Technical Appendix to "Leverage Restrictions in a Business Cycle Model",
Lawrence Christiano and Daisuke Ikeda

A Appendix A: Derivation of Financial Sector Equilibrium Conditions

This appendix derives the equilibrium conditions associated with the financial sector. The first subsection considers the conditions associated with the case where banker effort is observable. We then consider the unobservable effort case.

A.1 Observable Effort

The Lagrangian representation of the banker's problem in the observable effort rerpresentation of the problem is:

$$\max_{e,d,R_g^d,R_b^d} E_t\lambda_{t+1}\left\{p_t(e_t)\left[R_{t+1}^g(N_t+d_t)-R_{g,t+1}^d d_t\right]+(1-p_t(e_t))\left[R_{t+1}^b(N_t+d_t)-R_{b,t+1}^d d_t\right]\right\} \quad (29)$$
$$-\frac{1}{2}e_t^2$$
$$+E_t\left\{\mu_{t+1}\left[p_t(e_t)R_{g,t+1}^d d_t+(1-p_t(e_t))R_{b,t+1}^d d_t-R_t d_t\right]+\nu_{t+1}\left[R_{t+1}^b(N_t+d_t)-R_{b,t+1}^d d_t\right]\right\}$$

where μ_{t+1} is the Lagrange multiplier on (1) and $\nu_{t+1}\geq 0$ is the Lagrange multiplier on (2). Note that the constraints must be satisfied in each period $t+1$ state of nature, which is indicated by the fact that the multipliers, μ_{t+1} an ν_{t+1}, are contingent upon the realization of period $t+1$ uncertainty. The first order conditions associated with the banker problem are:

$$e: 0 = E_t\{\lambda_{t+1}p_t'(e_t)\left[\left(R_{t+1}^g-R_{t+1}^b\right)(N_t+d_t)-\left(R_{g,t+1}^d-R_{b,t+1}^d\right)d_t\right]-e_t$$
$$+\mu_{t+1}p_t'(e_t)\left(R_{g,t+1}^d-R_{b,t+1}^d\right)d_t\}$$
$$d: 0 = E_t\{\lambda_{t+1}\left[p_t(e_t)\left(R_{t+1}^g-R_{g,t+1}^d\right)+(1-p_t(e_t))\left(R_{t+1}^b-R_{b,t+1}^d\right)\right]$$
$$+\mu_{t+1}\left[p_t(e_t)R_{g,t+1}^d+(1-p_t(e_t))R_{b,t+1}^d-R_t\right]+\nu_{t+1}\left(R_{t+1}^b-R_{b,t+1}^d\right)\}$$
$$R_g^d: 0=-\lambda_{t+1}p_t(e_t)d_t+\mu_{t+1}p_t(e_t)d_t$$
$$R_b^d: 0=-\lambda_{t+1}(1-p_t(e_t))d_t+\mu_{t+1}(1-p_t(e_t))d_t-\nu_{t+1}d_t$$
$$\mu: p_t(e_t)R_{g,t+1}^d d_t+(1-p_t(e_t))R_{b,t+1}^d d_t=R_t d_t$$
$$\nu: \nu_{t+1}\left[R_{t+1}^b(N_t+d_t)-R_{b,t+1}^d d_t\right]=0,\ \nu_{t+1}\geq 0,\ R_{t+1}^b(N_t+d_t)-R_{b,t+1}^d d_t\geq 0,$$

where "$x:$" in the first column indicates the first order condition with respect to the variable, x. In the R_g^d and R_b^d equations, we differentiate state by state. In the results reported above the density of the state does not appear. This reflects our assumption that the density is strictly positive over all states, so that we can divide through by that density. We make this assumption throughout. Adding the R_g^d and R_b^d equations, we obtain:

$$\mu_{t+1}=\lambda_{t+1}+\nu_{t+1}. \quad (30)$$

Substituting (30) back into the R_g^d equation, we find

$$\nu_{t+1}=0,$$

so that the cash constraint is non-binding. Substituting the latter two results back into the system of equations, they reduce to (4), (5) and (6) in the text. To see this, note that

$\mu_{t+1} = \lambda_{t+1}$ in the e equation results in a simple cancellation that implies (4). Equation (5) is derived in a similarly simple way. Finally, equation (6) is simply the μ equation repeated.

Now suppose we impose a leverage restriction, (14). This only affects the d equation above, since d_t is the only choice variable in the leverage restriction. As a result, our findings, $\nu_{t+1} = 0$ and $\mu_{t+1} = \lambda_{t+1}$ are unaffected. That is, the cash constraint remains non-binding and the effort equation remains as in (4). The only change implied by a binding leverage constraint is that the 0 in the d equation is replaced by the multiplier on the leverage constraint.

A.2 Unobservable Effort

Given the indicated set of contracts, the Lagrangian representation of the banker's problem now is:

$$\max_{\left(e_t, d_t, R^d_{g,t+1}, R^d_{b,t+1}\right)} E_t\lambda_{t+1}\left\{p_t\left(e_t\right)\left[R^g_{t+1}\left(N_t+d_t\right)-R^d_{g,t+1}d_t\right]+\left(1-p_t\left(e_t\right)\right)\left[R^b_{t+1}\left(N_t+d_t\right)-R^d_{b,t+1}d_t\right]\right\} \quad (31)$$
$$-\frac{1}{2}e_t^2$$
$$+E_t\mu_{t+1}\left[p_t\left(e_t\right)R^d_{g,t+1}d_t+\left(1-p_t\left(e_t\right)\right)R^d_{b,t+1}d_t-R_td_t\right]$$
$$+\eta_t\left(e_t-E_t\lambda_{t+1}p_t'\left(e_t\right)\left[\left(R^g_{t+1}-R^b_{t+1}\right)\left(N_t+d_t\right)-\left(R^d_{g,t+1}-R^d_{b,t+1}\right)d_t\right]\right)$$
$$+E_t\nu_{t+1}\left[R^b_{t+1}\left(N_t+d_t\right)-R^d_{b,t+1}d_t\right].$$

where η_t is the Lagrange multiplier on (7). Note that this multiplier is not contingent on the realization of the period $t+1$ state of nature since the constraint is on the effort level exerted by the banker in t. To understand the solution to this problem, consider the first order necessary conditions associated with the banker problem, (31):

$$\begin{aligned}
e &: E_t\lambda_{t+1}p_t'\left(e_t\right)\left[\left(R^g_{t+1}-R^b_{t+1}\right)\left(N_t+d_t\right)-\left(R^d_{g,t+1}-R^d_{b,t+1}\right)d_t\right] \\
&\quad -e_t+E_t\mu_{t+1}p_t'\left(e_t\right)\left(R^d_{g,t+1}-R^d_{b,t+1}\right)d_t \\
&\quad +\eta_t\left(1-E_t\lambda_{t+1}p_t''\left(e_t\right)\left[\left(R^g_{t+1}-R^b_{t+1}\right)\left(N_t+d_t\right)-\left(R^d_{g,t+1}-R^d_{b,t+1}\right)d_t\right]\right)=0 \\
d &: 0=E_t\lambda_{t+1}p_t\left(e_t\right)\left(R^g_{t+1}-R^d_{g,t+1}\right)+E_t\lambda_{t+1}\left(1-p_t\left(e_t\right)\right)\left(R^b_{t+1}-R^d_{b,t+1}\right) \\
&\quad +E_t\mu_{t+1}\left[p_t\left(e_t\right)R^d_{g,t+1}+\left(1-p_t\left(e_t\right)\right)R^d_{b,t+1}-R_t\right] \\
&\quad -\eta_tE_t\lambda_{t+1}p_t'\left(e_t\right)\left[\left(R^g_{t+1}-R^b_{t+1}\right)-\left(R^d_{g,t+1}-R^d_{b,t+1}\right)\right]+E_t\nu_{t+1}\left(R^b_{t+1}-R^d_{b,t+1}\right) \\
R^d_g &: -\lambda_{t+1}p_t\left(e_t\right)+\mu_{t+1}p_t\left(e_t\right)+\eta_t\lambda_{t+1}p_t'\left(e_t\right)=0 \\
R^d_b &: -\lambda_{t+1}\left(1-p_t\left(e_t\right)\right)+\mu_{t+1}\left(1-p_t\left(e_t\right)\right)-\eta_t\lambda_{t+1}p_t'\left(e_t\right)-\nu_{t+1}=0 \\
\mu &: R_t=p_t\left(e_t\right)R^d_{g,t+1}+\left(1-p_t\left(e_t\right)\right)R^d_{b,t+1} \\
\eta &: e_t=E_t\lambda_{t+1}p_t'\left(e_t\right)\left[\left(R^g_{t+1}-R^b_{t+1}\right)\left(N_t+d_t\right)-\left(R^d_{g,t+1}-R^d_{b,t+1}\right)d_t\right] \\
\nu &: \nu_{t+1}\left[R^b_{t+1}\left(N_t+d_t\right)-R^d_{b,t+1}d_t\right]=0,\ \nu_{t+1}\geq 0,\ \left[R^b_{t+1}\left(N_t+d_t\right)-R^d_{b,t+1}d_t\right]\geq 0.
\end{aligned}$$

Add the R^d_g and R^d_b equations, to obtain (30). To simplify the e equation, use (30) to substitute out μ_{t+1} :

$$\begin{aligned}
e &: E_t\lambda_{t+1}p_t'\left(e_t\right)\left[\left(R^g_{t+1}-R^b_{t+1}\right)\left(N_t+d_t\right)-\left(R^d_{g,t+1}-R^d_{b,t+1}\right)d_t\right]-e_t \\
&\quad +E_t\left[\lambda_{t+1}+\nu_{t+1}\right]p_t'\left(e_t\right)\left(R^d_{g,t+1}-R^d_{b,t+1}\right)d_t \\
&\quad +\eta_t\left(1-E_t\lambda_{t+1}p_t''\left(e_t\right)\left[\left(R^g_{t+1}-R^b_{t+1}\right)\left(N_t+d_t\right)-\left(R^d_{g,t+1}-R^d_{b,t+1}\right)d_t\right]\right)=0
\end{aligned}$$

or,

$$\begin{aligned} e &: E_t\lambda_{t+1}p_t'(e_t)\left(R_{t+1}^g - R_{t+1}^b\right)(N_t + d_t) - e_t + E_t\nu_{t+1}p_t'(e_t)\left(R_{g,t+1}^d - R_{b,t+1}^d\right)d_t \\ &+\eta_t\left(1 - E_t\lambda_{t+1}p_t''(e_t)\left[\left(R_{t+1}^g - R_{t+1}^b\right)(N_t + d_t) - \left(R_{g,t+1}^d - R_{b,t+1}^d\right)d_t\right]\right) = 0 \end{aligned}$$

Now, make use of $p_t'' = 0$ and the η equation to substitute out for e_t :

$$\begin{aligned} e : E_t\lambda_{t+1}p_t'(e_t)\left(R_{t+1}^g - R_{t+1}^b\right)(N_t + d_t) - E_t\lambda_{t+1}p_t'(e_t)\left[\left(R_{t+1}^g - R_{t+1}^b\right)(N_t + d_t) - \left(R_{g,t+1}^d - R_{b,t+1}^d\right)d_t\right] \\ +E_t\nu_{t+1}p_t'(e_t)\left(R_{g,t+1}^d - R_{b,t+1}^d\right)d_t + \eta_t = 0 \end{aligned}$$

or,

$$e : E_t\lambda_{t+1}p_t'(e_t)\left(R_{g,t+1}^d - R_{b,t+1}^d\right)d_t + E_t\nu_{t+1}p_t'(e_t)\left(R_{g,t+1}^d - R_{b,t+1}^d\right)d_t + \eta_t = 0$$

or,

$$e : E_t\left[\lambda_{t+1} + \nu_{t+1}\right]p_t'(e_t)\left(R_{g,t+1}^d - R_{b,t+1}^d\right)d_t + \eta_t = 0.$$

We now simplify the d equation. From the μ-condition, we delete the third term in d equation and obtain

$$\begin{aligned} 0 &= E_t\lambda_{t+1}p_t(e_t)\left(R_{t+1}^g - R_{g,t+1}^d\right) + E_t\lambda_{t+1}(1 - p_t(e_t))\left(R_{t+1}^b - R_{b,t+1}^d\right) \\ &-\eta_t E_t\lambda_{t+1}p_t'(e_t)\left[\left(R_{t+1}^g - R_{t+1}^b\right) - \left(R_{g,t+1}^d - R_{b,t+1}^d\right)\right] + E_t\nu_{t+1}\left(R_{t+1}^b - R_{b,t+1}^d\right) \end{aligned}$$

Use (30) to substitute out for μ_{t+1} in the R_g^d condition:

$$\nu_{t+1}p_t(e_t) + \eta_t\lambda_{t+1}p_t'(e_t) = 0$$

Substituting out η_t using R_g^d-condition,

$$\begin{aligned} -\lambda_{t+1}p_t(e_t) + \left[\lambda_{t+1} + \nu_{t+1}\right]p_t(e_t) + \eta_t\lambda_{t+1}p_t'(e_t) &= 0 \\ \nu_{t+1}p_t(e_t) + \eta_t\lambda_{t+1}p_t'(e_t) &= 0 \qquad (32) \end{aligned}$$

Note that this equation implies

$$\eta_t \leq 0.$$

This is to be expected. The interpretation of this may be seen from (31). The sign of η_t suggests that in the absence of the η constraint, i.e., if $\eta_t = 0$, then e_t would be set in a way that makes e_t greater than the object on the right of the minus sign in the incentive constraint. A negative value of η_t in the Lagrangian penalizes such a setting. But, we know from our analysis of the observable effort case (the only difference in this case is that the incentive constraint is absent), that e_t is greater than the object on the right of the minus sign in the η constraint in (31) when that constraint is ignored. But, (32) has another notable implication. Suppose, for simplicity, that from the point of view of t, there are two possible states of nature in $t + 1$, 1 and 2. Then,

$$\begin{aligned} \nu_{t+1}^1 p_t(e_t) + \eta_t\lambda_{t+1}^1 p_t'(e_t) &= 0 \\ \nu_{t+1}^2 p_t(e_t) + \eta_t\lambda_{t+1}^2 p_t'(e_t) &= 0 \end{aligned}$$

We assume that $\lambda_{t+1}^1, \lambda_{t+1}^2, p_t'(e_t), p_t(e_t) > 0$. Suppose the cash constraint is not binding in state of nature, 1, so that $\nu_{t+1}^1 = 0$. In that case, the first equation says that $\eta_t = 0$. But, the second equation then implies $\nu_{t+1}^2 = 0$ too. Thus, if the cash constraint is not binding in some state of nature for a particular date, then it must not be binding in the other state either. If it is binding is one state, $\nu_{t+1}^1 > 0$, then $\eta_t > 0$ and it is binding in the other state. Thus, it is either binding in all states at a particular date, or none. This is general. Note from

R_g^d-condition that

$$\eta_t = -\nu_{t+1} p_t(e_t) / [\lambda_{t+1} p_t'(e_t)],$$

which implies that there exists no solution such that $\nu_{t+1} = 0$ for some states of nature and $\nu_{t+1} > 0$ for others. Intuitively this is because a banker smooths inefficiency caused by $R_{g,t+1}^d - R_{b,t+1}^d > 0$ state by state. Suppose R_{t+1}^b is very low in one state and it is very high in another. Then, the cash constraint is binding in the low state so that $R_{g,t+1}^d - R_{b,t+1}^d > 0$. In the high state the banker sets $R_{b,t+1}^d$ high enough so that the cash constraint is binding and $R_{g,t+1}^d - R_{b,t+1}^d < 0$. By doing this the banker minimizes $E_t \lambda_{t+1} p_t'(e_t) \left(R_{g,t+1}^d - R_{b,t+1}^d\right) \geq 0$, which is, loosely speaking, the measure of inefficiency.

Substituting out for η_t in the revised d equation:

$$\begin{aligned} 0 &= E_t \lambda_{t+1} p_t(e_t) \left(R_{t+1}^g - R_{g,t+1}^d\right) + E_t \lambda_{t+1} (1 - p_t(e_t)) \left(R_{t+1}^b - R_{b,t+1}^d\right) \\ &\quad + E_t \nu_{t+1} p_t(e_t) \left[\left(R_{t+1}^g - R_{t+1}^b\right) - \left(R_{g,t+1}^d - R_{b,t+1}^d\right)\right] + E_t \nu_{t+1} \left(R_{t+1}^b - R_{b,t+1}^d\right) \end{aligned}$$

or,

$$\begin{aligned} 0 &= E_t \lambda_{t+1} \left[p_t(e_t) \left(R_{t+1}^g - R_{g,t+1}^d\right) + (1 - p_t(e_t)) \left(R_{t+1}^b - R_{b,t+1}^d\right)\right] \\ &\quad + E_t \nu_{t+1} \left[p_t(e_t) \left(R_{t+1}^g - R_{g,t+1}^d\right) + (1 - p_t(e_t)) \left(R_{t+1}^b - R_{b,t+1}^d\right)\right] \end{aligned}$$

or

$$0 = E_t (\lambda_{t+1} + \nu_{t+1}) \left[p_t(e_t) \left(R_{t+1}^g - R_{g,t+1}^d\right) + (1 - p_t(e_t)) \left(R_{t+1}^b - R_{b,t+1}^d\right)\right].$$

Then, using the μ-condition,

$$0 = E_t (\lambda_{t+1} + \nu_{t+1}) \left[p_t(e_t) R_{t+1}^g + (1 - p_t(e_t)) R_{t+1}^b - R_t\right].$$

Finally, we use (30) to substitute out for μ_{t+1} in the R_g^d equation, to obtain:

$$R_g^d : \nu_{t+1} p_t(e_t) + \eta_t \lambda_{t+1} p_t'(e_t) = 0.$$

The optimization conditions derived here are summarized in (8).

To gain intuition into this multiplier, consider the case, $\nu_{t+1} = 0$, so that the cash constraint is not binding and the R_g^d condition implies $\eta_t = 0$. Since $\lambda_{t+1} + \nu_{t+1} > 0$ the e-condition then implies that $R_{g,t+1}^d = R_{b,t+1}^d$ can be the solution (as long as it does not make the cash constraint binding). Combining this with the μ-condition then implies that

$$R_{g,t+1}^d = R_{b,t+1}^d = R_t \tag{33}$$

can be the solution. It then follows from the η-condition that:

$$e_t = E_t \lambda_{t+1} p_t'(e_t) \left[\left(R_{t+1}^g - R_{t+1}^b\right) (N_t + d_t)\right], \tag{34}$$

so that banker effort level is efficient.

Now consider the case, $\nu_{t+1} > 0$ for *all* states of nature. Then, the R_g^d-condition implies $\eta_t < 0$ and the e-condition, after substituting out ν_{t+1} using the the R_g^d-condition, implies

$$E_t \lambda_{t+1} p_t'(e_t) \left(R_{g,t+1}^d - R_{b,t+1}^d\right) d_t = -\frac{\eta_t}{1 - \frac{\eta_t p_t'(e_t)}{p_t(e_t)}} > 0.$$

The e-condition then shows that banker effort is suboptimal. By continuity, when ν_{t+1} is large the inefficiency of the banking system is great and when it is small, there inefficiency is smaller. We think of a 'crisis time' as one in which ν_{t+1} is large.

Given our constraints, we suspect that when the cash constraint is always binding, $\nu_{t+1} > 0$, all state contingent deposit returns $R^d_{g,t+1}, R^d_{b,t+1}$,are pinned down. To see why, consider the case in which there are two aggregate states possible in period $t+1$, given period t. Denote these by 1 and 2 and suppose they have probability, π_t and $1-\pi_t$, respectively. The μ equations are:

$$\begin{aligned} R_t &= p_t(e_t) R^{d,1}_{g,t+1} + (1 - p_t(e_t)) R^{d,1}_{b,t+1} \\ R_t &= p_t(e_t) R^{d,2}_{g,t+1} + (1 - p_t(e_t)) R^{d,2}_{b,t+1} \end{aligned}$$

and the ν equations are

$$\begin{aligned} R^{b,1}_{t+1}(N_t + d_t) - R^{d,1}_{b,t+1} d_t &= 0 \\ R^{b,2}_{t+1}(N_t + d_t) - R^{d,2}_{b,t+1} d_t &= 0 \end{aligned}$$

Given the time t realization of variables, this represents four equations in four unknowns. In general, for given $R_t, p_t(e_t)$ these variables are pinned down. If there are more states of nature, then these equations represent restrictions on the deposit returns. Either way, the state contingency in the returns does not appear to contribute directly to multiplicity of equilibria, at least when the case constraint is always binding. As a practical matter, we can solve the model assuming the cash constraint always binds. We can then inspect the multiplier and verify that it is always positive. If ever it is negative that means that the constraint as an inequality is in fact not binding.

Consider the issue of the relative magnitude of $R^d_{b,+1}$ and $R^d_{g,+1}$. We suspect that it will not be true across all states of nature that $R^d_{b,t+1} \leq R^d_{g,t+1}$. Consider a simple example. Suppose there is an aggregate state where $R^b_{t+1} = 0$. In that state, it must be that $R^d_{b,t+1} = 0$ too. In such a state, assuming $R_t > 0$, it must be that $R^d_{g,t+1} > R^d_{b,t+1}$. By itself, this spread induces a substantial inefficiency in the e decision (see the η equation). But, the spread affects the choice of e only by its expected value. If that spread is very large in some state then it does not induce a large inefficiency if it is sufficiently small in another state. We might even imagine that it could be negative in another state, $R^d_{b,t+1} > R^d_{g,t+1}$. In this case, creditors in effect subsidize bankers that make positive profits and tax the ones that lose. This obviously has a big positive incentive effect on e. This possibility should not be a problem for our maintained assumption that the cash constraint is non-binding in the g state. To see this, suppose that it is binding in the b state:

$$R^b_{t+1}(N_t + d_t) - R^d_{b,t+1} d_t = 0.$$

By construction, $R^g_{t+1} > R^b_{t+1}$ in all aggregate states. also, in the scenario we are discussing, $R^d_{g,t+1} < R^d_{b,t+1}$. Both guarantee that the cash constraint is not binding in the g state.

An interesting feature of the model is that it implies a non-trivial cross-sectional variance on the returns of banks. In any given period $t+1$ state of nature, the cross section mean of bank returns on equity is:

$$R^m_{t+1} = p_t(e_t)\left[\frac{R^g_{t+1}(N_t + d_t) - R^d_{g,t+1} d_t}{N_t}\right] + (1 - p_t(e_t))\left[\frac{R^b_{t+1}(N_t + d_t) - R^d_{b,t+1} d_t}{N_t}\right].$$

To determine the cross sectional standard deviation of bank equity returns, note that in the cross section, in any aggregate state, $p_t(e_t)$ banks each earn

$$\frac{R^g_{t+1}(N_t + d_t) - R^d_{g,t+1} d_t}{N_t}$$

return on equity. Similarly, $1 - p_t(e_t)$ banks earn a return on equity equal to

$$\frac{R^b_{t+1}(N_t + d_t) - R^d_{b,t+1}d_t}{N_t}.$$

Recall that if a random variable has a binomial distribution and takes on the value x^h with probability p and x^l with probability $1-p$, then the variance of that random variable is $p(1-p)\left(x^h - x^l\right)^2$. So, the period t cross-sectional standard deviation of bank returns is:

$$\begin{aligned} s^d_{t+1} &= \left[p_t(e_t)(1 - p_t(e_t))\right]^{1/2}\left[\frac{R^g_{t+1}(N_t + d_t) - R^d_{g,t+1}d_t}{N_t} - \frac{R^b_{t+1}(N_t + d_t) - R^d_{b,t+1}d_t}{N_t}\right] \\ &= \left[p_t(e_t)(1 - p_t(e_t))\right]^{1/2}\frac{R^g_{t+1}(N_t + d_t) - R^d_{g,t+1}d_t}{N_t}, \end{aligned}$$

taking into account our assumption that the cash constraint is binding for bad banks. Note that $p_t(e_t) > 1/2$ then the cross sectional standard deviation is decreasing in e_t.

B Appendix B: Scaling and Miscellaneous Variables

To solve our model, we require that the variables be stationary. To this end, we adopt a particular scaling of the variables. Because our model satisfies sufficient conditions for balanced growth, when the equilibrium conditions of the model are written in terms of the scaled variables, only the growth rates and not the levels of the stationary shocks appear. In this appendix we describe the scaling of the model that is adopted. In addition, we describe the mapping from the variables in the scaled model to the variables measured in the data.

Let

$$\begin{aligned} q_t &= \Upsilon^t \frac{P_{k',t}}{P_t}, y_{z,t} = \frac{Y_t}{z^+_t},\ i_t = \frac{I_t}{z^+_t \Upsilon^t},\ \tilde{w}_t \equiv \frac{W_t}{z^+_t P_t},\ p_{I,t} \equiv \frac{1}{\Upsilon^t \mu_{\Upsilon,t}}, P_{I,t} = \frac{P_t}{\Upsilon^t \mu_{\Upsilon,t}} \\ \bar{k}_t &= \frac{\bar{K}_t}{z^+_{t-1}\Upsilon^{t-1}},\ r^k_t = \Upsilon^t \tilde{r}^k_t,\ \mu^*_{z,t} = \frac{z^+_t}{z^+_{t-1}},\ c_t = \frac{C_t}{z^+_t},\ \lambda_{z,t} = \lambda_t z^*_t P_t \end{aligned}$$

where $\tilde{r}^k_t P_t$ denotes the nominal rental rate on capital. Also, $\tilde{r}^k_t$ denotes the real, unscaled, rental rate of capital. We do not work with this variable. The rate of inflation in the nominal wage rate is:

$$\pi_{w,t} \equiv \frac{W_t}{W_{t-1}} = \frac{\tilde{w}_t \mu^*_{z,t} \pi_t}{\tilde{w}_{t-1}}.$$

Consider gdp growth, according to the model.

$$\frac{Y^{gdp}_t}{z^+_t} \equiv y_t = c_t + \frac{i_t}{\mu_{\Upsilon,t}} + g_t,$$

or,

$$Y^{gdp}_t = y_t z^+_t,$$

so that

$$\begin{aligned} \Delta \log Y^{gdp}_t &= \log Y^{gdp}_t - \log Y^{gdp}_{t-1} = \log(y_t) - \log(y_{t-1}) + \log\left(z^+_t\right) - \log z^+_{t-1} \\ &= \log(y_t) - \log(y_{t-1}) + \log \mu^*_{z,t} \end{aligned}$$

Let N_t denote period t nominal net worth, so that

$$n_t = \frac{N_t}{P_t z_t^+}.$$

Then,

$$\Delta \log \frac{N_t}{P_t} = \log n_t - \log n_{t-1} + \log \mu_{z,t}^*.$$

Another variable is investment. There is an issue about what units to measure investment in. Investment times its relative price is given by:

$$inv_t \equiv \frac{I_t}{\Upsilon^t \mu_{\Upsilon,t}} = \frac{i_t z_t^+ \Upsilon^t}{\Upsilon^t \mu_{\Upsilon,t}} = \frac{i_t z_t^+}{\mu_{\Upsilon,t}},$$

so that:

$$\Delta \log inv_t \equiv \log inv_t - \log inv_{t-1} = \log i_t - \log i_{t-1} + \log \mu_{z,t}^* - \left(\log \mu_{\Upsilon,t} - \log \mu_{\Upsilon,t-1}\right).$$

The investment goods relative to consumption goods is given by

$$p_{I,t} \equiv \frac{1}{\Upsilon^t \mu_{\Upsilon,t}},$$

so that

$$\begin{aligned} \Delta \log p_{I,t} &= -t \log \Upsilon + (t-1) \log \Upsilon - \log \mu_{\Upsilon,t} + \log \mu_{\Upsilon,t-1} \\ &= -\log \Upsilon - \log \mu_{\Upsilon,t} + \log \mu_{\Upsilon,t-1}. \end{aligned}$$

Also,

$$\Delta \log C_t = \log c_t - \log c_{t-1} + \log \mu_{z,t}^*.$$

The growth rate of the real wage is:

$$\Delta \log \frac{W_t}{P_t} = \log \tilde{w}_t - \log \tilde{w}_{t-1} + \log \mu_{z,t}^*$$

C Appendix C: Dynamic Equations

Here, we display all the dynamic equilibrium conditions associated with the model.

C.1 Prices

The equations pertaining to prices are:

$$(1) p_t^* - \left[(1-\xi_p) \left(\frac{K_{p,t}}{F_{p,t}} \right)^{\frac{\lambda_f}{1-\lambda_f}} + \xi_p \left(\frac{\tilde{\pi}_t}{\pi_t} p_{t-1}^* \right)^{\frac{\lambda_f}{1-\lambda_f}} \right]^{\frac{1-\lambda_f}{\lambda_f}} = 0 \tag{35}$$

and

$$(2) E_t \left\{ \zeta_{c,t} \lambda_{z,t} y_{z,t} + \left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda_f}} \beta \xi_p F_{p,t+1} - F_{p,t} \right\} = 0, \tag{36}$$

where $\lambda_{z,t}$ denotes $\lambda_t z_t^* P_t$. Also,

$$(3)\zeta_{c,t}\lambda_{z,t}\lambda_f y_{z,t} s_t + \beta\xi_p \left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}}\right)^{\frac{\lambda_f}{1-\lambda_f}} K_{p,t+1} - K_{p,t} = 0. \tag{37}$$

Note that both these equations involve $F_{p,t}$. This reflects that a lot of equations have been substituted out. In particular, we have

$$(4)F_{p,t}\left[\frac{1-\xi_p\left(\frac{\tilde{\pi}_t}{\pi_t}\right)^{\frac{1}{1-\lambda_f}}}{1-\xi_p}\right]^{1-\lambda_f} = K_{p,t},\ \tilde{p}_t = \frac{K_{p,t}}{F_{p,t}},$$

where $\tilde{p}_t$ is the real price set by price-optimizing firms in period t. This is not a variable of direct interest in the analysis.

C.2 Wages

The demand for labor is the solution to the following problem:

$$\max W_t \overbrace{\left[\int_0^1 (h_{t,i})^{\frac{1}{\lambda_w}} di\right]^{\lambda_w}}^{=l_t} - \int_0^1 W_{t,i} h_{t,i} di,$$

where $W_{t,i}$ is the wage rate of $i-$type workers and W_t is the wage rate for homogeneous labor, l_t. The first order condition is:

$$h_{t,i} = l_t \left(\frac{W_t}{W_{t,i}}\right)^{\frac{\lambda_w}{\lambda_w - 1}}.$$

The wages of non-optimizing unions evolve as follows:

$$W_{j,t} = \tilde{\pi}_{w,t}\left(\mu_{z^*,t}\right)^{\iota_\mu}\left(\mu_{z^*}\right)^{1-\iota_\mu} W_{j,t-1},\ \tilde{\pi}_{w,t} \equiv \left(\pi_t^*\right)^{\iota_{w1}}\left(\pi_{t-1}\right)^{\iota_{w2}} \bar{\pi}^{1-\iota_{w1}-\iota_{w2}}, \tag{38}$$

Nominal wage growth, $\pi_{w,t}$, is:

$$\pi_{w,t} = \frac{\tilde{w}_t \mu_{z,t}^* \pi_t}{\tilde{w}_{t-1}},$$

where $\tilde{w}_t$ denotes the scaled wage rate:

$$\tilde{w}_t \equiv \frac{W_t}{z_t^* P_t}.$$

The labor input variable that we treat as observed is the sum over the various different types of labor:

$$\begin{aligned} h_t &= \int_0^1 h_{it} di \\ &= l_t W_t^{\frac{\lambda_w}{\lambda_w - 1}} \int_0^1 (W_{t,i})^{\frac{\lambda_w}{1-\lambda_w}} di \\ &= l_t W_t^{\frac{\lambda_w}{\lambda_w - 1}} \left(W_t^*\right)^{\frac{\lambda_w}{1-\lambda_w}}, \end{aligned}$$

where

$$\begin{aligned} W_t^* &\equiv \left[\int_0^1 (W_{t,i})^{\frac{\lambda_w}{1-\lambda_w}} di\right]^{\frac{1-\lambda_w}{\lambda_w}} \\ &= \left[(1-\xi_w)\tilde{W}_t + \int_{\xi_w \text{ monopolists that do not reoptimize}} \left(\tilde{\pi}_{w,t}\left(\mu_{z^*,t}\right)^{\iota_\mu}\left(\mu_{z^*}\right)^{1-\iota_\mu} W_{i,t-1}\right)^{\frac{\lambda_w}{1-\lambda_w}} di\right]^{\frac{1-\lambda_w}{\lambda_w}} \\ &= \left[(1-\xi_w)\tilde{W}_t + \xi_w\left(\tilde{\pi}_{w,t}\left(\mu_{z^*,t}\right)^{\iota_\mu}\left(\mu_{z^*}\right)^{1-\iota_\mu} W_{t-1}^*\right)^{\frac{\lambda_w}{1-\lambda_w}}\right]^{\frac{1-\lambda_w}{\lambda_w}}. \end{aligned}$$

Let $w_t^* \equiv W_t^*/W_t$, and use linear homogeneity:

$$w_t^* = \left[(1-\xi_w)\frac{\tilde{W}_t}{W_t} + \xi_w\left(\frac{\tilde{\pi}_{w,t}\left(\mu_{z^*,t}\right)^{\iota_\mu}\left(\mu_{z^*}\right)^{1-\iota_\mu}}{\pi_{w,t}} w_{t-1}^*\right)^{\frac{\lambda_w}{1-\lambda_w}}\right]^{\frac{1-\lambda_w}{\lambda_w}},$$

$\tilde{W}_t$ is the nominal wage set by the $1-\xi_w$ wage optimizers in the current period. Rewriting,

$$w_t^* = [(1-\xi_w) w_t^{\frac{\lambda_w}{1-\lambda_w}} + \xi_w\left(\frac{\tilde{\pi}_{w,t}\left(\mu_{z,t}^*\right)^{\iota_\mu}\left(\mu_z^*\right)^{1-\iota_\mu}}{\pi_{wt}} w_{t-1}^*\right)^{\frac{\lambda_w}{1-\lambda_w}}]^{\frac{1-\lambda_w}{\lambda_w}}, \tag{39}$$

where

$$w_t \equiv \frac{\tilde{W}_t}{W_t}. \tag{40}$$

We conclude:

$$h_t = l_t\left(w_t^*\right)^{\frac{\lambda_w}{1-\lambda_w}}. \tag{41}$$

For purposes of evaluating aggregate utility, it is also convenient to have an expression for the following:

$$\begin{aligned} &\int_0^1 h_{it}^{1+\sigma_L} di \\ = \;& l_t^{1+\sigma_L} W_t^{-\frac{\lambda_w(1+\sigma_L)}{1-\lambda_w}} \int_0^1 (W_{t,i})^{\frac{\lambda_w(1+\sigma_L)}{1-\lambda_w}} di \\ = \;& l_t^{1+\sigma_L} W_t^{-\frac{\lambda_w(1+\sigma_L)}{1-\lambda_w}} \ddot{W}_t^{\frac{\lambda_w(1+\sigma_L)}{1-\lambda_w}}, \end{aligned}$$

where

$$\ddot{W}_t \equiv \left[\int_0^1 (W_{t,i})^{\frac{\lambda_w(1+\sigma_L)}{1-\lambda_w}} di\right]^{\frac{1-\lambda_w}{\lambda_w(1+\sigma_L)}}.$$

Then,

$$\begin{aligned} \ddot{W}_t &= \left[\int_0^1 (W_{t,i})^{\frac{\lambda_w(1+\sigma_L)}{1-\lambda_w}} di\right]^{\frac{1-\lambda_w}{\lambda_w(1+\sigma_L)}} \\ &= \left[(1-\xi_w)\left(\tilde{W}_t\right)^{\frac{\lambda_w(1+\sigma_L)}{1-\lambda_w}} + \int_{\xi_w \text{ that change}} (W_{t,i})^{\frac{\lambda_w(1+\sigma_L)}{1-\lambda_w}} di\right]^{\frac{1-\lambda_w}{\lambda_w(1+\sigma_L)}} \\ &= \left[(1-\xi_w)\left(\tilde{W}_t\right)^{\frac{\lambda_w(1+\sigma_L)}{1-\lambda_w}} + \xi_w\left(\tilde{\pi}_{w,t}\left(\mu_{z^*,t}\right)^{\iota_\mu}\left(\mu_{z^*}\right)^{1-\iota_\mu}\ddot{W}_{t-1}\right)^{\frac{\lambda_w(1+\sigma_L)}{1-\lambda_w}}\right]^{\frac{1-\lambda_w}{\lambda_w(1+\sigma_L)}}. \end{aligned}$$

Divide by W_t and make use of the linear homogeneity of the above expression:

$$\frac{\ddot{W}_t}{W_t} = \left[(1-\xi_w) \left(\frac{\tilde{W}_t}{W_t} \right)^{\frac{\lambda_w(1+\sigma_L)}{1-\lambda_w}} + \xi_w \left(\frac{\tilde{\pi}_{w,t} \left(\mu_{z^*,t}\right)^{\iota_\mu} \left(\mu_{z^*}\right)^{1-\iota_\mu}}{\pi_{w,t}} \frac{\ddot{W}_{t-1}}{W_{t-1}} \right)^{\frac{\lambda_w(1+\sigma_L)}{1-\lambda_w}} \right]^{\frac{1-\lambda_w}{\lambda_w(1+\sigma_L)}}$$

Define

$$\ddot{w}_t = \frac{\ddot{W}_t}{W_t},$$

so that

$$\ddot{w}_t = \left[(1-\xi_w) (w_t)^{\frac{\lambda_w(1+\sigma_L)}{1-\lambda_w}} + \xi_w \left(\frac{\tilde{\pi}_{w,t} \left(\mu_{z^*,t}\right)^{\iota_\mu} \left(\mu_{z^*}\right)^{1-\iota_\mu}}{\pi_{w,t}} \ddot{w}_{t-1} \right)^{\frac{\lambda_w(1+\sigma_L)}{1-\lambda_w}} \right]^{\frac{1-\lambda_w}{\lambda_w(1+\sigma_L)}}, \quad (42)$$

using (40). We conclude

$$\begin{aligned} \int_0^1 h_{it}^{1+\sigma_L} di &= \left[l_t \left(\ddot{w}_t\right)^{\frac{\lambda_w}{1-\lambda_w}} \right]^{(1+\sigma_L)} \quad (43) \\ &= \left[h_t \left(\frac{\ddot{w}_t}{w_t^*} \right)^{\frac{\lambda_w}{1-\lambda_w}} \right]^{(1+\sigma_L)}. \end{aligned}$$

using (41).

The optimality conditions associated with wage-setting are characterized by:

$$(5) E_t\{\zeta_{c,t}\lambda_{z,t} \frac{(w_t^*)^{\frac{\lambda_w}{\lambda_w-1}} h_t \left(1-\tau_t^l\right)}{\lambda_w} + \beta\xi_w \left(\mu_{z^*}\right)^{\frac{1-\iota_\mu}{1-\lambda_w}} E_t \left(\mu_{z^*,t+1}\right)^{\frac{\iota_\mu}{1-\lambda_w}-1} \left(\frac{1}{\pi_{w,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}} \frac{\tilde{\pi}_{w,t+1}^{\frac{1}{1-\lambda_w}}}{\pi_{t+1}} F_{w,t+1} - F_{w,t}\} = (\quad (44)$$

and

$$(6)\ E_t\{\zeta_{c,t}\zeta_t \left[(w_t^*)^{\frac{\lambda_w}{\lambda_w-1}} h_t \right]^{1+\sigma_L} + \beta\xi_w \left(\frac{\tilde{\pi}_{w,t+1} \left(\mu_{z,t+1}^*\right)^{\iota_\mu} \left(\mu_z^*\right)^{1-\iota_\mu}}{\pi_{wt+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} K_{w,t+1} - K_{w,t}\} = 0.$$

$$(7)\ \frac{1}{\psi_L} \left[\frac{1-\xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \left(\mu_{z^*}\right)^{1-\iota_\mu} \left(\mu_{z^*,t}\right)^{\iota_\mu} \right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w} \right]^{1-\lambda_w(1+\sigma_L)} \tilde{w}_t F_{w,t} - K_{w,t} = 0$$

Optimization by households implies:

$$w_t = \left[\frac{\psi_L}{\tilde{w}_t} \frac{K_{w,t}}{F_{w,t}} \right]^{\frac{1-\lambda_w}{1-\lambda_w(1+\sigma_L)}},$$

so that, using (39):

$$w_t^* = \left[(1-\xi_w) \left[\frac{\psi_L}{\tilde{w}_t} \frac{K_{w,t}}{F_{w,t}} \right]^{\frac{\lambda_w}{1-\lambda_w(1+\sigma_L)}} + \xi_w \left(\frac{\tilde{\pi}_{w,t} \left(\mu_{z,t}^*\right)^{\iota_\mu} \left(\mu_z^*\right)^{1-\iota_\mu}}{\pi_{wt}} w_{t-1}^* \right)^{\frac{\lambda_w}{1-\lambda_w}} \right]^{\frac{1-\lambda_w}{\lambda_w}}.$$

We can replace $K_{w,t}/F_{w,t}$ with the expression implied by (7) above:

$$(8)\ \ w_t^* = \left[(1-\xi_w) \left(\frac{1-\xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \left(\mu_{z^*}\right)^{1-\iota_\mu} \left(\mu_{z^*,t}\right)^{\iota_\mu} \right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w} \right)^{\lambda_w} + \xi_w \left(\frac{\tilde{\pi}_{w,t} \left(\mu_{z,t}^*\right)^{\iota_\mu} \left(\mu_z^*\right)^{1-\iota_\mu}}{\pi_{wt}} w_{t-1}^* \right)^{\frac{\lambda_w}{1-\lambda_w}} \right]^{\frac{1-\lambda_w}{\lambda_w}}$$

C.3 Capital Utilization, Marginal Cost, Return on Capital, Investment, Monetary Policy

The first order necessary condition associated with the capital utilization decision is:

$$\frac{1}{\Upsilon^t} a'\left(u_t\right) = \tilde{r}_t^k,$$

or,

$$a'\left(u_t\right) = \Upsilon^t \tilde{r}_t^k = r_t^k,$$

after scaling. Making use of our assumed utilization cost function, this reduces to:

$$(9)\ r_t^k = r^k \exp\left(\sigma_a \left[u_t - 1\right]\right),$$

where

$$a\left(u_t\right) = \frac{r^k}{\sigma_a}\left[\exp(\sigma_a \left[u_t - 1\right]) - 1\right].$$

Also, r^k denotes the steady state value of r_t^k. The above restriction on the $a\left(u_t\right)$ function implies that $u = 1$ in a steady state. As a result, the steady state is independent of the capital adjustment costs.

Marginal cost is given by:

$$(10) r_t^k = \frac{\alpha \epsilon_t}{\left[1 + \psi_{k,t} R_t\right]} \left(\frac{\Upsilon \mu_{z,t}^* L_t \left(w_t^*\right)^{\frac{\lambda_w}{\lambda_w - 1}}}{u_t \bar{k}_t} \right)^{1-\alpha} s_t \qquad (45)$$

$$\tilde{w}_t = \frac{(1-\alpha)\,\epsilon_t}{\left[1 + \psi_{l,t} R_t\right]} \left(\frac{\Upsilon \mu_{z,t}^* L_t \left(w_t^*\right)^{\frac{\lambda_w}{\lambda_w - 1}}}{u_t \bar{k}_t} \right)^{-\alpha} s_t,$$

where $\psi_{k,t}$ and $\psi_{l,t}$ denote the fraction of the capital services and labor bills, respectively, that must be financed in advance. Combining the last two equations, we obtain the familiar expression for marginal cost:

$$(11)\ s_t = \frac{1}{\epsilon_t} \left(\frac{r_t^k \left[1 + \psi_{k,t} R_t\right]}{\alpha} \right)^{\alpha} \left(\frac{\tilde{w}_t \left[1 + \psi_{l,t} R_t\right]}{1-\alpha} \right)^{1-\alpha}, \qquad (46)$$

where $\psi_{k,t} = \psi_{l,t} = 0$. Resource constraint:

$$(12) a(u_t) \frac{\bar{k}_t}{\Upsilon \mu_{z,t}^*} + g_t + c_t + \frac{i_t}{\mu_{\Upsilon,t}} = y_{z,t} \qquad (47)$$

where g_t is an exogenous stochastic process and

$$(13)\bar{k}_{t+1} = \overbrace{\left[p_t\left(e_t\right)e^{g_t} + \left(1-p_t\left(e_t\right)\right)e^{b_t}\right]}^{\text{relevant only for financial friction model, drop in CEE version}} \left\{(1-\delta)\frac{1}{\mu_{z,t}^*\Upsilon}\bar{k}_t + \left[1 - S\left(\frac{\zeta_{i,t}\, i_t\mu_{z,t}^*\Upsilon}{i_{t-1}}\right)\right]i_t\right\}, \tag{48}$$

where i_t is investment scaled by $z_t^*\Upsilon^t$.

Equation defining the nominal non-state contingent rate of interest:

$$(14)E_t\{\beta\frac{1}{\pi_{t+1}\mu_{z,t+1}^*}\zeta_{c,t+1}\lambda_{z,t+1}R_t - \zeta_{c,t}\lambda_{z,t}\} = 0 \tag{49}$$

The derivative of utility with respect to consumption is,

$$(15)E_t\left[\zeta_{c,t}\lambda_{z,t} - \frac{\mu_{z,t}^*\zeta_{c,t}}{c_t\mu_{z,t}^* - bc_{t-1}} + b\beta\frac{\zeta_{c,t+1}}{c_{t+1}\mu_{z,t+1}^* - bc_t}\right] = 0, \tag{50}$$

where c_t denotes consumption scaled by z_t^*. The following capital first order first order condition is an equilibrium condition in CEE, but not in our model with financial frictions because in that model households do not accumulate capital:

$$(16)E_t\left\{-\zeta_{c,t}\lambda_{zt} + \frac{\beta}{\pi_{t+1}\mu_{z,t+1}^*}\zeta_{c,t+1}\lambda_{zt+1}R_{t+1}^k\right\} = 0. \tag{51}$$

In (51), R_{t+1}^k denotes the benchmark rate of return on capital:

$$(17)\ R_t^k = \frac{u_t r_t^k - a(u_t) + (1-\delta)q_t}{\Upsilon q_{t-1}}\pi_t$$

where q_t denotes the scaled market price of capital, $Q_{\bar{K}',t}$:

$$q_t = \Upsilon^t\frac{Q_{\bar{K}',t}}{P_t}.$$

Equation (17) holds in our financial friction model, as well as in CEE. The investment first order condition, (27)

$$\begin{aligned}(18)\ E_t\{\zeta_{c,t}\lambda_{zt}q_t\left[1 - S(\frac{\zeta_{i,t}\mu_{z,t}^*\Upsilon i_t}{i_{t-1}}) - S'(\frac{\zeta_{i,t}\mu_{z,t}^*\Upsilon i_t}{i_{t-1}})\frac{\zeta_{i,t}\mu_{z,t}^*\Upsilon i_t}{i_{t-1}}\right] \\ -\frac{\zeta_{c,t}\lambda_{zt}}{\mu_{\Upsilon,t}} + \frac{\beta\lambda_{zt+1}\zeta_{c,t+1}\zeta_{i,t+1}q_{t+1}}{\mu_{z,t+1}^*\Upsilon}S'(\frac{\zeta_{i,t+1}\mu_{z,t+1}^*\Upsilon i_{t+1}}{i_t})\left(\frac{\mu_{z,t+1}^*\Upsilon i_{t+1}}{i_t}\right)^2\} = 0,\end{aligned} \tag{52}$$

where i_t is scaled (by $z_t^*\Upsilon^t$) investment. The scaled representation of aggregagte output is:

$$(19)\ y_{z,t} \equiv \frac{Y_t}{z_t^*} = (p_t^*)^{\frac{\lambda_f}{\lambda_f - 1}}\left[\epsilon_t\left(\frac{u_t\bar{k}_t}{\mu_{z,t}^*\Upsilon}\right)^\alpha\left((w_t^*)^{\frac{\lambda_w}{\lambda_w - 1}}h_t\right)^{1-\alpha} - \phi\right]$$

The monetary policy rule:

$$\begin{aligned}(20)\ \log(1+R_t) &= (1-\tilde{\rho})\log(1+R) + \tilde{\rho}\log(1+R_{t-1}) \\ &+\frac{1-\tilde{\rho}}{1+R}\left[\tilde{a}_p\pi\log\frac{\pi_{t+1}}{\pi_t^*} + \tilde{a}_y\frac{1}{4}\log\frac{y_t}{y}\right] + x_t^p,\end{aligned} \tag{53}$$

where x_t^p is an iid monetary policy shock and y_t denotes scaled GDP:

$$(21)\;\; y_t = g_t + c_t + \frac{i_t}{\mu_{\Upsilon,t}}.$$

It's important not to confuse y_t and Y_t. The former is scaled GDP while the latter is unscaled gross output. Scaled gross output and scaled GDP are the same in steady state but different in the dynamics because u_t is potentially different from unity then.

C.4 Conditions Pertaining to Financial Frictions

First, consider the equilibrium conditions associated with the financial friction model with unobserved effort. Consider the following scaling:

$$\tilde{d}_t = \frac{d_t}{z_t^* P_t},\;\; \lambda_{z,t+1} = \lambda_{t+1} z_{t+1}^* P_{t+1},\;\; v_{z,t+1} = \nu_{t+1} z_{t+1}^* P_{t+1},\;\; \tilde{N}_t = \frac{N_t}{z_t^* P_t},\;\; \tilde{T}_t = \frac{T_t}{z_t^* P_t}$$

Consider the e equation:

$$e : E_t \left(\lambda_{t+1} + \nu_{t+1}\right) p_t' \left(e_t\right) \left(R_{g,t+1}^d - R_{b,t+1}^d\right) d_t + \eta_t = 0$$

or,

$$e : E_t \left(\lambda_{z,t+1} + \nu_{z,t+1}\right) \frac{1}{z_{t+1}^* P_{t+1}} p_t' \left(e_t\right) \left(R_{g,t+1}^d - R_{b,t+1}^d\right) z_t^* P_t \tilde{d}_t + \eta_t = 0$$

or,

$$e : E_t \left(\lambda_{z,t+1} + \nu_{z,t+1}\right) \frac{1}{\mu_{z^*,t+1} \pi_{t+1}} p_t' \left(e_t\right) \left(R_{g,t+1}^d - R_{b,t+1}^d\right) \tilde{d}_t + \eta_t = 0.$$

Now consider the d equation:

$$d : 0 = E_t \left(\lambda_{z,t+1} + \nu_{z,t+1}\right) \frac{1}{z_{t+1}^* P_{t+1}} \left[p_t \left(e_t\right) R_{t+1}^g + \left(1 - p_t \left(e_t\right)\right) R_{t+1}^b - R_t\right]$$

Multiply this equation by $z_t^* P_t$ to obtain:

$$d : 0 = E_t \left(\lambda_{z,t+1} + \nu_{z,t+1}\right) \frac{1}{\mu_{z^*,t+1} \pi_{t+1}} \left[p_t \left(e_t\right) R_{t+1}^g + \left(1 - p_t \left(e_t\right)\right) R_{t+1}^b - R_t\right]$$

In the case of the R_g^d equation, we can simply multiply by $z_{t+1}^* P_{t+1}$:

$$R_g^d : \nu_{z,t+1} p_t \left(e_t\right) + \eta_t \lambda_{z,t+1} p_t' \left(e_t\right) = 0.$$

Equation μ requires no adjustment:

$$\mu : R_t = p_t \left(e_t\right) R_{g,t+1}^d + \left(1 - p_t \left(e_t\right)\right) R_{b,t+1}^d$$

Next, consider equation η :

$$\eta : e_t = E_t \lambda_{z,t+1} \frac{1}{\mu_{z^*,t+1} \pi_{t+1}} p_t' \left(e_t\right) \left[\left(R_{t+1}^g - R_{t+1}^b\right) \left(\tilde{N}_t + \tilde{d}_t\right) - \left(R_{g,t+1}^d - R_{b,t+1}^d\right) \tilde{d}_t\right]$$

The ν equation is:

$$\nu : R_{t+1}^b \left(\tilde{N}_t + \tilde{d}_t\right) - R_{b,t+1}^d \tilde{d}_t = 0,$$

The law of motion for net worth is:

$$N_{t+1} = \gamma_{t+1} \left\{ p_t(e_t) \left[R^g_{t+1} (N_t + d_t) - R^d_{g,t+1} d_t \right] + (1 - p_t(e_t)) \left[R^b_{t+1} (N_t + d_t) - R^d_{b,t+1} d_t \right] \right\} + T_{t+1}$$

Divide by $z^*_{t+1} P_{t+1}$

$$\tilde{N}_{t+1} = \frac{\gamma_{t+1}}{\mu_{z^*,t+1}\pi_{t+1}} \left\{ p_t(e_t) \left[R^g_{t+1} \left(\tilde{N}_t + \tilde{d}_t \right) - R^d_{g,t+1} \tilde{d}_t \right] + (1 - p_t(e_t)) \left[R^b_{t+1} \left(\tilde{N}_t + \tilde{d}_t \right) - R^d_{b,t+1} \tilde{d}_t \right] \right\} + \tilde{T}_{t+1}$$

or,

$$\tilde{N}_{t+1} = \frac{\gamma_{t+1}}{\mu_{z^*,t+1}} \left\{ p_t(e_t) \frac{R^g_{t+1}}{\pi_{t+1}} \left(\tilde{N}_t + \tilde{d}_t \right) + (1 - p_t(e_t)) \frac{R^b_{t+1}}{\pi_{t+1}} \left(\tilde{N}_t + \tilde{d}_t \right) - \frac{R_t}{\pi_{t+1}} \tilde{d}_t \right\} + \tilde{T}_{t+1}$$

We also require equations to define the returns of good and bad entrepreneurs:

$$\begin{aligned} R^g_{t+1} &= e^{g_t} R^k_{t+1}, \\ R^b_{t+1} &= e^{b_t} R^k_{t+1} \end{aligned}$$

Finally, we have the market clearing condition for capital:

$$P_{k',t} \tilde{K}_{t+1} = N_t + d_t,$$

If we multiply both sides of this expression by $p_t(e_t) e^{g_t} + (1 - p_t(e_t)) e^{b_t}$, we obtain:

$$P_{k',t} \bar{K}_{t+1} = \left[p_t(e_t) e^{g_t} + (1 - p_t(e_t)) e^{b_t} \right] [N_t + d_t],$$

or

$$\frac{q_t P_t}{\Upsilon^t} \frac{z^+_t \Upsilon^t \bar{k}_{t+1}}{z^*_t P_t} = \left[p_t(e_t) e^{g_t} + (1 - p_t(e_t)) e^{b_t} \right] \left[\tilde{N}_t + \tilde{d}_t \right],$$

or

$$q_t \bar{k}_{t+1} = \left[p_t(e_t) e^{g_t} + (1 - p_t(e_t)) e^{b_t} \right] \left[\tilde{N}_t + \tilde{d}_t \right],$$

Collecting the equations for simplicity,

$$\begin{aligned}
e &: \quad E_t \left(\lambda_{z,t+1} + \nu_{z,t+1} \right) \frac{1}{\mu_{z^*,t+1}\pi_{t+1}} p'_t(e_t) \left(R^d_{g,t+1} - R^d_{b,t+1} \right) \tilde{d}_t + \eta_t = 0 \\
d &: \quad 0 = E_t \left(\lambda_{z,t+1} + \nu_{z,t+1} \right) \frac{1}{\mu_{z^*,t+1}\pi_{t+1}} \left[p_t(e_t) R^g_{t+1} + (1 - p_t(e_t)) R^b_{t+1} - R_t \right] \\
R^d_g &: \quad \nu_{z,t+1} p_t(e_t) + \eta_t \lambda_{z,t+1} p'_t(e_t) = 0 \\
\mu &: \quad R_t = p_t(e_t) R^d_{g,t+1} + (1 - p_t(e_t)) R^d_{b,t+1} \\
\eta &: \quad e_t = E_t \lambda_{z,t+1} \frac{1}{\mu_{z^*,t+1}\pi_{t+1}} p'_t(e_t) \left[\left(R^g_{t+1} - R^b_{t+1} \right) \left(\tilde{N}_t + \tilde{d}_t \right) - \left(R^d_{g,t+1} - R^d_{b,t+1} \right) \tilde{d}_t \right] \\
\nu &: \quad R^b_{t+1} \left(\tilde{N}_t + \tilde{d}_t \right) - R^d_{b,t+1} \tilde{d}_t = 0 \\
\tilde{N}_{t+1} &= \frac{\gamma_{t+1}}{\mu_{z^*,t+1}\pi_{t+1}} \left\{ p_t(e_t) \left[R^g_{t+1} \left(\tilde{N}_t + \tilde{d}_t \right) - R^d_{g,t+1} \tilde{d}_t \right] + (1 - p_t(e_t)) \left[R^b_{t+1} \left(\tilde{N}_t + \tilde{d}_t \right) - R^d_{b,t+1} \tilde{d}_t \right] \right\} + \\
q_t \bar{k}_{t+1} &= \left[p_t(e_t) e^{g_t} + (1 - p_t(e_t)) e^{b_t} \right] \left[\tilde{N}_t + \tilde{d}_t \right] \\
R^g_{t+1} &= e^{g_t} R^k_{t+1} \\
R^b_{t+1} &= e^{b_t} R^k_{t+1}
\end{aligned}$$

To go from the CEE model to the model with financial frictions, we drop equation (16) (and modify the capital accumulation equation, (13)) and we add the above 10 equations. So, there is a net addition of 9 equations. The additional 9 variables are

$$R^g_{t+1}, R^b_{t+1}, \tilde{d}_t, \tilde{N}_t, R^d_{g,t+1}, R^d_{b,t+1}, e_t, \nu_{z,t+1}, \eta_t.$$

C.5 Social Welfare Function

We now turn to developing an expression for the representative household's utility function

$$\begin{aligned} Util_t &= \zeta_{c,t}\left\{\log(z_t^+ c_t - b z_{t-1}^+ c_{t-1}) - \psi_L \int_0^1 \frac{h_{it}^{1+\sigma_L}}{1+\sigma_L} di\right\} \\ &= \zeta_{c,t}\left\{\log\left[z_t^+ (c_t - b\frac{z_{t-1}^+}{z_t^+} c_{t-1})\right] - \psi_L \int_0^1 \frac{h_{it}^{1+\sigma_L}}{1+\sigma_L} di\right\} \\ &= \zeta_{c,t}\left\{\log(c_t - \frac{b}{\mu^*_{z,t}} c_{t-1}) - \frac{\psi_L}{1+\sigma_L}\int_0^1 h_{it}^{1+\sigma_L} di\right\}, \end{aligned}$$

apart from a constant term. Using (43):

$$\frac{\psi_L}{1+\sigma_L}\int_0^1 h_{it}^{1+\sigma_L} di = \frac{\psi_L}{1+\sigma_L}\left[h_t\left(\frac{\ddot{w}_t}{w_t^*}\right)^{\frac{\lambda_w}{1-\lambda_w}}\right]^{(1+\sigma_L)},$$

so that

$$Util_t = \zeta_{c,t}\left\{\log(c_t - \frac{b}{\mu^*_{z,t}} c_{t-1}) - \frac{\psi_L}{1+\sigma_L}\left[h_t\left(\frac{\ddot{w}_t}{w_t^*}\right)^{\frac{\lambda_w}{1-\lambda_w}}\right]^{(1+\sigma_L)}\right\},$$

where $\ddot{w}_t$ is defined in (42) and w_t^* is defined in (8). Both these variables are unity in steady state.

D Appendix D: Calculating Steady State

Here, we discuss algorithms for computing the steady state of three versions of our model. The first three sections describe the equations of the model. The last three sections describe the algorithms.

D.1 Price and Wage Equations

This section pertains to equations (1)-(8) of the dynamical system in Appendix C. These equations are trivial in the case, $\pi = \bar{\pi}$. Equation (35) in steady state, is:

$$p^* = \left[\frac{(1-\xi_p)\left(\frac{1-\xi_p\left(\frac{\tilde{\pi}}{\pi}\right)^{\frac{1}{1-\lambda_f}}}{1-\xi_p}\right)^{\lambda_f}}{1-\xi_p\left(\frac{\tilde{\pi}}{\pi}\right)^{\frac{\lambda_f}{1-\lambda_f}}}\right]^{\frac{1-\lambda_f}{\lambda_f}}.$$

Note that, if $\pi = \bar{\pi}$ then $p^* = 1$. Equation (36):

$$F_p = \frac{\lambda_z (p^*)^{\frac{\lambda_f}{\lambda_f - 1}} \left[\left(\frac{k}{\mu_z^* \Upsilon} \right)^{\alpha} \left((w^*)^{\frac{\lambda_w}{\lambda_w - 1}} h \right)^{1-\alpha} - \phi \right]}{1 - \left(\frac{\tilde{\pi}}{\pi} \right)^{\frac{1}{1-\lambda_f}} \beta \xi_p},$$

assuming

$$\left(\frac{\tilde{\pi}}{\pi} \right)^{\frac{1}{1-\lambda_f}} \beta \xi_p < 1.$$

Equation (37) in steady state is:

$$F_p = \frac{\lambda_z \lambda_f (p^*)^{\frac{\lambda_f}{\lambda_f - 1}} \left[\left(\frac{k}{\mu_z} \right)^{\alpha} \left((w^*)^{\frac{\lambda_w}{\lambda_w - 1}} h \right)^{1-\alpha} - \phi \right] s}{\left[\frac{1 - \xi_p \left(\frac{\tilde{\pi}}{\pi} \right)^{\frac{1}{1-\lambda_f}}}{1 - \xi_p} \right]^{1-\lambda_f} \left[1 - \beta \xi_p \left(\frac{\tilde{\pi}}{\pi} \right)^{\frac{\lambda_f}{1-\lambda_f}} \right]}$$

Equating the preceding two equations:

$$s = \frac{1}{\lambda_f} \frac{\left[\frac{1 - \xi_p \left(\frac{\tilde{\pi}}{\pi} \right)^{\frac{1}{1-\lambda_f}}}{1 - \xi_p} \right]^{1-\lambda_f} \left[1 - \beta \xi_p \left(\frac{\tilde{\pi}}{\pi} \right)^{\frac{\lambda_f}{1-\lambda_f}} \right]}{1 - \left(\frac{\tilde{\pi}}{\pi} \right)^{\frac{1}{1-\lambda_f}} \beta \xi_p}. \tag{54}$$

In the case, $\pi = \bar{\pi}$, $s = 1/\lambda_f$. Equation (44) in steady state is:

$$F_w = \frac{\lambda_z \frac{(w^*)^{\frac{\lambda_w}{\lambda_w - 1}} h}{\lambda_w}}{1 - \beta \xi_w \tilde{\pi}_w^{\frac{1}{1-\lambda_w}} \frac{\left(\frac{1}{\pi} \right)^{\frac{\lambda_w}{1-\lambda_w}}}{\pi}},$$

as long as the condition,

$$\beta \xi_w \tilde{\pi}_w^{\frac{1}{1-\lambda_w}} \frac{\left(\frac{1}{\pi} \right)^{\frac{\lambda_w}{1-\lambda_w}}}{\pi} < 1,$$

is satisfied. Also

$$\tilde{\pi}_w = (\pi)^{\iota_{w,2}} \bar{\pi}^{1-\iota_{w,2}}.$$

Equation (**??**) is

$$F_w = \frac{\psi_L \left[(w^*)^{\frac{\lambda_w}{\lambda_w - 1}} h \right]^{1+\sigma_L}}{\left[\frac{1 - \xi_w \left(\frac{\tilde{\pi}_w}{\pi} \right)^{\frac{1}{1-\lambda_w}}}{1 - \xi_w} \right]^{1-\lambda_w(1+\sigma_L)} \tilde{w} \left[1 - \beta \xi_w \left(\frac{\tilde{\pi}_w}{\pi} \right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} \right]},$$

as long as

$$\beta \xi_w \left(\frac{\tilde{\pi}_w}{\pi} \right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} < 1.$$

Equating the two expressions for F_w, we obtain:

$$\tilde{w} = W \lambda_w \frac{\psi_L h^{\sigma_L}}{\lambda_z}, \tag{55}$$

where

$$W = (w^*)^{\frac{\lambda_w}{\lambda_w - 1}\sigma_L} \left[\frac{1 - \xi_w \left(\frac{\tilde{\pi}_w}{\pi}\right)^{\frac{1}{1-\lambda_w}}}{1 - \xi_w} \right]^{\lambda_w(1+\sigma_L)-1} \frac{1 - \beta\xi_w \left(\frac{\tilde{\pi}_w}{\pi}\right)^{\frac{1}{1-\lambda_w}}}{1 - \beta\xi_w \left(\frac{\tilde{\pi}_w}{\pi}\right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)}}, \tag{56}$$

which is unity in the case $\pi = \bar{\pi}$. In steady state, (**??**) reduces to:

$$w^* = \left[\frac{(1 - \xi_w) \left(\frac{1 - \xi_w \left(\frac{\tilde{\pi}_w}{\pi}\right)^{\frac{1}{1-\lambda_w}}}{1 - \xi_w} \right)^{\lambda_w}}{1 - \xi_w \left(\frac{\tilde{\pi}_w}{\pi}\right)^{\frac{\lambda_w}{1-\lambda_w}}} \right]^{\frac{1-\lambda_w}{\lambda_w}}, \tag{57}$$

which is unity when $\pi = \bar{\pi}$. According to the wage equation, the wage is a markup, $W\lambda_w$, over the household's marginal cost. Note that the magnitude of the markup depends on the degree of wage distortions in the steady state. These will be important to the extent that $\tilde{\pi}_w \neq \pi_w$.

In the case $\pi = \bar{\pi}$, we have

$$\tilde{\pi}_w, p^*, w^* = 1, \ \tilde{w} = \lambda_w \frac{\psi_L h^{\sigma_L}}{\lambda_z}, \ s = \frac{1}{\lambda_f}, \tag{58}$$

in addition to F_p, F_w, K_p, K_w which do not get used in the subsequent equations.

D.2 Other Non-Financial Equations

The marginal cost equation, (45) implies:

$$r^k = \frac{\alpha\epsilon}{[1 + \psi_k R]} \left(\frac{\Upsilon\mu_z^* h (w^*)^{\frac{\lambda_w}{\lambda_w - 1}}}{\bar{k}} \right)^{1-\alpha} s, \tag{59}$$

where w^* is determined by (57). In steady state, the capital accumulation equation, (48), is

$$\left[\frac{1}{p(e) e^g + (1 - p(e)) e^b} - (1 - \delta) \frac{1}{\mu_z^* \Upsilon} \right] \bar{k} = i, \tag{60}$$

or,

$$\left[1 - (1 - \delta) \frac{1}{\mu_z^* \Upsilon} \right] \bar{k} = i, \tag{61}$$

using (**??**). In steady state, the equation for the nominal rate of interest, (49), reduces to:

$$R = \frac{\pi\mu_z^*}{\beta}. \tag{62}$$

In steady state, the marginal utility of consumption, (50), is

$$\lambda_z = \frac{1}{c} \frac{\mu_z^* - b\beta}{\mu_z^* - b}. \tag{63}$$

Finally, the euler equation for investment, (52), reduces to

$$q = 1.$$

Also, equations (17) and (19) in the dynamic system reduce to:

$$(19)\ y_z = \epsilon_t \left(\frac{\bar{k}}{\mu_z^* \Upsilon}\right)^{\alpha} h_t^{1-\alpha} - \phi$$

$$(17)\ R^k = \frac{r^k + 1 - \delta}{\Upsilon}\pi \tag{64}$$

We compute ϕ to guarantee that firm profits are zero in a steady state where $\pi = \bar{\pi}$. Let h and $\bar{k}$ denote hours worked and capital in such a steady state. Also, let F denote gross output of the final good in that steady state. Write sales of final good firm as $F - \phi$. Real marginal cost in this steady state is $s = 1/\lambda_f$. Since this is a constant, the total costs of the firm are sF. Zero profits requires $sF = F - \phi$. Thus, $\phi = (1-s)\,F = F(1 - 1/\lambda_f)$, or,

$$(7)\phi = \left(\frac{\bar{k}}{\mu_z^* \Upsilon}\right)^{\alpha} (h)^{1-\alpha} \left(1 - \frac{1}{\lambda_f}\right). \tag{65}$$

The steady state version of the resource constraint, (**??**), is:

$$(8) c + g + \frac{i}{\mu_\Upsilon} = \left(\frac{\bar{k}}{\mu_z^* \Upsilon}\right)^{\alpha} h^{1-\alpha} - \phi, \tag{66}$$

where $p^* = w^* = 1$. The steady state real wage can be solved from (45):

$$(9)\tilde{w} = s\,(1-\alpha)\left[\frac{\Upsilon \mu_{z^*} h}{\bar{k}}\right]^{-\alpha}. \tag{67}$$

The steady state labor supply equation, (55), is:

$$(10) h = \left[\frac{\lambda_z}{W \lambda_w \psi_L}\tilde{w}\right]^{\frac{1}{\sigma_L}}, \tag{68}$$

where $W = 1$ when $\pi = \bar{\pi}$.

D.3 Financial Sector Equations

In steady state, the equilibrium conditions pertaining to financial friction are

$$\begin{aligned}
e &: \quad (\lambda_z + \nu_z)\frac{\bar{b}}{\mu_{z^*}\pi}\left(R_g^d - R_b^d\right)\tilde{d} + \eta = 0, \\
d &: \quad 0 = \left[p(e)\,e^g + (1-p(e))\,e^b\right] R^k - R, \\
R_g^d &: \quad \nu_z p(e) + \eta \lambda_z \bar{b} = 0, \\
\mu &: \quad R = p(e)\,R_g^d + (1-p(e))\,R_b^d, \\
\eta &: \quad e = \frac{\lambda_z \bar{b}}{\mu_{z^*}\pi}\left[\left(e^g - e^b\right) R^k \left(\tilde{N} + \tilde{d}\right) - \left(R_g^d - R_b^d\right)\tilde{d}\right], \\
\nu &: \quad e^b R^k \left(\tilde{N} + \tilde{d}\right) - R_b^d \tilde{d} = 0, \\
\tilde{N} &= \frac{\gamma}{\mu_{z^*}\pi} R\tilde{N} + \tilde{T}, \\
q\bar{k} &= \left[p(e)\,e^g + (1-p(e))\,e^b\right]\left(\tilde{N} + \tilde{d}\right),
\end{aligned}$$

where $\bar{b} = p'(e)$ and we have substituted out R^g and R^b by $e^g R^k$ and $e^b R^k$ respectively. We need

$$\gamma < \beta,$$

for the net worth accumulation equation to make sense (i.e., have a steady state). Those 8 equations are solved for 8 variables: $\tilde{d}, \tilde{N}, R_g^d, R_b^d, e, \nu_z, \eta, R^k$, conditional on values for λ_z and $\bar{k}$ and some calibration information. We simply impose:

$$p(e)\, e^g + (1 - p(e))\, e^b = 1. \tag{69}$$

We suspect that this is in the nature of a normalization. Denote bank leverage by L:

$$L \equiv \left(\tilde{N} + \tilde{d}\right) / \tilde{N}. \tag{70}$$

We calibrate

$$sd_b,\ E^b, L,$$

where sd_b is the cross-sectional standard deviation of the nominal return on bank equity and E^b is the corresponding cross-sectional mean. We will use these three objects and (69) to determine $\tilde{T}$, b, g, $\bar{a}$. But, we must assume a value for the exogenous parameters, $\bar{b}$.

The market clearing condition for capital implies:

$$L = \frac{q\bar{k}}{\tilde{N}} \frac{1}{p(e)\, e^g + (1 - p(e))\, e^b} = \frac{\bar{k}}{\tilde{N}}, \tag{71}$$

using (69) and the fact, $q = 1$. Conditional on L, this gives us an expression that determines net worth, $\tilde{N}$. Then, the law of motion for net worth (i.e., (13)) allows us to pin down $\tilde{T}$:

$$\tilde{T} = [1 - \gamma R/(\mu_{z^*}\pi)]\tilde{N}.$$

From the d-condition,

$$R^k = \frac{R}{p(e)\, e^g + (1 - p(e))\, e^b} = R, \tag{72}$$

using (69), so that we now have R^k.

From ν-condition,

$$R_b^d = e^b R^k \frac{\tilde{N} + \tilde{d}}{\tilde{d}} = e^b R \frac{L}{L-1}. \tag{73}$$

where we have substituted using (72).

We find it convenient to compute the spread, though this does not directly bear on the calibration objects. The interest rate spreads for banks is, using the μ-equation:

$$\text{spread}_b \equiv R_g^d - R = \frac{1 - p(e)}{p(e)}(R - R_b^d).$$

Combining this with (73):

$$\text{spread}_b = \frac{1 - p(e)}{p(e)} \left(1 - e^b \frac{L}{L-1}\right) R \tag{74}$$

Next, we derive the expression for the cross-sectional variance of return on bank equity. The return on bank equity when a firm finds a good entrepreneur and when a firm finds a bad entrepreneur are given by:

$$e^g RL - R_g^d(L-1) \text{ and } \overbrace{e^b RL - R_b^d(L-1)}^{\text{we assume this is binding,}=0},$$

respectively. Recall that in the case of the binomial distribution, if a random variable can be x^h with probability p and x^l with probability $1-p$, then its variance is $p(1-p)\left(x^h - x^l\right)^2$. We conclude that the cross sectional standard deviation of the return on bank equity is:

$$\begin{aligned} sd_b &= [p(e)(1-p(e))]^{1/2}\left[e^g RL - R_g^d(L-1) - \left(e^b RL - R_b^d(L-1)\right)\right] \\ &= [p(e)(1-p(e))]^{1/2}\left[\left(e^g - e^b\right)RL - \left(R_g^d - R_b^d\right)(L-1)\right] \end{aligned}$$

From μ-condition,

$$\begin{aligned} R_g^d - R_b^d &= \frac{R - R_b^d}{p(e)} = \frac{R - \frac{e^b R}{p(e)e^g + (1-p(e))e^b}\frac{\tilde{N}+\tilde{d}}{\tilde{d}}}{p(e)} \\ &= \frac{R}{p(e)}\left[1 - e^b\frac{\tilde{N}+\tilde{d}}{\tilde{d}}\right] = \frac{R}{p(e)}\left[1 - e^b\frac{L}{L-1}\right] = \frac{\text{spread}_b}{1-p(e)} \end{aligned} \tag{75}$$

since

$$\frac{\tilde{N}+\tilde{d}}{\tilde{d}} = \frac{\tilde{N}+\tilde{d}}{\tilde{N}}\frac{\tilde{N}}{\tilde{d}} = \frac{L}{L-1}.$$

Replace $R_g^d - R_b^d$ in the expression for sd_b we obtain

$$sd_b = [p(e)(1-p(e))]^{\frac{1}{2}} R\left[(e^g - e^b)L - \frac{L(1-e^b)-1}{p(e)}\right].$$

According to the d equation with $R = R^k$:

$$1 = p(e)\,e^g + (1-p(e))\,e^b = p(e)\left(e^g - e^b\right) + e^b.$$

Then, substituting this into the sd_b equation:

$$\begin{aligned} sd_b &= [p(e)(1-p(e))]^{\frac{1}{2}} R\left[\frac{1-e^b}{p(e)}L - \frac{L(1-e^b)-1}{p(e)}\right] \\ &= [p(e)(1-p(e))]^{\frac{1}{2}} R\frac{1}{p(e)}, \end{aligned}$$

or,

$$sd_b = \left[\frac{1-p(e)}{p(e)}\right]^{\frac{1}{2}} R. \tag{76}$$

Given sd_b, (76) determines $p(e)$. Then, (74) determines e^b given L. The probability of finding a good entrepreneur is (using (69)):

$$p(e) = \frac{1-e^b}{e^g - e^b}, \tag{77}$$

and so this can be solved for g. We now have R^k from (72), R_b^d from (73), R_g^d from (75), $\tilde{N}$ from (71), $\tilde{d}$ from (70). We still need v_z, η and e. In addition, we still require $\bar{a}$.

Consider the η-condition,

$$e = \frac{\lambda_z \bar{b}}{\mu_{z^*}\pi}\left[\left(e^g - e^b\right) R^k \left(\tilde{N} + \tilde{d}\right) - \frac{R}{p(e)}\left(1 - e^b \frac{L}{L-1}\right)\tilde{d}\right],$$

using (75) to solve out for $R_g^d - R_b^d$. Then,

$$\begin{aligned} e &= \frac{\lambda_z \bar{b}}{\mu_{z^*}\pi}\left[\left(e^g - e^b\right) R^k L - \frac{R}{p(e)}\left(1 - e^b \frac{L}{L-1}\right)(L-1)\right]\tilde{N} \\ &= \frac{\lambda_z \bar{b}}{\mu_{z^*}\pi}\left[\left(e^g - e^b\right) RL - \frac{R}{p(e)}\left(L - 1 - e^b L\right)\right]\tilde{N} \\ &= \frac{\lambda_z \bar{b}}{\mu_{z^*}\pi}\left[L - \frac{1}{p(e)}\frac{\left(L - 1 - e^b L\right)}{\left(e^g - e^b\right)}\right]\left(e^g - e^b\right) R\tilde{N} \end{aligned}$$

Using (77),

$$\begin{aligned} e &= \frac{\lambda_z \bar{b}}{\mu_{z^*}\pi}\left[L - \frac{e^g - e^b}{1 - e^b}\frac{L\left(1 - e^b\right) - 1}{\left(e^g - e^b\right)}\right]\left(e^g - e^b\right) R\tilde{N} \\ &= \frac{\lambda_z \bar{b}}{\mu_{z^*}\pi}\left[L - \frac{L\left(1 - e^b\right) - 1}{1 - e^b}\right]\left(e^g - e^b\right) R\tilde{N} \end{aligned}$$

or,

$$e = \frac{\lambda_z \bar{b}}{\mu_{z^*}\pi}\frac{e^g - e^b}{1 - e^b} R\tilde{N}, \tag{78}$$

which determines e. Next, we have

$$p(e) = \bar{a} + \bar{b}e, \tag{79}$$

which determines $\bar{a}$.

We still have the following two equations:

$$\begin{aligned} e &: \left(\lambda_z + \nu_z\right)\frac{1}{\mu_{z^*}\pi}\bar{b}\left(R_g^d - R_b^d\right)\tilde{d} + \eta = 0, & (80) \\ R_g^d &: \nu_z p(e) + \eta\lambda_z \bar{b} = 0. & (81) \end{aligned}$$

Equations (80)-(81) are two equations in ν_z, η. Now solve for η using (80):

$$\eta = -\left(\lambda_z + \nu_z\right)\frac{1}{\mu_{z^*}\pi}\bar{b}\left(R_g^d - R_b^d\right)\tilde{d},$$

and use this to substitute out for η in (81):

$$\nu_z p(e) - \left(\lambda_z + \nu_z\right)\frac{1}{\mu_{z^*}\pi}\bar{b}\left(R_g^d - R_b^d\right)\tilde{d}\lambda_z \bar{b} = 0,$$

or,

$$\begin{aligned} \nu_z &= \frac{\frac{\lambda_z}{\mu_{z^*}\pi}\left(\bar{b}\right)^2\left(R_g^d - R_b^d\right)\tilde{d}\lambda_z}{p(e) - \frac{1}{\mu_{z^*}\pi}\left(\bar{b}\right)^2\left(R_g^d - R_b^d\right)\tilde{d}\lambda_z} & (82) \\ \eta &= -\left(\lambda_z + \nu_z\right)\frac{1}{\mu_{z^*}\pi}\bar{b}\left(R_g^d - R_b^d\right)\tilde{d}. & (83) \end{aligned}$$

This completes the computations we set out to accomplish.

D.4 Steady State Algorithm, Unobserved Effort Equilibrium

Here is an algorithm. We specify a value for π and compute R using (62). From (72) we obtain R^k. From (64) we obtain r^k. From (59) we obtain $h/\bar{k}$. Solve (67) for $\tilde{w}$.

Combining (65) and (66):

$$c + g + \frac{i}{\mu_\Upsilon} = \left(\frac{\bar{k}}{h\mu_z^*\Upsilon}\right)^\alpha h\frac{1}{\lambda_f}.$$

Substituting out for i using (61) and dividing the result by h :

$$\frac{c}{h} + \frac{g}{h} + \frac{\left(1-(1-\delta)\frac{1}{\mu_z^*\Upsilon}\right)\frac{\bar{k}}{h}}{\mu_\Upsilon} = \left(\frac{\bar{k}}{h\mu_z^*\Upsilon}\right)^\alpha \frac{1}{\lambda_f}.$$

We specify that g is a given fraction, η_g, of steady state gross output or GDP (both are the same in steady state), so that :

$$\begin{aligned} g &= \eta_g\left(\frac{\bar{k}}{\mu_z^*\Upsilon}\right)^\alpha h^{1-\alpha}\frac{1}{\lambda_f} \\ \frac{g}{h} &= \eta_g\left(\frac{\bar{k}}{h\mu_z^*\Upsilon}\right)^\alpha \frac{1}{\lambda_f}. \end{aligned}$$

Then,

$$\frac{c}{h} = (1-\eta_g)\left(\frac{\bar{k}}{h\mu_z^*\Upsilon}\right)^\alpha \frac{1}{\lambda_f} - \frac{\left(1-(1-\delta)\frac{1}{\mu_z^*\Upsilon}\right)\frac{\bar{k}}{h}}{\mu_\Upsilon},$$

and c/h is now determined. From (63),

$$\lambda_z = \frac{1}{(c/h)\,h}\frac{\mu_z^*-b\beta}{\mu_z^*-b},$$

where h is yet to be determined. Substitute this expresion for λ_z into (55) to obtain:

$$(10)h = \left[\frac{1}{(c/h)\,h}\frac{\mu_z^*-b\beta}{\mu_z^*-b}\frac{1}{\lambda_w\psi_L}\tilde{w}\right]^{\frac{1}{\sigma_L}},$$

where W has been set to unity, reflecting $\pi = \bar{\pi}$. Solve the resulting expression for h :

$$h^{1+\frac{1}{\sigma_L}} = \left[\frac{1}{(c/h)}\frac{\mu_z^*-b\beta}{\mu_z^*-b}\frac{1}{\lambda_w\psi_L}\tilde{w}\right]^{\frac{1}{\sigma_L}},$$

or,

$$h = \left[\frac{1}{(c/h)}\frac{\mu_z^*-b\beta}{\mu_z^*-b}\frac{1}{\lambda_w\psi_L}\tilde{w}\right]^{\frac{1}{1+\sigma_L}},$$

where c/h is the object derived above.

Given $\bar{k}$ $(= h/(h/\bar{k}))$ and λ_z we can compute the financial variables:$E^b = p\,(e)\left[R^g L - R_g^d\,(L-1)\right]$

$$\tilde{d}, \tilde{N}, R_g^d, R_b^d, e, \nu_z, \eta$$

using the approach in the previous section. In particular, given sd_b, $p\,(e)$ is determined by

(76); given L (74) determines e^b. The expression (77) can be solved for e^g. Then, R_b^d can be solved from (73); R_g^d from (75); $\tilde{N}$ from (71) and $\tilde{d}$ from (70). Then, $\bar{a}$ and e can be solved using (**??**) and (**??**). Finally, ν_z and η can be solved using (82) and (83). At the end of the calculations we need to verify that

$$\nu_z > 0,\ p(e) > 1/2,\ c > 0,\ \tilde{d} > 0,\ \tilde{N} > 0,\ g > b,\ e > 0,\ \bar{k} > 0, R_g^d > R_b^d$$

Some of these tests are nearly redundant. For example, $R_g^d > R_b^d$ by the calibration (see (75)).

D.5 Steady State Algorithm, Unobserved Effort with Leverage Restriction

In this section we discuss the computation of equilibrium under a binding leverage restriction. Our algorithm does not impose any of the calibration restrictions that we imposed in the previous section, and so it must be a different one. In terms of the equilibrium conditions from the section on price and wage equations, we have the equations in (58), which we reproduce here:

$$\begin{aligned}(1)s &= 1/\lambda_f,\\ (2)\tilde{w} &= \lambda_w \frac{\psi_L h^{\sigma_L}}{\lambda_z}.\end{aligned}$$

In terms of the non-price and wage equations, we have (59) and (60):

$$\begin{aligned}(3)r^k &= \alpha\left(\frac{\Upsilon\mu_z^* h}{\bar{k}}\right)^{1-\alpha} s,\\ (4)i &= \left[\frac{1}{p(e)\,e^g + (1-p(e))\,e^b} - (1-\delta)\frac{1}{\mu_z^*\Upsilon}\right]\bar{k}\end{aligned}$$

We also have (62) and (63):

$$\begin{aligned}(5)R &= \frac{\pi\mu_z^*}{\beta},\\ (6)\lambda_z &= \frac{1}{c}\frac{\mu_z^* - b\beta}{\mu_z^* - b}.\end{aligned}$$

The other equations listed right after this are:

$$s, \tilde{w}, h, \lambda_z, r^k, \bar{k}, i, e, R, R^k$$

$$\begin{aligned}(7)R^k &= \frac{r^k + 1 - \delta}{\Upsilon}\pi\\ (8)c + g + \frac{i}{\mu_\Upsilon} &= \left(\frac{\bar{k}}{\mu_z^*\Upsilon}\right)^{\alpha} h^{1-\alpha} - \phi\end{aligned}$$

Here, ϕ and g are exogenous parameters. They are not calibrated in this section.

$$(9)\tilde{w} = s(1-\alpha)\left[\frac{\Upsilon\mu_{z^*}h}{\bar{k}}\right]^{-\alpha}.$$

The financial sector equations are:

$$\begin{aligned}
(10)e &: (\lambda_z+\nu_z)\frac{\bar{b}}{\mu_{z^*}\pi}\left(R_g^d-R_b^d\right)\tilde{d}+\eta=0, \\
(11)d &: \Lambda=(\lambda_z+v_z)\frac{1}{\mu_{z^*}\pi}\left(\left[p(e)e^g+(1-p(e))e^b\right]R^k-R\right), \\
(12)R_g^d &: \nu_z p(e)+\eta\lambda_z\bar{b}=0, \\
(13)\mu &: R=p(e)R_g^d+(1-p(e))R_b^d, \\
(14)\eta &: e=\frac{\lambda_z\bar{b}}{\mu_{z^*}\pi}\left[\left(e^g-e^b\right)R^k\left(\tilde{N}+\tilde{d}\right)-\left(R_g^d-R_b^d\right)\tilde{d}\right], \\
(15)\nu &: e^bR^k\left(\tilde{N}+\tilde{d}\right)-R_b^d\tilde{d}=0, \\
(16)\tilde{N} &= \frac{\gamma}{\mu_{z^*}\pi}\left\{\left[p(e)e^g+(1-p(e))e^b\right]R^k\left(\tilde{N}+\tilde{d}\right)-R\tilde{d}\right\}+\tilde{T}, \\
(17)\bar{k} &= \left[p(e)e^g+(1-p(e))e^b\right]\left(\tilde{N}+\tilde{d}\right) \\
(18)L\tilde{N} &= \tilde{N}+\tilde{d}.
\end{aligned} \tag{84}$$

We have the following 11 non-financial market unknowns (steady state inflation is always fixed at π):

$$c,s,\tilde{w},h,\lambda_z,r^k,\bar{k},i,e,R,R^k.$$

We have the following 7 additional financial market variables:

$$\nu_z,R_g^d,R_b^d,\eta,\tilde{d},\tilde{N},\Lambda.$$

Thus, we have 18 equations in 18 unknowns.

Here is an algorithm. It is a one-dimensional search for a value of $\tilde{N}$ that enforces equation (16). We now discuss how the other endogenous variables in (16) are computed.

Assign an arbitrary value to $0\leq p(e)\leq 1$. From this we can compute e using

$$p(e)=\bar{a}+\bar{b}e.$$

We compute $\bar{k}$ from (17) and i from (4). We then reduce (14) to one nonlinear equation in one unknown, h. To see this, given $\bar{k}$, (8) now defines c as a function of h :

$$c=\left(\frac{\bar{k}}{\mu_z^*\Upsilon}\right)^\alpha h^{1-\alpha}-\phi-\frac{i}{\mu_\Upsilon}-g$$

Similarly, (6) defines λ_z as a function of h. Substituting (3) into (7):

$$R^k=\frac{\alpha\left(\frac{\Upsilon\mu_z^*h}{\bar{k}}\right)^{1-\alpha}\frac{1}{\lambda_f}+1-\delta}{\Upsilon}\pi,$$

we obtain that R^k is a function of h. Substituting from (13) into (14), we obtain:

$$e=\frac{\lambda_z\bar{b}}{\mu_{z^*}\pi}\left[\left(e^g-e^b\right)R^k\left(\tilde{N}+\tilde{d}\right)-\frac{R\tilde{d}-R_b^d\tilde{d}}{p(e)}\right]$$

Substituting from (15):

$$e = \frac{\lambda_z \bar{b}}{\mu_{z^*}\pi} \left[\left(e^g - e^b\right) R^k \left(\tilde{N} + \tilde{d}\right) - \frac{R\tilde{d} - e^b R^k \left(\tilde{N} + \tilde{d}\right)}{p\left(e\right)} \right]$$

Note that the right hand side of this expression is a function of h alone. We adjust the value of h until this expression is satisfied.

We use (15) to compute

$$R_b^d = e^b R^k \frac{\tilde{N} + \tilde{d}}{\tilde{d}}.$$

We also have R_g^d from (13):

$$R_g^d = \frac{R - \left(1 - p\left(e\right)\right) R_b^d}{p\left(e\right)}.$$

We compute Λ from (11).

Solving for η from (10):

$$\eta = -\left(\lambda_z + \nu_z\right) \frac{\bar{b}}{\mu_{z^*}\pi} \left(R_g^d - R_b^d\right) \tilde{d}.$$

Substitute this into (12)

$$\nu_z p\left(e\right) - \left(\lambda_z + \nu_z\right) \frac{\bar{b}}{\mu_{z^*}\pi} \left(R_g^d - R_b^d\right) \tilde{d} \lambda_z \bar{b} = 0,$$

and solving this for ν_z, we obtain:

$$\nu_z = \frac{\lambda_z \frac{\bar{b}}{\mu_{z^*}\pi} \left(R_g^d - R_b^d\right) \tilde{d} \lambda_z \bar{b}}{p\left(e\right) - \frac{\bar{b}}{\mu_{z^*}\pi} \left(R_g^d - R_b^d\right) \tilde{d} \lambda_z \bar{b}}.$$

So that we have η and ν_z.

Finally, we solve (9) for $\tilde{w}$. We adjust $p\left(e\right)$ until (2) is satisfied. Thus, for an arbitrary choice of value for $\tilde{N}$ we compute $p\left(e\right)$ and h as described above. We adjust the value of $\tilde{N}$ until (16) is satisfied.

D.6 Steady State Algorithm, Observed Effort

In the observed effort case, the equilibrium conditions for the financial sector do not require computing R_g^d and R_b^d and the multipliers, η and v_z, are both zero. This means that we can ignore equations (10), (12), (13), (15) in (84). Thus, the financial sector equilibrium conditions in nonstochastic steady state are:

$$\begin{aligned}
(11)d &: \quad \Lambda = \lambda_z \frac{1}{\mu_{z^*}\pi}\left(\left[p\left(e\right)e^g + \left(1-p\left(e\right)\right)e^b\right]R^k - R\right) \\
(14)\eta &: \quad e = \frac{\lambda_z \bar{b}}{\mu_{z^*}\pi}\left(e^g - e^b\right)R^k\left(\tilde{N}+\tilde{d}\right), \\
(16)\tilde{N} &= \frac{\gamma}{\mu_{z^*}\pi}\left\{\left[p\left(e\right)e^g + \left(1-p\left(e\right)\right)e^b\right]R^k\left(\tilde{N}+\tilde{d}\right) - R\tilde{d}\right\} + \tilde{T}, \\
(17)\bar{k} &= \left[p\left(e\right)e^g + \left(1-p\left(e\right)\right)e^b\right]\left(\tilde{N}+\tilde{d}\right) \\
(18)L\tilde{N} &= \tilde{N}+\tilde{d}.
\end{aligned}$$

When leverage is unrestricted, then $\Lambda = 0$ and (18) simply defines leverage, L. When the leverage restriction is imposed and is binding, then L in (18) is exogenous and $\Lambda > 0$.

We have the following 11 non-financial market unknowns (steady state inflation is always fixed at π):

$$c, s, \tilde{w}, h, \lambda_z, r^k, \bar{k}, i, e, R, R^k.$$

When the leverage restriction is non-binding, we have the following 3 additional financial market variables:

$$\tilde{d}, \tilde{N}, L,$$

with the understanding, $\Lambda = 0$. In terms of equations, we have 9 non-financial market equations and the above 5 financial market equations. Thus, we have 14 unknowns and 14 equations. When the leverage restriction is binding, then there is an additional equation that assigns a value to L and there is an additional unknown, Λ.

Here is an algorithm for solving the observed effort steady state when the leverage constraint is nonbinding, $\Lambda = 0$. Combining (11) (with $\Lambda = 0$) and (16), we obtain $\tilde{N} = \frac{\gamma}{\mu_{z^*}\pi}R\tilde{N} + \tilde{T}$, so that

$$\tilde{N} = \frac{\tilde{T}}{1 - \frac{\gamma}{\mu_{z^*}\pi}R}. \tag{85}$$

So, we can compute $\tilde{N}$ immediately. Fix a value of $p\left(e\right)$. Then, using (11) with $\Lambda = 0$:

$$R^k = \frac{R}{p\left(e\right)e^g + \left(1-p\left(e\right)\right)e^b}. \tag{86}$$

Then, r^k is computed using (7), $h/\bar{k}$ is obtained from (3), and $\tilde{w}$ is computed from (9). Now fix a value for h, so that we have $\bar{k}$. We obtain $\tilde{d}$ from (17), c from (8) and λ_z from (6). Adjust h until (14) is satisfied. Adjust $p\left(e\right)$ until (2) is satisfied.

We must consider the possibility that the observed effort equilibrium has the property,

$$p\left(e\right) = 1, \; e \leq \frac{\lambda_z \bar{b}}{\mu_{z^*}\pi}\left(e^g - e^b\right)R^k\left(\tilde{N}+\tilde{d}\right), \tag{87}$$

so that (14) does not hold. Since (11) and (16) are satisfied, we can still compute $\tilde{N}$ using (85). Set $p\left(e\right) = 1$ and compute R^k using (86). We can compute r^k, $h/\bar{k}$ and $\tilde{w}$ using (7), (3) and (9), as before. Now fix a value for h, so that we have $\bar{k}$. We obtain $\tilde{d}$, c, λ_z from (17), (8) and (6), as before. Adjust h until (2) is satisfied. Finally, verify that the inequality in (87) is satisfied.

Now consider the case of a binding leverage constraint. We cannot compute $\tilde{N}$ as before. Also, equation (11) does not hold with $\Lambda = 0$, so that we do not have access to (86). A different algorithm is required. Consider the following one. Fix a value for $\tilde{N}$ and use (18) to compute

$\tilde{d}$. Fix $p(e)$. Use (17) to compute $\bar{k}$. Use (4) to compute i.

Fix h. Compute c from (8) and λ_z from (6). Compute r^k from (3) and R^k from (7). Adjust h until (14) is satisfied. Compute $\tilde{w}$ from (9). Adjust $p(e)$ until (2) is satisfied. Finally, adjust $\tilde{N}$ until (16) is satisfied.

Again, we must consider the possibility that $p(e) = 1$ and (14) does not hold. As before, fix a value for $\tilde{N}$ and use (18) to compute $\tilde{d}$. Set $p(e) = 1$ and compute $\bar{k}, i$ using (17) and (4). Fix a value for h. Compute c, λ_z, $r^k, R^k, \tilde{w}$ from (8), (6), (3), (7), (9). Adjust h until (2) is satisfied. Adjust $\tilde{N}$ until (16) is satisfied. Finally, we must verify (87).