Involuntary (Unlucky) Unemployment and the Business Cycle

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Background

• New Keynesian (NK) models receive lots of attention in central banks.

• People use these models to place structure on discussions about monetary policy.
  – Recent: Curdia-Woodford, Gertler-Kiyotaki.

• In recent years, push to introduce labor market variables like unemployment.
Why Unemployment?

• Viewed by some as a direct observation on the efficiency with which resources are used (output gap).
  – With a theory of unemployment, can investigate this in a structured way.

• Because of imperfect labor market insurance, unemployment viewed as an indicator of welfare cost of business cycles.
  – Can assess general presumption that Lucas cost of business cycles conclusion not robust to assumption of imperfect labor market insurance.
What We Do:

• Investigate a particular approach to modeling unemployment.

• Explore the implications for monetary equilibrium (DSGE) models.
  – Simple NK model without capital (CGG)
  – Standard empirical NK model (e.g., CEE, ACEL, SW)
    • Estimate the model.
    • Does well reproducing response of unemployment and labor force to three identified shocks.
Unemployment

• To be ‘unemployed’ in US data, must
  – be ‘willing and able’ to work.
  – recently, made concrete efforts to find a job.

• Our presumption: a person has lower utility when unemployed than when employed.
  – Some indicators of utility (health, suicide, subjective sense of well being, kids’ well being) deteriorate when people experience unemployment
Suicide and Unemployment in Japan


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• Current monetary DSGE models with ‘unemployment’:
  – Unemployed are the lucky ones.
  – Finding a job requires no effort.
  – US Census Bureau employee dropped into current monetary DSGE models would find zero unemployment.
What we do:

• We explore a simple prototype model of unemployment, which has two key features.

• Probability, $p(e)$, of securing a job is increasing in effort, $e$.
  – Essential component of empirical measure of unemployment

• Unemployed worse off than employed.
  – assume household search effort, $e$, is not publicly observable.

  – full insurance against household labor market outcomes is not possible.
  • under perfect consumption insurance, no one would make an effort to secure a job.
Outline

• Simple Clarida-Gali-Gertler (CGG) NK model
  – Information in unemployment on the output gap.

• CEE model
  – Evaluate model’s ability to match US macroeconomic data, including unemployment and labor force
  – No loss on standard macro variables.
  – Do fine on unemployment rate and labor force.
Clarida-Gali-Gertler Model

- **Goods Production:** \[ Y_t = \left[ \int_0^1 Y_{i,t}^{\lambda_f} \, di \right]^{\lambda_f}, \ 1 \leq \lambda_f < \infty. \]

- **Monopolists produce intermediate goods**
  - Technology: \[ Y_{i,t} = A_t h_{i,t} \]
  - Calvo sticky prices:
    \[ P_{i,t} = \begin{cases} P_{i,t-1} \text{ with prob. } \xi_p \\ \text{chosen optimally} \text{ with prob. } 1 - \xi_p \end{cases} \]
  - Enter competitive markets to hire labor.

- **Policy:** \[ \hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R) [r_\pi \hat{\pi}_t + r_y \hat{x}_t] + \epsilon_t \]
Households

• This is where the new stuff is...
Typical Household During Period

Draw privately observed, idiosyncratic shock, \( l \), from Uniform, \([0, 1]\), that determines utility cost of work:

\[
\begin{align*}
\log c_{t+1} &= \text{ex post utility in case household finds a job} \\
&= \log c_t - F - \alpha - l \alpha \\
\log c_t &= \text{ex post utility in case of unemployment} \\
&= \log c_{t-1} - \frac{1}{2} e_t^2 - \beta e_t \\
\end{align*}
\]
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Draw privately observed, idiosyncratic shock, \( l \), from Uniform, \([0, 1]\), that determines utility cost of work:

\[
F + \zeta_t (1 + \sigma_L) l^\sigma_L .
\]

After observing decide whether to join

Household that stays out of labor market does not work and has utility

\[
\log c_t \quad \text{out of labor force}
\]

After observing, decide whether to join the labor force or stay out.

Household that joins labor force tries to find a job by choosing effort, \( e_t \), and receiving

ex ante utility in case household finds a job

\[
\log ct w - F - \alpha L - 1^{\alpha L} - 2 e_t + \zeta_t/e - \zeta_t/(1 - p_\zeta)
\]

ex post utility in case of unemployment

\[
\log ct u - 1^{\alpha L} - 1^{\alpha L} - 2 e_t
\]

\[
p_\zeta = \mathcal{U} + \alpha et
\]
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Household that joins labor force tries to find a job by choosing effort, \( e \), and receiving ex ante utility

\[
p(e_t) \left[ \log(c_t^w) - F - \zeta_t (1 + \sigma_L) l^{\sigma_L} - \frac{1}{2} e_t^2 \right] + (1 - p(e_t)) \left[ \log(c_t^u) - \frac{1}{2} e_t^2 \right] = \eta + ae_t
\]
Household Insurance

• They need it:
  – Idiosyncratic work aversion.
  – Job-finding effort, $e$, may or may not produce a job.

• Assume households gather into large families, like in Merz and Andolfatto
  – With *complete* information:
    • Households with low work aversion told to make big effort to find work.
    • All households given same consumption.
    • *Not* feasible with private information.
  – With *private* information:
    • To give households incentive to look for work, must make them better off in case they find work.
    • Trade-off between incentives and insurance.
Optimal Insurance

• Relation of family to household: standard principal/agent relationship.
  – family receives wage from working households
  – family observes current period employment status of household.

• For family with given $C, h$:
  – allocates consumption: $c_t^w, c_t^{nw}$
  – $c_t^w/c_t^{nw}$ big enough to incentivize $h$.
  – must satisfy family resource constraint:
    \[ h_t c_t^w + (1 - h_t) c_t^{nw} = C_t. \]
Family Indirect Utility Function

- Utility: \( u(C_t, h_t, \xi_t) = \log(C_t) - z(h_t, \xi_t) \)
Family Indirect Utility Function

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• Where

\[
z(h_t, \zeta_t) = \log[h_t(e^{F+\zeta_t(1+\sigma_L)f(h_t,\zeta_t)^{\sigma_L}} - 1) + 1] \\
- \frac{a^2 \zeta_t^2 (1 + \sigma_L) \sigma_L^2}{2\sigma_L + 1} f(h_t, \zeta_t)^{2\sigma_L+1} - \eta \zeta_t \sigma_L f(h_t, \zeta_t)^{\sigma_L+1} .
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• Clarida-Gali-Gertler utility function:

\[
u(C_t, h_t, \zeta_t) = \log(C_t) - \zeta_t h_t^{1+\sigma_L} \]
Family Problem

\[ \max_{\{C_t,h_t,B_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t [\log(C_t) - z(h_t, \zeta_t)] \]

– Subject to:

\[ P_tC_t + B_{t+1} \leq B_tR_{t-1} + W_th_t + Transfers \text{ and profits}_t. \]
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– Subject to:

$$P_tC_t + B_{t+1} \leq B_tR_{t-1} + W_th_t + \text{Transfers and profits}_t.$$ 

- Family takes market wage rate as given and tunes incentives so that marginal cost of extra work equals marginal benefit:

$$C_tz_h(h_t, \zeta_t) = \frac{W_t}{P_t}.$$
Reduced Form Properties

• Model is observationally equivalent to standard NK model, when represented in terms of output, interest rate, inflation:

\[
\begin{align*}
\hat{\eta}_t &= \hat{\eta}_{t+1} + \alpha z_t x_t - \eta_{t+1} - R_t - \eta_{t+1} - R_t^*.
\end{align*}
\]
Reduced Form Properties

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\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{(1-\beta \xi_p)(1-\xi_p)}{\xi_p} (1 + \sigma_z) \hat{x}_t
\]

\[
\hat{x}_t = E_t \hat{x}_{t+1} - (\hat{R}_t - \hat{\pi}_{t+1} - \hat{R}_t^*).
\]

\[
\hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R)[r_{\pi} \hat{\pi}_t + r_y \hat{x}_t] + \varepsilon_t,
\]
Reduced Form Properties

\[ z \text{ function: disutility of labor for family} \]

‘curvature of disutility of labor’: \[ \sigma_z \equiv \frac{z_{hh} h}{z_h} \]

\[ \hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{(1-\beta \xi_p)(1-\xi_p)}{\xi_p} (1 + \sigma_z) \hat{x}_t \]

\[ \hat{x}_t = E_t \hat{x}_{t+1} - \left( \hat{R}_t - \hat{\pi}_{t+1} - \hat{R}_t^* \right). \]

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Unemployment Gap

• Can express everything in terms of unemployment gap:

\[ u_t^g = -\kappa^{okun} \hat{\chi}_t. \]
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\[ \kappa^{okun} = \frac{a^2 \zeta \sigma_L^2 m^{\sigma_L} (1 - u)}{1 - u + a^2 \zeta \sigma_L^2 m^{\sigma_L}} > 0. \]

actual rate of unemployment \hspace{1cm} \text{efficient level of unemployment}

\[ u_t^g = \underbrace{u_t}_\text{ efficient level of unemployment} - \underbrace{u_t^*_t}_\text{ actual rate of unemployment} \]
Unemployment Gap

• Can express everything in terms of unemployment gap:

\[ u^g_t = -\kappa^{okun} \hat{x}_t. \]

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\[ u^g_t = \underbrace{u_t}_\text{actual rate of unemployment} - \underbrace{u^*_t}_\text{non-accelerating rate of inflation level of unemployment, NAIRU} \]
Properties of the Model

• Calibrated model first...
Calibration of the Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>1.03^{-0.25}</td>
<td>Discount factor</td>
</tr>
<tr>
<td>$g_A$</td>
<td>1.0047</td>
<td>Technology growth</td>
</tr>
<tr>
<td>$\xi_p$</td>
<td>0.75</td>
<td>Price stickiness</td>
</tr>
<tr>
<td>$\lambda_f$</td>
<td>1.2</td>
<td>Price markup</td>
</tr>
<tr>
<td>$\rho_R$</td>
<td>0.8</td>
<td>Taylor rule: interest smoothing</td>
</tr>
<tr>
<td>$r_\pi$</td>
<td>1.5</td>
<td>Taylor rule: inflation</td>
</tr>
<tr>
<td>$r_y$</td>
<td>0.2</td>
<td>Taylor rule: output gap</td>
</tr>
<tr>
<td>$\eta_g$</td>
<td>0.2</td>
<td>Government consumption share on GDP</td>
</tr>
</tbody>
</table>

To parameterize preference and search function, set:

- labor force participation rate: $m=0.67$
- employment rate: $h=0.63$
- unemployment rate: $u=0.056$
Properties

• Replacement ratio

\[ \frac{c^{nw}}{c^w} = 0.18 \]

– Very low! In model with habit persistence in preferences, replacement ratio = 0.80.
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• Cost of business cycles (in % of consumption)
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Limited Information Model    Full Information Model

Technology Shock Only

0.52% 0.57%

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Properties

- **Replacement ratio**
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- **Cost of business cycles (in % of consumption)**

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<tr>
<td>Technology Shock Only</td>
<td></td>
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<tr>
<td>0.52%</td>
<td>0.57%</td>
</tr>
<tr>
<td>Government Spending Shock</td>
<td></td>
</tr>
<tr>
<td>0.11%</td>
<td>0.13%</td>
</tr>
<tr>
<td>Monetary Policy Shock Only</td>
<td></td>
</tr>
<tr>
<td>0.07</td>
<td>0.10</td>
</tr>
</tbody>
</table>
Using Unemployment to Estimate the Output Gap

• CTW, ‘Handbook chapter’

• **Estimated Simple CGG model:**
  – 2 observables: real GDP per capita growth, unemployment rate
  – 4 shocks: technology, labor supply, Phillips curve, monetary policy
  – Calibrate economic parameters
  – Estimate parameters of exogenous processes using: i) Bayesian moment matching estimation and ii) Bayesian full information estimation
Using Unemployment to Estimate the Output Gap

• What is the information content of the unemployment rate for the output gap?

• Experiment:
  – Examine impact of including unemployment on estimate of the output gap for the US economy.
Put our theory of unemployment into a medium-sized DSGE Model

- Habit persistence in preferences.
- Variable capital utilization.
- Investment adjustment costs.
- Wage setting frictions as in Erceg-Henderson-Levin.
- Impulse-response matching by Bayesian methods:
  - prices reoptimized on average every 2.7 quarters.
  - wages reoptimized on average every 4 quarters.
Finding

• Estimate VAR impulse responses of macro variables to two technology shocks and monetary policy shock (following Fisher and ACEL).

• Results:
  
  – Performance on standard macro variables same as CEE and ACEL.

  – Performance on unemployment and the labor force is fine.
Figure 4: Dynamic Responses of Labor Market Variables to Three Shocks

Unemployment Rate

Monetary Shock

Labor Force

Neutral Tech. Shock

Labor Force

Invest. Tech. Shock

VAR 95%  VAR Mean  Involuntary Unemployment Model
Micro Implications of Model

• Model: consumption premium higher in booms.
  – Have time series evidence on cross-household variance, \( V \), of log consumption.
  – Heathcote, Perri and Violante (2010) show \( V \) goes down in three of past 5 recessions.

\[
V_t = (1 - h_t) h_t \left( \log \left( \frac{c_t^w}{c_t^{nw}} \right) \right)^2.
\]

• Model: search intensity lower in recessions
  – Consistent with evidence on ‘discouraged workers’
Conclusion

• Integrated a model of involuntary (unlucky) unemployment into monetary DSGE model.

• Results:
  – Obtained a theory of Okun’s gap, NAIRU.
  – Able to match responses of unemployment and labor force to macro shocks.
  – Raises several further empirical questions.

• Why introduce unemployment?
  – A policy variable of direct interest.
  – Useful for estimating output gap (CTW, ‘handbook chapter’).
  – By bringing in more data, get a better read on unobserved shocks and may improve forecasts.