Assessing Structural VARs*
(Preliminary and Incomplete: Do Not Cite Without Authors' Permission.)

Lawrence J. Christiano[†] Martin Eichenbaum[‡] Robert Vigfusson[§]

May 8, 2005

Abstract

We argue that structural vector autoregressions (VARs) are useful as a guide to constructing and evaluating dynamic general equilibrium models. Given a minimal set of identifying assumptions, structural VARs allow the analyst to estimate the dynamic effects of economic shocks. We ask the following three questions. First, what are the bias properties of the estimator? Second, what are the bias properties of standard estimators of sampling uncertainty in the estimator? Third, are there easy to implement variants of standard procedures which improve the bias properties of response function estimators? We address these questions using data generated from a series of dynamic general equilibrium models. Based on our answers, we conclude that structural VARs do indeed provide valuable information for building empirically plausible models of aggregate fluctuations.

[*]The first two authors are grateful to the National Science Foundation for Financial Suppport.
[†]Northwestern University and NBER.
[‡]Northwestern University and NBER.
[§]Federal Reserve Board of Governors.

1. Introduction

We argue that structural vector autoregressions (VARs) are useful as a guide to constructing and evaluating dynamic general equilibrium models. Given a minimal set of identifying assumptions, structural VARs allow the analyst to estimate the dynamic effects of economic shocks. These estimated response functions provide a natural way to assess the empirical plausibility of a structural model.[1] To be useful, the response function estimators must have good statistical properties. To assess these properties, we ask the following three questions. First, what are the bias properties of the estimator? Second, what are the bias properties of standard estimators of sampling uncertainty? Third, are there easy to implement variants of standard procedures which improve the bias properties of response function estimators? We address these questions using data generated from a series of dynamic general equilibrium models. Based on our answers, we conclude that structural VARs do indeed provide valuable information for building empirically plausible models of aggregate fluctuations.

There are two important traditions for constructing dynamic general equilibrium models. One tradition focusses on at most a handful of key shocks, and deliberately abstracts from the smaller shocks.[2] A classic example is Kydland and Prescott ( ), who work with a model driven only by technology shocks, even though they take the position that these shocks only account for 70 percent of business cycle fluctuations. A conundrum confronted by this modeling tradition is how to empirically evaluate models (which contain only a subset of the shocks) with the data (which are driven by *all* the shocks).[3] Structural VARs have the potential to provide a resolution to this challenge, by allowing the analyst to assess the empirical performance of a model relative to a particular set of shocks.

A second tradition to building macroeconomic models incorporates large numbers of shocks to provide a complete characterization of the stochastic processes generating the data.[4] This tradition avoids the Kydland and Prescott conundrum. Still, for diagnostic purposes it is useful to assess the implications of these models for particular shocks to the

[1] Perhaps the first in this line of research is Sims (1989). There are many others, including Christiano, Eichenbaum and Evans (2005), Francis and Ramey (2001), Gali (1999), Rotemberg and Woodford (1997), and Del Negro, Schorfheide, Smets, and Wouters (2005).

[2] The view that aggregate dynamics are dominated by the effects of a few shocks only, appears to receive confirmation from the literature on factor models, such as Sargent and Sims, Quah and Sargent (1993), Reichlin, and Uhlig (1992).

[3] Aiyagari (1994) and Prescott (1991) draw attention to the challenge by pointing to a difficulty with the standard RBC strategy for evaluating a model. In this strategy, one compares the second moment properties of the data with the second moment properties of the model. Prescott famously asserted that if a model matches the data, then that is bad news for the model. The argument is that since good models leave some things out of the analysis, good models should *not* match the data. Of course, lots of models do not match the data. This raises the question: how can we use the data to differentiate between good and bad models? Structural VARs provide one possible answer.

[4] See, for example, Smets and Wouters (2003) and Christiano, Motto and Rostagno (2004).

economy.

In practice, the literature uses two types of identifying restrictions in structural VARs. Blanchard and Quah (1989), Gali (1999) and others have exploited the implications that many models have for the long-run effects of shocks.[5] Other authors have exploited short-run restrictions.[6] There is a growing literature that questions the ability of structural VARs to uncover the dynamic response of macroeconomic variables to structural shocks. This literature focuses on identification strategies that exploit long run restrictions. Perhaps the first critique of these strategies was provided by Sims (1972). Although this paper was written before the advent of VARs, it articulates clearly why we should be concerned about the accuracy of identification based on long-run restrictions. The problem is that this strategy requires a reliable estimate of the sum of coefficients in distributed lag regressions. These sums are hard to reliably estimate even if the individual coefficients are reasonably precisely estimated. Faust and Leeper (1997) and Pagan make important related critiques of identification strategies based on long run restrictions. More recently Erceg, Guerrieri and Gust (2004) and Chari, Kehoe and McGrattan (2005) (CKM) have also examined the reliability of VAR-based inference using long run identifying restrictions. CKM are particularly critical and argue that structural VARs are very misleading.

We examine the reliability of inference using structural VARs based on long-run and short-run identifying assumptions. Throughout, we suppose that the data generating mechanism corresponds to variants of a standard real business cycle model. We focus on the question, how do hours worked respond to a technology shock?

We find that structural VAR's perform remarkably well when identification is based on short run restrictions. This is comforting for the vast literature that has exploited short run identification schemes to identify the dynamic effects of shocks to the economy. Of course, one can question the particular short run identifying assumptions used in any given analysis. But our results strongly support the view that if the relevant short run assumptions are satisfied in the data generating mechanism, then standard structural VAR procedures reliably uncover and identify the dynamic effects of shocks to the economy.

Regarding identification based on long-run assumptions, we find that if technology shocks account for a substantial fraction of business cycle fluctuations in output (say, over 50 percent), then VAR based analysis is reliable. We do find some evidence of bias when the fraction of output variance accounted for by technology shocks is very small relative to estimates in the standard RBC literature. But, this bias can be eliminated in at least two ways.

[5] See, for example, Basu, Fernald, and Kimball (1999), Christiano, Eichenbaum and Vigfusson (2003, 2004), and Francis and Ramey (2001)

[6] This list is particularly long and includes at least Bernanke and Blinder (1992), Bernanke and Mihov (1995), Christiano and Eichenbaum (1992), Christiano, Eichenbaum and Evans (2005), Grilli and Roubini (1995), Hamilton (1997) and Sims and Zha (1995).

First, our examples suggest that if there are enough variables in the VAR, the analyst is not likely to be misled in practice. Specifically, we find that when the number of aggregate variables in the VAR exceeds the number of important driving shocks, then the bias in impulse response estimators is substantially reduced. In light of the widespread consensus that there are only a handful of important shocks driving aggregate fluctuations, our finding suggests that including a reasonably small number of variables in the VAR should be sufficient to reduce small sample bias even if technology shocks are relatively unimportant.

Second, we have developed an adjustment to the standard VAR estimation strategy that virtually eliminates small sample bias, even in the worst case scenario when technology shocks play only a small role in aggregate fluctuations and there is only a small number of variables in the VAR. The standard VAR strategy requires factorizing an estimate of the spectral density at frequency zero of the data. The estimator used for this purpose is the zero-frequency spectral density implicit in the estimated VAR itself. It is principally because the quality of this estimator deteriorates that distortions occur in the worst-case scenario. This observation motivates us to adjust the standard VAR estimator by instead working with a Newey-West non-parametric estimator of the frequency-zero spectral density. We find that the effects of our adjusted strategy are minor when we are not in the worst-case scenario. However, in the cases when standard VAR procedures entail some bias, then that bias is virtually eliminated by our adjusted estimator, in all the cases we consider.

Our conclusions regarding the value of identified VARs differ sharply from those recently reached by CKM. Part of CKM's analysis concerns the consequences of working with the first difference of hours worked, when the levels specification is the correct one. We have nothing to say about this here, since we assume that the econometrician does not commit this specification error.[7] CKM argue that even when hours worked enter the analysis in levels, VARs are essentially useless as a tool for estimating the dynamic impact on hours worked of technology shocks. We reach a different conclusion, for three reasons.

First, CKM do not consider identified VARs when identification is accomplished via short run restrictions. They only consider the case of long-run restrictions. Second, CKM reach their conclusions based on two examples. These turn out to constitute worst case scenarios for VARs, because they have two particular properties: (i) they have the feature that technology shocks account for only a very small proportion (23 and 17 percent, respectively) of output fluctuations and (ii), the number of variables included in the VAR is equal to the number of important shocks in the model. As discussed above, we too find evidence of bias in experiments sharing these properties. As indicated above, this bias is easily dealt with and does not reflect a fundamental flaw in structural VARs.

[7] We have explored the differencing issue elsewhere, in Christiano, Eichenbaum and Vigfusson (2003, 2004).

CKM conjecture that the distortions in their two examples are due to the omission of capital in the VAR. If true this would indeed be a fundamental problem because quarterly data on the stock of capital is poorly measured and is typically not included in VAR analyses. Fortunately, it turns out that CKM's conjecture is incorrect. This follows from three observations. First, there is virtually no bias when we use our adjusted VAR estimation procedure in their two examples. Second, even with standard VAR procedures bias is reduced when technology shocks play a more important role in aggregate fluctuations than they do in CKM's examples. Interestingly, there is essentially no bias at all when, consistent with the findings of Kydland and Prescott ( ), technology shocks account for 70 percent of the business cycle variance in output. Finally, in our analysis of short-run identification strategies, VAR inference is very reliable, even though capital is not included in the VAR.

There is a third reason why our conclusions differ from CKM's. The CKM strategy for implementing long-run restrictions is anomalous relative to the strategy used in the literature. CKM identify a positive technology shock as one that drives labor productivity up in the period of the shock. The standard approach identifies a positive technology shock as one that drives labor productivity up in the long run.[8] In practice, which identification strategy is taken rarely matters. For example, it makes not difference in a version of Kydland and Prescott's RBC model in which technology shocks account for 70 percent of business cycle fluctuations in output. However, in the two CKM examples, whether one adopts the standard or the CKM identification strategy makes a dramatic difference. The CKM choice leads to a bimodal small sample distribution in the response of hours to a technology shock. In about 20 percent of the artificial data sets generated from their model, CKM's procedure leads them to infer that hours worked falls sharply, while labor productivity falls *permanently*, in the wake of what they identify as a positive technology shock. The standard identification strategy would identify these responses as the consequences of a negative technology shock. CKM's anomalous identification strategy leads them to overstate sampling uncertainty by a factor of two, leading them to conclude that VARs are completely uninformative. In reality, this conclusion is an artifact of their choice of identification strategy.

The remainder of this draft is organized as follows. Section 2 presents the general equilibrium model used in our examples. Section 3 discusses our results. Section 4 compares our results to those of CKM. Finally, section 5 contains concluding comments.

2. A Simple Real Business Cycle Model

In this section, we display the real business cycle model used in our analysis. The model has the property that the only shock that affects labor productivity in the long run is a shock to

[8] See, for example, the RATS manual.

technology. This property lies at the core of an identification strategy used by Gali (1999) and others to identify the effects of a shock to technology. We also consider a variant of the model in which we impose additional timing restrictions on agents' actions. In particular, we assume that agents choose hours worked before the technology shock is realized. This assumption allows us to identify the effects of a shock to technology using 'short run restrictions', that is, restrictions on the variance-covariance matrix of the disturbances to a vector autoregression. Finally, we discuss several parameterizations of our model that are used in the experiments we perform.

2.1. The Model

The representative agent maximizes expected utility over per capita consumption, c_t, and per capita employment, l_t

$$E_0 \sum_{t=0}^{\infty} \left(\beta\left(1+\gamma\right)\right)^t \left[\log c_t + \psi \frac{\left(\bar{l} - l_t\right)^{1-\sigma}}{1-\sigma}\right],$$

subject to the budget constraint:

$$c_t + \left(1+\tau_{x,t}\right)\left[\left(1+\gamma\right)k_{t+1} - \left(1-\delta\right)k_t\right] \leq \left(1-\tau_{lt}\right)w_t l_t + r_t k_t + T_t.$$

Here, k_t denotes the per capita capital stock at the beginning of period t, w_t is the wage rate, r_t is the rental rate on capital, $\tau_{x,t}$ is an investment tax, τ_{lt} is the tax rate on labor, $\delta \in (0,1)$ is the depreciation rate on capital, γ is the net growth rate of the population, and T_t represents lump-sum taxes. Finally, $\sigma > 0$ is a curvature parameter.

The representative competitive firm's production function is:

$$y_t = k_t^{\alpha}\left(Z_t l_t\right)^{1-\alpha},$$

where Z_t is the time t state of technology and $\alpha \in (0,1)$. The stochastic processes for the shocks are:

$$\begin{aligned} \log z_t &= \mu_Z + \sigma_z \varepsilon_t^z \\ \tau_{lt+1} &= \left(1-\rho_l\right)\bar{\tau}_l + \rho_l \tau_{lt} + \sigma_l \varepsilon_{t+1}^l \\ \tau_{xt+1} &= \left(1-\rho_x\right)\bar{\tau}_x + \rho_x \tau_{xt} + \sigma_x \varepsilon_{t+1}^x, \end{aligned} \tag{2.1}$$

where $z_t = Z_t / Z_{t-1}$. In addition, ε_t^z, ε_t^d, and ε_t^x are independent random variables with mean zero and unit standard deviation. The parameters, σ_z, σ_l and σ_x are non-negative scalars. The constant, μ_Z, is the mean growth rate of technology, $\bar{\tau}_l$ is the mean labor tax rate, τ_x is the mean tax on capital. We restrict the autoregressive coefficients, ρ_l and ρ_x, to be less than unity in absolute value.

Finally, the resource constraint is:

$$c_t + (1+\gamma)\, k_{t+1} - (1-\delta)\, k_t \leq y_t.$$

We consider two versions of the model, differentiated according to timing assumption. In the *standard version,* all time t decisions are taken after the realization of the time t shocks. This is the conventional assumption in the real business cycle literature. For pedagogical purposes, we also consider a second version of the model, we call the *recursive version* of the real business cycle model. Here, the timing assumptions are as follows. First, τ_{lt} is observed, after which labor decisions are made. Next, the other shocks are realized. Then, agents make their investment and consumption decisions. Finally, labor, investment, consumption, and output occur. We first discuss the standard version of our model.

2.1.1. The Standard Version of the Model

The log-linearized policy rule for capital can be written as follows:

$$\log \hat{k}_{t+1} = \gamma_0 + \gamma_k \log \hat{k}_t + \gamma_z \log z_t + \gamma_l \tau_{lt} + \gamma_x x_t,$$

where $\hat{k}_t \equiv k_t / Z_{t-1}$. The policy rule for hours worked is:

$$\log l_t = a_0 + a_k \log \hat{k}_t + a_z \log z_t + a_l \tau_{lt} + a_x \tau_{xt}.$$

$>$From this expression, it is clear that all shocks have only a temporary impact on l_t and $\hat{k}_t$. Since ε_t^z is the only shock that has a permanent effect on Z_t, it follows that ε_t^z is the only shock that has a permanent impact on the level of the capital stock, k_t. Similarly, ε_t^z is the only shock that has a permanent impact on output and labor productivity, $a_t \equiv y_t / l_t$. Formally, this *exclusion restriction* is given by:

$$\lim_{j \to \infty} \left[E_t a_{t+j} - E_{t-1} a_{t+j} \right] = f\left(\varepsilon_t^z \text{ only}\right), \tag{2.2}$$

where in our linear approximation to the model solution, f is a linear function. The model also implies the *sign restriction* that f is an increasing function. In (2.2), E_t is the expectation operator, conditional on $\Omega_t = \left(\log \hat{k}_{t-s}, \log z_{t-s}, \tau_{l,t-s}, \tau_{x,t-s};\ s \geq 0\right)$. The exclusion and sign restrictions have been used by Gali (1999) and others to identify the dynamic impact on macroeconomic variables of a positive shock to technology.

In practice, researchers impose the exclusion and sign restrictions on a vector autoregression to compute ε_t^z and identify its dynamic effects on macroeconomic variables. To describe this procedure, denote the variables in the VAR by Y_t :

$$
\begin{aligned}
Y_{t+1} &= B(L) Y_t + u_{t+1}, \;\; E u_t u_t' = V, \\
B(L) &\equiv B_1 + B_2 L + ... + B_p L^{p-1}, \\
Y_t &= \begin{pmatrix} \Delta \log a_t \\ \log l_t \\ x_t \end{pmatrix},
\end{aligned}
\tag{2.3}
$$

where x_t is an additional vector of variables that may be included in the VAR. It is assumed that the fundamental economic shocks are related to u_t in the following way:

$$
u_t = C\varepsilon_t, \; E\varepsilon_t\varepsilon_t' = I, \; CC' = V, \tag{2.4}
$$

where the first element in ε_t is ε_t^z. It is easy to verify that:

$$
\lim_{j\to\infty} \tilde{E}_t[a_{t+j}] - \tilde{E}_{t-1}[a_{t+j}] = \tau \left[I - B(1)\right]^{-1} C\varepsilon_t, \tag{2.5}
$$

where τ is a row vector with all zeros, except unity in the first location. Here, $B(1)$ is the sum, $B_1 + ... + B_p$. Also, $\tilde{E}_t$ is the expectation operator, conditional on $\tilde{\Omega}_t = \{Y_t, ..., Y_{t-p+1}\}$. To compute the dynamic effects of ε_t^z, we require $B_1, ..., B_p$ and C_1, the first column of C.

The symmetric matrix, V, and the B_i's can be computed by an ordinary least squares regression. However, it is well known that the requirement $CC' = V$ is not sufficient to determined a unique value of C_1. Adding the exclusion and sign restrictions does uniquely determine C_1. These restrictions are:

$$
\text{exclusion restriction: } \left[I - B(1)\right]^{-1} C = \begin{bmatrix} \text{number} & \underset{1\times(N-1)}{0} \\ \text{numbers} & \text{numbers} \end{bmatrix},
$$

and

$$
\text{sign restriction: } (1,1) \text{ element of } \left[I - B(1)\right]^{-1} C \text{ is positive.}
$$

Although there are many matrices, C, that satisfy $CC' = V$ as well as the exclusion and sign restrictions, they all have the same C_1. To see this, first let $D \equiv \left[I - B(1)\right]^{-1} C$, so that

$$
DD' = \left[I - B(1)\right]^{-1} V \left[I - B(1)'\right]^{-1} = S_0, \tag{2.6}
$$

say. Note that S_0 (the spectral density of Y_t at frequency zero) can be computed directly from the VAR coefficients and the variance-covariance matrix of the VAR disturbances. The exclusion restrictions require that D have the following structure:

$$
D = \begin{bmatrix} \underset{1\times 1}{d_{11}} & \underset{1\times(N-1)}{0} \\ \underset{(N-1)\times 1}{D_{21}} & \underset{(N-1)\times(N-1)}{D_{22}} \end{bmatrix}.
$$

Then,

$$DD' = \left[\begin{array}{cc} d_{11}^2 & d_{11}D_{21}' \\ D_{21}d_{11} & D_{21}D_{21}' + D_{22}D_{22}' \end{array} \right] = \left[\begin{array}{cc} S_0^{11} & S_0^{21\prime} \\ S_0^{21} & S_0^{22} \end{array} \right],$$

say. The sign restriction is:

$$d_{11} > 0. \tag{2.7}$$

Then, the first column of D is uniquely determined by:

$$d_{11} = \sqrt{S_0^{11}},\ D_{21} = S_0^{21}/d_{11}$$

Finally, the first column of C is determined from:

$$C_1 = [I - B(1)]\, D_1. \tag{2.8}$$

2.1.2. The Recursive Version of the Model

In the recursive version of the model, the policy rule for labor involves $\log z_{t-1}$ and x_{t-1} because they help forecast $\log z_t$ and x_t :

$$\log l_t = a_0 + a_k \log \hat{k}_t + \tilde{a}_l \tau_{lt} + \tilde{a}_z' \log z_{t-1} + \tilde{a}_x' x_{t-1}.$$

Because labor is a state variable at the time the investment decision is made, the policy rule for $\hat{k}_{t+1}$ takes the following form:

$$\begin{aligned} \log \hat{k}_{t+1} &= \gamma_0 + \gamma_k \log \hat{k}_t + \tilde{\gamma}_z \log z_t + \tilde{\gamma}_l \tau_{lt} + \tilde{\gamma}_x x_t \\ &\quad + \tilde{\gamma}_z' \log z_{t-1} + \tilde{\gamma}_x' x_{t-1}. \end{aligned}$$

It is easy to verify that these policy rules satisfy the exclusion restriction, (2.2), and the sign restriction on ε_t^z. So, the long-run identification strategy outlined above can be rationalized in this model. An alternative procedure for identifying ε_t^z that does not rely on estimating long-run responses to shocks can also be rationalized. We refer to this as the 'short run' strategy, because it involves recovering ε_t^z using just the realized one-step-ahead forecast errors in labor productivity and hours, as well as the second moment properties of those forecast errors. According to the model, the error in forecasting a_t given Ω_{t-1}, denoted by $u_{\Omega,t}^a$, is a linear combination of ε_t^z and ε_t^l. The error in forecasting $\log l_t$ given Ω_{t-1}, $u_{\Omega,t}^l$, is proportional to ε_t^l. Specifically,

$$u_{\Omega,t}^a = \alpha_1 \varepsilon_t^z + \alpha_2 \varepsilon_t^l,\ u_{\Omega,t}^l = \gamma \varepsilon_t^l,$$

where $\alpha_1 > 0$, α_2 and γ are functions of the model parameters. It follows that $\alpha_1 \varepsilon_t^z$ is the error from regressing $u_{\Omega,t}^a$ on $u_{\Omega,t}^l$:

$$u_{\Omega,t}^a = \beta u_{\Omega,t}^l + \alpha_1 \varepsilon_t^z,\ \beta = \frac{cov(u_{\Omega,t}^a, u_{\Omega,t}^l)}{V\left(u_{\Omega,t}^l\right)},$$

where $cov(x, y)$ denotes the covariance between the random variables, x and y, and $V(x)$ denotes the variance of x. Recall, we normalize the standard deviation of ε_t^z to be unity. Consequently, the value of α_1 can be recovered as the positive square root of the variance of the forecast error in this regression:

$$\alpha_1 = \sqrt{V\left(u^a_{\Omega,t}\right) - \beta^2 V\left(u^l_{\Omega,t}\right)}.$$

In practice, we implement the previous procedure using the one-step-ahead forecast errors generated from a VAR. It is convenient to work with a version of (2.3) in which the variables in Y_t are ordered as follows:

$$Y_t = \begin{pmatrix} \log l_t \\ \Delta \log a_t \\ x_t \end{pmatrix},$$

where x_t is an additional vector of variables that may be included in the VAR. In addition, we write the vector of VAR one-step-ahead forecast errors, u_t as:

$$u_t = \begin{pmatrix} u_t^l \\ u_t^a \\ u_t^x \end{pmatrix}.$$

We identify the technology shock with the second element in e_t in (2.4). To compute the dynamic response of the variables in Y_t to the technology shock, we require $B_1, ..., B_q$ in (2.3) and the second column of the matrix, C, in (2.4). We obtain the elements of the second column of C in two steps. First, we identify the technology shock using:

$$\varepsilon_t^z = \frac{1}{\hat{\alpha}_1}\left(u_t^a - \hat{\beta} u_t^l\right),$$

where

$$\hat{\beta} = \frac{cov(u_t^a, u_t^l)}{V\left(u_t^l\right)},\ \hat{\alpha}_1 = \sqrt{V\left(u_t^a\right) - \hat{\beta}^2 V\left(u_t^l\right)},$$

where the indicated variances and covariances are obtained from V in (2.3). Second, to obtain C_2, the second column of C, we regress u_t on ε_t^z :

$$C_2 = \begin{pmatrix} \frac{cov(u^l, e_2)}{var(e_2)} \\ \frac{cov(u^a, e_2)}{var(e_2)} \\ \frac{cov(u^x, e_2)}{var(e_2)} \end{pmatrix} = \begin{pmatrix} 0 \\ \hat{\alpha}_1 \\ \frac{1}{\hat{\alpha}_1}\left(cov(u_t^x, u_t^a) - \hat{\beta} cov\left(u_t^x, u_t^l\right)\right) \end{pmatrix}.$$

This procedure for computing C_2 can be implemented by computing $CC' = V$, where C is the lower triangular Choleski decomposition of V, and taking the second column of that matrix. This is a convenient strategy because the Choleski decomposition can be computed using widely-available software.

2.2. Parameterizing the Model

We consider different versions of the RBC model that are distinguished by the nature of the exogenous shocks. For comparability we assume, as in CKM, that:

$$\begin{aligned} \beta &= 0.9722^{1/4},\ \theta = 0.35,\ \delta = 1-(1-.0464)^{1/4},\ \psi = 2.24,\ \gamma = 1.015^{1/4}-1 \\ \bar{l} &= 1300,\ \bar{\tau}_x = 0.3,\ \bar{\tau}_l = 0.27388,\ \mu_z = 1.016^{1/4}-1,\ \sigma = 1. \end{aligned}$$

We consider various parameterizations for the shocks. These parameterizations were chosen to illustrate the key factors determining the reliability of inference based on short run and long-run identification restrictions.. It is convenient to report, for each parameterization, the variance of HP-filtered output due to technology shocks.

KP Benchmark Specification

In the Kydland and Prescott specification, the technology shock process is the same as the one estimated by Prescott (1986):[9]

$$\log z_t = \mu_Z + 0.011738 \times \varepsilon_t^z.$$

Erceg, Guerrieri and Gust (2005) update Prescott's analysis and estimate σ_z to be 0.0148. To be conservative, we use Prescott's estimate because it attributes relatively less importance to technology shocks in aggregate fluctuations. Although he concentrates on technology shocks in his analysis, Prescott (1986) argues that other shocks also affect aggregate fluctuations. To maintain comparability with CKM, we specify τ_{lt} to be the other shock in this specification.

We estimate a law of motion for $\tau_{l,t}$ as follows. Combining the household and firm first order conditions for labor, and rearranging, we obtain:

$$\tau_{l,t} = 1 - \frac{c_t}{y_t}\frac{l_t}{\bar{l}-l_t}\frac{\psi}{1-\theta}.$$

Given our parameter values, we compute a time series for $\tau_{l,t}$, and estimate the following first order autoregressive representation:[10]

$$\tau_{l,t} = (1-0.9934)\times 0.2660 + 0.9934 \times \tau_{l,t-1} + 0.0062 \times \varepsilon_t^l.$$

Figure 1 depicts the time series on $\tau_{l,t}$, $l_t/(\bar{l}-l_t)$ and c_t/y_t.

[9] Prescott (1986) estimates that the standard deviation of the innovation to technology growth is 0.763 percent. However, he adopts a different normalization than we do, placing technology in front of the production function, rather than next to hours worked, as we do. Our standard deviation is 0.01174=0.00763/(1-.35).

[10] Consumption, c_t, is the sum of nondurables, services and government consumption. We measure the ratio, c_t/y_t, as the ratio of dollar quantities. Total hours worked, l_t, is nonfarm business hours worked divided by a measure of the population, aged 16 and older.

Let p denote the percent variance in HP-filtered, log output due to technology shocks in a model. Table 1 reports that $p = 73$ in the Kydland-Prescott specification.[11] This value of p is consistent with a key claim advanced by Kydland and Prescott, namely that technology shocks account for toughly 70% of the cyclical volatility of output. The finding that p is 73% is the reason we refer to this version of our model as the Kydland-Prescott specification. For reference, Table 1 reports other standard business cycle statistics for the KP specification.

In artificial data generated by the KP Benchmark specification, we fit bivariate VAR models, in which x_t in (2.3) is omitted.

CKM Benchmark Specification

CKM's benchmark specification has two shocks, z_t and τ_{lt}, which have the following time series representations:

$$\begin{aligned} \log z_t &= \mu_Z + \log z_t = \mu_Z + 0.00581 \times \varepsilon_t^z \\ \tau_{lt} &= (1 - \rho_l)\,\bar{\tau}_l + \rho_l \tau_{l,t-1} + 0.00764 \times \varepsilon_t^l,\;\; \rho_l = 0.93782. \end{aligned}$$

In sharp contrast to the Kydland and Prescott specification, the CKM benchmark specification implies that p is only 23 (see Table 1). CKM use this specification to criticize Gali (1999)'s work. It is ironic that CKM use this specification to to criticize Gali when their specification embodies his main hypothesis, namely, that technology shocks play only a very small role in business cycle fluctuations. Other business cycle implications of the CKM benchmark specification are reported in Table 1. Note the substantial negative business cycle correlation of productivity and hours worked, and the relatively high volatility of hours worked. This reflects that non-technology shocks are the primary driver of business cycle fluctuations in the CKM benchmark specification.

In artificial data generated by the CKM Benchmark specification, we fit bivariate VAR models, in which x_t in (2.3) is omitted.

Other Specifications

We consider a series of perturbations on the Benchmark KP and CKM specifications. We vary the values of σ and σ_l, both of which have an important quantitative effect on the contribution of technology shocks to the volatility of output. In artificial data generated by these versions of the models, we fit bivariate VARs in which x_t in (2.3) is omitted.

In addition, we add shocks and variables to the model. In the *Three Shocks, Two Important* specification, we add the tax rate shock with a very small variance:

$$\tau_{xt} = \bar{\tau}_x + 0.0001 \times \varepsilon_t^x$$

[11] The 73 percent figure is a population value, computed using the spectral integration approach described in Christiano (2002).

In this case, we work with a three-variable VAR, with

$$x_t = \log\left(\frac{i_t}{y_t}\right). \tag{2.9}$$

in (2.3), where i_t denotes gross investment. In the *Three Shocks, Three Important* specification, we adopt, for the sake of comparability, the specification of the tax rate shock used in CKM:

$$\tau_{xt} = (1-0.9)\,\bar{\tau}_x + 0.9 \times \tau_{x,t-1} + 0.01\varepsilon_t^x. \tag{2.10}$$

Here too, we work with a three-variable VAR, with x_t in (2.3) specified as in (2.9).

Finally, we also consider a *Four Shocks, Three Important* specification. In this case, we specify x_t in (2.3) as

$$x_t = \begin{pmatrix} \log\left(\frac{i_t}{y_t}\right) \\ \tau_{xt} + w_t \end{pmatrix},$$

where w_t is a Normally distributed *iid* measurement error with mean zero and standard deviation 0.0001.

3. Results

In this section we analyze the properties of VAR-based strategies for identifying the effects of a technology shock. Our basic strategy is to simulate artificial time series using variants of the economic model discussed above as the data generating process. By construction we know the actual response of hours worked to a technology shock. We then consider what an econometrician using VARs would find, on average over repeated small samples.

Throughout we focus on three key questions. First, would the econometrician be misled when conducting inference about dynamic response functions? Second, is there substantial bias associated with the estimated dynamic response functions of hours to a technology shock? Third, are there easy-to-implement modifications to standard econometric procedures that would eliminate bias when it emerges? The first subsection presents our answers to these questions when we use the recursive version of the model. The second subsection presents our results when we use the long-run properties of the standard version of the model to identify technology shocks.

3.1. Recursive Identification

Analysis of the KP Specification

We begin by discussing the results we obtained using variants of the KP specification as the data generating mechanism. Throughout we proceed as follows. Using the economic model as the data generating mechanism,we simulate 1000 data sets, each of length 180 observations. The shocks ε_t^z, ε_t^l and possibly ε_t^x are drawn from *i.i.d.* standard normal distributions.

On each data set we estimate a four lag VAR, Two or three variables are included in the VAR depending on the specification being analyzed. Given the estimated VAR, we calculate the dynamic response of hours to a technology shock based on the short-run identifying restriction and method discussed in section 2.1.2 above. The solid lines in Figure 2 are the average dynamic response function obtained over the 1000 synthetic data sets in the different specifications. The starred lines are the true dynamic response function of hours worked implied by the economic model that is being used as the data generating process. The grey areas in the figure are measures of the sampling uncertainty associated with the estimated dynamic response functions. We obtain these measures by first calculating the standard deviation of the points in the estimated impulse response functions across the 1000 synthetic data sets. The grey areas correspond to a two standard deviation band about the relevant solid black line. The dashed lines corresponds to the top 2.5% and bottom 2.5% of the estimated coefficients in the dynamic response functions across the 1000 synthetic data sets. To the extent that the dashed lines coincide with boundaries of the grey area, there is support for the notion that the coefficients of estimated impulse response functions are normally distributed.

An important question is whether an econometrician would correctly estimate the true uncertainty associated with the estimated dynamic response functions. To address this question we proceed as follows. For each synthetic data set and corresponding estimated impulse response function, we calculated the bootstrap standard deviation of each point in the impulse response function. Specifically, for a given synthetic data set, we estimate a VAR and use it as the data generating process to construct 200 synthetic data sets, each of length 180 observations. For each synthetic data set, we estimate a new VAR and impulse response function. We then calculate the standard deviation of the coefficients in the impulse response functions across the 200 data sets. Finally, we take the average of these standard deviation across the 1000 synthetic data sets that were generated using the economic model as the data generating process. The lines with 0's in Figure 2 correspond to a two standard deviation band about the solid black line and are a measure of the average standard deviations that a econometrician would construct.

The top left graph in Figure 2 exhibits the properties of the VAR estimator of the response of hours to a technology shock when the data are generated by the KP specification. The 2,1 graph in Figure 2 corresponds to the case when the data generating mechanism is the KP specification with $\sigma = 0.0001$. This case is of interest, because utility is roughly linear in leisure, corresponding to Hansen (1985)'s indivisible labor model. The 3,1 graph in Figure

2 shows what happens when σ is increased above its benchmark specification, to $\sigma = 1.24$. This case is of interest, because this parameterization gives rise to roughly the same Frisch elasticity used in the model studied by Erceg, Guerrieri and Gust (1004). The 4,1 graph in Figure 2 shows what happens in the three variable, three shock version of the model. In each case, the impact effect on hours worked and associated sampling variance is also reported, for convenience, in Table 2.

The first column of Figure 2 exhibits three striking features. First, regardless of which variant of the KP specification we work with, there is no evidence whatsoever of bias in the estimated impulse response functions. In all cases, the solid lines virtually coincide with the starred lines. Second, Figure 2 indicates that an econometrician would not be misled in inference using standard procedures for constructing confidence intervals. This conclusion reflects the fact that the average value of the econometrician's confidence interval (the line with the 0's) coincides closely to the actual range of variation in the impulse response function (the grey area). Third, there is no evidence against the view that the estimated coefficients of the impulse response functions are normally distributed: in all cases the boundaries of the grey area coincide closely with the dashed lines.

Analysis of the CKM Specification

The right hand column of Figure 2 reports our results when the data generating mechanism is given by variants of the CKM specification. The top right hand graph in Figure 2 corresponds to the CKM specification. The 2,2 and 2,3 graphs in Figure 2 correspond the CKM specification with $\sigma = 0.0001$ and $\sigma = 1.24$, respectively. Finally, the 4,1 graph in Figure 2 corresponds to the three variable, three shock version of the CKM specification.

Notice that the second column of Figure 2 contains the same striking features as the first column. First, there is no evidence whatsoever of bias in the estimated impulse response functions. Second, the average value of the econometrician's confidence interval coincides closely to the actual range of variation in the impulse response function (the grey area). Third, there is no evidence against the view that the estimated coefficients of the impulse response functions are normally distributed.

In sum, our analysis of the recursive identification scheme reveals that structural VAR's perform remarkably well. This is extremely comforting for the vast literature that has exploited recursive identification schemes to identify the dynamic effects of shocks to the economy. Of course, one can criticize the particular short run identifying assumptions used in any given analysis. But our results strongly support the view that if the relevant recursive assumptions are satisfied in the data generating mechanism, standard structural VAR procedures will reliably uncover and identify the dynamic effects of shocks to the economy.

Finally, note we did *not* include capital as a variable in the VAR. Despite this omission, the structural VAR procedure performs remarkably well. This demonstrates that, claims in CKM to the contrary, omitting the economically relevant state variable capital does not in and of itself pose a problem for inference using structural VAR's.

3.2. Long-run Identification

Analysis of the KP Specification

We begin by discussing results associated with variants of the KP specification. As above we use the economic model as the data generating mechanism to simulate 1000 data sets, each of length 180 observations. The shocks ε_t^z, ε_t^l and possibly ε_t^x are drawn from *i.i.d.* standard normal distributions. If w_t is included in the analysis, it is drawn from an i.i.d. normal distribution with mean zero and standard deviation 0.0001. On each data set we estimate a four lag VAR. Two, three or four variables are included in the VAR depending on the specification being analyzed. Given the estimated VAR, we calculate the dynamic response of hours to a technology shock based on the long-run identifying restriction and method discussed in section 2.1.1 above. The solid, dashed and dotted lines, as well as the grey areas in the Figure 3 are the analogs of the corresponding objects in Figure 2.

The top left graph in Figure 3 exhibits the properties of the VAR estimator of the response of hours to a technology shock, when the data are generated by the KP specification. Notice that there is virtually no bias in the estimate of the response of hours worked to a technology shock. While there is considerable sampling uncertainty in the estimator, the econometrician would not be misled with respect to inference. This is because the average value of the econometrician's confidence interval (the line with the 0's) coincides reasonably closely to the actual range of variation in the impulse response function (the grey area) (actually, there is some tendency to understate the degree of sampling uncertainty.)

Consider next the 2,1 graph in Figure 3. Here the data generating mechanism is the KP specification with $\sigma = 0.0001$. This case is of interest, because utility is roughly linear in leisure, corresponding to Hansen (1985)'s indivisible labor model. Note that the bias associated with the estimator increases slightly. Still, the bias is very small relative to the sampling uncertainty associated with the estimated impulse response function. As above, the figure indicates that the econometrician would not be misled about sampling uncertainty on average if the data were generated by this specification.

To understand the reason for the appearance of some (small) bias in this case, it is interesting to note that $p = 62$, which is somewhat smaller than the corresponding value of 73 in the benchmark KP specification (recall, p is the percent of the business cycle variance in output due to technology shocks). Reducing σ increases the response of hours worked to both technology and labor tax shocks. However, the impact on the response of hours worked to a labor tax shock is greater than the impact on the response to a technology shock. The 3,1 graph in Figure 3 shows what happens when σ is increased above its benchmark specification, to $\sigma = 1.24$. This case is of interest, because this parameterization gives rise to roughly the same Frisch elasticity used in the model studied by Erceg, Guerrieri, and Gust (2005). In this case the bias in the VAR-based estimator of the impulse response function almost disappears, and the sampling uncertainty shrinks drastically. To understand the reason for this, simply apply the discussion underlying the 2,1 graph in reverse. In the parameterization underlying the 3,1 graph, $p = 92$, so that technology shocks account for the vast majority of cyclical fluctuations in output.

The 4,1 graph in Figure 3 shows what happens in the three variable, three shock version of the model. In this case there is a noticeable degree of bias associated with the estimated impulse response function. Still, the bias is relatively small in relation to the sampling alternative. Moreover, the econometrician's estimated confidence interval is roughly correct, on average. To understand the appearance of bias, it is useful to note that in this case p falls to to the relatively low value of 57.

To understand better why there is bias in the 4,1 graph, it is useful to use a formula due to Sims (1972). This formula allows us to characterize the VAR parameter estimates that an econometrician would obtain in a large sample of data.[12] Denote these parameter estimates by $\hat{B}_1, ..., \hat{B}_q$ and $\hat{V}$. Then,

$$\hat{V} = V + \min_{\hat{B}_1,...,\hat{B}_q} \frac{1}{2\pi} \int_{-\pi}^{\pi} \left[B\left(e^{-i\omega}\right) - \hat{B}\left(e^{-i\omega}\right)\right] S_Y(\omega) \left[B\left(e^{i\omega}\right) - \hat{B}\left(e^{i\omega}\right)\right]' d\omega, \quad (3.1)$$

where $B\left(e^{-i\omega}\right)$ is $B(L)$ with L replaced by $e^{-i\omega}$.[13] Here, B and V are the parameters of the actual VAR representation of the data, and $S_Y(\omega)$ is the associated spectral density, at frequency ω. Also, $\hat{B}_1, ..., \hat{B}_q$ and $\hat{V}$ are the parameters of the $q - th$ order VAR fit by the econometrician to the data.[14] According to (3.1), an econometrician who estimates a VAR

[12] For additional discussion of the Sims formula, see Sargent (1979, page).

[13] The minimization is actually over the trace of the indicated integral.

[14] The derivation of this formula is straightforward. Suppose that the true VAR representation of the covariance stationary process, Y_t, is:

$$Y_t = B(L)Y_{t-1} + u_t,$$

where $B(L)$ is a possibly infinite-ordered matrix polynomial in non-negative powers of L and $Eu_t u_t' = V$. Suppose the econometrician contemplates a particular parameterization of $B(L)$, $\hat{B}(L)$. Let the fitted disturbances associated with this parameterization be denoted $\hat{u}_t$. Simple substitution implies:

$$\hat{u}_t = \left[B(L) - \hat{B}(L)\right] Y_{t-1} + u_t.$$

The two random variables on the right of the equality are orthogonal, so that the variance of $\hat{u}_t$ is just the variance of the sum of the two:

$$var(\hat{u}_t) = var\left(\left[B(L) - \hat{B}(L)\right] Y_{t-1}\right) + V.$$

in population, chooses the VAR lag matrices to minimize a quadratic form in the difference between the estimated and true lag matrices, where the quadratic form assigns greatest weight to the frequencies where the spectral density is the greatest. If the econometrician's VAR is correctly specified, then $\hat{B} = B$ and $\hat{V} = V$ and the estimator is consistent. If there is specification error, then $\hat{B} \neq B$ and $V > \hat{V}$.[15] In our context, there is specification error because the true VAR implied by our models has $q = \infty$, but the econometrician uses a finite value of q. In quarterly data, q is typically set to 4.

Recall from section 2.1.1 that there are two key ingredients to computing the impact effects of shocks: the estimate of the variance covariance matrix of VAR disturbances and (2.6), the spectral density of Y_t at frequency zero. The variance covariance matrix is likely to be estimated precisely. As (3.1) suggests, OLS works especially hard to get $\hat{V}$ down to its true value of V. However, (3.1) also indicates that we cannot expect to be so lucky when it comes to the spectral density at frequency zero. The crucial input to this object is the sum of the estimated VAR matrices. According to (3.1), there is no particular reason for the latter to be estimated precisely by ordinary least squares. The sum of the lag VAR matrices corresponds to $\omega = 0$ in (3.1) and least squares will pay attention to this only if $S_Y(\omega)$ happens to be relatively large in a neighborhood of $\omega = 0$. This reasoning suggests that estimation based on long-run restrictions may be improved if the zero-frequency spectral density in (2.6) is replaced by an estimator that is specifically designed for the task. With this in mind, we replace S_0 with a standard Newey-West estimator:

$$S_0 = \sum_{k=-(T-1)}^{T-1} g(k)\hat{C}(k), \; g(k) = \begin{cases} 1 - \frac{|k|}{r} & |k| \leq r \\ 0 & |k| > r \end{cases}, \tag{3.2}$$

and (after removing the mean from Y_t)

$$\hat{C}(k) = \frac{1}{T} \sum_{t=k+1}^{T} Y_t Y'_{t-k}.$$

We use essentially all possible covariances in the data by choosing a large value of r, $r = 150$.

The results in the right column in Figure 3 show what happens when we redo the results in the first column, adjusting the procedure by replacing S_0 in (2.6) with S_0 as defined in (3.2). Comparing the top two graphs in Figure 3, we see that the change results in a reduction in sampling uncertainty in the estimator, although it also introduces a slight negative bias. Note however that this bias is small relative to sampling uncertainty. As above, the econometrician's confidence interval coincides quite closely with the true sampling uncertainty on average, so that he would not misled with respect to inference.

Expression (3.1) in the text follows immediately.

[15] By $V > \hat{V}$, we mean that $V - \hat{V}$ is a positive definite matrix.

Comparing the two graphs in the second row of Figure 3, we see that the use of the Newey-West spectral density estimator has introduced a small downward bias, but it has also reduced the sampling variance of the estimator. In the third row, we see the same pattern: the Newey-West estimator introduces a downward bias and also reduces the sampling variance of the impulse response function estimator.

Finally, consider the fourth row of Figure 3. In this case, use of the Newey-West spectral density estimator introduces a noticeable improvement in the small sample properties of the estimated dynamic response function. The bias is substantially smaller and there is a significant reduction in sampling uncertainty. We conclude that the Newey-West spectral density estimator has a small impact when VAR-based estimation works well, and improves accuracy otherwise.

Analysis of the CKM Specification

We begin our analysis of the CKM specification with the results in Figure 4. Consider the left column first. Results based on the benchmark specification appear in the top left graph. Note that now there is substantial bias in the estimated dynamic response function. In the model, the contemporaneous response of hours to a one-standard-deviation technology shock is 0.13 percent, while the mean estimated response is 0.62 percent.[16] A key factor behind this bias is the fact that technology shocks play a very small role output fluctuations in this model (see Table 1). To document that this is so, the second and third rows of Figure 4 show what happens with $\sigma_l = 0.00764/2$ and $\sigma_l = 0.00764/3$, respectively. In these two cases, the percent business cycle variance in output, p, is $p = 54$ and $p = 73$, respectively. Note how the accuracy of the impulse response functions improves as p increases. The right column of graphs corresponds to the left column, except that estimation is always based on the Newey-West zero-frequency spectral density estimator. Note in particular the dramatic improvement in the quality of the estimator of the benchmark CKM model. Almost all of the bias has been removed. Clearly, when problems occur, they reflect the difficulty in accurately estimating the zero-frequency spectral density.

Consider Figure 5. This corresponds to Figure 2 in the analysis of the KP model. The first row of Figure 5 reproduces the first row of Figure 4, for convenience. The second row corresponds to indivisible labor case, $\sigma = 0.0001$. The third row corresponds to the low Frisch elasticity case, $\sigma = 1.24$. The fourth row corresponds to the case with three shocks and three variables. Note that in the indivisible labor case, the bias is now so large that it lies outside 95 percent sampling interval in the impact period. The reason for this can

[16] Although the results in the top left panel of Figure 4 are based on the same data generating mechanism used by CKM, the results are quite different. They report that there is virtually *no* bias (see their Figure 10). The reason for this is the anomalous identification strategy they use. We discuss this below.

be understood using the argument developed above. In particular, the reduction in σ has the effect of reducing p to 0.13. The distortion in the estimator reflects this very small role for technology shocks in the model. In the low Frisch elasticity case, $\sigma = 1.24$, p is much higher, at 62, and this explains why there is relatively little distortion in the estimator of the impulse response function.

Consider now the right column in Figure 5. As before, this is the analog of the left column, except that the calculations are done using the Newey-West estimator of the zero-frequency spectral density matrix. Note from the first row, the one corresponding to the benchmark CKM model, that bias has now been almost completely eliminated, with little change in overall sampling uncertainty. The second row poses an even greater challenge than the first row, since the CKM model with indivisible labor implies a very substantial bias. When we apply the zero-frequency spectral density estimator, this bias is virtually gone with only a very small increase in sampling uncertainty.

Now consider the results in Figure 6. The top left graph repeats, for convenience, the top left graph in Figure 4. The subsequent rows give support to the possibility that with enough data, even if technology shocks played as small a rose as they do in the CKM model, VARs might yet be useful, as long as there are 'enough' variables in the analysis. The examples in Figure 6 suggest the possibility that 'enough' means at least one more variable than there are important shocks. Comparing the left and right figures in the first row of Figure 6, we see the effect of adding a variable (log, i_t/y_t) in the VAR and an unimportant shock in the model. Note that the bias is reduced, although the sampling uncertainty remains large. Because the econometrician using standard methods would see that sampling uncertainty is large, we are less concerned about this. Still, we are a little concerned because there is some evidence that sampling uncertainty is somewhat underestimated, on average. The second row shows what happens when we start with the 2,1 model - this is the three variable and three important shock case considered by CKM - and add one variable in to the VAR and one unimportant shock. Note that the bias falls nearly in half (unfortunately, the circles were inadvertently omitted from the 2,2 graph - they will be included in the next draft). When we go to the second row, we add a second variable and a very small additional shock. These examples suggest that if there are enough variables in the analysis, in excess of the important shocks, VARs with long-run restrictions may work tolerably well, even in a world that poses as sharp a challenge for these methods as the benchmark CKM model. This is encouraging from the point of VAR analysis, because there is a widespread perception - fueled in part by factor analysis studies - that there are at most 2-4 important shocks driving the economy. These examples suggest that with 5 or more variables in the VAR, the kind of problems inherent in the CKM benchmark model may not be severe.

4. Relation to CKM

In the introduction, we discussed some of the reasons for the different conclusions about VARs reached here and in CKM. This section documents the part of our discussion that was not addressed in previous sections. In particular, in imposing the long run restrictions, we implement the sign restriction, (2.7), while CKM impose that the first element of C_1 in (2.8) is positive. In the KP model, we found that it makes no difference which sign restriction is implemented (compare the rows Benchmark KP and Benchmark KP, CKM Identification in Table 2). However, it makes a very sharp difference in the CKM model. Up to now, all of our analysis has worked with the standard sign restriction, (2.7). In this section we show the consequences of working with (2.8) instead of (2.7) in the benchmark CKM model.

Our analysis appears in Figure 7. The figure displays the output and productivity responses to a positive technology shock, across the simulations in our Monte Carlo study. The upper portion displays what happens when (2.7) is imposed. It provides information on the simulation results using the benchmark CKM model reported in the top left graph of Figure 4. Note from the 1,2 graph that in all cases, labor productivity eventually rises after a positive technology shock. However, in some Monte Carlo replications, (roughly 20 percent), the initial response of labor productivity is actually negative. These are indicated by the yellow lines in the 1,2 graph. The histogram of the magnitude of the contemporaneous impact effect on hours worked appears in the bar chart in the 1,1 graph. The yellow lines in the bar chart correspond to the yellow lines in the 1,2 graph. In particular, the realizations in which the productivity response was initially negative were realizations in which the initial hours response was the biggest.

Now consider the bottom row of graphs in Figure 7. They show what happens when the CKM identification strategy is implemented. With this strategy, the productivity responses that were initially negative in the upper row are now interpreted as responses to negative productivity shocks. Normalizing the shocks to be positive, the 2,2 graph shows that in these cases CKM's identification has the implication that technology initially drops after a positive technology shock, after which it goes into a permanent dive. The responses of hours worked in these cases are interpreted to be strongly negative. In effect, the CKM identification strategy truncates the large hours responses in the histogram in the 1,1 chart and shifts them into the strong negative region. This is why CKM conclude that structural VARs imply a bimodal distribution for the hours response to technology shock. Since their identification strategy is anomalous relative to the literature, we conclude that their finding is simply a curiosity.

5. Concluding Remarks

In this paper we have studied the properties of structural VARs for uncovering impulse response functions to shocks. For pedagogical purposes, we only considered very simple data generating processes, based on variants of a prototype RBC model. We find that with short run restrictions, structural VARs perform remarkably well. With long run restrictions we find that with one exception structural VARs work well. The exception is that they potentially perform less well when the shock under investigation (in our case, technology shocks) plays a small role in output fluctuations. Even in this case, when there are enough variables in the VAR, problems are greatly mitigated. Perhaps more importantly, we develop and implement a modified VAR procedure which leads to a drastic improvement in the properties of VAR estimators, even when technology shocks play a limited role in business cycle fluctuations and a small number of variables are included in the VAR.

References

[1] Aiyagari, S. Rao, 1994, 'On the Contribution of Technology Shocks to Business Cycles,' Federal Reserve Bank of Minneapolis Quarterly Review, vol. 18, no. 1, Winter, pp. 22-34.

[2] Basu, Susanto, John G. Fernald, and Miles S. Kimball. 1999. 'Are Technology Improvements Contractionary?' Manuscript.

[3] Bernanke, Ben S. and Alan S. Blinder (1992), 'The Federal Funds Rate and the Channels of Monetary Transmission', *American Economic Review,* Vol. 82, No. 4, pages 901 - 921.

[4] Bernanke, Ben S. and Ilian Mihov (1995), 'Measuring Monetary Policy', NBER Working Paper No. 5145.

[5] Blanchard, Olivier Jean, and Danny Quah, 1989, 'The Dynamic Effects of Aggregate Demand and Supply Disturbances,' *American Economic Review*, v. 79, issue 4, pp. 655-673.

[6] Chari, V.V., Patrick Kehoe and Ellen McGrattan, 2005, 'A Critique of Structural VARs Using Real Business Cycle Theory,' Federal Reserve Bank of Minneapolis Working Paper 631, February.

[7] Christiano, Lawrence J., 2002, 'Solving Dynamic Equilibrium Models by a Method of Undetermined Coefficients,' *Computational Economics*, October, Volume 20, Issue 1-2.

[8] Christiano, Lawrence J. and Martin Eichenbaum (1992), 'Identification and the Liquidity Effect of a Monetary Policy Shock', in *Political Economy, Growth and Business Cycles,* edited by Alex Cukierman, Zvi Hercowitz, and Leonardo Leiderman, Cambridge and London: MIT Press, pages 335 - 370.

[9] Christiano, Lawrence J., Martin Eichenbaum and Charles Evans, 2005, 'Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy,' *Journal of Political Economy.*

[10] Christiano, Lawrence J., Martin Eichenbaum and Robert Vigfusson, 2003, "What Happens After a Technology Shock?", National Bureau of Economic Research working paper w9819.

[11] Christiano, Lawrence J., Martin Eichenbaum and Robert Vigfusson, 2004, 'The Response of Hours to a Technology Shock: Evidence Based on Direct Measures of Technology,' *Journal of the European Economic Association.*

[12] Christiano, Lawrence J., Roberto Motto and Massimo Rostagno, 2004, 'The Great Depression and the Friedman-Schwartz Hypothesis,' *Journal of Money, Credit and Banking.*

[13] Erceg, Christopher J., Luca Guerrieri, and Christopher Gust, 2004, 'Can Long-Run Restrictions Identify Technology Shocks?', International Finance Discussion Papers Number 792, Federal Reserve Boad of Governors.

[14] Faust, Jon and Eric Leeper, 1997, 'When Do Long-Run Identifying Restrictions Give Reliable Results?', *Journal of Business and Economic Statistics*, July, vol. 15, no. 3.

[15] Francis, Neville, and Valerie A. Ramey, 2001, 'Is the Technology-Driven Real Business Cycle Hypothesis Dead? Shocks and Aggregate Fluctuations Revisited,' manuscript, UCSD.

[16] Hansen, Gary, 1985, Indivisible Labor and the Business Cycle,' *Journal of Monetary Economics*, November, 16(3): 309-27

[17] Gali, Jordi, 1999, 'Technology, Employment, and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations?' *American Economic Review*, 89(1), 249-271.

[18] Grilli, Vittorio, and Nouriel Roubini (1995), 'Liquidity and Exchange Rates: Puzzling Evidence from the G-7 Countries', New York University Solomon Brothers Working Paper No. S/95/31.

[19] Hamilton, James D. (1997), 'Measuring the Liquidity Effect', *American Economic Review,* Vol. 87, No. 1 pages 80 - 97.

[20] Prescott, Edward, 1991, 'Real Business Cycle Theory: What Have We Learned?', Revista de Analisis Economico, 6 (November), pp. 3-19.

[21] Quah, Danny and Thomas Sargent, 1993, 'A Dynamic Index Model for Large Cross Sections,' in Stock, James H., Watson, Mark W., eds. *Business cycles, indicators, and forecasting*, NBER Studies in Business Cycles, vol. 28. Chicago and London: University of Chicago Press, 285-306

[22] Rotemberg, Julio J. and Michael Woodford, 1997, 'An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy,' National Bureau of Economic Research *Macroeconomics Annual*.

[23] Del Negro, Marco, Frank Schorfheide, Frank Smets, and Raf Wouters, 2005, 'On the Fit and Forecasting Performance of New Keynesian Models,' manuscript.

[24] Sargent, Thomas, 1979, Macroeconomics.

[25] Sims, Christopher, 1989, 'Models and Their Uses,' *American Journal of Agricultural Economics* 71, May, pp. 489-494.

[26] Sims, Christopher, 1972, 'The Role of Approximate Prior Restrictions in Distributed Lag Estimation,' *Journal of the American Statistical Association*, vol. 67, no. 337, March, pp. 169-175.

[27] Sims, Christopher A. and Tao Zha (1995), 'Does Monetary Policy Generate Recessions?', Manuscript. Yale University.

[28] Smets, Frank, and Raf Wouters, 2003, 'An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area,' *Journal of European Economic Association* 1(5), pp. 1123-1175.

[29] Uhlig, Harald, 2002, 'What Moves Real GNP?', manuscript, Humboldt University, Berlin.

Table 1: Selected Business Cycle Statistics			
	US data	Kydland-Prescott Specification	CKM
σ_y	1.6	1.4	1.3
$\frac{\sigma_c}{\sigma_y}$	0.53	0.57	0.35
$\frac{\sigma_i}{\sigma_y}$	3.65	2.9	4.13
$\frac{\sigma_l}{\sigma_y}$	1.08	0.87	1.38
corr$(h, \frac{y}{h})$	-0.40	-0.21	-0.77
% variance due to technology		73	23

Note: σ_x - standard deviation of x; corr(x, y) correlation between x and y
Here, x and y have been logged first, and then HP-filtered

Table 2: Percent Contemporaneous Impact on Hours of One Standard Deviation Shock to Technology				
	Contribution of	Impact of Tech Shock On Hours Worked		
	Tech Shocks to	True Value	Standard VAR	Newey-West
Model Specification	Business Cycle		Mean (Std Dev)	Mean (Std Dev)
Kydland-Prescott Parameterization				
BenchmarkKP	73	0.28	0.32 (0.42)	0.11 (0.30)
Benchmark KP, CKM Identification	73	0.28	0.28 (0.43)	0.30 (0.42)
$\sigma = .0001$ ('indivisible labor')	62	0.41	0.55 (0.54)	0.17 (0.42)
$\sigma = 1.24$ (Frisch elasticity=0.63)	92	0.11	0.10 (0.18)	0.03 (0.13)
Three Shocks, Two Important	73	0.28	0.35 (0.43)	-0.04 (1.58)
Three Shocks, Three Important	57	0.27	0.42 (0.50)	0.21 (0.35)
Four Shocks, Three Important	57	0.28	0.43 (0.52)	0.09 (2.88)
Chari-Kehoe-McGrattan Paramerization				
CKM Benchmark	23	0.13	0.62 (0.37)	0.20 (0.40)
CKM Benchmark, CKM Identification	23	0.13	0.07 (0.71)	0.04 (0.71)
$\sigma = .0001$ ('indivisible labor')	13	0.20	1.23 (0.46)	0.38 (0.56)
$\sigma = 1.24$ (Frisch elasticity=0.63)	64	0.05	0.13 (0.16)	0.05 (0.16)
$\sigma^l/2$	54	0.14	0.24 (0.21)	0.10 (0.21)
$\sigma^l/3$	73	0.14	0.18 (0.14)	0.09 (0.14)
Three Shocks, Two Important	23	0.14	0.34 (0.55)	0.06 (1.53)
Three Shocks, Three Important	17	0.13	0.75 (0.44)	0.28 (0.43)
Four Shocks, Three Important	17	0.13	0.33 (0.61)	0.26 (2.63)

Figure 1 - The Labor Tax Wedge and Its Components

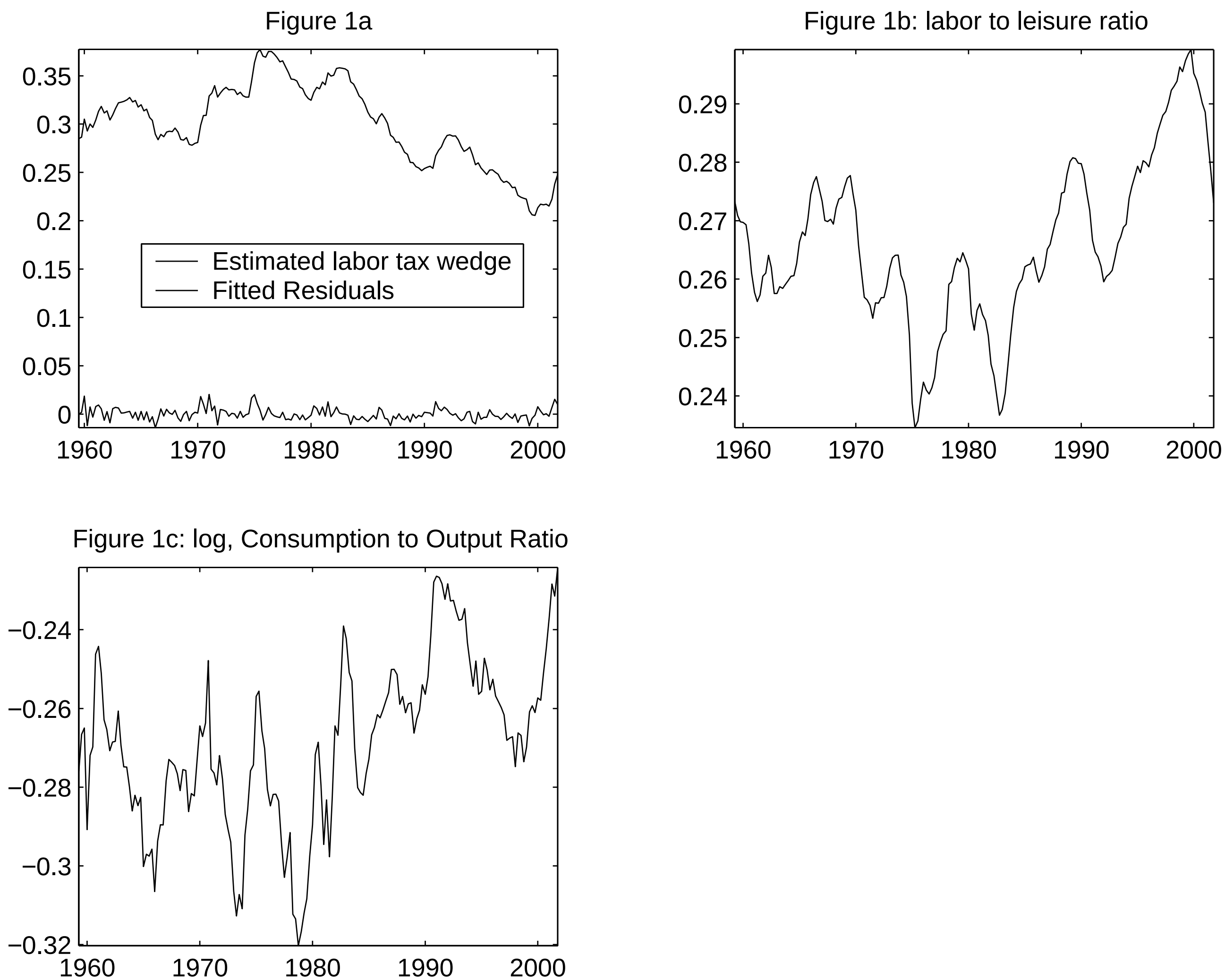
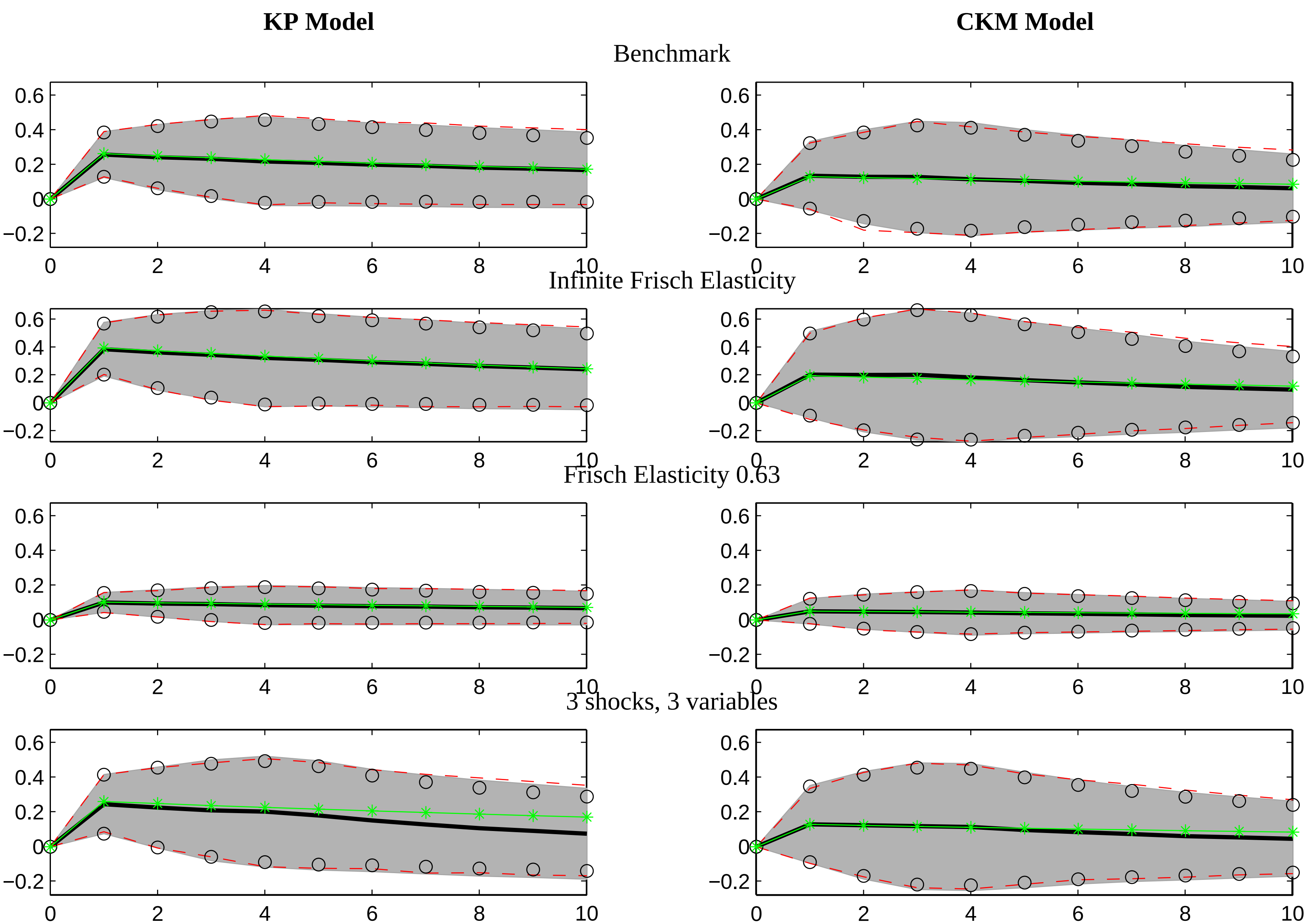


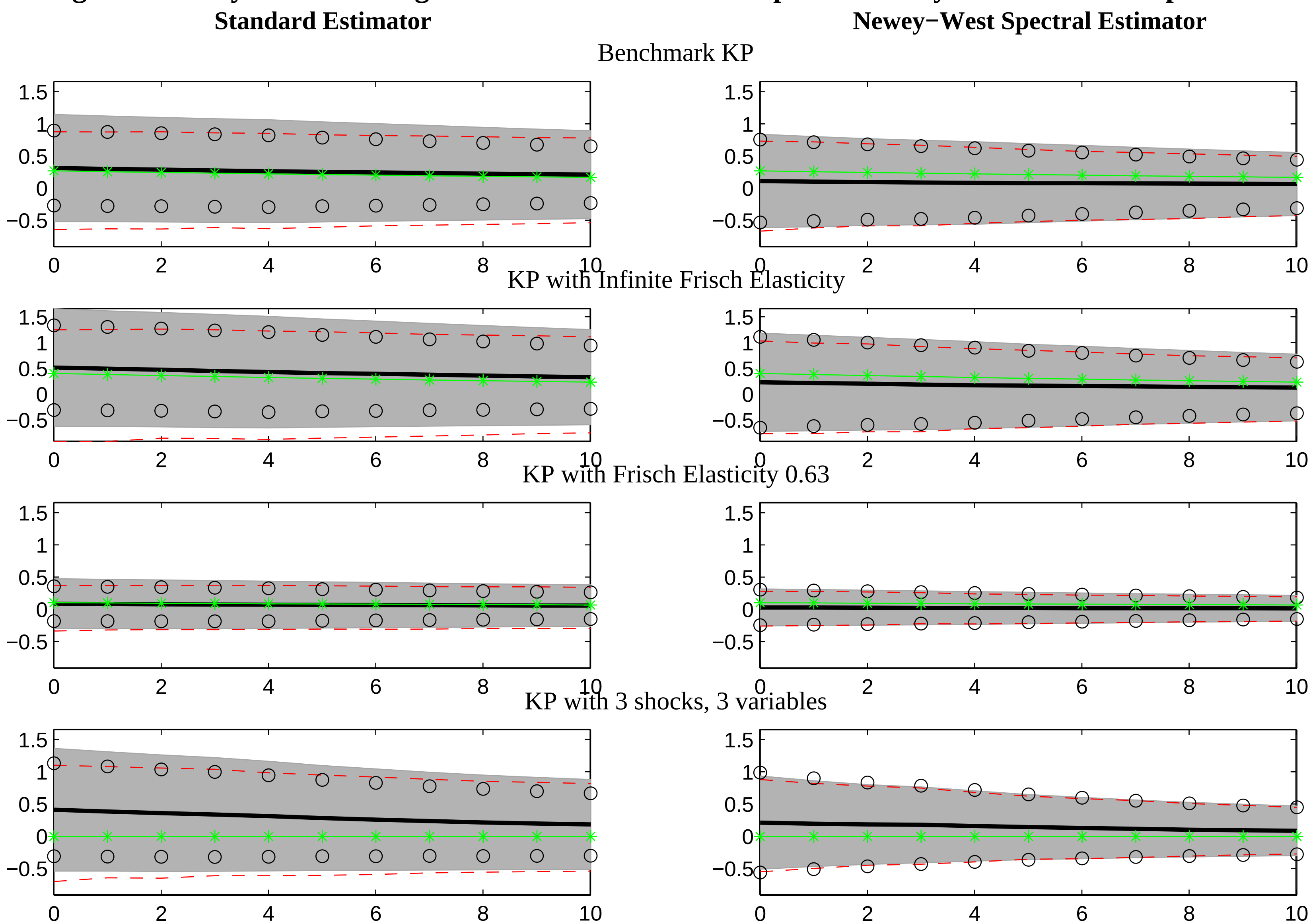
Figure 2: Analysis of Short−Run Identification Assumption



Note: hours response, in percent terms, to a 1.2 (KP) or 0.6 (CKM) percent innovation in technology, Z_t.

Solid line − mean response, Gray area − mean response plus/minus two standard errors,
Starred line − true response, Dashed line − 95.5 percent probability interval of responses,
Circles − average value of econometrician estimated plus/minus two standard errors.

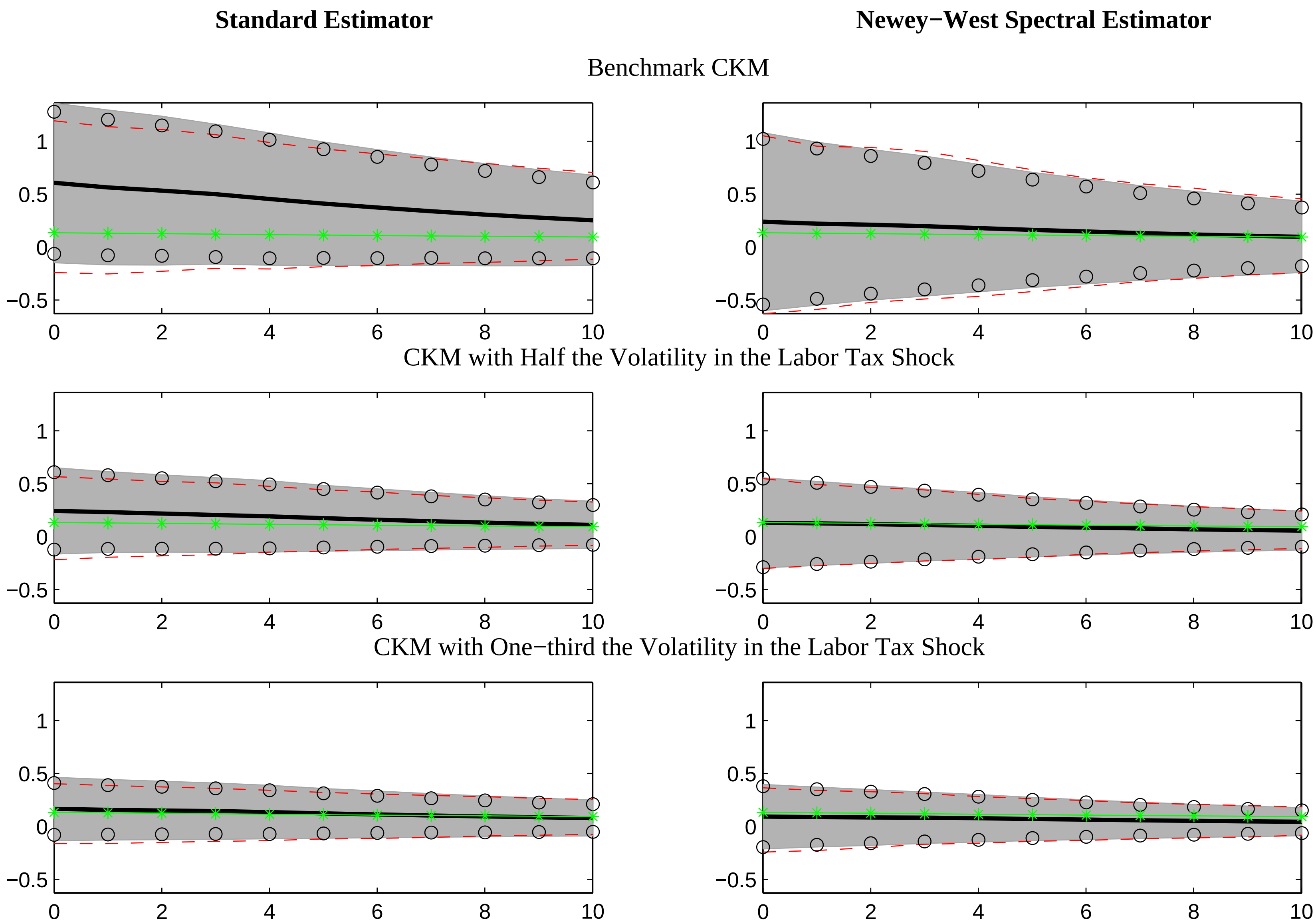
Figure 3: Analysis of the Long−Run Identification Assumption with Kydland−Prescott Specification



Note: hours response, in percent terms, to a 1.2 percent innovation in technology, Z_t.

Solid line − mean response, Gray area − mean response plus/minus two standard errors,
Starred line − true response, Dashed line − 95.5 percent probability interval of responses,
Circles − average value of econometrician estimated plus/minus two standard errors.

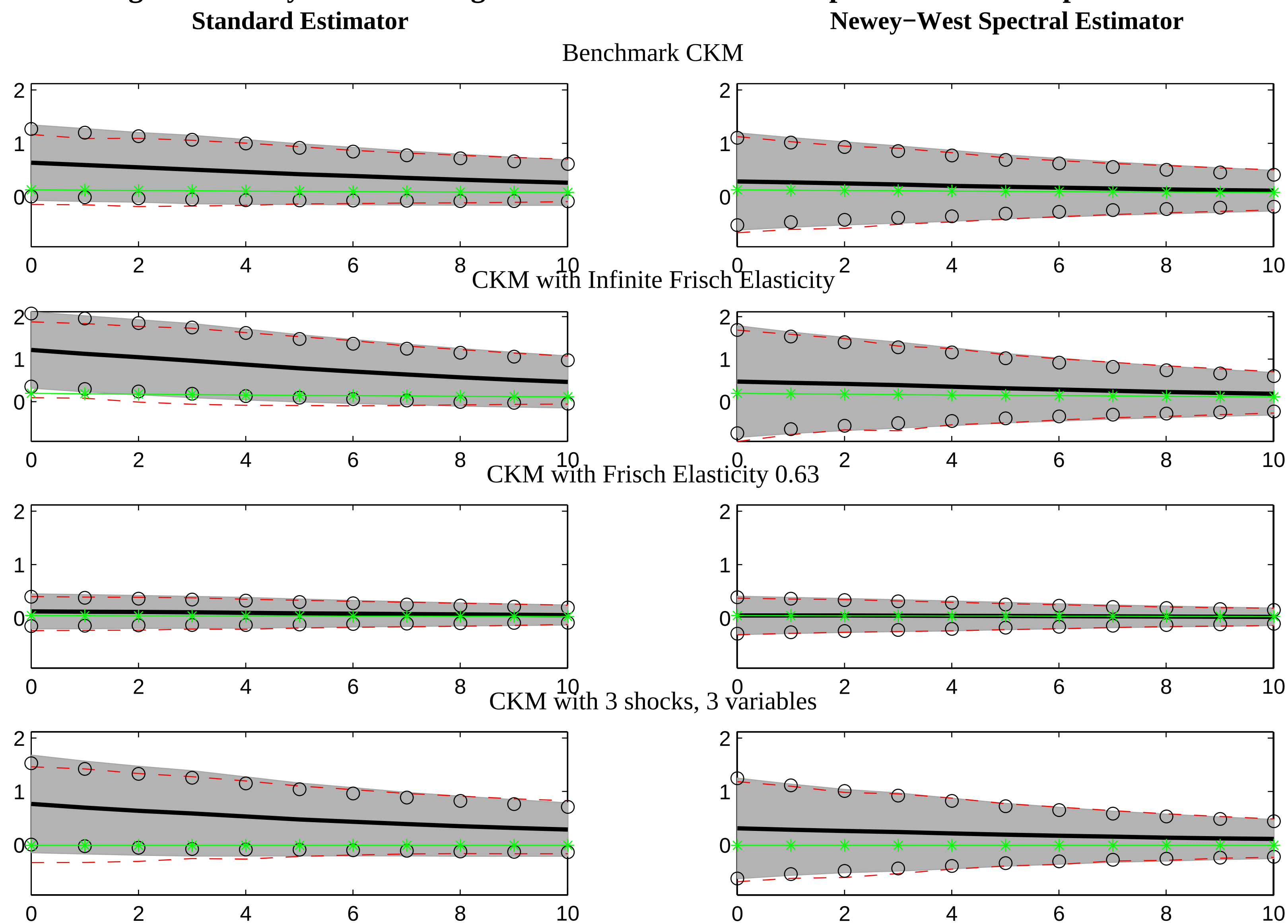
Figure 4: Analysis of the Long−Run Identification Assumption with CKM Specification



Note: hours response, in percent terms, to a 0.6 percent innovation in technology, Z_t.

Solid line − mean response, Gray area − mean response plus/minus two standard errors,
Starred line − true response, Dashed line − 95.5 percent probability interval of responses,
Circles − average value of econometrician estimated plus/minus two standard errors.

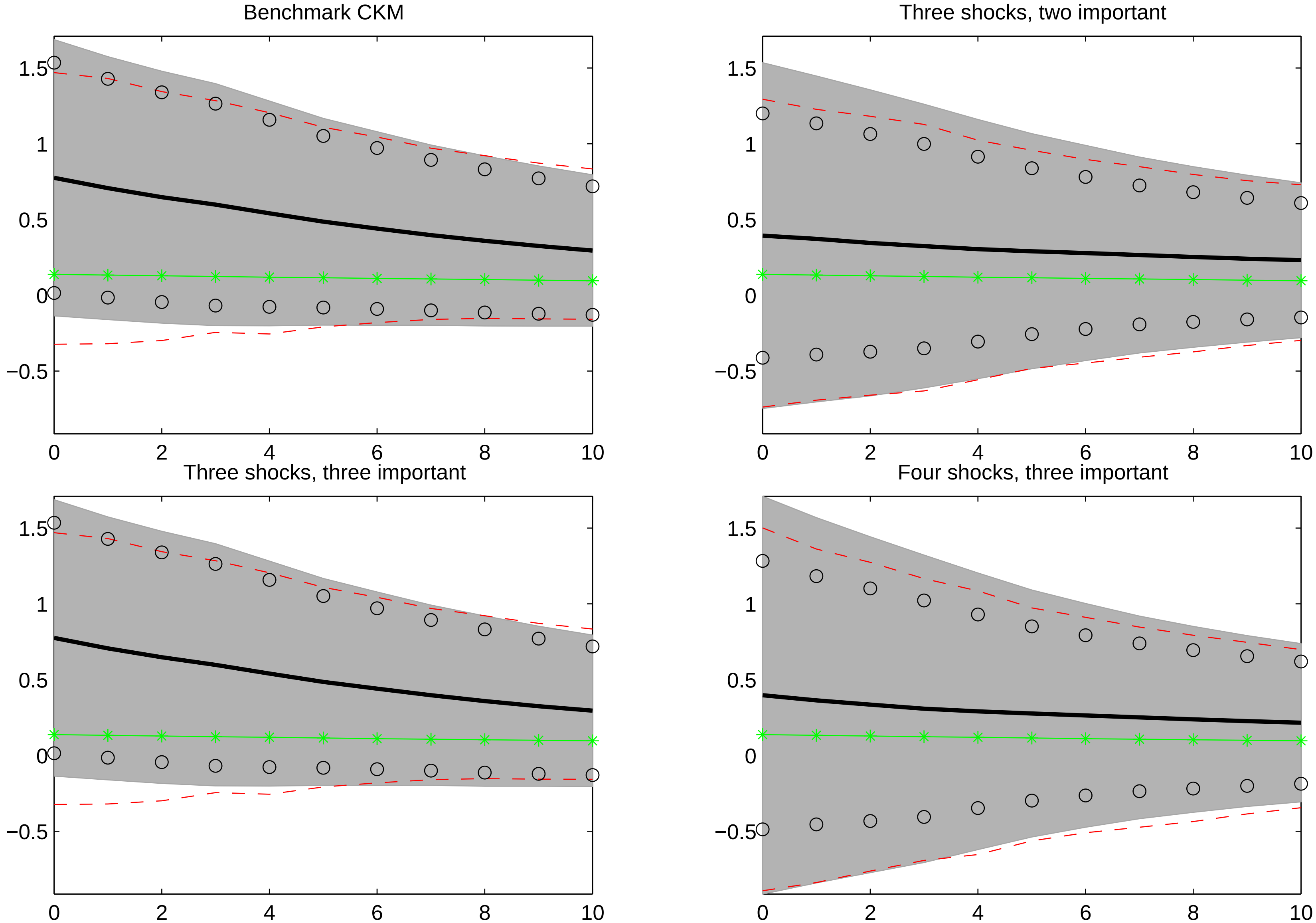
Figure 5: Analysis of the Long−Run Identification Assumption with CKM Specification



Note: hours response, in percent terms, to a 0.6 percent innovation in technology, Z_t.

Solid line − mean response, Gray area − mean response plus/minus two standard errors,
Starred line − true response, Dashed line − 95.5 percent probability interval of responses,
Circles − average value of econometrician estimated plus/minus two standard errors.

Figure 6: The Long−Run Identification Assumption: Adding Variables and Shocks to the CKM Benchmark

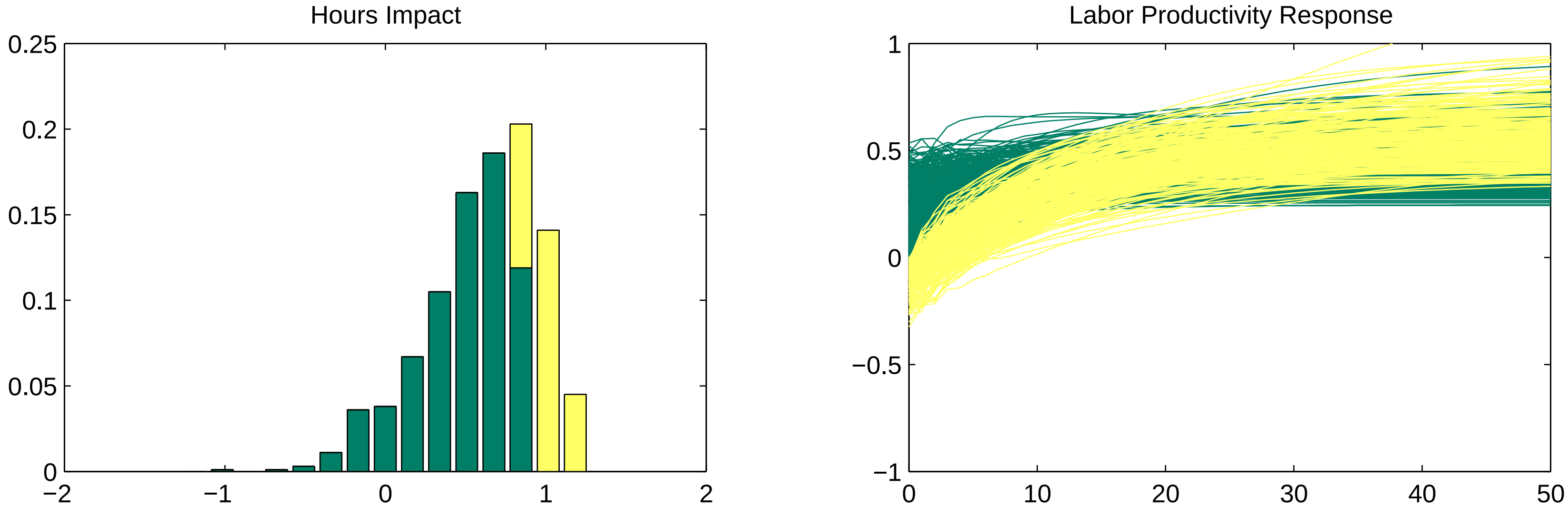


Note: hours response, in percent terms, to a 0.6 percent innovation in technology, Z_t.

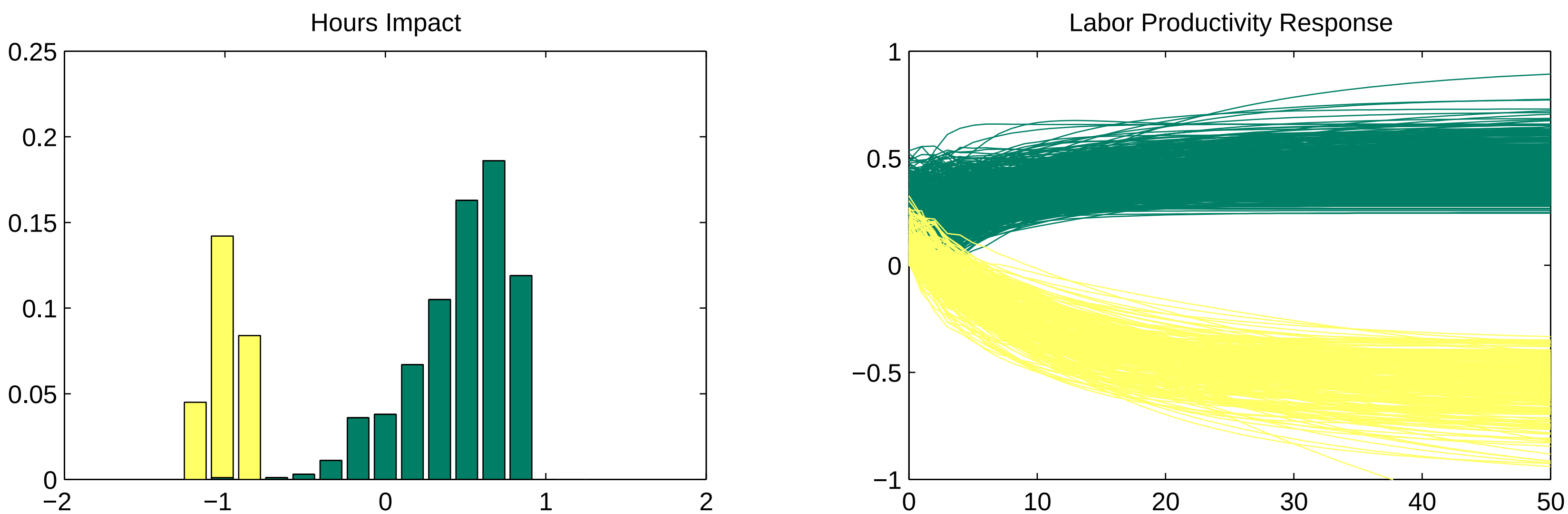
Estimation results for Standard VAR estimator.
Solid line − mean response, Gray area − mean response plus/minus two standard errors,
Starred line − true response, Dashed line − 95.5 percent probability interval of responses,
Circles − average value of econometrician estimated plus/minus two standard errors.

Figure 7: Implications of Two Different Long−Run Identification Strategies

A Positive Technology Shock Increases Labor Productivity in the Long Run (Standard Identification)



A Positive Technology Shock Increases Labor Productivity on Impact (CKM Identification)



Note: Responses, in percent terms, to a 0.6 percent innovation in technology, Z_t.
Estimation results for Standard VAR estimator using the Benchmark CKM model.