# Unemployment and Business Cycles[*]

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### Abstract

We develop and estimate a general equilibrium model that accounts for key business cycle properties of labor market variables. In sharp contrast to leading New Keynesian models, wages are not subject to exogenous nominal rigidities. Instead we derive inertial wages from our specification of how firms and laborers interact when negotiating wages. Our model outperforms the canonical Diamond-Mortensen-Pissarides model both in a statistical sense and in terms of the plausibility of the estimated structural parameter values. The model also outperforms an estimated sticky wage model.

# 1. Introduction

Employment and unemployment fluctuate a great deal over the business cycle. Macroeconomic models have difficulty accounting for this fact, see for example the classic real business cycle models of Kydland and Prescott (1982) and Hansen (1985). Models that build on the search-theoretic framework of Diamond (1982), Mortensen (1985) and Pissarides (1985) (DMP) also have difficulty accounting for the volatility of labor markets, see Shimer (2005). In both classes of models the problem is that real wages rise sharply in business cycle expansions, thereby limiting firms' incentives to expand employment. The proposed solutions for both classes of models depend on controversial assumptions, such as high labor supply elasticities or high replacement ratios.[1]

Empirical New Keynesian models have been relatively successful in accounting for the cyclical properties of employment. However, they do so by assuming that wage-setting is subject to nominal rigidities and employment is demand determined.[2] These assumptions prevent the sharp rise in wages that limits the employment responses in standard models. Empirical New Keynesian models have been criticized on at least three grounds. First, they do not explain wage inertia, they just assume it. Second, agents in the model would not choose the wage arrangements that are imposed upon them by the modeler.[3] Third, empirical New Keynesian models are inconsistent with the fact that many wages are constant for extended periods of time. In practice, these models assume that agents who do not reoptimize their wage simply index it to technology growth and inflation.[4] So, these models predict that all wages are always changing.

In this paper we develop and estimate a model that accounts for the response of key labor market variables like wages, employment, job vacancies and unemployment to identified monetary policy shocks, neutral technology shocks and capital-embodied technology shocks. In contrast to leading empirical New Keynesian models, we do not assume that wages are subject to nominal rigidities. Instead, we derive wage inertia as an equilibrium outcome. Like empirical New Keynesian models, we assume that price setting is subject to nominal (Calvo-style) rigidities. Guided by the micro evidence on prices, we assume that firms which

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[1]For discussions of high labor supply elasticities in real business cycle models, see, for example, Rogerson and Wallenius (2009) and Chetty, Guren, Manoli and Weber (2012). For discussions of the role of high replacement ratios in DMP models see, for example, Hagedorn and Manovskii (2008) and Hornstein, Krusell and Violante (2010).

[2]For example, Christiano, Eichenbaun and Evans (2005), Smets and Wouters (2003, 2007), Gali, Smets and Wouters (2012) assume that nominal wages are subject to Calvo frictions.

[3]This criticsm does not necessarily apply to a class of models initially developed by Hall (2005). We discuss these models in the conclusion.

[4]See, for example, Christiano, Eichenbaum and Evans (2005), Smets and Wouters (2007), Justiniano, Primiceri and Tambalotti (2010), Christiano, Trabandt and Walentin (2011), and Gali, Smets and Wouters (2012).

do not reoptimize their price must keep it unchanged, i.e. no price indexation.

We take it as given that a successful model must have the property that wages are relatively insensitive to the aggregate state of the economy. Our model of the labor market builds on Hall and Milgrom (2008) (HM).[5] In practice, by the time workers and firms sit down to bargain, they know there is a surplus to be shared if they can come to terms. So, rather than just going their separate ways in the wake of a disagreement, workers and firms continue to negotiate.[6] This process introduces a delay in the time required to make a deal. During this delay, firms and workers suffer various costs. HM's key insight is that if these costs are relatively insensitive to the aggregate state of the economy, then negotiated wages will inherit that insensitivity.

The contribution of this paper is to see whether a dynamic general equilibrium model which embeds this source of wage inertia can account for the key business cycle properties of labor markets. We show that it does. In the wake of an expansionary shock, wages rise by a relatively small amount, so that firms receive a substantial fraction of the rents associated with employment. Consequently, firms have a strong incentive to expand their labor force. In addition, the muted response of wages to aggregate shocks means that firms' marginal costs are relatively acyclical. This acyclicality enables our model to account for the inertial response of inflation even with modest exogenous rigidities in prices.

We estimate our model using a Bayesian variant of the strategy in Christiano, Eichenbaum and Evans (2005) that minimizes the distance between the dynamic response to three shocks in the model and the analog objects in the data. The latter are obtained using an identified vector autoregression (VAR) for 12 post-war quarterly U.S. times series that include key labor market variables. The particular Bayesian strategy that we use is the one developed in Christiano, Trabandt and Walentin (2011). We find the impulse response function methodology particularly useful at the basic model construction phase.

We contrast the empirical properties of our model with estimated versions of leading alternatives. The first alternative is a variant of our model where the labor market corresponds closely to the standard DMP model. The second alternative is a version of the standard New Keynesian sticky wage model of the labor market proposed in Erceg, Henderson and Levin (2000) (EHL). In light of our discussion of wage indexation above, there is no wage indexation in the sticky wage model that we consider.

We show that our model outperforms the DMP model in terms of econometric measures of model fit and in terms of the plausibility of the estimated structural parameter values. For example, in the estimated DMP model the replacement ratio of income for unemployed

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[5]For a paper that pursues a reduced form version of HM in a calibrated real business cycle model, see Hertweck (2006).

[6]This perspective on bargaining has been stressed in Rubinstein (1982), Binmore (1985) and Binmore, Rubinstein and Wolinsky (1986).

workers is higher than the upper bound suggested by existing microeconomic evidence. A different way to compare our model with the DMP version uses the procedures adopted in the labor market search literature. Authors like Shimer (2005) emphasize that the standard deviation of labor market tightness (vacancies divided by unemployment) is orders of magnitude higher than the standard deviation of labor productivity. We show that our model has no difficulty in accounting for the statistics that Shimer (2005) emphasizes.

Our paper is organized as follows. Section 2 describes the labor market of our model in isolation. In section 3 we integrate the labor market model into a simple New Keynesian model without capital. We use this model to exposit the intuition about how our model of the labor market works in a general equilibrium setting with sticky prices. Section 4 describes our empirical model. Section 5 describes our econometric methodology. In section 6, we present our empirical results. Section 7 contains concluding remarks.

## 2. The Labor Market

In this section we discuss our model of labor markets. The essential structure is similar to many papers in the literature. We assume there is a large number of identical and competitive firms that produce a homogeneous good using labor. Let $\vartheta_t$ denote the marginal revenue associated with an additional worker hired by a firm. In this section, we treat $\vartheta_t$ as an exogenous stochastic process. In the next section we embed the labor market in a general equilibrium model and determine the equilibrium process for $\vartheta_t$.

In our benchmark specification we assume that at the start of period $t$ a firm pays a fixed cost, $\kappa$, to meet a worker with probability one. We refer to this specification of the cost of meeting a worker as the *hiring cost specification*. Once a worker and firm meet they engage in bilateral bargaining. If bargaining results in agreement, as it always does in equilibrium, then the worker begins production immediately.

The number of workers employed in period $t$ is denoted $l_t$. The size of the labor force is fixed at unity. Towards the end of the period a randomly selected fraction, $1 - \rho$, of employed workers is separated from their firm. These workers join the ranks of the unemployed and search for work. So, at the end of the period there are $1 - \rho l_t$ workers searching for a job. In period $t + 1$ a random fraction, $f_{t+1}$, of searching workers meets with a firm and the complementary fraction remains unemployed. Thus, each of the $l_t$ employed workers in period $t$ remains with the same firm in period $t + 1$ with probability $\rho$, moves to another firm in period $t + 1$ with probability $(1 - \rho) f_{t+1}$ and is unemployment in period $t + 1$ with probability $(1 - \rho)(1 - f_{t+1})$. Our measure of unemployment in period $t$ is $1 - l_t$. We think of workers that change jobs between $t$ and $t + 1$ as job-to-job movements in employment. There are $(1 - \rho) f_{t+1} l_t$ workers of this type. With our specification, the job-to-job transition rate

is substantial and procyclical, consistent with the data (see, e.g., Shimer, 2005a.)[7] While controversial, the standard assumption that the job separation rate is acyclical has been defended on empirical grounds (see Shimer, 2005a).[8] Finally, we think of the time period as one quarter. This time period is relatively long from the point of view of the US labor market, where the median duration of unemployment is around 7 weeks in our sample.[9] This is why we adopt the assumption that workers meet firms, bargain and start work in the same period.

The value to a firm of employing a worker at the equilibrium real wage rate, $w_t$, is denoted $J_t$ which satisfies the following recursive relationship:

$$J_t = \vartheta_t - w_t + \rho E_t m_{t+1} J_{t+1}. \tag{2.1}$$

The wage, $w_t$, is the outcome of a bargaining process described below. Also, $m_{t+1}$ is the discount factor which in this section we assume is an exogenous stochastic process. When we embed the labor market in a general equilibrium model, we determine the equilibrium process for $m_t$. The presence of $\rho$ in (2.1) reflects that a worker matched with a firm in period $t$ remains matched in $t+1$ with probability $\rho$. Because there is free entry, firm profits must be zero:

$$\kappa = J_t. \tag{2.2}$$

The value to a worker of being matched with a firm that pays $w_t$ in period $t$ is denoted $V_t$ :

$$V_t = w_t + E_t m_{t+1} \left[ \rho V_{t+1} + (1 - \rho) \left( f_{t+1} V_{t+1} + (1 - f_{t+1}) U_{t+1} \right) \right]. \tag{2.3}$$

Here, $f_{t+1}$ denotes the probability that a worker searching for a job in period $t$ meets a firm in $t+1$. Although we use the same notation for each, the two $V_{t+1}$'s in (2.3) are conceptually distinct. The first $V_{t+1}$ is the value to a worker of being employed in the same firm it works for in period $t$, while the second $V_{t+1}$ is the value to a worker of being employed in another firm in $t+1$. The two values are the same in equilibrium. Finally, $U_{t+1}$ in (2.3) is the value of being an unemployed worker in period $t+1$.

The recursive representation of $U_t$ is:

$$U_t = D + E_t m_{t+1} \left[ f_{t+1} V_{t+1} + (1 - f_{t+1}) U_{t+1} \right]. \tag{2.4}$$

In (2.4), $D$ denotes goods received by unemployed workers from the government.[10]

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[7]For a recent paper that stresses the importance of job-to-job transitions, see Van Zandweghe (2010).

[8]For a different view, see Fujita and Ramey (2009).

[9]See the variable, LNU03008276, in the online database, Fred, provided by the Federal Reserve Bank of St. Louis.

[10]One can also interpret $D$ as the value of home production by unemployed workers, see Ravenna and Walsh (2008). See Aguiar, Hurst and Karabarbounis (2012) for evidence on the importance of home production by the unemployed.

The number of employed workers evolves as follows:

$$l_t = (\rho + x_t) \, l_{t-1}. \tag{2.5}$$

Here $x_t$ denotes the hiring *rate* so that the number of new hires in period $t$ is equal to $x_t l_{t-1}$. Note that the job finding rate is given by,

$$f_t = \frac{x_t l_{t-1}}{1 - \rho l_{t-1}}. \tag{2.6}$$

Here the numerator is the number of workers that are newly-hired at the beginning of time $t$, while the denominator is the number of workers who are searching for work at the end of time $t - 1$.

## 2.1. Wage Determination: Alternating Offer Bargaining

We assume that all workers and firms bargain over wages every period.[11] In bargaining over the current wage rate, a worker and a firm take as given the state-contingent wage process that will obtain in future periods as long as they are matched. Because hiring costs are sunk at the time of bargaining and because the expected duration of a match is independent of how long that match has already been in place, the bargaining problem of all workers is the same, regardless of how long they have been matched with a firm.

Consistent with Hall and Milgrom (2008), wages are determined according to the alternating offer bargaining protocol proposed in Rubinstein (1982) and Binmore, Rubinstein and Wolinsky (1986). When a firm and a worker meet, the firm makes a wage offer. The worker can accept the offer or reject it. If he accepts it, work begins immediately. If he rejects the offer, he can go to his outside option or he can make a counteroffer. In the latter case there is a probability, $\delta$, that negotiations break down. In that case the firm and the worker revert to their outside options. For the worker, the outside option is unemployment, which has value $U_t$. For the firm, the outside option has a value of zero. We only study model parameterizations in which workers who reject an offer prefer to make a counteroffer rather than go to the outside option.

In a similar way, when a worker makes an offer, a firm can accept the offer, it can reject the offer and go to the outside option, or it can reject the offer and plan to make a counteroffer. In the latter case there is a probability, $\delta$, that negotiations break down and no counteroffer is made. To actually make a counteroffer, the firm incurs a cost, $\gamma$. We only consider model parameterizations in which a firm chooses to make a counteroffer after rejecting an offer from the worker.

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[11] We discuss an alternative scenario below. In this scenario firms and workers bargain only once, when they first meet. At that time they bargain over the present discounted value of the wage.

Let $w_t$ denote the initial wage offered by the firm. We denote the worker's offer in the $i^{th}$ bargaining round by $w_t^{l(i)}$, where $i$ is odd. We denote the firm's offer in the $i^{th}$ bargaining round by $w_t^{f(i)}$, where $i$ is even and $w_t^{f(0)} \equiv w_t$. The sequence of offers across subsequent bargaining rounds is given by,

$$w_t, w_t^{l(1)}, w_t^{f(2)}, w_t^{l(3)}, w_t^{f(4)}, ... \tag{2.7}$$

If the horizon is finite, one can solve for this sequence by starting with the take-it-or-leave-it offer made by one of the parties in the last bargaining round and work backward to the first offer. In equilibrium the first offer, $w_t$, is accepted. However, the nature of the first offer is determined by the details of the later bargaining rounds in case agreement is not reached in the first bargaining round. When the $w_t^{f(i)}$ and $w_t^{l(i)}$ that solve a bargaining problem are functions of $i$, the solution to the bargaining problem is not stationary. Obviously, when the possible number of periods is finite, the solution to the bargaining problem is not stationary.

We suppose that the first few elements in the sequence, (2.7), that solves the bargaining problem is well approximated (perhaps because there is a sufficiently large number of bargaining rounds) by a stationary sequence of offers and counteroffers:

$$w_t^l, w_t, w_t^l, w_t, w_t^l, w_t, ...$$

Suppose that it is the firm's turn to make an offer. The firm would like to propose the lowest possible wage. However, there is no point for the firm to propose a wage that the worker would reject. So, the firm proposes a wage that just makes the worker indifferent between accepting it and rejecting it in favor of making a counteroffer. In the case of indifference, we assume that the worker agrees to the offer. So, the wage offered by the firm satisfies:

$$V_t = \delta U_t + (1 - \delta) \frac{V_t^l}{1 + r}, \tag{2.8}$$

where $V_t$ is defined in (2.3). The object on the right hand side of (2.8) is the worker's disagreement payoff, i.e. what he receives in case he rejects the firm's offer with the intention of making a counteroffer. The variable, $r$, is a within-period discount rate that captures the worker's impatience to enjoy the benefits of reaching agreement. Below, we make an analogous assumption about the firm's disagreement payoff. We assume, but always verify in practice, that the worker's disagreement payoff is no smaller than his outside option, $U_t$. The worker's disagreement payoff reflects our assumption that when a worker rejects an offer with the intention of making a counteroffer, there is a probability $\delta \in (0, 1)$ that both parties revert to their outside options.

The object, $V_t^l$, denotes the value of employment to a worker who makes a counteroffer, $w_t^l$, that is accepted by the firm. We show below that there is no reason for the worker to

consider the possibility that $w_t^l$ will be rejected by the firm in the next bargaining round. The condition that defines $V_t^l$ is:

$$V_t^l = w_t^l + E_t m_{t+1} \left[ \rho V_{t+1} + (1 - \rho) \left( f_{t+1} V_{t+1} + (1 - f_{t+1}) U_{t+1} \right) \right].$$  (2.9)

The term after the first plus sign in (2.9) is the same as the corresponding term in (2.3).

Now consider the problem of a worker who makes a wage offer to a firm. The worker wants the highest possible wage. But, there is no point for the worker to propose a wage that the firm will reject. So, the worker proposes a wage that makes the firm just indifferent between accepting it and rejecting it in favor of making a counter offer. In the case of indifference, we assume that the firm agrees to take the offer. So, the wage offered by a worker satisfies:

$$J_t^l = \delta \times 0 + (1 - \delta) \left[ -\gamma + \frac{1}{1+r} J_t \right].$$  (2.10)

Here $J_t^l$ denotes the value of a match to the firm that employs a worker at wage $w_t^l$:

$$J_t^l = \vartheta_t - w_t^l + \rho E_t m_{t+1} J_{t+1}.$$  (2.11)

The right side of (2.10) is the firm's disagreement payoff, i.e. what the firm receives if it rejects the worker's offer and intends to make a counteroffer. The presence of $J_{t+1}$ on the right side of (2.11) reflects our assumption that a firm which hires a worker at wage rate $w_t^l$ expects to employ him at the wage rate $w_{t+1}$ if the match survives into period $t + 1$. In (2.10), the 0 represents the surplus received by the firm if negotiations break down. In practice we must verify that the firm's disagreement payoff is no less than the value of its outside option, zero.

An equilibrium is a stochastic process for the following ten variables:

$$x_t, J_t, w_t, l_t, V_t, U_t, f_t, V_t^l, J_t^l, w_t^l,$$  (2.12)

that satisfy the ten equilibrium conditions, (2.1)-(2.6), (2.8), (2.9), (2.10), (2.11). We refer to such a stochastic process as an *alternating offer equilibrium*.

The equilibrium conditions exhibit a recursive structure that we exploit in our analysis. Equations (2.9) and (2.11) imply

$$V_t^l = V_t + w_t^l - w_t, \ \ J_t^l = J_t + w_t - w_t^l.$$  (2.13)

Use (2.13) to substitute out for $V_t^l$ in (2.8) and for $J_t^l$ in (2.10) to obtain two expressions for $w_t - w_t^l$. Using one of these to substitute out for $w_t - w_t^l$ in the other expression, we obtain:

$$V_t \left[ 1 - \frac{(1-\delta)}{1+r} \right] = \delta U_t + \frac{1-\delta}{1+r} \left( J_t \left[ 1 - \frac{(1-\delta)}{1+r} \right] + (1-\delta) \gamma \right).$$

Solving this for $J_t$ and rearranging, we obtain:

$$J_t = \frac{1+r}{1-\delta} \left[ V_t - \alpha U_t - \omega \right], \tag{2.14}$$

where

$$\alpha \equiv 1 - r\frac{1-\delta}{r+\delta}, \; \omega \equiv \frac{(1-\delta)^2}{r+\delta}\gamma. \tag{2.15}$$

We refer to (2.14) as an *alternating offer sharing rule*. We can use the seven equations (2.1)-(2.6) and (2.14) to determine the equilibrium values of the first seven variables in (2.12). The last three variables in (2.12) can then be determined using the two equations in (2.13) plus (2.8) and (2.10).

## 2.2. Alternating Offer Bargaining: Some Intuition

In our estimated business cycle model, wages are the outcome of an alternating offer bargaining process. A key finding of the paper is that the resulting negotiated wages are relatively insulated from general economic conditions. In this subsection, we use the value of unemployment, $U_t$, as the indicator of general economic conditions. Shocks that expand economic activity tend to simultaneously raise $U_t$. In what follows we provide intuition about how the parameters governing the alternating offer sharing rule, $\delta$, $\gamma$ and $r$, influence the responsiveness of the wage to $U_t$. To do so, we consider a bargaining session between a single worker and a single firm. We study the response of the wage negotiated by this firm-worker pair to a rise in $U_t$ experienced idiosyncratically by that pair. For convenience we assume the experiment occurs when the economy is in nonstochastic steady state. By this we mean a situation in which all aggregate shocks are fixed at their unconditional means, aggregate variables are constant and there is ongoing idiosyncratic uncertainty at the worker-firm level.

Let $i$ denote the particular worker-firm pair under consideration. Let $U^i$ denote the value of unemployment to the worker in the $i^{th}$ worker-firm pair. The variable, $w^i$ denotes the wage negotiated by the $i^{th}$ worker-firm pair. The object of interest is $w_U^i$, the elasticity of $w^i$ with respect to $U^i$, where

$$w_U^i \equiv \frac{d\log w^i}{d\log U^i} = \frac{U}{w}W_U^i, \; W_U^i \equiv \frac{dw^i}{dU^i}. \tag{2.16}$$

In what follows, we assume that firm and worker disagreement payoffs exceed the value of their outside options. We seek to understand how $w_U^i$ varies across alternative specifications of the bargaining environment. In (2.16), $w$ and $U$ denote the economy-wide average value of the wage rate and of the value of unemployment, respectively, in nonstochastic steady state.

## 2.2.1. A fall in $\gamma$

In this subsection we show that a fall in $\gamma$ raises $w_U^i$ and does not affect $W_U^i$. The basic argument is straightforward. A decrease in $\gamma$ raises the disagreement payoff of the firm, putting the worker in a weaker bargaining position. So, other things equal, a fall in $\gamma$ leads to a decrease in $w^i$. This decrease turns out to be the same, regardless of the value of $U^i$, so that $W_U^i$ is independent of $\gamma$.[12] It follows that $\gamma$ affects $w_U^i$ entirely through its effect on the economy-wide object, $U/w$. The zero profit condition of firms implies that the equilibrium value of $w$ is independent of the bargaining parameters. So, $\gamma$ affects $w_U^i$ only through its impact on $U$. For the reasons described in our discussion of $w^i$, a decrease in $\gamma$ places downward pressure on $w$. However, since equilibrium $w$ does not respond to $\gamma$, the value of $U$ must change to neutralize the downward pressure on $w$. A rise in $U$ places upward pressure on $w$ by increasing the worker's disagreement payoff and his bargaining power. This reasoning underlies the intuition for why a decrease in $\gamma$ leads to a rise in $U$ and $w_U^i$. We now show this result formally.

We begin by proving that $W_U^i$ is unaffected by $\gamma$. Define $\tilde{V}_t$ and $\tilde{J}_t$ as follows:

$$\tilde{V}_t \equiv E_t m_{t+1} \left[ \rho V_{t+1} + (1-\rho) \left( f_{t+1} V_{t+1} + (1 - f_{t+1}) U_{t+1} \right) \right] \qquad (2.17)$$

$$\tilde{J}_t \equiv \vartheta_t + \rho E_t m_{t+1} J_{t+1}. \qquad (2.18)$$

The variables, $\tilde{V}_t$ and $\tilde{J}_t$, are taken as given by each worker-firm pair. With this notation, in steady state the indifference conditions, (2.8) and (2.10) can be written:

$$w^i = -\tilde{V} + \delta U^i + \frac{1-\delta}{1+r} \left( w^{i,l} + \tilde{V} \right) \qquad (2.19)$$

$$w^{i,l} = \tilde{J} + (1-\delta)\gamma + \frac{1-\delta}{1+r} \left( w^i - \tilde{J} \right), \qquad (2.20)$$

where $\tilde{V}$ and $\tilde{J}$ are the steady state values of $\tilde{V}_t$ and $\tilde{J}_t$. Relation (2.19) indicates the wage offer, $w^i$, that a firm makes given its view about the worker's potential counteroffer, $w^{i,l}$. We refer to (2.19) as the firm's best response function. Similarly, we interpret relation (2.20) as giving the wage offer, $w^{i,l}$, that a worker makes given his view about the firm's potential counteroffer, $w^i$. We refer to (2.20) as the worker's best response function. The solution to the bargaining problem, $w^i$ and $w^{i,l}$, corresponds to the intersection of the best response functions.

In Figure 1, panel A we graph the best response functions, (2.19) and (2.20), with $w^i$ on the vertical axis and $w^{i,l}$ on the horizontal axis. The slope of the worker's best response function, taking into account that $w^{i,l}$ appears on the horizontal axis, is $(1+r)/(1-\delta) \geq 1$. The slope of the firm's best response function is $(1-\delta)/(1+r) \leq 1$.

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[12] That is, $0 = d(dw/d\gamma)/dU = d(dw/dU)/d\gamma$, or, $dW_U/d\gamma = 0$.

We consider the impact on $w^i$ of an increase, $\Delta U^i > 0$, in $U^i$. The firm's best response function shifts up in a parallel way by $\delta \Delta U^i$, while the worker's best response function is unaffected. The result is an increase in $w^i$ (see Panel A, Figure 1). Totally differentiating the best response functions, (2.19) and (2.20), setting $d\tilde{V} = d\tilde{J} = 0$ and evaluating the derivative, we obtain:

$$W^i_U = \frac{\delta (1+r)^2}{(r+\delta)(2+r-\delta)}. \tag{2.21}$$

It follows that $\gamma$ has no impact on $W^i_U$, a result that reflects the linearity of the best response functions.

The previous results imply that the sign of the impact of $\gamma$ on $w^i_U$ is completely determined by the sign of the impact of $\gamma$ on the aggregate value of unemployment, $U$. To determine the impact of $\gamma$ on $U$, we must solve for the steady state of the model. We now show that computing the steady state can be reduced to solving three equations in three unknowns, $w, w^l$, and $U$.

Combine (2.1) and (2.2) to obtain the first of our three equations:

$$\kappa = \frac{\vartheta - w}{1 - \rho\beta}, \tag{2.22}$$

where $\vartheta$ is the steady state value of $\vartheta_t$. We treat $\vartheta$ as an exogenous parameter. According to (2.22), the cost of meeting a worker, $\kappa$, must equal the expected present value of what the worker brings into the firm. The present value expression takes into account discounting, $\beta$, and the fact that the worker-firm match remains in place with probability $\rho$. From (2.22) we see that $w$ does not depend on the bargaining parameters, $\gamma$, $\delta$, and $r$.

We now show that $\tilde{V}$ can be expressed as a function of $U$. From (2.17),

$$\tilde{V} = \beta \left[ \rho V + (1-\rho)(fV + (1-f)U) \right].$$

Equations (2.2) and (2.14) imply that $V$ can be expressed as a function of $U$ :

$$V(U) = \alpha U + \omega + \frac{1-\delta}{1+r}\kappa.$$

This expression, together with the steady state version of (2.4), imply that $f$ can also be expressed as a function of $U$. We denote this function by $f(U)$. It follows that

$$\tilde{V}(U) = \beta \left[ \rho V(U) + (1-\rho)(f(U)V(U) + (1 - f(U))U) \right].$$

In steady state, (2.19) and (2.20) are satisfied for each $i$, so that:

$$w = -\tilde{V}(U) + \delta U + \frac{1-\delta}{1+r}\left(w^l + \tilde{V}(U)\right) \tag{2.23}$$

$$w^l = \tilde{J} + (1-\delta)\gamma + \frac{1-\delta}{1+r}\left(w - \tilde{J}\right), \tag{2.24}$$

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where $\tilde{J}$ is given by (2.2) and (2.18). Expressions (2.23) and (2.24) are the firm and worker best response functions conditional on a common value of unemployment, $U$, across all worker-firm pairs.

The steady state values of $w, w^l$, and $U$ are given by the solution to the relations (2.22), (2.23) and (2.24). The three equations are depicted in Figure 1, panel B. We start with an initial equilibrium, indicated by point a. A decrease in $\gamma$ shifts the worker best response function, (2.24), to the left (see Figure 1, panel B). Other things equal, this shift induces a fall in the wage rate (see point b). But, in steady state the wage rate must be equal to the value indicated by the horizontal line. The variable, $U$, moves the firms' best response function so that all three lines intersect at the same point. A change in $U$ affects the intercept,

$$-\tilde{V}(U) + \delta U + \frac{1-\delta}{1+r}\tilde{V}(U),$$

in the firm's best response function, (2.23). We have found that for reasonable parameter values, this intercept is increasing with $U$. We conclude that $U$ increases with a reduction in $\gamma$.

From (2.16) we conclude that a smaller value of $\gamma$ is associated with a larger value of $w_U^i$. So, in our model, smaller values of $\gamma$ are associated with increased sensitivity in the wage rate to general economic conditions.

### 2.2.2. An increase in $\delta$ and $r$

We now establish that $W_U^i$ is decreasing in $r$ and increasing in $\delta$. To understand the impact of $\delta$ on $W_U^i$ it is useful to first consider the extreme case where $\delta = 0$. When $\delta = 0$ the chance that a worker who is negotiating with a firm is exogenously sent to his outside option is zero. In this case $U^i$ does not enter the firm's best response function. Since it never enters the worker's best response function, it follows that $W_U^i = 0$ when $\delta = 0$. More generally an increase in $\delta$ directly raises the importance of $U^i$ in the worker's disagreement payoff, a force that makes $W_U^i$ increasing in $\delta$.

To consider the impact of $r$ on $W_U^i$ it is again useful to consider an extreme case. Suppose that the discount rate of the worker is very large. In this case, the weight on the worker's counteroffer in his disagreement payoff is essentially zero. So, when $U^i$ increases the firm's offer rises by exactly $\delta \Delta U^i$ (this is the case when the firm's best response function in panel A of Figure 1 is horizontal). When the worker's intra-period discount rate is smaller, then the worker's counteroffer receives positive weight in his disagreement payoff. Since the worker's best response function makes $w^{i,l}$ an increasing function of $w^l$, the worker's disagreement payoff rises by more than $\delta \Delta U^i$ when the worker's intra-period discount rate is smaller. This argument suggests that $W_U^i$ rises with a reduction in the household's intra-period discount

rate. A similar argument suggests that $W_U^i$ also increases with a reduction in the firm's intra-period discount rate. Taken together, these two arguments provide the basic intuition for why a fall in $r$ produces a larger value of $W_U^i$.

We now formalize the observations in the last two paragraphs. Straightforward differentiation of (2.21) implies

$$\frac{dW_U^i}{dr} = -\frac{2(1-\delta)^2}{(r+1)(r+\delta)(2+r-\delta)}W_U^i < 0, \qquad (2.25)$$

$$\frac{dW_U^i}{d\delta} = \left(\frac{1+r}{(r+\delta)(2+r-\delta)}\right)^2 [r(2+r-\delta) + \delta(r+\delta)] > 0.$$

Signing the response of $w_U^i$ to $\delta$ and $r$ is less straightforward than signing the response of $W_U^i$ to those parameters. In numerical experiments we found that $w_U^i$ is increasing in $\delta$. We found that the sign of the response in $w_U^i$ to an increase in $r$ is opposite to the sign of $dW_U^i/dr$. This reflects the fact that an increase in $r$ raises $U$ and this effect dominates the impact of a rise in $r$ on $W_U^i$.

## 2.3. Implications for Wage Rates

We have assumed that workers and firms bargain over the current wage rate in each period. An alternative arrangement is one in which each firm and worker pair bargain just once over the expected discounted value of the wage, $\Upsilon_t$ :

$$\Upsilon_t = w_t + \rho E_t m_{t+1} \Upsilon_{t+1}.$$

From (2.1) and (2.3) we see that the firm and worker do not care about the timing or size of any particular wage payment. Their interest in wages lies only in $\Upsilon_t$, the expected discounted value of the stream of wage payments while their match lasts. To see the implications of this observation, it is useful to rewrite (2.1) as follows:

$$J_t = \Theta_t - \Upsilon_t. \qquad (2.26)$$

Here, $\Theta_t$ is the present value of $\vartheta_t$ :

$$\Theta_t = \vartheta_t + \rho E_t m_{t+1} \Theta_{t+1}.$$

Equation (2.3) can similarly be written:

$$V_t = \Upsilon_t + M_t \qquad (2.27)$$

where $M_t$ denotes the expected present value of the utility experienced by the worker after match breaks up:

$$M_t = (1-\rho) E_t m_{t+1} [f_{t+1} V_{t+1} + (1-f_{t+1}) U_{t+1}] + \rho E_t m_{t+1} M_{t+1}.$$

Here, $V_{t+1}$ refers the value of employment at another firm.

An alternative approach to bargaining supposes that workers and firms bargain over $\Upsilon_t$ using the same protocols assumed above. We then obtain the same indifference conditions, (2.8) and (2.10). In addition, we obtain the same sharing rule that we derived under our assumption that worker-firm pairs bargain over the spot wage, (2.14). We conclude that the approach to bargaining described here and the one studied in the previous subsections lead to identical allocations, though possibly different wages. The approach to bargaining described here places no restrictions on the pattern of wages over dates and states of nature for a particular firm-worker pair, other than they must be consistent with the present discounted value of the wage rate they agreed on at the time they bargained. At one extreme, when workers and firms negotiate at time $t$ and agree on a value of $\Upsilon_t$, firms could simply pay the wage rate that is constant across all future states in which they remain matched and is consistent with $\Upsilon_t$. Under this decentralization, the cross-sectional distribution of wages would be very complicated, as the wage in any particular match would depend on the present discounted value the worker-firm pair in that match that was agreed on when they first met. This decentralization would have the interesting property that a worker's wage only changes when he changes employer and is constant otherwise. It would have this property, even though the allocations in the model coincide with what they would be if wages were negotiated in each period, in which case wages in the cross-section are all identical and all wages change in each period. From this point of view, the model has few testable implications for the wage rate.

## 3. Incorporating the Labor Market Model into a Simple Macroeconomic Framework

In this section we incorporate the labor market model of the previous section into the benchmark New Keynesian macroeconomic model. We use this framework to explore the intuition for how the alternating offer bargaining model of the labor market helps to account for the cyclical behavior of key macroeconomic variables.

### 3.1. Simple Framework

As in Andolfatto (1995) and Merz (1996), we assume that each household has a unit measure of workers. Because workers experience no disutility from working, they supply their labor inelastically to the labor market. An employed worker brings home the real wage, $w_t$. An unemployed worker receives $D$ goods in government-provided unemployment compensation. The latter is financed by lump-sum taxes paid by the household. Workers maximize their expected income, subject to the labor market arrangements described in the previous section.

By the law of large numbers, this strategy maximizes the total income of the household. Workers maximize expected income in exchange for perfect consumption insurance from the household. All workers have the same concave preferences over consumption. So, the optimal insurance arrangement involves allocating the same level of consumption, $C_t$, to each worker.

The household maximizes:

$$E_0 \sum_{t=0}^{\infty} \beta^t \ln(C_t)$$

subject to the sequence of budget constraints:

$$P_t C_t + B_{t+1} \leq W_t h_t + (1 - h_t) P_t D + R_{t-1} B_t - T_t.$$

Here $0 \leq h_t \leq 1$ denotes the fraction of the household's workers that is employed. In addition, $T_t$ denotes lump-sum taxes net of lump-sum profits and $B_{t+1}$ denotes purchases of bonds in period $t$. Finally, $R_{t-1}$ denotes the gross nominal interest rate on bonds purchased in the previous period.

A final homogeneous good, $Y_t$, is produced by competitive and identical firms using the following technology:

$$Y_t = \left[ \int_0^1 Y_{j,t}^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}}, \quad \epsilon > 1. \tag{3.1}$$

The representative firm chooses the specialized inputs, $Y_{j,t}$, to maximize profits:

$$P_t Y_t - \int_0^1 P_{j,t} Y_{j,t} dj,$$

subject to the production function. The firm's first order condition for the $j^{th}$ input is:

$$Y_{j,t} = \left( \frac{P_t}{P_{j,t}} \right)^{\epsilon} Y_t. \tag{3.2}$$

Following Ravenna and Walsh (2008), we say that the $j^{th}$ input good is produced by a monopolist *retailer*, with production function

$$Y_{j,t} = \exp(a_t) h_{j,t},$$

where $h_{j,t}$ is the quantity of the intermediate good purchased by the $j^{th}$ producer. This intermediate good is purchased in competitive markets at the after-tax price $(1 - \nu) P_t^h$ from a *wholesaler*. Here, $\nu$ represents a subsidy (financed by a lump-sum tax on households) which has the effect of eliminating the monopoly distortion in the steady state. That is, $1 - \nu = \boldsymbol{\lambda}_f$ where $\boldsymbol{\lambda}_f = (\epsilon - 1) / \epsilon$ denotes the steady state markup. In the retailer production function, $a_t$ denotes a technology shock that has the law of motion:

$$a_t - (\tau_1 + \tau_2) a_{t-1} + \tau_1 \tau_2 a_{t-2} = \varepsilon_t,$$

where $\varepsilon_t$ is the iid shock to technology and $|\tau_i| < 1, i = 1, 2$. For reasons discussed below, we adopt an AR(2) specification to allow for a hump-shaped response of technology to a shock. The monopoly producer of $Y_{j,t}$ sets $P_{j,t}$ subject to Calvo sticky price frictions. In particular,

$$P_{j,t} = \begin{cases} P_{j,t-1} & \text{with probability } \xi \\ \tilde{P}_t & \text{with probability } 1 - \xi \end{cases}. \tag{3.3}$$

Here, $\tilde{P}_t$ denotes the optimal price set by the $1 - \xi$ producers that have the opportunity to reoptimize. Note that we do not allow price indexation. So, the model is consistent with the observation that many prices remain unchanged for extended periods of time (see, Eichenbaum, Jaimovich and Rebelo (2011) and Klenow and Malin (2011)).

Let

$$s_t = \frac{\vartheta_t}{\exp(a_t)} \tag{3.4}$$

where $\vartheta_t = P_t^h/P_t$ so that $(1 - \nu)s_t$ denotes the retail firm's real marginal cost. Also, let

$$h_t = \int_0^1 h_{j,t} dj.$$

The wholesalers that produce $h_t$ correspond to the perfectly competitive firms modeled in the previous section. Recall that they produce $h_t$ using labor only and that labor has a fixed marginal productivity of unity. The total supply of the intermediate good is given by $l_t$ which equals the total quantity of labor used by the wholesalers. So, clearing in the market for intermediate goods requires

$$h_t = l_t. \tag{3.5}$$

We adopt the following monetary policy rule:

$$\ln(R_t/R) = \alpha \ln(R_{t-1}/R) + (1 - \alpha)\left[\phi_\pi \pi_t + \phi_y \log(l_t/l)\right] + \varepsilon_{R,t} \tag{3.6}$$

where $\pi_t = P_t/P_{t-1}$ denotes the gross inflation rate and $\varepsilon_{R,t}$ is a monetary policy shock.

## 3.2. Integrating the Labor Market into the Simple Framework

There are four points of contact between the model in this section and the one in the previous section. The first point of contact is the labor market in the wholesale sector where the real wage is determined as in section 2. The second point of contact is via $\vartheta_t$ in (3.4), which corresponds to the real price that appears in the previous section (see, e.g., (2.1)). The third point of contact occurs via the asset pricing kernel, $m_{t+1}$, which is now given by:

$$m_{t+1} = \frac{C_t}{C_{t+1}}. \tag{3.7}$$

The fourth point of contact is the resource constraint which specifies how the homogeneous good, $Y_t$, is allocated among the uses of goods in this economy. For our benchmark model, this constraint is given by

$$C_t + \kappa x_t l_{t-1} = Y_t, \tag{3.8}$$

where

$$Y_t = \exp(a_t) l_t. \tag{3.9}$$

Here $\kappa x_t l_{t-1}$ denotes the cost of generating new hires in period $t$. The expression on the right side of (3.9) is the production function for the final good. The absence of price distortions in this expression reflects Yun's (1996) result that these distortions can be ignored in (3.9) when linearizing about a nonstochastic steady state in which price distortions are absent.

From the perspective of the model in this section, the prices in the previous section correspond to real prices. So, $w_t$ and $w_t^l$ are to be interpreted as real wages, where conversion to real is accomplished using $P_t$. That is, workers and firms bargain over real wages according to the alternating wage offer arrangement described in section 2.

## 3.3. Quantitative Results in the Simple Model

This section displays the dynamic response of our simple model to monetary policy and technology shocks. In addition, we discuss the sensitivity of these responses to wage bargaining parameters, $\delta, \gamma$ and $r$. The first subsection below reports a set of baseline parameter values for the model. Impulse responses are presented in the second subsection.

### 3.3.1. Baseline Parameterization

Table 1 lists the baseline parameter values. We set the parameters of the monetary policy rule, (3.6), $\phi_\pi$, $\phi_y$, $\alpha$ equal to 1.5, 0.05 and 0.75, respectively. For convenience we assume that the steady state inflation rate, $\pi$, is equal to unity. We set the parameter that controls the degree of price stickiness, $\xi$, to 0.75. In addition, we assume that the elasticity of demand for the intermediate good, $\epsilon$, is equal to 6. This value implies a steady state markup of 1.2. We set the discount factor $\beta$ to $(1.03)^{-0.25}$.

We assume that the intra-period discount rate, $r$, is equal to the daily value implied by $\beta$, i.e., $r = \beta^{-4/365} - 1$. This way of calibrating $r$ is consistent with HM's assumption that the period between alternating offers is one day. As in Ravenna and Walsh (2008) we assume that $\rho = 0.9$ which implies a match survival rate that is consistent with both HM and Shimer (2012).[13] Finally, we set $\delta$, the probability that negotiations break down after an offer is rejected, to 0.65 percent. This value is roughly the same as the one used by HM.

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[13]Denote the probability that a worker separates from a job at a monthly rate by $1-\tilde{\rho}$. The probability that a person employed at the end of a quarter separates in the next three months is $(1-\tilde{\rho})+\tilde{\rho}(1-\tilde{\rho})+\tilde{\rho}^2(1-\tilde{\rho}) = (1-\tilde{\rho})(1+\tilde{\rho}+\tilde{\rho}^2)$. Shimer (2012) reports that $\tilde{\rho} = 1-0.034$, implying a quarterly separation rate of 0.0986.

We calibrate the remaining model parameters, $D$, $\kappa$ and $\gamma$ so that the model has three properties in steady state. The first property is a steady state unemployment rate, $1 - h$, of 5 percent. The second property is a value of 1 percent for the steady state ratio of hiring costs to gross output, i.e., $\kappa x h / Y = 0.01$. The third property is a steady state value of unemployment benefits relative to wages, $D/w$, equal to 0.4. The resulting values of $D$, $\kappa$ and $\gamma$ are 0.396, 0.1, and 0.0156, respectively.

Finally, we assume the parameters, $\tau_1$ and $\tau_2$, which govern the law of motion for technology are equal to 0.9 and 0.80, respectively. This specification implies that $a_t$ continues to rise for a while after a shock. This mimics a key property of the technology shock in our estimated DSGE model.

Table 2 summarizes the steady state properties of the simple model. Note that in conjunction with the other parameter values, the calibrated value of $\gamma$ is roughly equal to one and a half days of output in the model.[14] HM use a value of $\gamma$ that is roughly equal to one-quarter of a day's work. The estimated DSGE model in section (4) implies a value of $\gamma$ that is roughly equal to four day's output in the model. So the value of $\gamma$ that we use here is roughly half-way between HM's assumed value and our estimated value.

### 3.3.2. Impulse Responses

Figures 2 and 3 display the dynamic responses to monetary policy and technology shocks, respectively. We report results for the baseline parameterization. In addition, we display results for three other parameterizations, each of which changes the value of one parameter relative to the baseline case. In the first case, we lower $\gamma$ to 0.0143. In the second case we raise $\delta$ to 0.0075. Finally, in the third case, we raise $r$ to $(1.032)^{1/365} - 1$.

Figure 2 displays the dynamic responses of our baseline model and the three alternatives to a negative 25 annualized basis point monetary policy shock, $\varepsilon_{R,t}$. In the baseline model, real wages respond by a very small amount with the peak rise equal to 0.05 percent. Inflation also responds by only a small amount, with a peak rise of 0.04 percent (on annual basis). At the same, there is a substantial increase in consumption, which initially jumps by 0.2 percent. Finally, the unemployment rate is also very responsive, dropping 0.2 percentage points in the impact period of the shock.

We now consider the impact of reducing the value of $\gamma$. In terms of the steady state, consumption rises, unemployment falls, while inflation and the real wage are unaffected (see Figure 2). In terms of dynamics, Figure 2 shows that the dynamic responses of the real

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HM assume a similar value of 0.03 for the monthly separation rate. This value is also consistent with Walsh's (2003) summary of the empirical literature.

[14]Daily output is one quarter's production divided by 90 days. Steady state quarterly output is 0.95. So the value of daily output is 0.95/90 or 0.0105. The calibrated value of $\gamma$ is one and a half times this amount.

wage and inflation to the monetary policy shock are stronger than in the baseline case. At the same time, consumption and unemployment respond by less than in the baseline case. The basic intuition is the one that was emphasized above. In particular, with a lower value of $\gamma$ the real wage rises by more in the expansion, consistent with the intuition developed in subsection 2.2.1. Consistent with the intuition in the introduction, the stronger response of the real wage reduces the incentive of firms to hire workers, thus limiting the economic expansion. The larger rise in the real wage places upward pressure on the marginal costs of retailers, leading to higher inflation than in the baseline parameterization.

Consider next the effect of raising either $\delta$ or $r$. In both cases, steady state consumption increases and unemployment falls relative to the baseline case. Consistent with the intuition in section 2.2.2, a rise in $\delta$ increases the sensitivity of the real wage to the policy shock. As a result consumption and unemployment respond by less than in the baseline case while inflation responds by more. As we stressed above, these effects reflect that a higher value of $\delta$ makes the disagreement payoff of workers more sensitive to the value of their outside option, $U_t$. The impact of a rise in $r$ is qualitatively similar to the effects of a rise in $\delta$.

Figure 3 displays the dynamic responses of our baseline model and the three alternatives to a 0.1 percent innovation in technology. In the baseline model, real wages rise but by a relatively modest amount. Inflation also falls by a modest amount, with a peak decline of about one-quarter of one percent (on an annualized basis). Notice that unemployment falls by a substantial amount in the impact period of the shock, declining by one-quarter of one percent. Clearly our model is not subject to the problems of the standard DMP model highlighted by Shimer (2005).

The effect of lowering $\gamma$ is to make the real wage and inflation more responsive to the technology shock. While the response of consumption is not much affected, the decline in unemployment is muted relative to the baseline parameterization. As with the monetary policy shock, these results are broadly consistent with the intuition in subsection (2.2.1). Finally notice that the effect of raising $\delta$ is to exacerbate the impact of the technology shock on real wages, while muting its effect on the unemployment rate.

We conclude this section with an important caveat. The impact of perturbing $\gamma$ and $\delta$ on the response of different variables to monetary policy and technology shocks in the model economy is quite robust. But it is easy to find examples in which dynamic general equilibrium considerations overturn the simple static intuition regarding changes in $r$ highlighted in subsection (2.2.2). Indeed in Figures 2 and 3 a higher value of $r$ is associated with a larger initial rise in in real wages and a marginally smaller decline in the unemployment rate after an expansionary monetary policy and technology shock, respectively.

In sum, in this section we have shown that the alternating offer labor market model has the capacity to account for the cyclical properties of key labor market variables. In the next

section we analyze whether it actually provides an empirically convincing account of those properties. To that end we embed it in a medium-sized DSGE model which we estimate and evaluate.

# 4. An Estimated Medium-sized DSGE Model

In this section, we describe a medium-sized DSGE model similar to one in CEE, modified to include our labor market assumptions. The first subsection describes the problems faced by households and goods producing firms. The labor market is discussed in the second subsection and is a modified version of the labor market in the previous section. Among other things, the modifications include the requirement that firms post vacancies to hire workers. The third subsection specifies the law of motion of the three shocks to agents' environment. These include a monetary policy shock, a neutral technology shock and an investment-specific technology shock. The last subsection briefly presents a version of the model corresponding to the standard DMP specification of the labor market, i.e. wages are determined by a Nash sharing rule and firms face vacancy posting costs. In addition, we also examine a version of the model with sticky wages as proposed in EHL. These versions of the model represent important benchmarks for comparison.

## 4.1. Households and Goods Production

The basic structure of the representative household's problem is the same as the one in section 3.2). Here we allow for habit persistence in preferences, time varying unemployment benefits, and the accumulation of physical capital, $K_t$.

The preferences of the representative household are given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \ln\left(C_t - bC_{t-1}\right).$$

The parameter $b$ controls the degree of habit formation in household preferences. We assume $0 \le b < 1$. The household's budget constraint is:

$$P_t C_t + P_{I,t} I_t + B_{t+1} \le (R_{K,t}\Lambda_t - a(\Lambda_t)P_{I,t})K_t + (1 - h_t)\,P_t D_t + h_t W_t + R_t B_t - T_t \ . \quad (4.1)$$

As above, $T_t$ denotes lump taxes net of firm profits and $D_t$ denotes the unemployment compensation of an unemployed worker. In contrast to (2.4), $D_t$ is exogenously time varying to ensure balanced growth. In (4.1), $B_{t+1}$ denotes beginning-of-period $t$ purchases of a nominal bond which pays rate of return, $R_{t+1}$ at the start of period $t + 1$, and $R_{K,t}$ denotes the nominal rental rate of capital services. The variable, $P_{I,t}$, denotes the nominal price of an investment good. Also, $I_t$ denotes household purchases of investment goods.

The household owns the stock of capital which evolves according to

$$K_{t+1} = (1 - \delta_K) K_t + [1 - S(I_t/I_{t-1})] I_t.$$

The function, $S(\cdot)$, is an increasing and convex function capturing adjustment costs in investment. We assume that $S$ and its derivative are both zero along a steady state growth path. We discuss this function below. Capital services are given by $\Lambda_t K_t$ where $\Lambda_t$ denotes the utilization rate of capital. Analogous to CEE we assume that the household sells capital services in a perfectly competitive market. In (4.1), $R_{K,t} \Lambda_t K_t$ represents the household's earnings from supplying capital services. The increasing convex function $a(\Lambda_t)$ denotes the cost, in units of investment goods, of setting the utilization rate to $\Lambda_t$.

As in our simple macroeconomic model, we assume that a final good is produced by a perfectly competitive representative firm using the technology, (3.1). The final good producer buys the $j^{th}$ specialized input, $Y_{j,t}$, from a retailer who uses the following technology:

$$Y_{j,t} = (k_{j,t})^\alpha (z_t h_{j,t})^{1-\alpha} - \phi_t. \tag{4.2}$$

The retailer is a monopolist in the product market and competitive in the factor markets. Here $k_{j,t}$ denotes the total amount of capital services purchased by firm $j$. Also, $\phi_t$ represents an exogenous fixed cost of production which grows in a way that ensures balanced growth. The fixed cost is calibrated so that, along the balanced growth path, profits are zero. In (4.2), $z_t$ is a technology shock whose properties are discussed below. Finally, $h_{j,t}$ is the quantity of an intermediate good purchased by the $j^{th}$ retailer. This good is purchased in competitive markets at the price $P_t^h$ from a *wholesaler*, whose problem is discussed in the next subsection. Analogous to CEE, we assume that to produce in period $t$, the retailer must borrow $P_t^h h_{j,t}$ at the start of the period at the interest rate, $R_t$. The retailer repays the loan at the end of period $t$ when it receives its sales revenues. The $j^{th}$ retailer sets its price, $P_{j,t}$, subject to its demand curve, (3.2), and the Calvo sticky price friction:

$$P_{j,t} = \begin{cases} P_{j,t-1} & \text{with probability } \xi \\ \tilde{P}_t & \text{with probability } 1 - \xi \end{cases}.$$

Notice that we do not allow for automatic indexation of prices to either steady state or lagged inflation.

## 4.2. Wholesalers and the Labor Market

Each wholesaler employs a measure of workers. Let $l_{t-1}$ denote the representative wholesaler's labor force at the end of $t-1$. A fraction $(1-\rho)$ of these workers separate exogenously. So, the wholesaler has a labor force of $\rho l_{t-1}$ at the start of period $t$. At the beginning of period $t$ the firm selects its hiring rate, $x_t$, which determines the number of new workers that

it meets at time $t$. For our empirical model, we follow Gertler, Sala and Trigari (2008) by assuming that the firm's cost hiring is an increasing function of the hiring rate,

$$\kappa_t x_t^2 l_{t-1}/2. \tag{4.3}$$

The cost is denominated in units of the final consumption good. Here $\kappa_t$ is a process that exogenous to the firm and uncorrelated with the aggregate state of the economy. We include it to ensure balanced growth. When the cost of hiring new workers is linear in the number of new workers that the firm meets, $x_t l_{t-1}$, the labor market equilibrium conditions coincide with the ones derived for the hiring cost specification in the model of section (3).

To hire $x_t l_{t-1}$ workers, the firms must post $x_t l_{t-1}/Q_t$ vacancies, where $Q_t$ is taken as given by the firm and is described below. Posting vacancies is costless.

After setting $x_t$, the firm has access to $l_t$ workers (see (2.5)). Each worker in $l_t$ then engages in bilateral bargaining with a representative of the firm, taking the outcome of all other negotiations as given.[15] As above, the real wage rate $w_t$, i.e. $W_t/P_t$, denotes the equilibrium real wage that emerges from the bargaining process. As with the small model, we verify numerically that all bargaining sessions conclude successfully with the firm representative and worker agreeing to an employment contract. Thus, in equilibrium the representative wholesaler employs all $l_t$ workers with which it has met, at wage rate $w_t$.

In what follows, we derive various value functions and an expression for the firm's hiring decision. We then discuss alternating offer bargaining in the medium-sized model.

### 4.2.1. Value Functions and Hiring Decision

To describe the bargaining process we must define the values of employed and unemployed workers, $V_t$ and $U_t$. We must also define the value, $J_t$, assigned by the firm to employing a marginal worker that it is in contact with. We express each of $U_t$, $V_t$ and $J_t$ in units of the final good. The value of being an unemployed worker is given by (2.4) except that $D$ is replaced by $D_t$. The job finding rate is given by (2.6) where $x_{t+1}$ and $l_t$ denote the average value of the corresponding wholesaler specific variables. Individual workers view $x_{t+1}$ and $l_t$ as being exogenous and beyond their control.[16] As in (2.3), $V_{t+1}$ is the value of a worker that is employed at the equilibrium wage $w_{t+1}$ in period $t + 1$.

We now consider the value, $J_t$, assigned by the firm to employing the marginal worker in $l_t$ at the wage rate, $w_t$ :

$$J_t = \vartheta_t - w_t + E_t m_{t+1} F_{l,t+1} \left( l_t \right). \tag{4.4}$$

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[15]Alternatively we could assume that the workers are represented by a union, which bargains with a representative of the firm as a whole.

[16]Since wholesalers are identical, $x_{t+1}$ and $l_t$ are equal to the values chosen by the representative wholesaler.

Here, $\vartheta_t \equiv P_t^h / P_t$ is the real price of the intermediate good produced by a worker. Thus, $\vartheta_t - w_t$ represents the time $t$ flow profit associated with a marginal worker. The term, $F_{l,t+1}$, represents the contribution of a marginal worker to the wholesaler's time $t+1$ profit. The present discounted value of the representative wholesaler's profits beginning in $t+1$ is:

$$F_{t+1}(l_t) = \max_{x_{t+1}} \left\{ [\vartheta_{t+1} - w_{t+1}](\rho + x_{t+1}) l_t - 0.5\kappa_{t+1} x_{t+1}^2 l_t + E_{t+1} m_{t+2} F_{t+2}((\rho + x_{t+1}) l_t) \right\}.$$
(4.5)

Differentiating $F_{t+1}(l_t)$ with respect to $l_t$ and taking the envelope condition into account we obtain:

$$
\begin{aligned}
F_{l,t+1}(l_t) &= (\vartheta_{t+1} - w_{t+1})(\rho + x_{t+1}) - 0.5\kappa_{t+1} x_{t+1}^2 + E_{t+1} m_{t+2} F_{l,t+2}((\rho + x_{t+1}) l_t)(\rho + x_{t+1}) \\
&= J_{t+1}(\rho + x_{t+1}) - 0.5\kappa_{t+1} x_{t+1}^2,
\end{aligned}
$$
(4.6)

where $x_{t+1}$ is the hiring rate that solves the maximization problem in (4.5). The second equality in (4.6) makes use of (4.4). Using (4.6) to substitute out for $F_{l,t+1}(l_t)$, in (4.4) we conclude:

$$J_t = \vartheta_t - w_t + E_t m_{t+1} \left[ J_{t+1}(\rho + x_{t+1}) - 0.5\kappa_{t+1} x_{t+1}^2 \right].$$
(4.7)

This expression can be simplified using the first order condition for $x_t$. Maximizing the time $t$ version of (4.5) with respect to $x_t$ and using (4.6) we obtain:

$$\kappa_t x_t = \vartheta_t - w_t + E_t m_{t+1} \left[ J_{t+1}(\rho + x_{t+1}) - 0.5\kappa_{t+1} x_{t+1}^2 \right].$$
(4.8)

Combining (4.7) and (4.8), yields

$$J_t = \kappa_t x_t.$$
(4.9)

So, the value to a firm which has an initial labor of $l_{t-1}$ of employing a marginal worker is equal to the marginal cost of hiring the a worker. Using (4.9) to substitute out for period $t+1$ adjustment costs in (4.7), we obtain a useful recursive representation expression for $J_t$:

$$J_t = \vartheta_t - w_t + E_t m_{t+1} J_{t+1}(\rho + 0.5 x_{t+1}).$$
(4.10)

### 4.2.2. Wage Bargaining

The equilibrium wage rate, $w_t$, is the outcome of a version of the alternating offer bargaining process described in the simple model. The only difference is that the cost to a firm of making a counteroffer to an offer that it rejects is given by $\gamma_t$ instead of $\gamma$. The variable, $\gamma_t$, varies in an exogenous way to ensure that the model has a well defined balanced growth path. It is straightforward to show that we can summarize the outcome of the bargaining process with the analog to (2.14) where $\gamma$ is replaced by $\gamma_t$:

$$J_t = \frac{1+r}{1-\delta}\left[V_t - \alpha U_t - \omega_t\right],$$
(4.11)

23

where

$$\alpha \equiv 1 - r\frac{1-\delta}{r+\delta}, \quad \omega_t \equiv \frac{(1-\delta)^2}{\delta+r}\gamma_t.$$

The five key equilibrium conditions related to the labor market and wholesalers are (4.10), (4.9), (4.11), (2.6) and (2.4) with $D$ replaced by $D_t$. These equations reduce to the analogs of (2.2) and (2.1) when the cost of hiring new workers is linear in the number of new workers. In this case, the equilibrium conditions of the labor market are identical to what they are in our simple model except that $D$ and $\gamma$ are replaced by $D_t$ and $\gamma_t$.

## 4.3. Market Clearing, Monetary Policy and Functional Forms

The total amount of intermediate goods purchased by retailers from wholesalers is:

$$h_t \equiv \int_0^1 h_{j,t}dj.$$

Recall that the output of intermediate goods produced by wholesalers is equal to the the number of workers they employ. So the the supply of intermediate goods is $l_t$. It follows that as in the simple model, market clearing for intermediate goods requires $h_t = l_t$. The capital services market clearing condition is:

$$\Lambda_t K_t = \int_0^1 k_{j,t}dj.$$

Market clearing for final goods requires:

$$C_t + \frac{1}{\Psi_t}(I_t + a(\Lambda_t)K_t) + 0.5\kappa_t x_t^2 l_{t-1} + G_t = Y_t. \tag{4.12}$$

The right hand side of the previous expression denotes the quantity of final goods. The left hand side represents the various ways that final goods are used. Homogeneous output, $Y_t$, can be converted one-for-one into either consumption goods, goods used to hire workers or government goods, $G_t$. In addition, some of $Y_t$ is absorbed by capital utilization costs. Finally, $Y_t$ can be used to produce investment goods using a linear technology in which one unit of final goods is transformed into $\Psi_t$ units of $I_t$. Perfect competition in the production of investment goods implies

$$P_{I,t} = \frac{P_t}{\Psi_t}.$$

The asset pricing kernel, $m_{t+1}$, is constructed using the marginal utility of consumption, which we denote by $u_{c,t}$ :

$$u_{c,t} = \frac{1}{C_t - bC_{t-1}} - \beta b E_t \frac{1}{C_{t+1} - bC_t}.$$

Then,
$$m_{t+1} = \beta \frac{u_{c,t+1}}{u_{c,t}}.$$

We adopt the following specification of monetary policy:

$$\log R_t = \rho_R \log R_{t-1} + (1 - \rho_R)\left[\log R + r_\pi \log\left(\pi_t/\pi\right) + r_y \log\left(\mathcal{Y}_t/\mathcal{Y}\right)\right] + \sigma_R \varepsilon_{R,t}.$$

Here, $\pi$ denotes the monetary authority's target inflation rate. The steady state inflation rate in our model is equal to $\pi$. Also, $R$ and $\mathcal{Y}$ denote the steady values of $R_t$ and $\mathcal{Y}_t$. The shock, $\varepsilon_{R,t}$, is a unit variance, zero mean disturbance to monetary policy. The variable, $\mathcal{Y}_t$, denotes Gross Domestic Product (GDP):

$$\mathcal{Y}_t = C_t + \frac{I_t}{\Psi_t} + G_t.$$

We assume that $G_t$ grows in an exogenous way that is consistent with balanced growth. In terms of shocks, we assume that $\log(\mu_{z,t}) \equiv \log\left(z_t/z_{t-1}\right)$ and $\log(\mu_{\Psi,t}) \equiv \log\left(\Psi_t/\Psi_{t-1}\right)$ are AR(1) processes. The parameters that control the autocorrelations and standard deviations of both processes are denoted by $(\rho_z, \rho_\Psi)$ and $(\sigma_z, \sigma_\Psi)$, respectively.

Recall our model exhibits growth stemming from neutral and capital embodied technological progress. The variables $Y_t/\Phi_t, C_t/\Phi_t, w_t/\Phi_t$ and $I_t/(\Phi_t\Psi_t)$ converge to constants in nonstochastic steady state, where

$$\Phi_t = \Psi_t^{\frac{\alpha}{1-\alpha}} z_t.$$

is a weighted average of the sources of technological progress. If objects like the fixed cost of production, the cost of posting a vacancy, the cost to a firm of preparing a counter offer, government purchases, and transfer payments were constant, they would become irrelevant over time. To avoid this implication it is standard in the literature to suppose that such objects grow at the same rate as output, which in our case is given by $\Phi_t$.[17] An unfortunate implication of this assumption is that technology shocks of both types immediately affect the vector of objects $[\phi, \kappa, \gamma, D, G]'$. It seems hard to justify such an assumption. To avoid this problem, we proceed as in Christiano, Trabandt and Walentin (2011c) and Schmitt-Grohé and Uribe (2012) who assume that government purchases, $G_t$, are a distributed lag of unit root technology shocks, i.e. $G_t$ is cointegrated with $Y_t$ but has a smoother stochastic trend. In particular, we assume that

$$[\phi_t, \kappa_t, \gamma_t, D_t, G_t]' = [\phi, \kappa, \gamma, D, G]'\, \Omega_t.$$

where $\Omega_t$ denotes the distributed lag of past values of $\Phi_{t-1}$ defined by

$$\Omega_t = \Phi_{t-1}^\varsigma \Omega_{t-1}^{1-\varsigma}. \tag{4.13}$$

---

[17]See for ACEL (2011) and Justiniano, Primiceri, and Tambalotti (2010).

Here $0 < \varsigma \leq 1$ is a parameter to be estimated. Note that $\Omega_t$ grows at the same rate as $\Phi_t$. When $\varsigma$ is very close to zero, $\Omega_t$ is virtually completely unresponsive in the short-run to an innovation in either of the two technology shocks, a feature that we find very attractive on a priori grounds.

We assume that the cost of adjusting investment is assumed to take the following form:

$$S\left(I_t/I_{t-1}\right) = 0.5 \exp\left[\sqrt{S''}\left(I_t/I_{t-1} - \mu_\Phi \mu_\Psi\right)\right] + 0.5 \exp\left[-\sqrt{S''}\left(I_t/I_{t-1} - \mu_\Phi \mu_\Psi\right)\right] - 1.$$

Here, $\mu_\Phi$ and $\mu_\Psi$ denote the unconditional growth rates of $\Phi_t$ and $\Psi_t$. The value of $I_t/I_{t-1}$ in nonstochastic steady state is $\mu_\Phi \mu_\Psi$. In addition, $S''$ represents a model parameter that coincides with the second derivative of $S\left(\cdot\right)$, evaluated in steady state. It is straightforward to verify that $S\left(\mu_\Phi \mu_\Psi\right) = S'\left(\mu_\Phi \mu_\Psi\right) = 0$.

We assume that the cost associated with setting capacity utilization is given by

$$a(\Lambda_t) = 0.5\sigma_b \sigma_a \left(\Lambda_t\right)^2 + \sigma_b \left(1 - \sigma_a\right) \Lambda_t + \sigma_b \left(\left(\sigma_a/2\right) - 1\right)$$

where $\sigma_a$ and $\sigma_b$ are positive scalars. We normalize the steady state value of $\Lambda_t$ to one, which determines the value of $\sigma_b$ given an estimate of $\sigma_a$.

Finally, we discuss how vacancies are determined. We posit a standard matching function:

$$x_t l_{t-1} = \sigma_m \left(1 - \rho l_{t-1}\right)^\sigma \left(l_{t-1} v_t\right)^{1-\sigma}, \tag{4.14}$$

where $l_{t-1} v_t$ denotes the total number of vacancies and $v_t$ denotes the vacancy rate. Given $x_t, l_{t-1}$, (4.14) can be used to solve for $v_t$. Recall that we defined the total number of vacancies by $x_t l_{t-1}/Q_t$. As a result, we can solve for $Q_t$ using

$$Q_t = \frac{x_t}{v_t}. \tag{4.15}$$

The equilibrium of our model has a particular recursive structure. All the model variables, apart from $v_t$ and $Q_t$, can be solved for first. The other two variables can then be solved for using (4.14) and (4.15).

## 4.4. Alternative Labor Market Models

In this subsection we consider alternative labor market models that we include in our DSGE framework. The objective is to assess the relative empirical performance of these alternative models. First, we describe our version of the canonical DMP model which is characterized by search costs and a particular Nash sharing rule. Second, we describe the canonical sticky nominal wage model of Erceg, Henderson and Levin (2000).

### 4.4.1. The Canonical DMP Model

In this subsection, we describe the version of the medium-sized DSGE model which we refer to as the 'Nash sharing, Search' specification. To incorporate the Nash sharing rule into our DSGE model we simply set $\omega_t = 0$, $\alpha = 1$ and replace $(1+r)/(1-\delta)$ with $(1-\eta)/\eta$ in (4.11). Here, $\eta$ is the share of total surplus given to workers. Doing so we obtain the *Nash sharing rule*:

$$J_t = \frac{1-\eta}{\eta} \left[ V_t - U_t \right], \ 0 \le \eta \le 1. \tag{4.16}$$

We incorporate DMP-style search costs into our DSGE model as follows. We assume that vacancies are costly and that posting vacancies is the only action the firm takes to meet a worker. The probability that a vacancy results in a meeting with a worker is $Q_t$. The aggregate rate at which workers are hired, $x_t$, depends on aggregate vacancy rate, $v_t$, according to (4.15).

The cost of setting the vacancy rate to $v_t$ is given by[18]

$$0.5\kappa_t v_t^2 l_{t-1}. \tag{4.17}$$

The probability, $Q_t$, is determined by the matching function, (4.14).

Four changes are required to incorporate the search cost specification into the medium-sized DSGE model. Recall that there are five labor market equilibrium conditions, (4.10), (4.9), (4.11), (2.6) and (2.4) with $D$ replaced by $D_t$. First, (4.10) is replaced by

$$J_t = \vartheta_t - w_t + E_t m_{t+1} J_{t+1} \left( \rho + 0.5 v_{t+1} Q_{t+1} \right)$$

Second, (4.9) is be replaced by:

$$Q_t J_t = \kappa_t v_t. \tag{4.18}$$

So free entry and the zero-profit condition in the search cost specification imply that the expected return to posting a vacancy is equal to the marginal cost of doing so. Third, we add $Q_t$ and $v_t$ to the set of variables that must be solved and and add (4.15) and (4.14) to the list of equilibrium conditions used. The fourth change involves replacing the hiring cost term in (4.12) with the vacancy cost term (4.17) in the resource constraint. Doing so we obtain

$$C_t + \frac{1}{\Psi_t}(I_t + a(\Lambda_t)K_t) + 0.5\kappa_t v_t^2 l_{t-1} + G_t = Y_t. \tag{4.19}$$

---

[18] At a slight cost of creating confusion, we simplify the notation by not distinguishing between the economy-wide average values of a variable and its value for a particular firm. In (4.15), $v_t$ denotes the vacancy rate of the representative firm. At the same time, it is the economy-wide average values of $v_t$ that define $Q_t$. The distinction between the economy-wide average value of a variable and its value for the representative firm is crucial when deriving the first order conditions associated with the firm's decisions.

We conclude by discussing an important feature of the search cost specification. Define labor market tightness as

$$\Gamma_t = \frac{v_t l_{t-1}}{1 - \rho l_{t-1}}.$$ 

(4.20)

Relations (4.15) and (4.14) imply that $Q_t$ is given by,

$$Q_t = \sigma_m \Gamma_t^{-\sigma}$$

It follows that the probability of filling a vacancy is decreasing in labor market tightness. As discussed below, this feature of the model is important in understanding the empirical shortcoming of the standard DMP model.

### 4.4.2. The Canonical Sticky Wage Model

We now describe how to modify the medium-sized DSGE model to incorporate the sticky nominal wage framework of Erceg, Henderson and Levin (2000). We replace the wholesale production sector with the following environment. As in the medium-sized DSGE model, the final homogeneous good, $Y_t$, is produced by competitive and identical firms using the technology given by (3.1). The specialized inputs used in the production of $Y_t$ are produced by retailers using capital services and a homogeneous labor input. The final good producer buys the $j^{th}$ specialized input, $Y_{j,t}$, from a retailer who produces the input using the technology (4.2) As above the capital are purchased in competitive rental markers. In (4.2) $h_{jt}$ refers to the quantity of a homogeneous labor input that firm $j$ from'labor contractors'. These contractors produce the homogeneous labor input by combining a range of differentiated labor inputs, $h_{t,i}$, using the following technology:

$$h_t = \left[ \int_0^1 (h_{t,i})^{\frac{1}{\lambda_w}} \, di \right]^{\lambda_w}, \quad \lambda_w > 1.$$

(4.21)

Labor contractors are perfectly competitive and take the wage rate, $W_t$, of $h_t$ as given. They also take the wage rate, $W_{t,i}$, of the $i^{th}$ labor type as given. Profit maximization on the part of contractors leads to the labor demand curve:

$$h_{t,i} = \left( \frac{W_t}{W_{t,i}} \right)^{\frac{\lambda_w}{\lambda_w - 1}} h_t.$$

(4.22)

Substituting the (4.21) into (4.22) and rearranging, we obtain:

$$W_t = \left[ \int_0^1 W_{t,i}^{\frac{1}{1-\lambda_w}} \, di \right]^{1-\lambda_w}.$$

(4.23)

Specialized labor inputs are supplied by a large number of identical households. The representative household has many members corresponding to each type, $i$, of labor and

provides complete insurance to all of its members in return for their wage income. The household's budget constraint is given by (4.1) except that $D_t$ is equal to zero. This constraint reflects our assumption that the household owns the capital stock, sets the utilization rate and makes investment decisions are described in the alternating offer bargaining specification of the model.

Given our assumptions, it is optimal for the household to assign an equal amount of consumption to each of its members. The household's utility is given by:

$$\ln \left(C_t - bC_{t-1}\right) - A \int_0^1 \frac{h_{t,i}^{1+\psi}}{1+\psi} di. \tag{4.24}$$

Here $h_{t,i}$ denotes hours by the $i^{th}$ member of the household. The wage rate of the $i^{th}$ type of labor, $W_{t,i}$, is determined outside the representative household by a monopoly union that represents all $i$-type workers across all households.

In setting the wage rate the monopoly union face Calvo-type frictions. and . With probability $1 - \xi_w$ the union can optimize the wage $W_{t,i}$ and with probability, $\xi_w$, it cannot. In the latter case, the nominal wage rate according to:

$$W_{t,i} = W_{t-1,i}. \tag{4.25}$$

With this specification, the wage of each type $i$ of labor is the same in steady state. Note that wages are not indexed, so that the model implies that with probability $\xi_w$ the nominal wage is constant..

The union acts to maximize the welfare of its members. Because there is no state variable in the union's problem, all unions with the opportunity to re-optimize in a given period face the same problem. In particular, The union chooses the current value of the wage, $\tilde{W}_t$, to maximize:

$$E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \left[ v_{t+s}\tilde{W}_t h_{t+s} - A\frac{h_{t+s}^{1+\psi}}{1+\psi} \right]. \tag{4.26}$$

subject to its demand curve (4.22). For convenience we have dropped subscript $i$. Here, $h_{t+s}$ denotes the quantity of hours worked in period $t + s$. The variable $v_{t+s}$ denotes the marginal value assigned by the representative household to the wage.[19] The union treats $v_t$ as an exogenous constant. In the above expression, $\xi_w$ appears in the discounting because the union's period $t$ decision only impacts on future histories in which it cannot re-optimize its wage. We refer the reader to Christiano, Trabandt and Walentin (2011) for a detailed exposition of this model.

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[19] The object, $v_t$, is the multiplier on the household budget constraint in the Lagrangian representation of the problem.

# 5. Econometric Methodology

We estimate our model using a Bayesian variant of the strategy in Christiano, Eichenbaum and Evans (2005) that minimizes the distance between the dynamic response to three shocks in the model and the analog objects in the data. The latter are obtained using an identified VAR for post-war quarterly U.S. times series that include key labor market variables. The particular Bayesian strategy that we use is the one developed in Christiano, Trabandt and Walentin (2011) (CTW).

To facilitate comparisons, our analysis is based on the same VAR used in Christiano, Walentin and Trabandt (2011). The latter estimate a 14 variable VAR using quarterly data that are seasonally adjusted and cover the period 1951Q1 to 2008Q4. As in CTW, we identify the dynamic responses to a monetary policy shock by assuming that the monetary authority sees the contemporaneous values of all the variables in the VAR and the only variable that a monetary policy shock affects contemporaneously is the Federal Funds Rate. Also as in CTW, we make two assumptions to identify the dynamic response to the technology shocks: (i) the only shocks that affect labor productivity in the long-run are the innovations to the neutral technology shock, $\log z_t$, and the innovation to the investment specific technology shock, $\log \Psi_t$; and (ii) the only shock that affects the price of investment relative to consumption in the long-run is the innovation to $\log \Psi_t$. These identification assumptions are satisfied in our model.[20] Standard lag-length selection criteria lead CTW to work with a VAR with 2 lags.[21]

There is an ongoing debate over whether or not there is a break in the sample period that we use. Implicitly our analysis sides with those authors who argue that the evidence of parameter breaks in the middle of our sample period is not strong. See for example Sims and Zha (2006) who argue that the evidence is consistent with the idea that monetary policy rule parameters have been unchanged over the sample. See also Christiano, Eichenbaum and Evans (1999) argue that the evidence is consistent with the proposition that the dynamic effects of a monetary policy shock have not changed during this sample.

---

[20]The identification assumption for the monetary policy shock by itself imposes no restriction on the VAR parameters. Similarly, Fisher (2006) showed that the identification assumptions for the technology shocks when applied without simultaneously applying the monetary shock identification, also imposes no restriction on the VAR parameters. However, Altig, Christiano, Eichenbaum and Lindé (2011), showed that when all the identification assumptions are imposed at the same time, then there are restrictions on the VAR parameters. We found that the test of the overidentifying restrictions on the VAR fails to reject the null hypothesis that the restrictions are satisfied at the 5 percent critical level.

[21]See CTW for a sensitivity analysis with respect to the lag length of the VAR.

We include the following variables in the VAR,[22]

$$
\begin{pmatrix}
\Delta \ln(\text{relative price of investment}_t) \\
\Delta \ln(\text{real } GDP_t/\text{hours}_t) \\
\Delta \ln(GDP \text{ deflator}_t) \\
\text{unemployment rate}_t \\
\ln(\text{capacity utilization}_t) \\
\ln(\text{hours}_t) \\
\ln(\text{real } GDP_t/\text{hours}_t) - \ln(\text{real wage}_t) \\
\ln(\text{nominal } C_t/\text{nominal } GDP_t) \\
\ln(\text{nominal } I_t/\text{nominal } GDP_t) \\
\ln(\text{vacancies}_t) \\
\text{job separation rate}_t \\
\text{job finding rate}_t \\
\ln(\text{hours}_t/\text{labor force}_t) \\
\text{Federal Funds Rate}_t
\end{pmatrix} . \tag{5.1}
$$

Given an estimate of the VAR we can compute the implied impulse response functions to the three structural shocks. We stack the contemporaneous and 14 lagged values of each of these impulse response functions for 12 of the VAR variables in a vector, $\hat{\psi}$. We do not include the job separation rate or the labor force because our model assumes those variables are constant. We include these variables in the VAR to ensure the VAR-results are not driven by omitted variable bias.

The logic underlying our econometric procedure is as follows. Suppose that our structural model is true. Denote the true values of the model parameters by $\theta_0$. Let $\psi(\theta)$ denote the model-implied mapping from a set of values for the model parameters to the analog impulse responses in $\hat{\psi}$. Thus, $\psi(\theta_0)$ denotes the true value of the impulse responses whose estimates appear in $\hat{\psi}$. According to standard classical asymptotic sampling theory, when the number of observations, $T$, is large, we have

$$
\sqrt{T}\left(\hat{\psi} - \psi(\theta_0)\right) \overset{a}{\sim} N\left(0, W\left(\theta_0, \zeta_0\right)\right).
$$

Here, $\zeta_0$ denotes the true values of the parameters of the shocks in the model that we do not formally include in the analysis. Because we solve the model using a log-linearization procedure, $\psi(\theta_0)$ is not a function of $\zeta_0$. However, the sampling distribution of $\hat{\psi}$ is a function of $\zeta_0$. We find it convenient to express the asymptotic distribution of $\hat{\psi}$ in the following form:

$$
\hat{\psi} \overset{a}{\sim} N\left(\psi(\theta_0), V\right), \tag{5.2}
$$

where

$$
V \equiv \frac{W\left(\theta_0, \zeta_0\right)}{T}.
$$

---

[22] See section A of the technical appendix for Christiano, Walentin and Trabandt (2011) for details about the data.

For simplicity our notation does not make the dependence of $V$ on $\theta_0, \zeta_0, T$ explicit. We use a consistent estimator of $V$. Motivated by small sample considerations, that estimator has only diagonal elements (see CTW). The elements in $\hat{\psi}$ are graphed in Figures 4-6 (see the solid lines). The gray areas are centered, two-standard error bands computed using our estimate of $V$.

In our analysis we treat $\hat{\psi}$ as the observed data. We specify priors for $\theta$ and then compute the posterior distribution for $\theta$ given $\hat{\psi}$ using Bayes' rule. To use Bayes' rule, we require the likelihood of $\hat{\psi}$ given $\theta$. Our asymptotically valid approximation of this likelihood is motivated by (5.2):

$$f\left(\hat{\psi}|\theta, V\right) = \left(\frac{1}{2\pi}\right)^{\frac{N}{2}} |V|^{-\frac{1}{2}} \exp\left[-\frac{1}{2}\left(\hat{\psi} - \psi\left(\theta\right)\right)' V^{-1}\left(\hat{\psi} - \psi\left(\theta\right)\right)\right]. \qquad (5.3)$$

The value of $\theta$ that maximizes the above function represents an approximate maximum likelihood estimator of $\theta$. It is approximate for three reasons: (i) the central limit theorem underlying (5.2) only holds exactly as $T \to \infty$, (ii) our proxy for $V$ that is guaranteed to be correct only for $T \to \infty$, and (iii) $\psi\left(\theta\right)$ is calculated using a linear approximation.

Treating the function, $f$, as the likelihood of $\hat{\psi}$, it follows that the Bayesian posterior of $\theta$ conditional on $\hat{\psi}$ and $V$ is:

$$f\left(\theta|\hat{\psi}, V\right) = \frac{f\left(\hat{\psi}|\theta, V\right) p\left(\theta\right)}{f\left(\hat{\psi}|V\right)}. \qquad (5.4)$$

Here, $p\left(\theta\right)$ denotes the priors on $\theta$ and $f\left(\hat{\psi}|V\right)$ denotes the marginal density of $\hat{\psi}$ :

$$f\left(\hat{\psi}|V\right) = \int f\left(\hat{\psi}|\theta, V\right) p\left(\theta\right) d\theta.$$

The mode of the posterior distribution of $\theta$ can be computed by maximizing the value of the numerator in (5.4), since the denominator is not a function of $\theta$. The marginal density of $\hat{\psi}$ is required when we want an overall measure of the fit of our model and when we want to report the shape of the posterior marginal distribution of individual elements in $\theta$. To compute the marginal likelihood, we use the standard Laplace approximation. In our analysis we also find it convenient to compute the marginal likelihood based on a subset of the elements in $\hat{\psi}$. We describe our Laplace approximation procedure for doing this in Appendix A.1.

## 6. Results

In this section we present the empirical results for our model ('Alternating Offer, Hiring'). In addition we report results for a version of our model with the search cost specification

('Alternating Offer, Search'). We also display results for the Nash sharing model with search or hiring costs ('Nash Sharing, Hiring' and 'Nash Sharing, Search', respectively). The former specification is our version of the canonical DMP model. Finally, we report results for the sticky wage model ('Sticky Wages').

In the first three subsections we discuss results for the different models. In the final subsection we assess the models' ability to account for the statistics that Shimer (2005) used to evaluate standard DMP models.

We set the values for a subset of the model parameters a priori. These values are reported in Panel A of Table 3. We also set the steady values of five model variables and these are listed in Panel B of Table 3. The remaining model parameters are estimated subject to the restrictions implied by Table 3, and these parameters are reported in Table 4.

We now discuss the material in Table 3. We set $\beta$ so that the steady state real rate of interest is the same as it is in the small macro model. The depreciation rate on capital, $\delta_K$, is set to imply an annual depreciation rate of 10 percent. The values of $\mu_\Phi$ and $\mu_\Psi$ are determined by the sample average of per capita GDP and investment growth in our sample. We assume the monetary authority's inflation target is 2.5 percent and that the profits of intermediate good producers are zero in steady state. The rate at which vacancies create job-worker meetings, $Q$, is taken from den Haan, Ramey and Watson (2000) and Ravenna and Walsh (2008). We set the steady state unemployment rate to the average unemployment rate in our sample. This implies a steady state value of $l$ equal to 0.945. The assumed steady state value of the government consumption to gross output ratio corresponds roughly to the analogous sample average in our dataset.

## 6.1. The Estimated 'Alternating Offer, Hiring' Model

Table 4 presents prior and posterior distributions for all estimated objects in the different models. Table 5 shows the steady state values for key variables in the 'Alternating Offer, Hiring' model implied by the posterior mode of the estimated objects.

A number of features of the posterior modes of the estimated parameters in 'Alternating Offers, Hiring' model are worth noting. First, the posterior mode of $\xi$ implies a moderate degree of price stickiness, with prices changing on average roughly once every 2.5 quarters. This value lies within the range reported in the literature. For example, according to Nakamura and Steinsson (2012), the recent micro-data based literature finds that the price of the median product changes roughly every 1.5 quarters when sales are included, and every 3 quarters when sales are excluded. Notice that prices are perfectly flat in the interval of time between price re-optimization because there is no price indexation in our model. Second, the posterior mode of $\delta$ implies that there is a roughly 5% chance of an exogenous break up in negotiations in case a wage offer is rejected. Third, the posterior modes of our model

parameters, along with the assumption of a steady state unemployment rate equal to 5.5%, implies that it costs firms 2.5 days of production to prepare a counter offer during wage negotiations (recall that the key parameter governing this cost is $\gamma$). Fourth, the posterior mode of hiring costs as a percent of the wages of newly hired workers is equal to roughly 9%. Silva and Toledo (2009) report that, depending on the exact costs included, the value of this statistic is between 4 and 14 percent, a range that encompasses the corresponding statistic in our model. Fifth, the posterior mode for the replacement ratio is 0.77. There is an ongoing debate in the literature about the value of the replacement ratio in the microeconomic data. Based on a summary of the literature, Gertler, Sala and Trigari (2008) argue that a plausible range for this ratio is 0.4 to 0.7. The lower bound is based on studies of unemployment insurance benefits while the upper bound takes into account informal sources of insurance. Recently Aguiar, Hurst and Karabarbounis (2012) find that unemployed people increase the amount of time that they spend on home production by roughly 30 percent. Taking this fact into account one could rationalize a replacement rate of 0.77. Sixth, the posterior mode of the parameter $\varsigma$ which governs the responsiveness of the variables in the vector $[\phi_t, \kappa_t, \gamma_t, D_t, G_t]$ to technology shocks is close to zero. It follows that these variables are virtually unresponsive in the short-run to an innovation in either of the two technology shocks. But they are fully responsive in the long-run. Finally, the posterior modes of the parameters governing monetary policy are similar to those reported in the literature (see for example Justiniano, Primiceri, and Tambalotti (2010)). The remaining parameters are reasonably standard (see for example Christiano, Trabandt and Walentin (2011)).

The solid black lines in Figures 4, 5 and 6 present the impulse response functions to a monetary policy, neutral-technology and investment-specific technology shock, respectively, implied by the estimated VAR. The grey areas represent 95 percent probability intervals. The solid lines with the circles correspond to the impulse response functions of our model evaluated at the posterior mode of the structural parameters. From Figure 4 we see that the model does very well at reproducing the estimated effect of an expansionary monetary policy shock, including the sharp hump-shaped rise of real GDP and hours worked and the muted response of inflation. Notice that real wages respond even less than hours to the monetary policy shock. Even though the maximal rise in hours worked is roughly 0.15%, the maximal rise in real wages is only 0.05%. Significantly, the model accounts for the hump-shaped fall in the unemployment rate as well as the rise in the job finding rate and vacancies that occur after an expansionary monetary policy shock. One shortcoming of the model is that it understates the rise in the capacity utilization rate. Of course the sharp rise of capacity utilization in the estimated VAR may reflect the fact that our data on the capacity utilization rate pertains to the manufacturing sector which may overstate the average response across all sectors in the economy.

From Figure 5 we see that the model does a good job of accounting for the estimated effects of a neutral technology shock. Of particular note is that the model can reproduce the estimated sharp fall in the inflation rate that occurs a positive neutral technology shock, a feature of the data stressed in Altig, Christiano, Eichenbaum and Linde (2011) and Paciello (2009). Also, the model generates a sharp fall in the unemployment rate along with a large rise in job vacancies and the job finding rate. Finally, from Figure 6 we see that the model does a good job of accounting for the estimated response of the economy to an investment-specific technology shock.

### 6.2. The Estimated Sticky Wage Model

In this subsection we discuss the empirical properties of the sticky wage model and compare its performance to the 'Alternating Offer, Hiring' model. Table 3 provides the values for parameters of the 'Sticky Wages' model that were set a priori. Note that we set $\xi_w$ equal to 0.75 so that wages change on average once a year.[23] A number of features of the posterior mode of the model parameters are worth noting (see Table 4). First, the posterior mode of the coefficient on inflation in the Taylor rule, $r_\pi$, is substantially higher than the corresponding posterior mode in the 'Alternating Offer, Hiring' model. (2.06 versus 1.39). Second, the degree of price stickiness is higher than in the 'Alternating Offer, Hiring' model. In the sticky wage model prices are estimated to change on average roughly once a year.

Figures 4, 5 and 6 show that with two important exceptions, the sticky wage model does reasonably well at accounting for the estimated impulse response functions. The exceptions are that the model understates the response of inflation to a neutral technology shock and a monetary policy shock.

We would like to compare the fit of our baseline model with that of the sticky wage model. The marginal likelihood is a standard measure of fit. However, using it here is complicated by the fact that the two models do not address the same data. For example, the sticky wage model has no implications for vacancies and the job finding rate.[24] To obtain a measure of

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[23]We encountered numerical problems in calculating the posterior mode of model parameters when we did place a dogmatic prior on $\xi_w$. We suspect that part of the problem stems from indeterminacy of the equilibrium for various configurations of the parameter values. As Ascari, Benzoni and Castelnuovo (2011) stress, the range of parameter values for which the indeterminacy problem arises is substantially larger in sticky-wage models without indexation.

[24]Gali (2011) has shown how to derive implications for the unemployment rate from the sticky wage model. But, as stressed by Gali, Smets and Wouters (2012), the implications for unemployment of our sticky wage model are strongly counterfactual (see also Christiano, Trabandt and Walentin (2011c)). Gali, Smets and Wouters (2012) suggest a change to the utility function that improves the model's implications for unemployment (for a discussion, see Christiano (2012)). However implementing this change would take us away from our objective, which is to compare our baseline model with a standard representation of the sticky wage model. This is why we compare our baseline model and the sticky wage model on a set of impulse responses that exclude vacancy rates, the job finding rate and the unemployment rate.

fit based on a common dataset, we integrate out unemployment, the job finding rate and the vacancy rate from the marginal likelihood associated with our baseline model. The marginal likelihoods based on the impulse responses to the 9 remaining variables are reported in Table 4 (see 'Laplace, 9 variables'). According to the results reported there, the marginal likelihood for our baseline model is over 50 log points higher than it is for the sticky wage model. Our strategy for computing the marginal likelihood involves several approximations (see section 5). We suspect that these approximations cannot account for the enormous difference in the marginal likelihoods. On this basis, we conclude that there is substantial statistical evidence in favor of the 'Alternating Offer, Hiring' model relative to the sticky wage model.

## 6.3. The Canonical DMP Model

In this subsection, we compare the performance of the canonical DMP model with the 'Alternating Offer, Hiring' model. As discussed above, there are two key differences between these models: the assumption of hiring versus search costs and the way that wages are determined. To assess the importance of each difference we proceed as follows. First, we modify the baseline model by replacing the alternating offer bargaining specification with the Nash sharing rule of the canonical DMP model (see subsection 4.4.1). We consider two cases here corresponding to whether there are search costs (the canonical DMP model) or hiring costs. We refer to these cases as the 'Nash Sharing, Search' and 'Nash Sharing, Hiring' models, respectively. We compare the impulse response functions of the different models holding constant common parameters. Second, we re-estimate the different models to assess their statistical performance and the plausibility of the posterior mode of the structural parameters associated with the different models. Finally, we isolate the role of hiring versus search costs in the alternating offer model by considering a version of the alternating offer model in which hiring costs are replaced by search costs (Alternating Offer, Search' model). This version of the model is closest in spirit to Hall and Milgrom (2008) who assume that there are search costs rather hiring costs.

The solid black lines in Figures 7, 8 and 9 present the impulse response functions to a monetary policy, neutral technology and investment-specific technology shock, respectively, implied by the estimated VAR. The grey areas represent 95 percent probability intervals. The solid lines with the circles, the dashed lines, the dashed lines broken by dots and the thick solid line correspond to the impulse response functions of the 'Alternating Offer, Hiring', 'Nash Sharing, Search', 'Nash Sharing, Hiring' and 'Alternating Offer Search Costs' models. All impulse response functions are evaluated at the posterior mode of the structural parameters estimated for the 'Alternating Offer, Hiring' model. Conditional on the values of the other structural parameters, we calibrate the value of $\eta$ in the Nash models to obtain a steady state rate of unemployment equal to 5.5%. The resulting values of $\eta$ in the 'Nash

Sharing, Search' and 'Nash Sharing, Hiring' models are 0.44 and 0.78,respectively.

From Figure 7 we see that the responses of output, hours worked, job finding, unemployment, vacancies, consumption and investment to a monetary policy shock are weakest in the 'Nash Sharing, Search' model. That model gives rise to the strongest real wage and inflation responses. These findings are not surprising. The weak effects of a monetary policy shock in the 'Nash Sharing, Search' model are closely related to the Shimer (2005) critique of the conventional DMP model. These findings are also consistent with our discussion in the introduction where we argue that in the standard DMP model, the sharp rise in the wage rate that occurs after an expansionary shock mutes firms' incentive to expand employment.

If we compare the 'Nash Sharing, Search' and 'Nash Sharing, Hiring' models we see that switching from the search cost specification to the hiring cost specification improves the performance of the model. In particular, output, job findings, unemployment vacancies, consumption and investment exhibit stronger responses to a monetary policy shock while real wages and inflation exhibit weaker responses. The basic intuition here is that the search cost specification implies that yields on posting vacancies are countercyclical. This force mutes the effects of an expansionary monetary policy shock. The same message comes across if we compare 'Alternating Offer, Hiring' and 'Alternating Offer, Search' models.

From Figure 8 we see that the weakest output, hours worked, job findings, unemployment, vacancies, consumption and investment response to a neutral technology shock arises again in the 'Nash Sharing, Search' model. Again, consistent with Shimer (2005), job vacancies, the job findings and unemployment rate are essentially unresponsive to the shock. As in Figure 7 moving from a search cost specification to a hiring cost specification improves the performance of the model. Finally, Figure 9 shows that similar, but less dramatic conclusions emerge from considering an investment-specific technology shock.

We now consider the results of estimating the 'Nash Sharing, Search' and 'Nash Sharing, Hiring' models. Consider first the posterior mode of the estimated structural parameters (see Table 4). The key result here is that for the 'Nash Sharing, Search' and the 'Nash Sharing, Hiring' models the posterior mode of the replacement ratio is 0.98 and 0.93,respectively. The basic forces underlying the replacement ratio are as follows. As stressed above, the yield on posting vacancies is countercyclical. Other things equal this effect makes it difficult for the model to account for the cyclical properties of key labor market variables. So the likelihood function favors parameters values that lessen the importance of search costs (see the value of $s_h$ in Table 4). Other things equal, this change implies a counterfactually low steady state unemployment rate. To compensate, the estimation criterion moves to higher values for the replacement ratio. For the 'Nash Sharing, Hiring' model, a similar logic applies stemming from the sensitivity of wages to the state of the economy.

The high values of the replacement ratio and low search- and hiring costs enable the Nash

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sharing models to account for the response of unemployment to the three structural shocks that we consider. Indeed the impulse response functions of the 'Alternating Offer, Hiring' and the two Nash Sharing models are visually relatively similar. This finding is reminiscent of the Hagedorn and Manovskii (2008) argument that a high replacement ratio has the potential to boost the volatility of unemployment and vacancies in search and matching models.[25]

The 'Alternating Offer, Hiring' model does outperform all Nash models, based on the marginal likelihood. From Table 4 we see that the marginal likelihood for our baseline model is 21 and 27 log points higher than it is for the 'Nash Sharing, Search' and 'Nash Sharing, Hiring' models, respectively. We infer that, subject to the approximations that we have made in calculating the marginal likelihood function, there is substantial statistical evidence in favor of our benchmark model.

Finally, we investigate the relative importance of hiring versus search costs in our preferred model. To this end, we estimated the 'Alternating Offer, Search' model. From Table 4 we see that there are three significant changes in the posterior mode of the structural parameters relative to those of the 'Alternating Offer, Hiring' model. First, the posterior mode of the replacement ratio rises from 0.77 to 0.84. Second, the posterior mode of $\xi$ rises from 0.61 to 0.78 so that prices now change on average every 4.5 quarters. Both these changes move the model farther away from the relevant microeconomic evidence. Third, as might be anticipated from the discussion above, search costs are driven to a very low value in percent of GDP, 0.04%. In effect, the search part of the 'Alternating Offer, Search' model is driven out of the model. From Table 4 we see that the marginal likelihood for our baseline model is 4.5 log points higher than it is for the 'Alternating Offer, Search' model.[26] Taken together, these results imply that moving from search to hiring costs improves the empirical performance of the model. An additional reason to favor the hiring cost specification comes from the micro evidence in Yashiv (2000), Carlsson, Eriksson and Gottfries (2006) and Cheremukhin and Restrepo-Echavarria (2010).

More importantly, however, the improvement from moving from 'Nash Sharing' models to 'Alternating Offer' models is larger than the impact of moving from search to hiring costs in 'Alternating Offer' models (see for example the marginal likelihood values in Table 4).

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[25]In an estimation setting that is more similar to ours than to Hagedorn and Manovskii (2008), Gertler, Sala and Trigari (2008) also find that absent frictions that create wage rigidity, a high value of the replacement ratio is required.

[26]Using different models estimated on macro data of various countries, Christiano, Trabandt and Walentin (2011b), Furlanetto and Groshenny (2012a,b) and Justiniano and Michelacci (2011) also conclude that a hiring cost specification is preferred to a search cost specification.

## 6.4. The Cyclical Behavior of Unemployment and Vacancies

We have argued that our model can account for the estimated response of unemployment and vacancies to monetary policy, neutral and investment specific technology shocks. Our methodology is quite different than the one used in much of the relevant labor market search literature. In this subsection we show that our model does well even when we assess its performance using procedures adopted in that literature. Shimer (2005) considers a real version of the standard DMP model in which labor productivity shocks and the job separation rate are exogenous stationary stochastic processes. He argues that the shocks to the job separation rate cannot be very important because they lead to a positively sloped Beveridge curve.

Shimer (2005) deduces the models' implications for HP-filtered moments which he compares to the analog moments in U.S. data. The focus of his comparison is on the relative volatility of productivity and vacancies divided by unemployment. He also looks at the persistence of these variables and the correlation between them.[27] Shimer (2005) emphasizes that the model fails along the following key dimension: in U.S. data, the standard deviation of the ratio of vacancies to unemployment, $\sigma(v/u)$, is twenty times the standard deviation of labor productivity, $\sigma(Y/l)$.[28] We refer to the ratio $\sigma(v/u)/\sigma(Y/l)$ as the 'volatility ratio', so that Shimer (2005) reports the volatility ratio is 20 in the data. But in the standard DMP model analyzed by Shimer (2005), the volatility ratio is only roughly 2. This result is known as Shimer's critique of the canonical DMP model.

To be consistent with the spirit of Shimer (2005)'s analysis, we consider a version of our model in which the only source of uncertainty is a stationary neutral technology, $a_t$. This shock has the following stationary law of motion:

$$\ln a_t = 0.95 \ln a_{t-1} + \varepsilon_t.$$

In this case, the production function for intermediate goods production becomes:

$$Y_{j,t} = a_t(k_{j,t})^\alpha (z_t h_{j,t})^{1-\alpha} - \phi_t.$$

We choose the standard deviation of $\varepsilon_t$ so that the standard deviation of HP-filtered output in the model and the data are the same. This procedure implies that the standard deviation of $\varepsilon_t$ is equal to 0.004. We then simulate the model and deduce its implication for various moments of HP-filtered data. We perform calculations for the 'Alternating Offer, Hiring' and 'Nash Sharing, Search' models. We first simulate both models using the estimated posterior mode of the 'Alternating Offer, Hiring' model. We then consider the case when the estimated parameters of the 'Nash Sharing, Search' model are imposed

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[27]See Shimer (2005), Table 1, page 28. Hagedorn, and Manovskii (2008) consider the same statistics.

[28]Here, $\sigma(x)$ denotes the standard deviation of the time series variables, $x_t$, after it has been HP filtered.

Table 6 reports our results. The key finding is that the volatility ratio implied by the 'Alternating Offer, Hiring' model is 27.6, which effectively reproduces the analog statistic in the data. In this sense our model is *not* subject to Shimer's critique of the canonical DMP model. Notice that our model also accounts very well for the standard deviations and first order autocorrelations of vacancies and unemployment, as well the unconditional correlations between these variables and productivity. Table 6 also reports the implications of the 'Nash Sharing, Search' model. Consistent with Shimer (2005), this model generates a much smaller value of the volatility ratio, namely 13.6.

Interestingly, the volatility ratio implied by the 'Nash Sharing, Search' is still higher than the one reported in Shimer (2005) for the canonical DMP model. The difference in results reflects that our medium-sized DSGE model is considerably more complex than the model used by Shimer (2005). In particular, we have examined the case when habit formation, the working capital channel and physical capital are eliminated from our model ($\alpha = 0$). Further, we also suppose that firms change prices roughly once a quarter ($\xi = 0.1$). Under these assumptions - which brings our model as close as possible to the one studied by Shimer (2005) - it turns out that the 'volatility ratio' is equal to 16.1 in the 'Alternating Offer, Hiring' and only 2.9 in the 'Nash Sharing, Search' model.

Finally, we evaluate the implications of the 'Nash Sharing, Search' model using the posterior mode of the parameter estimates for that model. Among other things, the replacement ratio for this model is 0.98. Consistent with Hagedorn and Manovskii (2008), we find that this version of the model is able to account for the 'volatility ratio'. However, under this parameterization the model overstates the observed correlation between unemployment and productivity ($-0.3$ in the data and 0.05 in the model) and understates the correlation between vacancies and productivity (0.4 in the data and 0.09 in the model).

Viewed as a whole the results of this section corroborate our argument that the 'Alternating Offer, Hiring' model does well at accounting for the cyclical properties of key labor market variables and outperforms the competing models that we considered. The result obtains whether we assess the model using our impulse response methodology or use the statistics stressed in the relevant literature.

## 7. Conclusion

This paper constructs and estimates an equilibrium business cycle model which can account for the response of the U.S. economy to neutral and investment specific technology shocks as well as monetary policy shocks. The focus of our analysis is on how labor markets respond to these shocks. Significantly our model does not assume that wages are sticky. Instead we derive inertial wages from our specification of how firms and workers interact

when negotiating wages. We explained how this inertia could be interpreted as applying to the period-by-period wage, or to the present value the wage negotiated at the time a worker and firm first meet. It remains an open question which implications for optimal policy of existing DSGE models are sensitive to abandoning the sticky wage assumption. We leave the answer to this question to future research.

We have been critical of standard sticky wage models in this paper. Still, Hall (2005) describes one interesting line of defense for sticky wages. He introduces sticky wages into the DMP framework in a way that satisfies the condition that no worker-employer pair has an unexploited opportunity for mutual improvement (Hall, 2005, p. 50). A sketch of Hall's logic is as follows. In a model with labor market frictions, there is a gap between the reservation wage required by a worker to accept employment and the highest wage a firm is willing to pay an employee. This gap, or bargaining set, fluctuates with the shocks that affect the surplus enjoyed by the worker and employer. When calibrated based on aggregate data the fluctuations in the bargaining set are sufficiently small and the width of the set is sufficiently wide that an exogenously inertial wage rate can remain inside the set for an extended period of time. Shimer (2012a) pursues this idea in a calibrated model while Gertler, Sala and Trigari (2008) and Gertler and Trigari (2009) do so in an estimated, medium-sized DSGE model.[29] A concern about this strategy for justifying sticky wages is that the microeconomic shocks which move actual firms' bargaining sets are far more volatile than what the aggregate data suggest. As a result, it may be harder to use the preceding approach to rationalize sticky wages than had initially been recognized. An important task is to discriminate between the approach taken in this paper and the approach proposed in Hall (2005).

We wish to emphasize that our approach follows HM in assuming that the cost of disagreement in wage negotiations is relatively insensitive to the state of the business cycle. This assumption played a key role in the empirical success of our model. Assessing the empirical plausibility of this assumption using microeconomic data is a task that we leave to future research.

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## A. Appendix: Marginal Likelihood for a Subset of Data

We denote our data by the $N \times 1$ vector, $\hat{\psi}$. We decompose $\hat{\psi}$ into two parts:

$$\hat{\psi} = \left[ \begin{array}{c} \hat{\psi}_1 \\ \hat{\psi}_2 \end{array} \right],$$

where $\hat{\psi}_i$ is $N_i \times 1$, $i = 1, 2$ and $N_1 + N_2 = N$. We have a marginal likelihood for $\hat{\psi}$ :

$$f\left(\hat{\psi}\right) = \int f\left(\hat{\psi}|\theta\right) f\left(\theta\right) d\theta,$$

where $f\left(\hat{\psi}|\theta\right)$ denotes the likelihood of $\hat{\psi}$ conditional on the model parameters, $\theta$. Also, $f\left(\theta\right)$ denotes the priors. We seek the marginal likelihood of $\hat{\psi}_1$, which is defined as:

$$f\left(\hat{\psi}_1\right) = \int f\left(\hat{\psi}\right) d\hat{\psi}_2.$$

For this, we rely heavily on the Laplace approximation to $f\left(\hat{\psi}\right)$ :

$$\frac{f\left(\hat{\psi}|\theta^*\right) f\left(\theta^*\right)}{\left(\frac{1}{2\pi}\right)^{\frac{M}{2}} \left|g_{\theta^*\theta^*}\right|^{\frac{1}{2}}}, \tag{A.1}$$

where $g_{\theta^*\theta^*}$ denotes the second derivative of $\log f\left(\hat{\psi}|\theta\right) f\left(\theta\right)$ with respect to $\theta$, evaluated at the mode, $\theta^*$. Also, $M$ denotes the number of elements in $\theta$. Note that we can write the matrix $V$ as follows:

$$V = \begin{bmatrix} V_{11} & 0 \\ 0 & V_{22} \end{bmatrix},$$

Where $V_{11}$ is the upper $N_1 \times N_1$ block of $V$ and $V_{22}$ is the lower $N_2 \times N_2$ block. The zero's on the off-diagonal of $V$ reflect our assumption that $V$ is diagonal. Using this notation, we write our approximation to the likelihood (5.3) as follows:

$$
\begin{aligned}
f\left(\hat{\psi}|\theta^*\right) &= \frac{1}{(2\pi)^{\frac{N_1}{2}}} \left|V_{11}\right|^{-\frac{1}{2}} \exp\left[-\frac{1}{2}\left(\hat{\psi}_1 - \psi_1\left(\theta^*\right)\right)' V_{11}^{-1}\left(\hat{\psi}_1 - \psi_1\left(\theta^*\right)\right)\right] \\
&\quad \times \frac{1}{(2\pi)^{\frac{N_2}{2}}} \left|V_{22}\right|^{-\frac{1}{2}} \exp\left[-\frac{1}{2}\left(\hat{\psi}_2 - \psi_2\left(\theta^*\right)\right)' V_{22}^{-1}\left(\hat{\psi}_2 - \psi_2\left(\theta^*\right)\right)\right].
\end{aligned}
$$

Substituting this expression into (A.1), we obtain the following representation of the marginal likelihood of $\hat{\psi}$ :

$$
\begin{aligned}
f\left(\hat{\psi}\right) &= \left(\frac{1}{2\pi}\right)^{-\frac{M}{2}} \left|g_{\theta^*\theta^*}\right|^{-\frac{1}{2}} \frac{1}{(2\pi)^{\frac{N_1}{2}}} \left|V_{11}\right|^{-\frac{1}{2}} \exp\left[-\frac{1}{2}\left(\hat{\psi}_1 - \psi_1\left(\theta^*\right)\right)' V_{11}^{-1}\left(\hat{\psi}_1 - \psi_1\left(\theta^*\right)\right)\right] \\
&\quad \times \frac{1}{(2\pi)^{\frac{N_2}{2}}} \left|V_{22}\right|^{-\frac{1}{2}} \exp\left[-\frac{1}{2}\left(\hat{\psi}_2 - \psi_2\left(\theta^*\right)\right)' V_{22}^{-1}\left(\hat{\psi}_2 - \psi_2\left(\theta^*\right)\right)\right]
\end{aligned}
$$

Now it is straightforward to compute our approximation to $f\left(\hat{\psi}_1\right)$ :

$$
\begin{aligned}
f\left(\hat{\psi}_1\right) &= \int_{\hat{\psi}_2} f\left(\hat{\psi}\right) d\hat{\psi}_2 \\
&= (2\pi)^{\frac{M}{2}} \left|g_{\theta^*\theta^*}\right|^{-\frac{1}{2}} (2\pi)^{-\frac{N_1}{2}} \left|V_{11}\right|^{-\frac{1}{2}} \exp\left[-\frac{1}{2}\left(\hat{\psi}_1 - \psi_1\left(\theta^*\right)\right)' V_{11}^{-1}\left(\hat{\psi}_1 - \psi_1\left(\theta^*\right)\right)\right].
\end{aligned}
$$

Here, we have used

$$\int_{\hat{\psi}_2} \frac{1}{(2\pi)^{\frac{N_2}{2}}} \left|V_{22}\right|^{-\frac{1}{2}} \exp\left[-\frac{1}{2}\left(\hat{\psi}_2 - \psi_2\left(\theta^*\right)\right)' V_{22}^{-1}\left(\hat{\psi}_2 - \psi_2\left(\theta^*\right)\right)\right] d\hat{\psi}_2 = 1,$$

which follows from the fact that the integrand is a density function.

Table 1: Parameters and Steady State Values in the Small Macro Model

| Parameter | Value | Description |
|---|---|---|
| | *Panel A: Parameters* | |
| $\beta$ | $1.03^{-0.25}$ | Discount factor |
| $\xi$ | 0.75 | Calvo price stickiness |
| $\epsilon$ | 6 | Elasticity of demand parameter |
| $\alpha$ | 0.75 | Taylor rule: interest rate smoothing |
| $\phi_\pi$ | 1.5 | Taylor rule: inflation coefficient |
| $\phi_y$ | 0.05 | Taylor rule: employment coefficient |
| $\rho$ | 0.9 | Job survival probability |
| $1+r$ | $\beta^{(-4/365)}$ | Intra-period discounting |
| $\delta$ | 0.0065 | Prob. of bargaining session break-up |
| $\tau_1$ | 0.9 | Root 1 for AR(2) technology |
| $\tau_2$ | 0.8 | Root 2 for AR(2) technology |
| | *Panel B: Steady State Values* | |
| $400(\pi - 1)$ | 0 | Annual net inflation rate |
| $h$ | 0.95 | Employment |
| $\kappa x h/Y$ | 0.01 | Hiring cost to output ratio |
| $D/w$ | 0.4 | Replacement ratio |

Table 2: Small Model Steady States and Implied Parameters

| Variable | Model | Description |
|---|---|---|
| $C$ | 0.94 | Consumption |
| $Y$ | 0.95 | Gross output |
| $e^a$ | 1 | Steady state technology |
| $s$ | 1 | Marginal cost of retailers |
| $w$ | 0.99 | Market wage |
| $w^l$ | 1.01 | Counteroffer wage |
| $U$ | 129.53 | Value of unemployment |
| $V$ | 130.39 | Value of worker at market wage |
| $V^l$ | 130.40 | Value of worker at counteroffer wage |
| $J$ | 0.1 | Firm value at market wage |
| $J^l$ | 0.08 | Firm value at worker counteroffer wage |
| $u$ | 0.05 | Steady state unemployment rate |
| $f$ | 0.66 | Job finding rate |
| $\omega$ | 2.34 | Parameter in sharing rule |
| $D$ | 0.40 | Unemployment benefits |
| $\kappa$ | 0.1 | Hiring cost parameter |
| $90\gamma/Y$ | 1.50 | Days of lost production for firm |

Table 3: Non-Estimated Parameters and Calibrated Variables in the Medium-sized Model

| Parameter | Value | Description |
|---|---|---|
| *Panel A: Parameters* | | |
| $\delta_K$ | 0.025 | Depreciation rate of physical capital |
| $\beta$ | 0.9968 | Discount factor |
| $\rho$ | 0.9 | Job survival probability |
| $1+r$ | $1.03^{1/365}$ | Intra-period discounting (alternating offer bargaining model) |
| $\lambda_w$ | 1.2 | Wage markup parameter (sticky wage model) |
| $\xi_w$ | 0.75 | Wage stickiness (sticky wage model) |
| $400\log(\mu_\Phi)$ | 1.7 | Annual output per capita growth rate |
| $400\log(\mu_\Phi\mu_\Psi)$ | 2.9 | Annual investment per capita growth rate |
| *Panel B: Steady State Values* | | |
| $400(\pi-1)$ | 2.5 | Annual net inflation rate |
| $profits$ | 0 | Intermediate goods producers profits |
| $Q$ | 0.7 | Vacancy filling rate |
| $u$ | 0.055 | Unemployment rate |
| $G/Y$ | 0.2 | Government consumption to gross output ratio |

Table 4: Priors and Posteriors of Parameters for the Medium-sized Model

| | | | Prior | | Posterior | | | |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *Labor Market Model* | | | | | **Alternating Offer Bargaining** | | **Nash Sharing** | | **Sticky Wages**[a] |
| *Cost Specification* | | | | | Hiring | Search | Hiring | Search | - |
| *Model #* | | | | | $\mathcal{M}_1$ | $\mathcal{M}_2$ | $\mathcal{M}_3$ | $\mathcal{M}_4$ | $\mathcal{M}_5$ |
| | | Distr. | Mean,Std | | Mode,Std | Mode,Std | Mode,Std | Mode,Std | Mode,Std |
| *Price Setting Parameters* | | | | | | | | | |
| Price Stickiness | $\xi$ | B | 0.50,0.15 | | **0.61**,0.03 | **0.78**,0.02 | **0.77**,0.02 | **0.79**,0.02 | **0.72**,0.03 |
| Price Markup Parameter | $\lambda_f$ | G | 1.20,0.05 | | **1.42**,0.04 | **1.39**,0.05 | **1.35**,0.05 | **1.36**,0.13 | **1.22**,0.05 |
| *Monetary Authority Parameters* | | | | | | | | | |
| Taylor Rule: Smoothing | $\rho_R$ | B | 0.7,0.15 | | **0.87**,0.01 | **0.84**,0.02 | **0.80**,0.01 | **0.82**,0.02 | **0.79**,0.01 |
| Taylor Rule: Inflation | $r_\pi$ | G | 1.70,0.15 | | **1.39**,0.12 | **1.37**,0.11 | **1.32**,0.10 | **1.36**,0.12 | **2.07**,0.15 |
| Taylor Rule: GDP | $r_y$ | G | 0.10,0.05 | | **0.06**,0.02 | **0.04**,0.02 | **0.01**,0.01 | **0.01**,0.03 | **0.01**,0.01 |
| *Preferences and Technology* | | | | | | | | | |
| Consumption Habit | $b$ | B | 0.50,0.15 | | **0.83**,0.01 | **0.80**,0.02 | **0.79**,0.02 | **0.80**,0.02 | **0.71**,0.02 |
| Capacity Util. Adj. Cost | $\sigma_a$ | G | 0.50,0.30 | | **0.19**,0.06 | **0.06**,0.04 | **0.04**,0.03 | **0.03**,0.03 | **0.06**,0.04 |
| Investment Adj. Cost | $S''$ | G | 8.00,2.00 | | **14.58**,1.9 | **15.42**,2.2 | **17.18**,2.2 | **17.49**,2.8 | **5.69**,0.88 |
| Capital Share | $\alpha$ | B | 0.30,0.02 | | **0.23**,0.01 | **0.25**,0.02 | **0.26**,0.02 | **0.26**,0.02 | **0.30**,0.02 |
| Technology Diffusion | $\varsigma$ | B | 0.20,0.10 | | **0.002**,0.002 | **0.04**,0.01 | **0.10**,0.02 | **0.08**,0.07 | **0.06**,0.03 |
| *Labor Market Parameters* | | | | | | | | | |
| Prob. of Barg. Breakup | $\delta$ | G | 0.05,0.025 | | **0.055**,0.01 | **0.11**,0.02 | - | - | - |
| Replacement Ratio | $D/w$ | B | 0.40,0.15 | | **0.77**,0.03 | **0.84**,0.03 | **0.93**,0.01 | **0.98**,0.02 | - |
| Hiring-Search Cost/Y | $s_h$ | G | 0.50,0.30 | | **0.66**,0.10 | **0.04**,0.03 | **0.11**,0.08 | **0.13**,0.10 | - |
| Match. Function Param. | $\sigma$ | B | 0.50,0.10 | | **0.55**,0.03 | **0.55**,0.04 | **0.59**,0.03 | **0.55**,0.08 | - |
| Inv. Labor Supply Elast. | $\psi$ | G | 1.00,0.50 | | - | - | - | - | **0.68**,0.28 |
| *Shocks* | | | | | | | | | |
| Std. Monetary Policy | $\sigma_R$ | G | 0.65,0.05 | | **0.58**,0.03 | **0.62**,0.03 | **0.65**,0.03 | **0.64**,0.03 | **0.64**,0.03 |
| Std. Neutral Technology | $\sigma_{\mu_z}$ | G | 0.20,0.05 | | **0.10**,0.01 | **0.14**,0.02 | **0.15**,0.02 | **0.15**,0.02 | **0.18**,0.04 |
| Std. Invest. Technology | $\sigma_\Psi$ | G | 0.20,0.05 | | **0.15**,0.02 | **0.12**,0.02 | **0.10**,0.02 | **0.10**,0.04 | **0.18**,0.02 |
| AR(1) Neutral Technology | $\rho_{\mu_z}$ | B | 0.20,0.10 | | **0.12**,0.08 | **0.23**,0.11 | **0.41**,0.08 | **0.39**,0.23 | **0.46**,0.12 |
| AR(1) Invest. Technology | $\rho_\Psi$ | B | 0.75,0.15 | | **0.62**,0.07 | **0.72**,0.08 | **0.80**,0.06 | **0.81**,0.12 | **0.50**,0.07 |
| *Memo Items* | | | | | | | | | |
| Log Marginal Likelihood (Laplace, 12 Variables): | | | | | 291.4 | 287.1 | 284.6 | 282.8 | - |
| Log Marginal Likelihood (Laplace, 9 Variables[b]): | | | | | **324.4** | **318.9** | **297.3** | **302.9** | **270.3** |
| Posterior Odds - $\mathcal{M}_1 : \mathcal{M}_i$, $i = 1,..,5$ (9 Variables): | | | | | 1:1 | 233:1 | 6e11:1 | 2e9:1 | 3e23:1 |
| Implied Worker Surplus/Total Surplus: | | | | | 0.66 | 0.96 | 0.78 | 0.44 | - |

Notes: $s_h$ denotes the steady state hiring or search cost to gross output ratio (in percent). For model specifications where particular parameters are not relevant, the entries in this table are blank.

[a] Sticky wages as in Erceg, Henderson and Levin (2000).

[b] Common dataset across all models, i.e. when unemployment, vacancies and job finding rates are excluded.

Table 5: Medium-sized Model Steady States and Implied Parameters at Posterior Mode in Alternating Offer/Hiring Cost Model

| Variable | Value | Description |
|---|---|---|
| $K/Y$ | 6.52 | Capital to gross output ratio (quarterly) |
| $C/Y$ | 0.61 | Consumption to gross output ratio |
| $I/Y$ | 0.21 | Investment to gross output ratio |
| $l$ | 0.945 | Steady state labor input |
| $R$ | 1.014 | Gross nominal interest rate (quarterly) |
| $R^{\text{real}}$ | 1.0075 | Gross real interest rate (quarterly) |
| $mc$ | 0.704 | Marginal cost (inverse markup) |
| $\sigma_b$ | 0.036 | Capacity utilization cost parameter |
| $y$ | 1.05 | Gross output |
| $\phi/Y$ | 0.42 | Fixed cost to gross output ratio |
| $\sigma_m$ | 0.66 | Level parameter in matching function |
| $f$ | 0.63 | Job finding rate |
| $x$ | 0.1 | Hiring rate |
| $J$ | 0.15 | Value of firm at market wage |
| $J^l$ | 0.11 | Value of firm at workers counteroffer wage |
| $V$ | 256.47 | Value of worker at market wage |
| $V^l$ | 256.51 | Value of worker at counteroffer wage |
| $U$ | 256.19 | Value of unemployment |
| $v$ | 0.13 | Vacancy rate |
| $w$ | 0.835 | Market wage |
| $w^l$ | 0.87 | Counteroffer wage |
| $90\gamma/Y$ | 2.64 | Days of lost production for firm |

Table 6: Data vs. Medium-Sized Model With Stationary Neutral Technology Shock

| | $\sigma(u)$ | $\sigma(v)$ | $\sigma(v/u)$ | $\sigma(Y/l)$ | $\sigma(v/u)/\sigma(Y/l)$ |
|---|---|---|---|---|---|
| *Volatility Statistics* | | | | | |
| Data | 0.13 | 0.14 | 0.26 | 0.009 | 27.6 |
| Alternating Offer - Hiring Cost Model | 0.15 | 0.10 | 0.24 | 0.007 | 33.5 |
| Nash Sharing - Search Cost Model (DMP) | 0.05 | 0.05 | 0.09 | 0.007 | 13.6 |
| *First Order Autocorrelations* | | | | | |
| | $u$ | $v$ | $v/u$ | $Y/l$ | |
| Data | 0.86 | 0.90 | 0.89 | 0.70 | |
| Alternating Offer - Hiring Cost Model | 0.90 | 0.67 | 0.82 | 0.80 | |
| Nash Sharing - Search Cost Model (DMP) | 0.70 | 0.28 | 0.51 | 0.84 | |
| *Correlations* | | | | | |
| | $u,v$ | $u,v/u$ | $v,v/u$ | $u,Y/l$ | |
| Data | -0.91 | -0.98 | 0.98 | -0.28 | |
| Alternating Offer - Hiring Cost Model | -0.88 | -0.98 | 0.95 | -0.31 | |
| Nash Sharing - Search Cost Model (DMP) | -0.82 | -0.96 | 0.95 | 0.13 | |
| | $v,Y/l$ | $v/u,Y/l$ | | | |
| Data | 0.37 | 0.33 | | | |
| Alternating Offer - Hiring Cost Model | 0.42 | 0.36 | | | |
| Nash Sharing - Search Cost Model (DMP) | 0.04 | -0.05 | | | |

Notes: $u, v$ and $Y/l$ denote the unemployment rate, vacancies and labor productivity; $\sigma(\cdot)$ is the standard deviation of these variables. All data are in log levels and hp-filtered with smoothing parameter 1600. The sample period is 1951Q1 to 2008Q4. Data sources are the same as those used for the estimation of the medium-sized model. Similar to Shimer (2005), we simulate the model using a stationary neutral technology shock. See the main text for details.

# Figure 1: Alternating Offer Bargaining

**Panel A: Best response functions**
*Effect of an increase in $U^i$*



$w^i$

Worker best response
Slope: $(1+r)/(1-\delta)$

$\delta\Delta U^i > 0$

Firm best response
Slope: $(1-\delta)/(1+r)$

$w^{l,i}$
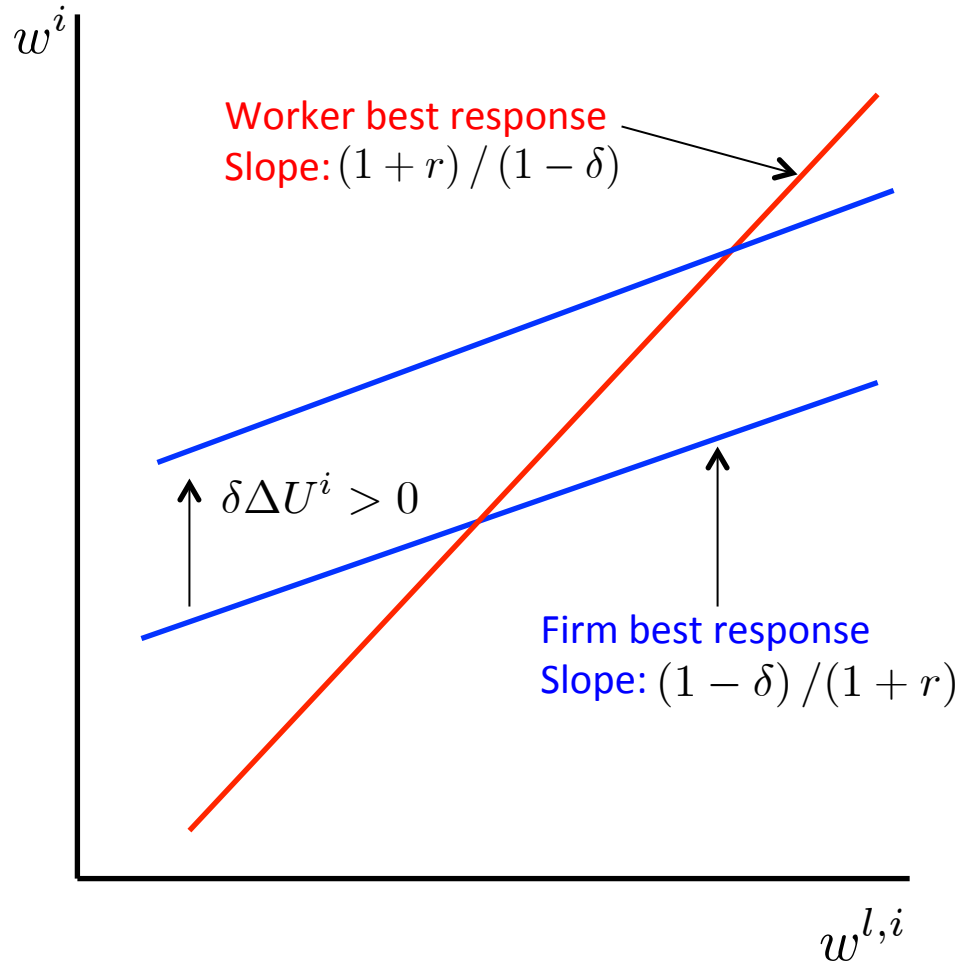
**Panel B: Labor Market in Steady State**
*Effects of a fall in $\gamma$*

$w$

Worker best response

Steady state wage:
$\vartheta - (1-\rho\beta)\kappa$

c

a

b

Firm best response

$w^l$
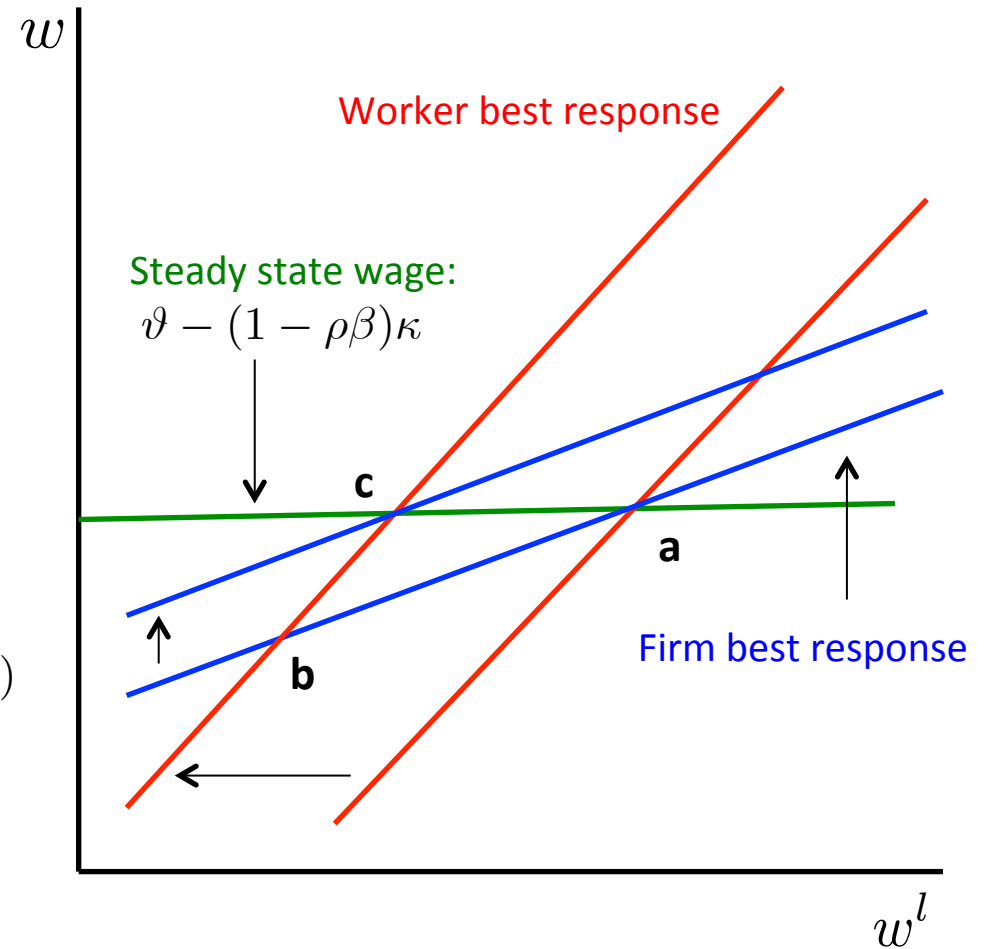
Figure 2: Simple Macro Model Responses to a 25 ABP Monetary Policy Shock

Baseline — Lower Firm Delay Cost, $\gamma$ — Higher Break-up Probability, $\delta$ — Higher Discount Rate, $r$

**Inflation (ABP)**

**Real Consumption (%)**

$C_{ss}=0.94$
$C_{ss}=0.987$
$C_{ss}=0.964$
$C_{ss}=0.965$

**Unemployment Rate (p.p.)**

$u_{ss}=0.05$
$u_{ss}=0.003$
$u_{ss}=0.026$
$u_{ss}=0.025$

**Real Wage (%)**

$w_{ss}=0.989$
$w_{ss}=0.989$
$w_{ss}=0.989$
$w_{ss}=0.989$
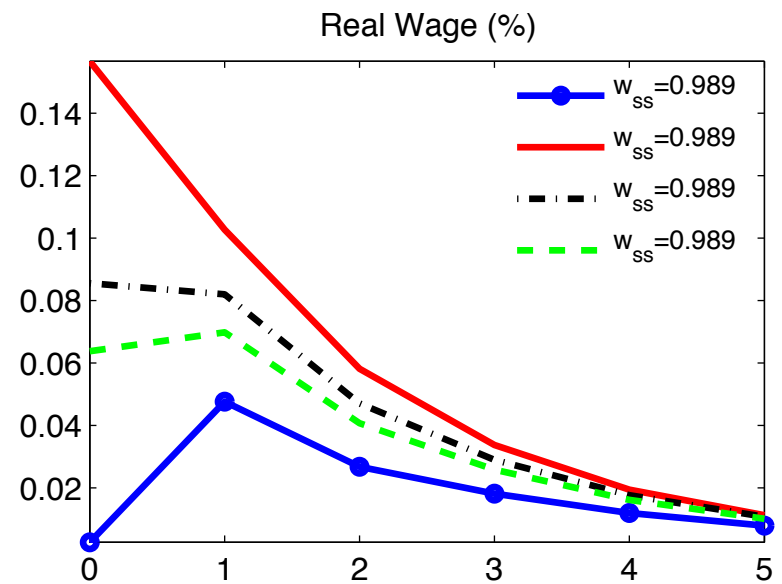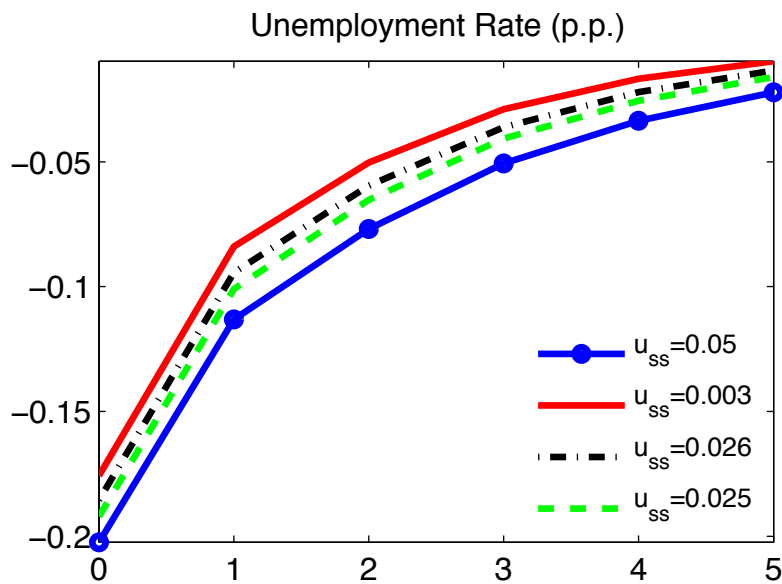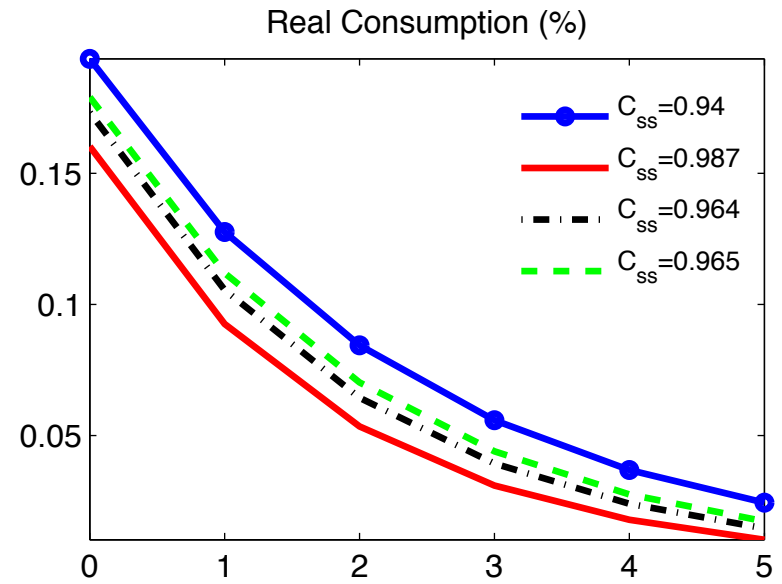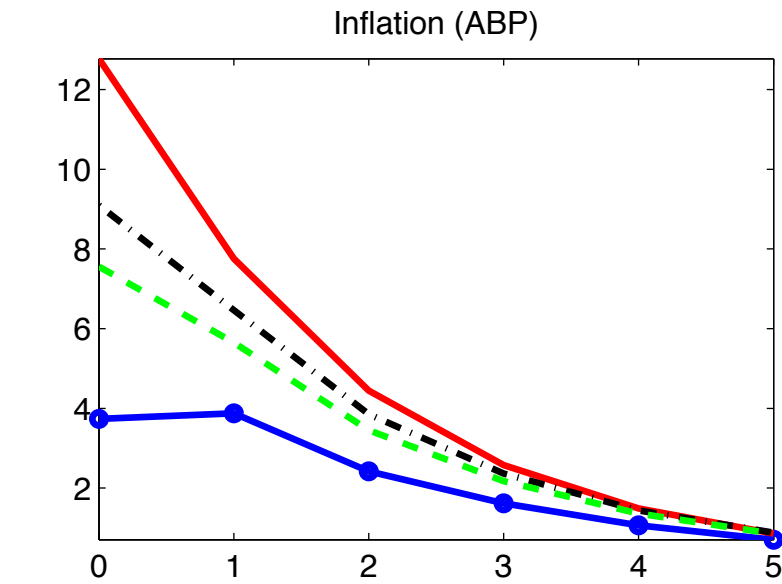
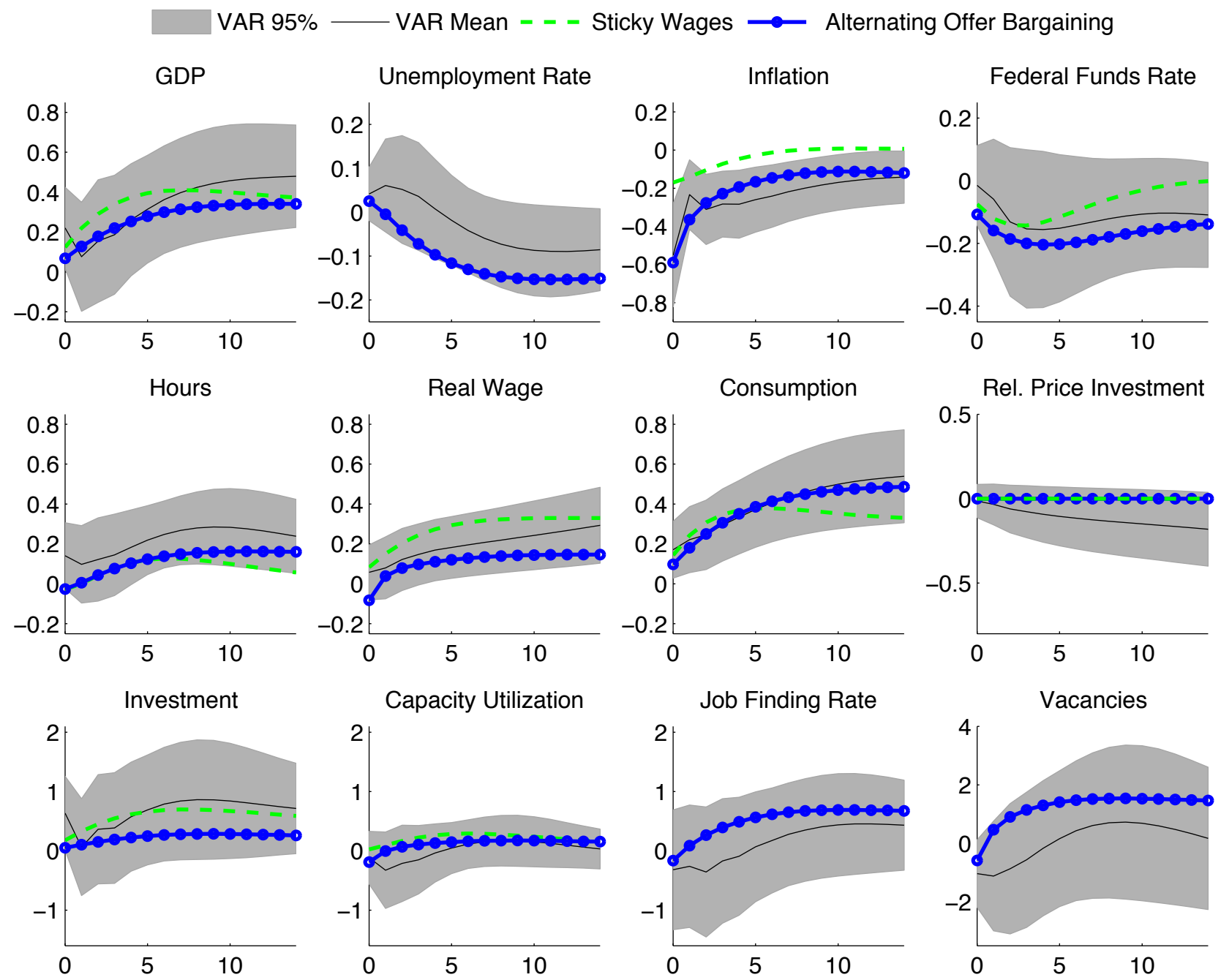Figure 3: Simple Macro Model Responses to a 0.1 Percent Technology Shock, AR(2)
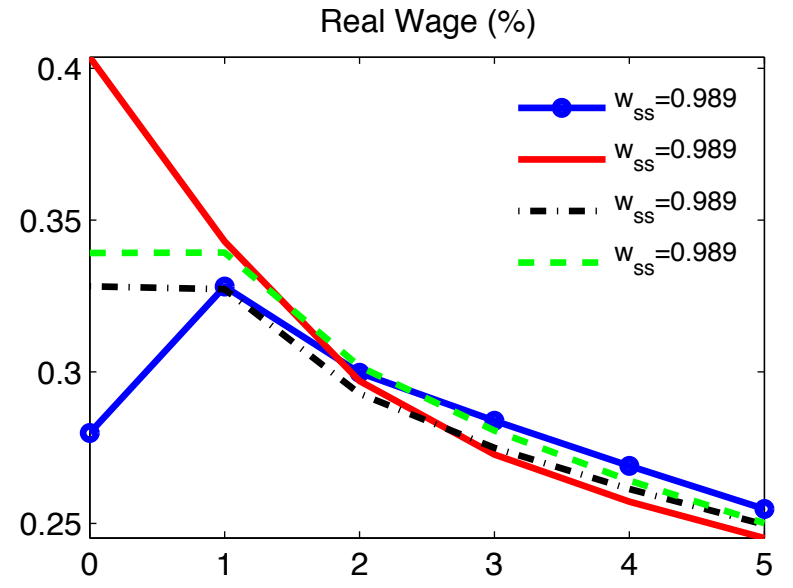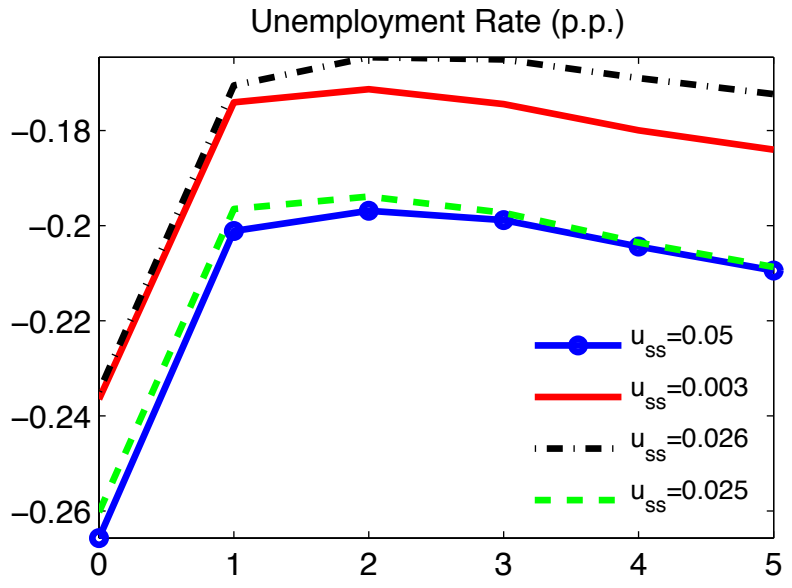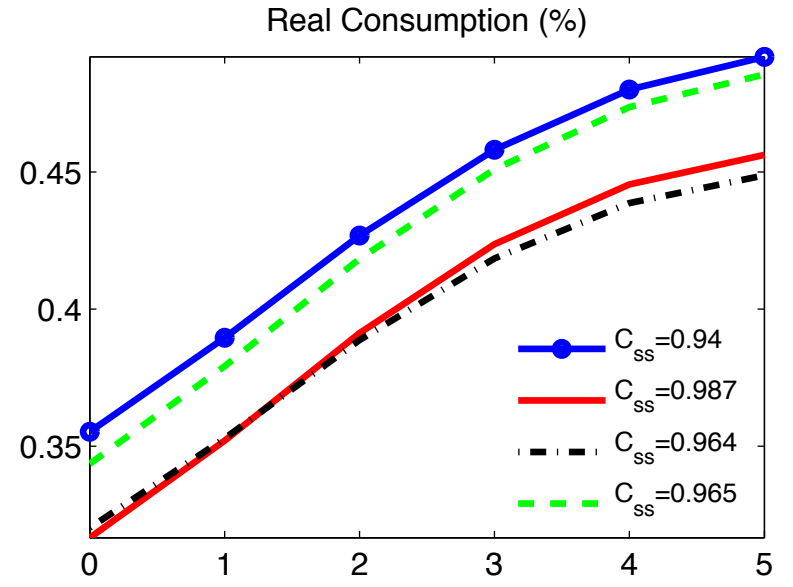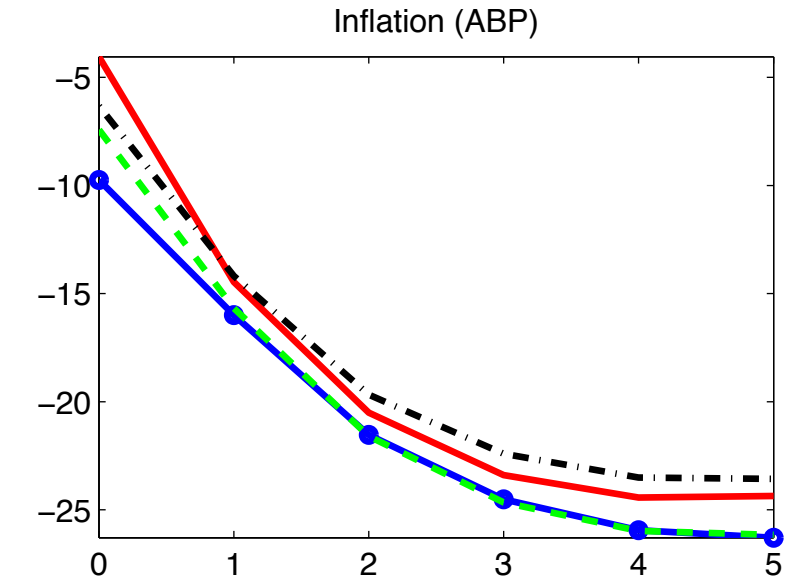
# Figure 4: Medium−Sized Model Impulse Responses to a Monetary Policy Shock
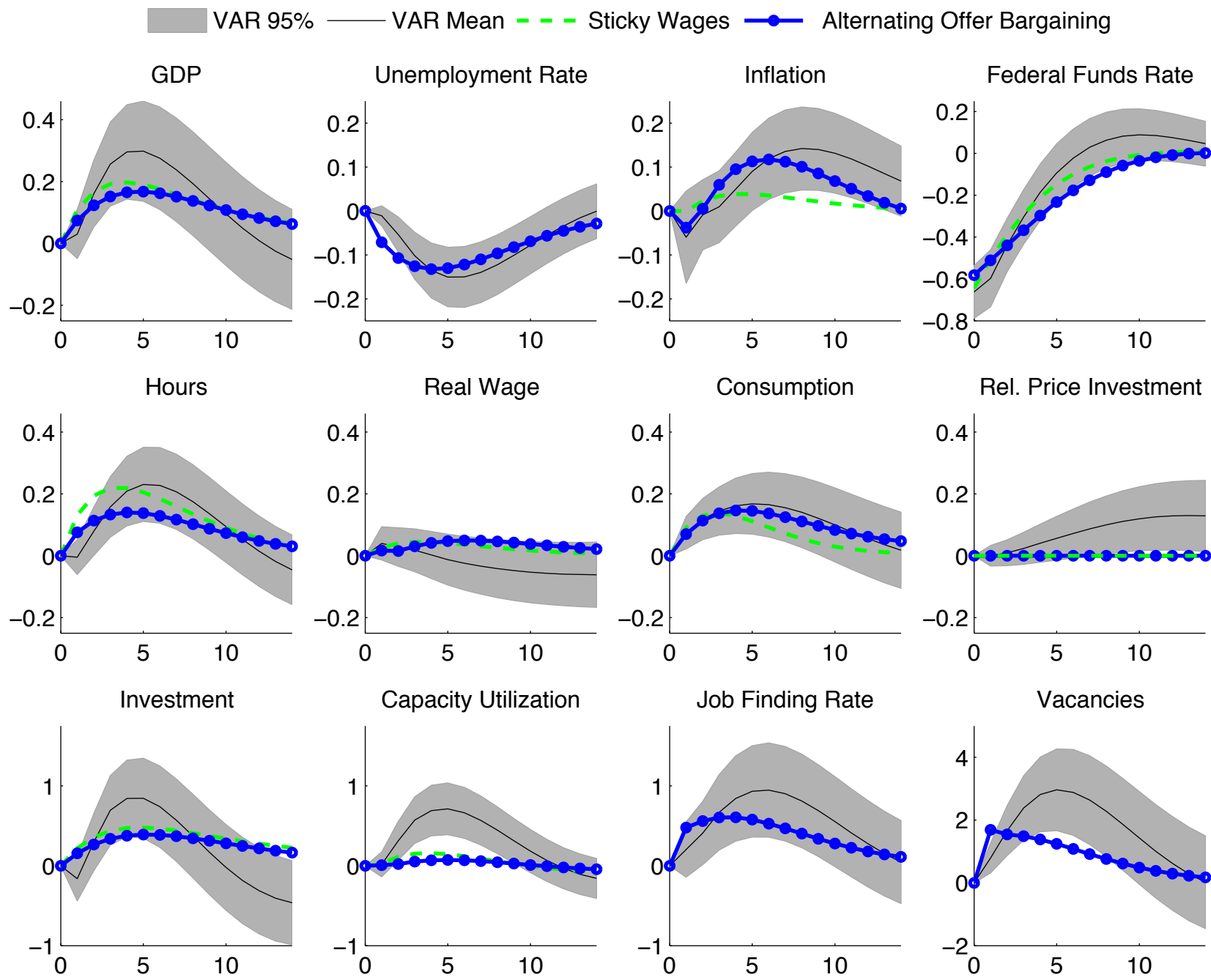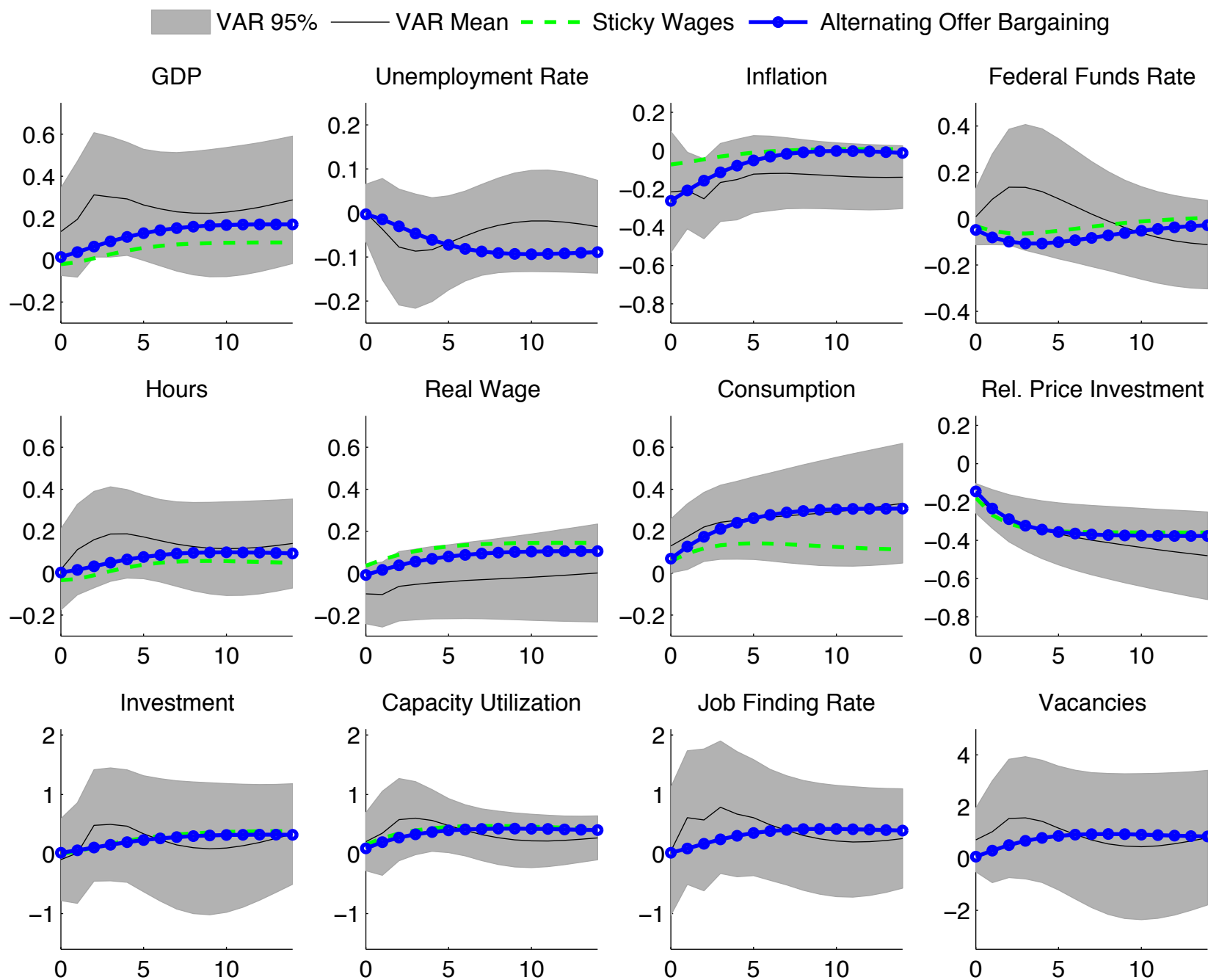


Notes: x−axis: quarters, y−axis: percent

Figure 5: Medium−Sized Model Impulse Responses to a Neutral Tech. Shock

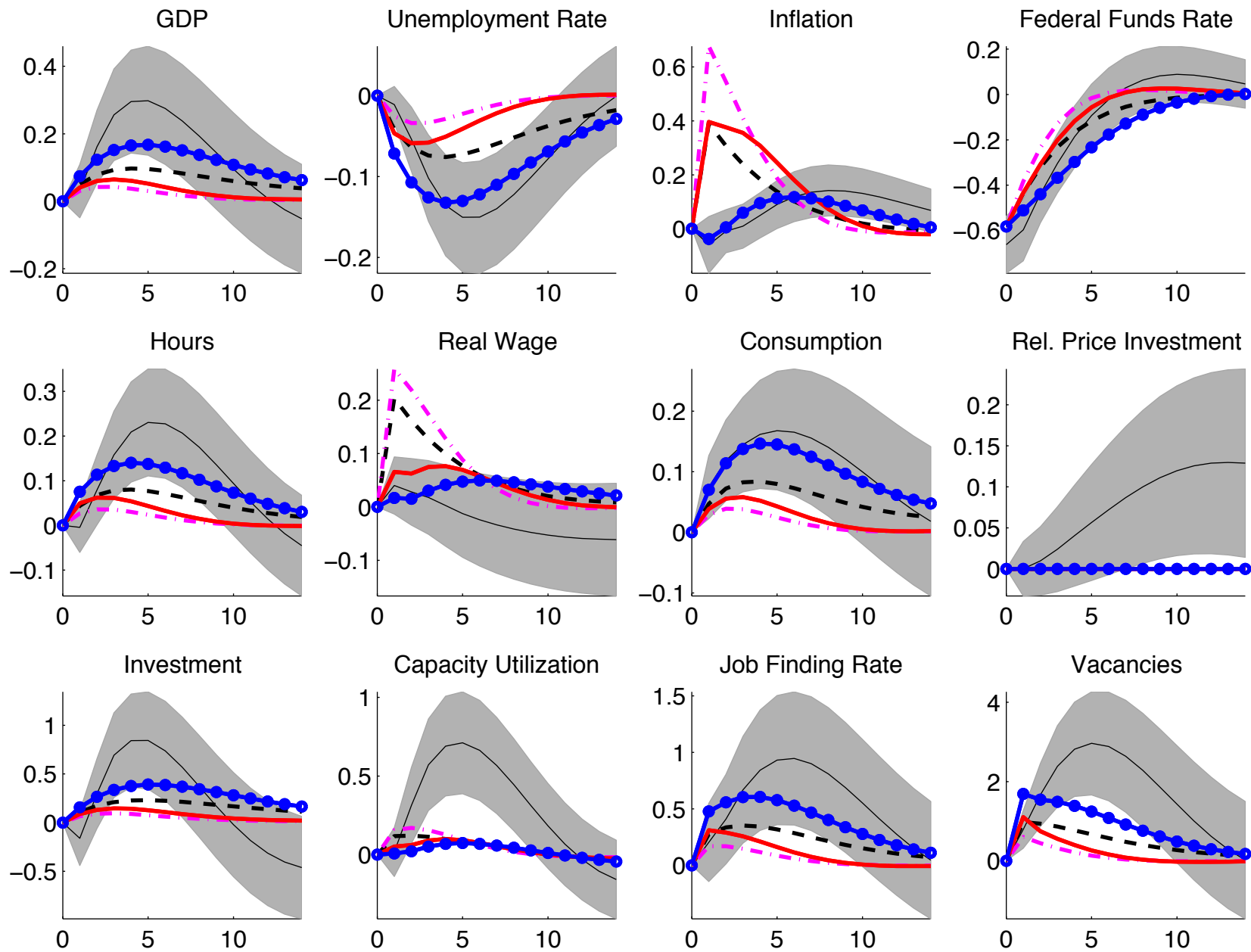Notes: x−axis: quarters, y−axis: percent

# Figure 6: Medium–Sized Model Responses to an Investment Specific Tech. Shock

VAR 95% ▬ VAR Mean − − − Sticky Wages ●—● Alternating Offer Bargaining



Notes: x–axis: quarters, y–axis: percent

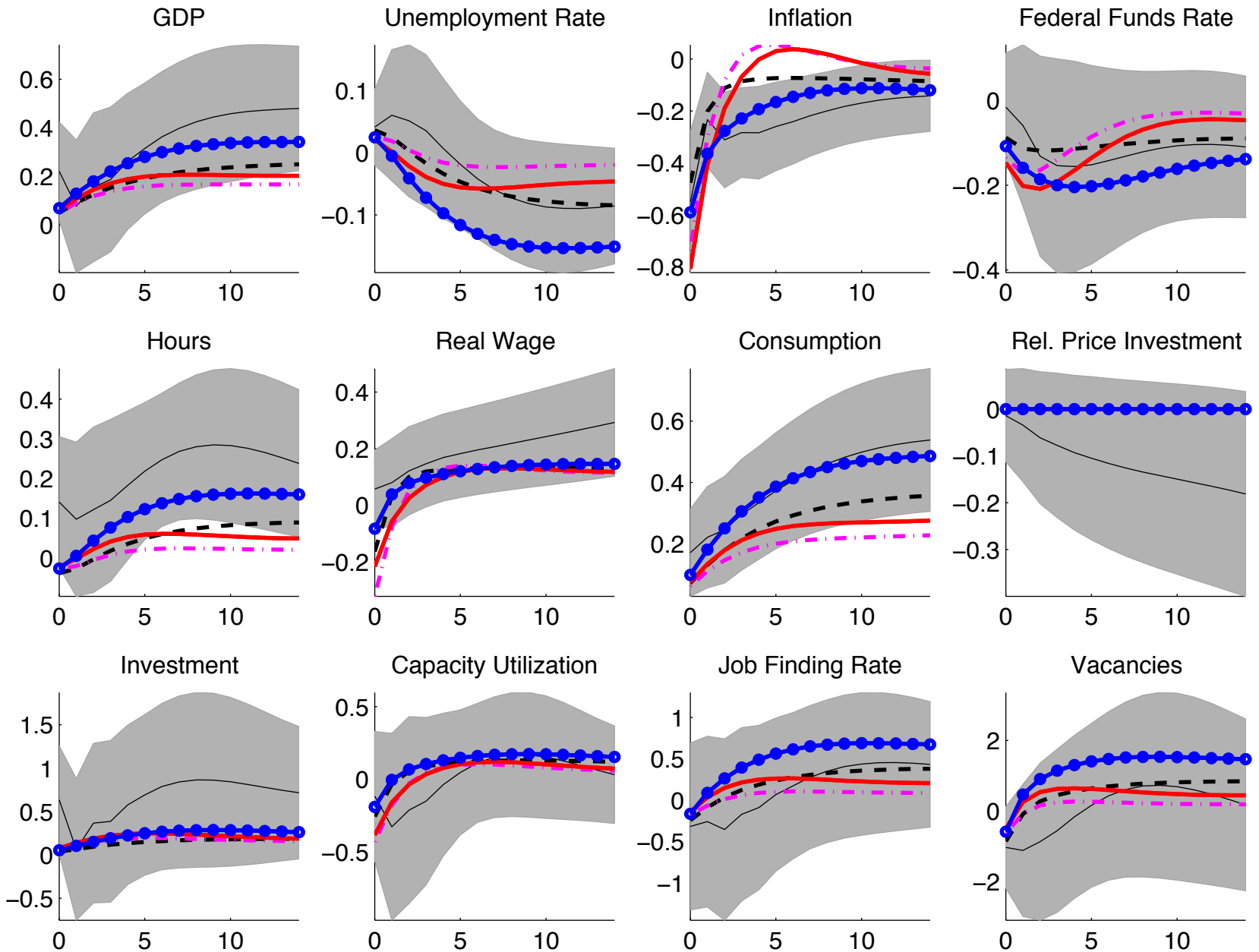Figure 7: Medium–Sized Model Impulse Responses to a Monetary Policy Shock

Legend: VAR 95% · VAR Mean · AltOfferHiring · AltOfferSearch · NashHiring · NashSearch

GDP · Unemployment Rate · Inflation · Federal Funds Rate · Hours · Real Wage · Consumption · Rel. Price Investment · Investment · Capacity Utilization · Job Finding Rate · Vacancies

Notes: x–axis: quarters, y–axis: percent

# Figure 8: Medium−Sized Model Impulse Responses to a Neutral Tech. Shock



Legend: VAR 95% | VAR Mean | AltOfferHiring | AltOfferSearch | NashHiring | NashSearch

Panels: GDP, Unemployment Rate, Inflation, Federal Funds Rate, Hours, Real Wage, Consumption, Rel. Price Investment, Investment, Capacity Utilization, Job Finding Rate, Vacancies

Notes: x−axis: quarters, y−axis: percent

# Figure 9: Medium−Sized Model Responses to an Investment Specific Tech. Shock
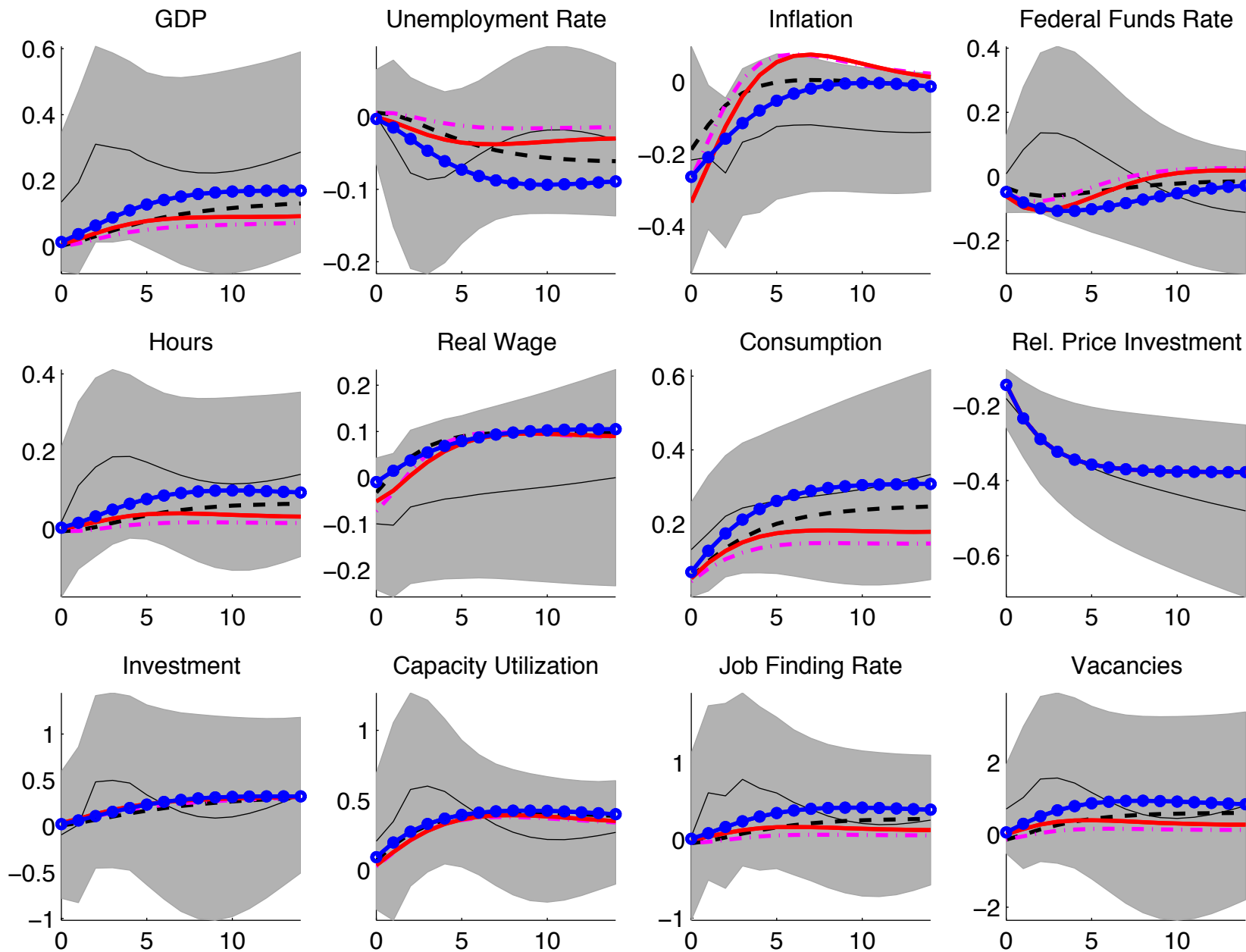
VAR 95% ■ — VAR Mean ● AltOfferHiring ━ AltOfferSearch ╍ NashHiring ╍·╍ NashSearch



Notes: x−axis: quarters, y−axis: percent