Involuntary Unemployment and the Business Cycle*

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July 14, 2018

Abstract

Can a model with limited labor market insurance explain standard macro- and labor market data *jointly*? We seek to construct a monetary model in which: i) the unemployed are worse off than the employed, i.e. unemployment is involuntary and ii) the labor force participation rate varies with the business cycle. To illustrate key features of our model, we start with the simplest possible New Keynesian framework with no capital. We then integrate the model into a medium sized DSGE model and show that the resulting model does as well as existing models at accounting for the response of standard macroeconomic variables to monetary policy shocks and two technology shocks. In addition, the model does well at accounting for the response of the labor force and unemployment rate to these three shocks.

Keywords: DSGE, unemployment, labor force participation, business cycles, monetary policy, Bayesian estimation.
JEL codes: E2, E3, E5, J2, J6

*Many people and seminar participants provided us with excellent comments for which we are very grateful. The views expressed in this paper are those of the authors and do not necessarily reflect those of Sveriges Riksbank.

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1. Introduction

Can a model with limited labor market insurance explain standard macro- and labor market data *jointly*? To answer this question, we seek to construct a monetary model in which: i) the unemployed are worse off than the employed, i.e. unemployment is involuntary and ii) the labor force participation rate varies with the business cycle. We investigate whether the resulting model fits standard real and nominal macro data and unemployment and labor force participation data in response to monetary policy and technology shocks.[1]

Recently, the unemployment rate and the labor force participation rate have been discussed prominently in the light of the Great Recession. A shortcoming of standard monetary dynamic stochastic general equilibrium (DSGE) models is that they are silent about these important variables. Work has begun on the task of introducing unemployment into monetary DSGE models. The Diamond-Mortensen-Pissarides search and matching approach of unemployment represents a leading framework and has been integrated into monetary models by a number of authors.[2]

However, the approaches taken to date have several important shortcomings. First, they assume the existence of perfect consumption insurance against labor market outcomes, so that consumption is the same for employed and non-employed households. With this kind of insurance, a household is delighted to be unemployed because it is an opportunity to enjoy leisure without a drop in consumption.[3] In other words, unemployment in these models is voluntary rather than involuntary. Second, it is generally assumed that labor force participation is constant and exogenous. This assumption is at odds with the business cycle properties of the labor force participation rate, especially in the recent downturn.[4] Moreover,

[1] We are interested in a monetary environment since it allows us to study the general equilibrium repercussions between e.g. unemployment, inflation and nominal interest rates. In addition, monetary models such as Christiano, Eichenbaum and Evans (2005, CEE) and Altig, Christiano, Eichenbaum and Linde (2004, ACEL) have proved to be useful to account for VAR-based evidence for real and nominal variables in response to monetary as well as technology shocks. The model features developed in CEE and ACEL have become standard ingredients in modern business cycle models, see e.g. Smets and Wouters (2003, 2007) and many others. Integrating our model of unemployment into such an environment therefore provides a useful empirical test for our approach to the labor market in general.

[2] Examples include Blanchard and Galí (2010), Christiano, Ilut, Motto and Rostagno (2008), Christiano, Trabandt and Walentin (2011b), Christoffel, Costain, de Walque, Kuester, Linzert, Millard, and Pierrard (2009), Christoffel, and Kuester (2008), Christoffel, Kuester and Linzert (2009), den Haan, Ramey and Watson (2000), Gertler, Sala and Trigari (2009), Groshenny (2009), Krause, Lopez-Salido and Lubik (2008), Lechthaler, Merkl and Snower (2009), Sala, Söderström and Trigari (2008), Sveen and Weinke (2008, 2009), Thomas (2008), Trigari (2009) and Walsh (2005).

[3] The drop in utility reflects that models typically assume preferences that are additively separable in consumption and labor or that have the King, Plosser, Rebelo (1988) form. Examples include all papers cited in the previous footnote.

[4] According to the CPS, the labor force participation rate has fallen by 3% in the relevant time period, from a peak of 66.4% in January 2007 to 63.6% in April 2012 (these numbers refer to population 16 years and over, seasonally adjusted).

it also appears important to restrict our models to be consistent with the endogenous choice of agents whether or not to participate in the labor market, see e.g. Veracierto (2008).[5]

To remedy these limitations, we pursue an approach to model the labor market that has not been used in the monetary DSGE literature. We believe that the approach – which we lay out in detail below – is interesting since the theory of unemployment developed here has the implication that the unemployed are worse off than the employed. Our approach follows the work of Hopenhayn and Nicolini (1997) and others, in which finding a job requires exerting a privately observed effort.[6] In this type of environment, the higher utility enjoyed by employed households is necessary for people to have the incentive to search for and keep jobs.[7] Moreover, our approach implies that households take an optimal decision whether or not to join the labor force. In other words, the labor force participation margin in our framework responds endogenously to business cycle shocks.

We define unemployment the way it is defined by the agencies that collect the data. To be officially unemployed a person must assert that she (i) has recently taken concrete steps to secure employment and (ii) is currently available for work.[8] To capture (i) we assume that people who wish to be employed must undertake a costly effort. Our model has the implication that a person who asserts (i) and (ii) enjoys more utility if she finds a job than if she does not, i.e., unemployment is involuntary. Empirical evidence appears to be consistent with the notion that unemployment is in practice more of a burden than a blessing.[9] For example, Chetty and Looney (2006) and Gruber (1997) find that US households suffer roughly a 10 percent drop in consumption when they lose their job. Also, there is a substantial literature which purports to find evidence that insurance against labor market outcomes is imperfect. An early example is Cochrane (1991). These observations motivate our third

[5] When allowing for endogenous participation, Veracierto (2008) finds that the canonical Diamond-Mortensen-Pissarides search model implemented in an RBC setting counterfactually implies i) procyclical unemployment and ii) labor force participation that is almost perfectly correlated with GDP.

[6] An early paper that considers unobserved effort is Shavell and Weiss (1979). Our approach is also closely related to the efficiency wage literature, as in Alexopoulos (2004). Our work is also related to Landais, Michaillat and Saez (2012) who study the cyclicality of optimal unemployment insurance in a real model with imperfect labor market insurance. However, the authors barely spell out the macro implications of their approach. In contrast to these authors, we study the implications of limited labor market insurance in a monetary model and, more importantly, examine the ability of the approach to explain actual macro- and labor market data in response to technology and monetary policy shocks quantitatively.

[7] Lack of perfect insurance in practice probably reflects other factors too, such as adverse selection. Alternatively, Kocherlakota (1996) explores lack of commitment as a rationale for incomplete insurance. Lack of perfect insurance is not necessary for the unemployed to be worse off than the employed (see Rogerson and Wright, 1988).

[8] See the Bureau of Labor Statistics website, http://www.bls.gov/cps/cps_htgm.htm#unemployed, for an extended discussion of the definition of unemployment, including the survey questions used to determine a household's employment status.

[9] There is a substantial sociological literature that associates unemployment with an increased likelihood of suicide and domestic violence.

defining characteristic of unemployment: (iii) a person looking for work is worse off if they fail to find a job than if they find one.[10]

To highlight the mechanisms in our model, we first introduce it into the simplest possible DSGE framework, the model presented by Clarida, Galí and Gertler (1999) (CGG). The CGG model has frictions in the setting of prices, but it has no capital accumulation and no wage-setting frictions. In our model, households gather into "families" for the purpose of partially ensuring themselves against bad labor market outcomes. We regard the "family" as a label or stand-in for all the various market and non-market arrangements that actual households have for dealing with idiosyncratic labor market outcomes.[11] In line with this view of the family, households are assumed to have no access to loan markets, while families have access to complete markets.

Each household experiences a privately observed shock that determines its aversion to work. Households that experience a sufficiently high aversion to work stay out of the labor force. The other households join the labor force and are employed with a probability that is an increasing function of a privately observed effort. The only thing about a household that is observed is whether or not it is employed. Although consumption insurance is desirable in our environment, perfect insurance is not feasible because everyone would claim high work aversion and stay out of the labor force.

For simplicity we suppose the wage rate is determined competitively so that firms and families take it as given.[12] Firms face no search frictions and hire workers up to the point where marginal costs and benefits are equated. But it is important to note that our modelling approach in principle could encompass these two elements, and that the friction that we emphasize – households have to make a job finding effort which is unobservable – might well be viewed as a complement to the currently dominating paradigm of wage bargaining and vacancy posting costs. At this point it is worth emphasizing that unemployment in our model is purely frictional. It is not generated by unions or other factors pushing up the

[10] Although all the monetary DSGE models that we know of fail (iii), they do not fail (ii). In these models there are workers who are not employed and who would say 'yes' in response to the question, 'are you currently available for work?'. Although such people in effect declare their willingness to take an action that reduces utility, they would in fact do so. This is because they are members of a large family insurance pool. They obey the family's instruction that they value a job according to the value assigned by the family, not themselves. In these models everything about the individual household is observable to the family, and it is implicitly assumed that the family has the technology necessary to enforce verifiable behavior. In our environment - and we suspect this is true in practice - the presence of private information makes it impossible to enforce a labor market allocation that does not completely reflect the preferences of the individual household.

[11] Alternative labels in this regard would be "a zero profit insurance company", "a social planner" or "a representative agent".

[12] One interpretation of our environment is that job markets occur on Lucas-Phelps-Prescott type islands. Effort is required to reach those islands, but a person who arrive at the island finds a perfectly competitive labor market. For recent work that uses a metaphor of this type, see Veracierto (2008).

general wage level to a point where supply exceeds demand. However, note too that our environment is flexible enough to allow for market power on labor markets as will be the case in the estimated medium-sized DSGE model, see section 4.

Although individual households face uncertainty as to who will work and who will not, families are sufficiently large that there is no uncertainty at the family level. Once the family sets incentives by allocating more consumption to employed households than to non-employed households, it knows exactly how many households will find work. The family takes the wage rate as given and adjusts employment incentives until the marginal cost (in terms of foregone leisure and reduced consumption insurance) of additional market work equals the marginal benefit. The firm and family first order necessary conditions of optimization are sufficient to determine the equilibrium wage rate.

Our theory of unemployment has interesting implications for the optimal variation of labor market insurance over the business cycle. In a boom more labor is demanded by firms. To satisfy the higher demand, the "family" provides households with more incentives to look for work by raising consumption for the employed, c_t^w, relative to consumption of the non-employed, c_t^{nw}. Conversely, in a recession, the consumption premium falls and thus the replacement ratio, c_t^{nw}/c_t^w, increases. Thus, our model implies a procyclical consumption premium – or equivalently – a countercyclical replacement ratio. Put differently, optimal labor market insurance is countercyclical in our model.

Our environment has a simple representative agent formulation, in which the representative agent has an indirect utility function that is a function only of market consumption and labor. As a result, our model is observationally equivalent to the CGG model when only the data addressed by CGG are considered. In particular, our model implies the three equilibrium conditions of the New Keynesian model: an IS curve, a Phillips curve and a monetary policy rule. The conditions can be written in the usual way, in terms of the output gap. The output gap is the difference between actual output and output in the efficient equilibrium: the equilibrium in which there are no price setting frictions and distortions from monopoly power are extinguished. Note, however, that the observational equivalence property breaks down when data for e.g. unemployment and the labor force are considered. In our model there is a simple relation between the output gap and the unemployment gap: the difference between actual and efficient unemployment.[13] The presence of this gap in our model allows us to discuss the microeconomic foundations of the non-accelerating inflation rate of unemployment (NAIRU). The NAIRU plays a prominent role in public discussions about the inflation outlook, as well as in discussions of monetary and labor market policies. In practice, these discussions leave the formal economic foundations of the NAIRU unspecified. This paper, in effect takes a step towards integrating the NAIRU into the formal quantitative

[13] This relationship is a formalization of the widely discussed Okun's law.

apparatus of monetary DSGE models.[14]

Next, we introduce our model of unemployment into a medium-sized monetary DSGE model that has been fit to actual data. In particular, we work with a version of the model proposed in Christiano, Eichenbaum and Evans (2005) (CEE). In this model there is monopoly power in the setting of wages, there are wage setting frictions, capital accumulation and other features.[15] We estimate and evaluate our model using the Bayesian version of the impulse response matching procedure proposed in Christiano, Trabandt and Walentin (2011a) (CTW). The impulse response methodology has proved useful in the basic model formulation stage of model construction, and this is why we use it here. The three shocks we consider are the same ones as in Altig, Christiano, Eichenbaum and Linde (2004) (ACEL). In particular, we consider VAR-based estimates of the impulse responses of macroeconomic variables to a monetary policy shock, a neutral technology shock and an investment-specific technology shock. Our model can match the impulse responses of standard macro variables as well as the standard model, i.e. the model in CEE and ACEL. However, our model also does a good job matching the responses of the labor force and unemployment to the three shocks.

Our paper emphasizes the importance of labor supply for the dynamics of unemployment and the labor force and is thereby related to Galí (2011). In his model, the presence of unemployment rests entirely on the assumption of market power in the labor market. By contrast, in our model unemployment reflects frictions that are necessary for people to find jobs. The existence of unemployment in our model does not require monopoly power. Moreover, Galí (2011) assumes i) that available jobs can be found without effort and ii) the presence of perfect labor market insurance which implies that the employed have lower utility than the non-employed, i.e. unemployment is voluntary from an individual households perspective. Moreover, Galí's theory of unemployment implies a drop of labor supply in response to an expansionary monetary policy shock.[16] The drop in labor supply is counterfactual, according to our VAR-based evidence. We estimate the standard model that contains Galí's theory of unemployment with and without imposing data for unemployment and the labor force. In both cases, our model of involuntary unemployment clearly outperforms the standard model in terms of data fit.

These results highlight another important implication of our work. In particular, it is in general not sufficient to account for the response of employment or total hours only to be able to draw conclusions about the unemployment rate. In particular, when the standard model is estimated without data on unemployment and the labor force, the fit of total

[14] For another approach to the NAIRU, see Blanchard and Galí (2010).

[15] The model of wage setting is the one proposed in Erceg, Henderson and Levin (2000).

[16] This drop in labor supply, or the labor force, is induced by the positive wealth effect. Galí (2011) and Galí, Smets and Wouters (2011) show that changes to the household utility function that offset wealth effects reduce the counterfactual implications of the standard model for the labor force.

hours of the model is in fact very good. By contrast, the implications of the model for unemployment and the labor force are disastrous. Conversely, when the standard model is estimated on unemployment and labor force data too, the fit of these two variables improves indeed somewhat. However the improvement of fit comes at the cost of not fitting total hours well. In other words, the standard model provides an example that is it not straightforward to account for the joint behavior of unemployment, labor force participation and total hours *together* with further real and nominal macroeconomic variables. By contrast, our model does a good job in doing so.

Finally, our model of unemployment has several interesting microeconomic implications. As mentioned above, the consumption premium is procyclical while the replacement ratio is countercyclical. Studies of the cross section variance of log household consumption are a potential source of evidence on the cyclical behavior of the premium. Evidence in Heathcote, Perri and Violante (2010) suggests indeed that the dispersion in log household non-durable consumption decreased in the 1980, 2001 and 2007 recessions. Thus, the observed cross sectional dispersion of consumption across households lends support to our model's implication that the consumption premium is procyclical. Another indication that the replacement ratio may be countercyclical indeed is the fact that the duration of unemployment benefits is routinely extended in recessions (e.g. in the US during the Great Recession). Second, our model predicts that high unemployment in recessions reflects the procyclicality of effort in job search. There is some evidence that supports this implication of the model. Data from the Bureau of Labor Statistics suggests that the number of "discouraged workers" jumped 70 percent from 2008Q1 to 2009Q1. In fact, the number of discouraged workers is only a tiny fraction of the labor force. However, to the extent that the sentiments of discouraged workers are shared by workers more generally, a jump in the number of discouraged workers could be a signal of a general decline in job search intensity in recessions.

The organization of the paper is as follows. The next section lays out our basic model of limited labor market insurance. Sections 3 and 4 proceed by integrating our model into the CGG and CEE models, respectively. After that, in section 5, we describe our estimation method. Section 6 reports the estimation results for our medium-sized model. Moreover, section 7 discusses some microeconomic implications of our model and examines evidence that provides tentative support for the model. The paper ends with concluding remarks.

2. Limited Labor Market Insurance

We begin by describing the physical environment of a typical household. Since households experience idiosyncratic uncertainty, there is a demand for insurance. Insurance cannot be perfect because of the presence of asymmetric information. We then describe an optimal insurance arrangement which respects our assumptions about publicly available information that balances the trade-off between incentive and insurance provision. Under the insurance arrangement, households band together into a large “family”. Note again, that “family” is merely a label that represents a stand-in for all the various market and non-market arrangements that actual households have for dealing with idiosyncratic labor market outcomes. The environment is sufficiently simple that we can obtain an analytic representation for the equally weighted utility of all the households in the family. This utility function corresponds to the preferences in a representative agent formulation of our economy. At the end of this section, we discuss some important implications of our basic model structure.

2.1. Households

A household can either work, or not.[17] At the start of the period, each household draws a privately observed idiosyncratic shock, l, from a stochastic process with support, $[0,1]$.[18] We assume the stochastic process for l exhibits dependence over time and that its invariant distribution is uniform. Thus, the distribution of l in the cross section of households at each date is uniform. A household's realized value of l determines its utility cost of working:

$$F + \varsigma_t (1 + \sigma_L) l^{\sigma_L}. \tag{2.1}$$

The parameters, ς_t, σ_L and F are common to all households.[19] In (2.1) we have structured the utility cost of employment so that σ_L affects its variance in the cross section and not its mean.[20] The object ς_t is potentially stochastic. In the CGG model in the next section, ς_t will be one of the shocks that result in a time-varying NAIRU. We interpret this shock as a stand-in for capturing structural changes in the economy that result in persistent movements in the labor force due to e.g. female labor force participation. Accounting for these longer-term structural changes endogenously is beyond the scope of this paper. Instead, we shall focus on the labor market at business cycle frequencies.

After drawing l, a household decides whether or not to participate in the labor market.

[17] In assuming that labor is indivisible, we follow Hansen (1985) and Rogerson (1988). The indivisible labor assumption has attracted substantial attention recently. See, for example, Mulligan (2001), and Krusell, Mukoyama, Rogerson, and Sahin (2008, 2009). The labor indivisibility assumption is consistent with the fact that most variation in total hours worked over the business cycle reflects variations in numbers of people employed, rather than in hours per person.

[18] A recent paper which emphasizes a richer pattern of idiosyncracies at the individual firm and household level is Brown, Merkl and Snower (2009).

[19] We have specified the utility cost of working such that different parameters affect the intercept, the slope and the curvature. This turns out to be especially convenient when taking the model to the data as we do in section 4. For example, the intercept F appears to be helpful to account for the gradual response of the labor force to a monetary policy shock.

[20] To see this, note that $\int_0^1 (1+\sigma_L) l^{\sigma_L} dl = 1$ and $\int_0^1 \left[(1+\sigma_L) l^{\sigma_L} - 1\right]^2 dl = \frac{\sigma_L^2}{1+2\sigma_L}$.

The probability that a household which participates in the labor market actually finds work is $p(e_{l,t})$. This probability is an increasing function of $e_{l,t}$, a level of effort that is privately observed to the household.[21] We find it convenient to adopt the following piecewise linear functional form for $p(e_{l,t})$:

$$p(e_{l,t}) = \eta + ae_{l,t},\ \ \eta, a \geq 0. \tag{2.2}$$

We adopt this simple linear representation in order to preserve analytic tractability. In our analysis, we only consider model parameterizations that imply $0 \leq p(e_{l,t}) \leq 1$ in equilibrium for $l \in [0,1]$.[22] The chosen functional form implies that $p(0) = \eta$. That is, even when no effort is exerted, households may still find a job with some positive probability. We believe that this property is not necessarily counterfactual. What is important, however, is that the probability of finding a job increases when *more* effort is exerted. Further, we abstract from aggregate variables such as e.g. the unemployment rate to affect the probability of finding a job in (2.2). We do so for two reasons. First, it allows us to preserve analytic tractability of our model. Second, extensions of this sort would surely be interesting, however, they would also fog up key issues addressed in this paper. Finally, we emphasize here that even though our functional form for $p(e_{l,t})$ is relatively simple, it turns out that the estimated medium-sized DSGE model is able to fit the responses of unemployment and the labor force to three identified shocks well. If anything, we expect suitable extensions of $p(e_{l,t})$ to result in an even better fit which we leave to future research.

Turning to household utility, consider first a household which has drawn an idiosyncratic work aversion shock, l, and chooses to participate in the labor market. This household has utility given by:[23]

$$\begin{aligned} & p(e_{l,t}) \overbrace{\left[\log\left(c_t^w\right) - F - \varsigma_t\left(1+\sigma_L\right) l^{\sigma_L} - \frac{1}{2}e_{l,t}^2\right]}^{\text{ex post utility of household that joins labor force and finds a job}} \\ & + \left(1-p(e_{l,t})\right) \overbrace{\left[\log\left(c_t^{nw}\right) - \frac{1}{2}e_{l,t}^2\right]}^{\text{ex post utility of household that joins labor force and fails to find a job}} . \end{aligned} \tag{2.3}$$

Here, $e_{l,t}^2/2$ is the utility cost associated with effort. In (2.3), c_t^w and c_t^{nw} denote the consump-

[21] In principle, we would still have a model of involuntary unemployment if we just made effort unobservable and allowed the household aversion to work, l, be observable. The manuscript focuses on the symmetric case where both e and l are not observed, and it would be interesting to explore the other case in future work.

[22] The specification of $p(e_{l,t})$ in (2.2) allows for probabilities greater than unity. We could alternatively specify the probability function to be $min\{\eta + ae_{l,t}, 1\}$. This would complicate some of the notation and the corner would have to be ignored anyway given the solution strategy that we pursue.

[23] The utility function of the household is assumed to be additively separable, as is the case in most of the DSGE literature. In the technical appendix, we show how to implement the analysis when the utility function is non-separable. The technical appendix is available online at:

http://sites.google.com/site/mathiastrabandt/home/downloads/CTWinvoluntary_techapp.pdf tion of employed and non-employed households, respectively. These are outside the control of an individual household and are determined in equilibrium given the arrangements which we describe below. Our notation reflects that in our environment, an individual household's consumption can only be dependent on its current employment status and labor type because these are the only household characteristics that are publicly observed. For example, we do not allow household consumption allocations to depend upon the history of household reports of l. We make the latter assumption to preserve tractability.

In case the household chooses non-participation in the labor market, its utility is simply:

$$\log\left(c_t^{nw}\right). \tag{2.4}$$

A non-participating household does not experience any disutility from work or from exerting effort to find a job.

We now characterize the effort and labor force participation decisions of the household. Because households' work aversion type and effort choice are private information, their effort and labor force decisions are privately optimal conditional on c_t^{nw} and c_t^{w}. In particular, the household decides its level of effort and labor force participation by comparing the magnitude of (2.4) with the maximized value of (2.3).

Consider a household that has decided to participate in the labor force. It selects effort $e_{l,t} \geq 0$ to maximize (2.3) subject to (2.2). This leads to the following optimality condition:

$$e_{l,t} = \max\left\{a\left(\log\left[\frac{c_t^w}{c_t^{nw}}\right] - F - \varsigma_t\left(1+\sigma_L\right)l^{\sigma_L}\right), 0\right\}. \tag{2.5}$$

The corresponding probability of finding a job is:

$$p\left(e_{l,t}\right) = \eta + a^2 \max\left\{\log\left[\frac{c_t^w}{c_t^{nw}}\right] - F - \varsigma_t\left(1+\sigma_L\right)l^{\sigma_L}, 0\right\}. \tag{2.6}$$

Expression (2.5) is intuitive. Households that participate in the labor force but have high work aversion, l, apply relatively little effort to find a job. Also, effort is greater the higher is the incentive to work, c_t^w/c_t^{nw}.

Collect the terms in $p\left(e_{l,t}\right)$ in (2.3) and then substitute out for $p\left(e_{l,t}\right)$ using $p\left(e_{l,t}\right)$ in (2.6). We then find that the utility of a household that draws work aversion index, l, and chooses to participate in the labor force is:

$$\begin{aligned}
&\left[\eta + a^2 \max\left\{\log\left[\frac{c_t^w}{c_t^{nw}}\right] - F - \varsigma_t\left(1+\sigma_L\right)l^{\sigma_L}, 0\right\}\right] \\
&\times\left[\log\left(\frac{c_t^w}{c_t^{nw}}\right) - F - \varsigma_t\left(1+\sigma_L\right)l^{\sigma_L}\right] + \log\left(c_t^{nw}\right) \\
&-\frac{1}{2}\left[\max\left\{a\left(\log\left[\frac{c_t^w}{c_t^{nw}}\right] - F - \varsigma_t\left(1+\sigma_L\right)l^{\sigma_L}\right), 0\right\}\right]^2.
\end{aligned} \tag{2.7}$$

The household makes its labor participation choice depending on which yields higher utility, (2.7) or (2.4). Let m_t denote the value of l for which a household is indifferent between participating and not participating in the labor force:

$$\log\left[\frac{c_t^w}{c_t^{nw}}\right] = F + \varsigma_t\,(1+\sigma_L)\,m_t^{\sigma_L}. \tag{2.8}$$

For households with $1 \geq l \geq m_t$, (2.7) is smaller than (2.4). They choose to be out of the labor force. For households with $0 \leq l < m_t$ (2.7) is greater than (2.4), and they strictly prefer to be in the labor force. The object, m corresponds to the labor force participation rate: the fraction of the population that chooses to join the labor force. According to (2.8), the higher is the cost of working for households, i.e., the bigger is ς_t, the smaller must be the replacement rate, $r_t = c_t^{nw}/c_t^w$, to induce a given number of households, m, to participate in the labor market. In other words, the trade-off between incentive provision and insurance is tilted towards the incentive part in this case.

Further, consider a household with aversion to work, l, which participates in the labor force. For such a household the ex post utility of finding work minus the ex post utility of not finding work is $\Delta(l) = \log[c_t^w/c_t^{nw}] - F - \varsigma_t\,(1+\sigma_L)\,l^{\sigma_L}$. Condition (2.8) guarantees that, with one exception, $\Delta(l) > 0$. That is, among households that participate in the labor force, those that find work are strictly better off than those that do not. The exceptional case is the marginal household with $m = l$. The ex post utility enjoyed by the marginal household is the same, whether its job search is successful or not.

2.2. Insurance Arrangement

We now consider the best possible insurance arrangement that households can make, given our information assumptions. In our environment, there is clearly a need for insurance. Households are subject to two sources of idiosyncratic uncertainty: the private cost of working, l, and the uncertainty of finding employment for households that participate in the labor market. Given our assumption of separability between consumption and leisure, under perfect insurance all households would enjoy the same level of consumption, regardless of their realized value of l and of whether or not they find employment (i.e., $c_t^{nw} = c_t^w$). This arrangement is not possible in our environment because of our assumption that work aversion and effort are privately observed. Under the first-best insurance arrangement, households would have no incentive to participate in the labor market and if they did, they would then have no incentive to exert effort in finding work. Instead, we consider the optimal insurance arrangement in our private information environment.

We suppose that households gather together into families. Individual households have no access to credit or insurance markets other than through their arrangements with the family.

In part, we view the family construct as a stand-in for the market and non-market arrangements that actual households use to insure against idiosyncratic labor market experiences. In part, we are following Andolfatto (1996) and Merz (1995), in using the family construct as a technical device to prevent the appearance of difficult-to-model wealth dispersion among households. Families have sufficiently many members that there is no idiosyncratic family-level labor market uncertainty.

2.3. Indirect Utility Function

We now derive the representative family's utility, $u(C_t, h_t; \varsigma_t)$, as a function of family aggregate employment, h_t, and family aggregate consumption, C_t. In this section, we do not discuss the determination of the values of h_t and C_t. The determination of these values is pursued in the general equilibrium analyses of the following two sections in this paper.

The number of employed households, h_t, is, using our uniform distribution assumption:

$$h_t = \int_0^{m_t} p(e_{l,t})\, dl. \tag{2.9}$$

After making use of (2.6) and (2.8) and rearranging,

$$h_t = m_t \eta + a^2 \varsigma_t \sigma_L m_t^{\sigma_L + 1}. \tag{2.10}$$

Note that the right side is equal to zero for $m_t = 0$. In addition, the right side of (2.10) is unbounded above and monotonically increasing in m_t. As a result, for any value of $h_t \geq 0$ there exists a unique value of $m_t \geq 0$ that satisfies (2.10), which we express as follows:

$$m_t = f(h_t; \varsigma_t), \tag{2.11}$$

where f is monotonically increasing in h_t.

Evidently, $\bar{p}_t$ is the probability associated with the household having the least aversion to work, $l = 0$. Setting $l = 0$ in (2.6) and imposing (2.8):

$$\bar{p}_t = \eta + \varsigma_t a^2 (1 + \sigma_L) m_t^{\sigma_L}. \tag{2.12}$$

We require $\bar{p}_t \leq 1$ for all t. We assume that model parameters have been chosen to guarantee this condition holds.

From (2.9) and the fact that $p(e_{l,t})$ is strictly decreasing in l, we see that $h_t < m_t \bar{p}_t$. It then follows per $\bar{p}_t \leq 1$ that $h_t < m_t$, so that the unemployment rate, u_t,

$$u_t = \frac{m_t - h_t}{m_t}, \tag{2.13}$$

is strictly positive.

Suppose the family has decided to send h_t to work and consume C_t. The constraints on the family's choice of these variables is discussed in the next two sections, when we insert the model discussed here into two different general equilibrium environments. For now, we take h_t and C_t as exogenously determined. The family that wants a level of employment, h_t, must set the labor force, m_t, to the level indicated by (2.11). To ensure that m_t households have the incentive to enter the labor force requires setting the consumption premium of work, c_t^w/c_t^{nw}, as indicated by (2.8). In setting the consumption premium, the household must satisfy the following resource constraint:

$$h_t c_t^w + (1 - h_t)\, c_t^{nw} = C_t. \tag{2.14}$$

Substituting out for m_t in (2.10) and (2.8) from (2.11), we obtain a single-valued mapping from h_t and C_t to c_t^w and c_t^{nw}. Solving for c_t^{nw}:

$$c_t^{nw} = \frac{C_t}{h_t \left(e^{F + \varsigma_t (1+\sigma_L) m_t^{\sigma_L}} - 1 \right) + 1}. \tag{2.15}$$

By setting c_t^w and c_t^{nw} according to (2.8) the family incentivizes the m_t households with the least work aversion to participate in the labor force. Note that there is no reason to describe a family optimization problem for selecting $c^{nw}$and c^w, since there is only one value for these variables that satisfies the resource constraint, (2.14), and the incentive compatibility constraint, (2.8).

Imposing (2.8) on (2.7), we find that the ex ante utility of households which draw $l \leq m_t$ is:

$$\eta \varsigma_t (1+\sigma_L)\left(m_t^{\sigma_L} - l^{\sigma_L}\right) + \frac{1}{2} a^2 \varsigma_t^2 (1+\sigma_L)^2 \left(m_t^{\sigma_L} - l^{\sigma_L}\right)^2 + \log\left(c_t^{nw}\right). \tag{2.16}$$

Integrating the utility, (2.16), of the m_t households in the labor force and the utility, (2.4), of the $1 - m_t$ households not in the labor force, we obtain:

$$\int_0^{m_t} \left[\eta \varsigma_t (1+\sigma_L)\left(m_t^{\sigma_L} - l^{\sigma_L}\right) + \frac{1}{2} a^2 \varsigma_t^2 (1+\sigma_L)^2 \left(m_t^{\sigma_L} - l^{\sigma_L}\right)^2 \right] dl + \log\left(c_t^{nw}\right). \tag{2.17}$$

Evaluating the integral, and making use of (2.11) and (2.15), we obtain the equally weighted utility of households within the family, i.e. the family indirect utility function:

$$u\left(C_t, h_t; \varsigma_t\right) = \log\left(C_t\right) - z\left(h_t; \varsigma_t\right), \tag{2.18}$$

where

$$\begin{aligned} z\left(h_t; \varsigma_t\right) &= \log\left[h_t \left(e^{F + \varsigma_t (1+\sigma_L) f(h_t;\varsigma_t)^{\sigma_L}} - 1 \right) + 1 \right] \\ &\quad - \frac{a^2 \varsigma_t^2 (1+\sigma_L)\sigma_L^2}{2\sigma_L + 1} f\left(h_t; \varsigma_t\right)^{2\sigma_L + 1} - \eta \varsigma_t \sigma_L f\left(h_t; \varsigma_t\right)^{\sigma_L + 1}. \end{aligned} \tag{2.19}$$

In (2.19) the function, f, is defined in (2.11). It is easy to extend our analysis to a broader set of preferences such as habit formation in consumption, see section 4.

2.4. Implications of Our Basic Model Structure

We now briefly discuss expression (2.18) as well as implications of our basic model structure.

First, note that the derivation of the family utility function, (2.18), involves no explicit maximization problem even though the resulting insurance arrangement is optimal given our information assumption. This is because the family incentive and resource constraints, (2.8) and (2.14), are sufficient to determine c_t^w and c_t^{nw} conditional on h_t and C_t. In general, the constraints would not be sufficient to determine the household consumption allocations, and the family problem would involve non-trivial optimization.[24]

Second, we can see from (2.18) that our model is likely to be characterized by a particular observational equivalence property. To see this, note that although the agents in our model are in fact heterogeneous, C_t and h_t are chosen as if the economy were populated by a representative agent with the utility function specified in (2.18). A model such as CGG, which specifies representative agent utility as the sum of the log of consumption and a constant elasticity disutility of labor is indistinguishable from our model, as long as data on the labor force and unemployment *are not used*. This is particularly obvious if, as is the case here, we only study the linearized dynamics of the model about steady state. In this case, the only properties of a model's utility function that are used are its second order derivative properties in nonstochastic steady state.[25]

Third, our model and the standard CGG model are distinguished by the following two features: i) our model addresses a larger set of time series than the standard model does and ii) in our model the representative agent's utility function is a reduced form object. With respect to the utility function, its properties are determined by i) the details of the technology of job search, and ii) the cross-sectional variation in preferences with regard to attitudes about market work. As a result, the basic structure of the utility function in our model can in principle be informed by time use surveys and studies of job search.[26]

Fourth, we gain insight into the determinants of the unemployment rate in the model, by substituting out h_t in (2.13) using (2.10):

$$u_t = 1 - \eta - a^2 \varsigma_t \sigma_L m_t^{\sigma_L}. \tag{2.20}$$

[24] One example of non-trivial optimization is the case of full information which is available in the technical appendix.

[25] This observational equivalence result reflects our simplifying assumptions. These assumptions are primarily driven by the desire for analytic tractability, so that the economics of the environment are as transparent as possible. Presumably, a careful analysis of microeconomic data would lead to different functional forms and the resulting model would then not be observationally equivalent to the standard model.

[26] A similar point was made by Benhabib, Rogerson and Wright (1991). They argue that a representative agent utility function of consumption and labor should be interpreted as a reduced form object, after non-market consumption and labor activities have been maximized out. From this perspective, construction of the representative agent's utility function can in principle be guided by surveys of how time in the home is used.

According to (2.20), a rise in the labor force is associated with a proportionately greater rise in employment, so that the unemployment rate falls. This greater rise in employment reflects that an increase in the labor force requires raising employment incentives, and this simultaneously generates an increase in search intensity. From (2.9) we see that h_t is linear in m_t if search intensity is held constant, but that h_t/m_t increases with m_t if search intensity increases with m_t. That search intensity indeed does increase in m_t can be seen by substituting (2.8) into (2.6). It is important to note that the theory developed here does not imply that the empirical scatter plot of the unemployment rate against the labor force lies rigidly on a negatively sloped line. Equation (2.20) shows that disturbances in ς_t (or in the parameters of the search technology, (2.2)) would make the scatter of u_t versus m_t resemble a shotgun blast rather than a line. A similar observation can be made about the relationship between h_t and m_t in the context of (2.10).

Fifth, our theory of unemployment implies a procyclical consumption premium – or equivalently – a countercyclical replacement ratio. So see this, combine equations (2.8) and (2.20) to obtain:

$$\log\left[\frac{c_t^w}{c_t^{nw}}\right] = \vartheta - \frac{1+\sigma_L}{a^2\sigma_L}u_t \tag{2.21}$$

where $\vartheta = \left[Fa^2\sigma_L + (1+\sigma_L)(1-\eta)\right]\left[a^2\sigma_L\right]^{-1} > 0$. Suppose a boom results in a fall of the unemployment rate as it will be the case in the models discussed in the two sections below. According to (2.21), the equilibrium consumption premium, c_t^w/c_t^{nw} increases. The boom results in more labor demanded by firms. In order to satisfy the higher demand, the "family" provides households with more incentives to look for work by raising consumption for the employed, c_t^w, relative to consumption of the non-employed, c_t^{nw}. Conversely, in a recession, the consumption premium falls and thus the replacement ratio c_t^{nw}/c_t^w increases. In other words, our model implies that households are provided with more insurance in a recession, i.e. optimal labor market insurance is countercyclical.

3. An Unemployment-based Phillips Curve

To highlight the mechanisms in our model of unemployment in a monetary environment, we embed it into the framework with price setting frictions, flexible wages and no capital analyzed in CGG. Despite the presence of heterogenous households in our environment, the model has a representative agent representation. As a result, the linearized equilibrium conditions of the model can be written in the same form as those in CGG. It turns out that the output gap is proportional to what we call the unemployment gap, the difference between the actual and efficient rates of unemployment. As a result, the Phillips curve can also be expressed in terms of the unemployment gap. We also discuss further implications of the theory developed here for e.g. the NAIRU.

3.1. Households and Family

The economy is populated by a large number of identical families each of which is composed of a large number of ex ante identical households. The preferences and assumptions about information are identical to those in the previous section. The representative family's optimization problem is:

$$\max_{\{C_t,h_t,B_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t, h_t; \varsigma_t\right), \ \beta \in (0,1), \tag{3.1}$$

subject to

$$P_t C_t + B_{t+1} \leq B_t R_{t-1} + W_t h_t + \text{Transfers and profits}_t. \tag{3.2}$$

Again, C_t, h_t denote family consumption and market work, respectively. In addition, B_{t+1} denotes the quantity of a nominal bond purchased by the family in period t. Also, R_t denotes the one-period gross nominal rate of interest on a bond purchased in period t. Finally, W_t denotes the competitively determined nominal wage rate. The family takes W_t as given and makes arrangements to set h_t so that the relevant marginal conditions are satisfied.

The necessary conditions for optimization are:

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\pi_{t+1}} \tag{3.3}$$

$$C_t z_h\left(h_t; \varsigma_t\right) = \frac{W_t}{P_t}. \tag{3.4}$$

Here, π_{t+1} is the gross rate of inflation from t to $t+1$. The expression to the left of the equality in (3.4) is the family's marginal cost in consumption units of providing an extra unit of market employment. This marginal cost takes into account the need for the family to provide appropriate incentives to increase employment. A cost of the incentives, which involves increasing the consumption differential between employed and non-employed households, is that consumption insurance to family members is reduced.

3.2. Goods Production and Price Setting

Production is standard in our model. Accordingly, we suppose that a final good, Y_t, is produced using a continuum of inputs as follows:

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{1}{\lambda_f}} di\right]^{\lambda_f}, \ 1 \leq \lambda_f < \infty. \tag{3.5}$$

The good is produced by a competitive, representative firm which takes the price of output, P_t, and the price of inputs, $P_{i,t}$, as given. The first order necessary condition associated with optimization is:

$$\left(\frac{P_t}{P_{i,t}}\right)^{\frac{\lambda_f}{\lambda_f-1}} Y_t = Y_{i,t}. \tag{3.6}$$

A useful result is obtained by substituting out for Y_{it} in (3.5) from (3.6):

$$P_t = \left[\int_0^1 (P_{i,t})^{\frac{-1}{\lambda_f-1}} di\right]^{-(\lambda_f-1)}. \tag{3.7}$$

Each intermediate good is produced by a monopolist that uses the production function $Y_{i,t} = A_t h_{i,t}$ where A_t is an exogenous stochastic process whose growth rate, $g_{A,t} = \frac{A_t}{A_{t-1}}$, is stationary. The marginal cost of the i^{th} firm is, after dividing by P_t:

$$s_t = (1-\nu)\frac{W_t}{A_t P_t} = (1-\nu)\frac{C_t z_h(h_t; \varsigma_t)}{A_t}, \tag{3.8}$$

after using (3.4) to substitute out for W_t/P_t. Here, ν is a subsidy designed to remove the effects, in steady state, of monopoly power. To this end, we set $1-\nu = 1/\lambda_f$. Further, monopolists are subject to Calvo price frictions. In particular, a fraction ξ_p of intermediate good firms cannot change price, $P_{i,t} = P_{i,t-1}$, and the complementary fraction, $1-\xi_p$, set their price optimally, $P_{i,t} = \tilde{P}_t$. The i^{th} monopolist that has the opportunity to reoptimize its price in the current period is only concerned about future histories in which it cannot reoptimize its price. This leads to the following problem:

$$\max_{\tilde{P}_t} E_t \sum_{j=0}^{\infty} (\xi_p \beta)^j \upsilon_{t+j} \left[\tilde{P}_t Y_{i,t+j} - P_{t+j} s_{t+j} Y_{i,t+j}\right], \tag{3.9}$$

subject to (3.6). In (3.9), υ_t is the multiplier on the representative family's time t flow budget constraint, (3.2), in the Lagrangian representation of its problem. Intermediate good firms take υ_{t+j} as given. The nature of the family's preferences, (2.18), implies $\upsilon_{t+j} = 1/(P_{t+j}C_{t+j})$.

3.3. Market Clearing, Aggregate Resources and Equilibrium

Clearing in the loan market requires $B_{t+1} = 0$. Clearing in the market for final goods requires:

$$C_t + G_t = Y_t, \tag{3.10}$$

where G_t denotes government consumption. We model government consumption as $G_t = g_t N_t$, where $\log g_t$ is a stationary stochastic process independent of any other shocks in the system, such as A_t. The variable, N_t, ensures that the model exhibits balanced growth, and has the law of motion $N_t = A_t^{\gamma} N_{t-1}^{1-\gamma}$, $0 < \gamma \leq 1$. The extreme case, $\gamma = 1$, implies the specification adopted in Christiano and Eichenbaum (1992). That model implies, implausibly, that G_t responds immediately to a shock in A_t. With γ close to zero, G_t is proportional to a long average of past values of A_t, and the immediate impact of a disturbance in A_t on G_t is arbitrarily small. For any admissible value of γ, the stationary variable $n_t = N_t/A_t$ converges in nonstochastic steady state. Also, the law of motion of n_t is $n_t = (n_{t-1}/g_{A,t})^{1-\gamma}$.

The relationship between aggregate output of the final good, Y_t, and aggregate employment, h_t, is given by (see Yun, 1996):

$$Y_t = p_t^* A_t h_t, \tag{3.11}$$

where

$$p_t^* \equiv \left(\frac{P_t^*}{P_t}\right)^{\frac{\lambda_f}{\lambda_f - 1}}, \; P_t^* = \left[\int_0^1 P_{i,t}^{\frac{\lambda_f}{1-\lambda_f}} di\right]^{\frac{1-\lambda_f}{\lambda_f}}. \tag{3.12}$$

The model is closed once we specify time series representations for the shocks as well as how monetary policy is conducted. A sequence of markets equilibrium is a stochastic process for prices and quantities which satisfies market clearing and optimality conditions for the agents in the model.

3.4. Log-Linearizing the Private Sector Equilibrium Conditions

It is convenient to express the equilibrium conditions in linearized form relative to the efficient equilibrium. We define the efficient equilibrium as the one in which $\pi_t = 1$ for all t, monopoly power does not distort the level of employment, and there are no price frictions. We refer to the equilibrium in our market economy with sticky prices as simply the equilibrium, or the actual equilibrium when clarity requires special emphasis.

3.4.1. The Efficient Equilibrium

In the efficient equilibrium, the marginal cost of labor and the marginal product of labor are equated i.e., $C_t z_h(h_t; \varsigma_t) = A_t$. The resource constraint in the efficient equilibrium is $C_t + G_t = A_t h_t$, which, when substituted into the previous expression implies:

$$(h_t^* - g_t n_t)\, z_h(h_t^*; \varsigma_t) = 1, \tag{3.13}$$

where the '$*$' indicates an endogenous variable in the efficient equilibrium. Evidently, the efficient level of employment, h_t^*, fluctuates in response to disturbances in g_t and ς_t. It also responds to disturbances in $g_{A,t}$ in the plausible case, $\gamma < 1$. The level of work in the nonstochastic steady state of the efficient equilibrium coincides with the level of work in the nonstochastic steady state of the actual equilibrium. This object is denoted by h in both cases. The values of all variables in nonstochastic steady state coincide across actual and efficient equilibria.

Linearizing (3.13) about steady state,

$$\hat{h}_t^* = \frac{\frac{\eta_g}{1-\eta_g}\left(\hat{g}_t + \hat{n}_t\right) - \sigma_\varsigma \hat{\varsigma}_t}{\frac{1}{1-\eta_g} + \sigma_z} = \frac{\eta_g}{1+\left(1-\eta_g\right)\sigma_z}\left(\hat{g}_t + \hat{n}_t\right) - \frac{1-\eta_g}{1+\left(1-\eta_g\right)\sigma_z}\sigma_\varsigma \hat{\varsigma}_t, \qquad (3.14)$$

where

$$\sigma_z \equiv \frac{z_{hh}h}{z_h},\ \sigma_\varsigma \equiv \frac{z_{h\varsigma}\varsigma}{z_h}, \qquad (3.15)$$

and η_g denotes the steady state value of G_t/Y_t. Further,

$$\hat{n}_t = (1-\gamma)\left(\hat{n}_{t-1} - \hat{g}_{A,t}\right). \qquad (3.16)$$

In (3.15), z_{ij} denotes the cross derivative of z with respect to i and j $(i, j = h, \varsigma)$, evaluated in steady state and z_h denotes the derivative of z with respect to h, evaluated in steady state. We follow the convention that a hat over a variable denotes percent deviation from its steady state value.

The object, σ_z, is a measure of the curvature of the function, z, in the neighborhood of steady state. Also, $1/\sigma_z$ is a consumption-compensated elasticity of family labor supply in steady state. Although $1/\sigma_z$ bears a formal similarity to the Frisch elasticity of labor supply, there is an important distinction. In practice the Frisch elasticity refers to a household's willingness to change its labor supply on the intensive margin in response to a wage change. In our environment, all changes in labor supply occur on the extensive margin.

The efficient rate of interest, R_t^*, is derived from (3.3) with consumption and inflation set at their efficient rates:

$$R_t^* = \left(\beta E_t\left[\frac{h_t^* - g_t n_t}{g_{A,t+1}\left(h_{t+1}^* - g_{t+1}n_{t+1}\right)}\right]\right)^{-1}.$$

Linearizing the efficient rate of interest expression about steady state, we obtain:

$$\hat{R}_t^* = E_t\hat{g}_{A,t+1} + \frac{1}{1-\eta_g}E_t\left(\hat{h}_{t+1}^* - \hat{h}_t^*\right) - \frac{\eta_g}{1-\eta_g}E_t\left[\left(\hat{g}_{t+1} + \hat{n}_{t+1}\right) - \left(\hat{g}_t + \hat{n}_t\right)\right], \qquad (3.17)$$

where $\hat{h}_{t+1}^*$, $\hat{h}_t^*$ are defined in (3.14).

3.4.2. The Actual Equilibrium

We turn now to the linearized equilibrium conditions in the actual equilibrium. The monetary policy rule (displayed below) ensures that inflation and, hence, price dispersion, is zero in the steady state. Yun (1996) showed that under these circumstances, p_t^* in (3.11) is unity to first order, so that

$$\frac{C_t}{A_t} = \frac{h_t A_t - g_t n_t A_t}{A_t} = h_t - g_t n_t. \tag{3.18}$$

Linearizing (3.8) about the non-stochastic steady state equilibrium and using (3.18), we obtain:

$$\hat{s}_t = \left(\frac{1}{1-\eta_g} + \sigma_z\right)\hat{h}_t - \left[\frac{\eta_g}{1-\eta_g}\left(\hat{g}_t + \hat{n}_t\right) - \sigma_\varsigma \hat{\varsigma}_t\right] = \left(\frac{1}{1-\eta_g} + \sigma_z\right)\left(\hat{h}_t - \hat{h}_t^*\right),$$

using (3.14). Then,

$$\hat{s}_t = \left(\frac{1}{1-\eta_g} + \sigma_z\right)\hat{x}_t, \tag{3.19}$$

where $\hat{x}_t$ denotes the output gap, the percent deviation of actual output from its value in the efficient equilibrium:

$$\hat{x}_t \equiv \hat{h}_t - \hat{h}_t^*. \tag{3.20}$$

Condition (3.7), together with the necessary conditions associated with (3.9) leads (after linearization about a zero inflation steady state) to:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{\left(1-\beta\xi_p\right)\left(1-\xi_p\right)}{\xi_p}\left(\frac{1}{1-\eta_g} + \sigma_z\right)\hat{x}_t. \tag{3.21}$$

The derivation of (3.21) is standard so that we shall leave out the details here.

The family's intertemporal Euler equation, (3.3), after using (3.18), can be expressed as follows:

$$1 = \beta E_t \frac{h_t - g_t n_t}{\left(h_{t+1} - g_{t+1} n_{t+1}\right) g_{A,t+1}} \frac{R_t}{\pi_{t+1}}.$$

Linearize this around steady state, to obtain:

$$\hat{h}_t = \eta_g \left[\left(\hat{g}_t + \hat{n}_t\right) - E_t\left(\hat{g}_{t+1} + \hat{n}_{t+1}\right)\right] + E_t \hat{h}_{t+1} + \left(1-\eta_g\right) E_t \hat{g}_{A,t+1} - \left(1-\eta_g\right)\left(\hat{R}_t - E_t \hat{\pi}_{t+1}\right).$$

Use (3.17) to solve out for $\left(1-\eta_g\right)\hat{g}_{A,t+1}$ in the preceding expression, to obtain:

$$\hat{x}_t = E_t \hat{x}_{t+1} - \left(1-\eta_g\right) E_t\left(\hat{R}_t - \hat{\pi}_{t+1} - \hat{R}_t^*\right). \tag{3.22}$$

Expression (3.22) is the standard representation of the New Keynesian IS curve, expressed in terms of the output gap, $\hat{x}_t$, and the efficient rate of interest, $\hat{R}_t^*$.

The model is closed with the assumption that monetary policy follows a Taylor rule of the following form:

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + \left(1-\rho_R\right)\left[r_\pi \hat{\pi}_t + r_y \hat{x}_t\right] + \varepsilon_t, \tag{3.23}$$

where ε_t is an iid monetary policy shock. The equilibrium conditions of the log-linearized system are (3.14), (3.16), (3.17), (3.21), (3.22), and (3.23). These conditions determine the equilibrium stochastic processes, $\hat{h}_t^*$, $\hat{n}_t$, $\hat{R}_t^*$, $\hat{R}_t$, $\hat{\pi}_t$ and $\hat{x}_t$ as a function of the exogenous stochastic processes, $\hat{g}_{A,t}$, $\hat{g}_t$, $\hat{\varsigma}_t$ and ε_t. The first three stochastic processes enter the system via the efficient rate of interest and employment as indicated in (3.14) and (3.17), and the monetary policy shock enters via (3.23). The variables, $\hat{h}_t$, can be solved using (3.14) and (3.20).

The model parameters that enter the equilibrium conditions are γ, η_g, σ_z, σ_ς, $\hat{\varsigma}_t$, ξ_p and β. Consistent with the observational equivalence discussion in subsection (2.4), there is no way, absent observations on unemployment and the labor force, to tell whether these parameters are the ones associated with CGG or with our involuntary unemployment model. Thus, relative to time series on the six variables, $\hat{R}_t^*$, $\hat{R}_t$, $\hat{\pi}_t$, $\hat{x}_t$, $\hat{h}_t$, and $\hat{h}_t^*$, our model and the standard CGG model are observationally equivalent.

3.4.3. The Unemployment Rate Phillips Curve

We can solve for the labor force and unemployment from (2.13) and (2.10). Linearizing (2.10) about steady state, we obtain

$$\hat{m}_t = \frac{1-u}{1-u+a^2\varsigma\sigma_L^2 m^{\sigma_L}}\hat{h}_t - \delta_\varsigma \hat{\varsigma}_t, \tag{3.24}$$

where $\delta_\varsigma = \frac{\eta}{1-u+a^2\varsigma\sigma_L^2 m^{\sigma_L}} > 0$. Linearizing (2.20):

$$du_t = -a^2\varsigma\sigma_L m^{\sigma_L}\left[\sigma_L \hat{m}_t + \hat{\varsigma}_t\right],$$

where $du_t \equiv u_t - u$ and u_t is a small deviation from steady state unemployment, u. Substituting from (3.24),

$$u_t = u - \kappa^{okun}\hat{h}_t - a^2\varsigma\sigma_L m^{\sigma_L}\left(1-\sigma_L\delta_\varsigma\right)\hat{\varsigma}_t, \tag{3.25}$$

where

$$\kappa^{okun} = \frac{a^2\varsigma\sigma_L^2 m^{\sigma_L}(1-u)}{1-u+a^2\varsigma\sigma_L^2 m^{\sigma_L}} > 0.$$

The analogous equation holds in the efficient equilibrium, with $\hat{h}_t$ replaced by $\hat{h}_t^*$:

$$u_t^* = u - \kappa^{okun}\hat{h}_t^* - a^2\varsigma\sigma_L m^{\sigma_L}\left(1-\sigma_L\delta_\varsigma\right)\hat{\varsigma}_t. \tag{3.26}$$

Here, the notation reflects that the steady states in the actual and efficient equilibria coincide. In (3.26), u_t^* denotes unemployment in the efficient equilibrium, i.e., the efficient rate of unemployment.

Let u_t^g denote the unemployment gap. Subtracting (3.26) from (3.25), we obtain:

$$u_t^g \equiv u_t - u_t^* = -\kappa^{okun}\hat{x}_t. \tag{3.27}$$

Note that the unemployment gap is the level deviation of the unemployment rate in the actual equilibrium from the efficient rate. The notation is chosen to emphasize that (3.27) represents the model's implication for Okun's law. In particular, a one percentage point rise in the unemployment rate above the efficient rate is associated with a $1/\kappa^{okun}$ percent fall in output relative to its efficient level. The general view is that $1/\kappa^{okun}$ is somewhere in the range, 2 to 3.

The model can be rewritten in terms of the unemployment gap instead of the output gap. Substituting (3.27) into (3.21), (3.22) and (3.23), respectively, we obtain:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} - \kappa u_t^g \tag{3.28}$$

$$u_t^g = \kappa^{okun} E_t u_{t+1}^g + \kappa^{okun} \left(\hat{R}_t - E_t \hat{\pi}_{t+1} - \hat{R}_t^* \right) \tag{3.29}$$

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R) [r_\pi \hat{\pi}_t - \frac{r_y}{\kappa^{okun}} u_t^g] + \varepsilon_t \tag{3.30}$$

where $\left(1-\beta\xi_p\right)\left(1-\xi_p\right)/\xi_p(1+\sigma_z)/\kappa^{okun}$.

3.5. Implications of the Model

The above model has several interesting implications that are worth emphasizing at this point.

First, our theory of unemployment is tractable enough so that it can be integrated easily into the standard New Keynesian monetary model. In fact, the model can be summarized by the often referred to three equations: a NK Phillips curve, a NK IS curve and a Taylor rule. However, equation (3.28) is the expression Stock and Watson (1999) refer to as the unemployment rate Phillips curve. Thus, in contrast to e.g. CGG, our approach implies that we can write the model in terms of the unemployment gap rather than the output gap. Interestingly, our model implies a tight relationship between the unemployment gap and the output gap, see equation (3.27). Using this relationship, it is straightforward to summarize our model by the standard set of variables such as the output gap, inflation and the nominal interest rate. When doing so, observe that the model becomes observationally equivalent to the CGG model. However, we shall emphasize here that the observational equivalence breaks down as soon as further data such as the unemployment rate and the labor force are considered in our model. Moreover, equation (3.27) is likely to have an interesting empirical implication. Suppose the natural rate of unemployment, u_t^* is a slowly moving object. Then, observable unemployment rate data is likely to be helpful to estimate the underlying output gap which is one of the key variables monetary policy makers are concerned about.

Second, we can relate the theory derived here to the idea of a non-accelerating inflation rate of unemployment (NAIRU). One interpretation of the NAIRU focuses on the first difference of inflation. From (3.28) it is evident that a negative value of u_t^g does not predict an acceleration of inflation in the sense of predicting a positive value for $\beta E_t \hat{\pi}_{t+1} - \hat{\pi}_t$. An alternative interpretation of the NAIRU focuses on the level of inflation, rather than its change. Under this interpretation, u_t^* in the theory developed here is a NAIRU.[27] That is, a shock that drives u_t below u_t^* is expected to be followed by a higher level of inflation and a shock that drives u_t above u_t^* is expected to be followed by a lower level of inflation. Thus, u_t^* in the theory derived here is a NAIRU if one adopts the level interpretation of the NAIRU.[28] Appendix A.1 contains a further in-depth discussion of the implications of the model with respect to the NAIRU.

4. Integrating Unemployment into a Medium-Sized DSGE Model

Our representation of the standard DSGE model is a version of the medium-sized DSGE model in CEE or Smets and Wouters (2003, 2007). The first section below describes how we introduce our model of involuntary unemployment into the standard model. The last section derives the standard model as a special case of our model.

4.1. Final and Intermediate Goods

A final good is produced by competitive firms using (3.5). The i^{th} intermediate good is produced by a monopolist with the following production function:

$$Y_{i,t} = \left(z_t H_{i,t}\right)^{1-\alpha} K_{i,t}^{\alpha} - z_t^+ \phi, \tag{4.1}$$

where $K_{i,t}$ denotes capital services used for production by the i^{th} intermediate good producer. Also, $\log\left(z_t\right)$ is a technology shock whose first difference has a positive mean and $z_t^+ \phi$ denotes the amount of production that is sunk (or lost) each period. The economy has two sources of growth: the positive drift in $\log\left(z_t\right)$ and a positive drift in $\log\left(\Psi_t\right)$, where Ψ_t is the state of an investment-specific technology shock discussed below. The object, z_t^+, in (4.1) is defined as $z_t^+ = \Psi_t^{\frac{\alpha}{1-\alpha}} z_t$. Along a non-stochastic steady state growth path, Y_t / z_t^+ and $Y_{i,t}/z_t^+$ converge to constants. The two shocks, z_t and Ψ_t, are specified to be unit root processes in order to be consistent with the assumptions we use in our VAR analysis to identify the dynamic response of the economy to neutral and capital-embodied technology shocks. The two shocks have the following time series representations:

$$\begin{aligned} \Delta \log z_t &= \mu_z + \varepsilon_t^n,\ E\left(\varepsilon_t^n\right)^2 = \left(\sigma_n\right)^2 && (4.2) \\ \Delta \log \Psi_t &= \mu_\psi + \rho_\psi \Delta \log \Psi_{t-1} + \varepsilon_t^\psi,\ E\left(\varepsilon_t^\psi\right)^2 = \left(\sigma_\psi\right)^2. && (4.3) \end{aligned}$$

[27]In his discussion of the NAIRU, Stiglitz (1997) appears to be open to either the first difference or level interpretation of the NAIRU.

[28]Interestingly, u_t^* is a NAIRU under the first difference interpretation if one adopts the price indexation scheme proposed in CEE, see appendix A.1 for the details.

Our assumption that the neutral technology shock follows a random walk with drift matches closely the finding in Smets and Wouters (2007) who estimate $\log z_t$ to be highly autocorrelated. The direct empirical analysis of Prescott (1986) also supports the notion that $\log z_t$ is a random walk with drift.

In (4.1), $H_{i,t}$ denotes homogeneous labor services hired by the i^{th} intermediate good producer. Intermediate good firms must borrow the wage bill in advance of production, so that one unit of labor costs is given by $W_t R_t$, where R_t denotes the gross nominal rate of interest. Intermediate good firms are subject to Calvo price-setting frictions. With probability ξ_p the intermediate good firm cannot reoptimize its price, in which case it is assumed to set its price according to $P_{i,t} = \bar{\pi} P_{i,t-1}$, where $\bar{\pi}$ is the steady state inflation rate. With probability $1-\xi_p$ the intermediate good firm can reoptimize its price. The i^{th} intermediate good producer's profits are:

$$E_t \sum_{j=0}^{\infty} \beta^j \upsilon_{t+j} \{ P_{i,t+j} Y_{i,t+j} - s_{t+j} P_{t+j} Y_{i,t+j} \},$$

where s_t denotes the marginal cost of production, denominated in units of the homogeneous good. The object, s_t, is a function only of the costs of capital and labor as in e.g. CEE. In the firm's discounted profits, $\beta^j \upsilon_{t+j}$ is the multiplier on the family's nominal period $t+j$ budget constraint. The equilibrium conditions associated with this optimization problem are standard so that we shall not display them here.

We suppose that the homogeneous labor hired by intermediate good producers is itself produced by competitive labor contractors. Labor contractors produce homogeneous labor by aggregating different types of specialized labor, $j \in (0,1)$, as follows:

$$H_t = \left[\int_0^1 (h_{t,j})^{\frac{1}{\lambda_w}} dj \right]^{\lambda_w}, \; 1 \leq \lambda_w < \infty. \tag{4.4}$$

Labor contractors take the wage rate of H_t and $h_{t,j}$ as given and equal to W_t and $W_{t,j}$, respectively. Profit maximization by labor contractors leads to the following first order necessary condition:

$$W_{j,t} = W_t \left(\frac{H_t}{h_{t,j}} \right)^{\frac{\lambda_w - 1}{\lambda_w}}. \tag{4.5}$$

Equation (4.5) is the demand curve for the j^{th} type of labor.

4.2. Family and Household Preferences

We integrate the model of unemployment in the previous section into the Erceg, Henderson and Levin (2000) (EHL) model of sticky wages used in the standard DSGE model. Each type, $j \in [0,1]$, of labor is assumed to be supplied by a particular family of households. The j^{th} family resembles the single representative family in the previous section, with one exception. The exception is that the unit measure of households in the j^{th} family is only able to supply the j^{th} type of labor service. Each household in the j^{th} family has the utility cost of working, (2.1), and the technology for job search, (2.2). The five parameters of these functions are: $F, \varsigma_t, \sigma_L, a, \eta$, where the first three pertain to the cost of working and the last two pertain to job search. In the analysis of the empirical model, the preference shock, ς_t, is constant. We assume that these parameters are identical across families.

In order that the representative family in the current section have habit persistence in consumption, we change the way consumption enters the additive utility function of the household. In particular, we replace $\log(c_t^{nw})$ and $\log(c_t^w)$ everywhere in the previous section with $\log\left(c_{j,t}^{nw} - bC_{t-1}\right)$, $\log\left(c_{j,t}^{w} - bC_{t-1}\right)$, respectively. Here, C_{t-1} denotes the family's previous period's level of consumption. When the parameter, b, is positive, then each household in the family has habit in consumption. Also, $c_{j,t}^{nw}$ and $c_{j,t}^{w}$ denote the consumption levels allocated by the j^{th} family to non-employed and employed households within the family. Although families all enjoy the same level of consumption, C_t, for reasons described momentarily each family experiences a different level of employment, $h_{j,t}$. Because employment across families is different, each type j family chooses a different way to balance the trade-off between the need for consumption insurance and the need to provide work incentives. For the j^{th} type of family with high $h_{j,t}$, the premium of consumption for working households to non-working households must be high. It is easy to verify that the incentive constraint in the version of the model considered here is the analog of (2.8):

$$\log\left[\frac{c_{j,t}^{w} - bC_{t-1}}{c_{j,t}^{nw} - bC_{t-1}}\right] = F + \varsigma\left(1+\sigma_L\right) m_{j,t}^{\sigma_L},$$

where $m_{j,t}$ solves the analog of (2.10):

$$h_{j,t} = m_{j,t}\eta + a^2 \varsigma \sigma_L m_{j,t}^{\sigma_L+1}. \tag{4.6}$$

Consider the j^{th} family that enjoys a level of family consumption and employment, C_t and $h_{j,t}$, respectively. It is readily verified that the utility of this family, after it efficiently allocates consumption across its member households subject to the private information constraints, is given by:

$$u\left(C_t - bC_{t-1}, h_{j,t}\right) = \log\left(C_t - bC_{t-1}\right) - z\left(h_{j,t}\right), \tag{4.7}$$

where the z function in (4.7) is defined in (2.19) with ς_t replaced by ς. The j^{th} family's discounted utility is:

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t - bC_{t-1}, h_{j,t}\right). \tag{4.8}$$

Note that this utility function is additively separable, like the utility functions assumed for the households. Additive separability is convenient because perfect consumption insurance at the level of families implies that consumption is not indexed by labor type, j. As implied by our results below, this simplification appears not to have come at a cost in terms of accounting for aggregate data.[29]

4.3. The Family Problem

The j^{th} family is the monopoly supplier of the j^{th} type of labor service. The family understands that when it arranges work incentives for its households so that employment is $h_{j,t}$, then $W_{j,t}$ takes on the value implied by the demand for its type of labor, (4.5). The family therefore faces the standard monopoly problem of selecting $W_{j,t}$ to optimize the welfare, (4.8), of its member households. It does so, subject to the requirement that it satisfies the demand for labor, (4.5), in each period. We follow EHL in supposing that the family experiences Calvo-style frictions in its choice of $W_{j,t}$. In particular, with probability $1 - \xi_w$ the j^{th} family has the opportunity to reoptimize its wage rate. With the complementary probability, the family must set its wage rate according to the following rule:

$$W_{j,t} = \tilde{\pi}_{w,t} W_{j,t-1} \tag{4.9}$$

$$\tilde{\pi}_{w,t} = (\pi_{t-1})^{\kappa_w} (\bar{\pi})^{(1-\kappa_w)} \mu_{z^+}, \tag{4.10}$$

where $\kappa_w \in (0,1)$. Note that in a non-stochastic steady state, non-optimizing families raise their real wage at the rate of growth of the economy. Because optimizing families also do this in steady state, it follows that in the steady state, the wage of each type of family is the same.

In principle, the presence of wage setting frictions implies that families have idiosyncratic levels of wealth and, hence, consumption. However, we follow EHL in supposing that each family has access to perfect consumption insurance. At the level of the family, there is no private information about consumption or employment. The private information and associated incentive problems all exist among the households inside a family. Because of the additive separability of the family utility function, perfect consumption insurance at the level of families implies equal consumption across families. We have used this property of the equilibrium to simplify our notation and not include a subscript, j, on the j^{th} family's consumption. Of course, we hasten to add that although consumption is equated across families, it is not constant across households.

The j^{th} family's period t budget constraint is as follows:

$$P_t \left(C_t + \frac{1}{\Psi_t} I_t \right) + B_{t+1} \leq W_{t,j} h_{t,j} + X_t^k \bar{K}_t + R_{t-1} B_t + a_{jt}. \tag{4.11}$$

[29] Still, it would be interesting to explore the implications of non-separable utility. The technical appendix to this paper derives (4.7) for two non-separable specifications of utility for households. Moreover, Guerron-Quintana (2008) shows how to handle the fact that family consumption is now indexed by j.

Here, B_{t+1} denotes the quantity of risk-free bonds purchased by the family, R_t denotes the gross nominal interest rate on bonds purchased in period $t-1$ which pay off in period t, and a_{jt} denotes the payments and receipts associated with the insurance on the timing of wage reoptimization. Also, P_t denotes the aggregate price level and I_t denotes the quantity of investment goods purchased for augmenting the beginning-of-period $t+1$ stock of physical capital, $\bar{K}_{t+1}$. The price of investment goods is P_t/Ψ_t, where Ψ_t is the unit root process with positive drift specified in (4.3). This is our way of capturing the trend decline in the relative price of investment goods.[30]

The family owns the economy's physical stock of capital, $\bar{K}_t$, sets the utilization rate of capital and rents the services of capital in a competitive market. The family accumulates capital using the following technology:

$$\bar{K}_{t+1} = (1-\delta)\,\bar{K}_t + \left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right) I_t. \tag{4.12}$$

Here, S is a convex function, with S and S' equal to zero on a steady state growth path.[31] The function has one free parameter, its second derivative in the neighborhood of steady state, which we denote simply by S''.

For each unit of $\bar{K}_{t+1}$ acquired in period t, the family receives X_{t+1}^k in net cash payments in period $t+1$,

$$X_{t+1}^k = u_{t+1}^k P_{t+1} r_{t+1}^k - \frac{P_{t+1}}{\Psi_{t+1}} a(u_{t+1}^k), \tag{4.13}$$

where u_t^k denotes the rate of utilization of capital. The first term in (4.13) is the gross nominal period $t+1$ rental income from a unit of $\bar{K}_{t+1}$. The family supply of capital services in period $t+1$ is $K_{t+1} = u_{t+1}^k \bar{K}_{t+1}$. It is the services of capital that intermediate good producers rent and use in their production functions, (4.1). The second term to the right of the equality in (4.13) represents the cost of capital utilization, $a(u_{t+1}^k)P_{t+1}/\Psi_{t+1}$. This function is constructed so that the steady state value of utilization is unity, and $u(1) = u'(1) = 0$. The function has one free parameter, which we denote by σ_a. Here, $\sigma_a = a''(1)/a'$ and corresponds to the curvature of u in steady state.[32]

The family's problem is to select sequences, $\left\{C_t, I_t, u_t^k, W_{j,t}, B_{t+1}, \bar{K}_{t+1}\right\}$, to maximize (4.8) subject to (4.5), (4.9), (4.10), (4.11), (4.12), (4.13) and the mechanism determining when wages can be reoptimized. The equilibrium conditions associated with this maximization problem are standard and are available in a technical appendix.

[30] We suppose that there is an underlying technology for converting final goods, Y_t, one-to-one into C_t and one to Ψ_t into investment goods. These technologies are operated by competitive firms which equate price to marginal cost. The marginal cost of C_t with this technology is P_t and the marginal cost of I_t is P_t/Ψ_t. We avoid a full description of this environment so as to not clutter the presentation, and simply impose these properties of equilibrium on the family budget constraint.

[31] In particular, we assume $S\left(\frac{I_t}{I_{t-1}}\right) = \frac{1}{2}\left\{\exp\left[\sqrt{S''}\left(\frac{I_t}{I_{t-1}} - \mu_{z^+}\mu_\Psi\right)\right] + \exp\left[-\sqrt{S''}\left(\frac{I_t}{I_{t-1}} - \mu_{z^+}\mu_\Psi\right)\right] - 2\right\}$.

[32] We assume the following functional form: $a(u_t^k) = 0.5\sigma_b\sigma_a(u_t^k)^2 + \sigma_b(1-\sigma_a)u_t^k + \sigma_b((\sigma_a/2)-1)$.

4.4. Aggregate Resource Constraint, Monetary Policy and Equilibrium

Goods market clearing dictates that the homogeneous output good is allocated among alternative uses as follows:

$$Y_t = G_t + C_t + \tilde{I}_t. \tag{4.14}$$

Here, C_t denotes family consumption, G_t denotes exogenous government consumption and $\tilde{I}_t$ is a homogenous investment good which is defined as follows:

$$\tilde{I}_t = \frac{1}{\Psi_t}\left(I_t + a\left(u_t^k\right)\bar{K}_t\right). \tag{4.15}$$

As discussed above, the investment goods, I_t, are used by the families to add to the physical stock of capital, $\bar{K}_t$, according to (4.12). The remaining investment goods are used to cover maintenance costs, $a\left(u_t^k\right)\bar{K}_t$, arising from capital utilization, u_t^k. Finally, Ψ_t in (4.15) denotes the unit root investment specific technology shock with positive drift discussed after (4.1).

We suppose that monetary policy follows a Taylor rule of the following form:

$$\log\left(\frac{R_t}{R}\right) = \rho_R \log\left(\frac{R_{t-1}}{R}\right) + (1-\rho_R)\left[r_\pi \log\left(\frac{E_t\pi_{t+1}}{\pi}\right) + r_y \log\left(\frac{gdp_t}{gdp}\right)\right] + \frac{\varepsilon_{R,t}}{4R}, \tag{4.16}$$

where $\varepsilon_{R,t}$ is an iid monetary policy shock. As in CEE and ACEL, we assume that period t realizations of ε_R are not included in the period t information set of households and firms. Further, gdp_t denotes scaled real GDP defined as:

$$gdp_t = \frac{G_t + C_t + I_t/\Psi_t}{z_t^+}, \tag{4.17}$$

and gdp denotes the nonstochastic steady state value of gdp_t. We adopt the model of government spending suggested in Christiano and Eichenbaum (1992), i.e. $G_t = gz_t^+$. Finally, lump-sum transfers are assumed to balance the government budget.

An equilibrium is a stochastic process for prices and quantities having the property that the family and firm problems are satisfied, and goods, capital and labor markets clear.

4.5. Aggregate Labor Force and Unemployment in Our Model

We now derive our model's implications for unemployment and the labor force. At the level of the j^{th} family, unemployment and the labor force are defined in the same way as in the previous section, except that the endogenous variables now have a j subscript (the parameters and shocks are the same across families). Thus, the j^{th} family's labor force, $m_{j,t}$, and total employment, $h_{j,t}$, are related by (2.10) (or, (4.6)). We linearize the latter expression as in (3.24):

$$\hat{m}_{j,t} = \frac{1-u}{1-u+a^2\varsigma\sigma_L^2 m^{\sigma_L}}\hat{h}_{j,t}, \tag{4.18}$$

though we ignore $\hat{\varsigma}_t$. Also, u and m denote the steady state values of unemployment and the labor force in the j^{th} family. Because we have made assumptions which guarantee that each family is identical in steady state, we drop the j subscripts from all steady state labor market variables (see the discussion after (4.9)).

Aggregate household hours and the labor force are defined as $h_t \equiv \int_0^1 h_{j,t}dj$ and $m_t \equiv \int_0^1 m_{j,t}dj$. Totally differentiating gives $\hat{h}_t = \int_0^1 \hat{h}_{j,t}dj$ and $\hat{m}_t \equiv \int_0^1 \hat{m}_{j,t}dj$. Using the fact that, to first order, type j wage deviations from the aggregate wage cancel, we obtain:

$$\hat{h}_t = \hat{H}_t. \tag{4.19}$$

That is, to a first order approximation, the percent deviation of aggregate household hours from steady state coincides with the percent deviation of aggregate homogeneous hours from steady state. Integrating (4.18) over all j :

$$\hat{m}_t = \int_0^1 \hat{m}_{j,t}dj = \frac{1-u}{1-u+a^2\varsigma\sigma_L^2 m^{\sigma_L}}\hat{H}_t.$$

Aggregate unemployment is defined as follows:

$$u_t \equiv \frac{m_t - h_t}{m_t},$$

so that

$$du_t = \frac{h}{m}\left(\hat{m}_t - \hat{h}_t\right).$$

Here, du_t denotes the deviation of unemployment from its steady state value, not the percent deviation.

4.6. The Standard Model

We derive the utility function used in the standard model as a special case of the family utility function in our involuntary unemployment model. In part, we do this to ensure consistency across models. In part, we do this as a way of emphasizing that we interpret the labor input in the utility function in the standard model as corresponding to the number of people working, not, say, the hours worked of a representative person. With our interpretation, the curvature of the labor disutility function corresponds to the (consumption compensated) elasticity with which people enter or leave the labor force in response to a change in the wage rate. In particular, this curvature does not correspond to the elasticity with which the typical person adjusts the quantity of hours worked in response to a wage change. Empirically, the latter elasticity is estimated to be small and it is fixed at zero in the model.

Another advantage of deriving the standard model from ours is that it puts us in position to exploit an insight by Galí (2011). In particular, Galí (2011) shows that the standard model already has a theory of unemployment implicit in it. The monopoly power assumed by EHL has the consequence that wages are on average higher than what they would be under competition. The number of workers for which the wage is greater than the cost of work exceeds the number of people employed. Galí suggests defining this excess of workers as unemployed. The implied unemployment rate and labor force represent a natural benchmark to compare with our model.

Notably, deriving an unemployment rate and labor force in the standard model does not introduce any new parameters. Moreover, there is no change in the equilibrium conditions that determine non-labor market variables. Galí's insight in effect simply adds a block recursive system of two equations to the standard DSGE model which determine the size of the labor force and unemployment. Although the unemployment rate derived in this way does not satisfy all the criteria for unemployment that we described in the introduction, it nevertheless provides a natural benchmark for comparison with our model. An extensive comparison of the economics of our approach to unemployment versus the approach implicit in the standard model appears in the appendix A.2 of this paper.

We suppose that the family has full information about its member households and that households which join the labor force automatically receive a job without having to expend any effort. As in the previous subsections, we suppose that corresponding to each type j of labor, there is a unit measure of households which gather together into a family. At the beginning of each period, each household draws a random variable, l, from a uniform distribution with support, $[0,1]$. The random variable, l, determines a household's aversion to work according to (2.1), with $F=0$. The fact that no effort is needed to find a job implies $m_{t,j}=h_{t,j}$. Households with $l \leq h_{t,j}$ work and households with $h_{t,j} \leq l \leq 1$ take leisure. The type j family allocation problem is to maximize the utility of its member households with respect to consumption for non-working households, $c_{t,j}^{nw}$, and consumption of working households, $c_{t,j}^{w}$, subject to (2.14), and the given values of $h_{t,j}$ and C_t. In Lagrangian form, the problem is:

$$u\left(C_t - bC_{t-1}, h_{j,t}\right) = \max_{c_{t,j}^{w}, c_{t,j}^{nw}} \int_0^{h_{t,j}} \left[\log\left(c_{t,j}^{w} - bC_{t-1}\right) - \varsigma\left(1+\sigma_L\right) l^{\sigma_L}\right] dl$$

$$+ \int_{h_{t,j}}^{1} \log\left(c_{t,j}^{nw} - bC_{t-1}\right) dl + \lambda_{j,t}\left[C_t - h_{t,j}c_{t,j}^{w} - \left(1-h_{t,j}\right)c_{t,j}^{nw}\right].$$

Here, $\lambda_{j,t}>0$ denotes the multiplier on the resource constraint. The first order conditions imply $c_{t,j}^{w} = c_{t,j}^{nw} = C_t$. Imposing this result and evaluating the integral, we find:

$$u\left(C_t - bC_{t-1}, h_{j,t}\right) = \log\left(C_t - bC_{t-1}\right) - \varsigma h_{t,j}^{1+\sigma_L}. \tag{4.20}$$

The problem of the family is identical to what it is in section 4.3, with the sole exception that the utility function, (4.7), is replaced by (4.20).

A type j household that draws work aversion index l is defined to be unemployed if the following two conditions are satisfied:

$$(a)\ l > h_{j,t},\ (b)\ \upsilon_t W_{j,t} > \varsigma l^{\sigma_L}. \tag{4.21}$$

Here, υ_t denotes the multiplier on the budget constraint, (4.11), in the Lagrangian representation of the family optimization problem. Expression (a) in (4.21) simply says that to be unemployed, the household must not be employed. Expression (b) in (4.21) determines whether a non-employed household is unemployed or not in the labor force. The object on the left of the inequality in (b) is the value assigned by the family to the wage, $W_{j,t}$. The object on the right of (b) is the fixed cost of going to work for the l^{th} household. Galí (2011) suggests defining households with l satisfying (4.21) as unemployed. This approach to unemployment does not satisfy properties (i) and (iii) in the introduction. The approach does not meet the official definition of unemployment because no one is exercising effort to find a job. In addition, the existence of perfect consumption insurance implies that unemployed workers enjoy higher utility that employed workers.

We use (4.21) to define the labor force, l_t^*, in the standard model. With l_t^* and aggregate employment, h_t, we obtain unemployment as follows

$$u_t = \frac{l_t^* - h_t}{l_t^*},$$

or, after linearization about steady state:

$$du_t = \frac{h}{l^*}\left(\hat{l}_t^* - \hat{h}_t\right).$$

Here, $h < l^*$ because of the presence of monopoly power. The object, $\hat{h}_t$ may be obtained from (4.19) and the solution to the standard model. We now discuss the computation of the aggregate labor force, l_t^*. We have $l_t^* \equiv \int_0^1 l_{j,t}^* dj$, where $l_{j,t}^*$ is the labor force associated with the j^{th} type of labor and is defined by enforcing (b) in (4.21) at equality. After linearization,

$$\hat{l}_t^* \equiv \int_0^1 \hat{l}_{j,t}^* dj.$$

We compute $\hat{l}_{j,t}^*$ by linearizing the equation that defines $l_{j,t}^*$. After scaling we obtain

$$\psi_{z^+,t}\bar{w}_t \mathring{w}_{j,t} = \varsigma\left(l_{j,t}^*\right)^{\sigma_L}, \tag{4.22}$$

where $\psi_{z^+,t} \equiv \upsilon_t P_t z_t^+$, $\bar{w}_t \equiv \frac{W_t}{z_t^+ P_t}$, $\mathring{w}_{j,t} \equiv \frac{W_{j,t}}{W_t}$. Linearizing (4.22) about steady state and integrating the result over all $j \in (0,1)$ yields $\hat{\psi}_{z^+,t} + \widehat{\bar{w}}_t + \int_0^1 \widehat{\mathring{w}}_{j,t} dj = \sigma_L \hat{l}_t^*$. It is straightforward to show that the integral in the above expression is zero, so that:

$$\sigma_L \hat{l}_t^* = \hat{\psi}_{z^+,t} + \widehat{\bar{w}}_t.$$

5. Estimation Strategy

We estimate the parameters of the model in the previous section using the impulse response matching approach applied by Rotemberg and Woodford (1997), CEE, ACEL and other papers. We apply the Bayesian version of that method proposed in CTW. To promote comparability of results across the two papers and to simplify the discussion here, we use the impulse response functions and associated probability intervals estimated using the 14 variable, 2 lag vector autoregression (VAR) estimated in CTW. Here, we consider the response of 11 variables to three shocks: the monetary policy shock, $\varepsilon_{R,t}$ in equation (4.16), the neutral technology shock, ε_t in equation (4.2), and the investment specific shock, ε_t^{Ψ} in equation (4.3).[33] Nine of the eleven variables whose responses we consider are the standard macroeconomic variables displayed in Figures 1-3. The other two variables are the unemployment rate and the labor force which are shown in Figure 4. The VAR is estimated using quarterly, seasonally adjusted data covering the period 1952Q1 to 2008Q4.

The assumptions that allow us to identify the effects of our three shocks are the ones implemented in ACEL and Fisher (2006). To identify the monetary policy shock we suppose all variables aside from the nominal rate of interest are unaffected contemporaneously by the policy shock. We make two assumptions to identify the dynamic response to the technology shocks: (i) the only shocks that affect labor productivity in the long run are the two technology shocks and (ii) the only shock that affects the price of investment relative to consumption is the innovation to the investment specific shock. All these identification assumptions are satisfied in our model. Details of our strategy for computing impulse response functions imposing the shock identification are discussed in ACEL.

Let $\hat{\psi}$ denote the vector of impulse responses used in the analysis here. Since we consider 15 lags in the impulses, there are in principle 3 (i.e., the number of shocks) times 11 (number of variables) times 15 (number of lags) = 495 elements in $\hat{\psi}$. However, we do not include in $\hat{\psi}$ the 10 contemporaneous responses to the monetary policy shock that are required to be zero by our monetary policy identifying assumption. Taking the latter into account, the vector $\hat{\psi}$ has 485 elements. To conduct a Bayesian analysis, we require a likelihood function for our 'data', $\hat{\psi}$. For this, we use an approximation based on asymptotic sampling theory. In particular, when the number of observations, T, is large, we have

$$\sqrt{T}\left(\hat{\psi}-\psi\left(\theta_0\right)\right) \overset{a}{\sim} N\left(0, W\left(\theta_0, \zeta_0\right)\right). \tag{5.1}$$

Here, θ_0 and ζ_0 are the parameters of the model that generated the data, evaluated at their true values. The parameter vector, θ_0, is the set of parameters that is explicit in our model,

[33] The VAR in CTW also includes data on vacancies, job findings and job separations, but these variables do not appear in the models in this paper and so we do not include their impulse responses in the analysis.

while ζ_0 contains the parameters of stochastic processes not included in the analysis. In (5.1), $W(\theta_0, \zeta_0)$ is the asymptotic sampling variance of $\hat{\psi}$, which - as indicated by the notation - is a function of all model parameters. We find it convenient to express (5.1) in the following form:

$$\hat{\psi} \overset{a}{\sim} N\left(\psi\left(\theta_0\right), V\left(\theta_0, \zeta_0, T\right)\right), \tag{5.2}$$

where $V(\theta_0, \zeta_0, T) \equiv \frac{W(\theta_0, \zeta_0)}{T}$. We treat $V(\theta_0, \zeta_0, T)$ as though it were known. In practice, we work with a consistent estimator of $V(\theta_0, \zeta_0, T)$ in our analysis (for details, see CTW). That estimator is a diagonal matrix with only the variances along the diagonal. An advantage of this diagonality property is that our estimator has a simple graphical representation.

We treat the following object as the likelihood of the 'data', $\hat{\psi}$, conditional on the model parameters, θ :

$$\begin{aligned} f\left(\hat{\psi}|\theta, V\left(\theta_0, \zeta_0, T\right)\right) &= \left(\frac{1}{2\pi}\right)^{\frac{N}{2}} \left|V\left(\theta_0, \zeta_0, T\right)\right|^{-\frac{1}{2}} \\ &\times \exp\left[-\frac{1}{2}\left(\hat{\psi} - \psi\left(\theta\right)\right)' V\left(\theta_0, \zeta_0, T\right)^{-1}\left(\hat{\psi} - \psi\left(\theta\right)\right)\right]. \end{aligned} \tag{5.3}$$

The Bayesian posterior of θ conditional on $\hat{\psi}$ and $V(\theta_0, \zeta_0, T)$ is:

$$f\left(\theta|\hat{\psi}, V\left(\theta_0, \zeta_0, T\right)\right) = \frac{f\left(\hat{\psi}|\theta, V\left(\theta_0, \zeta_0, T\right)\right) p\left(\theta\right)}{f\left(\hat{\psi}|V\left(\theta_0, \zeta_0, T\right)\right)}, \tag{5.4}$$

where $p(\theta)$ denotes the priors on θ and $f\left(\hat{\psi}|V\left(\theta_0, \zeta_0, T\right)\right)$ denotes the marginal density of $\hat{\psi}$:

$$f\left(\hat{\psi}|V\left(\theta_0, \zeta_0, T\right)\right) = \int f\left(\hat{\psi}|\theta, V\left(\theta_0, \zeta_0, T\right)\right) p\left(\theta\right) d\theta.$$

As usual, the mode of the posterior distribution of θ can be computed by simply maximizing the value of the numerator in (5.4), since the denominator is not a function of θ. The marginal density of $\hat{\psi}$ is required when we want an overall measure of the fit of our model and when we want to report the shape of the posterior marginal distribution of individual elements in θ. We do this using the MCMC algorithm.

6. Estimation Results for the Medium-sized Model

The first subsection discusses model parameter values. We then show that our model of involuntary unemployment does well at accounting for the dynamics of unemployment and the labor force. Fortunately, the model is able to do this without compromising its ability to account for the dynamics of standard macroeconomic variables.

6.1. Parameters

Parameters whose values are set a priori are listed in Table 1. We found that when we estimated the parameters, κ_w and λ_w, the estimator drove them to their boundaries. This is why we simply set λ_w to a value near unity and we set $\kappa_w = 1$. The steady state value of inflation (a parameter in the monetary policy rule and the price and wage updating equations), the steady state government consumption to output ratio, and the growth rate of investment-specific technology were chosen to coincide with their corresponding sample means in our data set.[34] The growth rate of neutral technology was chosen so that, conditional on the growth rate of investment-specific technology, the steady state growth rate of output in the model coincides with the corresponding sample average in the data. We set $\xi_w = 0.75$, so that the model implies wages are reoptimized once a year on average. We did not estimate this parameter because we found that it is difficult to separately identify the value of ξ_w and the curvature of family labor disutility. Finally, to ensure that we only consider parameterizations that imply an admissible probability function, $p(e_l)$, we simply fix the maximal value of this probability in steady state, $\bar{p}$, to 0.97 (see (2.12)).

The parameters for which we report priors and posteriors are listed in Table 2. We report results for two estimation exercises. In the first exercise we estimate the standard DSGE model discussed in section 4.6. In this exercise we only use the impulse responses of standard macroeconomic variables in the likelihood criterion, (5.3). In particular, we do not include the impulse responses of the unemployment rate or the labor force when we estimate the standard DSGE model.[35] Results based on this exercise appear under the heading, 'standard model'. In the second exercise we estimate our model with involuntary unemployment and we report those results under the heading, 'involuntary unemployment model'.

We make several observations about the parameters listed in Table 2. First, the results in the last two columns are similar. This reflects that the two models (i) are observationally equivalent relative to the impulse responses of standard macroeconomic variables and (ii) no substantial adjustments to the parameters are required for the involuntary unemployment model to fit the unemployment and labor force data.

Second, the list of household parameters contains one endogenous parameter, the curvature of utility, σ_z, defined in (3.15). Moreover, the list seems to be missing the structural parameters of the search technology and disutility of labor. We begin by explaining this in the context of the involuntary unemployment model. Throughout the estimation, we fix the steady state unemployment rate, u, at its sample average, 0.056, and the steady state labor

[34] In our model, the relative price of investment goods represents a direct observation of the technology shock for producing investment goods.

[35] Subection 6.3 discusses the implications when unemployment and the labor force are included in the estimation in the standard model.

force participation rate, m, at a value of 2/3. For given values of the four objects, σ_z, $\bar{p}$, u and m, we can uniquely compute values for:[36] F, ς, a, η.

This is why σ_z is included in the list of estimated parameters in Table 2, while the four parameters listed above are not. The parameter, σ_L, appears in Table 2 because it is distinct from σ_z and separately identifiable. We apply an analogous treatment to household parameter values in the case of the standard model. In particular, throughout estimation we fix the steady state level of hours worked, h, to the value implicit in the u and m used for the involuntary unemployment model. We choose the value of ς so that conditional on the other standard model parameter values, steady state hours worked coincides with h. Since $\sigma_z = \sigma_L$ in the standard model (see (4.20)), we only report estimation results for σ_z in Table 2.

Turning to the parameter values themselves, note first that the degree of price stickiness, ξ_p, is modest. The implied time between price reoptimizations is a little less than 3 quarters. The amount of information in the likelihood, (5.3), about the value of ξ_p is reasonably large. The posterior standard deviation is roughly an order of magnitude smaller than the prior standard deviation and the posterior probability interval is half the length of the prior probability interval. Generally, the amount of information in the likelihood about all the parameters is large in this sense. An exception to this pattern is the coefficient on inflation in the Taylor rule, r_π. There appears to be relatively little information about this parameter in the likelihood. Note that σ_z is estimated to be quite small, implying a consumption-compensated labor supply elasticity for the family of around 8. Such a high elasticity would be regarded as empirically implausible if it were interpreted as the elasticity of supply of hours by a representative agent. However, as discussed above, this is not our interpretation.

Table 3 reports steady state properties of the two models, evaluated at the posterior mean of the parameters. According to the results, the capital output ratio is a little lower than the empirical value of 12 typically reported in the real business cycle literature. The consumption replacement ratio, c^{nw}/c^{w}, is a novel feature of our model, that does not appear in standard monetary DSGE models. The replacement ratio is estimated to be roughly 80 percent. This coincides with the value used for calibration by Landais, Michaillat and Saez (2012). It is higher than the estimates of Hamermesh (1982) but somewhat lower than the empirical estimate of 90 percent reported by Chetty and Looney (2006) and Gruber (1997) and mentioned in the introduction. Also, our consumption replacement ratio appears to be higher than the number reported for developed countries in OECD (2006). However, the replacement ratios reported by OECD pertain to income, rather than consumption.[37] So,

[36] For details, see section E.4.2 in the technical appendix to this paper.

[37] The income replacement ratio for the US is reported to be 54 percent in Table 3.2, which can be found at http://www.oecd.org/dataoecd/28/9/36965805.pdf.

they are likely to underestimate the consumption concept relevant for us.

Not surprisingly, our model's implications for the consumption replacement ratio is very sensitive to the habit persistence parameter, b. If we set the value of that parameter to zero, then our model's steady state replacement ratio drops to 20 percent. Essentially, habit persistence adds curvature to the utility function and thereby increases households desire for insurance, i.e. a higher replacement ratio.

6.2. Impulse Response Functions of Non-labor Market Variables

Figures 1-3 display the results of the indicated macroeconomic variables to our three shocks. In each case, the solid black line is the point estimate of the dynamic response generated by our estimated VAR. The grey area is an estimate of the corresponding 95% probability interval.[38] Our estimation strategy selects a model parameterization that places the model-implied impulse response functions as close as possible to the center of the grey area, while not suffering too much of a penalty from the priors. The estimation criterion is less concerned about reproducing VAR-based impulse response functions where the grey areas are the widest.

The thick solid line and the line with solid squares in the figures display the impulse responses of the standard model and the involuntary unemployment model, respectively, at the posterior mean of the parameters. Note in Figures 1-3 that in many cases only one of these two lines is visible. Moreover, in cases where a distinction between the two lines can be discerned, they are nevertheless very close. This reflects that the two models account roughly equally well for the impulse responses to the three shocks. This is a key result. Expanding the standard model to include unemployment and the labor force does not produce a deterioration in the model's ability to account for the estimated dynamic responses of standard macroeconomic variables to monetary policy and technology shocks.

Consider Figure 1, which displays the response of standard macroeconomic variables to a monetary policy shock. Note how the model captures the slow response of inflation. Indeed, the model even captures the 'price puzzle' phenomenon, according to which inflation moves in the 'wrong' direction initially. This apparently perverse initial response of inflation is interpreted by the model as reflecting the reduction in labor costs associated with the cut in the nominal rate of interest.[39] It is interesting that the slow response of inflation is accounted

[38] We compute the probability interval as follows. We simulate 2,500 sets of impulse response functions by generating an equal number of artificial data sets, each of length T, using the VAR estimated from the data. Here, T denotes the number of observations in our actual data set. We compute the standard deviations of the artificial impulse response functions. The grey areas in Figures 1-5 are the estimated impulse response functions plus and minus 1.96 times the corresponding standard deviation.

[39] For a defense, based on firm-level data, of the existence of this 'working capital' channel of monetary policy, see Barth and Ramey (2001).

for with a fairly modest degree of wage and price-setting frictions. The model captures the response of output and consumption to a monetary policy shock reasonably well. However, there is a substantial miss on capacity utilization. Also, the model apparently does not have the flexibility to capture the relatively sharp rise and fall in the investment response, although the model responses lie inside the grey area. The relatively large estimate of the curvature in the investment adjustment cost function, S'', reflects that to allow a greater response of investment to a monetary policy shock would cause the model's prediction of investment to lie outside the grey area in the initial and later quarters. These findings for monetary policy shocks are broadly similar to those reported in CEE, ACEL and CTW.

Figure 2 displays the response of standard macroeconomic variables to a neutral technology shock. Note that the models do reasonably well at reproducing the empirically estimated responses. The dynamic response of inflation is particularly notable. The estimation results in ACEL suggest that the sharp and precisely estimated drop in inflation in response to a neutral technology shock is difficult to reproduce in a model like the standard monetary DSGE model. In describing this problem for their model, ACEL express a concern that the failure reflects a deeper problem with sticky price models. Perhaps the emphasis on price and wage setting frictions, largely motivated by the inertial response of inflation to a monetary shock, is shown to be misguided by the evidence that inflation responds rapidly to technology shocks.[40] Our results suggest a far more mundane possibility. There are two differences between our model and the one in ACEL which allow it to reproduce the response of inflation to a technology shock more or less exactly without hampering its ability to account for the slow response of inflation to a monetary policy shock. First, as discussed above, in our model there is no indexation of prices to lagged inflation. ACEL follows CEE in supposing that when firms cannot optimize their price, they index it fully to lagged aggregate inflation. The position of our model on price indexation is a key reason why we can account for the rapid fall in inflation after a neutral technology shock while ACEL cannot. We suspect that our way of treating indexation is a step in the right direction from the point of view of microeconomic data. Micro observations suggest that individual prices do not change for extended periods of time. A second distinction between our model and the one in ACEL is that we specify the neutral technology shock to be a random walk (see (4.2)), while in ACEL the growth rate of the estimated technology shock is highly autocorrelated. In ACEL, a technology shock triggers a strong wealth effect which stimulates a surge in demand that

[40] The concern is reinforced by the fact that an alternative approach, one based on information imperfections and minimal price/wage setting frictions, seems like a natural one for explaining the puzzle of the slow response of inflation to monetary policy shocks and the quick response to technology shocks (see Maćkowiak and Wiederholt, 2009, Mendes, 2009, and Paciello, 2009). Dupor, Han and Tsai (2009) suggest more modest changes in the model structure to accommodate the inflation puzzle.

places upward pressure on marginal cost and thus inflation.[41]

Figure 3 displays dynamic responses of macroeconomic variables to an investment-specific shock. The evidence indicates that the two models, parameterized at their posterior means, do well in accounting for these responses.

6.3. Impulse Response Functions of Unemployment and the Labor Force

Figure 4 displays the response of unemployment and the labor force to our three shocks. The key thing to note is that the model has no difficulty accounting for the pattern of responses. The probability bands are large, but the point estimates suggest that unemployment falls about 0.2 percentage points and the labor force rises a small amount after an expansionary monetary policy shock. The model roughly reproduces this pattern. In the case of each response, the model generates opposing movements in the labor force and the unemployment rate. This appears to be consistent with the evidence.

As discussed in section 4.6 above, Galí (2011) points out that the standard model has implicit in it a theory of unemployment and the labor force. Figure 5 adds the implications of the standard model for these variables to the impulses displayed in Figure 4 when data for unemployment and the labor force are not part of the dataset used in the standard model. Note that the impulses implied by the standard model are so large that they distort the scale in Figure 5. Consider, for example, the first panel of graphs in the figure, which pertain to the monetary policy shock. The standard model predicts a massive fall in the labor force after an expansionary monetary policy shock. The reason is that the rise in aggregate consumption (see Figure 1) reduces the value of work by reducing υ_t in (4.21). The resulting sharp drop in labor supply strongly contradicts our VAR-based evidence which suggests a small rise. Given the standard model's prediction for the labor force, it is not surprising that the model massively over-predicts the fall in the unemployment rate after a monetary expansion.

An alternative approach to deduce the implications of the standard model for unemployment and the labor force is to impose the corresponding VAR impulse responses in the estimation. When doing so, the unemployment rate and labor force responses are virtually flat after the monetary shock, which is counterfactual with respect to the VAR evidence. Basically, the estimation procedure selects parameters such that the unemployment rate and labor force do not fall as much as displayed in Figure 5.[42] However, selecting parameters

[41] An additional, important, factor accounting for the damped response of inflation to a monetary policy shock (indeed, the perverse initial 'price puzzle' phenomenon) is the assumption that firms must borrow in advance to pay for their variable production costs. But, this model feature is present in both our model and ACEL as well as CEE.

[42] Note that the standard model is not able to generate a rise in the labor force after an expansionary monetary policy shock. Thus, the "best" response possible in that model is a zero labor force response to the monetary shock.

in the standard model to basically shut down the responses of unemployment and the labor force comes at a heavy cost: the fit of all other macroeconomic data deteriorates sharply in this case. See Figures A1 to A4 in the appendix for the details. Therefore, our model of involuntary unemployment outperforms the standard model when both models face the same dataset including unemployment and the labor force. Quantitatively, the log data density at the posterior mean for our model is -75.1 while the one for the standard model is -294.1. See section A.3 for an in-depth discussion of the underlying estimation results for the standard model in this case.

The failure of the standard model raises a puzzle. Why does our involuntary unemployment model do so well at accounting for the unemployment rate and the labor force? The puzzle is interesting because the two models share essentially the same utility function at the level of the household. One might imagine that our model would have the same problem with wealth effects. In fact, it does not have the same problem because there is a connection in our model between the labor force and employment that does not exist in the standard model. In our model, the increased consumption premium from holding a job that occurs in response to an expansionary monetary policy shock simultaneously encourages households to search for work more intensely, and to substitute into the labor force.

The standard model's prediction for the response of the unemployment rate and the labor force to neutral and investment-specific technology shocks is also strongly counterfactual. The problem is always the same, and reflects the operation of wealth effects on labor supply.

The problems in Figure 5 with the standard model motivate Galí (2011) and Galí, Smets and Wouters (2011) to modify the household utility function in the standard model in ways that reduce wealth effects on labor. In effect, our involuntary unemployment model represents an alternative strategy for dealing with these wealth effects. Our model has the added advantage of being consistent with all three characteristics (i)-(iii) of unemployment described in the introduction.

7. Further Evidence in Favour of Our Model

Our model of unemployment has several interesting microeconomic implications that deserve closer attention. The model implies that the consumption premium of employed workers over the non-employed, c_t^w/c_t^{nw}, is procyclical or, equivalently, the replacement ratio, c_t^{nw}/c_t^w, is countercyclical. Although Chetty and Looney (2006) and Gruber (1997) report that there is a premium on average, we cannot infer anything about the cyclicality of the premium from the evidence they present. Studies of the cross section variance of log household consumption are a potential source of evidence on the cyclical behavior of the premium. To see this, let V_t denote the variance of log household consumption in the period t cross section in our model:[43]

$$V_t = (1 - h_t)\, h_t \left(\log \left(\frac{c_t^w}{c_t^{nw}} \right) \right)^2 .$$

According to this expression, the model posits two countervailing forces on the cross-sectional dispersion of consumption, V_t, in a recession. First, for a given distribution of the population across employed and non-employed households (i.e., holding h_t fixed), a decrease in the consumption premium leads to a decrease in consumption dispersion in a recession. Second, holding the consumption premium fixed, consumption dispersion increases as people move from employment to non-employment with the fall in h_t.[44] These observations suggest that (i) if V_t is observed to drop in recessions, this is evidence in favor of the model's prediction that the consumption premium is procyclical and (ii) if V_t is observed to stay constant or rise in recessions then we cannot conclude anything about the cyclicality of the consumption premium. Evidence in Heathcote, Perri and Violante (2010) suggests that the US was in case (i) in three of the previous five recessions.[45] In particular, they show that the dispersion in log household non-durable consumption decreased in the 1980, 2001 and 2007 recessions.[46] We conclude tentatively that the observed cross sectional dispersion of consumption across households lends support to our model's implication that the consumption premium is procyclical. In addition, the fact that the duration of unemployment benefits routinely are extended in recessions (e.g. in the US) is an indication that the income premium is procyclical empirically.

Another interesting implication of the model is its prediction that high unemployment in recessions reflects the procyclicality of effort in job search. There is some evidence that supports this implication of the model. The Bureau of Labor Statistics (2009) constructs a measure of the number of discouraged workers. These are people who are available to work and have looked for work in the past 12 months, but are not currently looking because they believe no jobs are available. This statistic has only been gathered since 1994, and so it covers just two recessions. However, in both the recessions for which we have data, the number of discouraged workers increased substantially. For example, the number of discouraged workers jumped 70 percent from 2008Q1 to 2009Q1. In fact, the number of discouraged workers is only a tiny fraction of the labor force. However, to the extent that the sentiments of discouraged workers are shared by workers more generally, a jump in the

[43] Strictly speaking, this formula is correct only for the model in the third section of this paper. The relevant formula is more complicated for the model with capital because that requires a non-trivial aggregation across households that supply different types of labor services. To see how we derived the formula in the text, note that the cross sectional mean of log household consumption is $E_t = h_t \log(c_t^w) + (1 - h_t)\log(c_t^{nw})$ so that $V_t = h_t (\log c_t^w - E_t)^2 + (1 - h_t)(\log c_t^{nw} - E_t)^2 = h_t (1 - h_t)(\log c_t^w - \log c_t^{nw})^2$.

[44] This statement assumes that the empirically relevant case applies, i.e. $h_t > 1/2$.

[45] Of course, we cannot rule out that the drop in V_t in recessions has nothing to do with the mechanism in our model but rather reflects some other source of heterogeneity in the data.

[46] A similar observation was made about the 2007 recession in Parker and Vissing-Jorgensen (2009).

number of discouraged workers could be a signal of a general decline in job search intensity in recessions. But, this is an issue that demands a more careful investigation.

Interestingly, Shimer (2004) reports evidence that search effort may be acyclical or even countercyclical. In his work, the number of different search methods that the unemployed use are counted at different stages of the business cycle. We interpret Shimer's finding as reflecting an *extensive* margin of search, i.e. how many alternative search methods are being used. By contrast, our model emphasizes the *intensive* margin of job search, i.e. how intensely one particular method of search is being used by the unemployed. Therefore, our model is not necessarily at odds with the evidence provided by Shimer.

8. Concluding Remarks

We constructed a model in which households must make an effort to find work. Because effort is privately observed, perfect insurance against labor market outcomes is not feasible. To ensure that people have an incentive to find work, workers that find jobs must be better off than people who do not work. With additively separable utility, this translates into the proposition that employed workers have higher consumption than the non-employed. We integrate our model of unemployment into a standard monetary DSGE model and find that the model's ability to account for standard macroeconomic variables is not diminished. At the same time, the new model appears to account well for the dynamics of variables like unemployment and the labor force.

The theory of unemployment developed here has interesting implications for the optimal variation of labor market insurance over the business cycle. In a boom more labor is demanded by firms. To satisfy the higher demand, households are provided with more incentives to look for work by raising consumption for the employed, c_t^w, relative to consumption of the non-employed, c_t^{nw}. Conversely, in a recession, the consumption premium falls and thus the replacement ratio, c_t^{nw}/c_t^w, increases. Thus, our model implies a procyclical consumption premium – or equivalently – a countercyclical replacement ratio. Put differently, optimal labor market insurance is countercyclical in our model.

The empirical results highlight an important implication of our work. In particular, it is in general not sufficient to account for the response of employment or total hours only to be able to draw conclusions about the unemployment rate. In particular, when the standard model is estimated without data on unemployment and the labor force, the fit of total hours of the model is in fact very good. By contrast, the implications of the model for unemployment and the labor force are disastrous. Conversely, when the standard model is estimated on unemployment and labor force data too, the fit of these two variables improves indeed somewhat. However the improvement of fit comes at the cost of not fitting total hours well. In other words, the standard model provides an example that is it not straightforward to account for the joint behavior of unemployment, labor force participation and total hours *together* with further real and nominal macroeconomic variables. By contrast, our model does a good job in doing so.

We leave it to future research to quantify the various ways in which the new model may contribute to policy analysis. In part, we hope that the model is useful simply because labor market data are of interest in their own right. But, we expect the model to be useful even when labor market data are not the central variables of concern. An important input into policy analysis is the estimation of 'latent variables' such as the output gap and the efficient, or 'natural', rate of interest. Other important inputs into policy analysis are forecasts of inflation and output. By allowing one to systematically integrate labor market information into the usual macroeconomic dataset, our model can be expected to provide more precise forecasts, as well as better estimates of latent variables.[47] We also believe, in line with Veracierto (2008), that confronting models with labor market data such as unemployment and the labor force provides an important test for any business cycle model.

[47] For an elaboration on this point, see Basistha and Startz (2004).

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Table 1: Non-Estimated Parameters in the Medium-sized Model

Parameter	Value	Description
α	0.25	Capital share
δ	0.025	Depreciation rate
β	0.999	Discount factor
π	1.0083	Gross inflation rate
η_g	0.2	Government consumption to GDP ratio
κ_w	1	Wage indexation to π_{t-1}
λ_w	1.01	Wage markup
ξ_w	0.75	Wage stickiness
$\overline{p}$	0.97	Max, $p(e)$
μ_n	1.0041	Gross neutral tech. growth
μ_ψ	1.0018	Gross invest. tech. growth

Table 3: Medium-sized Model Steady State at Posterior Mean for Parameters

Variable	Standard Model	Involuntary Unemp. Model	Description
$p_{k'}k/y$	7.73	7.73	Capital to GDP ratio (quarterly)
c/y	0.56	0.56	Consumption to GDP ratio
i/y	0.24	0.24	Investment to GDP ratio
$H = h$	0.63	0.63	Steady state labor input
c^{nw}/c^w	1.0	0.81	Replacement ratio
R	1.014	1.014	Gross nominal interest rate (quarterly)
R^{real}	1.006	1.006	Gross real interest rate (quarterly)
r^k	0.033	0.033	Capital rental rate (quarterly)
u	0.077	0.056	Unemployment rate
m	-	0.67	Labor force (involuntary unemployment model)
l^*	0.68	-	Labor force (standard model)
ς	1.98	1.95	Slope, labor disutility
F	-	0.75	Intercept, labor disutility
a	-	0.52	Slope, $p(e)$
η	-	0.75	Intercept, $p(e)$

Table 2: Priors and Posteriors of Parameters for the Medium-sized Model

Parameter		Prior		Posterior[a]	
		Distribution [bounds]	Mean, Std.Dev. [5% and 95%]	Mean, Std.Dev. [5% and 95%]	
				Standard Model	**Involuntary Unemp. Model**
Price setting parameters					
Price Stickiness	ξ_p	Beta	0.50, 0.15	0.63, 0.04	0.64, 0.04
		[0, 0.8]	[0.23, 0.72]	[0.57, 0.70]	[0.58, 0.70]
Price Markup	λ_f	Gamma	1.20, 0.15	1.15, 0.07	1.36, 0.09
		[1.01, ∞]	[1.04, 1.50]	[1.03, 1.26]	[1.21, 1.50]
Monetary authority parameters					
Taylor Rule: Int. Smoothing	ρ_R	Beta	0.80, 0.10	0.87, 0.02	0.89, 0.01
		[0, 1]	[0.62, 0.94]	[0.85, 0.90]	[0.87, 0.91]
Taylor Rule: Inflation Coef.	r_π	Gamma	1.60, 0.15	1.49, 0.11	1.47, 0.11
		[1.01, 4]	[1.38, 1.87]	[1.30, 1.66]	[1.30, 1.65]
Taylor Rule: GDP Coef.	r_y	Gamma	0.20, 0.10	0.06, 0.03	0.06, 0.02
		[0, 2]	[0.07, 0.39]	[0.02, 0.10]	[0.02, 0.09]
Household parameters					
Consumption Habit	b	Beta	0.75, 0.15	0.76, 0.02	0.79, 0.02
		[0, 1]	[0.47, 0.95]	[0.73, 0.79]	[0.76, 0.81]
Power, labor disutility[b]	σ_L	Uniform	10.0, 5.77	–	7.40, 0.47
		[0, 20]	[1.00, 19.0]	–	[6.61, 8.14]
Inverse labor supply elast.[b]	σ_z	Gamma	0.30, 0.20	0.13, 0.03	0.13, 0.02
		[0, ∞]	[0.06, 0.69]	[0.08, 0.17]	[0.09, 0.17]
Capacity Adj. Costs Curv.	σ_a	Gamma	1.00, 0.75	0.34, 0.09	0.30, 0.09
		[0, ∞]	[0.15, 2.46]	[0.18, 0.48]	[0.15, 0.44]
Investment Adj. Costs Curv.	S''	Gamma	12.00, 8.00	15.63, 3.28	20.26, 4.06
		[0, ∞]	[2.45, 27.43]	[10.5, 20.7]	[14.0, 26.6]
Shocks					
Autocorr. Invest. Tech.	ρ_ψ	Uniform	0.50, 0.29	0.60, 0.08	0.59, 0.08
		[0, 1]	[0.05, 0.95]	[0.48, 0.72]	[0.47, 0.71]
Std.Dev. Neutral Tech. Shock	σ_n	Inv. Gamma	0.20, 0.10	0.22, 0.02	0.22, 0.02
		[0, ∞]	[0.10, 0.37]	[0.19, 0.25]	[0.19, 0.24]
Std.Dev. Invest. Tech. Shock	σ_ψ	Inv. Gamma	0.20, 0.10	0.16, 0.02	0.16, 0.02
		[0, ∞]	[0.10, 0.37]	[0.12, 0.19]	[0.12, 0.20]
Std.Dev. Monetary Shock	σ_R	Inv. Gamma	0.40, 0.20	0.45, 0.03	0.43, 0.03
		[0, ∞]	[0.21, 0.74]	[0.39, 0.50]	[0.38, 0.48]

[a] Based on standard random walk metropolis algorithm. 150 000 draws, 30 000 for burn-in, acceptance rate 26%.
[b] In the case of the baseline model, σ_z and σ_L coincide. In the case of the involuntary unemployment model these two parameters are different.

Figure 1: Dynamic Responses of Non−Labor Market Variables to a Monetary Policy Shock

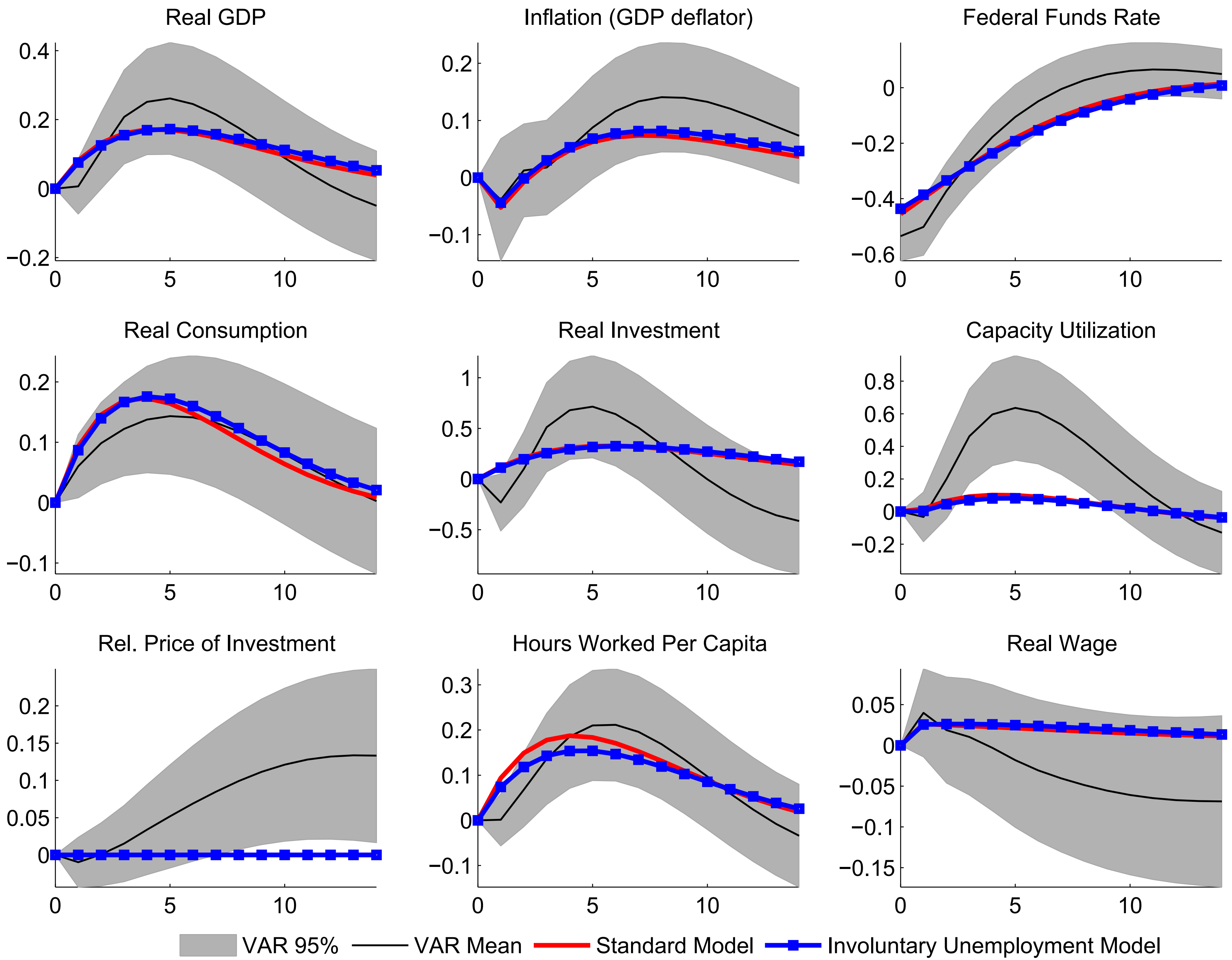


Figure 2: Dynamic Responses of Non−Labor Market Variables to a Neutral Technology Shock

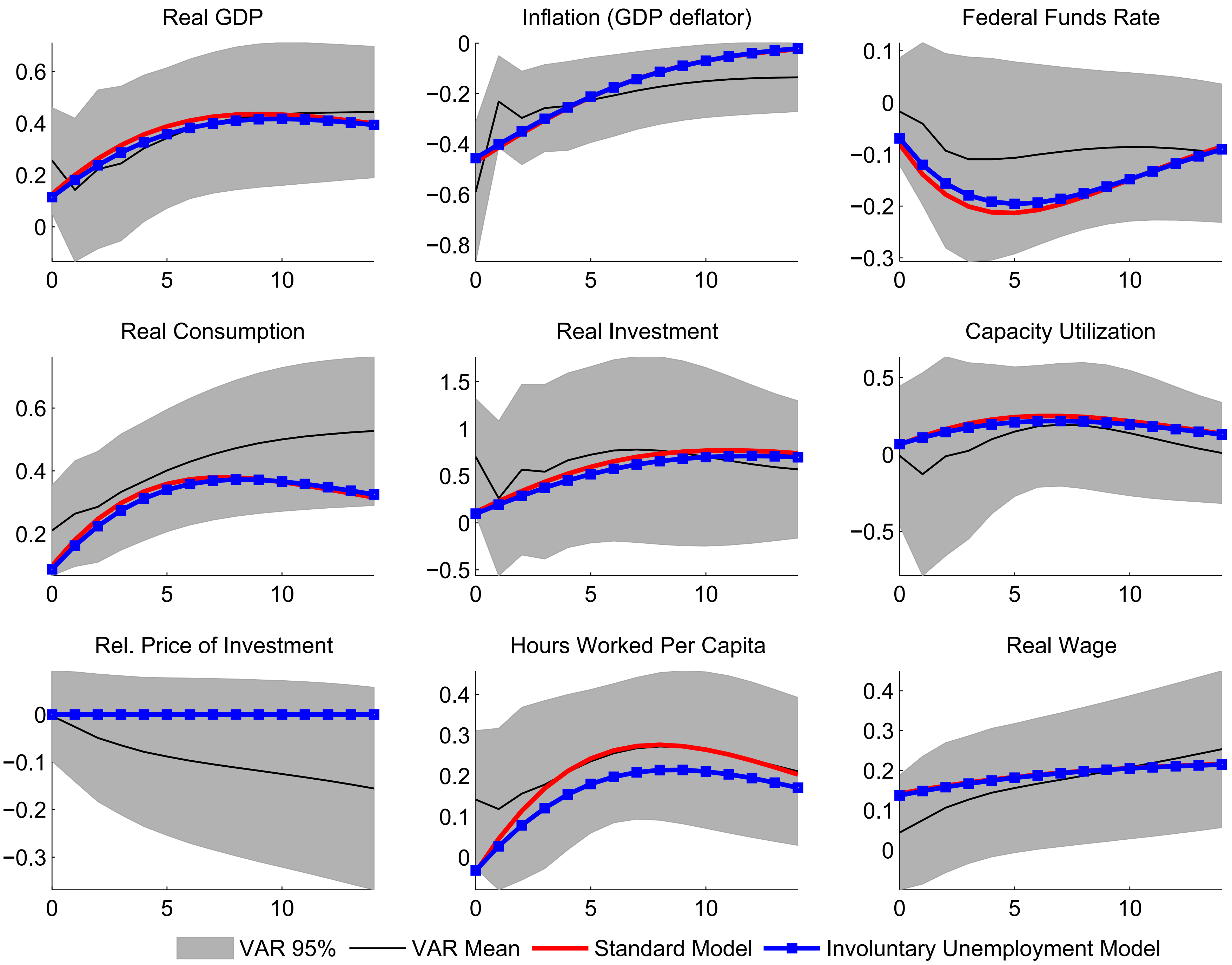


Figure 3: Dynamic Responses of Non−Labor Market Variables to an Investment Specific Technology Shock

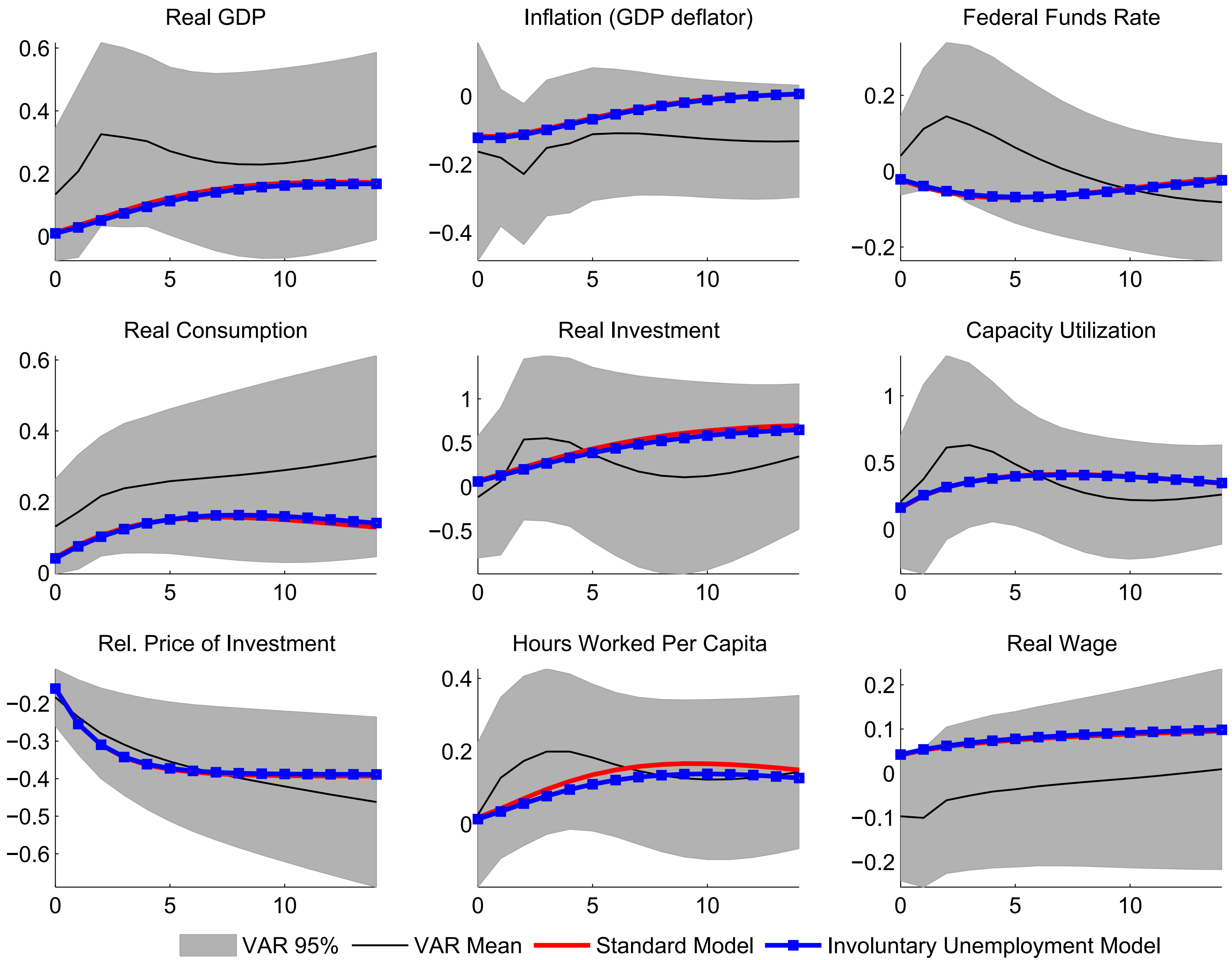


Figure 4: Dynamic Responses of Labor Market Variables to Three Shocks

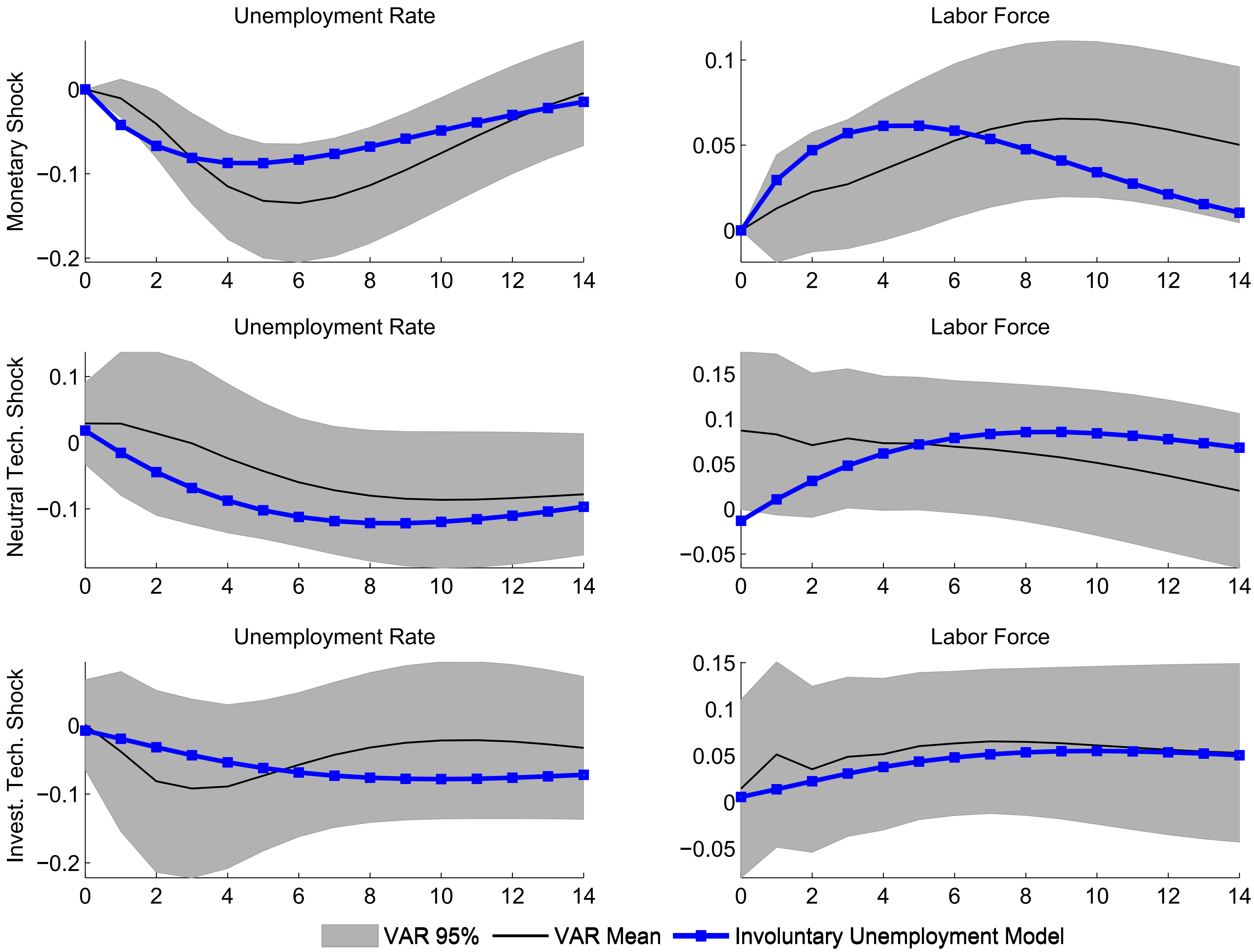
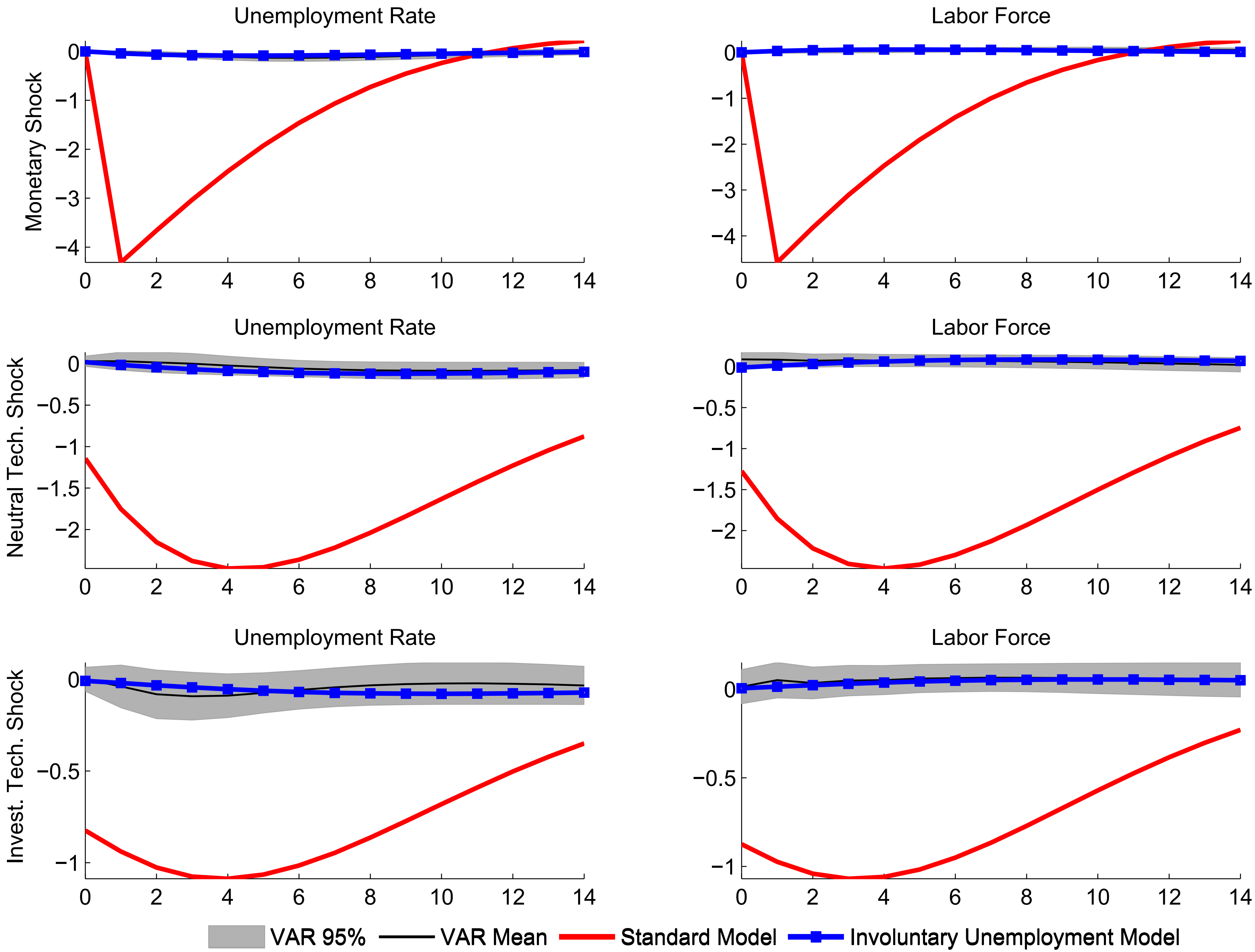


Figure 5: Dynamic Responses of Labor Market Variables to Three Shocks



A. Appendix

A.1. Implications of the Model for the NAIRU

We can relate the theory derived in this paper to the idea of a non-accelerating inflation rate of unemployment (NAIRU).[48] One interpretation of the NAIRU focuses on the first difference of inflation. Under this interpretation, the NAIRU is a level of unemployment such that whenever the actual unemployment rate lies below it, inflation is predicted to accelerate and whenever the actual unemployment rate is above it, inflation is predicted to decelerate. Consider the CGG model with our theory of unemployment developed in section 3. The efficient level of unemployment, u_t^*, does not in general satisfy this definition of the NAIRU. From (3.28) it is evident that a negative value of u_t^g does not predict an acceleration of inflation in the sense of predicting a positive value for $\beta E_t \hat{\pi}_{t+1} - \hat{\pi}_t$.

On the contrary, according to the unemployment rate Phillips curve, (3.28), a negative value of u_t^g creates an anticipated deceleration in inflation.[49] Testing this implication of the data empirically is difficult, because u_t^* is not an observed variable. However, some insight can be gained if one places upper and lower bounds on u_t^*. For example, suppose $u_t^* \in (4, 8)$. That is, the efficient unemployment rate in the postwar US was never below 4 percent or above 8 percent. In the 593 months between February 1960 and July 2009, the unemployment rate was below 4 percent in 52 months and above 8 percent in 42 months. Of the months in which unemployment was above its upper threshold, the change in inflation from that month to three months later was negative 79 percent of the time. Of the months in which unemployment was below the 4 percent lower threshold, the corresponding change in inflation was positive 67 percent of the time. If one accepts our assumption about the bounds on u_t^*, these results lend empirical support to the proposition that there exists a NAIRU in the first difference sense. They also represent evidence against the model developed here.[50]

[48] For an alternative approach to the NAIRU, see e.g. Blanchard and Galí (2010).

[49] In their discussion of the NAIRU, Ball and Mankiw (2002) implicitly reject (3.28) as a foundation for the notion that u_t^* is a NAIRU. Their discussion begins under a slightly different version of (3.28), with $\beta E_t \hat{\pi}_{t+1}$ replaced by $E_{t-1} \hat{\pi}_t$. They take the position that u_t^* in this framework is a NAIRU only when monetary policy generates the random walk outcome, $E_{t-1} \hat{\pi}_t = \hat{\pi}_{t-1}$. In this case, a negative value of u_t^g is associated with a deceleration of current inflation relative to what it was in the previous period. Ball and Mankiw argue that the random walk case is actually the relevant one for the US in recent decades.

[50] Our bounds test follows the one implemented in Stiglitz (1997) and was executed as follows. Monthly observations on the unemployment and the consumer price index were taken from the Federal Reserve Bank of St. Louis' online data base, FRED. We worked with the raw unemployment rate. The consumer price index was logged, and we computed a year-over-year rate of inflation rate, π_t. The percentages reported in the text represent the fraction of times that $u_t < 4$ and $\pi_{t+3} - \pi_t > 0$, and the fraction of times that $u_t > 8$ and $\pi_{t+3} - \pi_t < 0$.

An alternative interpretation of the NAIRU focuses on the level of inflation, rather than its change. Under this interpretation, u_t^* in the theory developed here is a NAIRU.[51] To see this, one must take into account that the theory (sensibly) implies that inflation returns to steady state after a shock that causes u_t^g to drop has disappeared. That is, the eventual effect on inflation of a negative shock to u_t^g must be zero. That a negative shock to u_t^g also creates the expectation of a deceleration in inflation then implies that inflation converges back to steady state from above after a negative shock to u_t^g. That is, a shock that drives u_t below u_t^* is expected to be followed by a higher level of inflation and a shock that drives u_t above u_t^* is expected to be followed by a lower level of inflation.[52]

Thus, u_t^* in the theory derived here is a NAIRU if one adopts the level interpretation of the NAIRU and not if one adopts the first difference interpretation.[53] Interestingly, u_t^* is a NAIRU under the first difference interpretation if one adopts the price indexation scheme proposed in CEE, in which non-optimizing firms set $P_{i,t} = \pi_{t-1}P_{i,t-1}$. In this case, $\hat{\pi}_t$ and $\hat{\pi}_{t+1}$ in (3.28) are replaced by their first differences. Retracing the logic of the previous two paragraphs establishes that with price indexation, u_t^* is a NAIRU in the first difference sense. Under our assumptions about the bounds on u_t^*, price indexation also improves the empirical performance of the model on the dimensions emphasized here.

It is instructive to consider the implications of the theory for the regression of the period $t+1$ inflation rate on the period t unemployment and inflation rates. In the very special case that u_t^* is a constant, the regression coefficient on u_t would be κ and other variables would not add to the forecast.[54] However, these predictions depend crucially on the assumption that u_t^* is constant. If it is stochastic, then u_t^* is part of the error term. Since u_t^* is expected to be correlated with all other variables in the model, then adding these variables to the

[51] In his discussion of the NAIRU, Stiglitz (1997) appears to be open to either the first difference or level interpretation of the NAIRU.

[52] A quick way to formally verify the convergence properties just described is to consider the following example. Suppose the monetary policy shock, ε_t, is an iid stochastic process. Let the response of the endogenous variables to ε_t be given by $u_t^g = u_\varepsilon \varepsilon_t$, $\hat{R}_t = R_\varepsilon \varepsilon_t$, $\hat{\pi}_t = \pi_\varepsilon \varepsilon_t$, where u_ε, R_ε and π_ε are undetermined coefficients to be solved for. Substituting these into the equations that characterize equilibrium and imposing that the equations must be satisfied for every realization of ε_t, we find: $u_\varepsilon = \frac{\kappa^{okun}}{1+\kappa^{okun}\kappa r_\pi + r_y}$, $\pi_\varepsilon = -\kappa u_\varepsilon$, $R_\varepsilon = \frac{1}{\kappa^{okun}} u_\varepsilon$. According to these expressions, a monetary policy shock drives u_t^g and R_t in the same direction. Thus, a monetary policy shock that drives the interest rate down also drives the unemployment gap down. The same shock drives current inflation up.

[53] Note further that we have abstracted from sticky wages similar to CGG. Adding imperfections in wage setting to the model complicates the analysis with respect to the NAIRU considerably.

[54] In our model, u_t^* is constant only under very special circumstances. For example, it is constant if government spending is zero and the labor preference shock, ς_t, is constant. However, as explained after (3.13), u_t^* is a function of all three shocks when government spending is positive and $\gamma < 1$.

forecast equation is predicted to improve fit.

A.2. Relationship of Our Work to Galí (2011)

In this section, we shall discuss the relationship of our work to Galí (2011) beyond those remarks made in the introduction and in section 4.6. Our paper emphasizes labor supply in its explanation of the dynamics of unemployment and the labor force. Galí adopts a similar perspective. To better explain our model, it is useful to compare its properties with those of Galí's model. Galí demonstrates that with a modest reinterpretation of variables, the standard DSGE model already contains a theory of unemployment. In particular, one can define the unemployed as the difference between the number of people actually working and the number of people that would be working if the marginal cost of work were equated to the wage rate. This difference is positive and fluctuating in the standard DSGE model because of the presence of wage-setting frictions and monopoly power. In effect, unemployment is a symptom of social inefficiency. People inflict unemployment upon themselves in the quest for monopoly profits. By contrast, in our model unemployment reflects frictions that are necessary for people to find jobs. The existence of unemployment does not require monopoly power. This point is dramatized by the fact that we introduce our model in the CGG framework, in which wages are set in competitive labor markets. At the same time, the logic of our model does create a positive relationship between monopoly power and unemployment. In our model, the employment contraction resulting from an increase in the monopoly power of unions produces a reduction in the incentives for households to work. Households' response to the reduced incentives is to allocate less effort to search, implying higher unemployment. So, our model shares the prediction of Galí's model that unemployment should be higher in economies with more union monopoly power. However, our model has additional implications that could differentiate it from Galí's. Ours implies that in economies with more union power both the labor force and the consumption premium for employed workers over non-employed workers are reduced. Galí's model predicts that with more union monopoly power, the labor force will be larger. The exact amount by which the labor force increases depends on the strength of wealth effects on leisure.

Other important differences between our model of unemployment and Galí's is that the latter fails to satisfy characteristics (i) and (iii) above. The model assumes that the available jobs can be found without effort. Because the model does not satisfy (i), unemployment does not meet the official US definition of unemployment. In addition, the presence of perfect insurance in Galí's model implies that the employed have lower utility than the non-employed, violating (iii).

There are more differences between ours and Galí's theory unemployment. In standard

DSGE models, labor supply plays little role in the dynamics of standard macro variables like consumption, output, investment, inflation and the interest rate. The reason is that the presence of wage setting frictions reduces the importance of labor supply. This is why the New Keynesian literature has been relatively unconcerned about all the old puzzles about income effects on labor and labor supply elasticities that were a central concern in the real business cycle literature. However, we show that these problems are back in full force if one adopts Galí's theory of unemployment. This is because labor supply corresponds to the labor force in that theory. To see how this brings back the old problems, we study the standard DSGE model's predictions for unemployment and the labor force in the wake of an expansionary monetary policy shock. Because that model predicts a rise in consumption, the model also predicts a decline in labor supply, as the income effect associated with increased consumption produces a fall in the value of work. The drop in labor supply is counterfactual, according to our VAR-based evidence. In addition, the large drop in the labor force leads to an counterfactually large drop in unemployment in the wake of an expansionary monetary policy shock.

Galí (2011) and Galí, Smets and Wouters (2011) show that changes to the household utility function that offset wealth effects reduce the counterfactual implications of the standard model for the labor force. In effect, our paper proposes a different strategy. We preserve the additively separable utility function that is standard in monetary DSGE models, and our model nevertheless does not display the labor force problems in the standard DSGE model. This is because in our model the labor force and employment have a strong tendency to comove. In our model, the rise in employment in the wake of an expansionary monetary policy shock is accomplished by increasing people's incentives to work. The additional incentives not only encourage already active households to intensify their job search, but also to shift into the labor force. More generally, the analysis highlights the fact that modeling unemployment requires thinking carefully about the determinants of the labor force.[55]

A.3. Estimating the Standard Model on Unemployment and Labor Force

In this section, we complement the discussion in section 6.3 when the standard model is also estimated on data for the unemployment rate and the labor force. In this case, the dataset used in the estimation of our involuntary unemployment model and the standard model is identical. Interestingly, there are four parameters that take on very different values at the posterior mean compared to the parameter estimates reported in Table 2 when the standard model is not estimated on unemployment and the labor force. These parameters are, the

[55] Our argument complements the argument in Krusell, Mukoyama, Rogerson, and Sahin (2009), who also stress the importance of understanding employment, unemployment and the labor force.

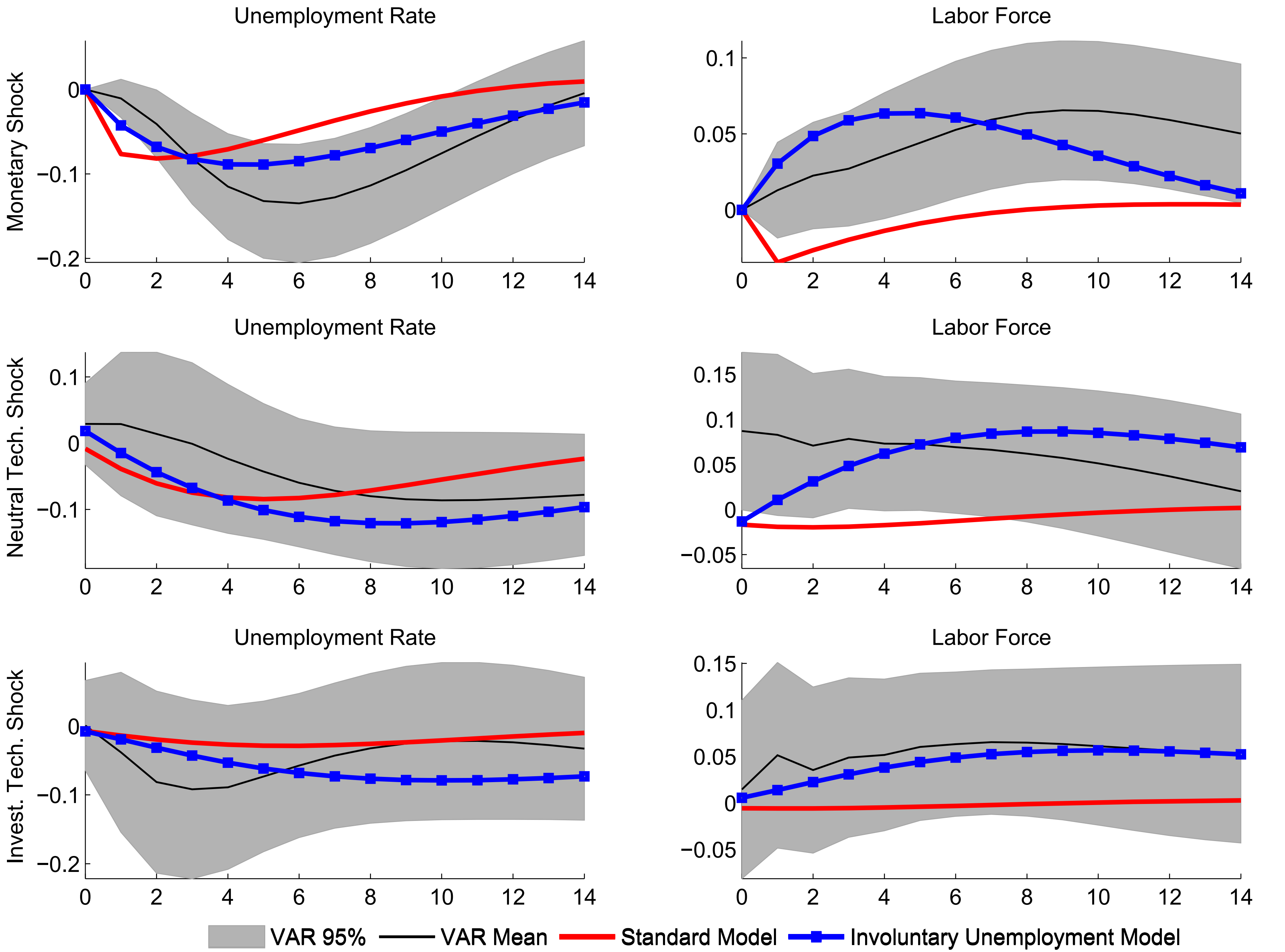
inverse labor supply elasticity, σ_L, the steady state gross price and wage markups, λ^f and λ^w, and the curvature of capacity adjustment costs, σ_a. All other parameters listed in Tables 1 and 2 are affected only very little when the additional labor market data are taken on board in the estimation of the standard model.

For convenience, let's repeat the equation from section 6.3 that determines the reaction of the labor force in the standard model, $\hat{l}_t^* = \frac{\hat{\psi}_{z+,t}+\widehat{\bar{w}}_t}{\sigma_L}$, where $\hat{l}_t^*$, $\hat{\psi}_{z^+,t}$ and $\widehat{\bar{w}}_t$ denote the labor force, marginal utility of consumption and the real wage, respectively. In the wake of an expansionary monetary policy shock, marginal utility of consumption falls much more than the real wage increases. Thus the labor force falls in the standard model while it rises according to the VAR. The only way the standard model can come close to the VAR responses is to drive σ_L to infinity and thereby shut down the response of the labor force. Setting σ_L to infinity, however, implies a zero labor supply elasticity and will therefore be harmful to the model in replicating the VAR responses for e.g. total hours. Thus, the estimation needs to balance the "miss" of the model for the labor force and e.g. total hours. It does so by selecting a posterior mean of $\sigma_L = 15.4$ which is much higher than the value of about 0.13 reported in Table 2. Note that a value of σ_L as high as 15.4 relative to 0.13 generates a steady state unemployment rate close to zero when all other parameters are held fixed. In other words, the labor supply curve becomes essentially vertical. To enable maximum comparability with the model versions estimated in Table 2, we impose the same steady state unemployment rate of 5.6 percent in this experiment too. To do so, we need to set the gross wage markup $\lambda^w = 2.43$ at the posterior mean. The higher values of σ_L and λ^w imply that marginal costs rise much more steeply in response to e.g. an expansionary monetary policy shock. To at least partly offset this, the estimation selects a higher steady state gross price markup of $\lambda^f = 2.44$ and a lower curvature of capacity adjustment costs of $\sigma_a = 0.04$, compared to Table 2.

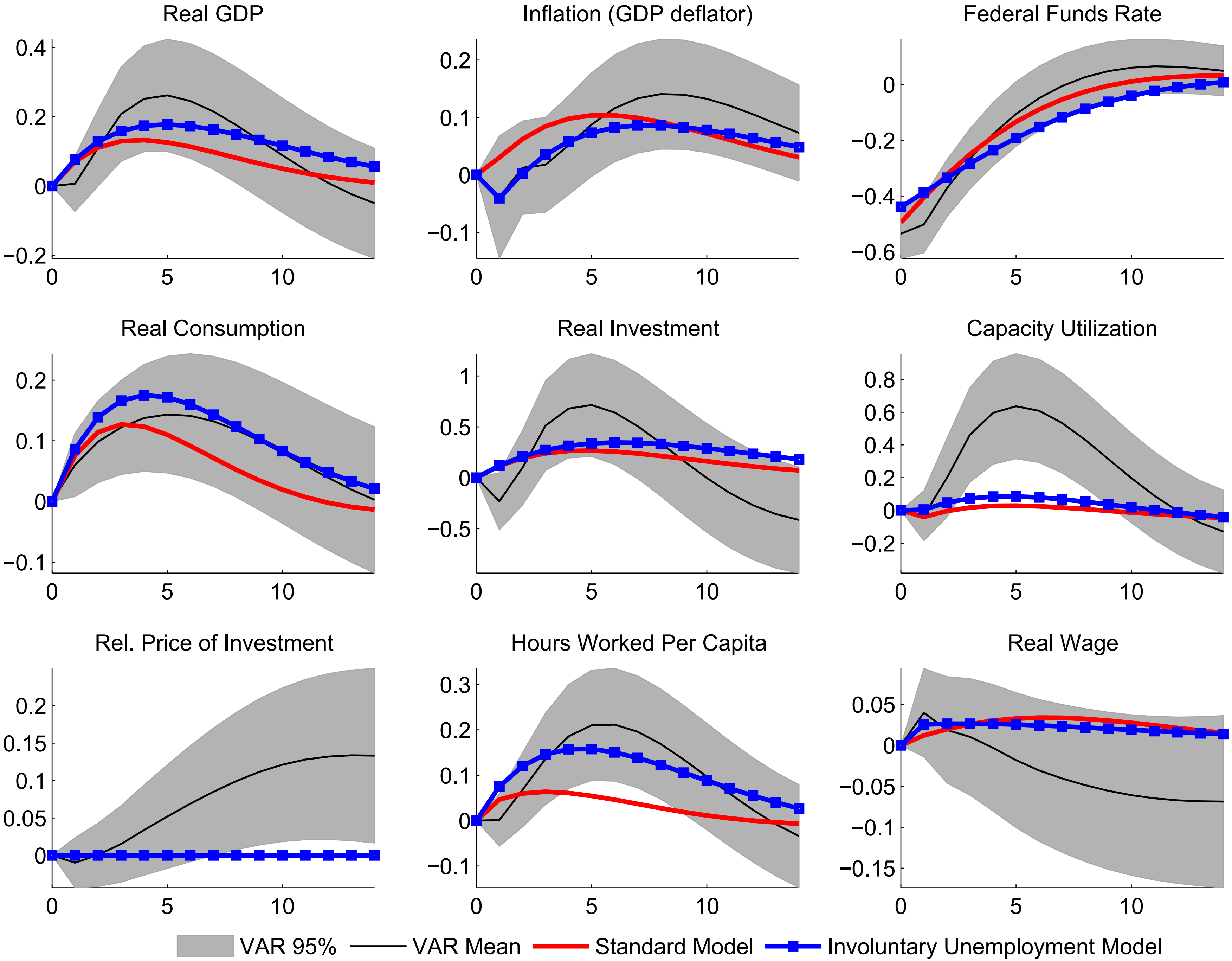
Figures A1 to A4 show the responses of the model to the two technology shocks and to the monetary shock. Indeed, the standard model now delivers a worse fit for the standard macro variables. Still, the fit for unemployment and the labor force is not satisfactory. In terms of fit, the log data density at the posterior mean for our model is -75.1 while the one for the standard model is -294.1. Overall, it turns out that our model of involuntary unemployment clearly outperforms the standard model when both models face the same dataset including unemployment and the labor force.

A.4. Appendix Figures

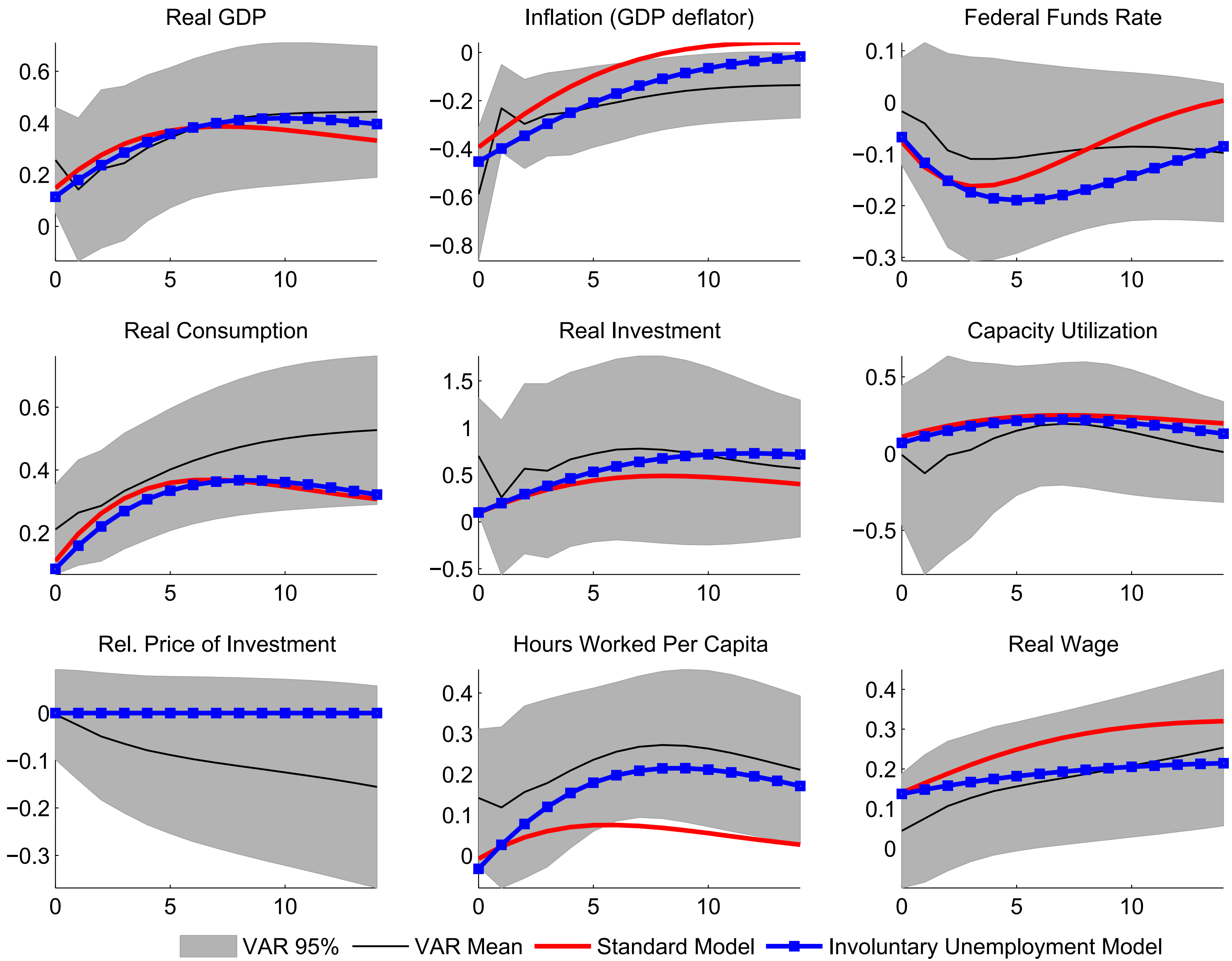
Appendix Figure A1: Dynamic Responses of Labor Market Variables to Three Shocks When Unemployment Rate and Labor Force Data are Included in the Estimation of the Standard Model



Appendix Figure A2: Dynamic Responses of Non-Labor Market Variables to Monetary Policy Shock When Unemployment Rate and Labor Force Data are Included in the Estimation of the Standard Model



Appendix Figure A3: Dynamic Responses of Non-Labor Market Variables to a Neutral Technology Shock When Unemployment Rate and Labor Force Data are Included in the Estimation of the Standard Model



Appendix Figure A4: Dynamic Responses of Non-Labor Market Variables to an Investment Specific Technology Shock When Unemployment Rate and Labor Force Data are Included in the Estimation of the Standard Model