# Assessing Structural VARs

by
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- Another tradition:
  - Build macro models with large numbers of shocks - complete characterization of DGP.
  - Avoids KP conundrum.

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- Important early contribution - Sims (1972)
  - Emphasizes Difficulty of Estimating Sum of Coefficients in Distributed lag Regressions.
  - This is at Heart of Difficulty in With Long-Run Identification
  - See also Faust and Leeper and Pagan.

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- More recently EGG and CKM examine reliability of VAR-based inference using long run identifying restrictions.
  - CKM are exceedingly critical.

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  - Look at Long Run Restrictions and Short Run Restrictions

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- We address these questions using data generated from dynamic GE models.
  - Look at Long Run Restrictions and Short Run Restrictions

- Our conclusion:
  - Structural VARs provide valuable information for building empirically plausible models of aggregate fluctuations.

# Findings for Short Run Restrictions

- DGP: variants of a standard real business cycle model augmented by timing restrictions..
  - Focus on response of hours to technology shock.

- Conclusion:
  - VAR's perform *remarkably* well
  - Virtually no bias - either in point estimates or estimates of sampling uncertainty.

- Very comforting for vast literature that uses short run restrictions to identify consequences of shocks to economy.

# Findings for long run restrictions

- When technology shocks account for a substantial fraction of business cycle fluctuations in output, VAR based analysis is reliable.
  - Some evidence of bias when tech shocks play much smaller role relative to estimates in standard RBC literature.

- First way to eliminate bias:
  - When number of variables in VAR exceeds number of important driving shocks, bias in impulse response estimators is substantially reduced.
    - Widespread consensus: only a handful (e.g., 3-4) of important shocks drive aggregate fluctuations

- Second way to eliminate bias:
  - Integrate Newey-West non-parametric estimator of zero-frequency spectral density

# Outline

- RBC Models
  - Various Parameterizations Considered
  - Standard Version (Long Run Restrictions)
  - Recursive Version (Short Run Restrictions)

- Structural VAR and the Identification Problem

- Short Run Restrictions Approach to Identification

- Long Run Restrictions Approach to Identification

- Reconciling with CKM

- Concluding Comments

# DGP: A Generic RBC Model

- Preferences:

$$E\left\{\sum_{t=0}^{\infty}\left(\beta\left(1+\gamma\right)\right)^{t}\left[\log c_t+\psi\frac{\left(\bar{l}-l_t\right)^{1-\sigma}}{1-\sigma}\right]|\Omega_0\right\}.$$

- Constraints:

$$c_t+\left(1+\tau_{x,t}\right)\left[\left(1+\gamma\right)k_{t+1}-\left(1-\delta\right)k_t\right]\ \leq\ \left(1-\tau_{lt}\right)w_t l_t+r_t k_t.$$

$$c_t+\left(1+\gamma\right)k_{t+1}-\left(1-\delta\right)k_t\ \leq\ k_t^{\alpha}\left(Z_t l_t\right)^{1-\alpha}.$$

- Shocks:

$$\begin{aligned}
\log\left(Z_t\right) &= \mu_Z+\log\left(Z_{t-1}\right)+\sigma_z\varepsilon_t^z,\\
\tau_{lt+1} &= \left(1-\rho_l\right)\bar{\tau}_l+\rho_l\tau_{lt}+\sigma_l\varepsilon_{t+1}^d,\\
\tau_{xt+1} &= \left(1-\rho_x\right)\bar{\tau}_x+\rho_x\tau_{xt}+\sigma_x\varepsilon_{t+1}^x.
\end{aligned}$$

# Parameterizing the Model

- As in CKM we assume

$$
\begin{aligned}
\beta &= 0.9722^{1/4},\ \theta = 0.35,\ \delta = 1 - (1 - .0464)^{1/4}, \\
\psi &= 2.24,\ \gamma = 1.015^{1/4} - 1, \bar{l} = 1300, \\
\bar{\tau}_x &= 0.3,\ \bar{\tau}_l = 0.27388,\ \mu_z = 1.016^{1/4} - 1,\ \sigma = 1.
\end{aligned}
$$

- Different versions of the RBC model, distinguished by the nature of exogenous shocks.

*KP Specification*

- Technology shock process (Prescott (1986))

$$\log z_t \;=\; \mu_Z + 0.011738 \times \varepsilon_t^z.$$

- EGG (2005) update Prescott's analysis, estimate $\sigma_z$ to be $0.0148$.
  – To be conservative, we use Prescott's estimate.

- Law of motion for $\tau_{l,t}$ as follows.
  – Household / firm FONC's imply:

$$\tau_{l,t} \;=\; 1 - \frac{c_t}{y_t}\frac{l_t}{\bar{l} - l_t}\frac{\psi}{1-\theta}.$$

$$\tau_{l,t} \;=\; (1 - 0.9934) \times 0.2660 + 0.9934 \times \tau_{l,t-1} + .0062 \times \varepsilon_t^l.$$

- Percent of variance in HP-filtered, log output due to technology shocks is 73%.
  – Consistent with key claim of KP.

*CKM Benchmark Specification*

$$\begin{aligned} \log z_t &= \mu_Z + \log z_t = \mu_Z + 0.00581 \times \varepsilon_t^z \\ \tau_{lt} &= (1-\rho_l)\,\bar{\tau}_l + \rho_l \tau_{l,t-1} + 0.00764 \times \varepsilon_t^l,\ \rho_l = 0.93782. \end{aligned}$$

- Percent of variance in HP-filtered, log output due to technology shocks is only 23%.
- Irony:
  - CKM use this specification to criticize Gali (1999).
  - Embodies Gali's main hypothesis that technology shocks play only a very small role in business cycle fluctuations.

*Other Specifications*

- Vary $\sigma$ and $\sigma_l$
  - Important quantitative effect on contribution of technology shocks to volatility of output.
- *Three Shocks*, *Two Important* Specification
  - Additional (Unimportant) Shock, Capital Tax Rate

$$\tau_{xt} = \bar{\tau}_x + 0.0001 \times \varepsilon_t^x$$

  - Three Variables in VAR Analysis:

$$a_t \equiv \log\left(\frac{y_t}{l_t}\right), \ \log l_t, \ \log\left(\frac{c_t}{y_t}\right)$$

**Parameterizing the Model ...**

- *Three Shocks, Three Important* Specification
  - As in CKM:

$$\tau_{xt} = (1-0.9)\,\bar{\tau}_x + 0.9 \times \tau_{x,t-1} + 0.01\varepsilon_t^x.$$

  - Three Variables in VAR Analysis
- *Four Shocks, Three Important* Specification
  - Capital Tax as in CKM
  - Four Variables in VAR Analysis

$$a_t \equiv \log\left(\frac{y_t}{l_t}\right),\ \log l_t,\ \log\left(\frac{c_t}{y_t}\right),\ \tau_{xt} + w_t$$

$$w_t \backsim N(0, 0.0001)$$

# Two Versions of Model

- Differentiated by timing assumptions.

- *Standard version*
  - All time $t$ decisions taken after realization of the time $t$ shocks.

- *Recursive version*
  - First, $\tau_{lt}$ is observed. Then, labor decision made.
  - Second, other shocks are realized.
  - Then, agents make their investment and consumption decisions.
  - Finally, labor, investment, consumption, and output occur

# Estimating the Effects of a Positive Technology Shock in VAR

- VAR:

$$X_t = B_1 X_{t-1} + B_2 X_{t-2} + ... + B_p X_{t-p} + u_t,$$

$$E u_t u_t' = V,\ u_t = C e_t,\ E e_t e_t' = I,\ CC' = V$$

$$X_t = \begin{pmatrix} \Delta \log a_t \\ \log l_t \\ x_t \end{pmatrix},\ C = [C_1 \vdots C_2 \vdots C_3],\ \varepsilon_t = \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{pmatrix},\ a_t = \frac{Y_t}{l_t}$$

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- Impulse Response to Positive Technology Shock (Say, $\varepsilon_{1t} = \varepsilon_t^z$):

$$X_t - E_{t-1} X_t \; = \; C_1 \varepsilon_{1t}, \; E_t X_{t+1} - E_{t-1} X_{t+1} = B_1 C_1 \varepsilon_{1t}$$

$$E_t X_{t+2} - E_{t-1} X_{t+2} \; = \; B_1^2 C_1 \varepsilon_{1t} + B_2 C_1 \varepsilon_{1t}$$

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- Need: $B_1, ..., B_p, C_1$.

# Identification Problem

- From Applying OLS To Both Equations in VAR, We 'Know':

$$B_1, ..., B_p,\ V$$

- Problem, Need first Column of $C$, $C_1$
- Restrictions (Bivariate Case): three equations in four unknowns

$$CC' = V$$

- Identification Problem:

Not Enough Restrictions to Pin Down $C_1$

- Need More Restrictions

# The Recursive Version of the Model

- First, $\tau_{lt}$ is observed. Then, labor decision made. Consequently,

$$u^l_{\Omega,t} \;=\; \gamma\varepsilon^l_t,\; u^l_{\Omega,t} \equiv P\left[l_t|\Omega_{t-1}\right]$$

- Second, other shocks are realized, so

$$u^a_{\Omega,t} \;=\; \alpha_1\varepsilon^z_t + \alpha_2\varepsilon^l_t,\; u^a_{\Omega,t} \equiv P\left[a_t|\Omega_{t-1}\right]$$

- Regression:

$$u^a_{\Omega,t} \;=\; \beta u^l_{\Omega,t} + \alpha_1\varepsilon^z_t,\; \beta = \frac{cov(u^a_{\Omega,t}, u^l_{\Omega,t})}{V\left(u^l_{\Omega,t}\right)},$$

- Perform Analogous Calculations in VAR

**The Recursive Version of the Model ...**

$$
\begin{aligned}
Y_t &= \begin{pmatrix} \log l_t \\ \Delta \log a_t \\ x_t \end{pmatrix}, \\
u_t &= \begin{pmatrix} u_t^l \\ u_t^a \\ u_t^x \end{pmatrix} \\
u_t &= C\varepsilon_t, E\varepsilon_t\varepsilon_t' = I,\ CC' = V
\end{aligned}
$$

$$\varepsilon_{2t} \sim \varepsilon_t^z$$

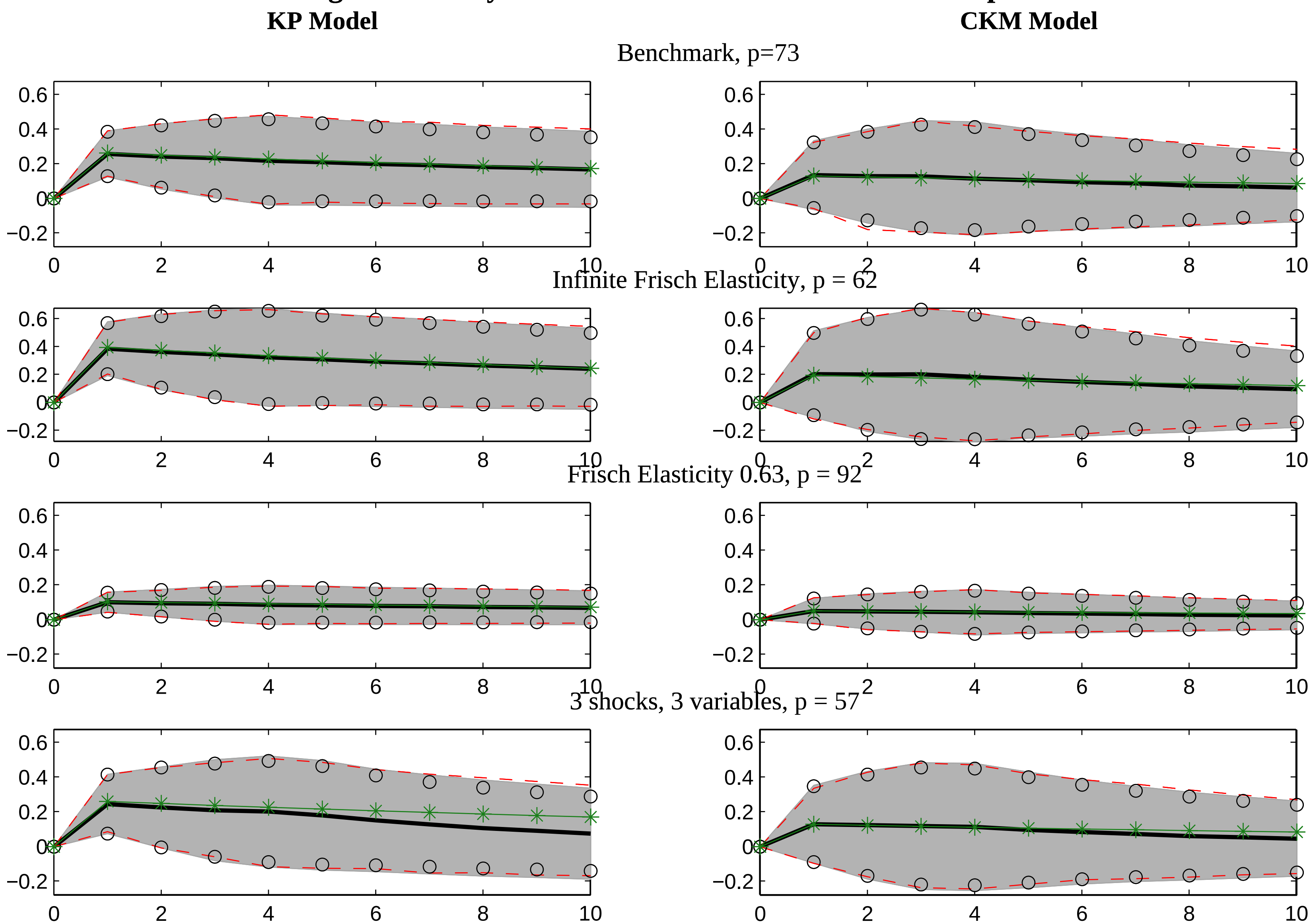
- For Response of $Y_t$ to $\varepsilon_t^z$ , need $B_1, ..., B_q$ and second column of $C$.
  - Compute $CC' = V$, where $C$ is lower triangular Choleski decomposition of $V$.
  - Take second column of $C$.
- Potential Source of Specification Error: Differences Between One-Step-Ahead Forecast Errors in Model and VAR.

# Experiments

- Simulate 1000 data sets, each of length 180 observations, using GE model as DGP.
  - Shocks $\varepsilon_t^z$, $\varepsilon_t^l$ and possibly $\varepsilon_t^x$ are drawn from $i.i.d.$ standard normal distributions.

- Estimate a four lag VAR.
  - Report Mean Impulse Response Function over 1000 synthetic data sets.
  - Measure of sampling uncertainty associated with the estimated dynamic response functions.
    - Calculate standard deviation of points in estimated impulse response functions across the 1000 synthetic data sets (Grey Area).
    - Also calculate middle 95% of the estimated coefficients in dynamic response functions across the 1000 synthetic data sets (Red lines).
  - Report Mean of Econometrician's Confidence Interval

**Figure 2: Analysis of Short−Run Identification Assumption**




Note: hours response, in percent terms, to a 1.2 (KP) or 0.6 (CKM) percent innovation in technology, $Z_t$.

Solid line − mean response, Gray area − mean response plus/minus two standard errors,
Starred line − true response, Dashed line − 95.5 percent probability interval of responses,
Circles − average value of econometrician estimated plus/minus two standard errors.

# Summary of Findings with Short Run Restrictions

- No evidence of bias in the estimated impulse response functions.

- An econometrician wouldn't be misled in inference using standard procedures for constructing confidence intervals.

- SVAR's perform remarkably well.
  - Absent specification error, standard structural VAR procedures reliably uncover and identify the dynamic effects of shocks to the economy.

- We did *not* include capital as a variable in the VAR.
  - Claims in CKM to contrary, omitting economically relevant state variable capital does not in and of itself pose a problem for inference using structural VAR's.

# Long-Run Restrictions

- Two Key Properties of Model:
  - Exclusion Restriction:

$$\lim_{j\to\infty}\left[E_t a_{t+j} - E_{t-1} a_{t+j}\right] \;=\; f(\varepsilon_t^z \text{ only})$$

  - Sign Restriction:

$$f \text{ increasing in } \varepsilon_t^z$$

- Exploit Analogous Properties in VAR to Identify Technology Shocks and their Effects

# Applying Analogous Restrictions to VAR

- Note:

$$
\begin{aligned}
\tilde{E}_t[a_{t+1}] - \tilde{E}_{t-1}[a_{t+1}] &= \tilde{E}_t[\Delta a_{t+1} + \Delta a_t] - \tilde{E}_{t-1}[\Delta a_{t+1} + \Delta a_t] \\
&= \left[\tilde{E}_t \Delta a_{t+1} - \tilde{E}_{t-1} \Delta a_{t+1}\right] + \left[\Delta a_t - \tilde{E}_{t-1} \Delta a_t\right]
\end{aligned}
$$

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- Then ($p = 1$)

$$\tilde{E}_t[a_{t+1}] - \tilde{E}_{t-1}[a_{t+1}] \; = \; (1,0)\,[B + I]\,C\varepsilon_t$$

$$\tilde{E}_t[a_{t+2}] - \tilde{E}_{t-1}[a_{t+2}] \; = \; (1,0)\left[B^2 + B + I\right]C\varepsilon_t$$

$$\tilde{E}_t[a_{t+j}] - \tilde{E}_{t-1}[a_{t+j}] \; = \; (1,0)\left[B^j + B^{j-1} + ... + B^2 + B + I\right]C\varepsilon_t$$

# Applying Analogous Restrictions to VAR

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as $j \longrightarrow \infty$:

$$\begin{aligned}
&\lim_{j\to\infty} \tilde{E}_t[a_{t+j}] - \tilde{E}_{t-1}[a_{t+j}] \\
&= \lim_{j\to\infty} (1,0)\left[... + B^j + B^{j-1} + ... + B^2 + B + I\right]C\varepsilon_t \\
&= (1,0)\,[I-B]^{-1}\,C\varepsilon_t
\end{aligned}$$

**Applying Analogous Restrictions to VAR ...**

- As $j \to \infty$ (for arbitrary $p$) :

$$\lim_{j\to\infty} \tilde{E}_t[a_{t+j}] - \tilde{E}_{t-1}[a_{t+j}] \ = \ (1, 0, ..., 0)\,[I - B(1)]^{-1}\, C\varepsilon_t$$

$$B(1) \ = \ B_1 + B_2 + ... + B_p$$

- $\tilde{E}_t$ ~ Expectation, Conditional on Information Set in VAR
  - Potential Specification Error
    - ∗ Too Few Variables in VAR
    - ∗ Too Few Lags in VAR

**Applying Analogous Restrictions to VAR ...**

- The VAR:

$$X_t \; = \; B_1 X_{t-1} + B_2 X_{t-2} + ... + B_p X_{t-p} + u_t$$

- Identification: Solve for $C$ Such that -

$$\text{(exclusion restriction)} \; [I - B(1)]^{-1} C = \begin{bmatrix} \text{number} & 0, ..., 0 \\ \text{numbers} & \text{numbers} \end{bmatrix}$$

$$\text{(sign restriction)} \; (1,1) \; \text{element of} \; [I - B(1)]^{-1} C \text{ is } positive$$

$$CC' = V$$

- There Are Many $C$ That Satisfy These Constraints. All Have the Same $C_1$.

# Standard Algorithm for Computing $C_1$

- Step 1: Compute Lower Triangular Choleski Decomposition, $D$

$$DD' = [I - B(1)]^{-1} V [I - B(1)']^{-1} = S_0 \text{ ('Spectral Density of } X_t \text{ at Frequency Zero')}$$

subject to $D(1,1) > 0$.

- Step 2: Solve

$$C = [I - B(1)] D.$$

- Remark: this $C$ Satisfies all Restrictions

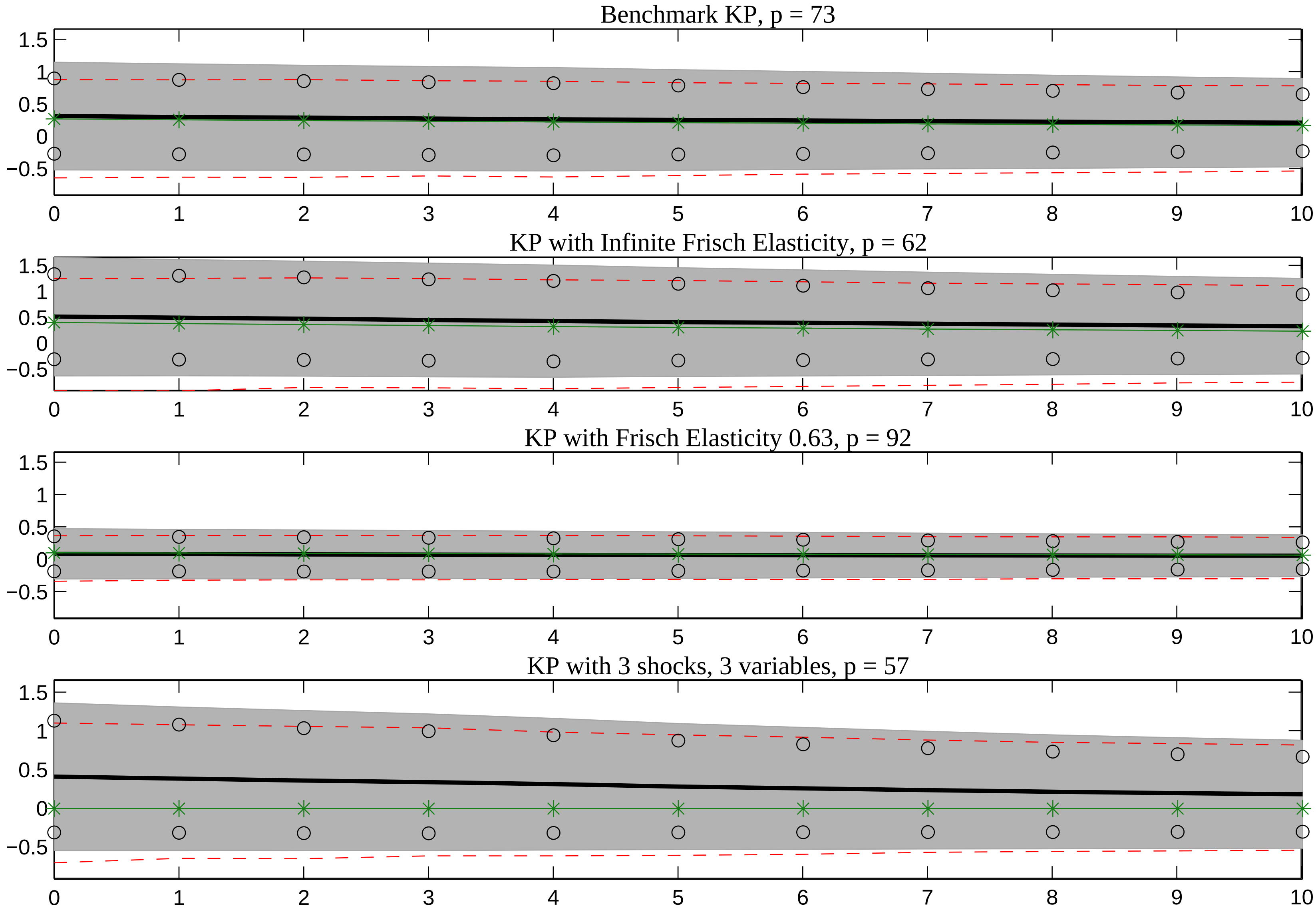
$$CC' = [I - B(1)] DD' [I - B(1)'] = V$$

$$\text{(exclusion restriction)} \quad [I - B(1)]^{-1} C = \begin{bmatrix} x & 0, ..., 0 \\ \text{numbers} & \text{numbers} \end{bmatrix}$$

(sign restriction) $x > 0$

**Figure 3a: Analysis of the Long−Run Identification Assumption with Kydland−Prescott Specification**

**Standard Estimator**

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# Long Run Restrictions: KP Specification

- Virtually no bias in point estimates.
- Considerable sampling uncertainty, but econometrician wouldn't be misled with respect to inference.

- Hansen Indivisible Labor model, $\sigma = 0.0001.$
  - Bias associated with estimator increases (very) slightly.
    - Percent of variance in HP-filtered, log output due to technology shocks is 62%.
  - Econometrician wouldn't be misled about sampling uncertainty.

- EGG: $\sigma = 1.24$ (Frisch elasticity = 0.63)
  - Bias almost disappears, and the sampling uncertainty shrinks drastically.
  - Percent of variance in HP-filtered, log output due to technology shocks is 92%.

- Three variable, three shock version of model.
  - Noticeable degree of bias associated with the estimated impulse response function.
    - But relatively small in relation to the sampling variation.
    - Econometrician's estimated confidence interval is roughly correct, on average.
    - Percent of variance in HP-filtered, log output due to technology shocks is 57%

# Why Does Bias Appear in Last Case?

- Sims (1972) : can characterize the VAR parameter estimates econometrician would obtain in large sample ($\hat{B}_1, ..., \hat{B}_q$ and $\hat{V}$)

$$\hat{V} = V + \min_{\hat{B}_1,...,\hat{B}_q} \frac{1}{2\pi} \int_{-\pi}^{\pi} \left[ B\left(e^{-i\omega}\right) - \hat{B}\left(e^{-i\omega}\right) \right] S_Y(\omega) \left[ B\left(e^{i\omega}\right) - \hat{B}\left(e^{i\omega}\right) \right]' d\omega$$

  – $S_Y(\omega)$ is associated spectral density, at frequency $\omega$.

- Econometrician chooses VAR lag matrices to minimize a quadratic form in difference between estimated and true lag matrices
  – Assigns greatest weight to frequencies where spectral density is greatest.
  – If there's specification error, then $\hat{B} \neq B$ and $V > \hat{V}$.
- Specification error:
  – Model Implies $q = \infty$, But Econometrician uses $q = 4$.
  – Model May Call for More Variables in Analysis.

**Why Does Bias Appear in Last Case? ...**

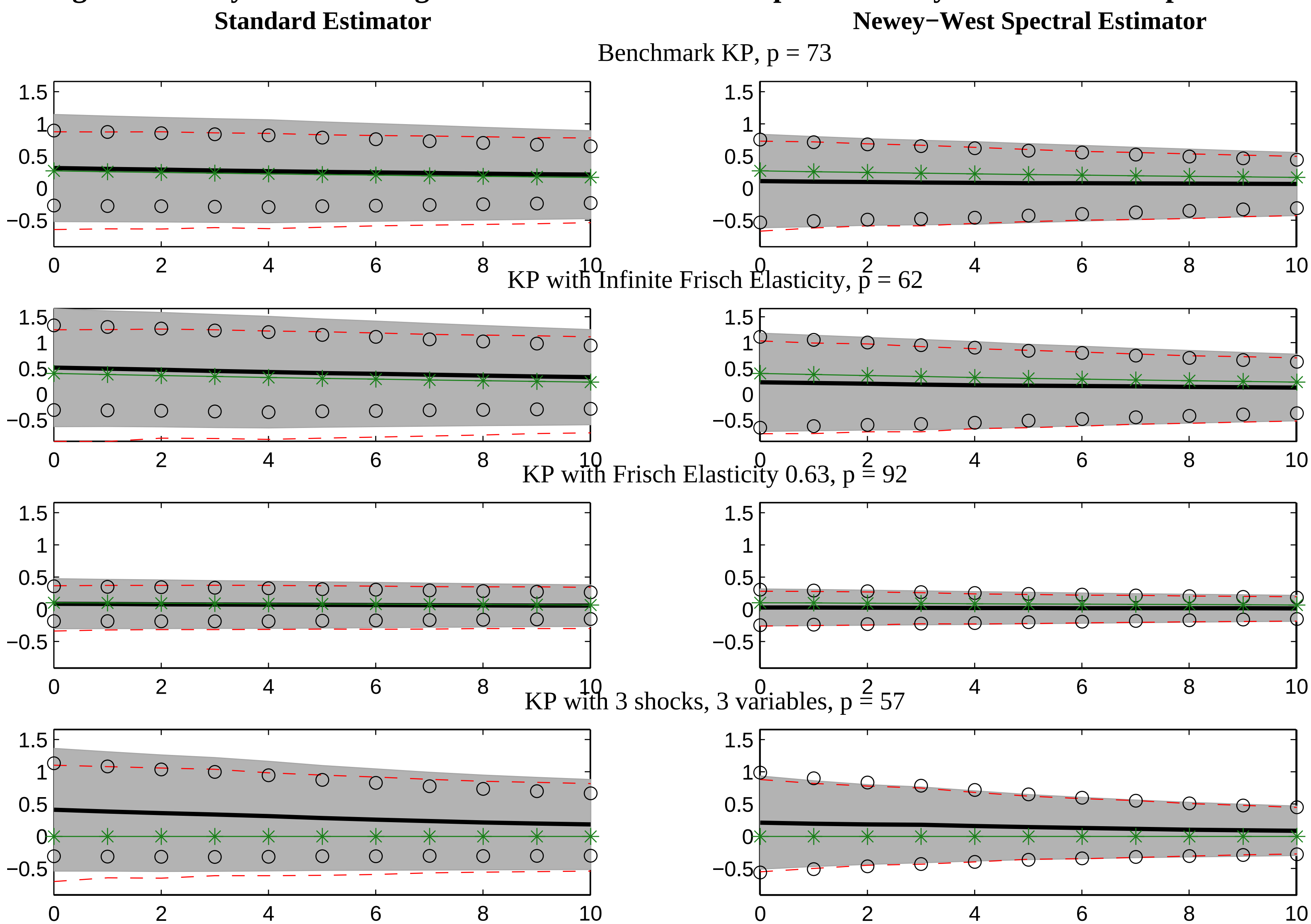
- Two key ingredients to computing impact effects of shocks:
  - Estimate of variance covariance matrix, $V$, of VAR disturbances and spectral density of $Y_t$ at frequency zero, $S_0$.
  - $V$ Estimated Precisely.
  - Problem with spectral density at frequency zero.
    - Standard VAR approach uses sum of estimated VAR matrices.
    - No particular reason for this to be estimated precisely by ordinary least squares.
    - Sum of lag VAR matrices corresponds to $\omega = 0$ and least squares will pay attention to this only if $S_Y(\omega)$ happens to be relatively large in a neighborhood of $\omega = 0$.
- Replace $S_0$ with Newey-West estimator:

$$S_0 = \sum_{k=-(T-1)}^{T-1} g(k)\hat{C}(k), g(k) = \left[1 - \frac{k}{r}\right]$$

where $\hat{C}(k)$ Sample Estimate of $EY_tY'_{t-k}$, $g(k) = 0$ for $k > r$ ($r = 150$).

- Figure 3
  - Bias is reduced
  - Less sampling uncertainty.

**Figure 3: Analysis of the Long−Run Identification Assumption with Kydland−Prescott Specification**

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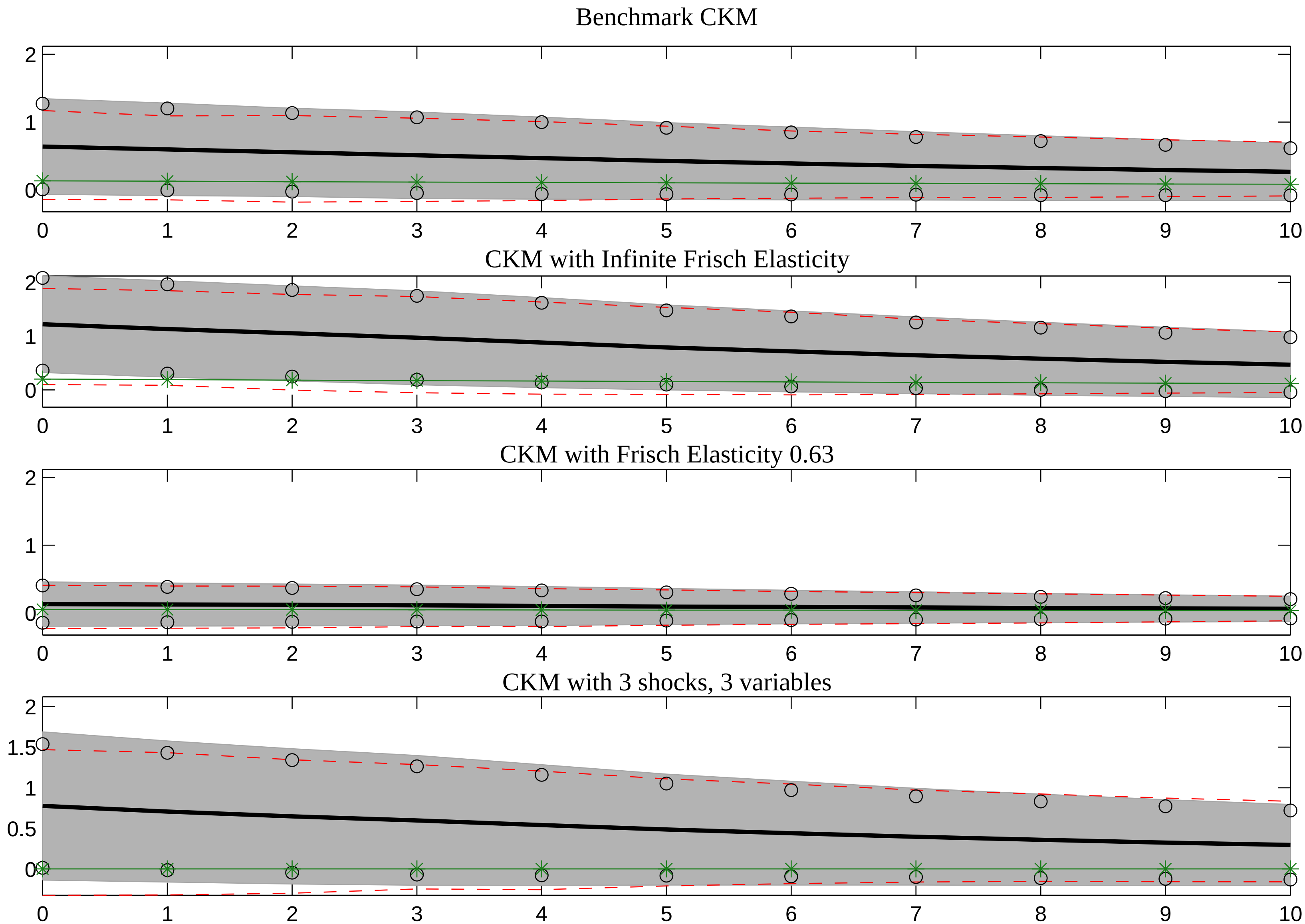
# CKM Long Run Results

- Benchmark CKM: substantial bias

## Figure 5a: Analysis of the Long−Run Identification Assumption with CKM Specification

**Standard Estimator**




Note: hours response, in percent terms, to a 0.6 percent innovation in technology, $Z_t$.

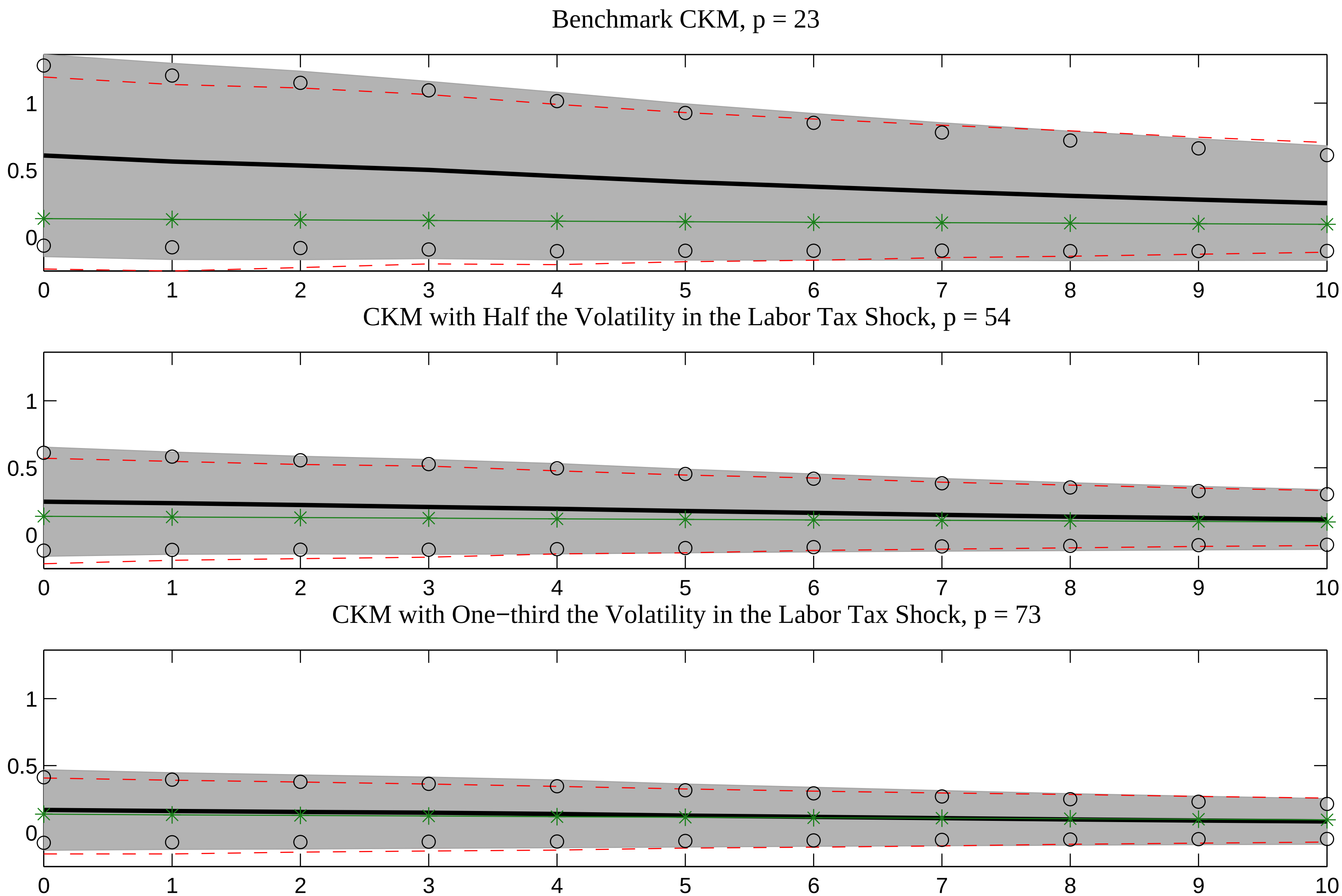
Solid line − mean response, Gray area − mean response plus/minus two standard errors,
Starred line − true response, Dashed line − 95.5 percent probability interval of responses,
Circles − average value of econometrician estimated plus/minus two standard errors.

**CKM Long Run Results ...**

- Key Difference Between CKM and KP Model: Fraction of Variance Due to Technology Very Small (23%)

## Figure 4a: Analysis of the Long−Run Identification Assumption with CKM Specification

### Standard Estimator

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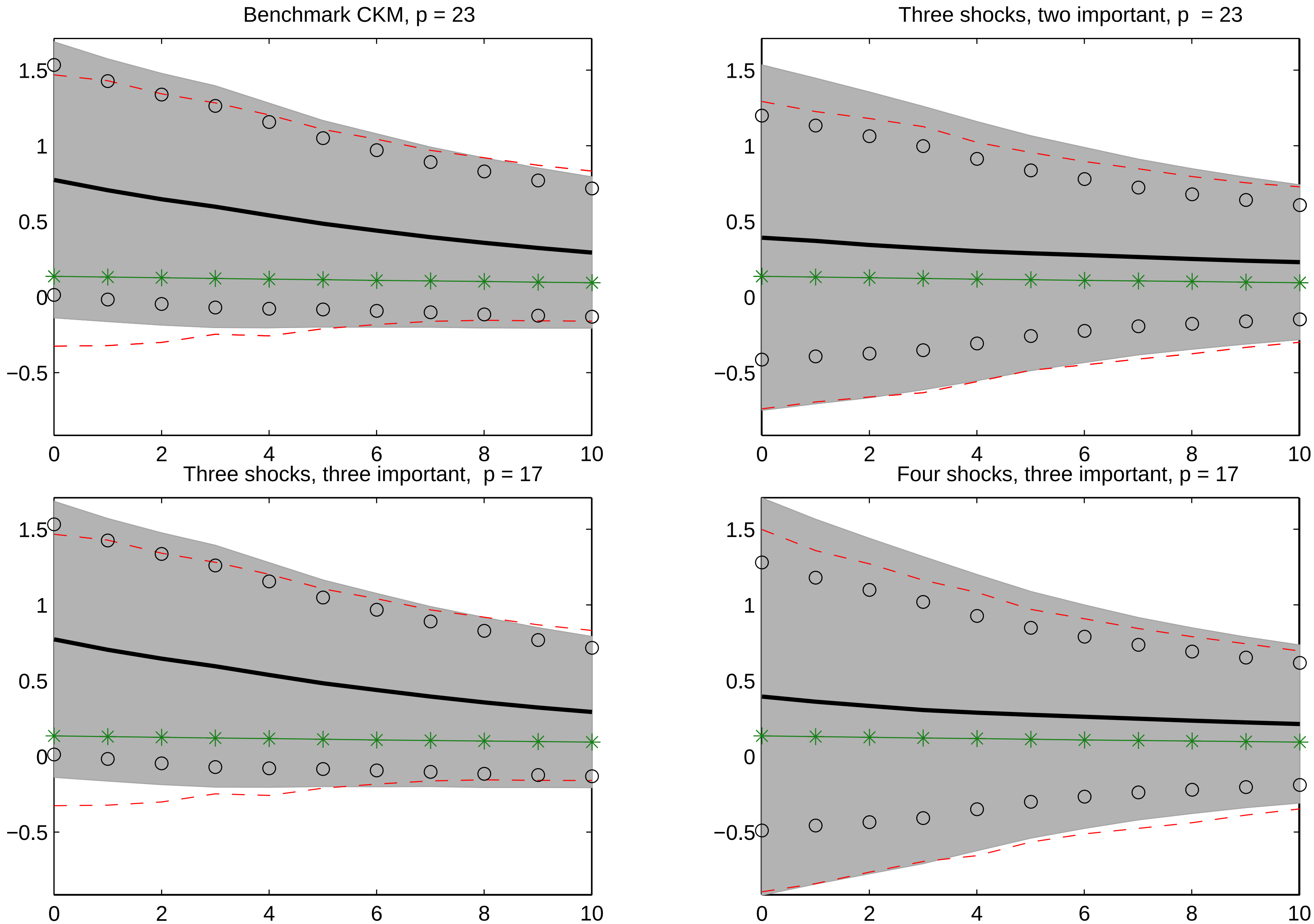
**CKM Long Run Results ...**

- Distortions in CKM Model Reduced if you
  - Have One More Variable Than Important Shocks

**Figure 6: The Long−Run Identification Assumption: Adding Variables and Shocks to the CKM Benchmark**



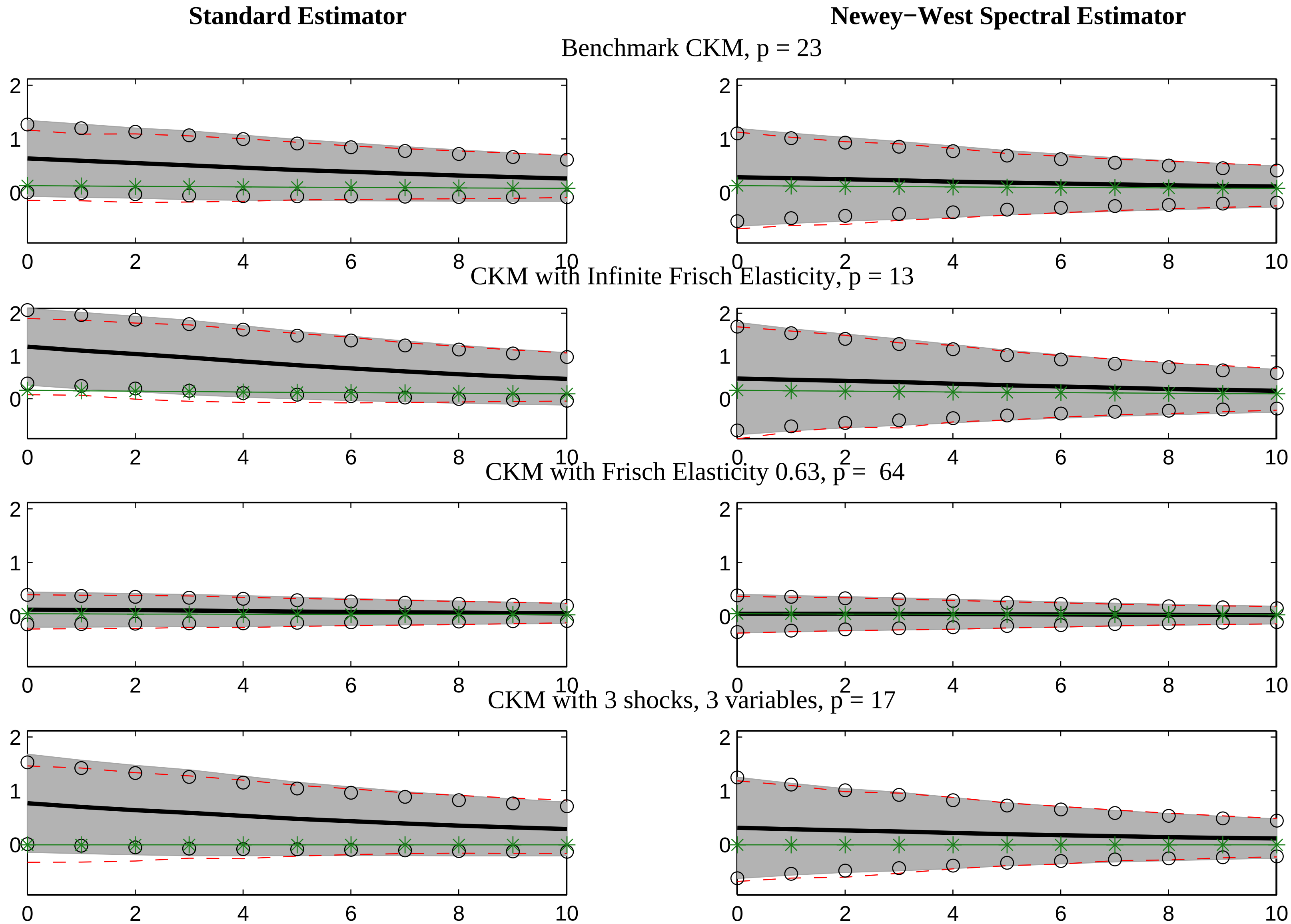

Note: hours response, in percent terms, to a 0.6 percent innovation in technology, $Z_t$.

Estimation results for Standard VAR estimator.
Solid line − mean response, Gray area − mean response plus/minus two standard errors,
Starred line − true response, Dashed line − 95.5 percent probability interval of responses,
Circles − average value of econometrician estimated plus/minus two standard errors.

**CKM Long Run Results ...**

- Distortions in CKM Model Also Reduced if you
  - Adopt Newey-West Estimator of Spectrum at frequency zero.

**Figure 5: Analysis of the Long−Run Identification Assumption with CKM Specification**

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# Key Lessons of the RBC Model Analysis

- With Short Run Exclusion Restrictions, VAR Analysis Highly Accurate

- With Long Run Exclusion Restrictions:

  - If Technology Shocks Important, Then Inference with VARs Reliable

  - Biases Could Occur When Technology Shocks Less Important. Then,

    * Use 5-6 Variables in VAR
    * If Can't Use More Variables and Worried About Possibility that Technology Shocks Not Important, Use Spectral Estimator.

# Why is Analysis with Short Run Restrictions So Much More Precise than with Long-Run Restrictions

- The Finding is Certainly Intuitively Appealing
  - Seems Like it Would be Tough to Find, in 50 Years' Data 'Only Shock that Has a Long-Run Effect on Productivity'
  - Shocks in Short Run Restrictions Equivalent to Regression Disturbances.

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- The VAR:

$$
\begin{aligned}
X_t &= B_1 X_{t-1} + B_2 X_{t-2} + \ldots + B_p X_{t-p} + u_t,\ E u_t u_t' = V \\
u_t &= C e_t,\ CC' = V.
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$$

- Short Run Restrictions:
  - To Obtain Impact Effect of Shock, $C_1$
    - Require Good Estimate of $V$
    - That's *Exactly* What OLS Does!
  - To Obtain Dynamic Effects of Shock:
    - Require Good Estimates of $B_j$, first few $j$'s

## Why is Analysis with Short Run Restrictions So Much More Precise than with Long-Run Restrictio

- Long Run Restrictions:
  - To Obtain Impact Effect of Shock, $C_1$
    - Require Good Estimate of $V$ *and*

$$B(1) = \sum_{j=1}^{p} B_j$$

    - OLS Provides Relatively Little Information About $B(e^{-i\omega})$, for $\omega \approx 0$.

# Relationship of Our Findings to Chari-Kehoe-McGrattan

- CKM Say Nothing About Short Run Restrictions.

- CKM Consider The Consequences other Specification Errors, Such as First Differencing. We do not Consider that Here (However, see Christiano, Eichenbaum and Vigfusson, NBER Working Papers W10254 and W9819).

- CKM Overstate the Degree of Sampling Uncertainty in Estimate of Response of Hours Worked.

  – Reflects a Non-Standard Way of Implementing Long Run Restrictions

**Relationship of Our Findings to Chari-Kehoe-McGrattan ...**

- Impact of Shocks on Forecast of Productivity in Long-Run:

$$\lim_{j\to\infty} E_t[a_{t+j}] - E_{t-1}[a_{t+j}] = (1,0)\,[I - B(1)]^{-1}\,C_1 e_{1t}$$

- Standard Implementation of Long Run Restrictions:

  $(1,1)$ Element of $[I - B(1)]^{-1}\,C_1$ Must Be Positive (Long Run Effect)
  Sign of $C_{11}$ (Impact Effect of Technology Shock) unrestricted
  Could Lead to Contemporaneous Drop in Productivity

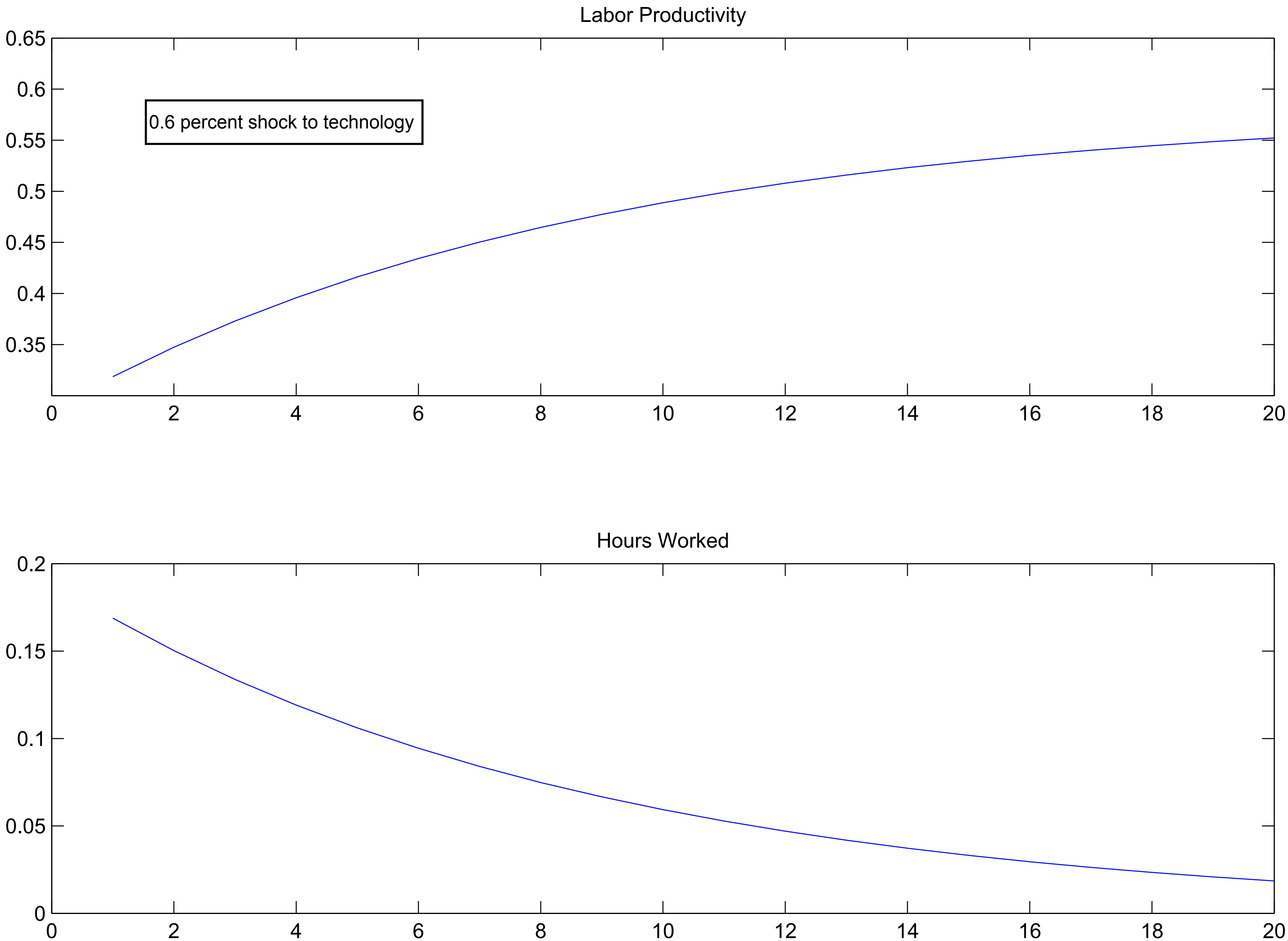
- CKM Sign Restriction:

CKM Sign Restriction: $C_{11} > 0,$
$(1,1)$ Element of $[I - B(1)]^{-1}\,C$ unrestricted
'Positive Technology Shock Leads to Contemporaneous Rise in Productivity'
Positive Technology Shock Could Lead to Permanent *Reduction* in Productivity
(This Pattern is Impossible in CKM DGP)

**Relationship of Our Findings to Chari-Kehoe-McGrattan ...**

- Benchmark CKM Model
  - Initial Hours Worked Response Strong, Productivity Response Weak (Fig 3)

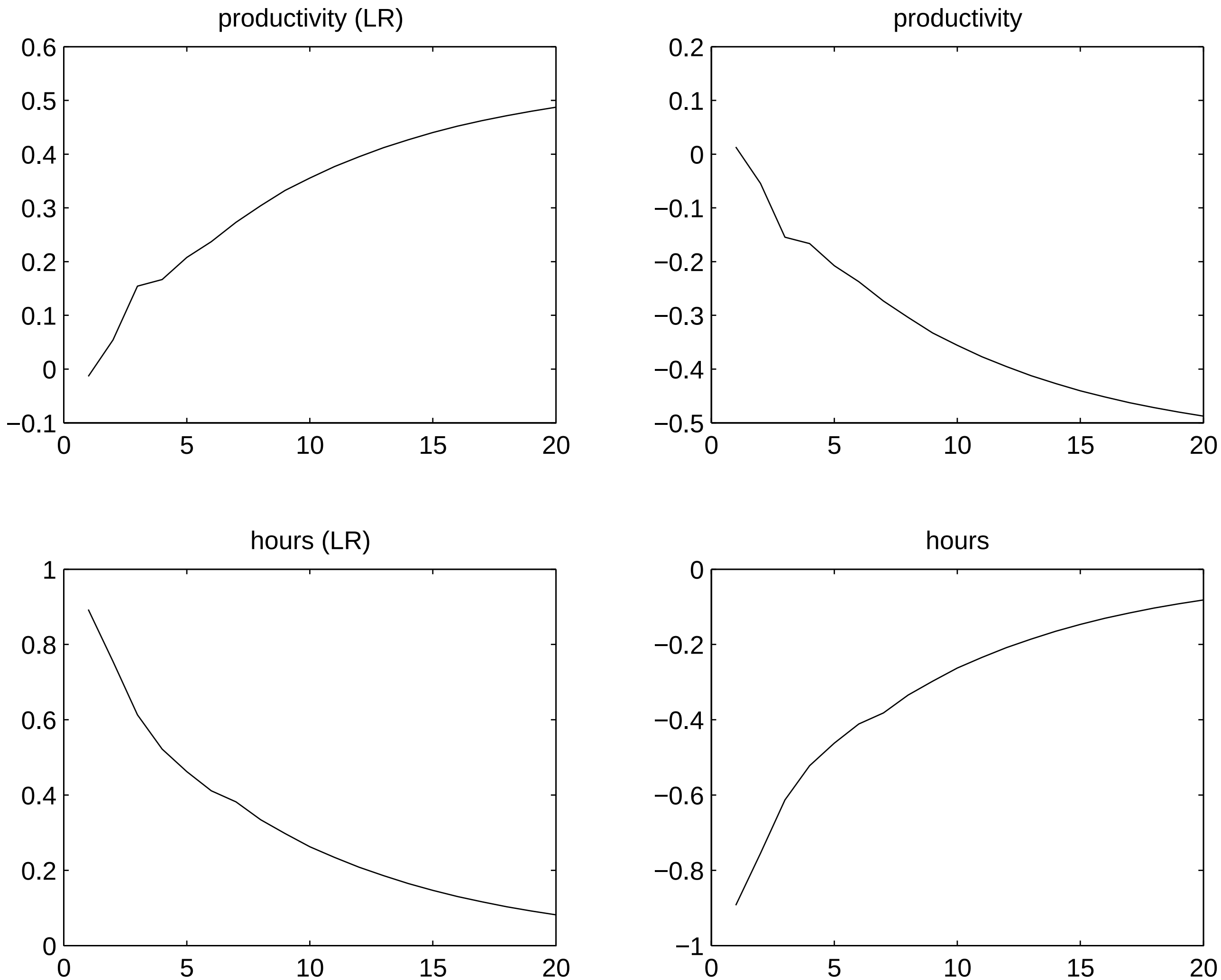
Response of Hours Worked and Labor Productivity in Benchmark CKM Example



**Relationship of Our Findings to Chari-Kehoe-McGrattan ...**

- Benchmark CKM Model
  - Initial Hours Worked Response Strong, Productivity Response Weak (Fig 3)
  - Consequences of CKM Sign Restriction
    - In Some Samples, Hours Response Very Strong and Initial Productivity Small Negative
    - CKM Algorithm Calls this a Negative Technology Shock
    - In These Cases, Their Procedure In Effect Multiplies the Hours Response by -1 (Fig 3a).

# Figure 3a: Example of Mistaken Inference



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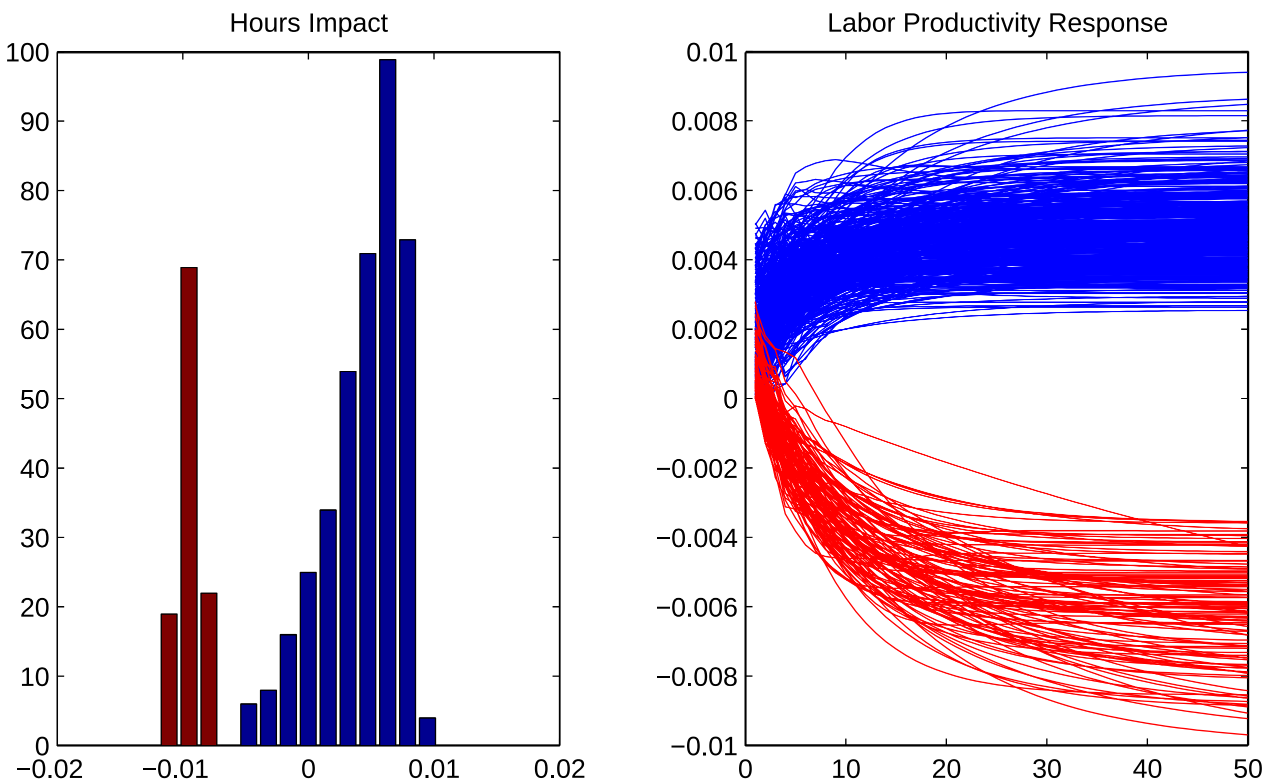
  - Is this a Theoretical Curiosum? No...It Happens 23% Of Time.
    - Leads to Pronounced Bimodal Distribution of Impact Effects (upper panel, Fig 4)



Hours Impact
100
90
80
70
60
50
40
30
20
10
0
−0.02
−0.01
0
0.01
0.02
Labor Productivity Response
0.01
0.008
0.006
0.004
0.002
0
−0.002
−0.004
−0.006
−0.008
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0
10
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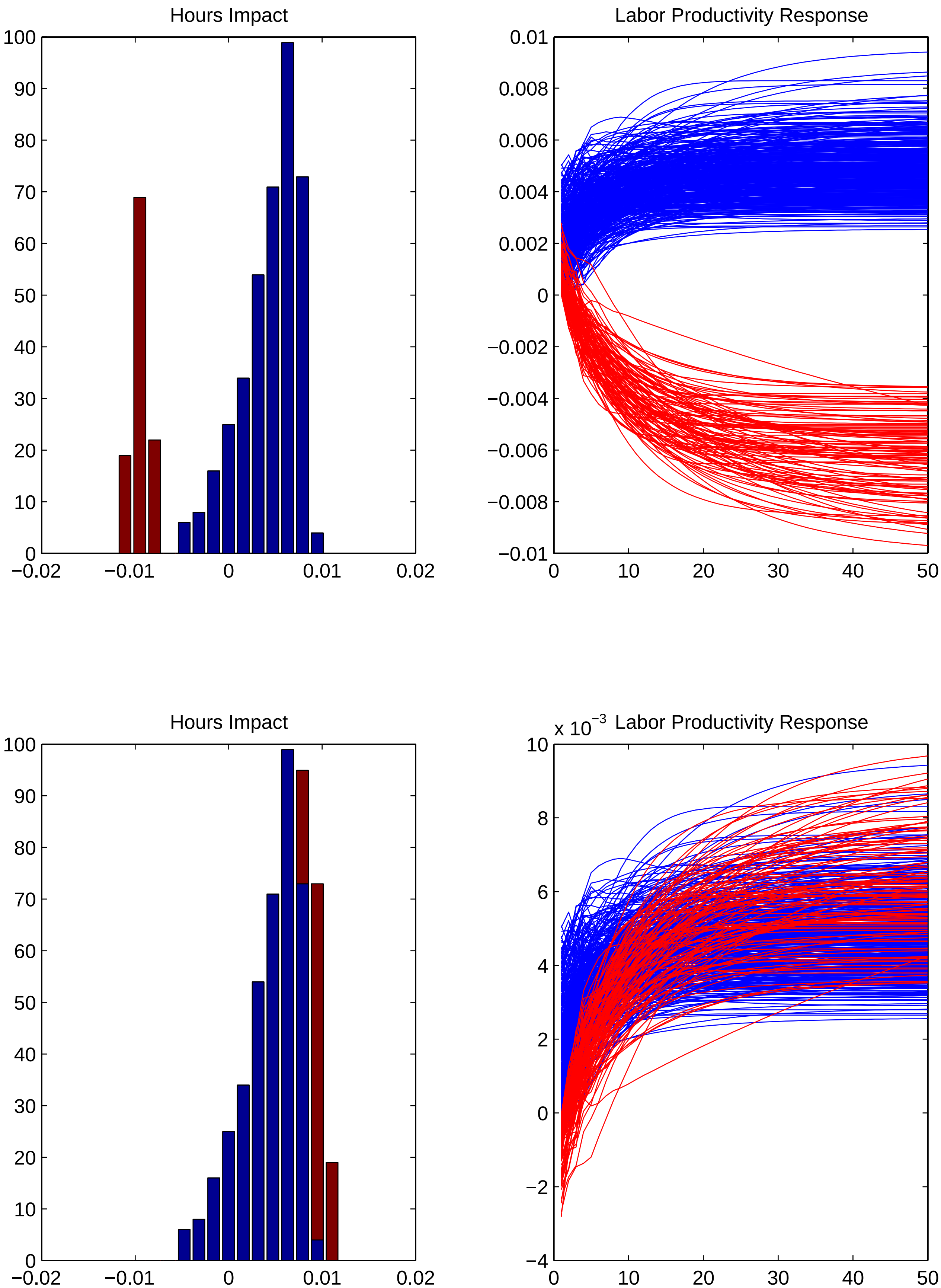
    - Actual Distribution of Impact Effects Not Bimodal (lower panel, Fig 4).



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x 10−3 Labor Productivity Response
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20
30
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50

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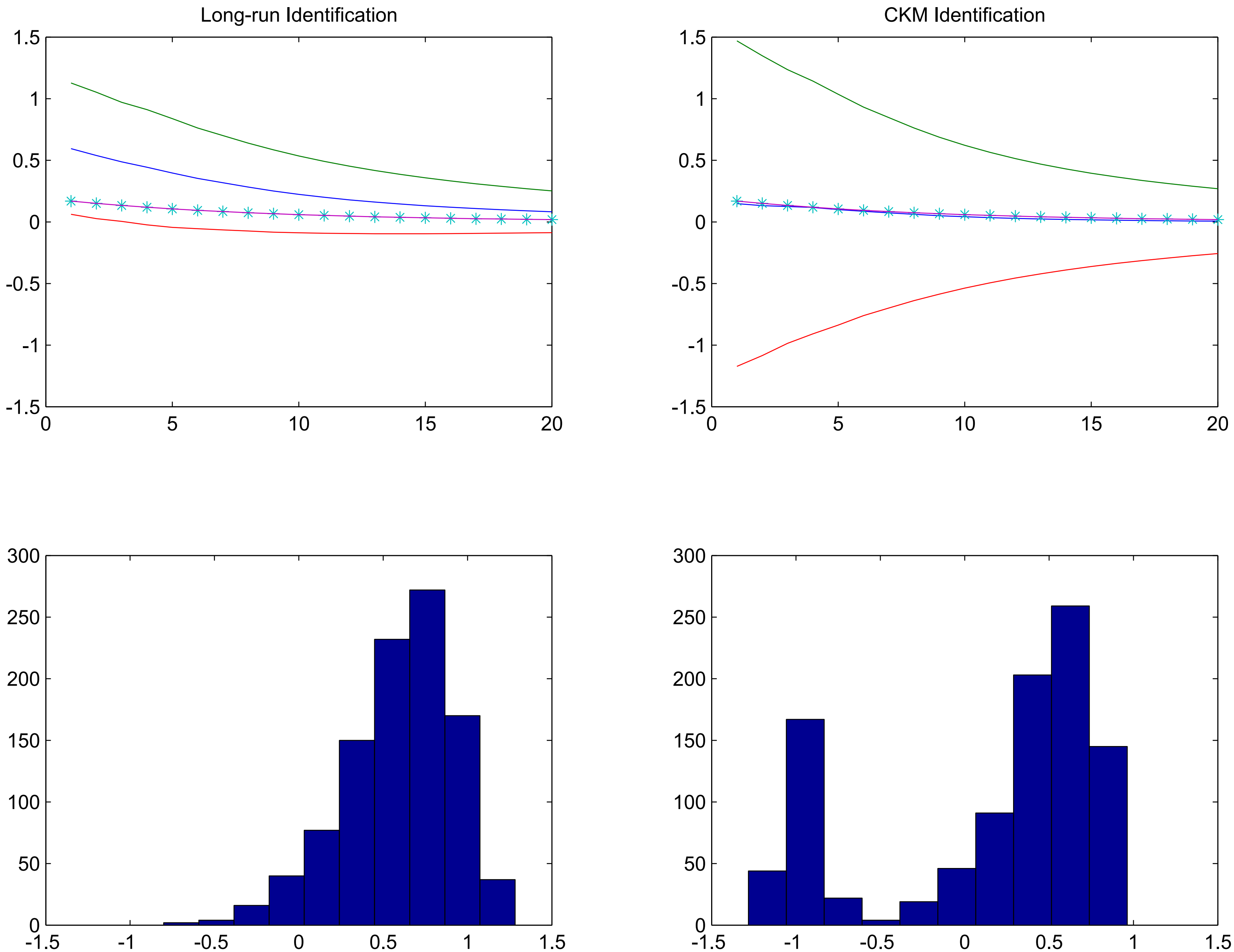
    - Actual Distribution of Impact Effects Not Bimodal (lower panel, Fig 4).
  - How Does this Affect Standard Errors?
    - Leads CKM to Substantially Overstate Sampling Uncertainty (Fig. 5)

# Impact of CKM Sign Restriction on Sampling Uncertainty



# Conclusion

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- With short run restrictions, VAR's perform remarkably well.

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- With long run restrictions,with one exception VAR's also perform well.
  - Potentially perform less well when shock under investigation isn't very important.
  - When there are enough variables in VAR, problems are greatly mitigated.
- Develop and implement a modified VAR approach
  - Leads to drastic improvement even when technology shocks play a limited role in aggregate fluctuations and a small number of variables are included in VAR.