"LAND-PRICE DYNAMICS AND MACROECONOMIC FLUCTUATIONS" by ZHENG LIU, PENGFEI WANG, AND TAO ZHA (Econometrica, 2013)

Lawrence Christiano and Yuta Takahashi

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This note derives the equilibrium conditions that were solved for Christiano's discussion of the above paper.

The household solves

$$\max E \sum_{t=0}^{\infty} \beta_h^t \left[\log(c_{h,t} - \gamma^h c_{h,t-1}) - \frac{\rho}{1+\psi} N_t^{1+\psi} + \varphi_t \log(L_t^h) \right]$$

$$s.t. c + q_{l,t}(L_t^h - L_{t-1}^h) + \frac{B_t}{R_t} = z w_t N_t + B_{t-1}.$$

Writing this in Lagrangian form, we obtain:

$$maxE \sum_{t=0}^{\infty} \beta_h^t \{ log(c_{h,t} - \gamma^h c_{h,t-1}) - \frac{\rho}{1+\psi} N_t^{1+\psi} + \varphi_t \log(L_t^h)$$

$$+\lambda_{h,t} \left[w_t N_t + B_{t-1} - \left(c_{h,t} + q_{l,t} \left(L_t^h - L_{t-1}^h \right) + \frac{B_t}{R_t} \right) \right] \}$$

Then, the fonc w.r.t. c_{ht}, N_t, L_t^h, B_t are the following 4 equations:

$$u'\left(c_{h,t}\right) = \lambda_{h,t}$$

$$\rho N_t^{\psi} = \lambda_{h,t} w_t$$

$$\frac{\varphi_t}{L_t^h} + \beta_h \lambda_{h,t+1} q_{l,t+1} = \lambda_{h,t} q_{l,t}$$

$$\lambda_{h,t} \frac{1}{R_t} = \beta_h \lambda_{h,t+1}$$

The representative entrepreneur's problem is

$$maxE \sum_{t=0}^{\infty} \beta_e^t \{ log \left(c_t^e - \gamma^e c_{t-1}^e \right)$$

$$+\lambda_{e,t}\left[z_t\left(\left(L_{t-1}^e\right)^{\phi}K_{t-1}^{1-\phi}\right)^{\alpha}N_t^{1-\alpha}+\frac{B_t}{R_t}-\left(c_t^e+q_{l,t}\left(L_t^e-L_{t-1}^e\right)+q_{k,t}K_t^{\delta}+\frac{I_t}{Q_t}+w_tN_t+B_{t-1}\right)\right]$$

$$+\nu_t\left[(1-\delta)K_{t-1}+\left(1-f\left(\frac{I_t}{I_{t-1}}\right)\right)I_t+K_t^{\delta}-K_t\right]$$

$$+\mu_t\left[\theta_t\left(q_{l,t+1}L_t^e+q_{k,t+1}K_t\right)-B_t\right]\},$$

where

$$f(x)=\frac{\Omega}{2}(x-1)^2$$

The first order conditions with respect to $c_t^e,\ N_t,\ L_t^e,\ K_t,\ I_t,\ B_t,\ K_t^{\delta}$ are the following 7 equations:

$$u'\left(c_t^e\right)=\lambda_{e,t}$$

$$MP_{l,t}=w_t$$

$$\lambda_{e,t}q_{l,t}=\beta\lambda_{e,t+1}\left[MP_{L^e,t+1}+q_{l,t+1}\right]+\mu_t\theta_tq_{l,t+1}$$

$$\nu_t=\beta_e\left[\lambda_{e,t+1}MP_{K,t+1}+\nu_{t+1}(1-\delta)\right]+\mu_t\theta_tq_{k,t+1}$$

$$\lambda_{e,t}\frac{1}{Q_t}=\nu_t\left[1-f_t-f_t'\frac{I_t}{I_{t-1}}\right]+\beta_e\nu_{t+1}f_{t+1}'\left(\frac{I_{t+1}}{I_t}\right)^2$$

$$\lambda_{e,t}\frac{1}{R_t}=\beta_e\lambda_{e,t+1}+\mu_t$$

$$\lambda_{e,t}q_{k,t}=\nu_t$$

Marker clearing and technology corresponds to the following three equations (we take $K_t^{\delta}=0$ as given):

$$c_t^h+c_t^e+I_t=F_t$$

$$L_t^e+L_t^h=L$$

$$F_t=z_t\left(\left(L_{t-1}^e\right)^{\phi}K_{t-1}^{1-\phi}\right)^{\alpha}N_t^{1-\alpha}$$

Consider the budget constraints:

$$F_t+\frac{B_t}{R_t}=c_t^e+q_{l,t}\left(L_t^e-L_{t-1}^e\right)+q_{k,t}K_t^{\delta}+\frac{I_t}{Q_t}+w_tN_t+B_{t-1}$$

$$w_tN_t+B_{t-1}=c_{h,t}+q_{l,t}\left(L_t^h-L_{t-1}^h\right)+\frac{B_t}{R_t}$$

Add the two budget constraints:

$$F_t+\frac{B_t}{R_t}+w_tN_t+B_{t-1}=c_t^e+q_{l,t}\left(L_t^e-L_{t-1}^e\right)+q_{k,t}K_t^{\delta}+\frac{I_t}{Q_t}+w_tN_t+B_{t-1}+c_{h,t}+q_{l,t}\left(L_t^h-L_{t-1}^h\right)+\frac{B_t}{R_t}$$

or,

$$F_t = c_t^e + \frac{I_t}{Q_t} + c_{h,t}$$

which is Walras' law. Given that we have the resource constraint, having in addition both budget constraints is redundant. Thus, we include only one budget constraint. In addition we have the following two constraints:

$$(1-\delta) K_{t-1} + \left(1 - f\left(\frac{I_t}{I_{t-1}}\right)\right) I_t = K_t$$

$$\theta_t \left(q_{l,t+1} L_t^e + q_{k,t+1} K_t\right) = B_t$$

and we have a total of 17 equations. We have the following 17 unknowns:

$$q_{k,t}, q_{l,t}, c_t^e, c_t^n,\ N_t,\ L_t^e,\ L_t^h,\ K_t,\ I_t,\ B_t, N_t, \lambda_{e,t}, \lambda_{h,t}, \mu_t, \nu_t, w_t, R_t$$