Understanding the Fiscal Theory of the Price Level*

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Abstract

We review the fiscal theory of the price level. We place special emphasis on the theory's implications for the feasibility and desirability of price stability.

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1. Introduction

Price stability is an important goal of public policy. To reach this goal, two key questions must be addressed:

- How can price stability be achieved?
- How much price stability is desirable?

Standard monetarist doctrine offers a simple answer to the first question: make sure the central bank has an unwavering commitment to price stability. Recently, a group of economists has begun to rethink the foundations of this doctrine. This has given rise to an alternative view, under which a tough, independent central bank is *not* sufficient to guarantee price stability. Under this view, price stability requires not only an appropriate monetary policy, but also an appropriate fiscal policy.[1] Because fiscal policy receives so much attention in this new view about the determinants of the price level, Michael Woodford has called it the Fiscal Theory of the Price Level (FTPL).[2]

Monetarist doctrine also recognizes that *both* fiscal and monetary policy must be selected in the appropriate way if price stability is to be achieved. However, this doctrine holds that if the central bank is tough, this will automatically *compel* the fiscal authorities to adopt an appropriate fiscal policy.[3] The FTPL denies this. It says that, unless special steps are taken to ensure that appropriate fiscal policies are taken, the goal of price stability may remain elusive regardless of how tough and independent the central bank is.

The FTPL has important implications for the way central banks conduct business. The conventional view has fostered the notion that central bankers should stay away from the fiscal authorities, in order to reduce the likelihood of being pressured into making poor monetary policy decisions. The FTPL has the implication that central bankers with a mandate to foster price stability need to do more than just make sure that their own house is in order. They must also convince the fiscal authority to adopt the appropriate fiscal policy.

[1]Indeed Cochrane (2000) appears to go so far as to say that monetary policy may be essentially *irrelevant* to price determination. He takes this position because, in his view, government-provided transactions assets are a vanishing component of all the financial assets that are traded.

[2]Papers that advocate the FTPL include Benhabib, Schmitt-Grohe and Uribe (2000), Cochrane (1998, 2000), Dupor (2000), Leeper (1991), Sims (1994, 1999), Woodford (1994, 1995, 1996, 1997, 1998a, 1999). For critical reviews of the FTPL, see Buiter (1999), Carlstrom and Fuerst (2000), Kocherlakota and Phelan (1999) and McCallum (1998).

[3]The classic statement is Sargent and Wallace (1981). See especially the last paragraph of their paper.

The FTPL literature also draws attention to the second question, which is both important and difficult. Sims (1999) and Woodford (1998) point out that allowing the price level to fluctuate with unexpected shocks to the government budget constraint produces public finance benefits.[4] For example, if a bad fiscal shock like a war or natural disaster drives up the price level, this is equivalent to taxing holders of the government's nominal liabilities. This tax promotes efficiency to the extent that it permits keeping labor tax rates smooth. In practice, this benefit is likely to be mitigated by whatever distortionary costs may be associated with price instability.[5] Cochrane (1999) is mindful of these costs when, in his analysis, he simply takes it for granted that complete price stability is a fundamental social objective.[6] Sims (1999) claims that the public finance benefits overwhelm the distortionary costs associated with volatile prices, and so he conjectures that complete price stability is non-optimal. A convincing answer to the second question awaits a quantitative study which carefully balances benefits and costs.

This review explains the FTPL and elaborates on its implications for the two questions listed above, as well as for other issues. In the remainder of this introduction, we provide an overview of the analysis. We first discuss the crucial assumption that differentiates the FTPL from the conventional view. Next, we summarize some of the key issues that must be confronted in any assessment of the FTPL. We then briefly describe other issues addressed by the FTPL literature. Finally, we emphasize the connection between the FTPL and the traditional Ramsey literature on optimal monetary and fiscal policy.

What Distinguishes the FTPL?

The difference between the conventional view and the FTPL does not lie in any error of logic.[7] Instead, they differ on how they view a particular equation, the government's intertemporal budget equation. That equation says that the value of the government debt is equal to the present discounted value of future government tax revenues net of expenditures (i.e., surpluses), where both the debt and surpluses are denominated in units of goods. This

[4]For previous discussions, see Chari, Christiano and Kehoe (1994), Judd (1989) and Lucas and Stokey (1983).

[5]See Woodford (1998, pp. 59-60), who elaborates on this point.

[6]Cochrane (1999) emphasizes the need for some type of government security whose payoff fluctuates in the right way with shocks to the government budget constraint, but which does not generate the sort of distortionary costs associated with a fluctuating price level.

[7]There are authors who are concerned about the possibility that FTPL may be logically incoherent (Buiter, 1999). We partially address these concerns, by displaying a class of economic environments in which the FTPL is logically sound.

equation is expressed as follows:

$$\frac{B}{P} = \text{present value of future surpluses}, \tag{1.1}$$

where B is the outstanding nominal debt of the government, and P is the price level. The conventional view holds that this equation is a constraint on the government's tax and expenditure policy.[8] That is, policy must be set so that the right hand side equals the left, whatever the value of P is. According to this view, when something happens to disturb (1.1), the government must alter its expenditures or its taxes to restore equality. Advocates of the FTPL argue that there is nothing inherent to government or to society that requires governments to treat this equation as a constraint on policy. In their view, the intertemporal budget equation is instead an equilibrium condition. According to advocates of the FTPL, when something happens that threatens to disturb this equation, then the market clearing mechanism moves the price level, P, to restore equality.

The assumption that government policy is not calibrated to satisfy the intertemporal budget equation for all P has been called the non-Ricardian assumption by Woodford. Another way of stating this assumption is that if the real value of government debt were to grow explosively, no adjustments to fiscal and monetary policy would be made to keep it in line.[9]

Assessing the FTPL

To evaluate the FTPL, it is useful to focus on the following positive and normative issues. Is the non-Ricardian assumption empirically plausible?[10] Does the FTPL provide a compelling explanation of episodes of high inflation? Does the FTPL provide useful input into the design of socially efficient policies?

[8] In our notion of taxes we include seignorage revenues and taxes on the return to government debt, i.e., default.

[9] Technically, we are exploiting the equivalence between the intertemporal budget equation and a certain transversality condition. This equivalence is discussed later in the text, and in the appendix.

The FTPL does not anticipate that exploding debt would ever be observed. The idea is that as long as there is absolutely no doubt about the government's commitment to not adjusting fiscal policy in the face of an exploding debt, then prices will respond in such a way that the debt will not explode in the first place.

[10] Assessing the empirical plausibility of the non-Ricardian assumption poses a special challenge, because it cannot be done based on time series alone. For further discussion, see section 2.5 below.

It is clear that the non-Ricardian assumption is *not* a good characterization of policy in *all* times and places. Often governments do seem ready to adjust fiscal policy when the debt gets large. One example is the United States in the 1980s and 1990s, when government debt started to increase and there was considerable pressure for some combination of a tax increase and an expenditure decrease to bring the debt back into line.[11] Another example is the Maastricht treaty, in which the members of the European Union formally record their intention to adjust fiscal policy in the event that the debt grows too large. A third example is the International Monetary Fund (IMF). That organization uses an array of sanctions and rewards to encourage member countries to keep their debt in line by suitably adjusting their fiscal policy.

But, for the FTPL to be an interesting positive theory, it is not necessary that it always holds. As emphasized by Woodford (1998a), it may provide a useful characterization of actual policies in some contexts, even if it does not in others. For example, the period of the 1960s and 1970s is a time when the government budget constraint is essentially absent from standard macroeconomic models, and plays little role in Keynesian policy analysis (Sargent (1987, p. 112)). As a result, it is perhaps reasonable to suppose that the non-Ricardian assumption held for that period.[12] Another example is provided by Loyo (1999), who argues that Brazilian policy in the late 1970s and early 1980s was non-Ricardian. Loyo argues that the FTPL provides a compelling explanation of the high inflation experienced by Brazil in this period.[13]

Even if in practice policy has never been non-Ricardian, the FTPL might still be interesting as a normative theory. There are two reasons why this might be so. First, as discussed later, optimal policies might themselves be non-Ricardian.[14] Second, the FTPL could serve as useful input to policy design, even if non-Ricardian policies are in practice, *bad.* To see why these policies might be bad, consider legislators living in a non-Ricardian regime. Understanding that tax cuts or increases in government spending do not necessarily have to be paid by higher taxes later on, they may be tempted to embrace policies which imply too much spending and too much debt. Restricting fiscal policy by placing a limita-

[11] Woodford (1998a) acknowledges that the political reaction to growing debt in the 1980s and 1990s (as well as other considerations) indicates that US policy is probably *not* well characterized as non-Ricardian during the past two decades (see also section 3.4 below). He argues that earlier episodes in US postwar history - for example, the 1965-1979 period - might be characterized in this way.

[12] See Woodford (1998a) for an elaboration on the argument that US policy was non-Ricardian in the 1960s and 1970s.

[13] One does wonder whether it makes sense to assume that a theory, like the FTPL, which focuses on the long-run properties of fiscal policy, holds over some subperiods and not others.

[14] See Sims (1999) and Woodford (1998).

tion on government debt may be an effective way to deal with this kind of problem.[15] By establishing the logical possibility of non-Ricardian policy, the FTPL in effect implies that such policies could occur, in the absence of specific measures to rule them out. As a result, the FTPL can be used to articulate a rationale for the type of debt limitations imposed by the IMF and by the Maastricht treaty.

Other Issues Addressed by the FTPL

Although we stress the implications of the FTPL for the two questions cited above, they are not the exclusive or even primary focus of the literature. The FTPL has plenty to offer, even for people with *no* interest in our two questions. Advocates of the FTPL emphasize the value of their framework for understanding price level determination in settings in which traditional quantity-theoretic reasoning breaks down. This can happen, for example, if the monetary authority adopts a policy of pegging the rate of interest, so that the supply of money responds passively to demand. This case deserves emphasis because interest rate targets are thought to play an important role in monetary policy in practice (Taylor (1993)).[16] Another scenario of potential interest occurs when private transactions involve no use of government-provided money. This paper presents examples to illustrate the interest rate pegging and cashless economy scenarios.

Frank Ramsey and the FTPL

We also use our cashless economy example to point out the parallels between the FTPL and the traditional literature on optimal monetary and fiscal policy inspired by Frank Ramsey (1927) and re-introduced into macroeconomics by Lucas and Stokey (1983).[17] In the Ramsey

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[15] Chari and Kehoe (1999) describe a model in which countries form a monetary union and, absent debt constraints, the result is excessive debt. Woodford (1996) argues that a union without debt constraints is likely to end up with excessive price volatility. His reasoning uses the kind of logic surveyed in this paper. He notes that if policy is non-Ricardian, then fiscal shocks must show up as shocks to the price level, regardless of monetary policy (we call this Woodford's *Really* Unpleasant arithmetic in section 3.3 below). He argues that price level instability arising from this source is likely to be excessive in a monetary union that adopts a non-Ricardian policy. It is precisely because he thinks that a non-Ricardian policy is a realistic possibility - a possibility which he thinks in this case is bad - that he approves of the explicit debt restrictions incorporated into the Maastricht treaty. For another, similar, discussion of the potential dangers of non-Ricardian policy, see Woodford (1998, p. 60).

[16] For a recent analysis of the case in which the interest rate is not pegged, but is allowed to move around with variations in the state of the economy, see Benhabib, Schmitt-Grohe and Uribe (2000).

[17] These parallels are emphasized in Sims (1999) and Woodford (1998).

literature, government 'policy' is a sequence of actions (tax rates, expenditures, etc.) indexed by the date and (in models with uncertainty) by the realized value of shocks. Because these policies are not functions of past prices, there do exist prices such that the debt explodes and households refuse to buy it. Such possibilities are of no concern in the Ramsey literature because the government is viewed as selecting its policy before prices are determined, and it is taken for granted that only equilibrium prices occur.[18] In equilibrium, demand equals supply in all markets, including markets for government debt.[19] We think of non-Ricardian policies as corresponding to the type of policies contemplated in the Ramsey literature.

In the Ramsey literature, there is a concern that policies may not be time-consistent, in the sense that it is not consistent with the government's incentives to actually implement them in real time. We think that the concerns about time consistency in the Ramsey literature may also apply to the FTPL. To see why, consider again the situation in which the price level rises when there is a bad shock to the government budget constraint. In a world like this, private agents may suspect that governments will frequently resort to high prices as an easy way to renege on debt. In this case, the policy would backfire, with agents refusing to accumulate government debt in the first place. To avoid this outcome, it is necessary to credibly convince potential holders of government debt that they will receive subsidies when good things happen to the government constraint.[20] However, there may be times and places where the institutional and other social structures needed to achieve the required degree of credibly do not exist.

The remainder of the paper is organized as follows. Section 2 makes most of our points in a one-period environment. By adopting such a simple setup we are able to get to the basic ideas without technical complications. At the same time, there are some issues that simply

[18]Implicitly, we are taking a particular stand on what constitutes 'government policy' in a Ramsey-optimal policy setting. Strictly speaking, all that Ramsey theory tells us is what government actions are taken in the best equilibrium. There may well be many different government policy rules which result in the same Ramsey equilibrium, where a policy rule specifies government actions as a function of the realization of exogenous and endogenous variables. Different policy rules imply different policy actions out of equilibrium. The distinction between the government's policy rule and the Ramsey equilibrium outcomes is clarified in Woodford (1998,1999). He computes Ramsey equilibrium outcomes and then searches for Taylor (1993)-like interest rate rules which support those outcomes as an equilibrium. In this paper, we take the position that the government's policy actions in a Ramsey equilibrium *are* the government's policy rule.

[19]In Ramsey theory and in the FTPL, there exist market prices where government policy commits it to doing infeasible things. For example, there may be prices where it commits to paying for goods with money financed from new debt issues which no one buys. By focusing on equilibrium prices only, standard practice ignores these possibilities. Bassetto (2000) argues that this is a mistake, and proposes alternative equilibrium concepts to deal with the problem.

[20]See Chari, Christiano and Kehoe (1991) for a detailed analysis.

cannot be discussed in a one period environment. We defer these to section 3. Section 4 presents a simple model for thinking about the desirability of price fluctuations under the FTPL in an environment with no government-provided money. Section 5 provides a summing-up.

2. A One-Period Economy

As noted in the introduction, the key defining characteristic of the FTPL is the non-Ricardian assumption on fiscal policy. The best way to understand this assumption and quickly get to the heart of the FTPL is to go to a one period model.[21] That's what we do here.

We begin with the conventional wisdom, the classic Sargent and Wallace (1981) (SW) analysis. We then explain how the FTPL differs from this conventional wisdom: the SW analysis adopts a Ricardian view and the FTPL adopts the non-Ricardian view about policy. We explore various interpretations of these assumptions and conclude that the non-Ricardian assumption requires that the government is able to commit in advance to its policy actions. We then ask how we can go about assessing the empirical plausibility of the non-Ricardian assumption. In the final subsection, we explain how the FTPL can be used to study the price level in a world where government provided fiat money has vanished.

2.1. Sargent and Wallace's Unpleasant Monetarist Arithmetic

Suppose that in the morning of the only day in this model, private agents are holding a given amount of government debt, b. Here and throughout this paper, we assume that government debt is non-negative: agents cannot borrow from the government. In the SW model, the debt is fixed in real terms. It is a commitment to pay a fixed real amount of goods, say corn.

Here is the government's budget constraint:

$$b' + s^f + s^m = b.$$

The left and right side of this equation summarizes the sources and uses, respectively, of corn to the government. The first source of funds, b', is corn the government receives from any households who purchase new debt in the evening. The second term, s^f, is taxes net of spending, and the third term is seignorage arising from government supplied fiat currency. The term, b, on the right side of the budget constraint is the principle and interest on the past government debt.

[21] We are grateful to Marco Bassetto for suggesting this to us.

Optimizing households will obviously never choose $b' > 0$ and they are constrained from setting $b' < 0$ by assumption. So, household optimization implies that b' must be zero. Imposing this, we get the intertemporal government budget equation:

$$b = s^f + s^m. \tag{2.1}$$

The main conclusions of the SW analysis may be understood from this equation. Suppose a 'loose' fiscal policy is adopted, i.e., s^f is reduced. Then, simple arithmetic dictates that the monetary authority must increase s^m. Under normal circumstances, this translates into an increase in inflation.[22] In a multiperiod model there is some discretion about timing. The rise in inflation can occur later, sooner, or it could be spread out over time. But, whatever the timing, if the fiscal authority reduces s^f, the arithmetic drives one to the view that inflation must at some point go up. This is the source of the famous title to the Sargent and Wallace paper, 'Some Unpleasant Monetarist Arithmetic'.

The same arithmetic suggests a solution to the inflation problem: design central banks so that they can credibly commit to not 'caving in' to an irresponsible fiscal authority who sets s^f low. Governments around the world have sought to implement this by making central banks independent and directing them to assign a high priority to inflation. The notion is that with the monetary authority completely committed to a fixed value for s^m, the arithmetic forces the fiscal authority to adopt the fiscal policy consistent with *it*. This is the basis for the current conventional view that to achieve a stable price level, it is sufficient to have a tough, independent central bank which is focused on prices.

2.2. The Fiscal Theory

According to advocates of the FTPL the SW framework may not be the one that is relevant for economies like the US. In practice government debt is a commitment to deliver a certain amount of the domestic *currency*, not goods. This creates new possibilities.

[22]By normal, we mean that the economy is on the 'right' side of the Laffer curve. Here is a brief explanation. Seignorage is the nominal increase in the stock of money, divided by the price level. A convenient formula for this becomes available if we temporarily reinterpret our model as a multiperiod model in a steady state. Let the demand for real balances be $m = \exp(-\alpha\pi)$, where $\alpha > 0$, π is the gross inflation rate from this period to the next, and m is the stock of money, divided by the price level. Then, seignorage is just $m(1 - 1/\pi) = \exp(-\alpha\pi)(1 - 1/\pi)$. For inflation rates below $\pi^* = (\alpha - \sqrt{\alpha^2 + 4\alpha})/(2\alpha)$, seignorage is increasing in inflation, and for inflation rates above this point, seignorage is decreasing. The 'right' side of the Laffer curve refers to inflation rates below π^*.

Let's redo the previous analysis with b replaced by B, nominal debt. This is the government's budget constraint:

$$B' + P\left(s^f + s^m\right) = B. \tag{2.2}$$

As before, optimizing households will not buy any government debt in the evening. With demand at zero, the only equilibrium can be at $B' = 0$. Equation (2.2) with $B' = 0$ is the government's intertemporal budget equation:

$$B = P\left(s^f + s^m\right). \tag{2.3}$$

Now, P is an endogenous variable. If the fiscal authorities make s^f small, there is now no arithmetic that compels the monetary authorities to raise s^m. If the monetary authorities hold fast to s^m while the fiscal authorities reduce s^f, then this equation can continue to be satisfied as long as P jumps. This is what advocates of the FTPL expect would happen.

2.3. Interpreting Ricardian and non-Ricardian Fiscal Policy

At this point, we clarify two key concepts. Fiscal and monetary policy are said to be *non-Ricardian* if $s \equiv s^f + s^m$ is chosen in a way that does not guarantee that the intertemporal budget equation, (2.3), is satisfied for all possible prices. In contrast, s is a *Ricardian* fiscal policy if it is chosen in a way that guarantees that the intertemporal budget equation is satisfied no matter what P is realized. In our single period model, the only way this can happen is if s is a particular function of the price level, $s(P) = B/P$. The assumption that fiscal and monetary policy are non-Ricardian is what defines the FTPL.

How are we to interpret non-Ricardian policy? In principle, two interpretations are possible. The first one may seem like the natural one initially. However, on further reflection it makes no sense. Under this interpretation, the government is simply unconcerned about the intertemporal budget equation when it chooses s. This could reflect either that it is not aware of its existence, or it simply does not care. If the government were completely unconcerned with intertemporal budget balance, then it is impossible to understand why we have any taxes. Absent concerns that stem from the existence of the intertemporal budget equation, borrowing is always a more appealing option than raising taxes because the latter produce deadweight losses. But, if governments didn't raise taxes, s would be negative and there would be no positive P that satisfies the intertemporal budget equation. So, if we adopt this interpretation of non-Ricardian policy the apparent existence of equilibrium is a puzzle. This interpretation deserves no further consideration.

Is it possible to reconcile the notion that the government cares about intertemporal budget balance with the notion that s is set exogenously, and not as a function of P? The answer is 'yes', if we imagine that the government commits itself to s in advance, before P is determined. We illustrate this in two ways. The first is based on the parable of the Walrasian auctioneer, who helps the economy find the equilibrium price level. Under the non-Ricardian assumption, the government announces s before the Walrasian auctioneer conducts the auction to find the market-clearing price level. When the government selects s, it understands fully that households will buy zero B' in equilibrium. However, because of its first mover advantage, the government knows that it can in effect force the Walrasian auctioneer to pick P so that $P = B/s$.

Our second illustration is based on an analogy drawn from everyday life. A pedestrian at a cross-walk who wants automobile traffic to stop will sometimes step onto the street making a show of being unconcerned about the oncoming cars. Is it that such a person really doesn't care about the prospect of being struck and killed? Of course not. They expect that oncoming automobiles, seeing the commitment to cross regardless of consequences, will stop rather than suffer the horror of an accident. Under a non-Ricardian fiscal policy, the government adopts an approach analogous to that of the pedestrian. The government's 'policy' is simply an action, s. In principle, a P could occur which would put the government in the fiscally explosive situation where it offers debt that the market refuses to absorb, i.e., where $B' > 0$. However, if the market is completely convinced of the government's commitment to s then, like the automobile that stops for the pedestrian, the market will generate a P which guarantees that debt is not excessive (in this case, 'excessive' simply means greater than zero). The non-Ricardian government is banking on the idea that the market abhors non-equilibrium P as much as drivers abhor hitting pedestrians.[23]

Although the use of the word 'commitment' in this context is consistent with the technical economics literature, it may nevertheless generate confusion because it has various meanings in everyday language. By saying that the government has commitment, we mean only that it 'moves first', before prices are set. We do not necessarily mean to imply that the government's

[23] As the analogy suggests, there is a potential for trouble if commitment is not credible. If the oncoming traffic is not completely convinced about the pedestrian's commitment (say, drivers believe the pedestrian is sneaking glances at the oncoming traffic, ready to make adjustments in case something goes wrong), then miscalculations can lead to tragic collision. We argue (see especially section 4.4 below) that something like this is possible if private agents are not completely convinced of the government's commitment to a non-Ricardian fiscal policy. In this case, markets might produce the 'wrong' prices, leading to excessive government debt in the sense that the private sector refuses to purchase it. This seems to be the kind of outcome that concerns Buiter (1999). He refers to the 'painful' fiscal adjustments that must be made when the 'Ricardian reality dawns' and the private sector refuses to buy government debt. We do not mean to suggest that catastrophe will *always* occur if there is uncertainty about government policy. As section 3.3 shows, the non-Ricardian assumption is perfectly consistent with a stochastic fiscal policy.

motives are laudable, or that its ability to move first reflects strength of character on the part of government policy makers. For example, a government that is perpetually in gridlock because legislators cannot reach agreement acts with commitment in our usage of the term.[24]

Now consider Ricardian policies. For the purpose of our analysis, we do not need to take a position on the relationship between these policies and the government's ability to commit. Still, we suspect that Ricardian policies are consistent with any degree of commitment.

Although they don't use this language, it seems fair to say that SW adopt a Ricardian specification of policy. To see this, note first that if we are to think of their analysis as applying to a realistic modern economy, then we *must* think of the real government debt, b, in their model (i.e., b in (2.1)) as B/P. With different values of P, the value of b changes, leading to adjustments in $s^m + s^f$ under the SW analysis. This is why we interpret SW as adopting the assumption that policy is Ricardian.

2.4. Is the FTPL *Sensible*? An Analogy Between the FTPL and Microsoft

Under the FTPL the price level is determined by (2.3) or (1.1). A reasonable question at this point is, 'does this have any sensible interpretation?' At first glance, this way of determining P may seem like some sort of accounting gimmickry without any substantive interest. But, this is not the case. As emphasized in Cochrane (2000), the price of Microsoft shares is determined in this way too! Under the FTPL, the relationship of the government to its bond holders is somewhat like that between Microsoft and its equity holders.

The managers of Microsoft also work to set aside real output for equity holders. Microsoft's motives for doing this are different than the government's. But, that is not the point. The point is that Microsoft does not calibrate its dividend stream to guarantee that the present value formula for its stock price holds, for all possible values of the stock price. Instead, the mechanism operates in precisely the opposite direction. Market traders forecast what Microsoft is going to generate for them, they then calculate the ratio of that to the number of shares outstanding, and that's the stock price! Advocates of the FTPL argue that the determination of the price level in an actual economy works in exactly the same way. The government does not calibrate s to ensure that its present value budget equation, (2.3) holds for all P. Instead, bond holders figure out how many goods, s, the government is setting aside for them and calculate the price level as the ratio of B to s.

[24] In private communication, Christopher Sims pointed out that our notion of 'commitment' encompasses '...the commitment of two pedestrians who enter a crosswalk while engaged in a fist fight. Few doubt that they are not watching traffic'.

2.5. Is the non-Ricardian Assumption Empirically Plausible?

In assessing the FTPL as a positive theory for a particular time period, a key issue to confront is the plausibility of the non-Ricardian assumption. A simple examination of the time series data won't help. Under both the non-Ricardian and the Ricardian assumptions, we expect to see $s = B/P$. The only direct way to distinguish between the two assumptions is to see what s looks like when the economy is out of equilibrium. According to the Ricardian assumption, s would adjust with P to preserve $s = B/P$. According to the non-Ricardian assumption s is like a utility function parameter. Its value would remain unchanged, so that s would not equal B/P out of equilibrium. This may sound like an easy thing to check: just compare s and B/P out of equilibrium. The problem is that, according to the theories considered here, only equilibrium values of s are recorded in the data.[25]

We don't think this means there is *no* way to choose between the non-Ricardian and the Ricardian assumptions. In fact, we think there are two ways to go. One is to try to extrapolate what is reasonable out of equilibrium behavior based on what we see in equilibrium.[26] Another way is to view the FTPL as a starting point for a natural set of auxiliary assumptions which do restrict time series data, and then test those assumptions.[27] If the non-Ricardian assumption leads us to a useful set of theories in this way, this would tip the balance in favor of that assumption. We now discuss these two approaches.

Extrapolating Out of Equilibrium From Equilibrium

According to the non-Ricardian assumption, the government's policy is a commitment to a particular action, s. Under the Ricardian assumption, policy is a strategy for setting s as a function of the real debt. If governments directly recorded in writing what their policy is, this would help us to discriminate between the two assumptions. There are two examples where this seems to have happened, and, with one important caveat, the results appear to favor the Ricardian over the non-Ricardian assumption. The Maastricht treaty requires the members of the European Union to have a policy of adjusting their fiscal variables

[25] It is perhaps obvious that this result continues to hold, even if there are multiple periods and there is uncertainty.

[26] There are examples of models in which the equilibrium time series data contain information about what happens out of equilibrium. For example, in Green and Porter (1984) limited information has the consequence that things happen in equilibrium which are observationally equivalent to agents' having deviated from the equilibrium. Even though agents don't actually deviate in the equilibrium, they must nevertheless be punished as though they had deviated as a credible signal to all of what would happen if a deviation really did occur. In this sense, the events in equilibrium provide evidence of what would happen out of equilibrium.

[27] For a thorough discussion of this strategy, see Woodford (1998a).

in case their real debt should get large. The IMF works in the same way, by bringing pressure to bear on its members to adjust fiscal variables if their real debt gets out of hand. We think it is fair to say that if somehow a non-equilibrium P were called out, these two institutional arrangements would work to generate an adjustment in s. Casual examination of the (admittedly, equilibrium!) time series data suggests the same. In practice, when the size of the debt gets large, political pressures tend to come into play to adjust the surplus to bring the debt back into line. This happened in the US in the late 1980s and 1990s, when the federal debt began to grow significantly, producing political support for raising taxes and/or reducing spending.

Now, for the caveat mentioned in the previous paragraph. The examples cited do suggest that the non-Ricardian assumption may be implausible as a characterization of current policy in Europe, the US and some emerging market economies. However, as emphasized in the introduction, they do *not* establish that the non-Ricardian assumption is implausible for *all* times and places.

Is the non-Ricardian Assumption a Good Starting Point?

A different approach to assessing the empirical value of the non-Ricardian assumption asks how good a platform it is for developing useful testable restrictions. There is no space to pursue this here, beyond mentioning that there is interesting work underway. In particular, Canzoneri, Cumby and Diba (1997), Cochrane (1998,1999), Loyo (1999) and Woodford (1998a) have pursued the assumption of statistical exogeneity of the government surplus. This is not an implication of the non-Ricardian assumption *per se*, but that assumption does naturally lead one to it.

This approach can perhaps best be understood using an analogy attributed by Cochrane (1998) to Benjamin Friedman. Consider the equation of exchange,

$$MV = PY,$$

where M is money, V is velocity, and Y is output. This equation as it stands has no testable implications, since without additional assumptions it is just a definition of V. Still, if incorporating simple, plausible, assumptions converts it into a theory that helps us understand data better, then we say the equation of exchange is empirically useful.[28] In a similar way, it may turn out that the non-Ricardian assumption is a good starting point for identifying simple auxiliary assumptions that convert the FTPL into a useful, testable theory. If so, this would help vindicate the non-Ricardian assumption as a useful empirical assumption.

[28] An example of such an assumption is the specification that V has a simple functional relationship to the nominal rate of interest.

Although our inclination is to be skeptical of the non-Ricardian assumption, the FTPL is still very much in its infancy. It remains to be seen where the FTPL takes us and what observations it helps us to explain. The initial results are promising, though not uncontroversial. Cochrane (1998,1999) and Woodford (1998a) argue that the version of the FTPL that assumes a statistically exogenous surplus process is helpful for understanding the dynamics of US inflation in the 1970s, and Loyo (1999) argues that it is useful for understanding the high inflation experienced by Brazil in the 1980s.

There is another literature, started by Calvo (1978), Kydland and Prescott (1977) and Barro and Gordon (1983), which posits that the absence of commitment in government policy can account for the high inflation episodes mentioned in the previous paragraph. One way to assess the FTPL is to compare its ability to account for experiences like these with that of the time consistency literature. It is not obvious what the outcome of this comparison would be. McCallum (1997), among others, has argued that time inconsistency is *not* a useful explanation of episodes of high inflation. Ireland (1998) has argued the other way, that absence of commitment *is* useful.[29]

2.6. The Price Level in a World with No Government-Provided Money

An important virtue that some advocates of the FTPL claim for their theory is that it provides a way of thinking about the price level that works even in a world where the demand and supply for government fiat money is nonexistent. Cochrane (1998, 2000) argues that this is of interest because, to a first approximation, we have already reached that point.[30]

The basic pieces of the argument are already in place. Under the non-Ricardian assumption that $s^f + s^m$ is exogenous, (2.3) determines the price level. Note that this conclusion was reached without reference to money or whether it is even present in the economy. That's the tip off for the result to come: the price level can be pinned down, even if there is *no* government provided money in the economy.

To see this, imagine that trade in the economy is carried out by barter. Equivalently, one could think of a scenario in which trades are financed with the exchange of financial claims on privately held assets. These trades could even be denominated in 'dollars'. The key thing

[29] For further discussion, see Albanesi, Chari and Christiano (1999) and Christiano and Gust (2000a,b).

[30] According to Woodford (1998, 1998a, 1998b), the assumption that money demand and supply are literally nonexistent is too extreme. He prefers to analyze what he calls a 'cashless limit'. This is an economy in which the demand for money is so small that seignorage is negligible and can, to a first approximation, be safely ignored in the government's budget constraint (both the flow budget constraint and the intertemporal budget equation). However, the demand for money is sufficiently large that the central bank can still control the rate of interest.

here is that government-provided money ('dollars') simply does not exist.

What is nominal, dollar denominated government debt in this world with no dollars? It obviously is not a pledge to deliver government-provided money, because there is none! Instead, it is a pledge to deliver B dollars worth of goods to the bearer of B. The formal obligation leaves open exactly how many goods B dollars corresponds to, because the price level is left unspecified. In this sense, it is like real-world US government debt.[31] The price level that *is* realized is determined by the government's fiscal decisions. The fiscal decisions result in the real surplus, s, which is what the government actually has available for paying off the bond holders. With the amount of goods available to pay the bond holders equal to s, and the nominal value of debt equal to B, the natural definition of the price level is $P = B/s$.

At this point, it may seem like the price level in a world with no government-provided money is a useless appendage, with no meaningful function. In section 4 below we shall see that the price level in such an economy does have a potentially important role to play, helping to implement an efficient fiscal policy.

3. The Fiscal Theory In General Equilibrium

This section addresses various issues which we cannot address in the one period example. The first issue we confront is one that we expect any reader who makes it through the previous section to be concerned about. That section shows how an equation not usually used in the context of price determination, the government's intertemporal budget equation, can pin down the price level. This naturally leads to the question: 'But, we already *have* a theory of the price level. If we adopt the non-Ricardian assumption on policy won't the price level be *overdetermined*?' Well, it might be. That depends on how we flesh out the rest of the economy. If the price level were overdetermined, this would imply that there is no equilibrium except in the fortuitous case where the government happened to pick just the right value for s. It is fair to say that if this were the situation for all reasonable ways of modeling the rest of the economy, then the FTPL would be in trouble. In this case, it would not be a logically coherent macroeconomic model. But, this turns out not to be the case. Below, we will flesh out the rest of the economy in what appears to be a reasonable way, and we will see that the price level is uniquely determined. We do this in the context of a very simple example. The issue is addressed more rigorously in the appendix.

[31] The US government also offers indexed debt, which does correspond to a commitment to deliver a specific amount of a basket of goods. Indexed debt is small in the government's portfolio of liabilities.

We then turn to an issue which we think is of greater concern. We display evidence which suggests that to use the FTPL one has to take the non-Ricardian assumption *very* seriously. Seemingly tiny departures from that assumption in the direction of allowing for some sensitivity in the surplus to the real debt causes the FTPL's ability to pin down the price level to collapse.

The third subsection shows how shocks to fiscal policy impact on the price level in the FTPL. We show that it is possible for the monetary authority to control the average inflation rate in this case. However, according to an important result due to Woodford, it is impossible for the central bank to set the variance of inflation to zero when there is instability in fiscal policy.

The final subsection revisits the ability of the central bank to control average inflation under the FTPL. It shows that conventional views about how to control average inflation could actually spark an explosive hyperinflation. So, while it is feasible for the central bank to control average inflation under the FTPL, the method for doing so must be designed with care.

3.1. Is the Price Level Overdetermined in the FTPL?

We begin this subsection by providing a general discussion of the issues involved in determining the price level. We then turn to a specific example in which the price level is uniquely determined by the FTPL.

General Discussion

It is easy to find examples of the FTPL where the price level is overdetermined. Recall the equation of exchange, which was discussed above. We reproduce it here, for convenience:

$$MV = PY.$$

In traditional, old-fashioned monetarism, V is assumed to be a number fixed by technology, Y is determined exogenously, and monetary policy takes the form of a choice of M. In this model, P is obviously determined by the equation of exchange. If the rest of the economy were characterized by these assumptions, then a logically coherent FTPL would be impossible.[32]

[32]Finite horizon models in which the simple quantity equation holds in the last period, but velocity is an increasing function of the nominal interest rate in the previous periods, also have the property that the price level is overdetermined (see Buiter (1999).) The reasoning is similar to what we have in the text. In those models, the equilibrium price level must satisfy a first order difference equation. All the equilibrium prices are then pinned down by the fact that the price level is pinned down in the last period. The likelihood that this price level coincides with the one produced by the intertemporal budget equation of the government seems small. The most likely outcome is that the price level is overdetermined in this model.

But, each of the assumptions of traditional, old-fashioned monetarism have been rejected on empirical grounds. First, V exhibits very substantial fluctuations in the data. The assumption that V is fixed has been replaced in modern models by an increasing function of the nominal rate of interest. Another property of the modern theory is that expected inflation plays an important role in determining R. With these two features, it can be shown that a logically coherent FTPL is possible. To see this, note that these changes cause expected future P's to enter the equation of exchange via V. This creates the possibility that there are many P processes that can satisfy this equation, leaving room for the non-Ricardian assumption to pin down one of them. This possibility is illustrated in a completely worked example in the appendix.[33]

Second, the assumption that Y is exogenous has been questioned. There is general agreement that, at least the short run movements in Y are influenced by movements in V, P and M. When models are constructed that capture this, one finds that expected future P's enter into the determination of Y. As in the example of the previous paragraph, the result can be that there are multiple P processes that can satisfy the equation of exchange. Again, this leaves room for the non-Ricardian assumption to pin down one of them.[34]

Third, there is a nearly universal consensus that exogenous M represents a poor characterization of monetary policy. For example, Taylor (1993) has argued that, in practice, monetary policy is best thought of as a rule for setting the rate of interest. In this case, M becomes an endogenous variable. We can see in the equation of exchange that if R is the exogenous policy variable, as opposed to M, then V is pinned down. But, there are now *two* endogenous variables, M and P, in this equation. Generally, under these circumstances, P and M are not pinned down. There is, in a sense, a missing equation. Again, there is room for the FTPL to fit in.

An Example

Next, we present a simple, multiperiod model economy in which the price level is uniquely determined in the FTPL. There is no last period, and time is indexed by $t = 0, 1, 2, \ldots$.

[33]Examples like this are also presented in Brock (1975), Obstfeld and Rogoff (1983) and Matsuyama (1991). An example taken from Woodford (1994) appears in the appendix to this paper.

[34]The cash in advance model displayed in Christiano, Eichenbaum and Evans (1998) is an example of this. Because this is a cash in advance model, velocity is fixed and factors discussed in the previous paragraph are ruled out. That paper shows that for various different specifications of the monetary policy rule for selecting M_t, the model has a continuum of equilibria. If the non-Ricardian assumption were adopted in this model, the equililbrium would be pinned down. For other examples like this, see Carlstrom and Fuerst (1998).

Suppose that output, Y, is the same for each date, t. Money demand depends on the rate of interest:

$$\frac{M_t}{P_t} = AR_t^{-\alpha},\ \alpha > 0. \tag{3.1}$$

The parameter, A, captures other things (like income) which also impact on money demand, but which are here assumed to be constant. Also, M_t is the stock of money at the beginning of period t, P_t is the price level during period t and R_t is the nominal rate of interest on government bonds held from the beginning of period t to the beginning of period $t+1$. The Fisher equation holds:

$$1 + r = (1 + R_t)\frac{P_t}{P_{t+1}}. \tag{3.2}$$

The expression on the right is the real rate of interest on bonds paying nominal rate of return, R_t. The term, $r > 0$, is the rate at which households discount future utility. This pins down the real rate of interest.

A reasonable specification of monetary policy is that the central bank targets the nominal rate of interest. For purposes of the exposition, we adopt an extreme version of this, in which it pegs the rate of interest to a constant, $R > 0$. As we will see below, it accomplish this by supplying whatever money the private economy demands at this rate of interest. The interest rate peg pins down seignorage:

$$s_t^m \equiv \frac{M_t - M_{t-1}}{P_t} = \frac{M_t}{P_t} - \frac{P_{t-1}}{P_t}\frac{M_{t-1}}{P_{t-1}}.$$

Imposing the money demand and Fisher equations, (3.1) - (3.2), and the policy rule, $R_t = R$, we find[35]

$$s^m = AR^{-\alpha}\frac{R - r}{1 + R},\ t = 0, 1, 2, \ldots\ . \tag{3.3}$$

Consistent with the FTPL, we assume that the primary budget surplus, s_t^f, is non-Ricardian. We adopt the simplest such policy, one in which s_t^f is simply a constant, s^f. So, net government revenues from all sources, excluding interest payments, is

$$s_t = s = s^f + s^m > 0. \tag{3.4}$$

To complete the description of the government, we present the period t budget constraint. We assume that the government debt is composed of one-period discount bonds. That is,

[35]Implicitly, we have assumed that the interest rate peg was in place in period -1 too. This is done for simplicity. A more rigorous treatment, which does not make this assumption, appears in the appendix.

the amount of borrowing in period t is $B_{t+1}/(1+R)$ and the amount paid in period $t+1$ is B_{t+1}. The period t government budget constraint is:

$$\frac{B_{t+1}}{1+R} + P_t s = B_t, \ t = 0, 1, 2, ... \tag{3.5}$$

The terms on the left of the equality are the government's sources of funds and the terms on the right are the uses of funds to pay off the debt.[36] It is convenient to rewrite this expression in real terms, i.e., in terms of $b_t \equiv B_t/P_t$. Dividing (3.5) by P_t, taking into account the Fisher equation, (3.2), and rearranging, we obtain:

$$b_{t+1} = (1+r)(b_t - s). \tag{3.6}$$

Finally, we develop the multiperiod analog of $B' = 0$ in the previous section. Recall the logic that we used there. First, $B' > 0$ is not optimal since households could increase utility by raising consumption and financing it with a reduction of B'. Second, a negative value of B' is also not optimal, since we removed it from the feasible set by assumption. We continue to assume that holdings of government bonds must be non-negative. That is, households only lend to the government. They do not borrow from it.

The analog of $B' = 0$ in this setting is

$$\lim_{T \to \infty} \frac{B_T}{(1+R)^T} = 0. \tag{3.7}$$

That household optimization implies this condition is established using the same type of reasoning used to establish $B' = 0$. The limit cannot be positive, for otherwise households could increase utility by reducing their holdings of government debt. To see this, suppose that the limit *is* positive. This implies that, eventually at least, the government debt grows at the rate of interest. That is, $B_t = B_{t^*}(1+R)^{t-t^*}, \ t \geq t^*$ for some t^*. But, at this point, the government is engaged in a Ponzi scheme with households. The principle and interest on debt coming due is financed entirely out of newly issued debt, forever. Under these circumstances, the household can do better by simply saying 'no' to the Ponzi game. It can do so by consuming the principle and interest on debt coming due in one period and then never again holding any more government debt. The household is better off doing this because the action allows it a one-time increase in consumption without generating a need to reduce consumption at any other date. An optimizing household would not pass up

[36] An alternative representation, which has some theoretical advantages, expresses the government's budget equation in terms of its total nominal liabilities, $B_t + M_{t-1}$. We work with this alternative representation in the appendix.

an opportunity like this. This is why household optimization implies that the above limit cannot be positive. The limit cannot be negative either, because we don't allow $B_t < 0$. Condition (3.7) is called the transversality condition.

It is convenient for us to express the transversality condition in real terms, after substituting out for the nominal rate of interest from the Fisher equation, (3.2). Using that equation, we find[37]

$$(1+R)^t = (1+r)^t \frac{P_t}{P_0},\ t = 1, 2,\ \ldots\ ,$$

so that $B_T/(1+R)^T = P_0 b_T/(1+r)^T$. Then, the transversality condition can be written

$$\lim_{T \to \infty} \frac{b_T}{(1+r)^T} = 0,\ b_T = \frac{B_T}{P_T}. \tag{3.8}$$

We have now stated the entire model. The part coming from the household is given by (3.1), (3.2), (3.8) and the condition, $B_t \geq 0$. The government is summarized by its policy, (3.4), and its flow budget constraint, (3.6). We ask, does this economy uniquely determine the price level? To see that it does, note first that the money demand equation and the government policy of pegging the rate of interest have the effect of pinning down real balances, but not M or P separately. Double M and P, and those equations remained satisfied. The same is true of the Fisher equation. Double P at all dates, and that continues to hold too. So, the level of the money stock and the price level are not pinned down. It turns out that the non-Ricardian specification of government policy, together with the household's transversality condition are enough to pin down the price level uniquely.

To see that the price level is uniquely determined, consider Figure 1, which graphs the government budget equation, $b' = (1+r)(b-s)$. The vertical axis measures b' and the horizontal axis measures b. The 45 degree line is also included in the figure for convenience. Note how the intercept for the budget equation is negative, and how it cuts the 45 degree line from below. Its slope is steeper than that of the 45 degree line because of our assumption, $r > 0$. The diagram can be used to see what happens to b over time for any initial value of

[37]For example, for $t = 2$

$$(1+R)^2 = \left[(1+R)\frac{P_1}{P_2}\right]\left[(1+R)\frac{P_0}{P_1}\right]\frac{P_2}{P_0} = (1+r)^2 \frac{P_2}{P_0}.$$

b.[38] Denote the value of b where the budget equation intersects the 45 degree line by b^* :

$$b^* = \frac{1+r}{r} s = \sum_{t=0}^{\infty} \frac{s}{(1+r)^t}. \tag{3.9}$$

As the last equality indicates, b^* is the present value of the future surpluses.

Now, the value of b in period 0, b_0, is an endogenous variable. Although the nominal debt, B_0, is predetermined at date 0, the price level is not. Consider three possibilities. Suppose $0 \leq b_0 < b^*$. The figure indicates that b quickly spirals into the negative zone, violating non-negativity constraint on the household's holdings of debt.[39] Now consider $b_0 > b^*$. In this case, the diagram indicates that the debt diverges to plus infinity. To see what the growth rate of the debt converges to, divide (3.6) by b_t :

$$\frac{b_{t+1}}{b_t} = (1+r)(1 - \frac{s}{b_t}).$$

Since, as b_t grows, s becomes relatively small, the growth rate of b_t eventually converges to $1 + r$. At this point, the debt has become so large that s is, by comparison, insignificantly small. Essentially, the government is now running a Ponzi scheme. For the reasons given above, it is not in the household's interest to participate in this scheme (technically, the household's transversality condition, (3.8), is violated). Since the households will not hold this debt, we conclude that $b_0 > b^*$ do not correspond to equilibria.

This leaves only $b_0 = b^*$ to consider. Since the level of real debt is fixed in this case, the transversality condition is now trivially satisfied. So, only $P_0 = B_0/b^*$ is consistent with equilibrium. We conclude that this version of the FTPL is an internally consistent theory of the price level.

3.2. Is the FTPL *Fragile*?

The assumptions underlying economists' theories are, at best, only approximations. We don't think of them as being exactly true. This is why we trust theories more if we have reason to believe their central implications will not change if we alter the assumptions a little. But, if key implications evaporate with small changes, particularly changes that are

[38] To see this, specify an initial value for b on the horizontal axis. Then, do the following: go in a vertical direction to the budget line; move horizontally to the 45 degree line; move vertically to the budget line, and so on.

[39] Implicitly, we have ruled out the possibility that negative b implies a negative P. This cannot be, since P_0 is positive for $0 \leq b_0 < b^*$. The Fisher equation (3.2) then pins down the price path for P_t, and this cannot produce a negative P_t if $P_0 > 0$.

arguably in the direction of greater empirical plausibility, then there is reason for concern. In this case, we say a theory is fragile.

Here, we describe one concern about the possible fragility of the FTPL, based on Canzoneri, Cumby and Diba (1997). We show that small, plausible perturbations on non-Ricardian policy lead to a collapse in the FTPL's ability to pin down the price level.[40] Consider the following alternative to the canonical non-Ricardian policy of simply setting s to a constant. Suppose $s = \varepsilon b$, where $0 < \varepsilon \leq 1$. With this policy, $b' = (1+r)(1-\varepsilon)b$, or

$$\frac{b_t}{(1+r)^t} = (1-\varepsilon)^t \frac{B_0}{P_0},$$

so that the transversality condition is satisfied for all $P_0 > 0$. Clearly, this is a Ricardian policy. With this policy, the FTPL does not pin down the price level. Now, this policy may appear to be a very *big* perturbation on the policy, $s_t = s$. Perhaps so, but there are close cousins to it in which the perturbation appears to be much smaller.

Consider the following alternative to the canonical non-Ricardian policy:

$$s_t = \begin{cases} -\frac{\xi}{1+r} + \frac{1+r-\gamma}{1+r} b_t & b_t > \bar{b} \\ s & b_t \leq \bar{b} \end{cases}, \tag{3.10}$$

where

$$\begin{aligned} 0 &\leq \gamma < 1, \\ \frac{\xi}{1-\gamma} &> \frac{1+r}{r} s, \\ \frac{1+r}{r} s &< \bar{b} < \frac{\xi}{1-\gamma}. \end{aligned}$$

In this case, as long as the real debt is below some upper bound, $\bar{b}$, then the policy is what we have been analyzing. But, as soon as b_t exceeds $\bar{b}$ a switch occurs and fiscal policy is adjusted to bring the debt back into line.

Although the algebraic representation of this policy may seem forbidding, it is quite easy to analyze it with the help of a graph like the one in Figure 2. Note that for $b_t > \bar{b}$, the real debt evolves according to the following equation:

$$b_{t+1} = \gamma b_t + \alpha.$$

[40] Woodford (1998) argues that there may be a local sense in which the FTPL's ability to pin down the price level survives the sort of perturbations we consider here. He also discusses versions of the model with learning which may survive these perturbations.

If it followed this equation forever, the real debt would eventually converge to $\tilde{b} = \xi/(1-\gamma)$. This equation, as well as (3.6), are graphed in Figure 2.

A simulation of the real debt under the fiscal policy in (3.10) is exhibited in Figure 2. The simulation is initiated with the indicated value of b_0. Note how the real debt initially follows the steep line with slope $1+r > 1$ until it passes $\bar{b}$, when it starts to follow the flatter line with slope $\gamma < 1$. All paths with $b_0 \geq b^*$ are consistent with the transversality condition because they all converge to a finite value, either b^* or $\tilde{b}$. So, with the given change in policy, the FTPL loses its ability to pin down the price level.

This perturbation on the non-Ricardian policy seems like a realistic one. At low levels of debt fiscal policy is exogenous, just like in the canonical non-Ricardian policy. But, if the debt should get out of line, then fiscal policy adjusts to bring it under control. This rings true in light of the US experience in the 1980s and 1990s and in light of the provisions of the Maastricht treaty which limit the real debts of the member countries in the European Union.

3.3. The FTPL with Stochastic Fiscal Policy

Up to now, we have illustrated non-Ricardian fiscal policy with $s_t = s$, a constant. But, the essence of the non-Ricardian fiscal policy is simply that s_t is not calibrated to satisfy the intertemporal budget equation for all prices. It is compatible with a much larger class of specifications for s_t than $s_t = s$. Here, we study non-Ricardian s_t's which are random. We use this to make three points.

First, we show that Barro's (1979) famous policy of absorbing fiscal shocks by raising taxes in the future can be represented as non-Ricardian fiscal policy.[41] As we explain below, this is an important example, in part because it helps clarify the definition of non-Ricardian policy, as it is used in the FTPL. This requires clarification, because there are other meanings one might mistakenly attach to the word, 'non-Ricardian', based on economists' everyday usage of the word, 'Ricardian'.

Second, we show that in general - unless policy is of the form advocated by Barro - fiscal shocks cause the inflation rate to fluctuate randomly about its average. The average value of inflation is determined by the value of the monetary authority's interest rate peg.

Third, we describe an important result due to Woodford (1996, 1998). He shows that, under the FTPL, if there is instability in fiscal policy then this *must* impact on the price level, no matter how committed the monetary authority is to price stability.[42] We call this

[41] That the Barro policy can also be represented as Ricardian seems obvious, and so we do not discuss this.

[42] Implicitly, we have in mind non-Ricardian policies *other* than the type advocated by Barro

striking result Woodford's *Really* Unpleasant Arithmetic, to contrast it with SW's famous title. Woodford's arithmetic is even tougher than SW's. As noted above, SW argue that if the central bank is weak, then the fiscal authorities can push it into producing price instability. However, this pessimistic ('unpleasant') conclusion of SW is balanced by their optimism that if the central bank just hangs tough, then the problem of price stability is solved. Looking at this from the perspective of the FTPL, Woodford argues that it doesn't matter how tough the central bank is. It still cannot stabilize the price level.[43]

3.3.1. Random Fiscal Policy

Suppose the surplus obeys the following first order autoregressive representation:

$$s_{t+1} = (1-\rho)s + \rho s_t + \varepsilon_{t+1}. \tag{3.11}$$

Here, ε_{t+1} is an iid, white noise process, independent of s_{t-j}, $j \geq 0$. A positive realization of ε_t induces a change in the date t government surplus and in the expected value of future government surpluses. Let ψ_j denote this effect at date $t+j$, for $j \geq 0$:

$$\psi_j \varepsilon_t = E_t s_{t+j} - E_{t-1} s_{t+j},\ j \geq 0,\ \psi_0 \equiv 1. \tag{3.12}$$

Here, E_t denotes the expectation operator, conditional on information available at date t ($E_t s_t = s_t$). When the surplus has the time series representation, (3.11), then

$$\psi_j = \rho^j.$$

Of course, (3.12) applies more generally, even when s_t does not have the time series representation given in (3.11). We can define the present value of the impact of ε_t on current and expected future surpluses as follows:

$$\psi\left(\frac{1}{1+r}\right)\varepsilon_t \equiv \varepsilon_t + \frac{\psi_1}{1+r}\varepsilon_t + \frac{\psi_2}{(1+r)^2}\varepsilon_t + ...$$

In the case of (3.11), this is:

$$\psi\left(\frac{1}{1+r}\right) = \frac{1+r}{1+r-\rho}.$$

When $\rho = 0$, so that s_t is iid, then the present value term is just unity. In this case, the effect of an innovation in the surplus is limited to the current surplus only. As ρ increases

(1979).

[43]But, recall our second point, that the central bank *can* control the average inflation rate.

above zero, then the present value term increases to take into account the future effects of an innovation. Negative values of ρ cause the present value terms to fall, as innovations in the current surplus generate expected reductions in the future surplus.

It is interesting to compare the type of fiscal policy considered in (3.11) with the sort of fiscal policy advocated in Barro (1979). Barro argues that a negative shock to government finances (due to, say, a war) should be met by a large increase in debt, coupled with a constant increase in the labor tax rate that is sufficient to pay off the interest and principal on that debt over time. In particular, Barro (1979) advocates fiscal policies of the form:

$$\psi\left(\frac{1}{1+r}\right) = 0.$$

Examples include:[44]

$$\psi_0 = 1,\ \psi_1 = -\left(1+r\right),\ \text{or, } \psi_0 = 1,\ \psi_i = -\left(\frac{1+r}{2}\right)^i,\ i \geq 1.$$

That is,

$$s_t = s + \varepsilon_t - \left(1+r\right)\varepsilon_{t-1},\ \text{or, } s_t = s + \varepsilon_t - \sum_{i=1}^{\infty}\left(\frac{1+r}{2}\right)^i \varepsilon_{t-i}.$$

These examples are useful in part because they help head off potential misunderstandings about the definition of ‘non-Ricardian’ policy. In everyday discussion, the word ‘Ricardian’ is used in a variety of senses. Sometimes economists use it to refer to a policy in which a current tax cut is financed by increases in future taxes that are large enough in present value to match the current cut.[45] It is clear from the preceding discussion that this type of policy can be part of a non-Ricardian regime.

Under the type of fiscal policy just discussed, the price level is insulated from fiscal shocks. Shocks to the real primary surplus are financed by appropriate movements in the opposite direction later on. We will show in the next subsection, that when $\psi \neq 0$, then surplus shocks are at least partially financed by movements in the price level.

3.3.2. Inflation With Random Fiscal Policy

We continue to assume that policy pegs $R_t = R$, so that the seignorage component of s_t is the constant value given in (3.3). As a result, the random nature of s_t in (3.11) reflects

[44] See, for example, Woodford (1998), footnote 18.
[45] One example is Hayashi (1989).

randomness in fiscal policy. The Fisher equation still holds, although it must be adjusted to take into account uncertainty:

$$1 + r = (1 + R)E_t \frac{P_t}{P_{t+1}},$$

where E_t is the conditional expectation, given information available at time t. This expression shows that the central bank controls the expected rate of deflation through its choice of R. This translates into control over the average rate of deflation by the fact, $E\left[E_t\left(P_t/P_{t+1}\right)\right] = E\left(P_t/P_{t+1}\right).$

Imposing the suitably adjusted version of the household's transversality condition, (3.8), on the government's flow budget equation, the intertemporal budget equation is now[46]:

$$\frac{B_t}{P_t} = E_t \sum_{j=0}^{\infty} \frac{s_{t+j}}{(1+r)^j} = s\left(\frac{1+r}{r}\right)\left(\frac{1-\rho}{1+r-\rho}\right) + \left(\frac{1+r}{1+r-\rho}\right) s_t. \tag{3.13}$$

We now have a completely specified theory of the price level and inflation. One way to understand this is to see how the model can be used to simulate a sequence of prices, for a given realization of primary surpluses. Thus, suppose we have a time series, s_0, $s_1, ...,$ s_T from (3.11), and an initial level of the nominal debt, B_0.[47] Then, P_0 is computed by evaluating (3.13) at $t = 0$. After this, B_1 is computed from the government's flow budget

[46]To see how this is derived, consider first the expression to the right of the first equality in (3.13). Note from the Fisher equation:

$$\frac{1}{P_{t+j}(1+R_{t+j})} = E_{t+j}\left[\frac{1}{(1+r)P_{t+j+1}}\right], \text{ all } t, j \geq 0.$$

Then, the flow budget constraint of the government can be written

$$\frac{B_{t+j}}{P_{t+j}} = s_{t+j} + \frac{B_{t+j+1}}{P_{t+j}(1+R_{t+j})} = s_{t+j} + \frac{1}{1+r} E_{t+j} \frac{B_{t+j+1}}{P_{t+j+1}},$$

or, after applying the law of iterated mathematical expectations:

$$E_t \frac{B_{t+j}}{P_{t+j}} = E_t s_{t+j} + \frac{1}{1+r} E_t \frac{B_{t+j+1}}{P_{t+j+1}} \text{ (**)}$$

Substitute this, for $j = 0$, into the period t flow budget constraint of the government:

$$\begin{aligned} \frac{B_t}{P_t} &= s_t + \frac{1}{1+r} \frac{B_{t+1}}{P_t(1+R)} \\ &= s_t + \frac{1}{1+r} E_t \frac{B_{t+1}}{P_{t+1}}, \end{aligned}$$

Applying (**) repeatedly to this expression, for $j = 1, 2, ...$ results in the expression to the right of the first equality in (3.13), if we apply the transversality condition, $\lim_{T\to\infty} E_0 b_T / (1+r)^T = 0$.

To obtain the expression to the right of the second equality in (3.13), first solve (3.11) to find:

$$E_t \frac{s_{t+j}}{(1+r)^j} = s\left\{\left(\frac{1}{1+r}\right)^j - \left(\frac{\rho}{1+r}\right)^j\right\} + \left(\frac{\rho}{1+r}\right)^j s_t,$$

for $j = 0, 1, 2, ...$ Then substitute this into (3.13) and apply the geometric sum formula.

[47]It should be obvious how this procedure could be adapted to accommodate any other time series representation for s_t.

equation, $B_{t+1} = (1+R)(B_t - P_t s_t)$, for $t = 0$. A sequence, P_0, $P_1, ...,$ P_T is obtained by performing these calculations in sequence, for $t = 0, 1, 2, ..., T$.

As noted before, the interest rate peg guarantees that in these simulations the expected rate of inflation (actually, *deflation* to be technically correct) is constant. As a result, the rate of inflation itself will be approximately uncorrelated over time. This latter feature is an artifact of the constant interest rate peg. If the interest rate rule were instead dependent on past interest rates and/or past inflation, then presumably persistence would appear in the model's inflation process.[48]

One can obtain further insight into (3.13) by subtracting $E_{t-1}B_t/P_t$ from it:

$$\begin{aligned} \frac{B_t}{P_t} - E_{t-1}\left(\frac{B_t}{P_t}\right) &= \left(\frac{1+r}{1+r-\rho}\right)(s_t - E_{t-1}s_t) \\ &= \psi\left(\frac{1}{1+r}\right)\varepsilon_t \end{aligned} \tag{3.14}$$

This says that a date t shock in the primary surplus induces a contemporaneous change in the real value of the debt equal to the present value of the shock.[49] Since B_t is predetermined at time t, the change is brought about entirely by a change in the price level.[50]

Fiscal policies like (3.14) underscore the fact that movements in the price level are an alternative to Barro's way of financing shocks to the primary surplus. A jump in the price level acts like a capital levy on the holders of government bonds, and that helps to finance government spending just as surely as the sort of taxes that are included in the primary surplus. In section 4 below, we describe an environment in which the efficient fiscal policy is of this type.

3.3.3. Woodford's *Really* Unpleasant Arithmetic

Woodford's argument that instability in fiscal policy must impact on the price level is a simple proof by contradiction. Thus, suppose that the monetary authority can perfectly

[48]Loyo (1999) emphasizes this in his discussion of the persistent rise in inflation observed in Brazil in the 1980s.

[49]It is perhaps obvious that the analysis of price determination under the FTPL is similar to the analysis of consumption in the permanent income hypothesis. For a review which uses much of the same algebra used here, see Christiano (1987).

[50]To see this, divide both sides of (3.14) by B_t and, taking into account, $E_{t-1}B_t = B_t$:

$$\frac{1}{P_t} - E_{t-1}\left(\frac{1}{P_t}\right) = \frac{1}{B_t}\psi\left(\frac{1}{1+r}\right)\varepsilon_t.$$

stabilize inflation and the price level. This implies that $P_{t+1} = P_t$, so that the nominal rate of interest fixed and equal to the real rate. This in turn implies that seignorage, s^m, is zero. As a result, $s_t = s_t^f$. Now, suppose fiscal policy is stochastic, with $\psi(1/(1+r)) \neq 0$. Then, according to (3.14), P_t responds to innovations in s_t. But, that contradicts our assumption that P_t is constant. It follows that with shocks to fiscal policy, it may not be feasible for the monetary authority to insulate the price level from those shocks.

In interpreting this, it is important to bear in mind the fact that the monetary authority *can* control the expected rate of inflation in the FTPL. For Woodford's *Really* Unpleasant Arithmetic to be unpleasant in fact, requires that shocks to the realized price level have socially inefficient consequences. In many economic environments, this is not the case, and only the expected inflation rate matters (see, e.g., Chari, Christiano and Kehoe (1991)). Shocks to the realized price level are costly in environments where there are nominal rigidities and where agents are heterogeneous.[51]

3.4. The FTPL and the Control of Average Inflation

The previous subsection showed that the monetary authority can control the average rate of inflation by pegging the nominal interest rate to an appropriate value.[52] In policy discussions about inflation, it is sometimes suggested that inflation can be controlled more effectively with an interest rate rule that responds aggressively to inflation. In this section, we show that such a monetary policy could in fact lead to disaster, if fiscal policy were non-Ricardian.

Suppose that the monetary authority adjusts the interest rate according to the following rule:

$$1 + R_t = \alpha_0 + \alpha_1 \pi_t, \ \pi_t = P_t / P_{t-1}.$$

The monetary authority implements this rule by adjusting the money supply so that money demand is satisfied at the targeted rate of interest. An 'aggressive' interest rate rule is one in which α_1 is large. For example, Taylor (1993) has argued that α_1 should be around 1.5. This means that if inflation rises by 1 percentage point, then the central bank raises the nominal rate of interest by 1.5 percentage points. According to conventional wisdom, an aggressive interest rate rule like this is a good way to keep inflation under control. As we shall see, this is not necessarily the case if policy is non-Ricardian.

We suppose that the rest of the economy corresponds to the one in the example in section 3.1. As in that model economy, we assume that there is no uncertainty, since that is not

[51] See Woodford (1996) for an environment with endogenous production and sticky prices. With sticky prices, shocks to the aggregate price level distort the allocation of resources across the production of different goods. They also distort aggregate output.

[52] See section 3.3.2 above.

essential to the analysis here. Combining the interest rate rule with the Fisher equation, (3.2), we obtain the following expression, which must hold in equilibrium:

$$\pi_{t+1} = \frac{\alpha_0}{1+r} + \frac{\alpha_1}{1+r}\pi_t.$$

Consider an aggressive interest rate rule, with $\alpha_1/(1+r) > 1$. The relationship between π_{t+1} and π_t is depicted in Figure 3. Note that there is a particular inflation rate, π^*, such that if $\pi_t = \pi^*$, then $\pi_{t+1} = \pi^*$ too. However, if the initial inflation rate should be greater than π^*, then π_t goes to plus infinity, as t gets large. This possibility is illustrated in Figure 3, where inflation starts with π_0 in period 0 and then explodes.

As in the model economy in section 3.1, the initial price level is determined by fiscal policy according to the intertemporal budget equation, (3.9). Technically, s is now no longer a constant. This is because the variations in inflation cause seignorage to vary over time too. However, we assume that seignorage revenues are small enough to ignore, so that s is composed only of s^f. We continue to assume that s^f is constant.[53] So, the price level in period 0 is determined by $P_0 = B_0/b^*$, where B_0 is the initial nominal debt and b^* is defined in (3.9).

With P_0 determined by the intertemporal budget equation, and P_{-1} determined by history, π_0 is uniquely pinned down. But, there is no way to rule out the possibility that this value of π_0 lies to the right of π^*, in which case an exploding inflation occurs.[54]

One way to gain intuition into the mechanics of this exploding inflation focuses on the government's budget constraint, (3.5). From that equation, we see that a higher nominal interest rate leads to a more rapid increase in the nominal debt, B_{t+1}. But, under the assumption that the outlook for the fiscal primary surplus does not change, this means that the real value of the debt remains constant. With the nominal debt growing more quickly and its real value constant, inflation must rise. The central bank's monetary policy responds to the rise in inflation by driving the interest rate up even further, and this leads to an additional increase in inflation. This circular, self-reinforcing process is what leads to the explosion in inflation.

The possibility just outlined, whereby an aggressive interest rate rule leads to an exploding inflation, may seem peculiar. Recognizing this appearance, Loyo (1999) refers to this as a 'tight money paradox'. According to the model, if instead of being aggressive, the central bank adopted a more accommodating stance by choosing a value of α_1 substantially less

[53]In effect, we assume the economy is in the 'cashless limit' discussed by Woodford (1998, 1998a, 1998b), and defined in an earlier footnote.

[54]This situation is one in which fiscal policy and monetary policy are both *active*, in the sense defined by Leeper (1991). Our analysis is consistent with Leeper's, which concludes that for almost all values of fiscal policy, s^f, there does not exist a stationary equilibrium inflation rate.

than unity, then exploding inflation could not occur. We saw this in the previous subsection, were it was shown that with $\alpha_1 = 0$ inflation fluctuates around a constant value. It is easy to confirm using the type of logic in Figure 3, that the same is true for $0 < \alpha_1 < 1$.

Relative to a simple monetarist perspective, the result that adopting an aggressive stance against inflation by increasing α_1 may convert a stable inflation into an exploding inflation is certainly a paradox.[55] However, we have just seen that it can occur in a coherent economic model. Moreover, Loyo (1999) argues that the model captures the basic forces driving the takeoff in inflation in Brazil in the early 1980s. Although we are skeptical that tough monetary policy was the cause of Brazil's high inflation, the hypothesis certainly does seem intriguing.

The exploding inflation scenario just described has been used by Woodford (1998a, pp. 399-400) to understand the nature of fiscal policy in the US in the past two decades. He observes that (i) econometric estimates of the Fed's policy rule in the 1980s and 1990s place α_1 substantially above unity (see Clarida, Gali and Gertler (1998)) and (ii) there is no evidence of instability in US inflation. He concludes from (i) and (ii) that policy in the US during this time, must not be non-Ricardian.

4. The FTPL and the Optimal Degree of Price Instability

The FTPL literature has also drawn attention to the possibility that some price instability may be desirable when there are unavoidable shocks to the government budget constraint (Sims (1999), Woodford (1998)). When there is nominal government debt, unanticipated shocks to the price level act like capital levies on bond holders. The idea is that it is efficient to absorb unanticipated shocks with capital levies, rather than by changing distortionary taxes.

We illustrate these observations in the simplest possible model.[56] Relative to our one-period model in section 2, the model incorporates two complications that are essential for the purpose at hand. First, we need to take into account possible distortionary effects on the bond accumulation decision arising from price level instability. For this reason, we adopt a two-period setup. The bond accumulation decision is taken in the first period, and the government spending shock and price level uncertainty occurs in the second period. Second,

[55] Tight money paradoxes also exist in environments with Ricardian fiscal policy. For example, Sargent and Wallace (1981) showed, in such an environment, a tight monetary policy may lead to an immediate rise in inflation. See Kocherlakota and Phelan (1999) for a discussion of a tight money paradox in the FTPL.

[56] The example illustrates the results on the desirability of tax smoothing and volatile prices reported in Chari, Christiano and Kehoe (1991).

the model must capture the notion that taxes are distortionary. Accordingly, we assume that labor supply is endogenous and taxes are raised using a proportional tax on labor income.

The basic setup of the model is an example of the FTPL because government policy, the choice of labor tax rates, is non-Ricardian. We illustrate how advocates of the FTPL study the optimal degree of price stability by examining the 'best' equilibrium of such a model (see, e.g., Sims (1999) and Woodford (1998)). This equilibrium is what the literature on optimal fiscal and monetary policy (see, e.g., Lucas and Stokey (1983)) calls the Ramsey equilibrium.

The first subsection below describes the model. To simplify the analysis, the model does not include money. A consequence of this is that the model constitutes another illustration of price determination in an economy with no government-provided money. The next subsection characterizes the best (i.e., the Ramsey) equilibrium in this economy. We then present a numerical example which illustrates the role of price instability in bringing about efficient resource allocation in the model. We assess the results in a summary section.

4.1. The Model

The economy is composed of firms, households and a government. Households and firms interact in competitive markets. The government must finance an exogenously given level of expenditures. It does so by levying a distortionary tax rate on labor and possibly also by issuing debt. There is no uncertainty in the first period. However, there is uncertainty in the second period's level of government spending. Spending could be high or low, with probability 1/2 each, with the uncertainty being resolved at the beginning of the second period. Consistent with the non-Ricardian assumption, the government commits to its policies before the first period. Trade occurs by barter, and there is no money in the model.[57]

Firms have access to a linear production technology:

$$y = n,\ y^h = n^h,\ y^l = n^l,$$

where y and n denote output and labor in the first period and y^i, n^i denote output and labor in the second period, $i = h, l$. The superscript h or l indicates the second period when government spending is high or low, respectively. The linearity in the production function guarantees that the real wage is always unity in equilibrium, and so from here on we simply impose this and do not refer to firms anymore.

[57]We could imagine that there is 'inside money', and that trade is accomplished through an efficient exchange of iou's.

Preferences of households over consumption and labor over the two periods take the following form:

$$U(x) = c - \frac{1}{2}n^2 + \frac{1}{2}\beta\left\{\left[c^h - \frac{1}{2}\left(n^h\right)^2\right] + \left[c^l - \frac{1}{2}\left(n^l\right)^2\right]\right\},\ 0 \leq \beta \leq 1, \tag{4.1}$$

where c denotes consumption in the first period and β is the discount rate, with $\beta = (1+r)^{-1}$. Similarly, c^i denotes consumption in the second period, conditional on the realization of government spending, $i = h, l$. Also, β is the discount rate of the household, and the number '1/2' before β corresponds to the probability of the h or l state of the world. Finally,

$$x = (c, c^h, c^l, n, n^h, n^l). \tag{4.2}$$

The linear-quadratic structure of preferences is chosen to ensure that the analysis is simple. The household's period 1 budget constraint is:

$$\frac{B'}{1+R} + Pc \leq B + P(1-\tau)n, \tag{4.3}$$

where P is the period 1 price level, B is the nominal bonds that the household inherits from the past, and R is the nominal rate of interest. Also, τ denotes the tax rate on labor and B' denotes bonds acquired from the government. The household's budget constraint in the second period, conditional on the realization of uncertainty, is

$$P^h c^h \leq B' + P^h(1-\tau^h)n^h,\ P^l c^l \leq B' + P^l(1-\tau^l)n^l. \tag{4.4}$$

Again, superscripts indicate the realization of the exogenous government spending shock. There is no government-supplied money in this economy.

The household maximizes utility by choice of non-negative values for B', c, c^h, c^l, n, n^h, n^l. It must also respect the budget constraints just specified, and it takes prices and the interest rate as given and beyond its control. The Euler equations associated with the household's optimal choice of labor and bonds are:

$$n = 1-\tau,\ n^h = 1-\tau^h,\ n^l = 1-\tau^l,\ \frac{1}{(1+R)\,P} = \frac{1}{2}\beta\left(\frac{1}{P^h} + \frac{1}{P^l}\right). \tag{4.5}$$

The last of these says that the expected gross real rate of return on bonds must be $1/\beta$. That this is true, independent of the intertemporal pattern of consumption, reflects our assumption that utility is linear in consumption.

The government's budget constraints in the first and second periods are:

$$\begin{aligned} \frac{B'}{1+R}+P\tau n &\geq B+Pg \\ P^h\tau^h n^h &\geq B'+P^h g^h \\ P^l\tau^l n^l &\geq B'+P^l g^l. \end{aligned} \tag{4.6}$$

Here, g denotes government consumption in the first period, and g^i denotes period 2 government consumption, $i=h,l$. Inspection of the equations of our model reveals that R appears everywhere as $(1+R)/P^h$, $(1+R)/P^l$, or $B'/(1+R)$.[58] Thus, we cannot pin down each of R, P^h, P^l, and B' separately. For this reason, we adopt the normalization, $R=0$, from here on. Government policy is a vector of three numbers, π, where

$$\pi=\left(\tau,\tau^h,\tau^l\right).$$

This is a non-Ricardian policy. There is no set of values for π having the property that the government's intertemporal budget equation (see below) is satisfied for all prices.

Combining the government and household budget equations, we obtain the resource constraints:

$$c+g\leq n,\ c^h+g^h\leq n^h,\ c^l+g^l\leq n^l. \tag{4.7}$$

There are 10 variables to be determined in equilibrium:

$$P,P^h,P^l,B',c,c^h,c^l,n,n^h,n^l.$$

They are determined by the three household budget constraints, (4.3)-(4.4), evaluated with a strict equality; the four household Euler equations, (4.5); and the three resource constraints, (4.7). These 10 equations, together with the requirement, $P,P^h,P^l>0$, characterize equilibrium (if one exists!) associated with a given government policy. The mapping from π to these variables is single-valued. We denote the function relating the last six of these variables to π by

$$x(\pi),$$

where x is defined in (4.2).

[58]The statement is obviously true in the case of the household Euler equation in (4.5). To see that it is also true of (4.3)-(4.4) and (4.6), replace B' by $\tilde{B}'=B'/(1+R)$ in those equations, and then divide the period 2 budget constraints by $1+R$.

4.2. The Ramsey Equilibrium

The Ramsey equilibrium is the one associated with the policies, π, which solve the problem

$$\max_{\pi} U(x(\pi)),$$

subject to the requirement that prices be strictly positive, $B' \geq 0$ and the elements in x be non-negative.[59] The Ramsey equilibrium is easy to compute in this model economy.

After substituting out for the endogenous variables in terms of π in (4.7) and (4.5), we find that the utility function has the following representation:

$$U(x(\pi)) = -\tau^2 - \frac{1}{2}\beta\left[\left(\tau^h\right)^2 + \left(\tau^l\right)^2\right] + \kappa, \tag{4.8}$$

where κ is a constant.[60] To complete the statement of the Ramsey problem, we need a simple representation of the restrictions placed on π by the requirement that prices be positive. But, before we do this, we must confront a technical issue.

It is well known in the literature on Ramsey equilibria that it is efficient to, in effect, renege on the initial nominal debt, B, by selecting policies which produce an infinite first period price level. Allowing this to happen here would plunge us into exotic mathematical issues, while distracting us from the central focus of the example. The focus of the example is on the desirability of letting prices in the second period react to the realization of government spending in that period. With this in mind, we simply fix the period 1 price level at $P = 1$. Since the nominal debt, B, is given from the past, it follows that we have fixed the initial real debt. It is important to emphasize, however, that we do *not* fix the *second* period price levels.[61]

The restriction placed on π of the requirement, $P = 1$, is easy to determine. We do so by expressing the government's first period intertemporal budget equation in terms of

[59] See Bizer and Judd (1989), Chari, Christiano and Kehoe (1991), Judd (1989), Lucas and Stokey (1983), among others.

[60] Here,

$$\kappa = 2\left[\frac{1}{2} - g + \frac{1}{2}\beta\left(1 - g^h - g^l\right)\right].$$

To see how we obtain this expression, note that $c - (1/2)n^2$ is $y - g - (1/2)n^2$ after using the resource constraint, $y = c+g$. Imposing $n = 1-\tau$ then yields that $c-(1/2)n^2$ is $(1/2)(1-\tau^2)-g$.

[61] Chari, Christiano and Kehoe (1991) confront the same problem. They deal with it by setting the initial debt to zero. In our context that creates a problem because it leaves us without an ability to pin down P.

π. To obtain this, combine the household's intertemporal Euler equation, (4.5), with the government's budget constraints, (4.6):

$$B \leq \tau(1-\tau) - g + \frac{1}{2}\beta\left[\tau^h(1-\tau^h) - g^h + \tau^l(1-\tau^l) - g^l\right], \tag{4.9}$$

where we have imposed $P = 1$. The restrictions on second period prices are obtained from the intertemporal government budget equations that obtain in those periods:

$$\tau^h(1-\tau^h) - g^h \geq 0,\ \tau^l(1-\tau^l) - g^l \geq 0. \tag{4.10}$$

The Ramsey problem, modified to incorporate the restriction, $P = 1$, is set up in Lagrangian form as follows:

$$\begin{aligned}
&\max_{\tau,\tau^h,\tau^l} -\tau^2 - \frac{1}{2}\beta\left[\left(\tau^h\right)^2 + \left(\tau^l\right)^2\right] \\
&+\lambda\left\{\tau(1-\tau) - g + \frac{1}{2}\beta\left[\tau^h(1-\tau^h) - g^h + \tau^l(1-\tau^l) - g^l\right] - B\right\} \\
&+\mu^h\left[\tau^h(1-\tau^h) - g^h\right] + \mu^l\left[\tau^l(1-\tau^l) - g^l\right],
\end{aligned}$$

where λ, μ^h, $\mu^l \geq 0$ are Lagrange multipliers.[62] The necessary and sufficient conditions associated with the maximum are the inequality constraints on the multipliers, λ, μ^h, $\mu^l \geq 0$, the inequality constraints in (4.9)-(4.10), the 'complementary slackness' conditions:

$$\begin{aligned}
0 &= \lambda\left\{\tau(1-\tau) - g + \frac{1}{2}\beta\left[\tau^h(1-\tau^h) - g^h + \tau^l(1-\tau^l) - g^l\right] - B\right\}, \\
0 &= \mu^h\left[\tau^h(1-\tau^h) - g^h\right], \\
0 &= \mu^l\left[\tau^l(1-\tau^l) - g^l\right],
\end{aligned} \tag{4.11}$$

and the three first order conditions corresponding to τ, τ^h, τ^l. After rearranging, these are:

$$\lambda = \frac{2\tau}{1-2\tau},$$

[62] Our model differs from Sims' (1999) model in two respects. First, ours has only two periods, while Sims' model has an infinite horizon. Second, we model agents at the level of preferences and technology, while Sims adopts a reduced form representation analogous to the one in Barro (1979). Our reduced form utility function coincides with Sims', but our budget constraint does not. Sims (1999) models taxes as lump sum taxes in the budget constraint, whereas we take into account the distortionary effects of taxation. For example, Sims would have τ in the budget constraint, rather than the $\tau(1-\tau)$ that appears in ours. The conclusions of the analysis are not sensitive to these differences.

$$
\begin{aligned}
\mu^h &= \beta\left[\frac{\tau^h}{1-2\tau^h} - \frac{\tau}{1-2\tau}\right] \\
\mu^l &= \beta\left[\frac{2\tau^l}{1-2\tau^l} - \frac{2\tau}{1-2\tau}\right].
\end{aligned} \tag{4.12}
$$

We solve the (constrained) Ramsey problem by finding multipliers, λ, μ^h, μ^l, and policies, τ, τ^h, τ^l, that satisfy these conditions.

Once the Ramsey policies have been identified, n, n^h, n^l are obtained from (4.5) and c, c^h, c^l are obtained from (4.7). Then, B', P^h and P^l are obtained by solving (4.6). Several qualitative features of the solution are immediately apparent. First, the weak inequality in (4.9) is satisfied as a strict equality.[63] This is not surprising. If it were otherwise, then taxes would be higher than necessary and, given the form of preferences, this is counterproductive. Also, the fact that the period 0 intertemporal budget equation is satisfied as a strict equality means that it would have been optimal to inflate away the initial debt by setting $P = \infty$, if we had not imposed the requirement that the government pay off B with $P = 1$.[64] Second, ignoring the requirements imposed by the non-negativity constraints on prices in the second period, then the optimal outcome is $\tau = \tau^h = \tau^l$. To see this, note that the first order conditions in this case are (4.12) with $\mu^h \equiv \mu^l \equiv 0$. Then, it is obvious by inspecting the second two equations in (4.6) that $P^h > P^l$ as long as $g^h > g^l$. Third, in practice the constant tax rate policy is not necessarily feasible, since it may conflict with the requirement that prices be positive. But, in this case, the price fluctuations across states of nature are even more extreme. Suppose, for example, that the constant tax rate policy is inconsistent with the first of the two inequalities in (4.10). Then, $\mu^h > 0$ and $\tau^h > \tau$ and, by (4.11), $\tau^h(1-\tau^h) - g^h = 0$. The latter implies that the government inflates away the debt completely in state h, with $P^h = \infty$. To ensure that households still have an incentive in the first period to accumulate debt, (4.5) indicates that P^l must satisfy $P^l = (\beta/2)P(1+R) = \beta/2$ in this case. That is, the real rate of return on debt into state l must be high.

[63] Here is a proof by contradiction. Thus, suppose the weak inequality in (4.9) were a strict inequality. Then, by the first expression in each of (4.11) and (4.12), we have $\lambda = 0$ and $\tau = 0$. The strict inequality in (4.9) then implies that at least one of the weak inequalities in (4.10) is strict. That implies, by (4.11), that the associated multiplier is zero. But then, (4.12) implies that the associated tax rate is zero. But, this contradicts that the primary surplus is non-negative in that period.

[64] In that case, the constraint would have been (4.11) without the term, $-B$.

4.3. A Numerical Example

This section studies a numerical example to illustrate the properties of P^h and P^l in the Ramsey equilibrium. A natural benchmark to consider is the no debt equilibrium: the one in which τ is selected so that $B' = 0$ and τ^h and τ^l are selected so the constraints in (4.10) are satisfied as exact equalities. With this as a benchmark, we then move on to evaluate the Ramsey equilibrium in which $B' > 0$ and to consider P^h and P^l.

To see how taxes are determined in the benchmark equilibrium, consider Figure 4. That graphs $\tau n = \tau(1 - \tau)$ for $\tau \in (0, 1)$. Note that we have a single-peaked Laffer curve in our model economy. The horizontal lines indicate the revenue requirements in the first and second periods. We have assumed that the first period revenue requirement, $B + g$, is 0.20. The second period revenue requirement is $g^h = 0.15$ when government spending is high and $g^l = 0.05$ when government spending is low. The benchmark equilibrium requires that τ, τ^h and τ^l be set as indicated on the horizontal axis. In particular, $\tau = 0.28$, $\tau^h = 0.18$ and $\tau^l = 0.05$, after rounding. The value of (4.8) in this equilibrium is -0.0941, ignoring κ and setting $\beta = 0.97$. Note how very uneven the tax rates are over time and over states of nature.

Now consider the Ramsey tax rates. We proceed under the conjecture, subsequently verified, that they are optimally chosen to be a constant, τ^*, across dates and states. We also use the fact, established in the previous subsection, that the constraint, (4.9), is binding. There are two constant tax rates that solve (4.9) evaluated as a strict equality. Given preferences, (4.8), we go with the lower one, $\tau^* = 0.19$, after rounding. To verify that this solves the Ramsey problem, we need to confirm that (4.10) are satisfied. Indeed they are, with $\tau^*(1 - \tau^*) - g^h = 0.0008$ and $\tau^*(1 - \tau^*) - g^l = 0.10$.

Solving the first expression in (4.6), we find that $B' = 0.05$. In addition, we find from the second two expressions in (4.6) that $P^h = 64.67$ and $P^l = 0.49$. Essentially, the government renegs on the debt in period h and pays an attractive 100% rate of return in state l. Finally, the utility of this equilibrium is -0.0674. The results just described, plus the consumption and labor allocations are summarized in Table 1. The key thing to note is that by issuing debt, it is possible to stabilize employment and consumption across dates. By issuing the debt in nominal terms, and allowing the price level to fluctuate, it is possible to make the payoff on that debt state contingent, in real terms.

Table 1: Two Equilibria

Variable	Benchmark, No Debt ($B' = 0$)	Ramsey
P	1	1
P^h	NA	64.67
P^l	NA	0.49
n	0.72	0.82
n^h	0.82	0.82
n^l	0.95	0.82
c	0.52	0.62
c^h	0.67	0.67
c^l	0.90	0.77
τ	0.28	0.19
τ^h	0.18	0.19
τ^l	0.05	0.19
$g + B$	0.20	0.20
g^h	0.15	0.15
g^l	0.05	0.05
B'	0.0	0.05
utility	-0.0941	-0.0674

Note: NA ~ 'not applicable'.

4.4. Summary

We have described a model in which it is part of an efficient fiscal program to issue nominal debt and then allow the price level to fluctuate. Although we demonstrated this finding in an economy with no government-provided money, this feature of our model plays no fundamental role in the result. The same result was obtained in Chari, Christiano and Kehoe (1991) and Woodford (1998) using models in which there is money.

In our model, the equilibrium is equivalent to one in which the government issues debt in the first period whose payoff is denominated in real terms, and where the payoff is explicitly contingent on the realization of government spending in the second period.[65] When seen from this perspective, the natural question to ask is, 'why not just use the state contingent debt strategy, rather than go to the trouble of issuing nominal debt and allow the state contingency to arise because of fluctuations in the price level?'

To address this question, one needs to invoke considerations that are not in the model. One potential advantage of the nominal debt strategy is that it is likely to have lower costs

[65]Lucas and Stokey (1983) emphasized the desirability of this type of debt.

of administration and information acquisition. This is because the appropriate response of the real payoff on the debt to shocks is achieved automatically as a by-product of the price fluctuations generated in the market clearing process (Sims (1999), Woodford (1998)).

But, there are reasons that this may provide an overly optimistic view on the nominal debt strategy. For example, if there are some sticky prices, then fluctuations in the price level could distort resource allocations. In addition, price volatility may interfere with private contracts by inducing reallocations of wealth among private agents. Presumably, a version of the Ramsey problem which incorporates those costs would still exhibit price fluctuations. But, those fluctuations are likely to be smaller.[66] The information problem of designing a fiscal system which properly balanced benefits and costs would presumably be very great, reducing the cost advantages of the nominal debt strategy that we alluded to above.

There is another reason to question the advantages of *both* the nominal and real-state contingent debt strategies. Unless the government has substantial ability to commit to its policies, either strategy could very well backfire. This possibility can be seen in the example. Recall how we pointed out that it is efficient in the first period for the government to inflate away the debt. But, note that when time moves forward one period, then the second period becomes the first period. When that time arrives, it is again in the government's interest to inflate away the debt! Households who understand this in the first period may well choose not to accumulate *any* debt in the first place.[67] Now, this case was excluded in our analysis because of the assumption that policy is non-Ricardian: the policy is just a sequence of numbers (tax rates) through time, and the possibility of adjusting them ex post is ruled out. But, is this a realistic assumption? Does it assume that governments have more commitment power than they actually have? The literature on Ramsey policy has generally concluded

[66]It would be very interesting to investigate this question in quantitative models. There is a possibility that the efficient degree of volatility in prices would be reduced to zero if price volatility introduced distortions. Chari, Christiano and Kehoe (1994) argue that in principle there is a variety of ways to achieve state-contingency in fiscal policy. If there were costs to using the price level for this, then the efficient thing to do would be to use one of the other ways. Only if there were costs associated with *all* ways of achieving state-contingency in fiscal policy, would some volatility in prices be desirable.

[67]A recent paper by Sims (1999) considers a proposal that Mexico adopt the US dollar as its national currency. He criticizes the proposal on the grounds that, with the Mexican national debt denominated in another currency, the Mexican government loses the fiscal benefits of the policy described in the text. That is, it would not be able to periodically renege on and periodically subsidize holders of its debt through fluctuations in the Mexican price level. Our point here is that giving up this option may not be very costly to Mexico, if the Mexican government lacks credibility. Indeed, giving up the option may be a *good* thing. In the absence of credibility, attempts to use the option may lead to the disasterous situation in which everyone just refuses to buy Mexican government debt.

that the answer is 'yes', and has moved on to equilibrium concepts that do not presume so much commitment power.

But, in principle one can also make the case that the degree of commitment needed for the policy to work is *not* implausibly large. This might be so if the required price fluctuations occurred automatically, in a way that legislatures have difficulty interfering with. For example, Judd (1989) has suggested that price movements in the US economy correspond roughly to what an efficient fiscal program requires. He notes that good shocks to the government budget constraint, like technology shocks, tend to produce a negative shock to the price level, generating transfers to holders of government bonds. Similarly, bad shocks like a jump in government spending due to war or natural disaster, tend to drive the price level up, taxing government bond holders.

So, our point is not that the degree of commitment required for the volatile price strategy is necessarily too great. Our point is only that commitment is a fundamental assumption of the volatile price strategy. In the absence of commitment, the strategy is likely to backfire.

5. Conclusion

What insights does the FTPL provide into the two questions about price stability cited at the beginning of this paper? Conventional wisdom holds that if there is no doubt about the central bank's commitment to low and stable inflation, then that is what will happen.[68] According to the FTPL, this overstates the control that a central bank actually has. Still, it remains an open question just how severe the limitations on the central bank's power are. It is possible that these limitations are not very great for modern developed economies. In the FTPL models that we studied, the central bank can determine the *average* rate of inflation. It is the variance of inflation that it cannot perfectly control. The problem is that it cannot eliminate the impact on the price level of shocks to fiscal policy. But, in a modern Western economy the stock of outstanding nominal government liabilities is quite large - Judd's (1989) estimate for the US puts it at one year's GDP. This means that a relatively small change in the price level can absorb a fairly large fiscal policy shock.[69] So, in practice it may be that the conventional answer to the first question may be roughly the right one, even under the FTPL.

Regarding the second question, Sims (1999) has stressed the potential benefits of price volatility.[70] Variations in the price level in response to fiscal shocks have the effect of taxing

[68]See Sargent and Wallace (1986), last paragraph.

[69]This point has also been stressed by Sims (1999).

[70]Woodford (1998) has made a similar suggestion. The result has also been obtained in Chari, Christiano and Kehoe (1991).

holders of nominal government liabilities. Under certain circumstances, this can be used to enhance the overall efficiency of government fiscal and monetary policy. But, this result also raises questions. It is obtained for an environment with few of the frictions observed in actual economies, that make price volatility costly. Whether the result would survive the introduction of a realistic set of frictions, and a realistic set of alternative methods for dealing with fiscal shocks is unclear at this time.

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A. The Logical Coherence of Fiscal Theory

Potentially, an important concern regarding the FTPL has to do with its internal logical consistency. When the FTPL uses the intertemporal government budget equation to pin down the price level, is that price level consistent with the price level determined by the rest of the economy? In some cases, the answer is 'no'.[71] Do these cases warrant a general conclusion that the FTPL is not logically coherent? We think not. Enough interesting examples can be constructed in which the fiscal theory *is* logically coherent. One example is given in the body of the paper. The point is also illustrated in several articles of a special issue in *Economic Theory* in 1994. In this appendix, we present another example.

The model we work with is the cash-credit good model of Lucas and Stokey (1983). We do the analysis for a range of parameter values which includes the empirically plausible ones, according to the estimates reported in Chari, Christiano and Kehoe (1991). We skip detailed proofs in certain places, though never without providing the intuition for the argument. The reader who wishes to see an extensive and rigorous treatment of the properties of the equilibria of this model should consult Woodford (1994). In effect, this appendix presents an extended example to illustrate his Propositions 2 and 10 at the level of an advanced undergraduate or first year graduate economics course.

We first consider the case where monetary policy is characterized by a constant money growth rate. We show that the model has a unique equilibrium when the non-Ricardian assumption is adopted. We then consider the case where the monetary authority pegs the rate of interest. Again, (like the example in the text) the model has a unique equilibrium when a non-Ricardian assumption is adopted. When the non-Ricardian assumption is *not* adopted, then the model fails to exhibit a unique equilibrium. In this case, the model reproduces the classic Sargent and Wallace (1975) result that the price level is indeterminate. So, from a technical standpoint, the non-Ricardian assumption is a device that can eliminate the 'price level indeterminacy' associated with interest rate pegging that Sargent and Wallace analyzed.[72]

The first section below describes the agents of the model, and defines equilibrium. The following section addresses the case when monetary policy is characterized by constant money growth and the final section addresses the interest rate pegging case.

[71] A simple example of this, in which the traditional quantity theory holds, was presented in the body of the paper. In the model of Obstfeld and Rogoff (1983), there is a countable set of equilibria. The Ricardian assumption does not sit comfortably with that model because the likelihood that the fiscal and monetaty authorities would pick a fiscal policy that would be consistent with one of those equilibria seems slim. Another example appears in Buiter (1999).

[72] This is how Kocherlakota and Phelan (1999) interpret the FTPL.

A.1. The Lucas-Stokey Cash-Credit Good Model

A.1.1. Households

The Household Problem and Constraints

The model abstracts from differences between households by assuming they are all identical. In addition, households are assumed to live infinitely long. This could be interpreted, following Barro (1974), as reflecting that each household actually lives a finite amount of time, but cares in a particular way about its offspring.

The preferences of the representative household are as follows:

$$\sum_{t=0}^{\infty} \beta^t u(c_t),\ u(c) = \log(c),\ 0 < \beta < 1,$$
$$c = [(1-\sigma)c_1^{\nu} + \sigma c_2^{\nu}]^{\frac{1}{\nu}},$$

where $0 < \sigma < 1$ and c_t denotes consumption services. In our analysis, we restrict ν to the range $0 < \nu < 1$. Chari, Christiano and Kehoe (1991) argue that this is the empirically relevant case. Based on post-war US data, their point estimates are $\sigma = 0.57$ and $\nu = 0.83$.

Consumption services are generated by the acquisition of two market-produced goods, as indicated. One of these, c_{1t}, is called a 'cash good' and the other, c_{2t}, is a 'credit good'. To purchase the cash good, households need to set aside cash in advance.

To make the notion 'in advance' precise, the model adopts a particular timing setup. It is assumed that each period is divided into two parts. In the first part (the 'morning'), the household participates in an asset market, and in the second part (the 'afternoon') the household participates in a goods market. The cash that households need to purchase c_{1t} in the afternoon of a given day must be set aside at the end of asset market trading that same morning. They hold these balances idle until the morning of the following day, when actual payment is due. Credit goods work a little differently. For these, the household has no need to accumulate cash in advance. The household simply pays for the goods with cash in the next morning's asset market.

Be careful not to be misled by the labels on these goods. It is not that one can be bought 'on credit' in the normal sense, and the other not. Both goods are paid in cash the morning after the purchase. No credit is being offered by the seller in either case. The difference between the goods is simply that in the case of one (the cash good), the household has to forfeit interest. This is because to buy it, the household has to carry idle cash balances in its pocket throughout the afternoon. From the point of view of the seller the goods are actually completely the same. The terms of the transaction are identical: cash only, to be delivered the morning after the sale.

The distinction between cash and credit goods may at first seem artificial. In fact, it is a clever device for capturing the notion that transactions in some goods are more cash intensive than in others. It will give rise to a demand for money, one that is a function of the rate of interest.

We assume that the marginal rate of transformation in production is unity between the two goods. Because of this, the price of the two goods, P_t, is identical in equilibrium. Also, in any equilibrium it must be that $R_t \geq 0$ and $P_t > 0$. Market clearing is impossible if either of these two conditions fails to be satisfied.

Let A_t denote the household's financial assets at the end of asset market trading. In the first period, $t = 0$, this is simply a given number, A_0. The household can allocate A_t as follows:

$$M_t^d + \frac{B_{t+1}^d}{1+R_t} + T_t \leq A_t, \; t = 0, 1, 2, \; ..., \tag{A.1}$$

where M_t^d denotes money balances; B_{t+1}^d denotes government debt which costs $B_{t+1}^d/(1+R_t)$ today and pays off B_{t+1}^d in the next period's asset market; and T_t denotes lump sum taxes. The reason the household does not simply set M_t^d to zero is that, as noted above, if it wishes to consume cash goods, it must set cash aside in advance:

$$P_t c_{1t} \leq M_t^d.$$

Assets at the beginning of the next period are:

$$A_{t+1} = M_t^d + P_t(y - c_{1t} - c_{2t}) + B_{t+1}^d. \tag{A.2}$$

Here, M_t^d is the cash balances carried into the previous period's goods market; $P_t y$ is receipts from the sale of y in the previous period's goods market; $P_t \left(c_{1t} + c_{2t}\right)$ represents the bill goods purchased in the previous period's goods market; and B_{t+1}^d is receipts from government debt purchased in the previous period's asset market.

For purposes of gaining intuition, it is useful to follow a suggestion in Woodford (1994) and write (A.1) and (A.2) in a slightly different form. Define 'spending':

$$S_t = P_t c_{1,t} + \frac{P_t c_{2,t}}{1+R_t} + \left(1 - \frac{1}{1+R_t}\right)\left(M_t^d - P_t c_{1,t}\right).$$

In this measure of spending, excess holdings of money balances above what is needed for the cash in advance constraint has a positive price, if $R_t > 0$. Also, the relative 'prices' on $c_{1,t}$ and $c_{2,t}$ accurately reflect that the former involves sacrificing interest earnings. Define 'income', I_t:

$$I_t = \frac{P_t y}{1+R_t} - T_t.$$

Divide both sides of (A.2) by $1 + R_t$, and then substitute out for $B^d_{t+1}/(1 + R_t)$ using (A.1) to obtain:

$$A_{t+1} \leq (1 + R_t)(A_t + I_t - S_t). \tag{A.3}$$

So, the accumulation of the household assets obeys the usual simple equation one finds in a non-monetary, single good model economy.

Some sort of lower bound constraint must be placed on A_t to ensure that the household has a bounded consumption set. We impose that the current value of assets must eventually be non-negative:

$$\lim_{T \to \infty} q_T A_T \geq 0, \tag{A.4}$$

where

$$q_T = \frac{1}{(1 + R_0) \times (1 + R_1) \ldots (1 + R_{T-1})},\ q_0 = 1.$$

It is easy to verify that (A.3)-(A.4) are equivalent to the usual single present-value budget constraint for consumption, S_t, and income, I_t.[73]

We suppose that at each date the household chooses $c_{1,t+j}$, $c_{2,t+j} \geq 0$, M^d_{t+j}, B^d_{t+1+j}, $j \geq 0$, to maximize its utility subject to the restrictions just described and taking A_t, R_{t+j}, P_{t+j}, $j \geq 0$, as given and beyond its control.[74]

Necessary and Sufficient Conditions for Household Optimization

The household first order conditions are as follows:

$$\frac{u_{2,t}}{P_t} = \beta \frac{u_{1,t+1}}{P_{t+1}}, \tag{A.5}$$

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[73]To see this, note first that by recursive substitution, (A.3) implies

$$q^T A_T \leq A_0 + \sum_{t=0}^{T-1} q_t (I_t - S_t).$$

Driving $T \to \infty$, we obtain:

$$\sum_{t=0}^{\infty} q_t S_t \leq A_0 + \sum_{t=0}^{\infty} q_t I_t.$$

This shows that (A.3)-(A.4) imply the standard single-equation budget constraint. To establish the reverse, simply show that if $\{S_t, I_t\}$, $t = 0, 1, \ldots$ satisfy the budget constraint, then they also satisfy (A.3)-(A.4). That the present value of income is finite will be a feature of equilibrium. Otherwise, demand would be unbounded and no equilibrium could exist.

[74]It is easy to verify that in any equilibrium, it must be that $R_t \geq 0$ and $P_t > 0$. Market clearing is impossible if either of these two conditions fails to be satisfied.

and

$$\frac{u_{1,t}}{u_{2,t}} = 1 + R_t. \tag{A.6}$$

Here, $u_{i,t}$ denotes the partial derivative of utility with respect to $c_{i,t}$, $i = 1, 2$. To understand why the first of these Euler equations is implied by household optimization, consider the following argument. Suppose the household reduces its purchases of credit goods in the period t goods market by one dollar and applies that dollar to additional cash goods consumption in the period $t + 1$ goods market. The cost of this is that credit good consumption today drops by $1/P_t$, which translates into an immediate fall in utility of $u_{2,t}/P_t$. This reduction of expenditures frees one dollar in the asset market next period, which can be applied towards the cash in advance constraint for purchasing $1/P_{t+1}$ units of the cash good in next period's goods market. The utility benefit of this, from the standpoint of period t, is $\beta u_{1,t+1}/P_{t+1}$. If the gain here exceeded the cost, obviously the household could not be optimizing, or we would have found a change in its plan which would improve utility. Similarly, if the gain were less than the cost, then the household could raise utility by increasing credit goods consumption in period t and reducing cash goods consumption in period $t+1$. Optimization requires that neither of these strategies be able to raise utility, and this is why the first Euler equation above is an implication of household optimization.

The second Euler equation above is also implied by household optimization. This is established by an argument similar to the one in the previous paragraph. The argument exploits the trade-off between cash and credit goods within the same period. The household can increase current period cash goods consumption by reducing its acquisition of government debt. This reduces its cash receipts in the next period's asset market, reducing the cash available for credit market good consumption today. Note that this Euler equation makes considerable sense: when R is high, it implies that u_1 is relatively high, so that c_1 is relatively low. This makes sense because high R raises the cost to households of purchasing c_1.

There is also a condition associated with the cash in advance constraint, which we write as follows:

$$R_t(P_t c_{1t} - M_t^d) = 0, \tag{A.7}$$

As noted above, only the case, $R_t \geq 0$, needs to be considered. Since, in addition, $P_t c_{1t} - M_t^d$ cannot be negative, (A.7) is a mathematically concise way of stating: if $R_t > 0$ it follows that $P_t c_{1t} = M_t^d$, while if $R_t = 0$, then all we know is $P_t c_{1t} \geq M_t^d$. The key thing, from the point of view of the analysis below, is that when $R_t > 0$, the cash in advance constraint holds as a strict equality. This makes sense. When the interest rate is positive, it is inconsistent with optimization to carry cash in the afternoon that is not absolutely necessary.

In addition to (A.5)-(A.7), the following transversality condition is also implied by household optimization:

$$\lim_{T\to\infty} q_T A_T = 0. \tag{A.8a}$$

The intuition for this condition is straightforward. To see that the limit cannot be positive suppose, on the contrary, that it is. In this case, A_t grows at a rate faster than the rate of interest. But, then it is feasible for households to increase spending in one date without reducing it in another. If this extra spending were financed by a loan, the power of compound interest would cause the resulting debt to spiral upward at a rate equal to the rate of interest. However, with total assets rising at an even greater rate, the household's net asset position would remain consistent with (A.4). The increase in consumption financed in this way raises utility because of nonsatiation, and so we have a contradiction. Thus, optimization implies that the above expression cannot be positive. It cannot be negative because of the restriction, (A.4).

For purposes of analysis, it is convenient to write the transversality condition in a different form. Combining (A.5) and (A.6), we find $u_{1,t} = \beta(1+R_t)u_{1,t+1}P_t/P_{t+1}$. Substituting this into the expression for q_t, we find

$$q_t = \left(\frac{P_0}{u_{1,0}}\right)\frac{\beta^T u_{1,t}}{P_t},\ t = 0, 1, 2,\ ... \tag{A.9}$$

After multiplying both sides of (A.8a) by the positive constant, $P_0/u_{1,0}$, the transversality condition reduces to:

$$\lim_{T\to\infty} \beta^T u_{1,T}\frac{A_T}{P_T} = 0. \tag{A.10}$$

It turns out that (A.5)-(A.10) are not just necessary for optimization. They are also sufficient. This is easily established with a suitably adjusted version of the proof to Stokey and Lucas, with Prescott (1989), Theorem 4.15.

A.1.2. Government

The government purchases no goods, and only participates in the asset market. Its sources of funds in the asset market are new debt issues, tax revenues and newly created money, $M_t^s - M_{t-1}^s$. It uses these funds to pay its outstanding debt obligations, B_t^s. Equating sources and uses of funds:

$$\frac{B_{t+1}^s}{1+R_t} + T_t + M_t^s - M_{t-1}^s = B_t^s.$$

At time t, the government takes M_{t-1}^s and B_t^s as given, $t = 0, 1, ...$. At date 0, $M_{-1}^s + B_0^s = A_0$. *Government policy* is a sequence of B_{t+1}^s's, T_t's, and M_t^s's that satisfy this flow budget constraint.

This budget constraint can also be written as follows:

$$\frac{A_{t+1}^s}{1+R_t} + T_t + \frac{R_t}{1+R_t}M_t = A_t^s,$$

where A_t^s measures total nominal assets, $A_t^s = B_t^s + M_{t-1}^s$, and $A_0^s = A_0$. Recursively substituting this expression forward, we find that for each fixed T :

$$q_T A_T^s + \sum_{t=0}^{T-1} q_t \left[T_t + \frac{R_t}{1+R_t} M_t \right] = A_0. \tag{A.11}$$

The presence of $R_t M_t/(1+R_t)$ reflects the interest costs the government saves when it issues money rather than bonds. The government's 'intertemporal budget equation' is the above expression, with $q_T A_T^s$ absent and $T-1$ replaced by ∞ :

$$\sum_{t=0}^{\infty} q_t \left[T_t + \frac{R_t}{1+R_t} M_t \right] = A_0. \tag{A.12}$$

The only restriction we have placed on government policy is that the flow budget constraint is satisfied for all possible values of prices, $\{q_t, P_t, R_t,\ t \geq 0\}$. That is, we require that (A.11) hold. But, no assumption has been made that (A.12) holds for all possible prices. Government policy is said to be *Ricardian* if (A.12) holds for all possible prices. Government policy is *non-Ricardian* if (A.12) holds only at the equilibrium prices. (We shall see that, at equilibrium prices, (A.12) must be satisfied regardless of whether government policy is Ricardian or non-Ricardian. This follows from (A.8a) and the fact that, in equilibrium, $A_t^s = A_t$.)

It turns out that (A.11) converges to (A.12) if, and only if,

$$q_T A_T^s \to 0. \tag{A.13}$$

According to this result, we can equivalently define a Ricardian policy as one that enforces (A.13) at all possible prices and a non-Ricardian policy as one that does not.

A.1.3. Firms

Firms in this economy are simple. They buy y from households and transform this into cash and credit goods. Given the assumed linearity of the production technology, the resource constraint turns out to have the following form:

$$c_{1t} + c_{2t} = y. \tag{A.14}$$

A.1.4. Equilibrium

A general equilibrium for this economy is a sequence of prices and interest rates, P_t and R_t, a sequence of consumptions, c_{1t}, c_{2t}, and a sequence of money supplies and bonds, M_{t+1} and B_{t+1} such that households optimize, the government flow budget constraint is satisfied, and markets clear. Bond market clearing requires

$$B_{t+1}^s = B_{t+1}^d = 0,$$

money market clearing requires

$$M_t^s = M_t^d = M_t,$$

say, for $t \geq 0$. These conditions imply that $A_{t+1} = M_t^d + B_{t+1}^d = A_{t+1}^s$. Goods market clearing corresponds to the resource constraint, (A.14).

A feature of equilibrium which will be useful in the analysis is

$$1 + R_t = \frac{1-\sigma}{\sigma} \frac{1}{w_t^{1-\nu}} \geq 1, \ w_t \equiv \frac{c_{1t}}{c_{2t}}. \tag{A.15}$$

We obtain this using (A.6) and our parametric form for the utility function. When $R_t > 1$, we can rewrite this to obtain the model's 'money demand' function. The binding cash in advance constraint, $c_{1t} = m_t$, and the resource constraint imply $w_t = m_t/(y - m_t)$. Solving (A.15) for m_t yields:

$$m_t = \frac{y}{1 + \left[\frac{\sigma}{1-\sigma}(1 + R_t)\right]^{\frac{1}{1-\nu}}}. \tag{A.16}$$

A.2. Constant Money Growth

Here, we consider the set of equilibria associated with a fixed money growth rate policy. We show that there is one equilibrium in which inflation is constant and equal to the money growth. There is also a continuum of equilibria with explosive inflation.

We suppose that the government sets $B_{t+1}^s = 0$ for all $t \geq 0$ by paying off the entire stock of debt in the first period. In addition, it sets $M_t^s = \mu M_{t-1}^s$, for $t = 0, 1, \ldots$, where $\mu \geq 1$. Money growth is accomplished by means of lump sum tax transfers. In particular:

$$\begin{aligned} T_0 &= B_0^s - (\mu - 1) M_{-1}^s, \\ T_t &= -(\mu - 1) M_{t-1}^s, \ t \geq 1. \end{aligned}$$

It is straightforward to verify that with this specification of policy, there are many price sequences such that (A.10) is satisfied. Technically, it does not fit into our formal definition of a Ricardian policy, because (A.10) is not satisfied for all prices. To see this, note that under this policy, A_t is composed only of the money supply. Then, (A.10) would be violated if the price level fell sufficiently rapidly. Still, for practical purposes it is not too misleading to think of this as a Ricardian policy.

It is useful to rewrite the household's dynamic Euler equation by multiplying both sides of (A.5) by M_t and using $M_{t+1} = \mu M_t$ to obtain:

$$u_{2,t} m_t = \frac{\beta}{\mu} u_{1,t+1} m_{t+1}, \; m_t \equiv \frac{M_t}{P_t}. \tag{A.17}$$

A sequence of prices and quantities represents an equilibrium if, and only if, (A.7)-(A.14) and $P_t > 0$, $R_t, c_{1t}, c_{2t} \geq 0$ are satisfied.

A.2.1. A Characterization Result for Equilibria

We will now simplify the equilibrium conditions to obtain a useful set of sufficient conditions for equilibria in which the cash in advance constraint binds. In this case (A.14) allows one to express (A.17) as a difference equation in m_t and m_{t+1} alone. Also, the fact that the cash in advance constraint binds implies that w_t in (A.15) can be written:

$$w_t = \frac{m_t}{y - m_t}.$$

Using this notation, (A.17) can be expressed as a difference equation in w_t and w_{t+1}:

$$a(w_t) = b(w_{t+1}), \tag{A.18}$$

where

$$\begin{aligned} a(w) &= \frac{\sigma w}{(1-\sigma) w^{\nu} + \sigma}, \\ a'(w) &= \frac{\sigma}{\left[(1-\sigma) w^{\nu} + \sigma\right]^2} \left[(1-\sigma)(1-\nu) w^{\nu} + \sigma\right]. \end{aligned} \tag{A.19}$$

and

$$\begin{aligned} b(w) &= \frac{\beta(1-\sigma)}{\mu} \frac{w^{\nu}}{(1-\sigma) w^{\nu} + \sigma} \\ b'(w) &= \frac{\beta(1-\sigma)}{\mu} \frac{w^{\nu-1} \nu \sigma}{\left[(1-\sigma) w^{\nu} + \sigma\right]^2}. \end{aligned} \tag{A.20}$$

Here, a' and b' represent the derivatives of a and b, respectively, with respect to w. The transversality condition reduces, in the present notation, to:

$$\lim_{T\to\infty}\beta^T b(w_T)=0. \tag{A.21}$$

We are now in a position to state our characterization result.

Proposition A.1. *Suppose $w_t \geq 0$, $t=0,1,2,...$, satisfies (A.18), (A.21) and (A.15). Then, w_t corresponds to an equilibrium.*

Proof: Write

$$\begin{aligned} m_t &= y\frac{w_t}{1+w_t},\ P_t=\frac{M_t}{m_t}, \\ R_t &= \frac{\sigma}{1-\sigma}\frac{1}{w_t^{1-\nu}},\ c_{2t}=\frac{y}{1+w_t},\ c_{1t}=c_{2t}w_t, \end{aligned}$$

and verify that all the equilibrium conditions are satisfied at these prices and quantities. QED

A.2.2. Multiple Equilibria With Ricardian, Constant Money Growth

We will use the characterization result to show that there is a continuum of equilibria in this economy when the money growth rate, μ, is constant and greater than unity. Note first from (A.18) that there is exactly one equilibrium with $w_t=w^*$ for all t:

$$w^*=\left[\frac{1-\sigma}{\sigma}\frac{\beta}{\mu}\right]^{\frac{1}{1-\nu}}. \tag{A.22}$$

It is easily verified that this satisfies the conditions of the characterization result. For example, substituting w^* into (A.15) yields the result that the interest rate is positive, with $1+R=\mu/\beta$. This is greater than unity by our assumptions on μ and β. Because real balances are constant in this equilibrium, the rate of inflation is equal to μ.

The intuition underlying (A.22) is straightforward. The relative quantity of cash goods consumed in the equilibrium (i.e., w^*) is increasing in $1-\sigma$, which is the relative weight in utility on these goods. It is decreasing in the money growth rate, μ, because increases in μ raises the nominal rate of interest, which increases the cost of the cash good. Finally, consider $\nu\to 1$. This is easiest to interpret when $\sigma=1/2$. In this case, the two consumption goods are perfect substitutes. Consequently, if the cash good is more expensive than the credit good, as is the case when $\mu \geq 1$, zero cash goods will be consumed, and $w^* = 0$ as $\nu \to 1$.

We now show that there are other equilibria too, in which inflation exceeds μ. To do this, we first study the properties of the functions, $a(w)$ and $b(w)$.

According to (A.19), $a(0) = 0$ and $a'(0) = 1$. Also, $a'(w) > 0$ for all $w \geq 0$. At the same time, (A.20) indicates that $b(0) = 0$, $b'(w) \to \infty$ as $w \to 0$, and $b'(w) > 0$ for $w > 0$. These observations establish that $a(w)$ and $b(w)$ coincide at $w = 0$, with b rising more steeply than a for small values of w.

From the discussion leading up to (A.22), we know that there is a unique value of $w > 0$, namely w^* in (A.22), where $a(w) = b(w)$. Since the two functions are continuous for $0 < w < w^*$, it follows that $b(w) > a(w)$ for w in this interval, and that a is steeper than b at $w = w^*$. The latter observation can be confirmed by direct differentiation, which yields:

$$\frac{a'(w^*)}{b'(w^*)} = \left(\frac{1-\sigma}{\sigma}\right)^{\frac{1}{1-\nu}} \left(\frac{1-\nu}{\nu}\right) \left(\frac{\beta}{\mu}\right)^{\frac{\nu}{1-\nu}} + \frac{1}{\nu} > 1.$$

The strict inequality reflects that the expression immediately after the equality is positive and that $1/\nu > 1$ because $0 < \nu < 1$.

Our results on the a and b functions are summarized in Figure A. Note how b rises above a, and then crosses once. Eventually, the two curves are parallel, since $a'(w)$ and $b'(w)$ both converge to zero as $w \to \infty$. We can use this diagram to study the set of equilibria for the model.

Consider an arbitrarily selected $w_0 < w^*$. To determine the value of w_1 implied by (A.18), draw a line in the vertical direction up to $a(w_0)$. Then, identify a w_1 such that $b(w_1)$ equals $a(w_0)$. This can be found by following a horizontal line to the left of $a(w_0)$, until it intersects the b curve. The properties of these curves that we have derived guarantee that such an intersection will occur for a positive value of w. With w_1 in hand, compute w_2 in the same way, and so on.

It should be clear that the sequence of w_t's computed in this way converges to 0. Along this path, $b(w_t) \geq 0$ and $b(w_t) \to 0$ as $t \to \infty$. Because b is bounded above along the path, (A.21) is satisfied. Also, using: (i) w_t declines monotonically; (ii) $R > 0$ at w^*; and (iii) $w_t > 0$ for a given t, (A.15) implies that $R_t > 0$ for each t. This establishes that the sequence just computed constitutes an equilibrium.[75] But, the same argument can be applied for each $0 < w_t < w^*$. In each of these equilibria there is a hyperinflation, as $w_t \to 0$.[76]

[75] Recall, in constructing (A.18) we assumed that the cash in advance constraint is binding. This assumption has been verified for $w_0 < w^*$.

[76] Our results would not be significantly affected if we allowed labor to be endogenous. Intro-

A.2.3. Unique Equilibrium With non-Ricardian, Constant Money Growth

The previous subsection showed that with a particular Ricardian policy, constant money growth results in a continuum of equilibria. Here is a particular non-Ricardian policy:

$$T_t = P_t s - \frac{R_t}{1+R_t} M_t,$$

where s is a positive constant. It is easy to verify that the set of equilibria under this policy is a strict subset of the set of equilibria analyzed in previous section. Thus, we conclude that with constant money growth, a non-Ricardian policy does not lead to an overdetermined price level.

A.3. Fixed Interest Rate Policies

This section considers two representations of policy in which the government pegs the nominal rate of interest to a constant value, $R > 0$. In the first representation, fiscal policy is Ricardian and there exists a continuum of equilibria. In the second, policy is non-Ricardian and the equilibrium, if it exists, is unique.

The fixed value of R pins down m (see (A.16)), c_1 and c_2:

$$c_1 = m, \; c_2 = y - c_1.$$

As a consequence, the marginal utility of the cash good is constant, so that

$$\frac{P_{t+1}}{P_t} = \beta(1+R), \text{ for all } t.$$

Consider the following two specifications of policy:

$$\begin{aligned} T_t &= -\frac{R}{1+R} m P_t + \varepsilon A_t \qquad \text{(A.23)} \\ T_t &= -\frac{R}{1+R} m P_t + d P_t. \end{aligned}$$

ducing labor as a third argument in the utility function has the effect of adding an extra Euler equation, $-u_3/u_2 = f'(l)$, where $f'(l)$ denotes the marginal product of labor, l, and u_3 denotes the marginal utility of labor. Feasibility restricts l to lie in some subspace, $l \in D$ (for example D might be the unit interval). Also, $y = f(l)$ denotes the production function. Combining the new Euler equation with the resource constraint produces a function, $l = F(w)$, where F has a nice analytical characterization with standard preferences and technology. To find an equilibrium, one would still start by looking for w_t's that solve the difference equation, $A(w_t) = B(w_{t+1})$. One would then have to verify $F(w) \in D$, in addition to the other conditions listed in the characterization result, before verifying that the candidate w_t's represent an equilibrium.

where d is a non-negative constant and $0 < \varepsilon \leq 1$. We will see that the first policy is Ricardian, while the second is not. These policies may initially appear strange, but we hope that in a moment it will become clear what the motivation for them is. Recall that to determine whether a policy is Ricardian or not requires determining whether (A.13) holds for all possible prices or whether it only holds for equilibrium prices.

To investigate this further, it is convenient to write the flow budget constraint in real terms:

$$\beta a_{t+1} + \tau_t + \frac{R}{1+R} m = a_t.$$

Here, $a_{t+1} = A_{t+1}/P_{t+1}$ and $\tau_t = T_t/P_t$. Substituting the first specification of policy in (A.23) into the flow budget constraint:

$$a_{t+1} = \frac{1-\varepsilon}{\beta} a_t \tag{A.24}$$

We seek to understand how $\tilde{a}_t \equiv \beta^t a_t$ evolves as $t \to \infty$. To see why, note that, by substituting from (A.9):

$$q_T A_T = P_0 \beta^T \frac{A_T}{P_T} = P_0 \tilde{a}_T. \tag{A.25}$$

Recall that a policy that makes $q_T A_T \to 0$ for all possible prices corresponds to a Ricardian policy and one that makes this happen only for the equilibrium prices is a non-Ricardian policy.

Multiplying both sides of (A.24) by β^{t+1}, we find

$$\tilde{a}_{t+1} = (1-\varepsilon)\tilde{a}_t = (1-\varepsilon)^t \tilde{a}_0,$$

or, using (A.25):

$$q_T A_T = (1-\varepsilon)^t A_0 \to 0.$$

This establishes that the first policy in (A.23) is Ricardian. The government's policy prevents the debt from exploding too fast, regardless of what happens. As a result, the intertemporal budget equation provides no useful restriction for pinning down prices.

Now consider the second policy in (A.23). For this policy, total real assets evolve as follows:

$$a_{t+1} = \frac{1}{\beta}\left(a_t - d\right).$$

The policy makes the evolution of total asset exogenous, while letting the private economy determine the breakdown of real assets between money and bonds so that it is consistent with the interest rate peg. Solve this for a_t and then multiply by β^t :

$$\beta^t a_t = a_0 - \frac{d}{1-\beta} + \frac{d}{1-\beta}\beta^t,$$

so that

$$\beta^t a_t \to \frac{A_0}{P_0} - \frac{d}{1-\beta},$$

where we have used $a_0 = A_0/P_0$. This is a non-Ricardian policy because $\beta^t a_t \to 0$ for only one value of P_0, the one that satisfies:

$$\frac{A_0}{P_0} = \frac{d}{1-\beta}.$$

We can now summarize our results for the interest rate peg. If it is accompanied by a Ricardian policy, then the price level is not pinned down by the intertemporal budget equation, nor by the rest of the model. The model only pins down M_t/P_t and P_{t+1}/P_t, but not the numerator and denominator terms. Under the non-Ricardian policy, the intertemporal budget equation supplies the extra equation needed. Once again, the price level is not overdetermined under the non-Ricardian policy.

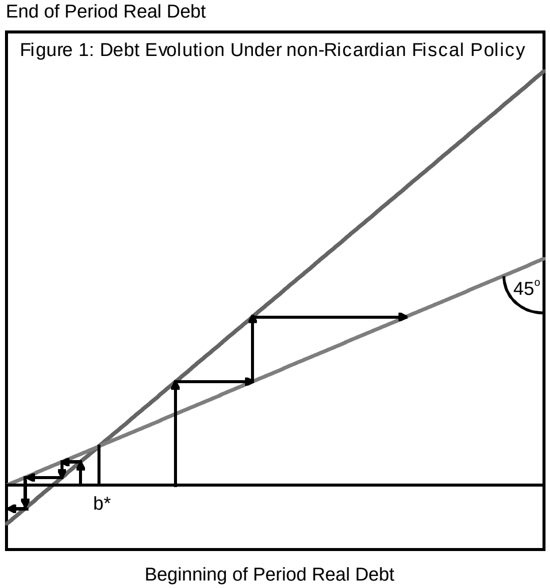

Figure 1: Debt Evolution Under non-Ricardian Fiscal Policy

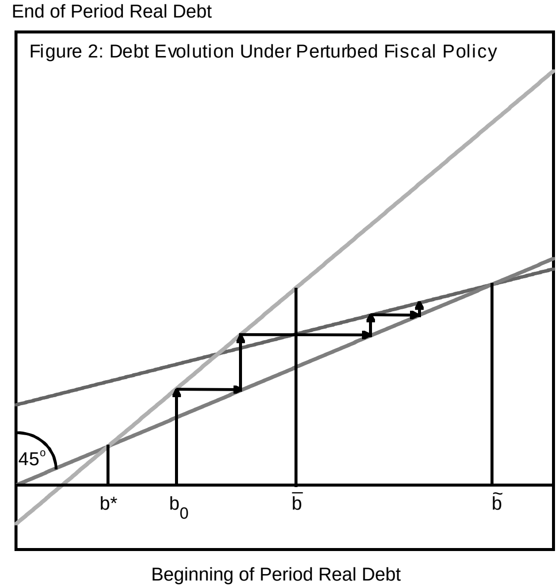

Figure 2: Debt Evolution Under Perturbed Fiscal Policy

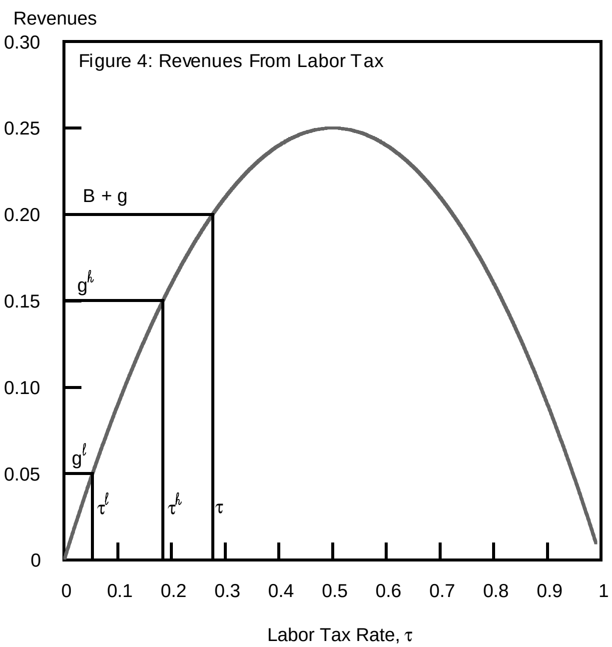

Figure 4: Revenues From Labor Tax

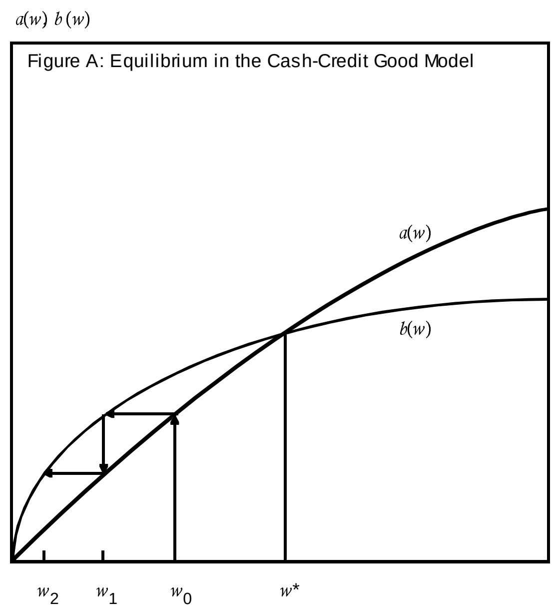

Figure A: Equilibrium in the Cash-Credit Good Model

Figure 3: Evolution of Inflation Under Aggressive Interest Rate Rule and non-Ricardian Policy