p_t\left(e_t\right)+\eta_t\lambda_{t+1}p_t'\left(e_t\right)=0\\
R^d_b &: -\lambda_{t+1}\left(1-p_t\left(e_t\right)\right)+\mu_{t+1}\left(1-p_t\left(e_t\right)\right)-\eta_t\lambda_{t+1}p_t'\left(e_t\right)-\nu_{t+1}=0\\
\mu &: R_t=p_t\left(e_t\right)R^d_{g,t+1}+\left(1-p_t\left(e_t\right)\right)R^d_{b,t+1}\\
\eta &: e_t=E_t\lambda_{t+1}p_t'\left(e_t\right)\left[\left(R^g_{t+1}-R^b_{t+1}\right)\left(N_t+d_t\right)-\left(R^d_{g,t+1}-R^d_{b,t+1}\right)d_t\right]\\
\nu &: \nu_{t+1}\left[R^b_{t+1}\left(N_t+d_t\right)-R^d_{b,t+1}d_t\right]=0,\ \nu_{t+1}\geq 0,\ \left[R^b_{t+1}\left(N_t+d_t\right)-R^d_{b,t+1}d_t\right]\geq 0.
\end{aligned}$$

Add the R^d_g and R^d_b equations, to obtain (A.2). To simplify the e equation, use (A.2) to substitute out μ_{t+1} :

$$\begin{aligned}
e &: E_t\lambda_{t+1}p_t'\left(e_t\right)\left[\left(R^g_{t+1}-R^b_{t+1}\right)\left(N_t+d_t\right)-\left(R^d_{g,t+1}-R^d_{b,t+1}\right)d_t\right]-e_t\\
&\quad +E_t\left[\lambda_{t+1}+\nu_{t+1}\right]p_t'\left(e_t\right)\left(R^d_{g,t+1}-R^d_{b,t+1}\right)d_t\\
&\quad +\eta_t\left(1-E_t\lambda_{t+1}p_t''\left(e_t\right)\left[\left(R^g_{t+1}-R^b_{t+1}\right)\left(N_t+d_t\right)-\left(R^d_{g,t+1}-R^d_{b,t+1}\right)d_t\right]\right)=0
\end{aligned}$$

or,

$$\begin{aligned} e &: E_t\lambda_{t+1}p_t'(e_t)\left(R_{t+1}^g - R_{t+1}^b\right)(N_t + d_t) - e_t + E_t\nu_{t+1}p_t'(e_t)\left(R_{g,t+1}^d - R_{b,t+1}^d\right)d_t \\ & +\eta_t\left(1 - E_t\lambda_{t+1}p_t''(e_t)\left[\left(R_{t+1}^g - R_{t+1}^b\right)(N_t + d_t) - \left(R_{g,t+1}^d - R_{b,t+1}^d\right)d_t\right]\right) = 0 \end{aligned}$$

Now, make use of $p_t'' = 0$ and the η equation to substitute out for e_t :

$$\begin{aligned} e : E_t\lambda_{t+1}p_t'(e_t)\left(R_{t+1}^g - R_{t+1}^b\right)(N_t + d_t) - E_t\lambda_{t+1}p_t'(e_t)\left[\left(R_{t+1}^g - R_{t+1}^b\right)(N_t + d_t) - \left(R_{g,t+1}^d - R_{b,t+1}^d\right)d_t\right] \\ +E_t\nu_{t+1}p_t'(e_t)\left(R_{g,t+1}^d - R_{b,t+1}^d\right)d_t + \eta_t = 0 \end{aligned}$$

or,

$$e : E_t\lambda_{t+1}p_t'(e_t)\left(R_{g,t+1}^d - R_{b,t+1}^d\right)d_t + E_t\nu_{t+1}p_t'(e_t)\left(R_{g,t+1}^d - R_{b,t+1}^d\right)d_t + \eta_t = 0$$

or,

$$e : E_t\left[\lambda_{t+1} + \nu_{t+1}\right]p_t'(e_t)\left(R_{g,t+1}^d - R_{b,t+1}^d\right)d_t + \eta_t = 0.$$

We now simplify the d equation. From the μ-condition, we delete the third term in d equation and obtain

$$\begin{aligned} 0 &= E_t\lambda_{t+1}p_t(e_t)\left(R_{t+1}^g - R_{g,t+1}^d\right) + E_t\lambda_{t+1}(1 - p_t(e_t))\left(R_{t+1}^b - R_{b,t+1}^d\right) \\ & -\eta_t E_t\lambda_{t+1}p_t'(e_t)\left[\left(R_{t+1}^g - R_{t+1}^b\right) - \left(R_{g,t+1}^d - R_{b,t+1}^d\right)\right] + E_t\nu_{t+1}\left(R_{t+1}^b - R_{b,t+1}^d\right) \end{aligned}$$

Use (A.2) to substitute out for μ_{t+1} in the R_g^d condition:

$$\nu_{t+1}p_t(e_t) + \eta_t\lambda_{t+1}p_t'(e_t) = 0$$

Substituting out η_t using R_g^d-condition,

$$\begin{aligned} -\lambda_{t+1}p_t(e_t) + \left[\lambda_{t+1} + \nu_{t+1}\right]p_t(e_t) + \eta_t\lambda_{t+1}p_t'(e_t) &= 0 \\ \nu_{t+1}p_t(e_t) + \eta_t\lambda_{t+1}p_t'(e_t) &= 0 \end{aligned} \tag{A.4}$$

Note that this equation implies

$$\eta_t \leq 0.$$

This is to be expected. The interpretation of this may be seen from (A.3). The sign of η_t suggests that in the absence of the η constraint, i.e., if $\eta_t = 0$, then e_t would be set in a way that makes e_t greater than the object on the right of the minus sign in the incentive constraint. A negative value of η_t in the Lagrangian penalizes such a setting. But, we know from our analysis of the observable effort case (the only difference in this case is that the incentive constraint is absent), that e_t is greater than the object on the right of the minus sign in the η constraint in (A.3) when that constraint is ignored. But, (A.4) has another notable implication. Suppose, for simplicity, that from the point of view of t, there are two possible states of nature in $t+1$, 1 and 2. Then,

$$\begin{aligned} \nu_{t+1}^1 p_t(e_t) + \eta_t\lambda_{t+1}^1 p_t'(e_t) &= 0 \\ \nu_{t+1}^2 p_t(e_t) + \eta_t\lambda_{t+1}^2 p_t'(e_t) &= 0 \end{aligned}$$

We assume that $\lambda_{t+1}^1, \lambda_{t+1}^2, p_t'(e_t), p_t(e_t) > 0$. Suppose the cash constraint is not binding in state of nature, 1, so that $\nu_{t+1}^1 = 0$. In that case, the first equation says that $\eta_t = 0$. But, the second equation then implies $\nu_{t+1}^2 = 0$ too. Thus, if the cash constraint is not binding in some state of nature for a particular date, then it must not be binding in the other state either. If it is binding is one state, $\nu_{t+1}^1 > 0$, then $\eta_t > 0$ and it is binding in the other state. Thus, it is either binding in all states at a particular date, or none. This is general. Note from

R_g^d-condition that

$$\eta_t = -\nu_{t+1} p_t(e_t) / [\lambda_{t+1} p_t'(e_t)],$$

which implies that there exists no solution such that $\nu_{t+1} = 0$ for some states of nature and $\nu_{t+1} > 0$ for others. Intuitively this is because a banker smooths inefficiency caused by $R_{g,t+1}^d - R_{b,t+1}^d > 0$ state by state. Suppose R_{t+1}^b is very low in one state and it is very high in another. Then, the cash constraint is binding in the low state so that $R_{g,t+1}^d - R_{b,t+1}^d > 0$. In the high state the banker sets $R_{b,t+1}^d$ high enough so that the cash constraint is binding and $R_{g,t+1}^d - R_{b,t+1}^d < 0$. By doing this the banker minimizes $E_t \lambda_{t+1} p_t'(e_t) \left(R_{g,t+1}^d - R_{b,t+1}^d\right) \geq 0$, which is, loosely speaking, the measure of inefficiency.

Substituting out for η_t in the revised d equation:

$$\begin{aligned} 0 &= E_t \lambda_{t+1} p_t(e_t) \left(R_{t+1}^g - R_{g,t+1}^d\right) + E_t \lambda_{t+1} (1 - p_t(e_t)) \left(R_{t+1}^b - R_{b,t+1}^d\right) \\ &\quad + E_t \nu_{t+1} p_t(e_t) \left[\left(R_{t+1}^g - R_{t+1}^b\right) - \left(R_{g,t+1}^d - R_{b,t+1}^d\right)\right] + E_t \nu_{t+1} \left(R_{t+1}^b - R_{b,t+1}^d\right) \end{aligned}$$

or,

$$\begin{aligned} 0 &= E_t \lambda_{t+1} \left[p_t(e_t) \left(R_{t+1}^g - R_{g,t+1}^d\right) + (1 - p_t(e_t)) \left(R_{t+1}^b - R_{b,t+1}^d\right)\right] \\ &\quad + E_t \nu_{t+1} \left[p_t(e_t) \left(R_{t+1}^g - R_{g,t+1}^d\right) + (1 - p_t(e_t)) \left(R_{t+1}^b - R_{b,t+1}^d\right)\right] \end{aligned}$$

or

$$0 = E_t (\lambda_{t+1} + \nu_{t+1}) \left[p_t(e_t) \left(R_{t+1}^g - R_{g,t+1}^d\right) + (1 - p_t(e_t)) \left(R_{t+1}^b - R_{b,t+1}^d\right)\right].$$

Then, using the μ-condition,

$$0 = E_t (\lambda_{t+1} + \nu_{t+1}) \left[p_t(e_t) R_{t+1}^g + (1 - p_t(e_t)) R_{t+1}^b - R_t\right].$$

Finally, we use (A.2) to substitute out for μ_{t+1} in the R_g^d equation, to obtain:

$$R_g^d : \nu_{t+1} p_t(e_t) + \eta_t \lambda_{t+1} p_t'(e_t) = 0.$$

The optimization conditions derived here are summarized in (9).

To gain intuition into this multiplier, consider the case, $\nu_{t+1} = 0$, so that the cash constraint is not binding and the R_g^d condition implies $\eta_t = 0$. Since $\lambda_{t+1} + \nu_{t+1} > 0$ the e-condition then implies that $R_{g,t+1}^d = R_{b,t+1}^d$ can be the solution (as long as it does not make the cash constraint binding). Combining this with the μ-condition then implies that

$$R_{g,t+1}^d = R_{b,t+1}^d = R_t \tag{A.5}$$

can be the solution. It then follows from the η-condition that:

$$e_t = E_t \lambda_{t+1} p_t'(e_t) \left[\left(R_{t+1}^g - R_{t+1}^b\right)(N_t + d_t)\right], \tag{A.6}$$

so that banker effort level is efficient.

Now consider the case, $\nu_{t+1} > 0$ for *all* states of nature. Then, the R_g^d-condition implies $\eta_t < 0$ and the e-condition, after substituting out ν_{t+1} using the the R_g^d-condition, implies

$$E_t \lambda_{t+1} p_t'(e_t) \left(R_{g,t+1}^d - R_{b,t+1}^d\right) d_t = -\frac{\eta_t}{1 - \frac{\eta_t p_t'(e_t)}{p_t(e_t)}} > 0.$$

The e-condition then shows that banker effort is suboptimal. By continuity, when ν_{t+1} is large the inefficiency of the banking system is great and when it is small, there inefficiency is smaller. We think of a 'crisis time' as one in which ν_{t+1} is large.

Given our constraints, we suspect that when the cash constraint is always binding, $\nu_{t+1} > 0$, all state contingent deposit returns $R^d_{g,t+1}, R^d_{b,t+1}$,are pinned down. To see why, consider the case in which there are two aggregate states possible in period $t+1$, given period t. Denote these by 1 and 2 and suppose they have probability, π_t and $1-\pi_t$, respectively. The μ equations are:

$$\begin{aligned} R_t &= p_t(e_t) R^{d,1}_{g,t+1} + (1-p_t(e_t)) R^{d,1}_{b,t+1} \\ R_t &= p_t(e_t) R^{d,2}_{g,t+1} + (1-p_t(e_t)) R^{d,2}_{b,t+1} \end{aligned}$$

and the ν equations are

$$\begin{aligned} R^{b,1}_{t+1}(N_t + d_t) - R^{d,1}_{b,t+1} d_t &= 0 \\ R^{b,2}_{t+1}(N_t + d_t) - R^{d,2}_{b,t+1} d_t &= 0 \end{aligned}$$

Given the time t realization of variables, this represents four equations in four unknowns. In general, for given $R_t, p_t(e_t)$ these variables are pinned down. If there are more states of nature, then these equations represent restrictions on the deposit returns. Either way, the state contingency in the returns does not appear to contribute directly to multiplicity of equilibria, at least when the case constraint is always binding. As a practical matter, we can solve the model assuming the cash constraint always binds. We can then inspect the multiplier and verify that it is always positive. If ever it is negative that means that the constraint as an inequality is in fact not binding.

Consider the issue of the relative magnitude of $R^d_{b,+1}$ and $R^d_{g,+1}$. We suspect that it will not be true across all states of nature that $R^d_{b,t+1} \leq R^d_{g,t+1}$. Consider a simple example. Suppose there is an aggregate state where $R^b_{t+1} = 0$. In that state, it must be that $R^d_{b,t+1} = 0$ too. In such a state, assuming $R_t > 0$, it must be that $R^d_{g,t+1} > R^d_{b,t+1}$. By itself, this spread induces a substantial inefficiency in the e decision (see the η equation). But, the spread affects the choice of e only by its expected value. If that spread is very large in some state then it does not induce a large inefficiency if it is sufficiently small in another state. We might even imagine that it could be negative in another state, $R^d_{b,t+1} > R^d_{g,t+1}$. In this case, creditors in effect subsidize bankers that make positive profits and tax the ones that lose. This obviously has a big positive incentive effect on e. This possibility should not be a problem for our maintained assumption that the cash constraint is non-binding in the g state. To see this, suppose that it is binding in the b state:

$$R^b_{t+1}(N_t + d_t) - R^d_{b,t+1} d_t = 0.$$

By construction, $R^g_{t+1} > R^b_{t+1}$ in all aggregate states. also, in the scenario we are discussing, $R^d_{g,t+1} < R^d_{b,t+1}$. Both guarantee that the cash constraint is not binding in the g state.

An interesting feature of the model is that it implies a non-trivial cross-sectional variance on the returns of banks. In any given period $t+1$ state of nature, the cross section mean of bank returns on equity is:

$$R^m_{t+1} = p_t(e_t)\left[\frac{R^g_{t+1}(N_t + d_t) - R^d_{g,t+1} d_t}{N_t}\right] + (1-p_t(e_t))\left[\frac{R^b_{t+1}(N_t + d_t) - R^d_{b,t+1} d_t}{N_t}\right].$$

To determine the cross sectional standard deviation of bank equity returns, note that in the cross section, in any aggregate state, $p_t(e_t)$ banks each earn

$$\frac{R^g_{t+1}(N_t + d_t) - R^d_{g,t+1} d_t}{N_t}$$

return on equity. Similarly, $1-p_t(e_t)$ banks earn a return on equity equal to

$$\frac{R^b_{t+1}(N_t+d_t)-R^d_{b,t+1}d_t}{N_t}.$$

Recall that if a random variable has a binomial distribution and takes on the value x^h with probability p and x^l with probability $1-p$, then the variance of that random variable is $p(1-p)\left(x^h-x^l\right)^2$. So, the period t cross-sectional standard deviation of bank returns is:

$$\begin{aligned}
s^d_{t+1} &= \left[p_t(e_t)(1-p_t(e_t))\right]^{1/2}\left[\frac{R^g_{t+1}(N_t+d_t)-R^d_{g,t+1}d_t}{N_t}-\frac{R^b_{t+1}(N_t+d_t)-R^d_{b,t+1}d_t}{N_t}\right] \\
&= \left[p_t(e_t)(1-p_t(e_t))\right]^{1/2}\frac{R^g_{t+1}(N_t+d_t)-R^d_{g,t+1}d_t}{N_t},
\end{aligned}$$

taking into account our assumption that the cash constraint is binding for bad banks. Note that $p_t(e_t)>1/2$ then the cross sectional standard deviation is decreasing in e_t.

B Appendix B: Scaling and Miscellaneous Variables

To solve our model, we require that the variables be stationary. To this end, we adopt a particular scaling of the variables. Because our model satisfies sufficient conditions for balanced growth, when the equilibrium conditions of the model are written in terms of the scaled variables, only the growth rates and not the levels of the stationary shocks appear. In this appendix we describe the scaling of the model that is adopted. In addition, we describe the mapping from the variables in the scaled model to the variables measured in the data.

Let

$$\begin{aligned}
q_t &= \Upsilon^t\frac{P_{k',t}}{P_t}, y_{z,t}=\frac{Y_t}{z_t^+},\ i_t=\frac{I_t}{z_t^+\Upsilon^t},\ \tilde{w}_t\equiv\frac{W_t}{z_t^+P_t},\ p_{I,t}\equiv\frac{1}{\Upsilon^t\mu_{\Upsilon,t}}, P_{I,t}=\frac{P_t}{\Upsilon^t\mu_{\Upsilon,t}} \\
\bar{k}_t &= \frac{\bar{K}_t}{z_{t-1}^+\Upsilon^{t-1}},\ r_t^k=\Upsilon^t\tilde{r}_t^k,\ \mu^*_{z,t}=\frac{z_t^+}{z_{t-1}^+},\ c_t=\frac{C_t}{z_t^+},\ \lambda_{z,t}=\lambda_t z_t^* P_t
\end{aligned}$$

where $\tilde{r}_t^kP_t$ denotes the nominal rental rate on capital. Also, $\tilde{r}_t^k$ denotes the real, unscaled, rental rate of capital. We do not work with this variable. The rate of inflation in the nominal wage rate is:

$$\pi_{w,t}\equiv\frac{W_t}{W_{t-1}}=\frac{\tilde{w}_t\mu^*_{z,t}\pi_t}{\tilde{w}_{t-1}}.$$

Consider gdp growth, according to the model.

$$\frac{Y_t^{gdp}}{z_t^+}\equiv y_t=c_t+\frac{i_t}{\mu_{\Upsilon,t}}+g_t,$$

or,

$$Y_t^{gdp}=y_tz_t^+,$$

so that

$$\begin{aligned}
\Delta\log Y_t^{gdp} &= \log Y_t^{gdp}-\log Y_{t-1}^{gdp}=\log(y_t)-\log(y_{t-1})+\log\left(z_t^+\right)-\log z_{t-1}^+ \\
&= \log(y_t)-\log(y_{t-1})+\log\mu^*_{z,t}
\end{aligned}$$

Let N_t denote period t nominal net worth, so that

$$n_t = \frac{N_t}{P_t z_t^+}.$$

Then,

$$\Delta \log \frac{N_t}{P_t} = \log n_t - \log n_{t-1} + \log \mu_{z,t}^*.$$

Another variable is investment. There is an issue about what units to measure investment in. Investment times its relative price is given by:

$$inv_t \equiv \frac{I_t}{\Upsilon^t \mu_{\Upsilon,t}} = \frac{i_t z_t^+ \Upsilon^t}{\Upsilon^t \mu_{\Upsilon,t}} = \frac{i_t z_t^+}{\mu_{\Upsilon,t}},$$

so that:

$$\Delta \log inv_t \equiv \log inv_t - \log inv_{t-1} = \log i_t - \log i_{t-1} + \log \mu_{z,t}^* - \left(\log \mu_{\Upsilon,t} - \log \mu_{\Upsilon,t-1}\right).$$

The investment goods relative to consumption goods is given by

$$p_{I,t} \equiv \frac{1}{\Upsilon^t \mu_{\Upsilon,t}},$$

so that

$$\begin{aligned}
\Delta \log p_{I,t} &= -t \log \Upsilon + (t-1) \log \Upsilon - \log \mu_{\Upsilon,t} + \log \mu_{\Upsilon,t-1} \\
&= -\log \Upsilon - \log \mu_{\Upsilon,t} + \log \mu_{\Upsilon,t-1}.
\end{aligned}$$

Also,

$$\Delta \log C_t = \log c_t - \log c_{t-1} + \log \mu_{z,t}^*.$$

The growth rate of the real wage is:

$$\Delta \log \frac{W_t}{P_t} = \log \tilde{w}_t - \log \tilde{w}_{t-1} + \log \mu_{z,t}^*$$

C Appendix C: Dynamic Equations

Here, we display all the dynamic equilibrium conditions associated with the model.

C.1 Prices

The equations pertaining to prices are:

$$(1) p_t^* - \left[(1-\xi_p) \left(\frac{K_{p,t}}{F_{p,t}} \right)^{\frac{\lambda_f}{1-\lambda_f}} + \xi_p \left(\frac{\tilde{\pi}_t}{\pi_t} p_{t-1}^* \right)^{\frac{\lambda_f}{1-\lambda_f}} \right]^{\frac{1-\lambda_f}{\lambda_f}} = 0 \tag{C.1}$$

and

$$(2) E_t \left\{ \zeta_{c,t} \lambda_{z,t} y_{z,t} + \left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda_f}} \beta \xi_p F_{p,t+1} - F_{p,t} \right\} = 0, \tag{C.2}$$

where $\lambda_{z,t}$ denotes $\lambda_t z_t^* P_t$. Also,

$$(3)\zeta_{c,t}\lambda_{z,t}\lambda_f y_{z,t} s_t + \beta\xi_p \left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}}\right)^{\frac{\lambda_f}{1-\lambda_f}} K_{p,t+1} - K_{p,t} = 0. \tag{C.3}$$

Note that both these equations involve $F_{p,t}$. This reflects that a lot of equations have been substituted out. In particular, we have

$$(4)F_{p,t}\left[\frac{1-\xi_p\left(\frac{\tilde{\pi}_t}{\pi_t}\right)^{\frac{1}{1-\lambda_f}}}{1-\xi_p}\right]^{1-\lambda_f} = K_{p,t},\ \tilde{p}_t = \frac{K_{p,t}}{F_{p,t}},$$

where $\tilde{p}_t$ is the real price set by price-optimizing firms in period t. This is not a variable of direct interest in the analysis.

C.2 Wages

The demand for labor is the solution to the following problem:

$$\max W_t \overbrace{\left[\int_0^1 (h_{t,i})^{\frac{1}{\lambda_w}} di\right]^{\lambda_w}}^{=l_t} - \int_0^1 W_{t,i} h_{t,i} di,$$

where $W_{t,i}$ is the wage rate of $i-$type workers and W_t is the wage rate for homogeneous labor, l_t. The first order condition is:

$$h_{t,i} = l_t \left(\frac{W_t}{W_{t,i}}\right)^{\frac{\lambda_w}{\lambda_w - 1}}.$$

The wages of non-optimizing unions evolve as follows:

$$W_{j,t} = \tilde{\pi}_{w,t}\left(\mu_{z^*,t}\right)^{\iota_\mu}\left(\mu_{z^*}\right)^{1-\iota_\mu} W_{j,t-1},\ \tilde{\pi}_{w,t} \equiv \left(\pi_t^*\right)^{\iota_{w1}}\left(\pi_{t-1}\right)^{\iota_{w2}} \bar{\pi}^{1-\iota_{w1}-\iota_{w2}}, \tag{C.4}$$

Nominal wage growth, $\pi_{w,t}$, is:

$$\pi_{w,t} = \frac{\tilde{w}_t \mu_{z,t}^* \pi_t}{\tilde{w}_{t-1}},$$

where $\tilde{w}_t$ denotes the scaled wage rate:

$$\tilde{w}_t \equiv \frac{W_t}{z_t^* P_t}.$$

The labor input variable that we treat as observed is the sum over the various different types of labor:

$$\begin{aligned} h_t &= \int_0^1 h_{it} di \\ &= l_t W_t^{\frac{\lambda_w}{\lambda_w - 1}} \int_0^1 (W_{t,i})^{\frac{\lambda_w}{1-\lambda_w}} di \\ &= l_t W_t^{\frac{\lambda_w}{\lambda_w - 1}} (W_t^*)^{\frac{\lambda_w}{1-\lambda_w}}, \end{aligned}$$

where

$$
\begin{aligned}
W_t^* &\equiv \left[\int_0^1 (W_{t,i})^{\frac{\lambda_w}{1-\lambda_w}} di\right]^{\frac{1-\lambda_w}{\lambda_w}} \\
&= \left[(1-\xi_w)\tilde{W}_t + \int_{\xi_w \text{ monopolists that do not reoptimize}} \left(\tilde{\pi}_{w,t}\left(\mu_{z^*,t}\right)^{\iota_\mu}\left(\mu_{z^*}\right)^{1-\iota_\mu} W_{i,t-1}\right)^{\frac{\lambda_w}{1-\lambda_w}} di\right]^{\frac{1-\lambda_w}{\lambda_w}} \\
&= \left[(1-\xi_w)\tilde{W}_t + \xi_w\left(\tilde{\pi}_{w,t}\left(\mu_{z^*,t}\right)^{\iota_\mu}\left(\mu_{z^*}\right)^{1-\iota_\mu} W_{t-1}^*\right)^{\frac{\lambda_w}{1-\lambda_w}}\right]^{\frac{1-\lambda_w}{\lambda_w}}.
\end{aligned}
$$

Let $w_t^* \equiv W_t^*/W_t$, and use linear homogeneity:

$$
w_t^* = \left[(1-\xi_w)\frac{\tilde{W}_t}{W_t} + \xi_w\left(\frac{\tilde{\pi}_{w,t}\left(\mu_{z^*,t}\right)^{\iota_\mu}\left(\mu_{z^*}\right)^{1-\iota_\mu}}{\pi_{w,t}} w_{t-1}^*\right)^{\frac{\lambda_w}{1-\lambda_w}}\right]^{\frac{1-\lambda_w}{\lambda_w}},
$$

$\tilde{W}_t$ is the nominal wage set by the $1-\xi_w$ wage optimizers in the current period. Rewriting,

$$
w_t^* = [(1-\xi_w)w_t^{\frac{\lambda_w}{1-\lambda_w}} + \xi_w\left(\frac{\tilde{\pi}_{w,t}\left(\mu_{z,t}^*\right)^{\iota_\mu}\left(\mu_z^*\right)^{1-\iota_\mu}}{\pi_{wt}} w_{t-1}^*\right)^{\frac{\lambda_w}{1-\lambda_w}}]^{\frac{1-\lambda_w}{\lambda_w}}, \tag{C.5}
$$

where

$$
w_t \equiv \frac{\tilde{W}_t}{W_t}. \tag{C.6}
$$

We conclude:

$$
h_t = l_t\left(w_t^*\right)^{\frac{\lambda_w}{1-\lambda_w}}. \tag{C.7}
$$

For purposes of evaluating aggregate utility, it is also convenient to have an expression for the following:

$$
\begin{aligned}
&\int_0^1 h_{it}^{1+\sigma_L} di \\
&= l_t^{1+\sigma_L} W_t^{-\frac{\lambda_w(1+\sigma_L)}{1-\lambda_w}} \int_0^1 (W_{t,i})^{\frac{\lambda_w(1+\sigma_L)}{1-\lambda_w}} di \\
&= l_t^{1+\sigma_L} W_t^{-\frac{\lambda_w(1+\sigma_L)}{1-\lambda_w}} \ddot{W}_t^{\frac{\lambda_w(1+\sigma_L)}{1-\lambda_w}},
\end{aligned}
$$

where

$$
\ddot{W}_t \equiv \left[\int_0^1 (W_{t,i})^{\frac{\lambda_w(1+\sigma_L)}{1-\lambda_w}} di\right]^{\frac{1-\lambda_w}{\lambda_w(1+\sigma_L)}}.
$$

Then,

$$
\begin{aligned}
\ddot{W}_t &= \left[\int_0^1 (W_{t,i})^{\frac{\lambda_w(1+\sigma_L)}{1-\lambda_w}} di\right]^{\frac{1-\lambda_w}{\lambda_w(1+\sigma_L)}} \\
&= \left[(1-\xi_w)\left(\tilde{W}_t\right)^{\frac{\lambda_w(1+\sigma_L)}{1-\lambda_w}} + \int_{\xi_w \text{ that change}} (W_{t,i})^{\frac{\lambda_w(1+\sigma_L)}{1-\lambda_w}} di\right]^{\frac{1-\lambda_w}{\lambda_w(1+\sigma_L)}} \\
&= \left[(1-\xi_w)\left(\tilde{W}_t\right)^{\frac{\lambda_w(1+\sigma_L)}{1-\lambda_w}} + \xi_w\left(\tilde{\pi}_{w,t}\left(\mu_{z^*,t}\right)^{\iota_\mu}\left(\mu_{z^*}\right)^{1-\iota_\mu}\ddot{W}_{t-1}\right)^{\frac{\lambda_w(1+\sigma_L)}{1-\lambda_w}}\right]^{\frac{1-\lambda_w}{\lambda_w(1+\sigma_L)}}.
\end{aligned}
$$

Divide by W_t and make use of the linear homogeneity of the above expression:

$$\frac{\ddot{W}_t}{W_t} = \left[(1-\xi_w) \left(\frac{\tilde{W}_t}{W_t} \right)^{\frac{\lambda_w(1+\sigma_L)}{1-\lambda_w}} + \xi_w \left(\frac{\tilde{\pi}_{w,t} \left(\mu_{z^*,t}\right)^{\iota_\mu} \left(\mu_{z^*}\right)^{1-\iota_\mu}}{\pi_{w,t}} \frac{\ddot{W}_{t-1}}{W_{t-1}} \right)^{\frac{\lambda_w(1+\sigma_L)}{1-\lambda_w}} \right]^{\frac{1-\lambda_w}{\lambda_w(1+\sigma_L)}}$$

Define

$$\ddot{w}_t = \frac{\ddot{W}_t}{W_t},$$

so that

$$\ddot{w}_t = \left[(1-\xi_w) \left(w_t \right)^{\frac{\lambda_w(1+\sigma_L)}{1-\lambda_w}} + \xi_w \left(\frac{\tilde{\pi}_{w,t} \left(\mu_{z^*,t}\right)^{\iota_\mu} \left(\mu_{z^*}\right)^{1-\iota_\mu}}{\pi_{w,t}} \ddot{w}_{t-1} \right)^{\frac{\lambda_w(1+\sigma_L)}{1-\lambda_w}} \right]^{\frac{1-\lambda_w}{\lambda_w(1+\sigma_L)}}, \tag{C.8}$$

using (C.6). We conclude

$$\begin{aligned} \int_0^1 h_{it}^{1+\sigma_L} di &= \left[l_t \left(\ddot{w}_t\right)^{\frac{\lambda_w}{1-\lambda_w}} \right]^{(1+\sigma_L)} \qquad \text{(C.9)} \\ &= \left[h_t \left(\frac{\ddot{w}_t}{w_t^*} \right)^{\frac{\lambda_w}{1-\lambda_w}} \right]^{(1+\sigma_L)}. \end{aligned}$$

using (C.7).

The optimality conditions associated with wage-setting are characterized by:

$$\begin{aligned} &(5) E_t\{\zeta_{c,t}\lambda_{z,t} \frac{\left(w_t^*\right)^{\frac{\lambda_w}{\lambda_w-1}} h_t \left(1-\tau_t^l\right)}{\lambda_w} \qquad \text{(C.10)} \\ &+\beta\xi_w \left(\mu_{z^*}\right)^{\frac{1-\iota_\mu}{1-\lambda_w}} E_t \left(\mu_{z^*,t+1}\right)^{\frac{\iota_\mu}{1-\lambda_w}-1} \left(\frac{1}{\pi_{w,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}} \frac{\tilde{\pi}_{w,t+1}^{\frac{1}{1-\lambda_w}}}{\pi_{t+1}} F_{w,t+1} - F_{w,t}\} = 0 \end{aligned}$$

and

$$(6)\ E_t\{\zeta_{c,t}\zeta_t \left[\left(w_t^*\right)^{\frac{\lambda_w}{\lambda_w-1}} h_t \right]^{1+\sigma_L} + \beta\xi_w \left(\frac{\tilde{\pi}_{w,t+1} \left(\mu_{z,t+1}^*\right)^{\iota_\mu} \left(\mu_z^*\right)^{1-\iota_\mu}}{\pi_{wt+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} K_{w,t+1} - K_{w,t}\} = 0.$$

$$(7)\ \frac{1}{\psi_L} \left[\frac{1-\xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \left(\mu_{z^*}\right)^{1-\iota_\mu} \left(\mu_{z^*,t}\right)^{\iota_\mu} \right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w} \right]^{1-\lambda_w(1+\sigma_L)} \tilde{w}_t F_{w,t} - K_{w,t} = 0$$

Optimization by households implies:

$$w_t = \left[\frac{\psi_L}{\tilde{w}_t} \frac{K_{w,t}}{F_{w,t}} \right]^{\frac{1-\lambda_w}{1-\lambda_w(1+\sigma_L)}},$$

so that, using (C.5):

$$w_t^* = \left[(1-\xi_w) \left[\frac{\psi_L}{\tilde{w}_t} \frac{K_{w,t}}{F_{w,t}} \right]^{\frac{\lambda_w}{1-\lambda_w(1+\sigma_L)}} + \xi_w \left(\frac{\tilde{\pi}_{w,t} \left(\mu_{z,t}^*\right)^{\iota_\mu} \left(\mu_z^*\right)^{1-\iota_\mu}}{\pi_{wt}} w_{t-1}^* \right)^{\frac{\lambda_w}{1-\lambda_w}} \right]^{\frac{1-\lambda_w}{\lambda_w}}.$$

We can replace $K_{w,t}/F_{w,t}$ with the expression implied by (7) above:

$$(8)\;\; w_t^* = \left[(1-\xi_w) \left(\frac{1-\xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \left(\mu_{z^*}\right)^{1-\iota_\mu} \left(\mu_{z^*,t}\right)^{\iota_\mu} \right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w} \right)^{\lambda_w} + \xi_w \left(\frac{\tilde{\pi}_{w,t} \left(\mu_{z,t}^*\right)^{\iota_\mu} \left(\mu_z^*\right)^{1-\iota_\mu}}{\pi_{wt}} w_{t-1}^* \right)^{\frac{\lambda_w}{1-\lambda_w}} \right]^{\frac{1-\lambda_w}{\lambda_w}} \tag{C.11}$$

C.3 Capital Utilization, Marginal Cost, Return on Capital, Investment, Monetary Policy

The first order necessary condition associated with the capital utilization decision is:

$$\frac{1}{\Upsilon^t} a'(u_t) = \tilde{r}_t^k,$$

or,

$$a'(u_t) = \Upsilon^t \tilde{r}_t^k = r_t^k,$$

after scaling. Making use of our assumed utilization cost function, this reduces to:

$$(9)\; r_t^k = r^k \exp\left(\sigma_a \left[u_t - 1\right]\right),$$

where

$$a(u_t) = \frac{r^k}{\sigma_a} \left[\exp(\sigma_a \left[u_t - 1\right]) - 1\right].$$

Also, r^k denotes the steady state value of r_t^k. The above restriction on the $a(u_t)$ function implies that $u = 1$ in a steady state. As a result, the steady state is independent of the capital adjustment costs.

Marginal cost is given by:

$$\begin{aligned} (10) r_t^k &= \frac{\alpha \epsilon_t}{\left[1+\psi_{k,t} R_t\right]} \left(\frac{\Upsilon \mu_{z,t}^* L_t \left(w_t^*\right)^{\frac{\lambda_w}{\lambda_w - 1}}}{u_t \bar{k}_t} \right)^{1-\alpha} s_t \\ \tilde{w}_t &= \frac{(1-\alpha)\epsilon_t}{\left[1+\psi_{l,t} R_t\right]} \left(\frac{\Upsilon \mu_{z,t}^* L_t \left(w_t^*\right)^{\frac{\lambda_w}{\lambda_w - 1}}}{u_t \bar{k}_t} \right)^{-\alpha} s_t, \end{aligned} \tag{C.12}$$

where $\psi_{k,t}$ and $\psi_{l,t}$ denote the fraction of the capital services and labor bills, respectively, that must be financed in advance. Combining the last two equations, we obtain the familiar expression for marginal cost:

$$(11)\; s_t = \frac{1}{\epsilon_t} \left(\frac{r_t^k \left[1+\psi_{k,t} R_t\right]}{\alpha} \right)^{\alpha} \left(\frac{\tilde{w}_t \left[1+\psi_{l,t} R_t\right]}{1-\alpha} \right)^{1-\alpha}, \tag{C.13}$$

where $\psi_{k,t} = \psi_{l,t} = 0$. Resource constraint:

$$(12) a(u_t) \frac{\bar{k}_t}{\Upsilon \mu_{z,t}^*} + g_t + c_t + \frac{i_t}{\mu_{\Upsilon,t}} = y_{z,t} \tag{C.14}$$

where g_t is an exogenous stochastic process and

$$(13)\bar{k}_{t+1} = \overbrace{\left[p_t\left(e_t\right)e^{g_t} + \left(1-p_t\left(e_t\right)\right)e^{b_t}\right]}^{\text{relevant only for financial friction model, drop in CEE version}} \left\{(1-\delta)\frac{1}{\mu^*_{z,t}\Upsilon}\bar{k}_t + \left[1 - S\left(\frac{\zeta_{i,t}\, i_t \mu^*_{z,t}\Upsilon}{i_{t-1}}\right)\right] i_t\right\}, \tag{C.15}$$

where i_t is investment scaled by $z^*_t\Upsilon^t$.

Equation defining the nominal non-state contingent rate of interest:

$$(14)E_t\{\beta\frac{1}{\pi_{t+1}\mu^*_{z,t+1}}\zeta_{c,t+1}\lambda_{z,t+1}R_t - \zeta_{c,t}\lambda_{z,t}\} = 0 \tag{C.16}$$

The derivative of utility with respect to consumption is,

$$(15)E_t\left[\zeta_{c,t}\lambda_{z,t} - \frac{\mu^*_{z,t}\zeta_{c,t}}{c_t\mu^*_{z,t} - bc_{t-1}} + b\beta\frac{\zeta_{c,t+1}}{c_{t+1}\mu^*_{z,t+1} - bc_t}\right] = 0, \tag{C.17}$$

where c_t denotes consumption scaled by z^*_t. The following capital first order first order condition is an equilibrium condition in CEE, but not in our model with financial frictions because in that model households do not accumulate capital:

$$(16)E_t\left\{-\zeta_{c,t}\lambda_{zt} + \frac{\beta}{\pi_{t+1}\mu^*_{z,t+1}}\zeta_{c,t+1}\lambda_{zt+1}R^k_{t+1}\right\} = 0. \tag{C.18}$$

In (C.18), R^k_{t+1} denotes the benchmark rate of return on capital:

$$(17)\ R^k_t = \frac{u_t r^k_t - a(u_t) + (1-\delta)q_t}{\Upsilon q_{t-1}}\pi_t$$

where q_t denotes the scaled market price of capital, $Q_{\bar{K}',t}$:

$$q_t = \Upsilon^t\frac{Q_{\bar{K}',t}}{P_t}.$$

Equation (17) holds in our financial friction model, as well as in CEE. The investment first order condition, (28)

$$\begin{aligned}(18)\ E_t\{\zeta_{c,t}\lambda_{zt}q_t\left[1 - S(\frac{\zeta_{i,t}\mu^*_{z,t}\Upsilon i_t}{i_{t-1}}) - S'(\frac{\zeta_{i,t}\mu^*_{z,t}\Upsilon i_t}{i_{t-1}})\frac{\zeta_{i,t}\mu^*_{z,t}\Upsilon i_t}{i_{t-1}}\right] \\ -\frac{\zeta_{c,t}\lambda_{zt}}{\mu_{\Upsilon,t}} + \frac{\beta\lambda_{zt+1}\zeta_{c,t+1}\zeta_{i,t+1}q_{t+1}}{\mu^*_{z,t+1}\Upsilon}S'(\frac{\zeta_{i,t+1}\mu^*_{z,t+1}\Upsilon i_{t+1}}{i_t})\left(\frac{\mu^*_{z,t+1}\Upsilon i_{t+1}}{i_t}\right)^2\} &= 0,\end{aligned} \tag{C.19}$$

where i_t is scaled (by $z^*_t\Upsilon^t$) investment. The scaled representation of aggregagte output is:

$$(19)\ y_{z,t} \equiv \frac{Y_t}{z^*_t} = (p^*_t)^{\frac{\lambda_f}{\lambda_f - 1}}\left[\epsilon_t\left(\frac{u_t\bar{k}_t}{\mu^*_{z,t}\Upsilon}\right)^{\alpha}\left((w^*_t)^{\frac{\lambda_w}{\lambda_w - 1}}h_t\right)^{1-\alpha} - \phi\right]$$

The monetary policy rule:

$$\begin{aligned}(20)\ \log(1+R_t) &= (1-\tilde{\rho})\log(1+R) + \tilde{\rho}\log(1+R_{t-1}) \\ &+\frac{1-\tilde{\rho}}{1+R}\left[\tilde{a}_p\pi\log\frac{\pi_{t+1}}{\pi^*_t} + \tilde{a}_y\frac{1}{4}\log\frac{y_t}{y}\right] + x^p_t,\end{aligned} \tag{C.20}$$

where x_t^p is an iid monetary policy shock and y_t denotes scaled GDP:

$$(21)\ \ y_t = g_t + c_t + \frac{i_t}{\mu_{\Upsilon,t}}.$$

It's important not to confuse y_t and Y_t. The former is scaled GDP while the latter is unscaled gross output. Scaled gross output and scaled GDP are the same in steady state but different in the dynamics because u_t is potentially different from unity then.

C.4 Conditions Pertaining to Financial Frictions

First, consider the equilibrium conditions associated with the financial friction model with unobserved effort. Consider the following scaling:

$$\tilde{d}_t = \frac{d_t}{z_t^* P_t},\ \lambda_{z,t+1} = \lambda_{t+1} z_{t+1}^* P_{t+1},\ v_{z,t+1} = \nu_{t+1} z_{t+1}^* P_{t+1},\ \tilde{N}_t = \frac{N_t}{z_t^* P_t},\ \tilde{T}_t = \frac{T_t}{z_t^* P_t}$$

Consider the e equation:

$$e : E_t \left(\lambda_{t+1} + \nu_{t+1}\right) p_t' \left(e_t\right) \left(R_{g,t+1}^d - R_{b,t+1}^d\right) d_t + \eta_t = 0$$

or,

$$e : E_t \left(\lambda_{z,t+1} + \nu_{z,t+1}\right) \frac{1}{z_{t+1}^* P_{t+1}} p_t' \left(e_t\right) \left(R_{g,t+1}^d - R_{b,t+1}^d\right) z_t^* P_t \tilde{d}_t + \eta_t = 0$$

or,

$$e : E_t \left(\lambda_{z,t+1} + \nu_{z,t+1}\right) \frac{1}{\mu_{z^*,t+1}\pi_{t+1}} p_t' \left(e_t\right) \left(R_{g,t+1}^d - R_{b,t+1}^d\right) \tilde{d}_t + \eta_t = 0.$$

Now consider the d equation:

$$d : 0 = E_t \left(\lambda_{z,t+1} + \nu_{z,t+1}\right) \frac{1}{z_{t+1}^* P_{t+1}} \left[p_t \left(e_t\right) R_{t+1}^g + \left(1 - p_t \left(e_t\right)\right) R_{t+1}^b - R_t\right]$$

Multiply this equation by $z_t^* P_t$ to obtain:

$$d : 0 = E_t \left(\lambda_{z,t+1} + \nu_{z,t+1}\right) \frac{1}{\mu_{z^*,t+1}\pi_{t+1}} \left[p_t \left(e_t\right) R_{t+1}^g + \left(1 - p_t \left(e_t\right)\right) R_{t+1}^b - R_t\right]$$

In the case of the R_g^d equation, we can simply multiply by $z_{t+1}^* P_{t+1}$:

$$R_g^d : \nu_{z,t+1} p_t \left(e_t\right) + \eta_t \lambda_{z,t+1} p_t' \left(e_t\right) = 0.$$

Equation μ requires no adjustment:

$$\mu : R_t = p_t \left(e_t\right) R_{g,t+1}^d + \left(1 - p_t \left(e_t\right)\right) R_{b,t+1}^d$$

Next, consider equation η :

$$\eta : e_t = E_t \lambda_{z,t+1} \frac{1}{\mu_{z^*,t+1}\pi_{t+1}} p_t' \left(e_t\right) \left[\left(R_{t+1}^g - R_{t+1}^b\right) \left(\tilde{N}_t + \tilde{d}_t\right) - \left(R_{g,t+1}^d - R_{b,t+1}^d\right) \tilde{d}_t\right]$$

The ν equation is:

$$\nu : R_{t+1}^b \left(\tilde{N}_t + \tilde{d}_t\right) - R_{b,t+1}^d \tilde{d}_t = 0,$$

The law of motion for net worth is:

$$N_{t+1} = \gamma_{t+1}\left\{p_t(e_t)\left[R^g_{t+1}(N_t+d_t) - R^d_{g,t+1}d_t\right] + (1-p_t(e_t))\left[R^b_{t+1}(N_t+d_t) - R^d_{b,t+1}d_t\right]\right\} + T_{t+1}$$

Divide by $z^*_{t+1}P_{t+1}$

$$\tilde{N}_{t+1} = \frac{\gamma_{t+1}}{\mu_{z^*,t+1}\pi_{t+1}}\left\{p_t(e_t)\left[R^g_{t+1}\left(\tilde{N}_t+\tilde{d}_t\right) - R^d_{g,t+1}\tilde{d}_t\right] + (1-p_t(e_t))\left[R^b_{t+1}\left(\tilde{N}_t+\tilde{d}_t\right) - R^d_{b,t+1}\tilde{d}_t\right]\right\} + \tilde{T}_{t+1}$$

or,

$$\tilde{N}_{t+1} = \frac{\gamma_{t+1}}{\mu_{z^*,t+1}}\left\{p_t(e_t)\frac{R^g_{t+1}}{\pi_{t+1}}\left(\tilde{N}_t+\tilde{d}_t\right) + (1-p_t(e_t))\frac{R^b_{t+1}}{\pi_{t+1}}\left(\tilde{N}_t+\tilde{d}_t\right) - \frac{R_t}{\pi_{t+1}}\tilde{d}_t\right\} + \tilde{T}_{t+1}$$

We also require equations to define the returns of good and bad entrepreneurs:

$$\begin{aligned} R^g_{t+1} &= e^{g_t}R^k_{t+1}, \\ R^b_{t+1} &= e^{b_t}R^k_{t+1} \end{aligned}$$

Finally, we have the market clearing condition for capital:

$$P_{k',t}\tilde{K}_{t+1} = N_t + d_t,$$

If we multiply both sides of this expression by $p_t(e_t)e^{g_t} + (1-p_t(e_t))e^{b_t}$, we obtain:

$$P_{k',t}\bar{K}_{t+1} = \left[p_t(e_t)e^{g_t} + (1-p_t(e_t))e^{b_t}\right][N_t+d_t],$$

or

$$\frac{q_tP_t}{\Upsilon^t}\frac{z_t^+\Upsilon^t\bar{k}_{t+1}}{z^*_tP_t} = \left[p_t(e_t)e^{g_t} + (1-p_t(e_t))e^{b_t}\right]\left[\tilde{N}_t+\tilde{d}_t\right],$$

or

$$q_t\bar{k}_{t+1} = \left[p_t(e_t)e^{g_t} + (1-p_t(e_t))e^{b_t}\right]\left[\tilde{N}_t+\tilde{d}_t\right],$$

Collecting the equations for simplicity,

$$\begin{aligned}
e &: \quad E_t(\lambda_{z,t+1}+\nu_{z,t+1})\frac{1}{\mu_{z^*,t+1}\pi_{t+1}}p_t'(e_t)\left(R^d_{g,t+1}-R^d_{b,t+1}\right)\tilde{d}_t + \eta_t = 0 \\
d &: \quad 0 = E_t(\lambda_{z,t+1}+\nu_{z,t+1})\frac{1}{\mu_{z^*,t+1}\pi_{t+1}}\left[p_t(e_t)R^g_{t+1} + (1-p_t(e_t))R^b_{t+1} - R_t\right] \\
R^d_g &: \quad \nu_{z,t+1}p_t(e_t) + \eta_t\lambda_{z,t+1}p_t'(e_t) = 0 \\
\mu &: \quad R_t = p_t(e_t)R^d_{g,t+1} + (1-p_t(e_t))R^d_{b,t+1} \\
\eta &: \quad e_t = E_t\lambda_{z,t+1}\frac{1}{\mu_{z^*,t+1}\pi_{t+1}}p_t'(e_t)\left[\left(R^g_{t+1}-R^b_{t+1}\right)\left(\tilde{N}_t+\tilde{d}_t\right) - \left(R^d_{g,t+1}-R^d_{b,t+1}\right)\tilde{d}_t\right] \\
\nu &: \quad R^b_{t+1}\left(\tilde{N}_t+\tilde{d}_t\right) - R^d_{b,t+1}\tilde{d}_t = 0 \\
\tilde{N}_{t+1} &= \frac{\gamma_{t+1}}{\mu_{z^*,t+1}\pi_{t+1}}\left\{p_t(e_t)\left[R^g_{t+1}\left(\tilde{N}_t+\tilde{d}_t\right) - R^d_{g,t+1}\tilde{d}_t\right] + (1-p_t(e_t))\left[R^b_{t+1}\left(\tilde{N}_t+\tilde{d}_t\right) - R^d_{b,t+1}\tilde{d}_t\right]\right\} \\
&\quad + \tilde{T}_{t+1} \\
q_t\bar{k}_{t+1} &= \left[p_t(e_t)e^{g_t} + (1-p_t(e_t))e^{b_t}\right]\left[\tilde{N}_t+\tilde{d}_t\right] \\
R^g_{t+1} &= e^{g_t}R^k_{t+1} \\
R^b_{t+1} &= e^{b_t}R^k_{t+1}
\end{aligned}$$

To go from the CEE model to the model with financial frictions, we drop equation (16) (and modify the capital accumulation equation, (13)) and we add the above 10 equations. So, there is a net addition of 9 equations. The additional 9 variables are

$$R^g_{t+1}, R^b_{t+1}, \tilde{d}_t, \tilde{N}_t, R^d_{g,t+1}, R^d_{b,t+1}, e_t, \nu_{z,t+1}, \eta_t.$$

C.5 Social Welfare Function

We now turn to developing an expression for the representative household's utility function

$$\begin{aligned} Util_t &= \zeta_{c,t}\left\{\log(z_t^+ c_t - b z_{t-1}^+ c_{t-1}) - \psi_L \int_0^1 \frac{h_{it}^{1+\sigma_L}}{1+\sigma_L} di\right\} \\ &= \zeta_{c,t}\left\{\log\left[z_t^+ (c_t - b\frac{z_{t-1}^+}{z_t^+} c_{t-1})\right] - \psi_L \int_0^1 \frac{h_{it}^{1+\sigma_L}}{1+\sigma_L} di\right\} \\ &= \zeta_{c,t}\left\{\log(c_t - \frac{b}{\mu^*_{z,t}} c_{t-1}) - \frac{\psi_L}{1+\sigma_L} \int_0^1 h_{it}^{1+\sigma_L} di\right\}, \end{aligned}$$

apart from a constant term. Using (C.9):

$$\frac{\psi_L}{1+\sigma_L}\int_0^1 h_{it}^{1+\sigma_L} di = \frac{\psi_L}{1+\sigma_L}\left[h_t \left(\frac{\ddot{w}_t}{w_t^*}\right)^{\frac{\lambda_w}{1-\lambda_w}}\right]^{(1+\sigma_L)},$$

so that

$$Util_t = \zeta_{c,t}\left\{\log(c_t - \frac{b}{\mu^*_{z,t}} c_{t-1}) - \frac{\psi_L}{1+\sigma_L}\left[h_t \left(\frac{\ddot{w}_t}{w_t^*}\right)^{\frac{\lambda_w}{1-\lambda_w}}\right]^{(1+\sigma_L)}\right\},$$

where $\ddot{w}_t$ is defined in (C.8) and w_t^* is defined in (8). Both these variables are unity in steady state.

D Appendix D: Calculating Steady State

Here, we discuss algorithms for computing the steady state of three versions of our model. The first three sections describe the equations of the model. The last three sections describe the algorithms.

D.1 Price and Wage Equations

This section pertains to equations (1)-(8) of the dynamical system in Appendix C. These equations are trivial in the case, $\pi = \bar{\pi}$. Equation (C.1) in steady state, is:

$$p^* = \left[\frac{(1-\xi_p)\left(\frac{1-\xi_p\left(\frac{\bar{\pi}}{\pi}\right)^{\frac{1}{1-\lambda_f}}}{1-\xi_p}\right)^{\lambda_f}}{1-\xi_p\left(\frac{\bar{\pi}}{\pi}\right)^{\frac{\lambda_f}{1-\lambda_f}}}\right]^{\frac{1-\lambda_f}{\lambda_f}}.$$

Note that, if $\pi = \bar{\pi}$ then $p^* = 1$. Equation (C.2):

$$F_p = \frac{\lambda_z (p^*)^{\frac{\lambda_f}{\lambda_f - 1}} \left[\left(\frac{k}{\mu_z^* \Upsilon} \right)^{\alpha} \left((w^*)^{\frac{\lambda_w}{\lambda_w - 1}} h \right)^{1-\alpha} - \phi \right]}{1 - \left(\frac{\tilde{\pi}}{\pi} \right)^{\frac{1}{1-\lambda_f}} \beta \xi_p},$$

assuming

$$\left(\frac{\tilde{\pi}}{\pi} \right)^{\frac{1}{1-\lambda_f}} \beta \xi_p < 1.$$

Equation (C.3) in steady state is:

$$F_p = \frac{\lambda_z \lambda_f (p^*)^{\frac{\lambda_f}{\lambda_f - 1}} \left[\left(\frac{k}{\mu_z} \right)^{\alpha} \left((w^*)^{\frac{\lambda_w}{\lambda_w - 1}} h \right)^{1-\alpha} - \phi \right] s}{\left[\frac{1 - \xi_p \left(\frac{\tilde{\pi}}{\pi} \right)^{\frac{1}{1-\lambda_f}}}{1 - \xi_p} \right]^{1-\lambda_f} \left[1 - \beta \xi_p \left(\frac{\tilde{\pi}}{\pi} \right)^{\frac{\lambda_f}{1-\lambda_f}} \right]}$$

Equating the preceding two equations:

$$s = \frac{1}{\lambda_f} \frac{\left[\frac{1 - \xi_p \left(\frac{\tilde{\pi}}{\pi} \right)^{\frac{1}{1-\lambda_f}}}{1 - \xi_p} \right]^{1-\lambda_f} \left[1 - \beta \xi_p \left(\frac{\tilde{\pi}}{\pi} \right)^{\frac{\lambda_f}{1-\lambda_f}} \right]}{1 - \left(\frac{\tilde{\pi}}{\pi} \right)^{\frac{1}{1-\lambda_f}} \beta \xi_p}. \tag{D.1}$$

In the case, $\pi = \bar{\pi}$, $s = 1/\lambda_f$. Equation (C.10) in steady state is:

$$F_w = \frac{\lambda_z \frac{(w^*)^{\frac{\lambda_w}{\lambda_w - 1}} h}{\lambda_w}}{1 - \beta \xi_w \tilde{\pi}_w^{\frac{1}{1-\lambda_w}} \frac{\left(\frac{1}{\pi} \right)^{\frac{\lambda_w}{1-\lambda_w}}}{\pi}},$$

as long as the condition,

$$\beta \xi_w \tilde{\pi}_w^{\frac{1}{1-\lambda_w}} \frac{\left(\frac{1}{\pi} \right)^{\frac{\lambda_w}{1-\lambda_w}}}{\pi} < 1,$$

is satisfied. Also

$$\tilde{\pi}_w = (\pi)^{\iota_{w,2}} \bar{\pi}^{1-\iota_{w,2}}.$$

Equating the two expressions for F_w, we obtain:

$$\tilde{w} = W \lambda_w \frac{\psi_L h^{\sigma_L}}{\lambda_z}, \tag{D.2}$$

where

$$W = (w^*)^{\frac{\lambda_w}{\lambda_w - 1} \sigma_L} \left[\frac{1 - \xi_w \left(\frac{\tilde{\pi}_w}{\pi} \right)^{\frac{1}{1-\lambda_w}}}{1 - \xi_w} \right]^{\lambda_w (1+\sigma_L) - 1} \frac{1 - \beta \xi_w \left(\frac{\tilde{\pi}_w}{\pi} \right)^{\frac{1}{1-\lambda_w}}}{1 - \beta \xi_w \left(\frac{\tilde{\pi}_w}{\pi} \right)^{\frac{\lambda_w}{1-\lambda_w} (1+\sigma_L)}}, \tag{D.3}$$

which is unity in the case $\pi = \bar{\pi}$. In steady state, (C.11) reduces to:

$$w^* = \left[\frac{(1-\xi_w)\left(\frac{1-\xi_w\left(\frac{\tilde{\pi}_w}{\pi}\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w}\right)^{\lambda_w}}{1-\xi_w\left(\frac{\tilde{\pi}_w}{\pi}\right)^{\frac{\lambda_w}{1-\lambda_w}}} \right]^{\frac{1-\lambda_w}{\lambda_w}}, \tag{D.4}$$

which is unity when $\pi = \bar{\pi}$. According to the wage equation, the wage is a markup, $W\lambda_w$, over the household's marginal cost. Note that the magnitude of the markup depends on the degree of wage distortions in the steady state. These will be important to the extent that $\tilde{\pi}_w \neq \pi_w$.

In the case $\pi = \bar{\pi}$, we have

$$\tilde{\pi}_w, p^*, w^* = 1,\ \tilde{w} = \lambda_w \frac{\psi_L h^{\sigma_L}}{\lambda_z},\ \ s = \frac{1}{\lambda_f}, \tag{D.5}$$

in addition to F_p, F_w, K_p, K_w which do not get used in the subsequent equations.

D.2 Other Non-Financial Equations

The marginal cost equation, (C.12) implies:

$$r^k = \frac{\alpha\epsilon}{[1+\psi_k R]}\left(\frac{\Upsilon\mu_z^* h\,(w^*)^{\frac{\lambda_w}{\lambda_w - 1}}}{\bar{k}}\right)^{1-\alpha} s, \tag{D.6}$$

where w^* is determined by (D.4). In steady state, the capital accumulation equation, (C.15), is

$$\left[\frac{1}{p\,(e)\,e^g + (1-p\,(e))\,e^b} - (1-\delta)\frac{1}{\mu_z^*\Upsilon}\right]\bar{k} = i, \tag{D.7}$$

or,

$$\left[1-(1-\delta)\frac{1}{\mu_z^*\Upsilon}\right]\bar{k} = i, \tag{D.8}$$

using (D.16) below. In steady state, the equation for the nominal rate of interest, (C.16), reduces to:

$$R = \frac{\pi\mu_z^*}{\beta}. \tag{D.9}$$

In steady state, the marginal utility of consumption, (C.17), is

$$\lambda_z = \frac{1}{c}\frac{\mu_z^* - b\beta}{\mu_z^* - b}. \tag{D.10}$$

Finally, the euler equation for investment, (C.19), reduces to

$$q = 1.$$

Also, equations (17) and (19) in the dynamic system reduce to:

$$(19)\ y_z = \epsilon_t \left(\frac{\bar{k}}{\mu_z^*\Upsilon}\right)^{\alpha} h_t^{1-\alpha} - \phi$$

$$(17)\ R^k = \frac{r^k + 1 - \delta}{\Upsilon}\pi \tag{D.11}$$

We compute ϕ to guarantee that firm profits are zero in a steady state where $\pi = \bar{\pi}$. Let h and $\bar{k}$ denote hours worked and capital in such a steady state. Also, let F denote gross output of the final good in that steady state. Write sales of final good firm as $F - \phi$. Real marginal cost in this steady state is $s = 1/\lambda_f$. Since this is a constant, the total costs of the firm are sF. Zero profits requires $sF = F - \phi$. Thus, $\phi = (1 - s)\, F = F(1 - 1/\lambda_f)$, or,

$$(7)\phi = \left(\frac{\bar{k}}{\mu_z^* \Upsilon}\right)^{\alpha} (h)^{1-\alpha} \left(1 - \frac{1}{\lambda_f}\right). \tag{D.12}$$

The steady state version of the resource constraint, (C.14), is:

$$(8)c + g + \frac{i}{\mu_\Upsilon} = \left(\frac{\bar{k}}{\mu_z^* \Upsilon}\right)^{\alpha} h^{1-\alpha} - \phi, \tag{D.13}$$

where $p^* = w^* = 1$. The steady state real wage can be solved from (C.12):

$$(9)\tilde{w} = s\,(1-\alpha) \left[\frac{\Upsilon \mu_{z^*} h}{\bar{k}}\right]^{-\alpha}. \tag{D.14}$$

The steady state labor supply equation, (D.2), is:

$$(10)h = \left[\frac{\lambda_z}{W \lambda_w \psi_L} \tilde{w}\right]^{\frac{1}{\sigma_L}}, \tag{D.15}$$

where $W = 1$ when $\pi = \bar{\pi}$.

D.3 Financial Sector Equations

In steady state, the equilibrium conditions pertaining to financial friction are

$$\begin{aligned}
e &: \quad (\lambda_z + \nu_z) \frac{\bar{b}}{\mu_{z^*} \pi} \left(R_g^d - R_b^d\right) \tilde{d} + \eta = 0, \\
d &: \quad 0 = \left[p\,(e)\, e^g + (1 - p\,(e))\, e^b\right] R^k - R, \\
R_g^d &: \quad \nu_z p\,(e) + \eta \lambda_z \bar{b} = 0, \\
\mu &: \quad R = p\,(e)\, R_g^d + (1 - p\,(e))\, R_b^d, \\
\eta &: \quad e = \frac{\lambda_z \bar{b}}{\mu_{z^*} \pi} \left[\left(e^g - e^b\right) R^k \left(\tilde{N} + \tilde{d}\right) - \left(R_g^d - R_b^d\right) \tilde{d}\right], \\
\nu &: \quad e^b R^k \left(\tilde{N} + \tilde{d}\right) - R_b^d \tilde{d} = 0, \\
\tilde{N} &= \frac{\gamma}{\mu_{z^*} \pi} R \tilde{N} + \tilde{T}, \\
q\bar{k} &= \left[p\,(e)\, e^g + (1 - p\,(e))\, e^b\right] \left(\tilde{N} + \tilde{d}\right),
\end{aligned}$$

where $\bar{b} = p'\,(e)$ and we have substituted out R^g and R^b by $e^g R^k$ and $e^b R^k$ respectively. We need

$$\gamma < \beta,$$

for the net worth accumulation equation to make sense (i.e., have a steady state). Those 8 equations are solved for 8 variables: $\tilde{d}, \tilde{N}, R_g^d, R_b^d, e, \nu_z, \eta, R^k$, conditional on values for λ_z and $\bar{k}$ and some calibration information. We simply impose:

$$p\,(e)\, e^g + (1 - p\,(e))\, e^b = 1. \tag{D.16}$$

We suspect that this is in the nature of a normalization. Denote bank leverage by L:

$$L \equiv \left(\tilde{N} + \tilde{d}\right)/\tilde{N}. \tag{D.17}$$

We calibrate

$$sd_b,\ \ E^b, L,$$

where sd_b is the cross-sectional standard deviation of the nominal return on bank equity and E^b is the corresponding cross-sectional mean. We will use these three objects and (D.16) to determine $\tilde{T}$, b, g, $\bar{a}$. But, we must assume a value for the exogenous parameters, $\bar{b}$.

The market clearing condition for capital implies:

$$L = \frac{q\bar{k}}{\tilde{N}} \frac{1}{p(e)\, e^g + (1 - p(e))\, e^b} = \frac{\bar{k}}{\tilde{N}}, \tag{D.18}$$

using (D.16) and the fact, $q = 1$. Conditional on L, this gives us an expression that determines net worth, $\tilde{N}$. Then, the law of motion for net worth (i.e., (14)) allows us to pin down $\tilde{T}$:

$$\tilde{T} = [1 - \gamma R/(\mu_{z^*}\pi)]\tilde{N}.$$

From the d-condition,

$$R^k = \frac{R}{p(e)\, e^g + (1 - p(e))\, e^b} = R, \tag{D.19}$$

using (D.16), so that we now have R^k.

From ν-condition,

$$R_b^d = e^b R^k \frac{\tilde{N} + \tilde{d}}{\tilde{d}} = e^b R \frac{L}{L - 1}. \tag{D.20}$$

where we have substituted using (D.19).

We find it convenient to compute the spread, though this does not directly bear on the calibration objects. The interest rate spreads for banks is, using the μ-equation:

$$\text{spread}_b \equiv R_g^d - R = \frac{1 - p(e)}{p(e)}(R - R_b^d).$$

Combining this with (D.20):

$$\text{spread}_b = \frac{1 - p(e)}{p(e)} \left(1 - e^b \frac{L}{L - 1}\right) R \tag{D.21}$$

Next, we derive the expression for the cross-sectional variance of return on bank equity. The return on bank equity when a firm finds a good entrepreneur and when a firm finds a bad entrepreneur are given by:

$$e^g RL - R_g^d(L - 1) \text{ and } \overbrace{e^b RL - R_b^d(L - 1)}^{\text{we assume this is binding,=0}},$$

respectively. Recall that in the case of the binomial distribution, if a random variable can be x^h with probability p and x^l with probability $1 - p$, then its variance is $p(1 - p)\left(x^h - x^l\right)^2$. We conclude that the cross sectional standard deviation of the return on bank equity is:

$$\begin{aligned} sd_b &= [p(e)(1 - p(e))]^{1/2} \left[e^g RL - R_g^d(L - 1) - \left(e^b RL - R_b^d(L - 1)\right)\right] \\ &= [p(e)(1 - p(e))]^{1/2} \left[\left(e^g - e^b\right) RL - \left(R_g^d - R_b^d\right)(L - 1)\right] \end{aligned}$$

From μ-condition,

$$R_g^d - R_b^d = \frac{R - R_b^d}{p(e)} = \frac{R - \frac{e^b R}{p(e)e^g + (1-p(e))e^b}\frac{\tilde{N}+\tilde{d}}{\tilde{d}}}{p(e)} \tag{D.22}$$

$$= \frac{R}{p(e)}\left[1 - e^b \frac{\tilde{N}+\tilde{d}}{\tilde{d}}\right] = \frac{R}{p(e)}\left[1 - e^b \frac{L}{L-1}\right] = \frac{\text{spread}_b}{1-p(e)}$$

since

$$\frac{\tilde{N}+\tilde{d}}{\tilde{d}} = \frac{\tilde{N}+\tilde{d}}{\tilde{N}}\frac{\tilde{N}}{\tilde{d}} = \frac{L}{L-1}.$$

Replace $R_g^d - R_b^d$ in the expression for sd_b we obtain

$$sd_b = [p(e)(1-p(e))]^{\frac{1}{2}} R \left[(e^g - e^b)L - \frac{L(1-e^b)-1}{p(e)}\right].$$

According to the d equation with $R = R^k$:

$$1 = p(e)\, e^g + (1 - p(e))\, e^b = p(e)\left(e^g - e^b\right) + e^b.$$

Then, substituting this into the sd_b equation:

$$\begin{aligned} sd_b &= [p(e)(1-p(e))]^{\frac{1}{2}} R \left[\frac{1-e^b}{p(e)}L - \frac{L(1-e^b)-1}{p(e)}\right] \\ &= [p(e)(1-p(e))]^{\frac{1}{2}} R \frac{1}{p(e)}, \end{aligned}$$

or,

$$sd_b = \left[\frac{1-p(e)}{p(e)}\right]^{\frac{1}{2}} R. \tag{D.23}$$

Given sd_b, (D.23) determines $p(e)$. Then, (D.21) determines e^b given L. The probability of finding a good entrepreneur is (using (D.16)):

$$p(e) = \frac{1-e^b}{e^g - e^b}, \tag{D.24}$$

and so this can be solved for g. We now have R^k from (D.19), R_b^d from (D.20), R_g^d from (D.22), $\tilde{N}$ from (D.18), $\tilde{d}$ from (D.17). We still need v_z, η and e. In addition, we still require $\bar{a}$.

Consider the η-condition,

$$e = \frac{\lambda_z \bar{b}}{\mu_{z^*}\pi}\left[\left(e^g - e^b\right) R^k \left(\tilde{N} + \tilde{d}\right) - \frac{R}{p(e)}\left(1 - e^b \frac{L}{L-1}\right)\tilde{d}\right],$$

using (D.22) to solve out for $R_g^d - R_b^d$. Then,

$$\begin{aligned} e &= \frac{\lambda_z \bar{b}}{\mu_{z^*}\pi}\left[\left(e^g - e^b\right) R^k L - \frac{R}{p(e)}\left(1 - e^b\frac{L}{L-1}\right)(L-1)\right]\tilde{N} \\ &= \frac{\lambda_z \bar{b}}{\mu_{z^*}\pi}\left[\left(e^g - e^b\right) RL - \frac{R}{p(e)}\left(L - 1 - e^b L\right)\right]\tilde{N} \\ &= \frac{\lambda_z \bar{b}}{\mu_{z^*}\pi}\left[L - \frac{1}{p(e)}\frac{\left(L-1-e^b L\right)}{\left(e^g - e^b\right)}\right]\left(e^g - e^b\right) R\tilde{N} \end{aligned}$$

Using (D.24),

$$\begin{aligned} e &= \frac{\lambda_z \bar{b}}{\mu_{z^*}\pi}\left[L - \frac{e^g - e^b}{1-e^b}\frac{L\left(1-e^b\right)-1}{\left(e^g-e^b\right)}\right]\left(e^g - e^b\right)R\tilde{N} \\ &= \frac{\lambda_z \bar{b}}{\mu_{z^*}\pi}\left[L - \frac{L\left(1-e^b\right)-1}{1-e^b}\right]\left(e^g - e^b\right)R\tilde{N} \end{aligned}$$

or,

$$e = \frac{\lambda_z \bar{b}}{\mu_{z^*}\pi}\frac{e^g - e^b}{1-e^b}R\tilde{N}, \tag{D.25}$$

which determines e. Next, we have

$$p\left(e\right) = \bar{a} + \bar{b}e, \tag{D.26}$$

which determines $\bar{a}$.

We still have the following two equations:

$$e \;:\; \left(\lambda_z + \nu_z\right)\frac{1}{\mu_{z^*}\pi}\bar{b}\left(R_g^d - R_b^d\right)\tilde{d} + \eta = 0, \tag{D.27}$$

$$R_g^d \;:\; \nu_z p\left(e\right) + \eta\lambda_z \bar{b} = 0. \tag{D.28}$$

Equations (D.27)-(D.28) are two equations in ν_z, η. Now solve for η using (D.27):

$$\eta = -\left(\lambda_z + \nu_z\right)\frac{1}{\mu_{z^*}\pi}\bar{b}\left(R_g^d - R_b^d\right)\tilde{d},$$

and use this to substitute out for η in (D.28):

$$\nu_z p\left(e\right) - \left(\lambda_z + \nu_z\right)\frac{1}{\mu_{z^*}\pi}\bar{b}\left(R_g^d - R_b^d\right)\tilde{d}\lambda_z \bar{b} = 0,$$

or,

$$\nu_z = \frac{\frac{\lambda_z}{\mu_{z^*}\pi}\left(\bar{b}\right)^2\left(R_g^d - R_b^d\right)\tilde{d}\lambda_z}{p\left(e\right) - \frac{1}{\mu_{z^*}\pi}\left(\bar{b}\right)^2\left(R_g^d - R_b^d\right)\tilde{d}\lambda_z} \tag{D.29}$$

$$\eta = -\left(\lambda_z + \nu_z\right)\frac{1}{\mu_{z^*}\pi}\bar{b}\left(R_g^d - R_b^d\right)\tilde{d}. \tag{D.30}$$

This completes the computations we set out to accomplish.

D.4 Steady State Algorithm, Unobserved Effort Equilibrium

Here is an algorithm. We specify a value for π and compute R using (D.9). From (D.19) we obtain R^k. From (D.11) we obtain r^k. From (D.6) we obtain $h/\bar{k}$. Solve (D.14) for $\tilde{w}$.

Combining (D.12) and (D.13):

$$c + g + \frac{i}{\mu_\Upsilon} = \left(\frac{\bar{k}}{h\mu_z^*\Upsilon}\right)^\alpha h\frac{1}{\lambda_f}.$$

Substituting out for i using (D.8) and dividing the result by h :

$$\frac{c}{h} + \frac{g}{h} + \frac{\left(1 - \left(1-\delta\right)\frac{1}{\mu_z^*\Upsilon}\right)\frac{\bar{k}}{h}}{\mu_\Upsilon} = \left(\frac{\bar{k}}{h\mu_z^*\Upsilon}\right)^\alpha \frac{1}{\lambda_f}.$$

We specify that g is a given fraction, η_g, of steady state gross output or GDP (both are the same in steady state), so that :

$$\begin{aligned} g &= \eta_g \left(\frac{\bar{k}}{\mu_z^* \Upsilon}\right)^\alpha h^{1-\alpha} \frac{1}{\lambda_f} \\ \frac{g}{h} &= \eta_g \left(\frac{\bar{k}}{h\mu_z^* \Upsilon}\right)^\alpha \frac{1}{\lambda_f}. \end{aligned}$$

Then,

$$\frac{c}{h} = (1-\eta_g)\left(\frac{\bar{k}}{h\mu_z^*\Upsilon}\right)^\alpha \frac{1}{\lambda_f} - \frac{\left(1-(1-\delta)\frac{1}{\mu_z^*\Upsilon}\right)\frac{\bar{k}}{h}}{\mu_\Upsilon},$$

and c/h is now determined. From (D.10),

$$\lambda_z = \frac{1}{(c/h)\,h}\frac{\mu_z^* - b\beta}{\mu_z^* - b},$$

where h is yet to be determined. Substitute this expresion for λ_z into (D.2) to obtain:

$$(10) h = \left[\frac{1}{(c/h)\,h}\frac{\mu_z^* - b\beta}{\mu_z^* - b}\frac{1}{\lambda_w \psi_L}\tilde{w}\right]^{\frac{1}{\sigma_L}},$$

where W has been set to unity, reflecting $\pi = \bar{\pi}$. Solve the resulting expression for h :

$$h^{1+\frac{1}{\sigma_L}} = \left[\frac{1}{(c/h)}\frac{\mu_z^* - b\beta}{\mu_z^* - b}\frac{1}{\lambda_w \psi_L}\tilde{w}\right]^{\frac{1}{\sigma_L}},$$

or,

$$h = \left[\frac{1}{(c/h)}\frac{\mu_z^* - b\beta}{\mu_z^* - b}\frac{1}{\lambda_w \psi_L}\tilde{w}\right]^{\frac{1}{1+\sigma_L}},$$

where c/h is the object derived above.

Given $\bar{k}$ $(= h/(h/\bar{k}))$ and λ_z we can compute the financial variables:

$$\tilde{d}, \tilde{N}, R_g^d, R_b^d, e, \nu_z, \eta$$

using the approach in the previous section. In particular, given sd_b, $p(e)$ is determined by (D.23); given L (D.21) determines e^b. The expression (D.24) can be solved for e^g. Then, R_b^d can be solved from (D.20); R_g^d from (D.22); $\tilde{N}$ from (D.18) and $\tilde{d}$ from (D.17). Then, $\bar{a}$ and e can be solved using (D.25) and (D.26). Finally, ν_z and η can be solved using (D.29) and (D.30). At the end of the calculations we need to verify that

$$\nu_z > 0,\ p(e) > 1/2,\ c > 0,\ \tilde{d} > 0,\ \tilde{N} > 0,\ g > b,\ e > 0,\ \bar{k} > 0, R_g^d > R_b^d$$

Some of these tests are nearly redundant. For example, $R_g^d > R_b^d$ by the calibration (see (D.22)).

D.5 Steady State Algorithm, Unobserved Effort with Leverage Restriction

In this section we discuss the computation of equilibrium under a binding leverage restriction. Our algorithm does not impose any of the calibration restrictions that we imposed in the previous section, and so it must be a different one. In terms of the equilibrium conditions from the section on price and wage equations, we have the equations in (D.5), which we reproduce here:

$$
\begin{aligned}
(1) s &= 1/\lambda_f, \\
(2) \tilde{w} &= \lambda_w \frac{\psi_L h^{\sigma_L}}{\lambda_z}.
\end{aligned}
$$

In terms of the non-price and wage equations, we have (D.6) and (D.7):

$$
\begin{aligned}
(3) r^k &= \alpha \left(\frac{\Upsilon \mu_z^* h}{\bar{k}} \right)^{1-\alpha} s, \\
(4) i &= \left[\frac{1}{p(e)\, e^g + (1 - p(e))\, e^b} - (1-\delta) \frac{1}{\mu_z^* \Upsilon} \right] \bar{k}
\end{aligned}
$$

We also have (D.9) and (D.10):

$$
\begin{aligned}
(5) R &= \frac{\pi \mu_z^*}{\beta}, \\
(6) \lambda_z &= \frac{1}{c} \frac{\mu_z^* - b\beta}{\mu_z^* - b}.
\end{aligned}
$$

The other equations listed right after this are:

$$
s, \tilde{w}, h, \lambda_z, r^k, \bar{k}, i, e, R, R^k
$$

$$
\begin{aligned}
(7) R^k &= \frac{r^k + 1 - \delta}{\Upsilon} \pi \\
(8) c + g + \frac{i}{\mu_\Upsilon} &= \left(\frac{\bar{k}}{\mu_z^* \Upsilon} \right)^{\alpha} h^{1-\alpha} - \phi
\end{aligned}
$$

Here, ϕ and g are exogenous parameters. They are not calibrated in this section.

$$
(9) \tilde{w} = s(1-\alpha) \left[\frac{\Upsilon \mu_{z^*} h}{\bar{k}} \right]^{-\alpha}.
$$

The financial sector equations are:

$$
\begin{aligned}
(10)e &: \quad (\lambda_z + \nu_z) \frac{\bar{b}}{\mu_{z^*}\pi} \left(R_g^d - R_b^d\right) \tilde{d} + \eta = 0, \\
(11)d &: \quad \Lambda = (\lambda_z + v_z) \frac{1}{\mu_{z^*}\pi} \left(\left[p(e)\, e^g + (1 - p(e))\, e^b\right] R^k - R\right), \\
(12)R_g^d &: \quad \nu_z p(e) + \eta \lambda_z \bar{b} = 0, \\
(13)\mu &: \quad R = p(e)\, R_g^d + (1 - p(e))\, R_b^d, \\
(14)\eta &: \quad e = \frac{\lambda_z \bar{b}}{\mu_{z^*}\pi} \left[\left(e^g - e^b\right) R^k \left(\tilde{N} + \tilde{d}\right) - \left(R_g^d - R_b^d\right) \tilde{d}\right], \\
(15)\nu &: \quad e^b R^k \left(\tilde{N} + \tilde{d}\right) - R_b^d \tilde{d} = 0, \\
(16)\tilde{N} &= \frac{\gamma}{\mu_{z^*}\pi} \left\{\left[p(e)\, e^g + (1 - p(e))\, e^b\right] R^k \left(\tilde{N} + \tilde{d}\right) - R\tilde{d}\right\} + \tilde{T}, \\
(17)\bar{k} &= \left[p(e)\, e^g + (1 - p(e))\, e^b\right] \left(\tilde{N} + \tilde{d}\right) \\
(18)L\tilde{N} &= \tilde{N} + \tilde{d}.
\end{aligned} \tag{D.31}
$$

We have the following 11 non-financial market unknowns (steady state inflation is always fixed at π):

$$c, s, \tilde{w}, h, \lambda_z, r^k, \bar{k}, i, e, R, R^k.$$

We have the following 7 additional financial market variables:

$$\nu_z, R_g^d, R_b^d, \eta, \tilde{d}, \tilde{N}, \Lambda.$$

Thus, we have 18 equations in 18 unknowns.

Here is an algorithm. It is a one-dimensional search for a value of $\tilde{N}$ that enforces equation (16). We now discuss how the other endogenous variables in (16) are computed.

Assign an arbitrary value to $0 \le p(e) \le 1$. From this we can compute e using

$$p(e) = \bar{a} + \bar{b}e.$$

We compute $\bar{k}$ from (17) and i from (4). We then reduce (14) to one nonlinear equation in one unknown, h. To see this, given $\bar{k}$, (8) now defines c as a function of h :

$$c = \left(\frac{\bar{k}}{\mu_z^* \Upsilon}\right)^{\alpha} h^{1-\alpha} - \phi - \frac{i}{\mu_\Upsilon} - g$$

Similarly, (6) defines λ_z as a function of h. Substituting (3) into (7):

$$R^k = \frac{\alpha \left(\frac{\Upsilon \mu_z^* h}{\bar{k}}\right)^{1-\alpha} \frac{1}{\lambda_f} + 1 - \delta}{\Upsilon} \pi,$$

we obtain that R^k is a function of h. Substituting from (13) into (14), we obtain:

$$e = \frac{\lambda_z \bar{b}}{\mu_{z^*}\pi} \left[\left(e^g - e^b\right) R^k \left(\tilde{N} + \tilde{d}\right) - \frac{R\tilde{d} - R_b^d \tilde{d}}{p(e)}\right]$$

Substituting from (15):

$$e = \frac{\lambda_z \bar{b}}{\mu_{z^*}\pi}\left[\left(e^g - e^b\right)R^k\left(\tilde{N}+\tilde{d}\right) - \frac{R\tilde{d} - e^b R^k\left(\tilde{N}+\tilde{d}\right)}{p\left(e\right)}\right]$$

Note that the right hand side of this expression is a function of h alone. We adjust the value of h until this expression is satisfied.

We use (15) to compute

$$R_b^d = e^b R^k \frac{\tilde{N}+\tilde{d}}{\tilde{d}}.$$

We also have R_g^d from (13):

$$R_g^d = \frac{R - \left(1 - p\left(e\right)\right)R_b^d}{p\left(e\right)}.$$

We compute Λ from (11).

Solving for η from (10):

$$\eta = -\left(\lambda_z + \nu_z\right)\frac{\bar{b}}{\mu_{z^*}\pi}\left(R_g^d - R_b^d\right)\tilde{d}.$$

Substitute this into (12)

$$\nu_z p\left(e\right) - \left(\lambda_z + \nu_z\right)\frac{\bar{b}}{\mu_{z^*}\pi}\left(R_g^d - R_b^d\right)\tilde{d}\lambda_z\bar{b} = 0,$$

and solving this for ν_z, we obtain:

$$\nu_z = \frac{\lambda_z \frac{\bar{b}}{\mu_{z^*}\pi}\left(R_g^d - R_b^d\right)\tilde{d}\lambda_z\bar{b}}{p\left(e\right) - \frac{\bar{b}}{\mu_{z^*}\pi}\left(R_g^d - R_b^d\right)\tilde{d}\lambda_z\bar{b}}.$$

So that we have η and ν_z.

Finally, we solve (9) for $\tilde{w}$. We adjust $p\left(e\right)$ until (2) is satisfied. Thus, for an arbitrary choice of value for $\tilde{N}$ we compute $p\left(e\right)$ and h as described above. We adjust the value of $\tilde{N}$ until (16) is satisfied.

D.6 Steady State Algorithm, Observed Effort

In the observed effort case, the equilibrium conditions for the financial sector do not require computing R_g^d and R_b^d and the multipliers, η and υ_z, are both zero. This means that we can ignore equations (10), (12), (13), (15) in (D.31). Thus, the financial sector equilibrium conditions in nonstochastic steady state are:

$$
\begin{aligned}
(11)d \quad &: \quad \Lambda = \lambda_z \frac{1}{\mu_{z^*}\pi}\left(\left[p(e)e^g + (1-p(e))e^b\right]R^k - R\right) \\
(14)\eta \quad &: \quad e = \frac{\lambda_z \bar{b}}{\mu_{z^*}\pi}\left(e^g - e^b\right)R^k\left(\tilde{N}+\tilde{d}\right), \\
(16)\tilde{N} \quad &= \quad \frac{\gamma}{\mu_{z^*}\pi}\left\{\left[p(e)e^g + (1-p(e))e^b\right]R^k\left(\tilde{N}+\tilde{d}\right) - R\tilde{d}\right\} + \tilde{T}, \\
(17)\bar{k} \quad &= \quad \left[p(e)e^g + (1-p(e))e^b\right]\left(\tilde{N}+\tilde{d}\right) \\
(18)L\tilde{N} \quad &= \quad \tilde{N}+\tilde{d}.
\end{aligned}
$$

When leverage is unrestricted, then $\Lambda = 0$ and (18) simply defines leverage, L. When the leverage restriction is imposed and is binding, then L in (18) is exogenous and $\Lambda > 0$.

We have the following 11 non-financial market unknowns (steady state inflation is always fixed at π):

$$
c, s, \tilde{w}, h, \lambda_z, r^k, \bar{k}, i, e, R, R^k.
$$

When the leverage restriction is non-binding, we have the following 3 additional financial market variables:

$$
\tilde{d}, \tilde{N}, L,
$$

with the understanding, $\Lambda = 0$. In terms of equations, we have 9 non-financial market equations and the above 5 financial market equations. Thus, we have 14 unknowns and 14 equations. When the leverage restriction is binding, then there is an additional equation that assigns a value to L and there is an additional unknown, Λ.

Here is an algorithm for solving the observed effort steady state when the leverage constraint is nonbinding, $\Lambda = 0$. Combining (11) (with $\Lambda = 0$) and (16), we obtain $\tilde{N} = \frac{\gamma}{\mu_{z^*}\pi}R\tilde{N} + \tilde{T}$, so that

$$
\tilde{N} = \frac{\tilde{T}}{1 - \frac{\gamma}{\mu_{z^*}\pi}R}. \tag{D.32}
$$

So, we can compute $\tilde{N}$ immediately. Fix a value of $p(e)$. Then, using (11) with $\Lambda = 0$:

$$
R^k = \frac{R}{p(e)e^g + (1-p(e))e^b}. \tag{D.33}
$$

Then, r^k is computed using (7), $h/\bar{k}$ is obtained from (3), and $\tilde{w}$ is computed from (9). Now fix a value for h, so that we have $\bar{k}$. We obtain $\tilde{d}$ from (17), c from (8) and λ_z from (6). Adjust h until (14) is satisfied. Adjust $p(e)$ until (2) is satisfied.

We must consider the possibility that the observed effort equilibrium has the property,

$$
p(e) = 1, \; e \leq \frac{\lambda_z \bar{b}}{\mu_{z^*}\pi}\left(e^g - e^b\right)R^k\left(\tilde{N}+\tilde{d}\right), \tag{D.34}
$$

so that (14) does not hold. Since (11) and (16) are satisfied, we can still compute $\tilde{N}$ using (D.32). Set $p(e) = 1$ and compute R^k using (D.33). We can compute r^k, $h/\bar{k}$ and $\tilde{w}$ using (7), (3) and (9), as before. Now fix a value for h, so that we have $\bar{k}$. We obtain $\tilde{d}$, c, λ_z from (17), (8) and (6), as before. Adjust h until (2) is satisfied. Finally, verify that the inequality in (D.34) is satisfied.

Now consider the case of a binding leverage constraint. We cannot compute $\tilde{N}$ as before. Also, equation (11) does not hold with $\Lambda = 0$, so that we do not have access to (D.33). A different algorithm is required. Consider the following one. Fix a value for $\tilde{N}$ and use (18) to compute $\tilde{d}$. Fix $p(e)$. Use (17) to compute $\bar{k}$. Use (4) to compute i.

Fix h. Compute c from (8) and λ_z from (6). Compute r^k from (3) and R^k from (7). Adjust h until (14) is satisfied. Compute $\tilde{w}$ from (9). Adjust $p(e)$ until (2) is satisfied. Finally, adjust $\tilde{N}$ until (16) is satisfied.

Again, we must consider the possibility that $p(e) = 1$ and (14) does not hold. As before, fix a value for $\tilde{N}$ and use (18) to compute $\tilde{d}$. Set $p(e) = 1$ and compute $\bar{k}, i$ using (17) and (4). Fix a value for h. Compute c, λ_z, $r^k, R^k, \tilde{w}$ from (8), (6), (3), (7), (9). Adjust h until (2) is satisfied. Adjust $\tilde{N}$ until (16) is satisfied. Finally, we must verify (D.34).