Comment on Cochrane, "Michelson-Morley, Fisher and Occam: The Radical Implications of Stable Quiet Inflation at the Zero Bound"

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1 Introduction

Cochrane's headline argument is straightforward. The US monetary authorities have kept short term interest rates relatively constant since 2009, suggesting that monetary policy became passive then. The standard New Keynesian model ('NK model') predicts that under passive monetary policy, sunspots should have appeared in 2009, raising the volatility of aggregate economic variables. But, Cochrane infers from the observed 'stable and quiet inflation' that the predicted sunspots never appeared. In effect, 2009 was a Michelson-Morley moment for the NK model. Cochrane recommends that the nearly universal assumption of active money, passive fiscal policy in the standard NK model be replaced by the reverse: passive money and active fiscal policy. There are other reasons to make this change, according to Cochrane. These include that the standard NK model entangles one in a "menagerie of policy paradoxes" and the standard NK model has many equilibria without any reasonable way to choose between them.

The paper wakes up many old debates that have never been fully settled, some that go back two decades. I respond to many of those challenges in my comment.

The new part of the paper replaces the standard specification of the fiscal theory of the price level, which assumes one-period debt, with an alternative specification in which the government issues debt of all maturities. Cochrane tests the version of the NK model with this adjusted fiscal theory against a common conjecture about what would happen if central banks were to raise the nominal rate of interest permanently. The conjecture is that such an interest rate hike would cause a transitory decline in inflation and output before eventually producing a rise in inflation equal to the rise in the interest rate, and no change in output. The conjecture seems sensible to me, but I do not (yet) see why it requires adopting the fiscal theory of the price level.

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The comment proceeds as follows. The next subsection pushes back on Cochrane's claim that the recent data represent a Michelson-Morley moment for the NK model. In section 3 I discuss Cochrane's adjusted fiscal theory and his conjecture about the effects of a permanent interest rate hike. Section 4 responds to Cochrane's remarks on the unique, bounded rational expectations equilibrium in the standard NK model. I also discuss the learnability of that equilibrium, the other equilibria in that model and the identifiability of the Taylor rule coefficient on inflation. All these are subjects that are raised by Cochrane in his paper, and in each case I push back on the position that he takes.

Section 5 addresses the "menagerie of policy paradoxes" that Cochrane asserts the standard NK model possesses. Finally, section 6 concludes.

2 A Michelson-Morley Experiment for the Standard NK Model?

Cochrane's Michelson-Morley argument is based on the premise that US monetary policy became passive, beginning in 2009. By contrast, the conventional view is that monetary policy in fact remained active.[1] Policy only seemed like an interest rate peg because the zero lower bound on the nominal rate of interest had become binding. The NK model predicted that sunspots could not occur during this period because policy was expected to resume an active stance against inflation in the future when the zero lower bound would once again cease to bind. Cochrane scoffs at this view as representing an ex post "...rescue by epicycles", presumably by bitter NK model enthusiasts unhappy about the outcome of the Michelson-Morley experiment.

But, 2009 was no Michelson-Morley experiment. What the economy would look like if something rammed it into the zero lower bound had already been envisioned long ago in Krugman (1998) and Woodford and Eggertsson (2003).[2] For example, the latter paper predicted that inflation and other variables would literally be constant as long as the zero lower bound lasted. Cochrane's suggestion that the relatively small amount of volatility observed after 2009 is an embarrassment requiring a patch to the NK model is a misrepresentation of the literature.

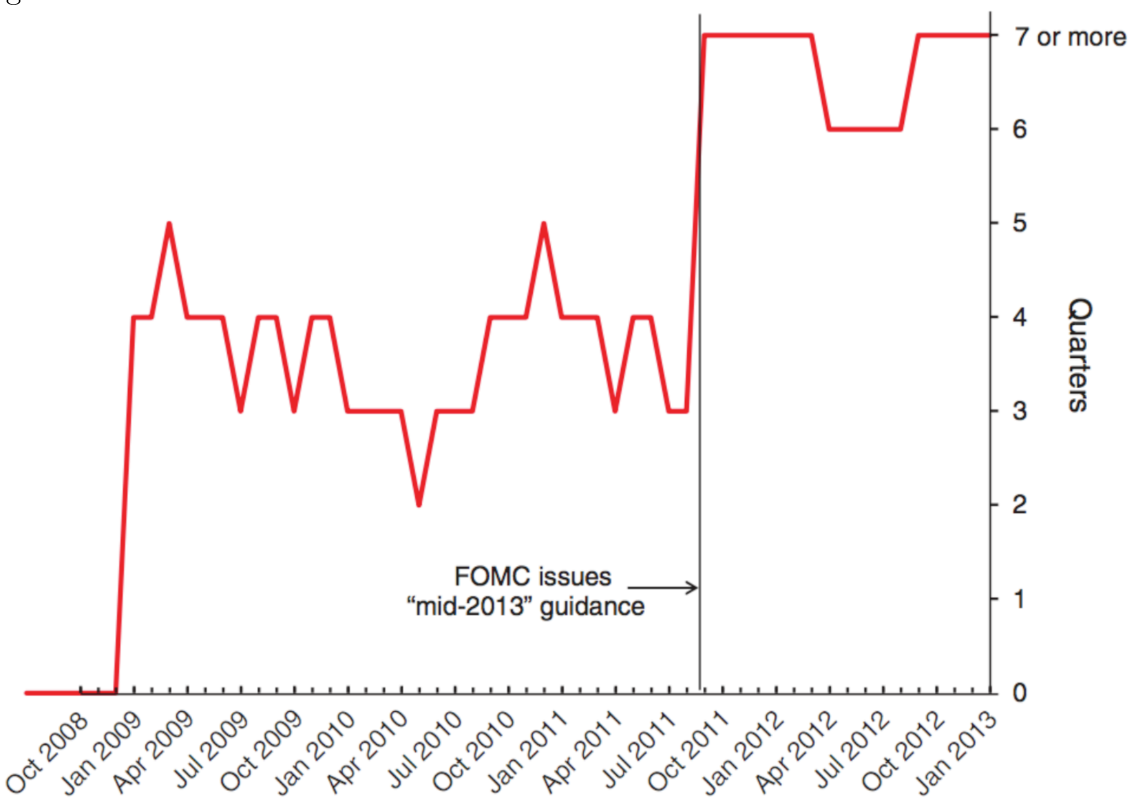
Consistent with the conventional view, the level of interest rates were expected to eventually return to normal levels. Consider, for example, Figure 1, taken from Swanson and Williams (2014). Figure 1 shows that throughout 2009-2011, professional forecasters consistently expected the interest rate to lift off from its lower bound within about 1 year. The forecasts resemble the prediction of Woodford and Eggertsson (2003), according to which the factors that put the economy into the zero lower bound were expected to end according to a Markov chain with constant transition probability.

The results in Figure 1 are top coded at 7 quarters. So, technically that figure is consistent with the view that 2011 monetary policy had become a peg at zero. In fact, other evidence in the Swanson and Williams paper (see, e.g., their Figure 5) suggests that financial market

[1] For a detailed quantitative analysis, see Christiano et al. (2015).

[2] For a discussion of the shocks that caused the Financial Crisis and Great Recession, and why they were not forecast, see Christiano et al. (2018).

Figure 1: Expected Number of Quarters Until First Federal Funds Rate Increase to 25 BP or Higher



Source: Swanson and Williams (2014), Figure 4.

participants continued to expect the interest rate to eventually lift off from its lower bound, after 2011.

It is true that it has taken longer than initially expected for rates to come unstuck from zero, but this provides no reason to doubt the conventional narrative.[3] Nor does Cochrane offer any reason to doubt that narrative.

Cochrane argues that there are other reasons, beyond his Michelson-Morley argument, to reject the NK model. I review three here.

3 The Response to a Permanent Rise in the Nominal Rate of Interest

Cochrane replaces the active monetary policy assumption in the NK model with the assumption that monetary policy is passive. In addition, he introduces an adjusted version of active fiscal policy in the model. In his adjustment, Cochrane replaces the usual assumption of one-period debt with the assumption that the government issues debt of all maturities.

This is an interesting adjustment because it has the effect of causing the time t nominal value of the outstanding debt - which is a state variable in the usual specification - to be a function of time t shocks. The dollar value of the outstanding debt is the dollar price of debt at each maturity, times the quantity of that debt. In each case, the price is the inverse of the nominal rate of interest that has the same duration as the associated debt. Thus, a rise in the rate of interest reduces the market value of the debt in dollar terms by reducing the price of debt at each maturity.[4] If the real interest rate does not change, then the real present value of surpluses does not change either. So, the fiscal theory then predicts a drop in the price level, to ensure that the real value of all outstanding debt remains unchanged. With sticky prices, the predicted price level drop in the adjusted theory becomes a slow fall, or a drop in inflation. Note that this implies a rise in the real rate of interest, creating a fall in the present value of government surpluses. This latter effect lessens the need for a fall in the price level. In the numerical simulations, inflation falls.

A substantial part of the paper is devoted to exploring whether, with the proposed adjustment, the fiscal theory can replicate a conjecture that Cochrane says is widespread among central bankers. The conjecture is that an immediate and permanent rise in the interest rate initially leads to a fall in inflation and output, but eventually causes inflation and the interest rate to rise by the same amount, leaving the real interest rate unchanged.

That a permanent rise in the nominal interest rate would ultimately raise the inflation rate by roughly the same amount is a feature shared by most models. So, the challenge is to see if Cochrane's adjusted fiscal theory model can produce a transitory fall in inflation and output in response to a permanent increase in the nominal rate of interest. In view of

[3]There is a second reason to doubt Cochrane's Michelson-Morley argument. Even if the US had adopted a peg in 2009, the NK model would have predicted only that a sunspot equilibrium was *possible*. The model makes not prediction that a sunspot equilibrium would necessarily have occurred.

[4]I would have liked to have seen more discussion of the mechanics by which the interest rate is changed. In particular, I presume the change would have been produced by an open market sale to the private sector of government debt taken from the central bank's holdings. This would increase the quantity of privately held debt and how can we be sure that the value of that debt would not have increased or stayed the same?

the results in the previous paragraph, it is perhaps not surprising that Cochrane's model can indeed produce the result. Because his version of the fiscal theory is incorporated into a version of the NK model, the transitory rise in the real interest rate causes output to be low during the transitory period of low inflation.

It would be interesting to explore whether the standard NK model can produce similar effects. Presumably, some version of the Erceg and Levin (2003) imperfect credibility argument would work. Thus, suppose that the monetary authority resolves to increase the inflation target permanently, so that with perfect credibility the standard NK model predicts an immediate and equal permanent jump in both inflation and the nominal rate of interest (see the top left panel in Cochrane's Figure 15). What adjustment would be required to get a transitory fall in inflation? One possibility is that when the monetary authority announces the plan to increase the nominal rate of interest, agents believe the increase is only temporary. In that case, the bottom right panel of Figure 15 suggests inflation would move in the opposite direction from the interest rate, i.e., down. The fall in inflation, coupled with the rise in the nominal rate of interest, implies a strong rise in the real rate of interest, which in turn would mean a substantial drop in output.

Why would the announcement of a permanent rise in the interest rate not be credible? One possibility is simply that most interest rate changes are in fact temporary, and permanent shifts are rarely, or never, observed. That such changes in the interest rate are so rarely observed is the reason that it is hard to evaluate the conjecture relative to the data.

But, I agree that Cochrane conducts an interesting model experiment. If someone where to ask me whether a permanent rise in the interest rate would create a short term recession, while leaving the real rate and output unaffected in the long run, I would probably answer 'yes'. If there were a follow-up question about which mechanism is more plausible, the fiscal theory or imperfect credibility, I would probably go with the latter. It is hard for me to let go of the skepticism I feel for the fiscal theory.[5]

4 Equilibrium Selection

4.1 Overview

The equilibrium conditions in the standard NK model with active monetary policy have many solutions. There exists a unique bounded solution that is local to steady state, and this is the one that is typically studied in the literature. In part, the reason for focusing on the locally bounded solution is that, being close to steady state, there is a hope that linearization methods for analyzing it provide acceptable accuracy. In addition, the choice to work with the locally bounded equilibrium reflects a perception that that equilibrium has the appealing characteristic of being learnable (see, e.g., McCallum (2009)).[6] But, Cochrane denies the McCallum result that the unique, locally bounded solution is learnable. If he is right, this would rob the bounded equilibrium of its appeal. The question arises: what is it that keeps the economy in the bounded equilibrium if it cannot be learned? Cochrane's

[5] I elaborate in Christiano and Fitzgerald (2000).

[6] For a more recent analysis, see Evans and McGough (2015).

answer is that the equilibrium exists because of a Central Bank threat to destabilize inflation, if the economy should ever diverge from the equilibrium.

I push back against Cochrane's position in the subsection below. I start by explaining why learnability of an equilibrium is appealing. I then summarize the argument for why the unique bounded solution to the NK model *is* learnable and why Cochrane is uncomfortable with that argument. I find Cochrane's position less than compelling. Moreover, if we interpret the bounded solution as the limit of a learning equilibrium, then there exists a very conventional interpretation for how government policy keeps the economy on track, an interpretation that does not involve destabilizing government interventions. In the course of his analysis, Cochrane also makes conjectures about the relationship between what he calls 'individual learnability' and learning (see especially, Cochrane (2009)). These conjectures seem incorrect to me, but at the very least they would benefit from further clarification.

Cochrane argues that the non-bounded solutions to the equilibrium conditions of the NK models also deserve attention. I do agree with Cochrane, but only up to a point. Cochrane seems to be suggesting that these other equilibria can be studied by analyzing the explosive paths in linearized equilibrium conditions.[7] This is not a problem in the very simple model that Cochrane sometimes uses (see section 4.2.1 below) because that model is linear. But, in models with sticky prices and non-constant consumption, not to mention investment, foreign trade, etc., the nonlinearities in equilibrium conditions are substantial, away from the steady state.[8] As a rule, linearized equilibrium conditions are a poor guide to understanding equilibrium paths outside of a neighborhood of steady state.[9] Moreover, in many cases models have equilibria that are not local to steady state, and which leave no trace at all in equilibrium conditions that have been linearized about steady state.[10] These possibilities and the need for government policy to select among equilibria are being examined closely in the literature. These studies may well end up supporting the current habit of focusing on the bounded equilibrium in the standard NK model. This is because the socially efficient equilibrium in the NK model is not far from the steady state, so that equilibria far away can be suboptimal. Some of the work on equilibria far from steady state involves the design of 'escape strategies' in government policy that act like deposit insurance in the case of bank runs and prevent bad equilibria from forming.[11]

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[7]When Cochrane (2009, p. 1112) makes statements like "the explosive equilibria are learnable", he seems to suggest that the exploding paths predicted by linearized equilibrium conditions should be treated as reasonable approximations to actual equilibria.

[8]See, for example, Christiano et al. (2017), for a discussion of the substantial nonlinearities arising from variable consumption, sticky prices and the lower bound on the interest rate.

[9]Stokey and Lucas (1989, exercise 6.7) provide an example of the limits of linearization methods for studying equilibrium trajectories in which the capital stock leaves the steady state. In the example, the linearized equilibrium conditions correctly characterize these trajectories in a neighborhood of steady state, but then become highly misleading as the actual system settles smoothly into a two-period cycle while the linearized system explodes in an oscillatory pattern towards plus and minus infinity. For examples like this in monetary models, see Christiano and Rostagno (2001) and the references they cite. The point is not that paths which explode away from steady state according to linearized equilibrium conditions are not valid equilibria. The point is that there is nothing to be learned about those equilibria by studying equilibrium conditions that have been linearized around steady state.

[10]For two examples, see Christiano and Harrison (1999) and Christiano et al. (2017).

[11]See, for example, Atkeson et al. (2010), Bassetto (2005), Christiano and Rostagno (2001) and Woodford (2003, sec. 4.3) to name just a few.

4.2 Learning

One motivation for the appeal of learnability is the perspective on rational expectations equilibrium adopted by Lucas (1978, p. 1437).[12] Lucas notes that rational expectations makes very strong assumptions about how much people know and about their willingness and capacity to perform sophisticated computations. He asserts that for rational expectations equilibrium to be interesting, it must be that "...as time passes..." it approximates reasonably well the behavior of actual people who behave quite differently, adopting "...'sensible' rules of thumb, revising these rules from time to time so as to claim observed rents". This is the perspective that motivates the learning literature, which focuses on whether deviations from rational expectations, coupled with common sense assumptions about learning, lead to convergence to a rational expectations equilibrium. When a rational expectations equilibrium has this convergence property, then we say that that equilibrium is *stable under learning*.

Cochrane's sense that the unique, locally bounded equilibrium in the NK model is not learnable greatly reduces the appeal of that equilibrium. It is one of several reasons that motivate his advice that the standard NK model be modified by replacing the assumption of active monetary policy with passive monetary policy and adopting the fiscal theory of the price level. Because the stakes are high and I want to document my claims in section 4.1, I need to review the learning argument, even though it can be found in other places (see, for example, Evans and McGough (2015)). To begin, I first describe the full set of rational expectations solutions to the model. In the second subsection below I turn to learning.

4.2.1 Rational Expectations Equilibrium Under the Taylor Principle, $\phi > 1$

I use the model with constant endowment and flexible prices studied in Cochrane (2009, 2011). The equilibrium conditions of the model are:

$$i_t = E_t \pi_{t+1} \tag{1}$$

$$i_t = \phi \pi_t + e_t \tag{2}$$

$$e_t = \rho e_{t-1} + \varepsilon_t, \; E\varepsilon_t^2 = \sigma_\varepsilon^2, \; 0 < \rho < 1, \tag{3}$$

for $t = 1, 2, ...$. Here, i_t denotes the nominal rate of interest, π_t denotes the inflation rate and e_t denotes a monetary policy shock with given initial condition, e_0. According to equation (1), the nominal rate of interest, adjusted for anticipated inflation, is constant (constant terms are ignored). Equation (2) is the monetary policy rule, where ϕ is the coefficient on inflation. In the standard NK model, the Taylor principle is satisfied:

$$\phi > 1.$$

Equation (3) is the given law of motion of the policy shock.

Applying the argument in Cochrane (2009), I now identify all possible stochastic processes for π_t and i_t that solve equations (1)-(3). Such a stochastic process is a candidate rational expectations equilibrium, and is an actual equilibrium if it satisfies additional equilibrium restrictions such as those implied by non-negativity of the interest rate or household transversality conditions.

[12] See also Evans and Honkapohja (2001).

Substituting out for i_t from (7) into (2), we obtain a single expression in π_t:

$$\pi_{t+1} = \phi\pi_t + e_t + \delta_{t+1}. \tag{4}$$

for $t = 1, 2, ...$. Here, I have used the convenient representation,

$$\pi_{t+1} = E_t\pi_{t+1} + \delta_{t+1},$$

where δ_{t+1} denotes a random variable with the property, $E_t\delta_{t+1} = 0$. Let

$$\psi \equiv \frac{1}{\phi - \rho}. \tag{5}$$

Add ψe_{t+1} to both sides of (4), use (5) and rearrange to obtain

$$z_{t+1} = \phi z_t + v_{t+1}, \tag{6}$$

where

$$z_t \equiv \pi_t + \psi e_t, \ v_t \equiv \psi\varepsilon_t + \delta_t \tag{7}$$

for $t = 1, 2, ...$. Solving (6)

$$z_t = \phi^t z_0 + v_t + \phi v_{t-1} + ... + \phi^{t-1} v_1. \tag{8}$$

A stochastic process that solves the model is defined by a choice of z_0 and of a stochastic process for $\{\delta_t\}$, subject only to $E_t\delta_{t+1} = 0$. A realization of π_t from one of these stochastic processes is constructed in the following way. First, draw a realization of δ_t's from the chosen stochastic process for $\{\delta_t\}$. Then, draw a realization from the stochastic process for the monetary policy innovation, $\{\varepsilon_t\}$. Using (8), we can now compute a realization for $\{z_t, e_t\}$, conditional on z_0 and e_0. Using e_t and z_t, for $t \geq 0$, we can compute a sequence of π_t using (7). We then obtain i_t using (2).

I summarize these observations in the form of a characterization result:

Proposition 1. *A solution to (1)-(3) is characterized by a choice of z_0 and of a stochastic process, $\{\delta_t\}$, with $E_t\delta_{t+1} = 0$.*

It is evident from equation (8) that almost all solutions are explosive. If $z_0 \neq 0$, the first term to the right of the equality in equation (8) explodes. If any $v_t \neq 0$, then z_{t+j} also explodes as j increases. We have the following definition:

Definition 2. A solution is *bounded* if it has the following properties:

$$E_0 z_t \rightarrow_{t\rightarrow\infty} 0$$
$$var_0(z_t) \rightarrow_{t\rightarrow\infty} < \infty.$$

It is evident that there is only one solution, i.e., specification of $(z_0, \{\delta_t\})$, which satisfies boundedness:

Proposition 3. *Suppose $\phi > 1$. The unique bounded solution corresponds to $z_0 = 0$ and $v_t = 0$ for all t and*

$$\delta_t = -\psi\varepsilon_t \tag{9}$$
$$\pi_t = \rho\pi_{t-1} - \psi\varepsilon_t, \tag{10}$$

where $\psi = 1/(\phi - \rho)$.

We refer to the unique bounded solution as a rational expectations equilibrium on the assumption that the additional equilibrium restrictions like non-negativity of the interest rate and transversality are satisfied. Many of the explosive solutions are also rational expectations equilibria.

It is easy to see that a person living in the unique bounded rational expectations equilibrium, possessing an infinite amount of data on π_t and i_t, would have no way to identify the value of ϕ. The autocorrelation of π_t identifies the value of ρ. The variance of the error term, $\psi\varepsilon_t$, in equation (10) involves both ϕ and σ_ε^2, so that neither can be separately identified. This is what Cochrane means when he states that the value of ϕ is not *individually learnable* in the bounded rational expectations equilibrium.

Although the lack of identification of ϕ may at first seem intriguing, it turns out that there is less there than meets the eye, for two reasons. First, lack of individual learnability for ϕ is not a property of NK models generally. An important class of such models adopts a particular recursiveness assumption. In those models it is assumed that the monetary policy shock, e_t, is iid and that the time t values of aggregate variables like output, prices and wages are determined before the time t realization of e_t.[13] Thus, if monetary policy has the Taylor rule representation adopted by Cochrane, then ϕ can be estimated by an ordinary least squares regression and so it is obviously individually learnable.[14]

Second, Cochrane asserts that there is an important link between individual learnability of ϕ and learnability of a rational expectations equilibrium. But, as I show below, there is in fact no such link (see also Evans and McGough (2015, section 4.2)). I turn to learnability in the next subsection.

4.2.2 Is the Bounded Rational Expectations Equilibrium Learnable when $\phi > 1$?

I assume that agents use regression analysis to learn about the stochastic law of motion of the variables in an equilibrium. I suppose that agents begin with an initial set of beliefs

[13]The assumption that e_t is iid is not restrictive in terms of observables, because in practice Taylor rules include the lagged interest rate as a right-hand variable. For analyses that adopt the recursiveness assumption, see , e.g., Sims (1986), Bernanke and Blinder (1992), Rotemberg and Woodford (1997) Christiano et al. (1999), Giannoni and Woodford (2004), Altig et al. (2011) and Christiano et al. (2015).

[14]Cochrane (2011, p. 601) discusses the recursiveness assumption in his review of Rotemberg and Woodford (1997) and Giannoni and Woodford (2004). However, he asserts that the approach is equivalent to assuming that wages, prices and output are fixed one period in advance. This is not actually an implication of the procedure because in principle, it allows non-monetary shocks to have an immediate impact on these variables. Moreover, these other shocks probably account for the lion's share of the variance in variables like wages, prices and output, so that there is substantial one-step-ahead uncertainty in these variables. Papers that have adopted the recursiveness assumption include Sims (1986), Bernanke and Blinder (1992), Christiano et al. (1999), Christiano et al. (2005), Altig et al. (2011) and Christiano et al. (2015).

about how inflation will be determined in the next period. I index their initial beliefs by $l = 0$. Agents maintain their beliefs during a period of time long enough that sampling uncertainty in regression coefficients can be ignored. Their beliefs affect the actual laws of motion of the economy. Eventually, agents pause and collect all data generated since the last time they updated their beliefs. They then run regressions on the data and use the results to update their beliefs. The updated beliefs are indexed by $l + 1$. This process continues indefinitely. I show that the process converges to the unique bounded rational expectations equilibrium.[15]

Following is a formal definition of the learning mechanism. I describe it constructively, according to how actual data in the learning environment are generated. I refer to data generated in this way as a *learning equilibrium*.

Agents' beliefs are described by two parameters, $(\hat{\rho}_l, \hat{\mu}_l)$, which determine the following perceived law of motion used at time t to forecast π_{t+1}:

$$\pi_{t+1} = (1 - \hat{\rho}_l)\,\hat{\mu}_l + \hat{\rho}_l \pi_t + \hat{v}_{l,t+1}, \tag{11}$$

where $\hat{v}_{t+1}$ is treated by agents as an iid process.[16] The subscript l takes on values, $l = 0, 1, 2, \ldots$. I assume that agents can start with essentially any initial belief. I impose only the following restriction on $(\hat{\rho}_0, \hat{\mu}_0)$:

$$\hat{\rho}_0 \neq \phi,$$

where $\phi > 1$. Equation (1), implies:

$$i_t = \hat{E}_t^l \pi_{t+1} = (1 - \hat{\rho}_l)\,\hat{\mu}_l + \hat{\rho}_l \pi_t,$$

where $\hat{E}_t^l$ denotes the expectation, conditional on time t information and beliefs, (11), indexed by l. The realized value of current inflation, π_t, adjusts to satisfy the monetary policy rule, equation (2):

$$(1 - \hat{\rho}_l)\,\hat{\mu}_l + \hat{\rho}_l \pi_t = \phi \pi_t + e_t,$$

or,

$$\pi_t = \frac{(1 - \hat{\rho}_l)\,\hat{\mu}_l - e_t}{\phi - \hat{\rho}_l}. \tag{12}$$

As I show below, excluding the isolated initial belief, $\hat{\rho}_0 = \phi$, ensures that the division in (12) is well defined for all $l = 0, 1, \ldots$.

Agents adhere to their beliefs, $\hat{\rho}_l$ and $\hat{\mu}_l$, for many periods. Then they stop to update their beliefs using all the data generated since the previous update. Their updated beliefs, $\hat{\rho}_{l+1}$ and $\hat{\mu}_{l+1}$ are the parameters of the actual law of motion for π_t induced by their perceived

[15] The learning mechanism I use is the same as the mapping from perceived to actual laws of motion studied in, for example, Evans and McGough (2015). A difference is that I assume the process proceeds in calendar time, with agents running regressions on observed data. Evans and McGough (2015) assume the learning process proceeds in 'notional time', presumably implemented by the agents themselves. The mathematics of the two approaches are the same. However, my calendar time approach allows me to make observations on the individual learnability of ϕ that I could not make if I adopted the Evans and McGough (2015) approach. The latter in effect assume that agents generate the 'observed' data in their heads. This requires that agents know the values of all structural parameters, something that my approach does not require.

[16] When agents perform regressions (see below) they will never see evidence that contradicts the iid assumption.

law of motion. The actual law of motion is obtained by multiplying π_t in (12) by $(1-\rho L)$, where L denotes the lag operator:

$$\pi_t = \rho\pi_{t-1} + (1-\rho)\frac{(1-\hat{\rho}_l)\,\hat{\mu}_l}{\phi-\hat{\rho}_l} - \frac{\varepsilon_t}{\phi-\hat{\rho}_l}. \tag{13}$$

The parameters, $(\hat{\mu}_{l+1}, \hat{\rho}_{l+1})$, of the new perceived law of motion are taken from the actual law of motion in equation (13). That is:

$$\hat{\mu}_{l+1} = \frac{1-\hat{\rho}_l}{\phi-\hat{\rho}_l}\hat{\mu}_l,\;\; \hat{\rho}_{l+1} = \rho. \tag{14}$$

Evidently, agents' beliefs about ρ converge in one step, with $\hat{\rho}_1 = \rho$. It takes more iterations for beliefs about μ to converge. Still, eventually they learn the true value of μ, which is zero, because, since $\phi > 1$,

$$0 \leq \frac{1-\rho}{\phi-\rho} < 1.$$

We have the following definition:

Definition 4. Consider a learning equilibrium with $(\hat{\rho}_0, \hat{\mu}_0)$ arbitrary, except that $\hat{\rho}_0 \neq \phi$. A rational expectations equilibrium is least squares learnable if the actual laws of motion in the learning equilibrium converge, as $l \to \infty$, to the laws of motion in the rational expectations equilibrium.

Since $(\hat{\mu}_l, \hat{\rho}_l) \to (0, \rho)$, where (10) is satisfied, we have:

Proposition 5. *The unique bounded solution under the Taylor principle is least squares learnable.*

Evans and McGough (2015) establish a result that is stronger in some respects than the one in Proposition 14. Their results apply when the perceived law of motion is a finite order autoregression, rather than the first order autoregression considered here. However, their result is 'local' in the sense that it assumes the initial perceived law of motion is inside a small neighborhood of the actual rational expectation law of motion. Evans and McGough (2015, Theorem 1) also obtain results for a subset of the non-bounded solutions to equations (1)-(3). The solutions they consider are the ones in which the time series representation of π_t in the actual solution is a second order autoregression with roots ϕ and ρ. In particular, they allow $z_0 \neq 0$, but they set $v_t = 0$ for all t in ((8)).[17] For their perceived laws of motion, they consider N^{th} order autoregressions, with $N \geq 2$. Their learning mechanism is the analog of the one used above, and they obtain a local non-stability result: the perceived law of motion diverges from the rational expectations law of motion when it starts in a small neighborhood of that law of motion.

The opening sentence of section 5 in Cochrane (2009) suggests, somewhat surprisingly, that he is aware of the argument underlying Proposition 5. Cochrane states that arriving at Proposition 5 is "easy", but nevertheless concludes that to accept the proposition "...is

[17] Allowing for $v_t \neq 0$ complicates the analysis by converting the iid error term in the autoregressive representation of π_t into a first oder moving average.

a mistake." Cochrane (2009) reports two reasons why it would be a mistake to accept Proposition 5. First, he says, "Agents must know that alternative equilibria will lead to explosions," but it was not clear to me how this justifies rejecting Proposition 5. Second, Cochrane (2009) says at the bottom of p. 1112, "And it deeply begs the question, how does the 'equilibrium' of a new Keynesian model work, in which agents are forecasting based on the same π_t that is being determined?". In effect, the forecasting rule converts the Fisher equation, (1), and the monetary policy rule, (2), into two simultaneous equations in the two unknowns, π_t and i_t. Perhaps I have spent too many years staring at equations of demand and supply, but I have a hard time feeling squeamish about equilibrium objects that are the solution to two simultaneous equations.

Not all the i's have been dotted and t's crossed yet, in the analysis of learning. But, it looks like the bounded rational expectations equilibrium is learnable and the other equilibria are not, when $\phi > 1$. That is, if we require that for an equilibrium to be interesting, it must be learnable, there is only one interesting equilibrium for the model in equations ((1))-(3).

4.2.3 Individual Learnability of ϕ When $\phi > 1$

Next, we turn to the individual learnability of ϕ in the learning equilibrium. Learning adds dynamics beyond what occurs in the rational expectations equilibrium, which can be exploited by agents interested in knowing the value of ϕ.

Consider the following definition:

Definition 6. A parameter is individually learnable in a learning equilibrium, if an agent can recover that parameter's value from observations in the learning equilibrium.

It is easy to see that in the rational expectations equilibrium with learning, the parameter, ϕ, is individually learnable, as in Definition 6. As before, let $\hat{\rho}_l, \hat{\mu}_l$, for $l = 0$ correspond to the initial beliefs about ρ and μ. Then, by (14) we have $\hat{\rho}_l = \rho$ for $l \geq 1$, and

$$\frac{\hat{\mu}_{l+1}}{\hat{\mu}_l} = \frac{1-\rho}{\phi-\rho},$$

for $l \geq 1$. Thus, we have, for $l \geq 1$,

$$\phi = \frac{1-\rho}{\frac{\hat{\mu}_{l+1}}{\hat{\mu}_l}} + \rho.$$

We conclude:

Proposition 7. *Consider a learning equilibrium with $(\hat{\rho}_0, \hat{\mu}_0)$ arbitrary, except that $\hat{\rho}_0 \neq \phi$. The parameter, ϕ, is individually learnable from data in the equilibrium.*

Cochrane argues that under-identification of ϕ in a bounded rational expectations equilibrium inhibits learning. This is not true. In fact, the bounded rational expectations equilibrium is learnable and the value of ϕ is learnable in that equilibrium.

4.2.4 How the Taylor Principle Guides Inflation Expectations When $\phi > 1$

In section 5.4 Cochrane discusses his view about how the Taylor principle works to stabilize inflation. He argues that the mechanism is completely different from the way it is described in undergraduate textbooks. There, higher inflation expectations lead to a rise in the interest rate which then moderates actual inflation by reducing aggregate demand. Cochrane (see especially Cochrane (2009, p. 1113)) argues that in the rational expectations equilibrium the Taylor principle works very differently, by threatening to destabilize the economy if people choose the 'wrong' inflation rate (note the explosion created in (8) by $\phi > 1$ if $z_0 \neq 0$). I agree with Cochrane that it is highly improbable that this is the way monetary policy affects a real-world economy. I do not know if Cochrane is right in his characterization of the rational expectations equilibrium.

But, he is definitely not right if we interpret the bounded rational expectations equilibrium as the tail end of a learning equilibrium. The conventional mechanism actually works reasonably well in the learning equilibrium. Suppose, for example, that expected inflation, $\hat{\mu}_l$, jumps in (14) for some l. The Taylor principle, $\phi > 1$, means that $\hat{\mu}_{l+1}$ rises by less than $\hat{\mu}_l$ does and subsequent expectations of inflation gradually return down to what they would have been, had $\hat{\mu}_l$ not jumped. The fact, $\phi > 1$, inserts a stationary root in the mechanism, promoting a return to the bounded rational expectations equilibrium if a perturbation to beliefs were to bump it off. No explosion or other drama.

5 Alleged Implausible Implications of the NK Model When Lower Bound on Nominal Rate of Interest is Binding

Cochrane suggests that some of the predictions made by the NK model when the lower bound on the nominal rate of interest is binding are probably counterfactual. He is particularly concerned with the model's implications that (i) good technology shocks reduce output and (ii) greater price flexibility imply a larger drop in output.

The proposition that (i) is an implication of the NK model when the lower bound is binding is too simple, to the point of being misleading. That proposition only applies when technology shocks are sufficiently temporary that the wealth effect can be ignored. For example, Christiano et al. (2014) show that when technology shocks are characterized by the degree of persistence assumed in the real business cycle literature, then a positive technology shock raises output. The intuition for this can be found in the interplay between the rate of return and wealth effects triggered by a technology shock. To understand the rate of return effect, recall that, other things the same, a positive technology shock reduces the marginal cost of production, placing downward pressure on inflation. With a binding lower bound on the interest rate, this raises the real rate of interest, reducing consumption and output.

Property (i) would hold if the rate of return effect were the whole story. But, when a positive technology shock is persistent, then there is also a wealth effect. Other things the same, the wealth effect stimulates consumption and output. When the log level of technology in the NK model has a scalar first order autoregressive representation with autocorrelation, 0.95, as in the real business cycle (RBC) literature, then the wealth effect dominates the rate of return effect in the NK model without capital (see Christiano et al. (2015)).

The empirical evidence favors the notion that technology shocks have persistent effects. An early analysis supporting this view appears in Prescott (1986). Updated data on US total factor productivity growth reported in Fernald (2014) has first order autocorrelation 0.20. This represents even more persistence than is assumed in the RBC literature. The nuclear disaster at Fukushima, Japan, is sometimes viewed as a example of a negative technology shock. That shock is best thought of as persistent because of the deep distrust in nuclear power that it spawned in the Japanese public. Finally, the literature on actual movements of technology reports a great deal of persistence. Technological improvements display a diffusion property, whereby an initial jump is followed by further increases (see .

Christiano et al. (2011) argue that property (ii) of the NK model is not as counterintuitive as it may seem at first. They draw attention to the work of De Long and Summers (1986), who argue that the idea is implicit in conventional macroeconomic views dating back at least to the 1920s.

6 Conclusion

In my discussion, I have pushed back against some of the objections raised by Cochrane against the standard NK model. By doing this, I do not mean to suggest that that model does not require further work. For example, much more work is required to fully understand the equilibrium multiplicity problems. I described some initial steps, but many more are required before we get to the bottom of that issue. Many other model features deserve close attention. For example, it is important to incorporate a structural interpretation of the reduced form price stickiness assumption. The forward guidance puzzle also deserves close attention.[18] A full list of important questions to investigate in the NK model is too long to describe here (see Christiano et al. (2018)).

The experiment that Cochrane explores in his paper, to study the effects of a permanent rise in the interest rate, is an interesting and important one. It is not an experiment for which we have data, but it is nevertheless relevant for policy discussions at this time. It is exactly the experiments for which we do not have historical evidence that must be done in models.

[18]See Farhi and Werning (2017), Angeletos and Lian (2017) and Mckay et al. (2017) for examples of papers that explore the problem.

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