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Government Policy, Credit Markets, and Economic Activity[*]

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1. Introduction

1.1. Preliminary Observations

The recession that began in late 2007 poses new challenges for macro-economic modeling. Asset values collapsed, initially in housing and then in equity [see Figure 5.1(a)]. In late 2008, interest rate spreads suddenly jumped to levels not seen in over seventy years [see Figure 5.1(b)].[1] There was widespread concern among policymakers that financial markets had become dysfunctional because of a deterioration in financial firm balance sheets associated with the fall in asset values.[2] These concerns were reinforced by the dramatic fall in investment in late 2008 [see Figure 5.1(c)], which suggested that a serious breakdown in the intermediation sector might have occurred. The U.S. Treasury and Federal Reserve (Fed) reacted forcefully. The Fed's actions had the effect of reducing the cost of funds to financial institutions. For example, the Federal Funds rate was driven to zero [see Figure 5.1(d)] and the interest rate on the three-month commercial paper of financial firms also fell sharply. In addition, the Fed took a variety of unconventional actions by acquiring various kinds of financial claims on financial and nonfinancial institutions. Standard macroeconomic models are silent on the rationale and on the effects of the Fed's unconventional monetary policies.

Still, there is casual evidence that suggests the Fed's unconventional monetary policy helped.[3] The Fed began to purchase financial assets in late 2008, and financial firm commercial paper spreads dissipated quickly thereafter. In March 2009 the Fed expanded its asset purchase program enormously and corporate bond spreads also began to come down [Figure 5.1(b)]. Soon, aggregate output began to recover and the National Bureau of Economic
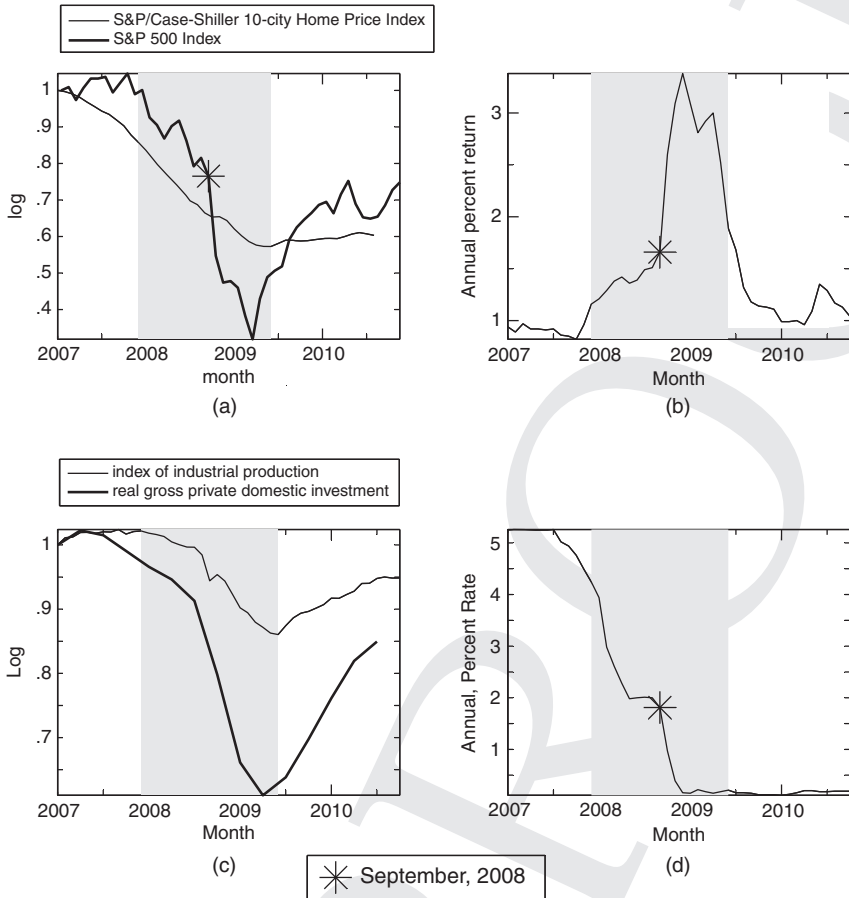
Figure 5.1. (a) Real equity and housing prices. (b) Spreads, BAA over AAA rated bonds. (c) Production and investment. (d) Federal funds rate

Research declared an end to the recession in June 2009 [Figure 5.1(c)]. Of course, it is difficult to say what part of the recovery (if any) was due to the Fed's policies, what part was due to the tax and spending actions in the American Recovery and Reinvestment Act of 2009, and what part simply reflects the internal dynamics of the business cycle. Many observers suppose that the Fed's policies had at least some effect.

These observations raise challenging questions for macroeconomics:

• What are the mechanisms whereby a deterioration in financial firm balance sheets causes a drop in financial intermediation and a jump in interest rate spreads?

- How do reductions in interest rate costs for financial firms and large-scale government asset purchases correct these financial market dysfunctions? What are the effects of these actions on economic efficiency?

The answers to these questions are important for determining which asset market program should be undertaken and on what scale. Traditional macroeconomic models used in policy analysis in central banks have little to say about these questions. Although our analysis is primarily motivated by events in the United States since 2007, the questions we ask have a renewed urgency because of recent events in Europe. There is a concern that a collapse in the market value of sovereign debt may, by damaging the balance sheets of financial firms, plunge that continent into a severe recession. Models are required that can be used to think about the mechanisms by which such a scenario could unfold.

We survey the answers to questions raised in the preceding two bullets from the perspective of four standard models borrowed from the banking literature and inserted into a general equilibrium environment. In each case, we drastically simplify the model environment so that we can focus sharply on the main ideas. Accordingly, the kind of details that are required for ensuring that models fit quarterly time series data well are left out. For example, the models have only two periods, most shocks are left out of the analysis, and we abstract from such things as labor effort, capital utilization, habit persistence, nominal variables, money, price, and wage-setting frictions. We also abstract from the distortionary effects of seigniorage and the other mechanisms by which governments and central banks acquire the purchasing power to finance their acquisition of private assets. We abstract from these complications by assuming that revenues are raised with nondistorting, lump-sum taxes. Finally, we make assumptions that allow us to abstract from the effects of changes in the distribution of income in the population. For the reasons described in Section 2, it is important to relax this assumption in more general analyses of unconventional monetary policy.

Ultimately, the questions just raised must be addressed in fully specified dynamic, stochastic general equilibrium (DSGE) models. Only then can we say with confidence which of the financial frictions subsequently discussed is quantitatively important. Similarly, we require a DSGE model if we are to quantify the magnitude of the required policy interventions. Work on the task of integrating financial frictions into DSGE models is well under way.[4] Our hope is that this chapter may be useful in this enterprise by providing a bird's-eye view of the qualitative properties of the different models, in terms of their implications for the questions just raised.

Our survey does not examine all models of financial frictions. For example, we do not review models that can be used to think about the effects of government asset purchases on a liquidity shortage (see, e.g., Moore, 2009; Kiyotaki & Moore, 2008).[5] Instead, we review models that are in the spirit of Mankiw (1986), Bernanke, Gertler, and Gilchrist (1999) (BGG), Gertler and Karadi (2009), and Gertler and Kiyotaki (2011) (GK[2]). We review four models. The first two feature moral hazard problems, and the third features adverse selection. The fourth model features asymmetric information and monitoring costs. The latter model resembles that of BGG closely, although we follow Nowobilski (2011) by assuming that the financial frictions apply to financial rather than to nonfinancial firms.

Our models capture in different ways the hypothesis that a drop in bank net worth caused the rise in interest rate spreads and the fall in investment and intermediation that occurred in 2007 and 2008.[6] In our first two models, these effects involve the operation of fundamental "nonlinearities." In particular, in these models there is a threshold level of bank net worth, such that when net worth falls below it, the equations that characterize equilibrium change. The second two models involve nonlinearities in the sense that the equations characterizing equilibrium are not linear. However, they are not characterized by the more fundamental type of nonlinearity found in the first two models.

Where possible, we use our four models to investigate the consequences for economic efficiency of the following tax-financed government interventions: (i) reductions in the cost of funds to financial firms, (ii) equity injections into financial firms, (iii) loans to financial and nonfinancial firms, and (iv) transfers of net worth to financial firms. Regarding (ii), we define an equity injection as a tax-financed transfer of funds to a bank in which all the resulting profits are repaid to the government. In the case of (iii), we define a government bank loan as a tax-financed commitment of funds that must be repaid on the same terms as those received by ordinary depositors.

All the models suggest that (i) helps to alleviate the dysfunctions triggered by a fall in net worth, though the precise mechanisms through which this happens vary. There is less agreement among the models in the case of (ii) and (iii). Whether these policies work depend on the details of the financial frictions.

All the models suggest that (iv) helps. This is perhaps not surprising, because (iv) in effect undoes what we assume to be the cause of the trouble. Still, this aspect of our analysis is best viewed as incomplete, for at least two reasons. First, our models are silent on why markets cannot achieve the

transfer of net worth to financial firms. Within the context of the models, there is no fundamental reason why it is that when funds are transferred to banks they must go in the form of credit and not net worth. We simply assume that the quantity of net worth in financial firms is fixed exogenously. We think there is some empirical basis for the assumption that bank net worth is hard to adjust quickly in response to a crisis, but whatever factors account for this observation should be incorporated into a full evaluation of (iv). Second, policy (iv) entails a redistribution of wealth and income among the population. Our models abstract from the effects of wealth redistribution.

Some policies are best analyzed in only a subset of our models. Examples of such policies include leverage restrictions on banks, as well as a policy of bailing out the creditors of banks experiencing losses on their portfolios. We study the first of these policies in only two of our models, the ones in Sections 4 and 6. We study creditor bailouts in Section 4.

1.2. Overview of the Model Analysis

1.2.1. Moral Hazard I: "Running Away" Model

We first describe a simplified version of the analysis in GK[2], which focuses on a particular moral hazard problem in the financial sector.[7] This problem stems from the fact that bankers have the ability to abscond with a fraction of the assets they have under management. A repeated version of the one-period model that we study provides a crude articulation of the post-2007 events. Before 2007, interest rate spreads were at their normal level (actually, zero according to the model), and the financial system functioned smoothly in that the first-best allocations were supported in equilibrium. Then, with the collapse in banking net worth, interest rate spreads jumped and financial markets became dysfunctional, in the sense that the volume of intermediation and investment fell below their first-best levels.

According to the model, banks respond to the decline in their own net worth by restricting the amount of deposits that they issue. Banks do so out of a fear that if they tried to maintain the level of deposits in the face of the decline in their net worth, depositors would lose confidence and take their money elsewhere.[8] Depositors would do so in the (correct) anticipation that a higher level of bank leverage would cause bankers to abscond with bank assets. From this perspective, a sharp cut in the cost of funds to banks calms the fears of depositors by raising bank profits and providing bankers with an incentive to continue doing business normally.

In the case of direct equity injections and loans, we follow GK[2] in assuming that the government can prevent banks from absconding with government funds.[9] Under these circumstances, it is perhaps not surprising that government equity injections and loans, (ii) and (iii), are effective. With the government taking over a part of the economy's intermediation activity, the amount of intermediation handled by the banking system is reduced to levels that can be handled efficiently with the reduced level of banking net worth. Of course, if the nature of the financial market frictions is not something that can be avoided by using the government in this way, then one suspects that (ii) and (iii) are less likely to be helpful. This is the message of our second model.

1.2.2. Moral Hazard II: Unobserved Banker Effort

Our second model captures moral hazard in banking in a different way. We suppose that bankers must exert a privately observed and costly effort to identify good investment projects. The problem here is not that bankers may abscond with funds. Instead, it is that bankers may exert too little effort to make sure that assets under management are invested wisely. Bankers must be given an incentive to exert the efficient amount of effort. One way to accomplish this is for bank deposit rates to be independent of the performance of bank portfolios, so that bankers receive the full marginal return from exerting extra effort. But bankers must have sufficient net worth of their own if the independence property of deposit rates is to be feasible. This is because we assume that bankers cannot hold a perfectly diversified portfolio of assets. As a result, bankers – even those that exert high effort – occasionally experience a low return on their assets. For deposit rates to be independent of the performance of banker portfolios, bankers with poorly performing portfolios must have sufficient net worth to pay the return on their deposits. We show that when bankers have a sufficiently high level of net worth, then bank deposit rates are independent of the performance of bank portfolios and equilibrium supports the efficient allocations.

Financial markets become dysfunctional when the banks whose assets perform poorly have too little net worth to cover their losses. Depositors in such banks must in effect share in the losses by receiving a low return. To be compensated for low returns from banks with poor assets, depositors require a relatively high return from banks with good assets. But when deposit rates are linked to the performance of bank assets in this way, bankers have less incentive to exert effort. Reduced effort by bankers pushes down the average return on bank assets and hence deposit rates for savers. With lower deposit

rates, household deposits – and hence investment – are reduced below their efficient levels.

Consider the implications for policy. Interest rate subsidies, policy (i), help by reducing the cost of funds to banks. This policy reduces banks' liabilities in the bad state and so increases the likelihood that deposit rates can be decoupled from bank asset performance. This result is of more general interest, because it conflicts with the widespread view that interest rate subsidies to banks cause them to undertake excessive risk. In our environment, an interest rate subsidy increases bankers' incentive to undertake effort, leading to a rise in the mean return on their portfolios and a corresponding reduction in variance. Interest rate subsidies have this effect by raising the marginal return on banker effort.

Government equity injections and loans, policies (ii) and (iii), have no effect in the model. Although the proof of this finding involves details, the result is perhaps not surprising. The government equity injections and bank loans that we consider do not offer any special opportunity to avoid financial frictions in the way that our first model of moral hazard does. It is not obvious (at least to us) what unique advantage the government has in performing intermediation when that activity involves a costly and hidden effort. Our hidden-effort model illustrates the general principle that the sources of moral hazard matter for whether a particular government asset purchase program is effective.

Our hidden-action model is well suited to studying the effects of leverage restrictions and bailouts of creditors to banks with poorly performing assets. We have previously noted that when net worth is low, it may not be possible for deposit rates to be decoupled from the performance of bank assets. Obviously, if the quantity of deposits were sufficiently low, then deposit rates could be fixed and independent of bank asset performance even if net worth is low. We show that when binding leverage restrictions are placed on banks when net worth is low, social welfare is increased.

1.2.3. Adverse Selection

Our third model focuses on adverse selection as a source of financial market frictions.[10] In our model, the portfolios of some banks are relatively risky in that these banks have a high probability of not being able to repay their creditors. Banks have access to credit markets. However, because bank creditors cannot assess a given bank's riskiness, all banks must pay the same interest rate for credit.[11] This interest rate must be high enough to take into account the bankers with high-risk portfolios that are likely not to

repay. As is the case in adverse-selection models, under these circumstances "good" bankers – those who could potentially acquire low-risk assets – find it optimal not to borrow at all. This reflects the fact that good bankers repay creditors with high probability, so that their expected profits from borrowing and acquiring securities are low. When the net worth of bankers drops, the adverse-selection effect driving out good bankers becomes stronger.

Because the rise in the interest rate spread on credit to banks drives away potentially good bankers, the quality of the assets on the balance sheet of banks seeking credit deteriorates. The result is a decline in the overall return on bank assets and thereby a fall in the equilibrium return on household saving. The reduction in saving in turn causes a fall in investment. We show that this fall in investment corresponds to an increase in the gap between the equilibrium level of investment and investment in the first-best equilibrium. In this sense, the decline in banker net worth makes the banking system more dysfunctional. For these reasons the adverse-selection model formalizes a perspective on the financial events since 2007 that is similar to the one captured by the models in the previous two sections.

We consider the policy implications of the adverse-selection model. A tax-financed transfer of net worth to bankers improves equilibrium outcomes. This is not surprising, because an increase in banker net worth reduces banks' dependence on external finance and hence reduces the adverse-selection distortions. Government policies that have the effect of subsidizing the cost of funds to bankers also improve equilibrium outcomes. The reason is that they raise the return on saving received by households and have the effect of reducing the gap between the equilibrium interest rate and the social return on loans.

1.2.4. Asymmetric Information and Monitoring Costs

Our fourth model of financial frictions focuses on asymmetric information and costly monitoring as the source of financial frictions. At this time, the costly state verification model is perhaps the most widely used model of financial frictions in macroeconomics.[12]

In the model, bankers combine their own net worth with loans to acquire the securities of firms with projects that are subject to idiosyncratic risk. We assume that a bank can purchase the securities of at most one firm, so that the asset side of bank balance sheets is risky. There are no financial frictions between a bank and the firm whose securities it purchases. The realization of uncertainty in a firm's project is observed by its bank, but can be observed by bank creditors only by their paying a monitoring cost. We assume that

creditors offer banks a "standard debt contract." The contract specifies a loan amount and an interest rate. The bank repays the loan with interest, if it can. If the securities of a bank are bad because the issuing firm has an adverse idiosyncratic shock then the bank declares bankruptcy, is monitored by its creditor, and loses everything.[13] Our characterization of the 2007–8 crisis follows the line explored with our other models by supposing that the crisis was triggered by a fall in bank net worth. In addition, our model also allows us to consider the idea that an increase in the cross-sectional dispersion of idiosyncratic shocks played a role.[14]

Our environment is sufficiently simple that we obtain an analytic characterization of the inefficiency of equilibrium. We show that in the model the marginal social return on credit to banks exceeds the average return, and it is the latter that is communicated to bank creditors by the market. Lending to banks is inefficiently low in the equilibrium because a planner prefers that the credit decision be made based on the marginal return on loans. The problem is exacerbated when the net worth of banks is low. Not surprisingly, we find that a policy of subsidizing bank interest rate costs improves welfare. Also, the optimal subsidy is higher when bank net worth is low. In addition, we study the effects of direct government loans to banks, but find that this has no impact on the equilibrium.

The rest of the chapter is organized as follows. Section 2 describes what we call the Barro-Wallace irrelevance proposition, which sets out a basic challenge that any model of government asset purchases must address. The following two sections describe the two models of moral hazard. Section 5 studies the model of adverse selection. Section 6 studies the model with asymmetric information and costly monitoring. The final section presents concluding remarks.

2. The Barro-Wallace Irrelevance Proposition

Any analysis of unconventional policy must confront a basic question. If the government acquires privately issued assets by levying taxes (either in the present or in the future), then the ownership of the asset passes from private agents to the government, which later reduces households' tax obligations as the asset bears fruit. The question any analysis of asset purchases by the government has to answer is why it makes a difference whether private agents hold assets themselves or the government holds them on taxpayers' behalf. In the simplest economic settings, households' intertemporal consumption opportunities are not affected by government asset purchases, so that such purchases are irrelevant for allocations and prices. We refer

to this irrelevance result as the Barro-Wallace irrelevance proposition, because it is closely related to the Ricardian equivalence result emphasized by Barro (1974) and extended by Wallace (1981) to open market operations.[15] Any analysis in which government asset purchases have real effects must explain what assumptions have been made to defeat the Barro-Wallace irrelevance result.

One way to defeat Barro-Wallace irrelevance builds on heterogeneity in the population. For example, suppose that a subset of the population has a special desire to hold a certain asset (e.g., thirty-year treasury bonds). If the government engages in a tax-financed purchase of that bond, then in effect the bond is transferred from the subset of the population that holds it initially to all taxpayers. Such a redistribution of assets among heterogeneous agents may change prices and allocations. This type of logic may be useful for interpreting the recent substantial changes that have occurred in the Federal Reserve's balance sheet.[16] We do not pursue this line of analysis further here.

There are other ways in which tax-financed purchases of private securities may have real effects. In the examples we explore, this can happen by changing the market rate of interest.

3. Moral Hazard I: "Running-Away" Model[17]

We construct a two-period model. In the first period, households make deposits in banks. Bankers combine these deposits with their own net worth and provide funds to firms. In the second period, households purchase the goods produced by firms by using income generated by bank profits and interest payments on bank deposits. The source of moral hazard is that bankers have an option to default by absconding with an exogenously fixed fraction of their total assets, leaving the rest to depositors. When a sufficiently large fraction of a bank's assets is purchased with bankers' own net worth, then a bank simply hurts itself by defaulting, and it chooses not to do so. We show that, when the net worth of banks is sufficiently large that the option to default is not relevant, then the equilibrium allocations correspond to the first-best efficient allocations. We refer to this scenario as a "normal time." When banks' net worth is sufficiently low, banks restrict the supply of deposits. Banks do this because they know that if they planned a higher level of deposits, depositors would rationally lose confidence and take their deposits elsewhere. With the supply of deposits reduced in this way, and no change in demand, the market-clearing interest rate on deposits is low. Because the return on bank assets is fixed by assumption, the result

is an increase in banks' interest rate spreads.[18] We refer to the situation in which bank net worth is so low that the banking system is dysfunctional and conducts too little intermediation as a "crisis time." Thus, the model articulates one view about what happened in the past few years: "a fall in housing prices and other assets caused a fall in bank net worth and initiated a crisis. The banking system became dysfunctional as interest rate spreads increased and intermediation and economic activity was reduced." In contemplating such a scenario, we imagine a version of our two-period model, repeated many times.

Government policy can push the economy out of crisis and back to normal by undoing the underlying cause of the problem. One way the government can do this is by purchasing bank assets. In the Gertler-Karadi and Gertler-Kiyotaki analysis, it is assumed that the government has the ability to prevent banks from absconding with bank assets financed by equity or deposit liabilities to the government. We show that sufficiently large government purchases of bank assets can restore the banking system to normal. In particular, government asset purchases cause interest rate spreads to disappear and total intermediation to return to its first-best level. Interest rate spreads disappear because government-financed purchases of assets induce a fall in household demand for deposits. If the government purchases are executed on a large-enough scale, the fall in the demand for deposits is sufficient to push the deposit interest rate back up to the efficient level where it equals banks' return on their funds. The logic of the Barro-Wallace irrelevance result does not hold in a crisis time because tax-financed government purchases of bank assets have an impact on the interest rate.

Another policy that can resolve a crisis is one in which the government provides tax-financed loans to firms. Under this policy the government returns the proceeds of its investment in firms to households in the form of lower taxes in the second period. Households understand that this government policy is a substitute for their bank deposits, and so they reduce the supply of deposits. With the supply and demand for bank deposits both reduced, the deposit interest rate rises back up and the interest rate spread is wiped out. Total intermediation returns to its normal level because, though household deposits are relatively low, this is matched by a corresponding increase in government provision of funds. In this way, tax-financed loans to nonfinancial business can resolve a crisis.

Finally, we show that a policy of subsidizing banks' cost of funds can push the economy out of a crisis. Such a policy works by increasing banks' profits during a crisis and so reducing their temptation to abscond with bank assets. Understanding that their depositors are aware of this, banks expand their deposits back to the first-best level.

We first describe the model. We then formally establish the properties of government policy just reviewed.

3.1. Model

There are many identical households, each with a unit measure of members. Some members are "bankers" and others are "workers." There is perfect insurance inside households, so that all household members consume the same amount c in period 1 and C in period 2. In period 1, workers are endowed with y goods and the representative household makes a deposit d in a bank subject to the period 1 budget constraint:

$$c + d \leq y.$$

The representative household's period 2 budget constraint is:

$$C \leq Rd + \pi.$$

Here, R represents the gross return on deposits and π denotes the profits brought home by bankers. The household treats π as lump sum transfers. The intertemporal budget constraint is constructed by using period 1 and period 2 budget constraints in the usual way:

$$c + \frac{C}{R} \leq y + \frac{\pi}{R}. \tag{3.1}$$

The representative household chooses c and C to maximize

$$u(c) + \beta u(C), \quad u(x) = \frac{x^{1-\gamma}}{1-\gamma}, \quad \gamma > 0, \tag{3.2}$$

subject to (3.1). The solution to the household problem is

$$c = \frac{y + \frac{\pi}{R}}{1 + \frac{(\beta R)^{\frac{1}{\gamma}}}{R}}, \quad d = y - c, \quad C = Rd + \pi. \tag{3.3}$$

We can see the basic logic of the Barro-Wallace irrelevance proposition from (3.1). Suppose the government raises taxes T in period 1, uses the proceeds to purchase T deposits, and gives households a tax cut RT in period 2. Periods 1 and 2 budget constraints are replaced by

$$c + d \leq y - T, \quad C \leq Rd + \pi + RT. \tag{3.4}$$

Using these two equations to substitute out for $d + T$, we obtain (3.1) and T is irrelevant for the determination of c and C. Deposits are determined residually by $d = y - T$. If the government increases T, then d drops by

the same amount. Of course, if we change the environment in some way, then the Barro-Wallace irrelevance proposition may no longer be true. This could happen, for example, if T affected R. To investigate this, we need to flesh out the rest of the model.

Bankers in period 1 are endowed with N goods. They accept deposits from households and purchase securities s from firms. Firms issue securities in order to finance the capital they use to produce consumption goods in period 2. Intermediation is crucial in this economy. If firms receive no resources from banks in period 1, then there can be no production, and therefore no consumption, in period 2.

We first consider the benchmark case in which there are no financial frictions and the banking sector helps the economy achieve the first-best allocations. We suppose that the gross rate of return on privately issued securities is technologically fixed at R^k. Bankers combine their own net worth N with the deposits received d to purchase securities s from firms. Firms use the proceeds from s to purchase an equal quantity of period 1 goods that they turn into capital. The quantity of goods produced by firms in period 2 using this capital is sR^k. Goods-producing firms make no profits, so sR^k is the revenue they pass back to the banks. Banks pay Rd on household deposits in period 2. Bankers solve the following problem:

$$\pi = \max_d [sR^k - Rd], \tag{3.5}$$

where $s = N + d$ and N is the banker's state.

An equilibrium is defined as follows:

Benchmark equilibrium: R, c, C, d, π such that

   (i) the household and firm problems are solved,
  (ii) the bank problem, (3.5), is solved,
 (iii) markets for goods and deposits clear, and
  (iv) $d, c, C > 0$.

Condition (iv) indicates that we consider only interior equilibria, both here and elsewhere in the chapter. A property of a benchmark equilibrium is $R = R^k$. To see this, suppose it were not so. If $R > R^k$ the bank would set $d = 0$ and if $R < R^k$ the bank would set $d = \infty$, neither of which is consistent with the equilibria that we study. Thus, in the benchmark case the interest rate faced by households in equilibrium coincides with the actual rate of return on capital. It is therefore not surprising that the first-best allocations are achieved in this version of the model. That is, the allocations

in the efficient, benchmark equilibrium coincide with the allocations that solve the following planning problem[19]:

$$\max_{c,\,C,\,k} u(c) + \beta u(C)$$
$$\text{subject to } c + k \leq y + N, \quad C \leq R^k k. \tag{3.6}$$

The interest rate spread in this economy is defined as $R^k - R$. In the benchmark equilibrium the interest rate spread is zero. This makes sense, because there are no costs associated with intermediation and there is no default. We summarize this result as follows:

Proposition 3.1: *A benchmark equilibrium has the following properties:*

(i) the interest rate spread, $R^k - R$, is zero,
(ii) d takes on its first-best value.

To gain intuition, it is useful to define the demand for *d* by banks and the supply of *d* by households. The supply of *d* is obtained by solving (3.1) for *d*, after substituting out for bank profits from (3.5). This provides an upward sloping curve in a diagram with *R* on the vertical axis and *d* on the horizontal. Also, a drop in bank net worth, *N*, induces a less than one-for-one increase in the supply of *d*, for consumption smoothing reasons. The bank demand for *d* is simply a horizontal line at $R = R^k$. Representing the demand and supply for *d* in this way provides an immediate graphical illustration of the observation that in equilibrium, $R = R^k$.

Note that in this economy, the Barro-Wallace irrelevance proposition is satisfied. Tax-financed government purchases of private assets have no impact on consumption or total intermediation, $d + T$.

We now introduce the moral hazard problem studied by Gertler-Karadi and Gertler-Kiyotaki. A bank has two options: "default" and "not default." Not defaulting means that a bank simply does what it does in the benchmark version of the model. In this case, the bank earns profits

$$\pi = R^k(N + d) - Rd. \tag{3.7}$$

The option to default means that the banker can take an exogenously fixed fraction θ of the assets and leave whatever is left for the depositors. A defaulting bank receives $\theta R^k(N + d)$, and its depositors receive $(1 - \theta)R^k(N + d)$. The bank chooses the no-default option if and only if doing so increases its profits:

$$(N + d)R^k - Rd \geq \theta(N + d)R^k. \tag{3.8}$$

By rearranging terms, we see that (3.8) is equivalent to

$$(1 - \theta)(N + d)R^k \geq Rd. \tag{3.9}$$

That is, a bank chooses the no-default option if and only if doing so reduces what depositors receive.

Each bank takes the interest rate on deposits as given, and sets its own level of deposits d. Banks are required to post their intended values of d at the start of the period, so that households can assess whether a bank will default. We consider symmetric equilibria in which no bank chooses to default and the d posted by banks satisfy (3.8). In such an equilibrium, an individual bank has no incentive to choose a level of deposits that violates (3.8) because depositors would in this case prefer to take their deposits to another bank, where they obtain a higher return [see (3.9)]. In this setting, the banker solves the following problem:

$$\pi = \max_d [R^k(N + d) - Rd], \text{ subject to (3.8)}. \tag{3.10}$$

Our formal definition of equilibrium in the case in which the banker has a default option is as follows:

Financial equilibrium: R, c, C, d, π such that

   (i) the household and firm problems are solved,
  (ii) the bank problem, (3.10), is solved,
 (iii) markets for goods and deposits clear, and
  (iv) $c, C, d > 0$.

It is useful to represent the equilibrium in a demand and supply diagram. As before, a bank's demand for deposits is the mapping from each possible R into a value of d that solves (3.10). For $R > R^k$ a bank maximizes profits by setting $d = 0$. For $R \leq (1 - \theta) R^k$, (3.8) does not constrain d and the bank would choose $d = \infty$. For $R = R^k$ a bank makes no profits on d. Combining this fact with a bank's incentive not to violate (3.8) implies that for $R = R^k$ a bank is indifferent over values of d such that $0 \leq d \leq N(1 - \theta)/\theta$. Finally, for $(1 - \theta)R^k < R < R^k$ a bank wishes to hold the largest amount of deposits that is consistent with (3.8). This implies $d = R^k N(1 - \theta)/(R - (1 - \theta)R^k)$.

Thus, in the presence of the financial friction, a bank's demand for d is no longer simply a horizontal line at $R = R^k$, with the demand for d infinite for $R < R^k$ and the demand for d zero for $R > R^k$. The demand for d is now a horizontal line at $R = R^k$, extending over the interval $0 \leq d \leq N(1 - \theta)/\theta$.
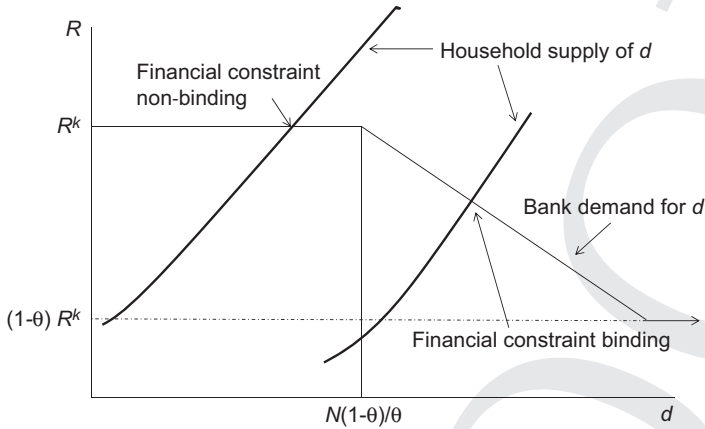
Figure 5.2. Interior, no default equilibrium.

For $(1 - \theta)R^k < R < R^k$ a bank's demand for deposits is finite and declining in R and it is infinite for $R \le (1 - \theta)R^k$. See Figure 5.2 for an illustration. This analysis implies that if a bank's net worth N falls, then the quantity of deposits it demands shifts left. The shift is substantial for the plausible case, $\theta < \frac{1}{2}$ (see Figure 5.2).

The supply of deposits is unaffected by the financial frictions. Equilibrium can again be represented as the intersection of the household's upward-sloped supply of d with the generally downward-sloping demand curve for banks just described. Now, of course, it is not necessary for equilibrium to imply $R = R^k$. This will occur only if the financial constraint is nonbinding, but $R < R^k$ if the constraint is binding (see Figure 5.2). The constraint will be binding if N is sufficiently low.

We summarize our results in the following proposition:

Proposition 3.2: *When (3.8) is nonbinding, the financial equilibrium allocations are first-best and the interest rate spread is zero. When (3.8) binds, then the equilibrium values of d and R are below their first-best levels and the interest rate spread is positive.*

A sequentially repeated version of this model economy provides a rough characterization of events before and after 2007. Suppose that N was large enough in the early period, so that the economy was operating at its efficient level and no part of actual spreads was due to the type of default considerations addressed here. Then, in late 2007 the net worth of banks

suddenly began to fall as a consequence of the collapse in housing prices. When the participation constraint began to bind, spreads opened up. The volume of intermediation – and the investment it supported – then collapsed.

The preceding scenario can be visualized using a diagram like the one in Figure 5.2. Suppose that the pre-2007 economy corresponded to the left of the two equilibria depicted there. With a drop in N, the demand for d by banks shifts left by a relatively large amount (we assume $\theta < 1$) and the supply of d by households shifts right by a relatively small amount (recall the results for supply derived earlier). With these shifts, the economy can end up in the constrained region where $R < R^k$ and intermediation d is smaller.

3.2. Implications for Policy

We now consider the effects of four kinds of tax-financed unconventional monetary policies: injections of equity into banks, deposits in banks, direct loans to firms, and subsidies to banks' cost of funds. In each case, the policy is financed by lump sum taxes T in the first period. In the case of the asset purchase policies, the government transfers the proceeds back to households in the form of a second-period tax reduction.

3.2.1. Equity Injections into Banks

In the case of an equity injection, the government transfers T to each bank. The government requires the banks to repay the earnings $R^k T$ on the assets financed by the equity. The government transfers the $R^k T$ back to households in period 2 in the form of a tax reduction.

We assume that, unlike the household, the government has the power to prevent the bank from absconding with any part of the assets financed by T. Thus, for a bank that receives an equity injection of T, the incentive to default is still the object on the right of the inequality in (3.8). An equity injection also has no impact on a bank's profits:

$$(N + T + d)R^k - Rd - R^k T = (N + d)R^k - Rd.$$

Thus, for a given level of deposits d, an equity injection has no effect on a bank's decision to default. However, the government's equity injection does affect the representative household's choice of d.

To understand how the representative household responds to the tax implications of an equity injection, a suitable adjustment of (3.3) implies that

$$c = \frac{y - T + \frac{\pi}{R} + \frac{R^k T}{R}}{1 + \frac{(\beta R)^{\frac{1}{\gamma}}}{R}}.$$

Note that T does not directly cancel in the numerator because the rate of interest enjoyed by the government when it does an equity injection is different from the household's rate of return on deposits when (3.8) binds and $R^k \neq R$. To understand the general equilibrium impact of T on c, it is necessary to substitute out for π (3.7):

$$c = \frac{y - T + \frac{R^k(N+d) - Rd}{R} + \frac{R^k T}{R}}{1 + \frac{(\beta R)^{\frac{1}{\gamma}}}{R}}.$$

The household's period 1 budget constraint implies $d = y - T - c$. Using this to substitute out for d in the preceding expression and rearranging, we obtain

$$c = \frac{R^k}{(\beta R)^{\frac{1}{\gamma}} + R^k}(N + y),$$

$$d = y - c - T. \tag{3.11}$$

Interestingly, the general equilibrium effect of T on consumption is nil, despite the difference between the government's and the household's interest rate. From the latter expression, we see that a rise in T has no impact on c and so it has a one-for-one negative impact on d. If (3.8) is nonbinding, then the equity injection is irrelevant. There is no impact on total intermediation $d + T$, and the interest rate spread remains unchanged at zero.

Now suppose that (3.8) is binding. Given R, the marginal fall in d with a rise in T reduces the right-hand side of (3.9) by R and reduces the left-hand side of (3.9) by $(1 - \theta)R^k$, thus making the incentive constraint less binding. This is because $R > (1 - \theta)R^k$; otherwise, (3.8) never binds. With T large enough, the incentive constraint ceases to bind altogether, and an analogous argument to the one leading up to proposition 3.2 establishes that the interest rate spread is eliminated, $R = R^k$, and total intermediation $T + d$ achieves its first-best level.

To see what level of T achieves the first-best, let d^* denote the level of deposits in a benchmark equilibrium [we can find d^* by solving (3.6) and

setting $d^* = k - N$]. Our assumption that (3.8) is strictly binding implies that

$$NR^k < \theta(N + d^*),$$

so that d^* is not part of a financial equilibrium. Set T to the value T^* that solves

$$NR^k = \theta(N + d^* - T^*). \tag{3.12}$$

We summarize the preceding results in the form of a proposition:

Proposition 3.3: *When* (3.8) *is nonbinding, tax-financed equity injections have no impact on total intermediation $d + T$ and on the interest rate spread $R^k - R$. When* (3.8) *binds, tax-financed equity injections reduce the interest rate spread and increase total intermediation. A sufficiently large injection restores spreads and total intermediation to their first-best level.*

We can express the equations of the model in words as follows. When N falls enough, the supply of deposits by banks decreases because the incentive constraint binds on the banks. This creates an interest rate spread by reducing the deposit rate (recall, the return on assets is fixed in this model). A tax-financed government purchase of assets causes the demand for deposits by households to decrease, pushing the deposit rate back up and reducing the interest rate spread. The decrease in deposits is somewhat offset by the rise in the deposit rate and this is why $d + T$ increases with the government intervention. The intervention is welfare improving because it pushes the economy back up to the first-best allocations.

3.2.2. Government Deposits in Banks and Loans to Firms

Suppose the government makes tax-financed deposits T in banks in period 1. In period 2 it returns the proceeds to households in the form of a tax cut in the amount RT. It is easy to verify that c and d are determined according to (3.11) in this case. As a result, total deposits $d + T$ are invariant to T for a given R.

If we assume that banks can as easily default on the government as on households, then total deposits $d + T$ enter the incentive constraint and the tax-financed deposits are irrelevant. However, suppose that the government can prevent banks from defaulting on any part of the government's deposits. In that case, the profits earned by banks on government deposits,

$(R^k - R)T$, are not counted in the incentive constraint, (3.8). With only household deposits in the incentive constraint, the analysis is identical to the analysis of equity injections.

Now consider the case in which the government makes tax-financed loans directly to firms. This case is formally identical to the case of tax-financed equity injections. For a given R, $d + T$ is invariant to T. However, because only d enters the incentive constraint, (3.8), the reduction in d that occurs with a rise in T relaxes the incentive constraint in case it is binding. This results in an increase in R and hence a rise in total intermediation. We summarize these results in the following proposition:

Proposition 3.4: *If the government can prevent bank defaults on its own bank deposits, then the effects of tax-financed government deposits in banks resemble the effects of equity injections summarized in proposition 3.3. Direct government loans to firms have the same effects as those of equity injections.*

3.2.3. Interest Rate Subsidies and Net Worth Transfers to Banks

We now consider a policy in which the government subsidizes the interest rate that banks pay on deposits. Suppose that the equilibrium is such that the incentive constraint, (3.8), is binding. As in the previous subsection, this implies that the first-best level of deposits [i.e., the one that solves (3.6) with "deposits" identified with $k - N$] violates (3.8):

$$(N + d^*)R^k - Rd^* < \theta(N + d^*)R^k, \tag{3.13}$$

when the deposit rate R is at its efficient level R^k. Let $\tau > 0$ be the solution to

$$(N + d^*)R^k - R^k(1 - \tau)d^* = \theta(N + d^*)R^k. \tag{3.14}$$

Note that there exists a unique value of $\tau > 0$ that solves this equation because the left-hand side is increasing in τ and the left-hand side exceeds the right-hand side when $\tau = 1$. To finance the transfer $\tau R^k d^*$ to banks the government levies taxes, $T = \tau R^k d^*$, on households in the second period. We now verify that this policy, together with $d = d^*$, $R = R^k$, and c, C at their first-best levels c^*, C^*, satisfies all the equilibrium conditions. Bank profits in the second period are

$$\pi = (N + d^*)R^k - R^k(1 - \tau), \quad d^* = (N + d^*)R^k - R^k d^* + R^k \tau d^*.$$

Total household income is

$$Rd + \pi - T = (N + d^*)R^k.$$

The latter result and the assumption that c^*, C^* solve (3.6) imply that the household problem is solved. The fact that the incentive constraint is satisfied implies that the bank problem, (3.10), is solved. We summarize these findings as follows:

Proposition 3.5: *Suppose* (3.8) *binds in equilibrium so that deposits are strictly below their first-best level in a financial equilibrium. Then, a subsidy to bank deposit liabilities at the rate defined by* (3.14) *ensures that the first-best allocations are supported as a financial equilibrium.*

Next, we consider the case in which taxes are levied on households in the first period and the proceeds are given to bankers as a supplement to their net worth. The net worth transfer is financed by taxes on households in period 1. Suppose the equilibrium is such that the incentive constraint, (3.8), is binding. This implies that the first-best level of deposits d^* violates (3.8) and that (3.13) is satisfied with R at its efficient level R^k. Let T denote the tax-financed transfer of net worth to bankers. The pretax level of banker net worth is N and after taxes it is $N + T$. We conjecture, and then verify, as for T sufficiently large, that the financial equilibrium has the property that deposits equal $d^* - T$, the incentive constraint is nonbinding, and c, C coincide with their first-best values. Let T be the solution to

$$(N + d^*)R^k - R^k(d^* - T) = \theta(N + d^*)R^k. \qquad (3.15)$$

Note that $N + d^*$ is unaffected under the tax policy and the conjecture about the equilibrium. A unique $T > 0$ that solves (3.15) is guaranteed to exist because the left-hand side is monotonically increasing in T and the left-hand side is assumed to be smaller than the right-hand side when $T = 0$.

To understand how the representative household responds to the tax-financed equity injection, a suitable adjustment of (3.3) implies that

$$c = \frac{y - T + \frac{\pi}{R}}{1 + \frac{(\beta R)^{\frac{1}{\gamma}}}{R}}.$$

Under our conjecture, $R = R^k$ and π is given by the expression on the left-hand side of the equality in (3.15). Substituting, we obtain (3.11), the level

of consumption in the first-best equilibrium. This verifies our conjecture about the period 1 level of consumption. It is straightforward to verify that the first-best level of period 2 consumption satisfies the period 2 household budget constraint. We summarize our findings as follows:

Proposition 3.6: *Suppose* (3.8) *binds in equilibrium so that deposits are strictly below their first-best level in a financial equilibrium. Then, a tax-financed transfer of net worth to bankers at a level defined in* (3.15) *ensures that the first-best allocations are supported as a financial equilibrium.*

4. Moral Hazard II: Unobserved Banker Effort

The basic framework of the model used here is similar to the one in the previous section. The difference lies in the source of moral hazard. We assume that bankers, to make a high return for their depositors, must exert an unobserved and costly effort. As in the case of the model in the previous section, the model used here can articulate the idea that the banking system supported efficient allocations prior to 2007, but then became dysfunctional as a consequence of a fall in bank net worth. As in the previous section, the fall in net worth pushes the economy against a nonlinearity, which causes an increase in interest rate spreads and a fall in intermediation and in the activities that intermediation supports.

Despite the similarities, there are some important differences between the models in terms of their implications for policy. For example, the model used here implies that equity injections into banks during a crisis have no impact on equilibrium allocations. The model of the previous section implies that injections of bank equity can move the economy to the efficient allocations. In addition, we use the model of this section to study a broader range of policy interventions. We consider the effects of government bailouts of the creditors of banks whose assets perform poorly. The model is also useful for thinking about the benefits of imposing leverage restrictions on banks.

The following section provides an intuitive summary of the analysis. After that comes the formal presentation.

4.1. Overview

There are two periods. There are a large number of households. Each household has many bankers and workers. Bankers are endowed in the first period

with their own net worth, and they combine this with deposits to acquire securities from firms.[20] There are a large number of firms, each having access to one investment project. The investment project available to some firms is a good one in that it has a high (fixed) gross rate of return. If these firms invest one unit of goods in period 1, they are able to produce R^g goods in period 2. The investment project available to other firms is bad, and we denote the gross rate of return on these investment projects by R^b, where $R^b < R^g$. The rates of return R^g and R^b are exogenous and technologically determined.

Empirically, we observe that some banks enjoy higher profits than others, and we interpret this as reflecting that banks cannot hold a fully diversified portfolio of assets. This could be because there are many different types of investment projects – differentiated according to industry, geographic location, etc. – and there are gains to specializing in the identification of good projects of a particular type. In the model, these observations are captured by the assumption that banks can purchase the securities of at most one firm. Similarly, a firm can issue securities to at most one bank. Production for a firm is costless, and the rate of return on bank securities is identical to the rate of return on the underlying investment.[21] The task of a banker is to exert an unobserved and costly effort e to identify a firm with a good project. The ex post rate of return on the banker's securities is observed, but this does not reveal the banker's effort. This is because e affects only the probability $p(e)$ that a banker identifies a good firm.

We define the efficient levels of effort and of intermediation as those that occur in competitive markets in the special case that the efforts exerted by bankers are fully observed. For a banker to have the incentive to exert the efficient level of effort when effort is not observed requires that he or she receive a reward that is linked in the right way to the performance of the securities. Let R_b^d and R_g^d denote the interest rate on bank deposits when the bank's securities pay R^b and R^g, respectively. We show that a banker sets effort to the efficient level when $R_b^d = R_g^d$, that is, when the cost of funds is independent of the performance of the securities. We characterize the situation in which $R_b^d = R_g^d$ as one in which the banker's creditors (i.e., the depositors) do not share in the losses when a banker's securities do not perform well. The banker exerts the efficient level of effort when $R_b^d = R_g^d$ because the banker fully internalizes the marginal benefit of increased effort.

For the arrangement $R_b^d = R_g^d$ to be feasible, it is necessary that the banker have a sufficiently large amount of net worth. Otherwise, the banker would not have enough funds to pay depositors in the probability $1 - p(e)$ event that the banker's loan turns out to be bad.[22] When net worth is too low in

this sense, then a bank's depositors share in the loss that occurs when their bank's securities generate a bad return. In this case, depositors must receive a relatively high return, $R_g^d > R_b^d$, in the good state as compensation. But, with this cross-state pattern in deposit rates, the banker does not fully capture the marginal product of increased effort. Thus, when banker net worth is not sufficiently high to permit an uncontingent deposit rate, the banker's incentive to exert effort is reduced. This reduced effort has a consequence that relatively more low-quality projects are funded. As a result, the overall rate of return on deposits falls and so the quantity of deposits falls too. With the fall in deposits, intermediation and investment are reduced.

We now briefly discuss the concept of the "interest rate spread." We can loosely think of the bad state as a bankruptcy state, a state that occurs with relatively low probability. For the purpose of defining the interest rate spread, we think of the "interest rate" paid by a bank on its source of funds as the rate it pays, R_g^d, when the good state is realized. This notion of the interest rate is similar to that of the face value of a bond, which specifies what the holder receives as long as nothing goes wrong with the issuing firm. Households are the ultimate source of funds for banks, and they receive an interest rate R that is risk free. This is so because the representative household is perfectly diversified across banks (he or she accomplishes this by using a mutual fund) and so he or she receives the average rate of return across all banks. With these considerations in mind, we define the interest rate spread as follows:

$$R_g^d - R. \tag{4.1}$$

When bank net worth is sufficiently high, then $R_g^d - R = 0$, so that the interest rate spread is zero. When net worth falls enough, then R_b^d must be low in the bad state and thus R_g^d must be relatively high in the good state. As a result, the interest rate spread is positive when bank net worth is low.

In sum, when bank net worth is high (we refer to this as normal times), then the interest rate spread is zero and effort and deposits are at their efficient levels. When bank net worth is low (a crisis), then there is a positive interest rate spread and deposits are below their efficient levels. In this sense, the financial system is dysfunctional when net worth is sufficiently low. From this perspective, the model implications are qualitatively similar to those of the model in the previous section.

Still, the economics of the two models differ. For example, in the model considered here, the interest rate spread compensates for the low returns paid by banks with bad investments. In principle, one could perform an empirical study to measure the bank losses that are reflected in the

high-risk spread. In the model of Section 3, the interest rate spread reflects a fear of out-of-equilibrium misbehavior by banks. As such, the fear is about something that does not actually happen.

The two models also differ in terms of their implications for policy. In the model of the previous section, equity injections have no effect in normal times and they improve the efficiency of the economy in a crisis. In the model here, equity injections in normal times are counterproductive because they reduce bankers' incentives to exert effort. The intuition is simple. We treat an equity injection as a "loan" from the government that must be repaid according to the actual return that the bank receives as a consequence of the government loan. The direct impact of this sort of loan on the bank is nil because it generates zero net cash flow regardless of whether the bank identifies a good or bad firm. However, there is a general equilibrium effect that matters. From the point of view of the household, an equity injection corresponds to a tax hike in the first period, followed by a tax reduction in the second period. Because this pattern of taxes satisfies part of the household's desire to save, the household responds by reducing his or her own deposits. With fewer deposits, the banker has less incentive to exert effort. With less effort, the average quality of bank securities falls. This produces a fall in the risk-free interest rate paid to households and causes them to save less. The net effect is that intermediation falls below its ideal level.

It turns out that in a crisis, an equity injection has *no* effect in the model. This is because in a crisis there is an additional positive effect from equity investments that cancels the negative effects in normal times that were discussed in the previous paragraph. Recall, the definition of a crisis time is that net worth is too low to permit a state-non-contingent interest rate on deposits. When household deposits with banks are reduced in response to an equity injection, it becomes possible to reduce the degree of state contingency in deposit rates. This is because, with lower deposits, the amount of money owed by banks in the bad state is smaller and more likely to be manageable with bank net worth. The reduced state contingency in deposit rates improves the incentive of banks to exert effort. This positive effect exactly cancels the negative effects that occur in a normal time.

We also investigate other policies. For example, we study the effects of placing tax-financed government deposits in banks during a crisis. Such a policy has no effect because, consistent with the Barro-Wallace proposition, households respond by reducing their deposits by the same amount. Subsidizing banks' cost of funds in a crisis is helpful, because this policy improves the likelihood that a bank can cover losses with its own net worth. Bailing out the creditors of banks whose loans perform badly is also welfare

increasing in a crisis. Finally, we find that leverage restrictions improve welfare in a crisis. The reason for this is that by forcing banks to reduce their level of deposits, a leverage restriction increases the likelihood that a bank can cover its losses with its own net worth, thus increasing its incentive to exert effort. This is welfare improving in a crisis, when banker effort is below its efficient level, absent government intervention.

The following subsections present the formal description of the model and the results, respectively.

4.2. Model

There are many identical households, each composed of many workers and bankers. The workers receive an endowment y in period 1, and the households allocate the endowment between period 1 consumption c and period 1 deposits in mutual funds d. All quantity variables are expressed in per-household-member terms. The gross rate of return on deposits is risk free and is denoted by R. The preferences of the representative household are as in the previous example, in (3.2). Optimality of the deposit decision is associated with the usual intertemporal Euler equation. This Euler equation and the first-period budget constraint are given by

$$u'(c) = \beta R u'(C), \tag{4.2}$$

$$c + d = y. \tag{4.3}$$

In the second period, households receive Rd and profits from their bankers π. In the interior equilibria that we study, the second-period budget constraint is satisfied as a strict equality:

$$C = Rd + \pi.$$

We impose the following restriction on the curvature parameter in the utility function [see (3.2)]:

$$0 < \gamma < 1. \tag{4.4}$$

The upper bound on γ ensures that the equilibrium response of d to R is positive, which we view as the interesting case.

Bankers receive an endowment N in the first period. They combine N with deposits received from mutual funds and buy securities that finance the investment of a firm. Firms are perfectly competitive and costless to operate, so the bank receives the entire return on its firm's investment project.

The probability $p(e)$ that the firm whose securities the bank buys are good is specified as follows:

$$p(e) = a + be, \quad b > 0, \tag{4.5}$$

so that $p'(e) = b$, $p''(e) = 0$. We consider only model parameter values that imply $0 < p(e) < 1$ in equilibrium.

The mean $m(e)$ and variance $V(e)$ of a bank's asset are given by

$$
\begin{aligned}
m(e) &= p(e)R^g + (1 - p(e))R^b, \\
V(e) &= p(e)(1 - p(e))(R^g - R^b)^2,
\end{aligned}
\tag{4.6}
$$

respectively. Note that

$$V'(e) = (1 - 2p(e))(R^g - R^b)^2 b.$$

This expression is negative for $p(e) > 1/2$. In our analysis, we assume that $p(e)$ satisfies this condition. Thus, when bankers increase effort, the mean of the return on their securities increases and the variance decreases.

Our primary interest is in the scenario with "financial frictions," in which the mutual fund does not observe the effort e made by the banker. To this end, it is of interest to first discuss the observable-effort version of the model in which e is observed by the mutual fund. Throughout, we assume that e is observed by the banker's own household. Absent this assumption, a banker would always set $e = 0$ because e is costly to the banker and because a banker's consumption while at home is independent of the return on the banker's portfolio.

4.3. Observable-Effort Benchmark

A loan contract between a banker and a mutual fund is characterized by four numbers (d, e, R_g^d, R_b^d). Here, R_g^d, R_b^d denote the gross returns on d paid by bankers whose firms turn out to be good and bad, respectively. All four elements of the contract are assumed to be directly verifiable to the mutual fund in the observable-effort version of the model. Throughout, we assume that sufficient sanctions exist so that verifiable deviations from a contract never occur.

The representative competitive mutual fund itself takes deposits d from households and commits to paying households a gross rate of return R. The mutual fund is competitive in that it treats R as exogenous. Because the representative mutual fund is perfectly diversified, its revenues from deposits d are $p(e)R_g^d d + [1 - p(e)]R_b^d d$. The mutual fund must repay

Rd to depositors, so that the profits of the mutual fund are $p(e)R_g^d d + [1 - p(e)]R_b^d d - Rd$. Because mutual funds are competitive, profits must be zero[23]:

$$p(e)R_g^d d + [1 - p(e)]R_b^d d = Rd. \tag{4.7}$$

We assume the banker's only source of funds for repaying the mutual fund is the earnings on his or her investment. In each state of nature the banker must earn enough to pay his or her obligation to the mutual fund in that state of nature:

$$R^g(N + d) - R_g^d d \geq 0, \quad R^b(N + d) - R_b^d d \geq 0. \tag{4.8}$$

In practice, these constraints will either never bind or they will bind only in the bad state of nature. Thus, an additional restriction on the menu of contracts (d, e, R_g^d, R_b^d) available to a bank is

$$R^b(N + d) - R_b^d d \geq 0.$$

The problem of the banker is to select a contract (d, e, R_g^d, R_b^d) from the menu defined by (4.7) and (4.8).

A banker's ex ante reward from a loan contract is

$$\lambda\big\{ p(e)\big[R^g(N + d) - R_g^d d\big] + (1 - p(e))\big[R^b(N + d) - R_b^d d\big]\big\} - \frac{1}{2}e^2, \tag{4.9}$$

where $e^2/2$ is the banker's utility cost of expending effort and λ denotes the marginal value of consumption for the household of the banker. In addition, d denotes the deposits issued by the banker and is distinct from the deposit decision of the banker's household. As part of the terms of the banker's arrangement with the household, the banker is required to seek a contract that maximizes (4.9). Throughout the analysis, we assume that the banker's household observes all the variables in (4.9) and that the household has the means to compel the banker to do what the household requires.

The Lagrangian representation of the banker's problem is

$$\max_{e,d,R_g^d,R_b^d} \lambda\big\{ p(e)\big[R^g(N+d) - R_g^d d\big] + (1 - p(e))\big[R^b(N+d) - R_b^d d\big]\big\} - \frac{1}{2}e^2$$
$$+ \mu\big[p(e)R_g^d d + (1 - p(e))R_b^d d - Rd\big] + \nu\big[R_b^d d - R^b(N + d)\big], \tag{4.10}$$

where μ is the Lagrange multiplier on (4.7) and $\nu \leq 0$ is the Lagrange multiplier on (4.8).

An interior equilibrium for this economy is as follows:

Observable-effort equilibrium: $c, C, e, d, R, \lambda, R_g^d, R_b^d$ such that

   (i) the household maximization problem is solved,
  (ii) mutual funds earn zero profits,
 (iii) the banker problem, (4.10), is solved,
 (iv) markets clear,
  (v) $c, C, d, e > 0$.

We now study the properties of this equilibrium.

The first-order conditions associated with the banker problem in equilibrium are

$$e: \ \lambda p'(e)\big[\big(R^g - R^b\big)(N+d) - \big(R_g^d - R_b^d\big)d\big]$$
$$- e + \mu\, p'(e)\big(R_g^d - R_b^d\big)d = 0,$$
$$d: \ \lambda\big\{p(e)\big(R^g - R_g^d\big) + (1-p(e))\big(R^b - R_b^d\big)\big\}$$
$$+ \mu\big[p(e)R_g^d + (1-p(e))R_b^d - R\big] + v\big(R_b^d - R^b\big) = 0,$$
$$R_g^d: \ -\lambda p(e)d + \mu\, p(e)d = 0,$$
$$R_b^d: \ -\lambda(1-p(e))d + \mu(1-p(e))d + vd = 0,$$
$$\mu: \ p(e)R_g^d d + (1-p(e))R_b^d d = Rd,$$
$$v: \ v\big[R_b^d d - R^b(N+d)\big] = 0, \quad v \le 0, \quad R_b^d d - R^b(N+d) \le 0,$$

where x indicates the first-order condition with respect to the variable x.
Adding the R_g^d and R_b^d equations, we obtain

$$\mu = \lambda - v. \tag{4.11}$$

Substituting (4.11) back into the R_g^d equation, we find

$$v = 0,$$

so that the cash constraint is nonbinding. Substituting the latter two results back into the system of equations, they reduce to

$$e: e = \lambda p'(e)(R^g - R^b)(N+d), \tag{4.12}$$
$$d: R = p(e)R^g + (1-p(e))R^b, \tag{4.13}$$
$$\mu: R = p(e)R_g^d + (1-p(e))R_b^d. \tag{4.14}$$

Note from (4.12) that in setting effort e, the banker looks only at the sum $N + d$ and not at how this sum breaks down into the component reflecting

the banker's own resources N and the component reflecting the resources d supplied by the mutual fund. By committing to care for d as if these were the banker's own funds, the banker is able to obtain better contract terms from the mutual fund. The banker is able to commit to the level of effort in (4.12) because e is observable to the mutual fund, and throughout the analysis we assume that all actions that are verifiable are enforceable.

The profits π brought home by the bankers in the representative household in period 2 are

$$\pi = p(e)\big[R^g(N+d) - R_g^d d\big] + (1 - p(e))\big[R^b(N+d) - R_b^d d\big] = RN, \tag{4.15}$$

using the zero profit condition of mutual funds. Thus, the representative household's second-period budget constraint is

$$C = R(N+d). \tag{4.16}$$

The five equilibrium conditions, (4.12), (4.13), (4.3), (4.2), and (4.16), can be used to determine values for

$$c, C, e, d, R.$$

Also,

$$\lambda = \beta u'(C). \tag{4.17}$$

Although the observable effort version of the model uniquely determines variables like c, C, d, and R, it does not uniquely determine the values of the state contingent return on deposits, R_g^d, R_b^d. These are restricted only by (4.14) and (4.8). For example, there is an equilibrium in which deposits have the following state contingent pattern: $R_g^d = R^g$, $R_b^d = R^b$. There may also be an equilibrium in which deposit rates are not state contingent, so that $R_g^d = R_b^d = R$. However, for the latter to be an equilibrium requires that N be sufficiently large. The equilibrium values of c, C, e, d, and λ are the same across all state-contingent returns on deposits that are consistent with (4.14) and (4.8).

4.4. Unobservable Effort

We now suppose that the banker's effort e is not observed by the mutual fund. Thus, whatever d, R_g^d, or R_b^d is specified in the contract, a banker always chooses e to maximize:

$$\lambda\big\{p(e)\big[R^g(N+d) - R_g^d d\big] + (1 - p(e))\big[R^b(N+d) - R_b^d d\big]\big\} - \frac{1}{2}e^2.$$

The first-order condition necessary for optimality is

$$e : e = \lambda p'(e)\left[(R^g - R^b)(N + d) - \left(R_g^d - R_b^d\right)d\right]. \qquad (4.18)$$

Note that $R_g^d > R_b^d$ reduces the banker's incentive to exert effort. This is because in this case the banker receives a smaller portion of the marginal increase in expected profits caused by a marginal increase in effort. Understanding that e will be selected according to (4.18), a mutual fund will offer only contracts (d, e, R_g^d, R_b^d) that satisfy not just (4.8), but also (4.18).

In light of the previous observations, the Lagrangian representation of the banker's problem is

$$\max_{\left(e,d,R_g^d,R_b^d\right)} \lambda\Big\{ p(e)\left[R^g(N + d) - R_g^d d\right]$$

$$+ (1 - p(e))\left[R^b(N + d) - R_b^d d\right]\Big\} - \frac{1}{2}e^2$$

$$+ \mu\left[p(e)R_g^d d + (1 - p(e))R_b^d d - Rd\right] \qquad (4.19)$$

$$+ \eta\Big(e - \lambda p'(e)\left[(R^g - R^b)(N + d) - \left(R_g^d - R_b^d\right)d\right]\Big)$$

$$+ v\left[R_b^d d - R^b(N + d)\right],$$

where η is the Lagrange multiplier on (4.18).

The equilibrium concept used here is as follows:

Unobservable-effort equilibrium: $c, C, e, d, R, \lambda, R_g^d, R_b^d$ such that

  (i) the household maximization problem is solved,
 (ii) mutual funds earn zero profits,
(iii) the banker problem, (4.19), is solved,
 (iv) markets clear, and
  (v) $c, C, d, e > 0$.

To understand the properties of this equilibrium, consider the first-order necessary conditions associated with the banker problem, (4.19):

$$e : \ \lambda p'(e)\left[(R^g - R^b)(N+d) - \left(R_g^d - R_b^d\right)d\right] - e + \mu p'(e)\left(R_g^d - R_b^d\right)d$$

$$+ \eta\Big(1 - \lambda p''(e)\left[(R^g - R^b)(N + d) - \left(R_g^d - R_b^d\right)d\right]\Big) = 0,$$

$$d : \ 0 = \lambda p(e)\left[R^g - R_g^d\right] + \lambda(1 - p(e))\left[R^b - R_b^d\right]$$

$$+ \mu\left[p(e)R_g^d + (1 - p(e))R_b^d - R\right]$$

$$- \eta \lambda p'(e)\big[(R^g - R^b) - \big(R_g^d - R_b^d\big)\big] + \nu\big(R_b^d - R^b\big),$$

$$R_g^d : \ -\lambda p(e) + \mu p(e) + \eta \lambda p'(e) = 0,$$

$$R_b^d : \ -\lambda(1 - p(e)) + \mu(1 - p(e)) - \eta \lambda p'(e) + \nu = 0, \tag{4.20}$$

$$\mu : \ R = p(e)R_g^d + (1 - p(e))R_b^d,$$

$$\eta : \ e = \lambda p'(e)\big[(R^g - R^b)(N + d) - \big(R_g^d - R_b^d\big)d\big],$$

$$\nu : \ \nu\big[R_b^d d - R^b(N + d)\big] = 0, \ \nu \leq 0, \ \big[R_b^d d - R^b(N + d)\big] \leq 0.$$

We refer to these equations – and their subsequent counterparts – by their names to the left of the colon. Add the R_g^d and R_b^d equations to obtain (4.11). After using (4.11) to substitute out for μ in (4.20), making use of (4.5), and rearranging, we obtain[24]

$$e : \ (\lambda - \nu)b\big(R_g^d - R_b^d\big)d + \eta = 0,$$

$$d : \ R = p(e)R^g + (1 - p(e))R^b,$$

$$R_g^d : \ \nu p(e) = \eta \lambda b, \tag{4.21}$$

$$\mu : \ R = p(e)R_g^d + (1 - p(e))R_b^d,$$

$$\eta : \ e = \lambda b\big[(R^g - R^b)(N + d) - \big(R_g^d - R_b^d\big)d\big],$$

$$\nu : \ \nu\big[R_b^d d - R^b(N + d)\big] = 0, \ \nu \leq 0, \quad \big[R_b^d d - R^b(N + d)\big] \leq 0.$$

We distinguish two cases. Equilibrium in a normal time corresponds to the case in which N is sufficiently large that the cash constraint is nonbinding, so that $\nu = 0$. Equilibrium in a crisis time corresponds to the case in which $\nu < 0$.

We first consider the properties of equilibrium in a normal time. Substituting $\nu = 0$ into the R_g^d equation, we deduce that in an interior equilibrium with $d, \lambda > 0$, the multiplier on the incentive constraint η is zero. With $\eta = 0$ and the fact that $\lambda - \nu > 0$, the e and μ equations imply that

$$R_g^d = R_b^d = R. \tag{4.22}$$

It then follows from the η equation that

$$e : e = \lambda b(R^g - R^b)(N + d). \tag{4.23}$$

Equations (4.23) and the μ equation in (4.21), together with the three household equilibrium conditions (4.3), (4.2), and (4.16), represent five

conditions. These conditions can be used to determine the following five variables:

$$c, C, e, d, R.$$

A notable feature of the equilibrium in a normal time is that the incentive constraint, (4.18), is nonbinding and the allocations are efficient in the sense that they coincide with the allocations in the version of the model in which effort is observable. The level of effort exerted by the banker in the unobservable effort equilibrium coincides with what it is in the observable effort equilibrium because the loan contract transfers the full marginal product of effort to the banker. This is accomplished by making the rate of interest on banker deposits not state contingent [see (4.22)]. The interest rate spread in this equilibrium [see (4.1)] is zero in a normal time. We state these results as a proposition:

Proposition 4.1: *When the cash constraint, (4.8), does not bind (i.e., $\nu = 0$), then the allocations in the unobserved effort equilibrium coincide with those in the observed effort equilibrium and the interest rate spread is zero.*

We now turn to the case in which the cash constraint is binding, so that $\nu < 0$ and

$$\nu : R_b^d d = R^b(N + d). \qquad (4.24)$$

In this case, the observed and unobserved effort equilibria diverge, because the cash constraint never binds in the observed effort equilibrium. The R_g^d equation in (4.21) implies $\eta < 0$, so that according to the e equation in (4.21),

$$R_g^d > R_b^d \qquad (4.25)$$

in an interior equilibrium with $d > 0$. It follows from the η equation in (4.21) that

$$e < \lambda b(R^g - R^b)(N + d).$$

That is, the banker in a crisis equilibrium exerts less effort, for a given $N + d$, than he or she does in the observed effort equilibrium. The reason is that with (4.25), the banker does not capture the full marginal return from effort. With reduced effort, equation d in (4.21) shows that equilibrium R

is smaller. Given (4.4), the household equilibrium conditions, (4.3), (4.2), and (4.16), imply a lower d, reinforcing the low e. We summarize these findings in the following proposition:

Proposition 4.2: *When the cash constraint, (4.8), binds (i.e., $v < 0$), then e, d, and R in the unobserved effort equilibrium are lower than they are in the observed effort equilibrium, and the interest rate spread is positive.*

4.5. Implications for Policy

In this section, we consider the impact of government deposits and equity injections into banks, and show that these are not helpful in a crisis. We then show that bank deposit rate subsidies and transfers of net worth to banks can solve the crisis completely by eliminating the interest rate spread and moving allocations to their efficient levels. Finally, we study the effects of bailing out the creditors of banks with poor-performing securities and the effects of leverage restrictions.

4.5.1. Government Deposits Into Mutual Funds

Consider the case in which the government raises taxes T and deposits the proceeds in the mutual fund. The household's period 1 budget constraint is given by

$$c + \tilde{d} = y, \tag{4.26}$$

where $\tilde{d}$ denotes $d + T$ and d denotes deposits placed by households in the mutual fund. The intertemporal condition, (4.2), is unaffected by the change. The household's second-period budget constraint is unaffected by the change, except that d is replaced by $\tilde{d}$. Similarly, the equilibrium conditions associated with the banker problem, (4.19), are unchanged, with the exception that d is replaced by $\tilde{d}$. In particular, if the government deposits taxpayer money into the mutual funds, taxpayers reduce their deposits by the same amount and there is no change. From the point of view of households in the economy, it is the same whether deposits are held in their capacity as taxpayers or directly in their own name. That is, the policy considered in this section does not overcome the Barro-Wallace irrelevance proposition. This conclusion makes use of the assumption we use throughout our analysis, that an equilibrium is interior. In the present context, this implies that T is

not so large that the constraint $d \geq 0$ is nonbinding. We summarize these results in the form of a proposition:

Proposition 4.3: *In an interior equilibrium, the level of tax-financed government deposits are irrelevant for the equilibrium levels of c, C, R, $d + T$, and e.*

4.5.2. Equity Injections Into Banks

In this section we adopt the same interpretation of equity injections as in Subsection 3.2.1. That is, the government raises taxes T and hands these over to the banks in period 1. The government requires that the banks repay the earnings they actually make on these funds in period 2. Under this policy, the expected profits of the bank are

$$p(e)\left[R^g(N + T + d) - R^d_g d - R^g T\right]$$
$$+(1 - p(e))\left[R^b(N + T + d) - R^d_b d - R^b T\right].$$

Note that taxes enter revenues symmetrically with deposits and the bank's own net worth. On the cost side, equity injections require that banks pay the government the actual rate of return on its securities. Thus, equity injections have no direct impact on bank profits, because they enter revenues and costs in exactly the same way. For the same reason, equity injections also do not change the banker's cash requirement in the bad state. That is, the bank requirement that revenues be no smaller than costs is, in the presence of equity injections,

$$R^b(N + d + T) \geq R^d_b d + R^b T,$$

so that T cancels from both sides and thus coincides with (4.8). We conclude that the banker's problem, (4.19), is completely unaltered by the presence of equity injections.

Now consider the household problem. The period 1 budget constraint is

$$c + d \leq y - T, \tag{4.27}$$

reflecting that equity injections T are financed with taxes on households. The government transfers the revenues from equity injections back to households in period 2. In this way, the period 2 household budget constraint is

$$C = Rd + p(e)\left[R^g(N + d) - R^d_g d\right] + (1 - p(e))\left[R^b(N + d) - R^d_b d\right]$$
$$+ \left[p(e)R^g + (1 - p(e))R^b\right] T$$
$$= R(N + d + T). \tag{4.28}$$

The last term on the right-hand reflects that the government's distribution of equity among banks is completely diversified. The intertemporal Euler equation, (4.2), is unchanged.

From the household problem we see that an increase in T induces an equal reduction in d for a given value of R. Note that although T does not enter the banker's problem, d does. Thus, it is possible that T has an indirect effect on the equilibrium.

Consider first the case in which the cash constraint in the bad state is not binding, $\nu = 0$. In this case, the problem solved by the banker's contract is given by (4.19) with $\nu = 0$, so that (4.22) and (4.23) are satisfied. In this case, increased equity injections for a given interest rate R reduce deposits and so reduce the banker's incentives to exert effort e [see (4.23)]. This in turn produces a fall in R, so that d falls some more. Thus, $d + T$ falls with a tax-financed equity injection in a normal time when the cash constraint is not binding. The intuition for this result is described in Section 4.1. We summarize this result in the form of a proposition:

Proposition 4.4: *If the cash constraint, (4.8), is not binding, then an equity injection produces a fall in effort e, the interest rate R, and total intermediation $d + T$.*

In a crisis time when the cash constraint in the bad state is binding, the fall in $d + T$ that occurs with an equity injection is offset by a second effect. The two cancel, and so equity injections are irrelevant in a crisis. The second effect occurs because a fall in deposits d loosens the cash constraint, (4.8), in the bad state. This relaxation of (4.8) requires an increase in the rate of return on deposits for banks in the bad state. The reduction in the state contingency of deposit rates enhances bankers' incentives to exert effort. As a result, the fraction of good projects that are identified is increased, so that the risk-free rate rises, leading to a rise in deposits. Formally, we have the following proposition.

Proposition 4.5: *If the cash constraint is binding, then an equity injection has no impact on consumption c, C, the interest rate R, and the volume of intermediation $d + T$.*

See Appendix A for a proof of this proposition.

We summarize our two propositions as follows. In a normal time, when the cash constraint is not binding, equity injections are counterproductive, as they lead to a reduction in effort by bankers. In crisis times, an equity

injection satisfies the Barro-Wallace irrelevance result, so that they have no impact on c, C, R, or $d + T$ as long as the cash constraint remains binding. Once equity injections reach a sufficient scale, then the cash constraint ceases to bind and proposition 4.4 is relevant. That is, equity injections that are large enough to render the cash constraint nonbinding are counterproductive in that they reduce effort.

4.5.3. Interest Rate Subsidies and Net Worth Transfers to Banks

As we have emphasized, the heart of the problem in a crisis is that banks with poor-performing securities do not have enough resources to fully absorb their losses. Equilibrium in this case requires that deposit rates covary positively with the return on the bank portfolio. But this positive covariance leads to welfare-reducing allocations by reducing banks' incentive to exert effort. State noncontingency in the banks' deposit rate is crucial if they are to have enough incentive to exert the efficient level of effort. This reasoning suggests two policies that can help solve the problem. First, by reducing the costs of their deposits, a tax-financed subsidy on banks' cost of funds makes it possible for banks to cover their losses in bad states and for bank deposit rates to be state noncontingent. Second, a tax-financed transfer of equity to banks also allows them to cover their losses in bad states with state-non-contingent deposit rates.

Consider first the case of interest rate subsidies. Suppose we have the allocations and returns in the observable effort equilibrium. The assumption that we are in a crisis implies that if $R = R_b^d = R_g^d$, where R solves (4.13), then the cash constraint, (4.8), is violated:

$$Rd > R^b(N + d). \tag{4.29}$$

Let τ solve

$$(1 - \tau)Rd = R^b(N + d). \tag{4.30}$$

A value of $\tau > 0$ is guaranteed to exist because the left-hand side of this expression is monotonically decreasing in τ and it is zero when $\tau = 1$. All the equilibrium conditions associated with the banker problem [see (4.21)] are satisfied, with $\nu = 0$. As a result, the banker exerts the level of effort that occurs in the observed effort equilibrium [see (4.23)]. The key thing is that state noncontingency of deposit rates causes the banker to exert effort as though the deposits belonged to the banker. The fact that the level of deposit rates is lower across the realized returns of its securities is irrelevant to the effort exerted by the banker.

It remains only to verify that the household decisions in the observable-effort equilibrium also solve their problem in the unobservable-effort equilibrium with an interest rate subsidy. The households' period 1 budget constraint, (4.3), is unaffected. The household's intertemporal Euler equation, (4.2), is also not affected. The only household equilibrium condition that requires attention is the second-period budget constraint, because the tax subsidy to banks is financed by period 2 taxes, $T = \tau Rd$,

$$C = Rd + \pi - T.$$

Bank profits π are higher under the interest rate subsidy than they are in the observable effort equilibrium. However, they are higher by exactly T. So, the assumption that the period 2 household budget constraint is satisfied in an observable-effort equilibrium implies that the allocations in that equilibrium also satisfy the preceding budget constraint with taxes. We summarize these findings as follows:

Proposition 4.6: *Suppose the cash constraint in an unobservable-effort equilibrium is binding. The interest rate subsidy, (4.30), financed by a period 2 tax on households, causes the allocations in the unobservable-effort equilibrium to coincide with those in the observable-effort equilibrium.*

The interest subsidy policy is of wider interest because it allows us to address a common view that interest rate subsidies to banks lead them to undertake excessive risk. In the environment here, an interest rate subsidy in a crisis induces bankers to exert greater effort e. This leads to a rise in the mean return on assets and a fall in their variance. Thus, this environment does not rationalize the common view about the impact of interest subsidies on risk taking by banks.

We now turn to tax-financed transfers of net worth to banks. Suppose again that the cash constraint is binding in the unobservable effort equilibrium. The government raises taxes T in period 1 and transfers the proceeds to banks. If the transfer is sufficiently large, then the cash constraint in the unobservable effort equilibrium ceases to bind. To establish this result, suppose we have the allocations in the observable-effort equilibrium in hand. The assumption that we are in a crisis implies that if $R = R_b^d = R_g^d$, where R solves (4.13), then the cash constraint, (4.8), is violated, as in (4.29).

We first consider the response of the observable-effort equilibrium to $T > 0$. Banks' pretax net worth is N and after taxes their net worth is $N + T$. We conjecture, and then verify, that with $T > 0$, deposits decline

one-for-one in the observable effort equilibrium and period 1 and period 2 consumption allocations do not change. Suppose that T satisfies

$$R(d - T) = R^b(N + d). \tag{4.31}$$

The value of T that satisfies this equation exists and is unique because the left-hand side is monotonically decreasing and continuous in T and it is zero when $T = d$. According to (4.31), the cash constraint is (marginally) nonbinding. It is easily verified that the household period 1 budget constraint and Euler equations in the observable-effort equilibrium are satisfied [see (4.3) and (4.2)]. It is also easily verified that households' second period income is invariant to T.[25] Finally, the bank equilibrium conditions, (4.12), (4.13), (4.14), are easily seen to be satisfied. We conclude that we have an observable-effort equilibrium. Because in addition the cash constraint is satisfied, it follows that we have an unobserved-effort equilibrium too. We summarize our finding as follows:

Proposition 4.7: *Suppose the cash constraint in an unobservable-effort equilibrium is binding. The net worth subsidy, (4.31), financed by a period 1 tax on households, causes the allocations in the unobservable-effort equilibrium to coincide with those in the observable effort equilibrium.*

4.5.4. Creditor Bailouts

In this subsection we explore another policy that can increase welfare in a crisis. This policy subsidizes bank creditors (i.e., the mutual funds) when their portfolios perform poorly (i.e., when banks earn R^b). This policy is helpful because it goes to the heart of the problem. The problem when net worth is too low is that creditors must share in the losses when bank portfolios perform poorly. Under these circumstances, creditors require R_g^d to be high to compensate them for the losses associated with the low R_b^d. This increase in $R_g^d - R_b^d$ causes bankers to reduce effort below the efficient level [recall (4.18)]. By subsidizing creditors in the bad state, $R_g^d - R_b^d$ is reduced and effort moves back in the direction of its efficient level. We explore the quantitative magnitude of these effects in this section.

Let R_b^d denote, as before, the bank's payment in the bad state. The amount the mutual fund actually receives is $(1 + \tau)R_b^d$. We assume that the bailout τR_b^d is financed by a lump-sum tax on households in the second period. The zero-profit condition of the mutual fund is

$$p(e)R_g^d d + (1 - p(e))(1 + \tau)R_b^d d = Rd.$$

With this change, the equilibrium loan contract is the (e, d, R_g^d, R_b^d) that solves the following analog of (4.19):

$$
\max_{\left(e,\, d,\, R_g^d,\, R_b^d\right)} \lambda\{ p(e)\left[R^g(N+d) - R_g^d d\right] + (1 - p(e))\left[R^b(N+d) - R_b^d d\right]\} - \frac{1}{2}e^2
$$

$$
+ \mu\left[p(e)R_g^d d + (1 - p(e))(1 + \tau)R_b^d d - Rd\right]
$$

$$
+ \eta\left(e - \lambda p'(e)\left[(R^g - R^b)(N + d) - \left(R_g^d - R_b^d\right)d\right]\right)
$$

$$
+ \nu\left[R_b^d d - R^b(N + d)\right]. \tag{4.32}
$$

Note that τ enters only the zero-profit condition of mutual funds. Because τ does not explicitly enter the banks' own profits, the incentive constraint on bank effort is not affected. For a detailed characterization of the loan contract and an algorithm for computing the equilibrium, see Section A.3 in Appendix A.

We compute the socially optimal value of τ in a numerical example. The social welfare function aggregates the utility of everyone in the household:

$$
u(c) + \beta u(C) - \frac{1}{2}e^2.
$$

We do the computations for a crisis situation, one in which N is sufficiently low that $\nu \neq 0$. We construct an example by first selecting an equilibrium with $\tau = 0$ in which the cash constraint is nonbinding, that is, $\nu = 0$. We then reduce N sufficiently so that the cash constraint is binding and we then compute equilibria for a range of values of τ.

We must assign values to the following parameters:

$$
\beta, \gamma, R^g, R^b, a, b, y, N,
$$

where a and b are the parameters of $p(e)$ [see (4.5)], and γ, β are parameters that govern household utility [see (3.2)]. We set $\beta = 0.97$, $\gamma = 0.9$, $a = 0.5$, and $N = 1$, and we set the other four parameters, R^g, R^b, b, and y to achieve $R = 1/\beta$ and the following three calibration targets:

$$
p(e) = 0.99, \quad V(e) = 0.0036, \quad \frac{d}{y} = 0.26,
$$

where $V(e)$ denotes the variance, across banks, of returns [see (4.6)]. The equilibrium associated with this parameterization is characterized by a nonbinding cash constraint. In this equilibrium, $R_b^d = R_g^d = R$ when $\tau = 0$. We verified numerically that $\tau = 0$ corresponds to a local maximum of the social welfare function.
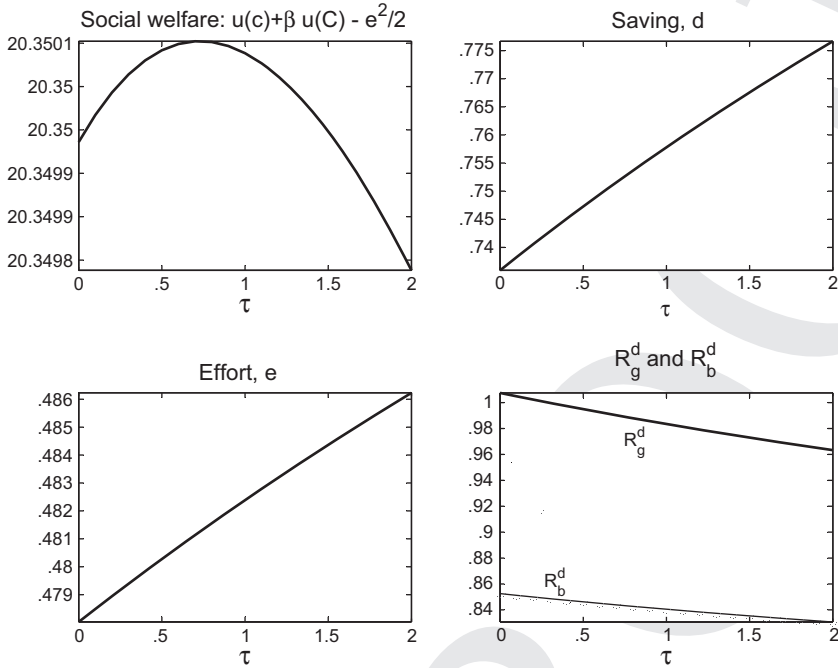
Figure 5.3. Hidden effort model properties, various τ.

We reduced the value of N to 0.70, in which case the cash constraint is binding. Figure 5.3 displays features of the equilibrium for values of $\tau \in (0, 2)$. The optimal value of τ is roughly 0.7282. Note that equilibrium effort e is increasing in τ. As indicated in the introduction to this section, this result reflects that $R_g^d - R_b^d$ is falling in τ. The rise in equilibrium effort gives rise to an increase in the return R generated by the financial system and hence produces a rise in deposits d.

4.5.5. Leverage Restrictions

In normal times, a binding leverage restriction on banks reduces welfare because the equilibrium is efficient. However, bankers make inefficiently low efforts in a crisis because their cost of funds is positively correlated with the performance of their assets. This correlation reflects that bankers' net worth is too low for them to insulate creditors from losses when banks experience a low return R^b. Obviously, if banks had a sufficiently low level of deposits when net worth was low, then bankers' net worth would be sufficient to cover losses. This raises the possibility that leverage restrictions may be welfare improving when net worth is low. However, recall that bank incentives are a

function not only of $R_g^d - R_b^d$, but also of the level of deposits d [see (4.18)]. So it is not so obvious, ex ante, that leverage restrictions are desirable in a crisis. For this reason, we investigate the desirability of leverage restrictions in a numerical example. In the example, we use the same parameter values as the ones in the previous subsection. We find that leverage restrictions indeed are desirable in crisis times.

We suppose that the government imposes a restriction on the equilibrium contract, which prohibits banks from exceeding a specified level of leverage $\bar{L}$:

$$\frac{N + d}{N} \leq \bar{L}.$$

This leads to the following alternative formulation of the problem solved by the equilibrium contract:

$$\max_{\left(e,\, d,\, R_g^d,\, R_b^d\right)} \lambda\big\{ p(e)\big[R^g(N + d) - R_g^d d\big] + (1 - p(e))\big[R^b(N + d) - R_b^d d\big]\big\}$$

$$- \frac{1}{2}e^2$$

$$+ \mu\big[p(e)R_g^d d + (1 - p(e))(1 + \tau)R_b^d d - Rd\big]$$

$$+ \eta\big(e - \lambda p'(e)\big[(R^g - R^b)(N + d) - (R_g^d - R_b^d)d\big]\big)$$

$$+ \nu\big[R_b^d d - R^b(N + d)\big] + \delta\big[\bar{L}N - (N + d)\big],$$

where $\delta \geq 0$ is the multiplier on the leverage constraint. We assume that the last two constraints are binding, so that $\delta > 0$, $\nu < 0$.

The nine panels in Figure 5.4 display selected characteristics of the equilibrium for a range of values of $\bar{L}$ and for two values of the bailout rate τ. The two values of τ are $\tau = 0$ and $\tau = 0.7282$, which are its optimal values when there are no leverage restrictions. When $\tau = 0$ and 0.7282, leverages in the absence of leverage restrictions are 2.0453 and 2.0684, respectively. The highest value of $\bar{L}$ reported in Figure 5.4 is 2.0453.

Consider the case $\tau = 0$ first. Note from Figure 5.4(b) that social welfare initially rises as $\bar{L}$ is reduced from $\bar{L} = 2.0453$. The optimal value of $\bar{L}$ is 1.9980. This represents a 2.3 percent cut in leverage, which translates into a reasonably substantial cut of roughly 5 percent in deposits d. Consistent with the intuition previously provided, the reduction in $\bar{L}$ reduces the state contingency in banks' costs of credit $R_g^d - R_b^d$ [see Figure 5.4(i)]. According to Figures 5.4(e) and (h), the fall in $R_g^d - R_b^d$ results in higher effort e, despite the lower level of deposits. As a result, the leverage restriction produces an increase in the cross-sectional average return on bank portfolios [Figure 5.4(d)], as well as a fall in the cross-sectional variance $V(e)$ in (4.6). To be
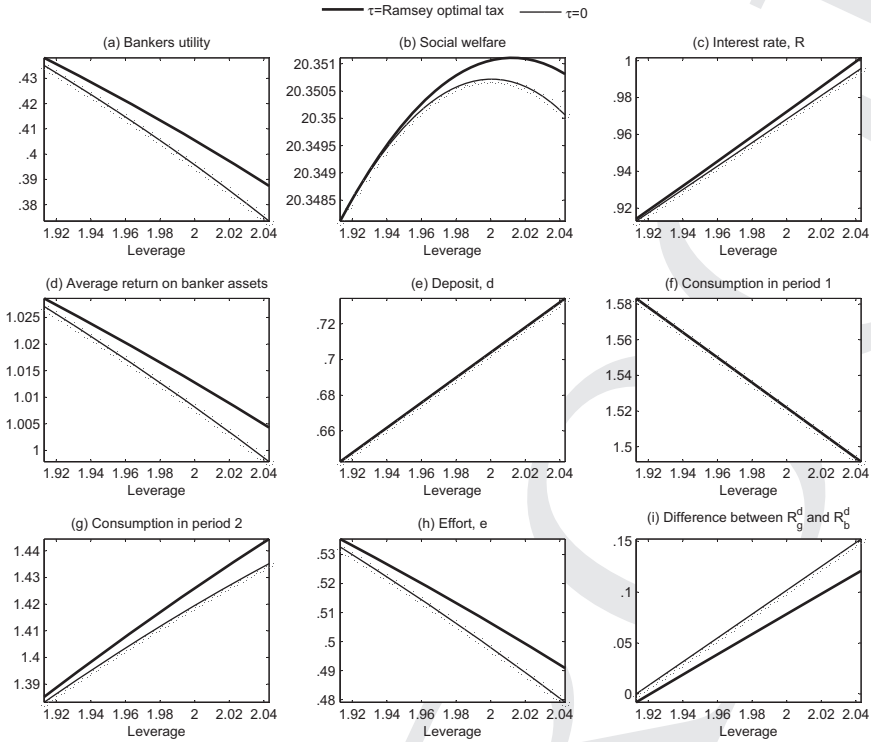
Figure 5.4. Hidden effort model properties for various restrictions and bailout rates, τ.

consistent with clearing in the market for deposits, the deposit rate R must fall as the leverage restriction becomes more binding [see Figure 5.4(c)]. With the fall in the deposit rate and the increase in the average return on assets, there is a rise in the net profits earned by banks on deposits. In the absence of government intervention, competition drives these profits to zero.[26] In effect, the government reduction in $\overline{L}$ causes the banking sector to behave as a monopsonist. Restricting $\overline{L}$ raises banker utility, (4.9), according to Figure 5.4(a).

We now turn to the case $\tau = 0.7282$. The results suggest that bailouts are, to some extent, a substitute for leverage restrictions. To see this, note that when τ is positive, then the optimal level of leverage is raised. This property of the model starkly contradicts conventional wisdom, which maintains that leverage restrictions are required to undo the bad side effects of bailout commitments.

The conventional wisdom on leverage can perhaps be paraphrased as follows: "bailouts reduce the incentive for creditors to play a socially important role in monitoring bankers, and this leads bankers to choose overly risky

portfolios." In our model, creditors have no ability to monitor bankers and so this monitoring channel is not present. However, from comparing the unobservable- and observed-effort versions of our model, we can conjecture what would happen if we modified our model so that creditors could decide, at a cost, whether and how much to monitor banks. Recall that in the observed effort version of our model studied in Section 4.3 creditors perfectly monitor the activities of the banker. In that model, even if net worth is so low that creditors must share in banker losses, the effort level of bankers is efficient. Thus, suppose net worth is low, so that the observable- and unobservable-effort equilibria differ. Suppose further that the economy is repeated twice, with the observable-effort equilibrium occurring at a first date and the unobservable-effort equilibrium occurring at the second date. Loosely, one can interpret this two-date economy (each date has two subperiods) as one in which creditors monitor in the first date but do not monitor in the second date. In this model, the effort level of bankers is inefficiently low in the second date (recall proposition 4.2) and the cross-sectional variance of their portfolios increases as a result (see (4.6)). This reasoning suggests to us that our model would be consistent with the conventional wisdom if creditor monitoring of bankers were endogenized. Of course, an important empirical question is whether in fact creditors do have the ability to monitor banks apart from observing the performance of banker securities.

The other results corresponding to the case $\tau = 0.7282$ are consistent with the idea that leverage complements bailouts in this model. For example, at every level of $\overline{L}$, banker effort is higher with $\tau > 0$ than with $\tau = 0$. Finally, note that the response of d [Figure 5.4(e)] and c [Figure 5.4(f)] are invariant to τ. This is because net worth and y are fixed in the figure and in this case leverage immediately determines d and c.

5. Adverse Selection

We consider an environment that is similar to the one in the previous sections, except that the friction now is adverse selection. As in the previous section, we capture the notion that banks do not hold a diversified portfolio of assets with the assumption that each bank can acquire the securities of at most one firm. Each firm in the economy has access to an investment project that requires a fixed input of resources to operate. Firms have no resources of their own and must rely on funding from a bank. Moreover, a firm can have a relationship with at most one bank.[27] A firm earns no rent from its investment project and turns over all revenues to the bank that holds its securities. Because a bank's own net worth is not sufficient

to finance the investment project of the firm whose securities it purchases, banks must also obtain deposits. Banks obtain deposits from mutual funds. Mutual funds are competitive and each one is perfectly diversified across banks. Finally, mutual funds are completely financed by risk-free deposits from households.

Because firm investment projects differ according to how risky they are, it follows that the asset side of the balance sheets of different banks differ in terms of their risk. We assume that the mutual funds that lend to banks cannot differentiate the high- and low-risk banks. To compensate for losses from deposits in the riskier banks the interest rate spread – the difference between the rate paid by banks to mutual funds and the rate paid by mutual funds on their risk-free deposits – must be positive.[28] The distortions associated with the interest rate spread imply that intermediation and investment are below their efficient levels. A drop in bank net worth aggravates the distortions because banks become more dependent on external finance.

We insert the banks and mutual funds into the type of general equilibrium environment considered in the previous sections of this chapter. When bank net worth falls, interest rate spreads jump and intermediation and investment drop. In this way the environment rationalizes the type of observations that motivated this chapter.

Consistent with the analysis of Mankiw (1986) and Bernanke and Gertler (1990), who consider a similar environment, we find that a subsidy to banks' cost of funds can ameliorate the problem.[29] Indeed, a suitable choice of the interest rate subsidy can make the market allocations coincide with the first-best efficient allocations. This is so, even though the subsidy policy does not require observing the riskiness of individual bank portfolios whereas our efficient allocations are those chosen by a benevolent planner who does observe those risks. A subsidy to banks, by reducing their dependence on external finance, can also improve allocations. We consider government deposits in banks, but these do not overcome the Barro-Wallace irrelevance result. That is, they have no effect on the allocations. Finally, we show that a tax-financed transfer of net worth to banks moves the allocations closer to first-best. The gains from doing this are greater in a crisis, because a decline in bank net worth increases the gap between equilibrium and first-best efficient allocations.

5.1. Model

The economy is populated by many large and identical households. The representative household has a unit measure of members composed of workers

and bankers. The measure of bankers is $e < 1$.[30] All these agents receive perfect consumption insurance from households. Workers and bankers receive endowments of y and N, respectively, at the start of the first period. Here, $y > 0$ is measured in household per capita terms. We find it convenient to measure $N < 1$ in banker per capita terms. Thus, in household per capita terms the quantity of banker net worth is eN. The conditions that characterize household optimization are as in the other parts of this chapter but are reproduced here for convenience:

$$c + d = y, \tag{5.1}$$

$$c^{-\gamma} = \beta R C^{-\gamma}, \quad \gamma > 0, \tag{5.2}$$

$$C = Rd + e\pi. \tag{5.3}$$

Here, c denotes first-period household consumption, d denotes deposits, and C denotes second-period consumption. These three variables are measured in household per capita terms. The object π denotes earnings, in banker per capita terms, brought home in period 2 by bankers. Finally, R denotes the gross rate of interest on household deposits in mutual funds. We now discuss the problems of firms, bankers, and mutual funds.[31]

Each banker meets a firm with an investment project characterized by two parameters, $\theta > 0$ and $p \in [0, 1]$, which are drawn independently from the cumulative distribution function (cdf), $F(\theta, p)$. These parameters are subsequently explained. The banker must then select between one of two options. The net worth can be deposited in a mutual fund and earn RN. Alternatively, the net worth can be combined with loans obtained from a mutual fund and securities can be purchased from the firm with which the banker is paired. From here on, we simplify the language by pretending that the investment project operated by a bank's firm is operated directly by the bank.

A banker's realized values of θ and p are known only to the bank and to the household to which the bank belongs. In particular, the mutual fund from which the banker obtains funds does not observe θ and p. The distribution F is known to all. All investment projects are indivisible and require an investment of one good in period 1. We explain the reason for our assumption that there is an upper bound on the scale of investment in the discussion of Proposition 5.3, which appears in Subsection 5.2.1. In period 2, the investment project yields θ goods with probability p and zero goods with probability $1 - p$. Our analysis is greatly simplified by placing the following restriction on F:

$$\theta p = \bar{\theta}, \tag{5.4}$$

where $\bar{\theta}$ is a nonrandom parameter known to all. Thus, each banker's investment project generates the same expected return, but differs in terms of riskiness. We characterize F by specifying a distribution for p and then setting $\theta = \bar{\theta}/p$. We assume that p is drawn from a uniform distribution with support $[0,1]$.

Because $N < 1$, a banker must obtain a loan from a mutual fund in order to operate any investment technology. We suppose that the mutual fund can observe only whether the banker's project succeeds or fails. In case the project succeeds, the mutual fund cannot tell ex post what that project's value of θ was. As a result, the payment made by the banker to the mutual fund can be contingent only on whether the banker's project is successful. We denote the interest rate paid by the banker in the event that the project succeeds by r. Because the banker has no resources in the event that the project fails, the interest rate in that event must be zero.

Bankers who choose not to activate their investment projects earn RN with certainty by depositing their net worth in mutual funds. For a banker who decides to operate a project with probability p earns $\theta - r(1 - N)$ and with probability $1 - p$ earns nothing. It is in the household's interest that each of his or her bankers makes the project activation decision with the objective of maximizing expected earnings. The law of large numbers and the fact that there are many bankers in each household guarantee that if each banker behaves in this way, the total resources brought home by all bankers in a family is maximized. Households are assumed to be able to compel bankers to maximize expected returns and not divert any profits because of the assumption that households observe everything (including θ and p) about their bankers. Thus, a given banker invests his or her net worth N in a project and borrows $1 - N$ if and only if the realized value of p satisfies

$$\bar{\theta} - pr(1 - N) \geq RN. \tag{5.5}$$

The values of p that satisfy (5.5) are as follows:

$$0 \leq p \leq \bar{p}(r), \quad \bar{p}(r) \equiv \frac{\bar{\theta} - RN}{r(1 - N)}. \tag{5.6}$$

The object $\bar{p}(r)$ in (5.6) summarizes several interesting features of the equilibrium. For example, note that when r increases, bankers with higher values of p decide not to activate their investment project [i.e., $\bar{p}(r)$ is decreasing in r]. The reason is that under our assumptions expected investment income is fixed at $\bar{\theta}$ whereas bankers with high-p projects are more likely to experience success and pay r to their mutual fund. As a result, the

expected return on investment is lower for bankers with less risky invest-
ment projects, that is, those with high p. Also, $\overline{p}(r)$ is the fraction of bankers
who invest:

$$\int_0^{\overline{p}(r)} dp = \overline{p}(r). \tag{5.7}$$

Here, we have used the implication of the uniform distribution that the den-
sity of bankers with each $p \in [0, 1]$ is unity. Similarly, because the quantity
of goods used in each investment project is unity, $\overline{p}(r)$ also corresponds
to the total quantity of goods invested by all bankers in a household. The
average value of p among the bankers who invest is denoted as $\Pi(r)$,
where

$$\Pi(r) = \frac{\int_0^{\overline{p}(r)} p\,dp}{\overline{p}(r)} = \frac{1}{2}\overline{p}(r). \tag{5.8}$$

Expression (5.8) reflects that, among the bankers who operate their invest-
ment technologies, the density of bankers with $p \in [0, \overline{p}(r)]$ is $1/\overline{p}(r)$.
Finally, we subsequently show that $\overline{p}(r)$ is inversely proportional to the
interest rate spread, the difference between the interest rate r paid by bankers
with successful projects and the risk-free rate R. We restrict our attention
to model parameterizations that imply the efficient allocations (see Sub-
section 5.2.1) and the equilibrium allocations are interior. This means the
usual nonnegativity constraints on quantities and also $0 < \overline{p}(r) < 1$.

We now turn to the mutual funds. Because each mutual fund is fully
diversified across bankers, its revenues are nonrandom. Because we also
assume that mutual funds are competitive, it follows that their profits must
be zero. A mutual fund's average earnings per unit of loan is $\Pi(r)r$. The
cost of a unit of deposits for a mutual fund is R, so that the each mutual
fund's zero-profit condition is

$$\Pi(r)r = R. \tag{5.9}$$

Much of the economics of the model is summarized in (5.9). For example,
multiplying (5.8) by r and using (5.9), we obtain a simple expression for
the interest rate spread:

$$\text{interest rate spread} = \frac{r}{R} = \frac{2}{\overline{p}(r)}. \tag{5.10}$$

According to this expression, the interest rate spread is at least 2 and can
be much higher. The intuition for (5.10) is simple. Suppose all bankers
activated their investment project, so that $\overline{p} = 1$. In this case, the average

probability of success is 1/2 [see (5.8)]. With half the bankers unable to pay, the ones that do pay must pay $2R$ if the mutual fund is to be able to pay R to its depositors.[32]

Interestingly, (5.9) determines the equilibrium rate of interest R in the model. To see this, substitute (5.6) into (5.8) to obtain

$$\Pi(r)r = \frac{1}{2}\frac{\overline{\theta} - RN}{1 - N} = R, \qquad (5.11)$$

where the second equality reflects (5.9). Evidently, R is determined exclusively by variables specific to the loan market and not by, for example, households' intertemporal preferences. That the zero-profit condition is compatible with only one R is a striking result, though well known in the literature on adverse selection. To understand the result requires understanding why mutual fund revenues per loan are independent of the interest rate r that they charge on a loan. Note that a higher r implies that mutual funds earn more revenues from bankers who borrow and repay their loan. However, this positive impact on revenues is canceled by an adverse-selection effect. Recall that when a bank raises r, bankers with a high probability of repaying their loan decide to become inactive.[33] As a result, the average probability that a banker repays the loan falls [see (5.6) and (5.7)]. In principle, this need not be a problem because the lower-probability bankers also enjoy a better outcome when they are successful. However, this is little comfort to the mutual funds in the model, because they must charge the same interest rate r to all borrowers. A fixed interest rate on loans prevents mutual funds from sharing in the huge payoffs experienced when low-p bankers are successful. This is why a mutual fund's revenues are independent of r.

Adverse selection also explains why the revenue function $\Pi(r)r$ is decreasing in R. As R increases, high-p bankers switch to being inactive, and this reduces the average p among borrowers from mutual funds, reducing mutual fund revenues per unit of loan extended.

Because R is determined by the zero-profit condition, in equilibrium the quantity of saving by households adjusts passively to the R that is implied by (5.11). If, for example, the supply of saving were perfectly elastic at an interest rate that is different from the one that solves (5.11), then a small perturbation in the variables (such as N) that determine $\Pi(r)r$ would have an enormous impact on intermediation. In a one-sector model such as ours, the notion that the supply of saving is highly inelastic seems implausible. However, in a multisector (or open-economy) version of the model, the situation would be different. Thus, suppose that the zero-profit condition in (5.9) pertained to mutual funds specializing in the supply of

funds to banks in a particular sector that is small enough that it takes the economy-wide deposit rate R, as given. In this case, the supply of funds to the particular sector could be expected to be perfectly elastic at the interest rate R. If a decrease in net worth N among the bankers of the given sector drives revenues per loan [i.e., the object to the right of the first equality in (5.11)] down, then a fall in N could cause intermediation in that sector to collapse entirely. We do not explore the multisector version of our model more here, though this would clearly be of interest.

Clearing in financial markets requires that the quantity of investment $e\bar{p}(r)$ equal the quantity of household deposits d plus the quantity of net worth eN in the hands of bankers:

$$e\bar{p}(r) = d + eN. \tag{5.12}$$

We now obtain a simplified expression for period 2 household income. Averaging earnings over all bankers, we obtain

$$
\pi = \int_0^{\bar{p}(r)} \{\bar{\theta} - pr(1-N)\}\, dp + \int_{\bar{p}(r)}^1 NR\, dp
$$
$$
= \bar{p}(r)[\bar{\theta} - \Pi(r)r(1-N)] + (1 - \bar{p}(r))NR. \tag{5.13}
$$

Adding $e\pi$ to household earnings on deposits yields the equilibrium expression for total household income in the second period:

$$Rd + e\bar{p}(r)[\bar{\theta} - \Pi(r)r(1-N)] + e(1 - \bar{p}(r))NR = e\bar{p}(r)\bar{\theta}.$$

The expression after the equality is obtained after substituting out for R and d using (5.9) and (5.12). The object $e\bar{p}(r)\bar{\theta}$ represents the total period 2 output from bankers' investment projects in household per capita terms. Replacing total household income with its equilibrium value of $e\bar{p}(r)\bar{\theta}$ in (5.3), we obtain the household's second-period budget constraint in equilibrium:

$$C = e\bar{p}(r)\bar{\theta}. \tag{5.14}$$

Consistent with Walras's law, (5.14) is also the second-period resource constraint.

We have the following definition of equilibrium:

Adverse-selection equilibrium: $c, C, d, r, R, \pi, \bar{p}(r)$ such that

   (i) c, C, d solve the household problem given R, π,
  (ii) mutual funds earn zero profits, and
 (iii) bankers maximize expected revenues.

An equilibrium is straightforward to compute for this economy. The five equilibrium conditions, (5.1), (5.2), (5.9), (5.12), and (5.14), as well as the definitions of π and $\bar{p}(r)$ in (5.13) [with $\Pi(r)$ defined in (5.8)] and (5.6), respectively, are sufficient to determine the seven equilibrium objects. Substitute (5.6) into (5.8) and solve the resulting expression for R:

$$R = \frac{\bar{\theta}}{2 - N}. \tag{5.15}$$

Combine (5.14) and (5.2) and use (5.6):

$$c = (\beta R)^{-\frac{1}{\gamma}} \frac{\bar{\theta} - RN}{r(1 - N)} \bar{\theta} e. \tag{5.16}$$

Use the latter expression and (5.12) to substitute out for c and d in (5.1). Solving the resulting expression for r, we obtain

$$r = 2eR \frac{(\beta R)^{\frac{-1}{\gamma}} \bar{\theta} + 1}{y + eN}, \tag{5.17}$$

with the understanding that R is determined by (5.15). With r in hand, c can be computed from (5.16), C from (5.2), d from (5.1), $\bar{p}$ from (5.6), and π from (5.13) and (5.8). In this way, all the equilibrium variables can be computed uniquely as long as the model parameters are such that an interior equilibrium, $\bar{p}(r) < 1$ and $c, C > 0$, exists.[34]

The ratio of equations (5.15) and (5.17) provides a convenient expression for the interest rate spread r/R:

$$\frac{r}{R} = 2e \frac{(\beta R)^{\frac{-1}{\gamma}} \bar{\theta} + 1}{y + eN}. \tag{5.18}$$

According to (5.15), R falls with a decrease in N. According to (5.18), this fact alone drives the spread up. The total effect of a decrease in N on the interest rate spread also involves the denominator in (5.18), and this drives the interest rate spread up too. We summarize these results as follows:

Proposition 5.1: *When an interior adverse selection equilibrium exists, it is unique and characterized by (5.15), (5.16), (5.17), and the observations thereafter. The interest rate spread, given by (5.18), rises with a reduction in N.*

5.2. Implications for Policy

Subsection 5.2.1 discusses a planner problem for our model economy. In addition, we use this subsection to explain why we assume the existence of an upper bound on the scale of bankers' projects. In the subsequent subsections, we show that two types of subsidy schemes improve the equilibrium allocations: a tax-financed transfer of net worth to bankers and a tax subsidy to mutual funds. Tax-financed government deposits with the mutual funds do not overcome the Barro-Wallace proposition. They have no effect because they do not affect the equilibrium interest rate on bank deposits. Households respond to the increase in taxes by reducing their deposits one-for-one with the increase in taxes and government deposits.

5.2.1. Efficient Allocations

We consider the allocations selected by a benevolent planner who observes a banker's p. We use these allocations as a benchmark from which to evaluate the adverse-selection equilibrium and various policy interventions studied in the subsequent subsections. Although here we assume the planner observes each bank's p, the policy interventions studied in subsequent subsections do not require that policymakers observe p.

The planner faces the period 1 resource constraint,

$$c + d \leq y. \tag{5.19}$$

To describe the planner's decisions about which and how many projects to activate and to derive the planner's period 2 resource constraint, we find it useful to describe the model environment using a particular figure.

Figure 5.5 arranges all the agents in the economy in the unit square. Each point in the square corresponds to a particular household (vertical dimension) and member of household (horizontal dimension). There is a unit measure of households and a unit measure of members of any given household. We suppose that the box is constructed in period 1, just after each banker has drawn his or her value of p. A horizontal line inside the box highlights one particular household. The points on the line to the left of e correspond to the bankers. The points to the right of e correspond to the workers. The bankers are ordered according to their value of p, from $p = 0$ to $p = 1$ passing from left to right. For any particular $p \in [0, 1]$, the banker with that investment project is indicated by the point pe on the horizontal axis. Each point on a vertical line through pe corresponds to the bankers
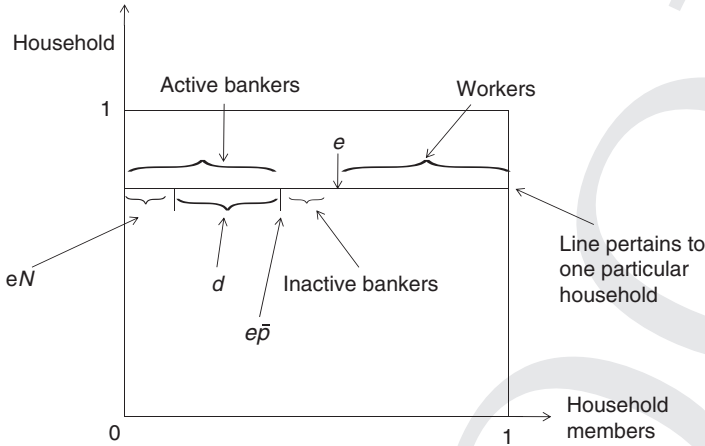
Figure 5.5. Agents in the adverse selection model.

with the given p in the cross section of households. Each of those bankers has the value of θ that is given by (5.4).

The planner must decide how many bankers in the interval 0 to e to activate. If the planner elects to activate a banker with a particular p, it instructs all the bankers in the cross section of households with that p to activate their project. The planner is indifferent about which projects (i.e., which p's) to activate. Each project is the same to the planner because each has the same mean productivity $\bar{\theta}$, and bankers suffer no cost to activate their project. As a result, there is no loss of generality in simply assuming that the planner selects bankers with p's extending from $p = 0$ to $p = \bar{p}$ for some $\bar{p} \leq 1$. This corresponds to the mass of bankers in the interval 0 to $e\bar{p}$ in the figure.

Consider a mass of bankers on an arbitrary interval Δ inside $[0, e]$. The resource cost of activating these bankers in the cross section of households is the area of the rectangle with base Δ inside the unit square. The latter area is just Δ itself. This reflects the assumption that there is a unit mass of households and that each project costs one unit of resources to activate. The available net worth N per banker is sufficient to operate the bankers corresponding to the interval 0 to eN. Because these resources have no alternative use, the planner applies them. Activating additional bankers is costly to the planner because this requires suppressing consumption in period 1. Suppose the planner considers activating an additional mass d of bankers. This corresponds to the bankers extending from the point eN to the

point $eN + d$ in the figure. Activating these bankers requires d resources. So, if the planner wishes to activate a measure $e\overline{p}$ of bankers, then $d + eN$ resources are needed, subject to

$$e\overline{p} \leq d + eN. \tag{5.20}$$

When the planner activates bankers from 0 to $e\overline{p}$, the total amount of goods available in period 2 is $e\overline{p}\overline{\theta}$. Thus the second-period resource constraint for the planner is

$$C \leq e\overline{p}\overline{\theta}. \tag{5.21}$$

The planner's problem is to solve

$$\max_{c,\, C,\, \overline{p},\, d} u(c) + \beta u(C),$$

subject to $0 \leq \overline{p} \leq 1$, (5.19), (5.20), (5.21), and $c, C \geq 0$. The unique interior solution is characterized by the first-order conditions evaluated at equality. Solving these, we obtain

$$c = \frac{y + eN}{(\beta\overline{\theta})^{\frac{1}{\gamma}} + \overline{\theta}}\,\overline{\theta}, \tag{5.22}$$

$$e\overline{p} = \frac{y + eN}{1 + \overline{\theta}(\beta\overline{\theta})^{-\frac{1}{\gamma}}}, \tag{5.23}$$

$$C = c(\beta\overline{\theta})^{\frac{1}{\gamma}}, \tag{5.24}$$

with d given by solving (5.19) with equality. It is convenient to compare these allocations with the allocations in the adverse-selection equilibrium. Substituting (5.17) into (5.16) and using (5.15), we obtain that first-period consumption in the equilibrium is

$$c = \frac{y + eN}{(\beta R)^{\frac{1}{\gamma}} + \overline{\theta}}\,\overline{\theta}.$$

Using (5.6) and making use of (5.15) and (5.17), we find that total resource use in the adverse-selection equilibrium is

$$c = \frac{y + eN}{1 + \overline{\theta}(\beta R)^{\frac{1}{\gamma}}}\,\overline{\theta}. \tag{5.25}$$

According to (5.2), second-period consumption in equilibrium is

$$C = c(\beta R)^{\frac{1}{\gamma}}.$$

Evidently, the sole factor preventing the equilibrium from replicating the planner's allocations is that the interest rate R is too low. In the adverse-selection equilibrium, R is $\bar{\theta}/(2 - N)$, but the social rate of return on investment is $\bar{\theta}$. With the market sending the wrong signal to households about the return on investment, saving and investment are too low. The more severe the problem, the smaller N is.

The ratio of investment in equilibrium to its first-best level is given by dividing (5.25) by (5.23):

$$\frac{1 + \bar{\theta}(\bar{\theta}\beta)^{-\frac{1}{\gamma}}}{1 + \bar{\theta}(R\beta)^{-\frac{1}{\gamma}}}.$$

From this expression, we see that equilibrium investment falls relatively more than the first-best level of investment when N decreases [here we have used the relation between R and N in (5.15)].

We summarize the preceding results in the form of a proposition:

Proposition 5.2: *The equilibrium household deposit rate R is less than the social return on investment $\bar{\theta}$, and R falls with a reduction in N. Equilibrium investment falls relatively more than the first-best level of investment with a reduction in N.*

In our adverse-selection model we suppose that there is an upper bound on the resources that bankers can invest in their projects. In Appendix B we consider a version of the model that does not impose an upper bound on the scale of banker projects. For that version of the model we find the following proposition:

Proposition 5.3: *Suppose banker projects have constant returns and can be operated at any scale. If there is an equilibrium, then (i) only bankers with the lowest value of p operate their projects, (ii) the aggregate profits of these bankers is zero, and (iii) the allocations in equilibrium coincide with the first-best efficient allocations and $R = \bar{\theta}$.*

To help ensure the existence of an equilibrium under the assumption of proposition 5.3, we modify the distribution of p slightly by supposing that it

has positive mass at the lower bound of its support and that the lower bound is a (very) small positive number. For our purposes, property (i) renders the equilibrium of this version of our model uninteresting. Also, we suspect that in reality investment projects have diminishing returns to scale. Although the diminishing returns to scale implicit in our upper bound assumption for investment projects is extreme, we find this assumption more interesting than the constant returns to scale alternative used in proposition 5.3. Intermediate scenarios are presumably also of interest, but we do not examine these here.

5.2.2. Interest Rate Subsidies

According to the analysis in previous subsections, the problem with the adverse-selection equilibrium is that the deposit rate R is too low. In addition, when net worth drops, the problem is aggravated as R falls even more and investment, relative to first-best, drops (see proposition 5.2). Consistent with the empirical phenomenon we seek to understand, the interest rate spread also rises with a drop in N (see proposition 5.1). The inefficiently low deposit rate R reflects that mutual funds do not recover the full return made possible by their loans. This suggests two direct ways to repair the market mechanism: subsidize mutual fund earnings or, equivalently, their cost of funds. We consider the latter here. We show that an appropriate interest rate subsidy can make the allocations in the adverse-selection equilibrium coincide with the first-best efficient allocations. Significantly, implementation of this policy does not require that the government observe any characteristics of the banks that borrow from mutual funds.

Denoting the pretax cost of funds to the mutual fund by R, the after-subsidy cost under our policy is $R/(1 + v) < R$. We suppose that this subsidy is financed by a lump-sum tax on households in period 2 in the amount

$$T = (1 - N)e\overline{p}(r)\left[R - \frac{R}{1 + v}\right].$$

Here, the terms in front of the square bracket represent the total amount of loans made by the mutual funds to the active banks. The mutual funds finance these loans with deposits taken from inactive bankers and households. The amount of the subsidy is $R - R/(1 + v)$ per unit of loans made. The household's second-period budget constraint, (5.3), is replaced by

$$C = Rd + e\pi - T.$$

Repeating the substitutions leading up to (5.14), taking account of the modified second-period budget constraint of the household, we find that (5.14) continues to hold.

The impact of the tax subsidy on the equilibrium value of R is determined by studying the appropriately modified mutual fund zero-profit condition, (5.9):

$$\Pi(r)r = \frac{R}{1+v}. \tag{5.26}$$

Substituting (5.6) and (5.8) into the latter expression and solving for R, we obtain

$$R = \frac{\overline{\theta}(1+v)}{2-N+Nv}.$$

Evidently, achieving $R = \overline{\theta}$ requires $v = 1$.

Recall that the seven conditions determining c, C, d, r, R, π, and $\overline{p}(r)$ are (5.1), (5.2), (5.9), (5.12), (5.14), (5.13), and (5.6). We have verified that (5.14) continues to hold. Apart from (5.14), the other conditions are obviously unaffected by T. The only equilibrium condition that must be adjusted is the mutual fund zero-profit condition, (5.9), which we replace by (5.26). With $v = 1$, the interest rate that solves (5.26) is the efficient one, $R = \overline{\theta}$. Given that the other equations are unaffected, it follows from the discussion in the previous subsection that the efficient allocations are supported by the subsidy policy. We summarize this result in a proposition:

Proposition 5.4: *With an interest cost subsidy, $v = 1$, the allocations in the adverse-selection equilibrium are efficient.*

5.2.3. Tax-Financed Transfers to Bankers

Here, we consider a policy of raising lump-sum taxes T on households in period 1 and transferring T/e to each banker. By setting

$$T = e(1-N), \tag{5.27}$$

the transfer ensures that each banker has enough funds to fully finance an investment project. With this policy the financial frictions are completely circumvented. To see this, note that the value of R that satisfies the mutual fund zero-profit condition, (5.9), is still the one in (5.15), except that N is replaced by the posttransfer level of banker's net worth. Because that is unity under the tax-transfer scheme, we have that $R = \overline{\theta}$, its value in the

efficient equilibrium. It is also straightforward to verify that c, C, and $\overline{p}$ satisfy (5.22), (5.23), and (5.24). We have assumed that model parameters imply that $\overline{p} \leq 1$. When $\overline{p} < 1$, then $d < 0$. That is, in this case some of the net worth transferred to bankers is recycled back to households through the loan market.

We summarize these results as follows:

Proposition 5.5: *Under the tax-transfer scheme in (tax), the allocations in the adverse-selection equilibrium are efficient.*

A problem with the tax-transfer scheme considered in this section is that it presses hard on a feature of the model in which we have little confidence. In particular, we assume that it is known how much net worth each banker has and how much he or she needs for his or her investment project. In practice, these assumptions may be difficult to assess. In addition, our environment abstracts from any problems associated with distributing wealth between bankers and other agents. These would have to be considered in case such a policy were actually implemented.

5.2.4. Tax-Financed Government Deposits in Banks

Suppose the government levies a lump-sum tax T on households in period 1, deposits the proceeds in a bank, and then transfers the earnings RT on the deposits to households in period 2. It is easy to see that this has no impact on allocations, as long as the policy does not drive the economy into a boundary. The equations that characterize an interior equilibrium are (5.1), (5.2), (5.9), (5.12), (5.14), (5.13), (5.6), and (5.8) (see the discussion after the definition of equilibrium in Section 5.1). The new policy simply requires replacing d with $d + T$ in (5.1), (5.9), and (5.12). If T is increased, then d falls by the same amount. Total investment in period 1, consumption in the two periods, and the interest rate R do not change. Although a drop in net worth makes the banking system more dysfunctional, a policy of tax-financed loans to banks has no impact.

6. Asymmetric Information and Monitoring Costs

This section presents a version of the model in the previous sections, in which the financial friction on the liability side of banks' balance sheets stems from an asymmetric information problem similar to the one studied in BGG.[35] As in the previous sections, we assume the trigger for the crisis is a fall

in the net worth of banks. Our environment is well suited to contemplating the effects of an increase in a particular type of microeconomic uncertainty, and so we consider this as an additional trigger for the crisis. Like the model in the previous section (though unlike our first two models), the model analyzed here does not display the nonlinearity of a sharp dichotomy between normal and crisis times. This is because equilibrium conditions of the model do not include equations that are satisfied as strict equalities for some values of the state and strict inequalities for other values of the state. As a result, we simply define a normal time as one in which banking net worth is high and a crisis time as one in which net worth is substantially lower.

We now provide a sketch of the model, which most closely resembles the setup in the previous section. There are two periods. A large number of workers and bankers with identical utility functions live in a representative, competitive household. Workers have an exogenous endowment y of income in the first period, and bankers possess N units of net worth. All variables are expressed in household per capita units. Households allocate y to first-period household consumption and to deposits in identical, competitive mutual funds. Each banker combines his or her net worth with a loan from a mutual fund to acquire the securities of one firm. The firm operates a technology perturbed by an idiosyncratic technology shock to build capital in period 1 and uses the capital in period 2 to produce goods. Because we assume markets are competitive and because firms contribute nothing themselves to building capital and selling goods, they enjoy zero earnings after paying the return on the security sold to their bank. Because a bank can invest in the securities of at most one firm, the asset side of a bank's balance sheet is risky. As in the previous section, we simplify the exposition by pretending that a firm's activity building capital and selling goods is undertaken directly by its bank.

A financial friction arises because the bank observes the realization of the idiosyncratic technology shock on the asset side of its balance sheet, whereas its mutual fund can observe the technology shock only by paying a monitoring cost. The loan received by banks from their mutual fund comes in the form of a standard debt contract. The contract specifies a loan amount and an interest rate. Banks with a sufficiently low realization of their idiosyncratic productivity shock are not able to repay their loan, and these banks must transfer whatever assets they have to their mutual fund after being monitored. The measure of microeconomic uncertainty referred to in the opening paragraph pertains to the variance across banks in their idiosyncratic technology shock. We consider a characterization of

the financial crisis that involves just a drop in bank net worth as one that also involves an increase in the cross-sectional variance of the technology shock. Household consumption in the second period is financed out of income on period 1 deposits as well as bank profits.

The model implies that a fall in bank net worth causes interest rate spreads – the cost of funds to banks minus the risk-free rate – to rise and investment to fall. We consider various policy responses. We find that a policy that subsidizes the cost of funds to banks is welfare improving in both normal and crisis times. Moreover, the optimal value of the subsidy is greater in crisis times, so that the model suggests a more aggressive policy stance then. We also find that, absent government intervention, bank leverage is too low. In this sense, the model does not include the kind of features that rationalize the current interest in imposing leverage restrictions on economic agents. The reason for our "underborrowing" result is that the marginal return on loans by mutual funds to banks is higher than the average return, whereas the economics of the model implies that bank creditors focus on the average return.

Next, we show that the Barro-Wallace irrelevance result applies for loans made by the government to banks. We also show that a sufficiently large tax-financed transfer of net worth to bankers allows the economy to support the first-best equilibrium outcomes. Because the model abstracts from the income distribution consequences of this type of policy, we think of the result only as illustrating the logic of the model. The latter result is reported in proposition C.4 in Appendix C.2.6.

Although the model framework into which we insert the BGG-type financial frictions in this section has the virtue of consistency with the other models in the paper, it has one potential drawback. In our framework, the price of capital is always unity, whereas in the literature (see, e.g., BGG or Christiano, Motto, & Rostagno, 2003, 2010, 2011), the price of capital is endogenous. Moreover, the recent literature on pecuniary externalities (see Bianchi, 2011; Korinek, 2011; Lorenzoni, 2008; Mendoza, 2010) raises the possibility that there is overborrowing when banker net worth is in part a function of the market price of an asset like capital (see, e.g., Bianchi, 2011; Mendoza, 2010). These findings motivate us to investigate the robustness of our underborrowing result to endogenizing the price of capital the way it is done in BGG. We do so by introducing curvature into the technology for converting consumption goods into capital. Because capital is the only source of net worth for bankers, the change introduces the type of pecuniary externality studied in the literature. We display numerical results that suggest that our underborrowing result in fact is robust to this change.

6.1. Banks and Mutual Funds

The typical banker takes a net worth N and approaches a mutual fund for a loan B. The bank combines its net worth and the loan to produce output in period 2, using the following production function:

$$\omega(N + B)r^k, \tag{6.1}$$

where r^k is a fixed parameter of technology. Also, ω is an idiosyncratic productivity shock,

$$\omega \sim F, \quad \int_0^\infty dF(\omega) = 1, \tag{6.2}$$

where F is the cdf of ω.

Before the realization of the bank's productivity shock, the bank and its mutual fund have the same information about ω. Both know the shock will be drawn from F. After the realization of the shock, the bank and the mutual fund are asymmetrically informed. The bank observes the realization of its ω, but the bank's mutual fund can observe the shock only by paying a monitoring cost. Townsend (1979) showed that under these circumstances a "standard debt contract" works well. Under such a contract, the bank pays the mutual fund an amount ZB in period 2 if it is able to do so. Given F, some banks experience such a low ω that they are not able to repay ZB. Under a standard debt contract, those banks are "bankrupt." The mutual fund associated with such a bank verifies bankruptcy by undertaking costly monitoring,

$$\mu\omega(N + B)r^k,$$

where $\mu > 0$ is a parameter. A bankrupt bank transfers everything it has, $\omega(N + B)r^k$, to its mutual fund.

The cutoff level of productivity $\overline{\omega}$ that separates the bankrupt and non-bankrupt banks is defined by

$$\overline{\omega}(N + B)r^k = ZB. \tag{6.3}$$

According to (6.3),

$$\overline{\omega} = \frac{ZB}{(N + B)r^k} = \frac{Z\frac{B}{N}}{\frac{(N+B)}{N}r^k} = \frac{Z}{r^k}\frac{L-1}{L},$$

where L denotes the leverage of the banker,

$$L \equiv \frac{N + B}{N}. \tag{6.4}$$

Evidently, as $L \to \infty$, $\overline{\omega}$ converges to a constant, that is, the value of ω that a bank with no net worth needs to be able to pay back its debt.

From the perspective of period 1, an individual bank's expected profits π in period 2 are given by

$$\pi = \int_{\overline{\omega}}^{\infty} \left[\omega(N+B)r^k - ZB \right] dF(\omega) = NLr^k \left(\int_{\overline{\omega}}^{\infty} [\omega - \overline{\omega}] dF(\omega) \right) \tag{6.5}$$

using (6.3). Expression (6.5) is written in compact notation as follows:

$$\pi = NLr^k \int_{\overline{\omega}}^{\infty} [\omega - \overline{\omega}] \, dF(\omega) = NLr^k (1 - \Gamma(\overline{\omega})), \tag{6.6}$$

where

$$\Gamma(\overline{\omega}) \equiv G(\overline{\omega}) + \overline{\omega} \left[1 - F(\overline{\omega}) \right], \quad G(\overline{\omega}) \equiv \int_0^{\overline{\omega}} \omega dF(\omega). \tag{6.7}$$

The bank maximizes (6.6) and remits all its profits to its household. The banker does so in exchange for perfect consumption insurance. It is in the interest of the household that each of its banks maximize expected returns because, by the law of large numbers, this implies that bankers as a group maximize the total resources brought home to the household in period 2. Households can observe everything about their own banks (including ω), and we make the (standard) assumption that what is observable is enforceable. That is, the household has the means to make sure that each of its bankers actually maximizes (6.6) and does not, for example, divert any proceeds toward private consumption.

From (6.6) and the observation about $\overline{\omega}$ for large L, we see that the banker's objective is unbounded above in L for any given Z and r^k. As a result, we cannot use the classic Marshallian demand and supply paradigm in which the banker takes Z as given and chooses a loan amount B. To describe our market arrangement, we must first discuss the situation of the banks' creditors, mutual funds.

There are a large number of identical and competitive mutual funds, each of which makes loans to banks and takes deposits from households. Because mutual funds are diversified, there is no risk on the asset side of their balance sheet. Although a mutual fund does not know whether any particular bank will fully repay its loan, the mutual fund knows exactly how much it will receive from its banks as a group. Because there is no risk on the asset side of the balance sheet, it is feasible for mutual funds to commit in period 1 to paying households a fixed and certain gross rate of interest R

on their deposits in period 2. Because mutual funds are competitive, they take R as given. A mutual fund that makes size B loans to each of a large number of banks earns the following per bank:

$$[1 - F(\overline{\omega})] ZB + (1 - \mu) \int_0^{\overline{\omega}} \omega dF(\omega) r^k (N + B).$$

Here, the term before the plus sign corresponds to the revenues received from banks that are not bankrupt, that is, those with $\omega \geq \overline{\omega}$. The term after the plus sign indicates receipts, net of monitoring costs, received from banks that cannot fully repay their loan. Because the cost of funds is RB, the mutual fund's zero-profit condition is [using (6.3)]

$$[1 - F(\overline{\omega})] \overline{\omega}(N + B) r^k + (1 - \mu) \int_0^{\overline{\omega}} \omega dF(\omega) r^k (N + B) = RB,$$

or

$$[\Gamma(\overline{\omega}) - \mu G(\overline{\omega})] \frac{r^k (N + B)}{B} = R. \tag{6.8}$$

An important feature of this environment is that the interest rate paid to households is proportional to the average return, $r^k(N + B)/B$, on loans and not, say, to the marginal return. We subsequently discuss the policy implications of this feature, evident from (6.8). Mutual funds are indifferent between loan contracts, as long as they satisfy the preceding zero-profit condition.

Expression (6.8) motivates the market arrangement that we adopt. Let the combinations of $\overline{\omega}$ and B that satisfy (6.8) define a "menu" of loan contracts that is available to banks for a given R.[36] It is convenient to express this menu in terms of L and $\overline{\omega}$. Rewriting (6.8), we obtain

$$L = \frac{1}{1 - \frac{r^k}{R} [\Gamma(\overline{\omega}) - \mu G(\overline{\omega})]}. \tag{6.9}$$

Banks take N, R, and r^k as given and select the contract $(L, \overline{\omega})$ from this menu, which maximizes expected profits, (6.6). Using (6.9) to substitute out for L in the banker's objective, the problem reduces to one of choosing $\overline{\omega}$ to maximize

$$N r^k \frac{1 - \Gamma(\overline{\omega})}{1 - \frac{r^k}{R} [\Gamma(\overline{\omega}) - \mu G(\overline{\omega})]}.$$

The first-order necessary condition for optimization problem is

$$\frac{1 - F(\overline{\omega})}{1 - \Gamma(\overline{\omega})} = \frac{\frac{r^k}{R}\left[1 - F(\overline{\omega}) - \mu\overline{\omega}F'(\overline{\omega})\right]}{1 - \frac{r^k}{R}\left[\Gamma(\overline{\omega}) - \mu G(\overline{\omega})\right]}, \tag{6.10}$$

which can be solved for $\overline{\omega}$ given R.[37] Given the solution for $\overline{\omega}$, L solves (6.9) and Z solves (6.3).

A notable feature of the contract is that L and Z are independent of net worth N. That is, if banks had different levels of net worth, the theory as stated predicts that each bank in the cross section receives a loan contract specifying the same leverage and rate of interest. This feature of the model reflects the assumption that all banks draw ω from the same distribution, F. In a more realistic setting, different banks would draw from different F's and they would receive different debt contracts.

6.2. Households and Government

In period 1 the household budget constraint is

$$c + B \leq y, \tag{6.11}$$

where c, B, and y denote consumption, deposits in mutual funds, and an endowment of output y, respectively. Expression (6.11) is also the period 1 resource constraint. The second-period budget constraint of the household is

$$C \leq (1 + \tau)RB + \pi - T,$$

where C denotes period 2 consumption, T denotes lump-sum taxes, τ denotes a subsidy on household saving, and π denotes the profits brought home by bankers.[38] Households maximize utility,

$$u(c) + \beta u(C), \tag{6.12}$$

subject to their periods 1 and 2 budget constraints. In practice, we assume that

$$u(c) = \frac{c^{1-\alpha}}{1 - \alpha}, \quad \alpha > 0.$$

The government's budget constraint is

$$T = \tau RB. \tag{6.13}$$

The second-period resource constraint is obtained from the household budget constraint by substituting out for π from (6.6), RB from (6.8), and T from (6.13), to obtain the second-period resource constraint:

$$C \le r^k(N + B)\left[1 - \mu G(\overline{\omega})\right]. \tag{6.14}$$

According to (6.14), period 2 consumption is no greater than total output, net of the output used up in monitoring by mutual funds.

6.3. Equilibrium

We define an equilibrium as follows:

Definition 6.1: *For given τ, a private sector equilibrium is $(C, c, R, \overline{\omega}, B, T)$ such that*

  (i) *the household problem is solved (see Section 6.2),*
 (ii) *the problem of the bank is solved (see Section 6.1),*
(iii) *mutual fund profits are zero (see Section 6.1),*
(iv) *the government budget constraint is satisfied (see Section 6.2), and*
 (v) *the first- and second-period resource constraints are satisfied.*

For convenience, we collect the equations that characterize a private sector equilibrium here:

Equation number	Equation	Economic description
	$C = c\left(\beta[1+\tau]R\right)^{\frac{1}{\alpha}}$	household first-order condition
(6.14)	$C = r^k[N+B][1-\mu G(\overline{\omega})]$	period 2 resource constraint
(6.11)	$c + B = y$	period 1 resource constraint
(6.10)	$\dfrac{1-F(\overline{\omega})}{1-\Gamma(\overline{\omega})} = \dfrac{\frac{r^k}{R}\left[1-F(\overline{\omega})-\mu\overline{\omega}F'(\overline{\omega})\right]}{1-\frac{r^k}{R}[\Gamma(\overline{\omega})-\mu G(\overline{\omega})]}$	contract efficiency condition
(6.9)	$\dfrac{N+B}{N} = \dfrac{1}{1-\frac{r^k}{R}[\Gamma(\overline{\omega})-\mu G(\overline{\omega})]}$	mutual fund zero-profit condition

$$\tag{6.15}$$

These equations represent five equations in five private sector equilibrium objects:

$$C, c, R, \overline{\omega}, B. \tag{6.16}$$

The equilibrium value of T can be backed out by imposing either the household or government budget constraint. Note too that the rate of interest on banks Z is determined from (6.3).

6.4. Implications for Policy

The first part of this section shows that the equilibrium in our economy is characterized by too little borrowing, so that subsidizing the cost of funds to banks is welfare improving. We then show that a policy of extending loans directly to banks fails to overcome the Barro-Wallace irrelevance result and so has no impact.

6.4.1. Subsidizing the Cost of Funds to Banks and Leverage Restrictions

Interest rate subsidies are desirable from a welfare point of view because they correct a particular inefficiency in the model economy. Households make their deposit decision by treating R as the marginal return on a deposit. However, the structure of financial markets is such that R corresponds to the average, not the marginal, return on loans to banks. Not surprisingly, a planner prefers that household deposit decisions be made based on the marginal return. We show that in our environment, the marginal return on loans is higher than the average return, so that the market signal received by the households, R, does not provide them with an appropriately strong incentive to save. An interest rate subsidy corrects this problem.

In the second subsection we turn to quantitative simulations. In view of the results in the previous paragraph, it is perhaps not surprising that restrictions on leverage in the model reduce welfare. We find that the market inefficiency in the model – the excess of the marginal return on loans over the average return – increases in a crisis time. As a result, the interest rate subsidy ought to be expanded substantially then. In addition, the efficient policy expands leverage by a greater percent in a crisis time than in normal times.

Qualitative Analysis. A private sector equilibrium is defined conditionally on a particular value of τ. If we treat τ as an undetermined variable, then the system is underdetermined: There are many equilibria, one for each possible value of τ. The Ramsey equilibrium is defined as the best of these equilibria in terms of social welfare, (6.12). That is, the Ramsey equilibrium solves

$$\max_{c,\, C,\, B,\, \bar{\omega},\, R,\, \tau} u(c) + \beta u(C),$$

subject to (6.14), (6.11), (6.10), (6.9), and the household intertemporal first-order condition. The latter is nonbinding as it can simply be used

to define τ without placing any limitation on the maximization problem. Making use of the latter observation, and substituting out for c and C using (6.14), (6.11), we can find the Ramsey equilibrium allocations by solving

$$\max_{B,\,\overline{\omega}} u(y - B) + \beta u(r^k[N + B][1 - \mu G(\overline{\omega})]), \qquad (6.17)$$

subject to (6.10), (6.9). It is convenient to further simplify the problem by solving (6.9) for r^k/R and using the result to substitute out for r^k/R in (6.10):

$$\frac{1 - F(\overline{\omega})}{1 - \Gamma(\overline{\omega})} = \frac{L(B) - 1}{[\Gamma(\overline{\omega}) - \mu G(\overline{\omega})]} \left[1 - F(\overline{\omega}) - \mu\overline{\omega}F'(\overline{\omega})\right], \quad (6.18)$$

where $L(B)$ denotes the private sector equilibrium level of leverage as a function of B:

$$L(B) = \frac{N + B}{N}. \qquad (6.19)$$

Equation (6.18) defines a mapping from B to $\overline{\omega}$. We denote this mapping by $\overline{\omega}(B)$.[39] When $\overline{\omega}(B)$ is substituted into (6.17), the Ramsey problem reduces to[40]

$$\max_{B} u(y - B) + \beta u(r^k[N + B][1 - \mu G(\overline{\omega}(B))]),$$

The first-order necessary condition for an (interior) optimum is

$$\frac{u'(c)}{\beta u'(C)} = r^k \left\{1 - \mu \left[G(\overline{\omega}(B)) + (N + B)G'(\overline{\omega}(B))\overline{\omega}'(B)\right]\right\} \equiv R^m,$$
$$(6.20)$$

say. The term R^m denotes the marginal social return on loans.

Once the Ramsey problem is solved, the remaining objects in Ramsey equilibrium can be found as follows. The cost of funds to mutual funds, R, is obtained by computing the average return on loans:

$$R = \frac{r^k L(B)}{L(B) - 1}[\Gamma(\overline{\omega}) - \mu G(\overline{\omega})].$$

Here, we have used (6.8) and (6.19). The saving subsidy τ is computed so that the household's saving decision is based on the marginal return on loans R^m and not on the average return R:

$$R^m = R(1 + \tau). \qquad (6.21)$$

Two features of (6.20) deserve emphasis. First, r^k is the return to loans in the first-best version of our economy. The first-best allocations are those

that solve the problem

$$\max_{c,\, C,\, B} u(c) + \beta u(C)$$
$$\text{subject to } c + B \le y, \, C = r^k[N + B].$$

This is the problem in which allocations are chosen by a planner who can observe each bank's productivity shock ω. In this problem, there are no monitoring costs. The object in braces in (6.20) represents a wedge introduced by the presence of asymmetric information. A second interesting feature of (6.20) is that the solution to the Ramsey problem implies that $\tau > 0$. This is because the intertemporal marginal rate of substitution in consumption is equated with the marginal return to loans in the Ramsey problem R^m, whereas in the private sector equilibrium with $\tau = 0$ it is equated to the average return on loans R [recall the discussion surrounding (6.8)].[41] Proposition C.1 in Section C.1 of Appendix C establishes that under a certain regularity condition, $R^m > R$, so that marginal return on loans exceeds the corresponding average return. Thus, by (6.21), $\tau > 0$. To define the regularity condition, let the hazard rate be denoted as follows:

$$h(\overline{\omega}) \equiv \frac{F'(\overline{\omega})}{1 - F(\overline{\omega})}. \tag{6.22}$$

The regularity condition is

$$\overline{\omega} h(\overline{\omega}) \text{ increasing in } \overline{\omega}. \tag{6.23}$$

BGG study (6.23) and argue that it is satisfied when F corresponds to the log-normal distribution.

Although we assume $\mu > 0$ in our model, it is interesting to consider the limiting case, $\mu \to 0$. In this case, $R = r^k$ according to (6.10). But (6.20) implies that $R^m = r^k$ when $\mu = 0$, so we conclude that in this case the average and marginal returns coincide. That is, if $\mu = 0$ then $\tau = 0$ in the Ramsey equilibrium.

We summarize the preceding results in the form of a proposition:

Proposition 6.2: *Suppose $\mu > 0$ and condition (6.23) holds. Then, the interest rate subsidy τ is positive in a Ramsey equilibrium. This reflects that (i) the household bases his or her saving decision on the after-tax average return on deposits, $(1 + \tau)R$, whereas the Ramsey planner wants the household to base his or her saving decision on the corresponding marginal return R^m and (ii) $R^m > R$. Suppose $\mu = 0$. Then $R = R^m$; $\tau = 0$ in a Ramsey equilibrium; and the allocations in a private sector equilibrium with $\tau = 0$ are first-best.*

Table 5.1. *Parameters of the asymmetric information model*

β	discount factor	0.97	α	relative risk aversion	1	r^k	return on capital	1.04
σ	standard deviation	0.37	μ	monitoring cost	0.2	y	household's endowment	3.11
N	banker's endowment	1						

Quantitative Analysis. We construct several numerical examples to illustrate the observations in the previous subsection. We suppose that the cumulative distribution of ω, denoted by F, is log-normal. This distribution has two parameters, $E \log \omega$ and

$$\sigma^2 \equiv \text{Var}(\log \omega).$$

The assumption $E\omega = 1$ implies that $E \log \omega = -\sigma^2/2$ so σ is the only free parameter in F. The baseline values of our model parameters are displayed in Table 5.1. These parameter values were chosen in part to ensure a bankruptcy rate of 4 percent, that is, $F(\overline{\omega}) = 0.04$, a leverage ratio $L = 2$, and $R = 1.01$ when $\tau = 0$. The value of $F(\overline{\omega})$ that we use is about one percentage point higher than what appears in the literature (see Christiano et al., 2010, for a review). That literature uses models that are specified at a quarterly frequency. Given our setting of β, we are tempted to interpret the period in the model as one year, in which case our specification of $F(\overline{\omega})$ is somewhat low relative to that in the literature. However, given that the model has only two periods, it is unclear how to compare the time dimension of the model with the quarterly time dimension in empirical models. In light of these considerations, we decided to use a relatively conventional value for $F(\overline{\omega})$ in our calibration. Our value of μ is also within the range used in the literature. We select values for σ, r^k, and $\overline{\omega}$ so that (6.8), (6.10), and the calibrated value of $F(\overline{\omega})$ are satisfied. The resulting value of σ, reported in Table 5.1, is within the range used in the literature.

As in other sections of the chapter, we capture the onset of the crisis with an exogenous drop in N.[42] The quantitative properties of the model are displayed in Table 5.2. Panel A in that table displays the properties of the model under the benchmark parameterization. This is our characterization of the economy in a normal time. Panels B and C display two representations of our model economy in a crisis time. Panel B corresponds to the case in which N is reduced. Our second representation of a crisis is that the drop in N is accompanied by a 20 percent rise in σ. We are interested in this

Table 5.2. *Properties of the asymmetric information model*

		Panel A: Baseline			Panel B: Drop in N			Panel C: Drop in N and rise in σ		
		$\tau = 0$	Ramsey	First best	$\tau = 0$	Ramsey	First best	$\tau = 0$	Ramsey	First best
Interest rate subsidy, 100τ		0	0.32	–	0	1.84	–	0	1.95	–
Financial variables										
$(R-1)100$	risk-free rate	1.00	0.98	–	−5.36	−5.45	–	−6.99	−7.08	–
$(Z/R-1)100$	spread	1.23	1.24	–	7.68	7.83	–	12.32	12.53	–
N/y	net worth	0.321	0.321	–	0.161	0.161	–	0.161	0.161	–
$F(\bar{\omega})\,100$	bankruptcy rate	4.00	4.03	–	20.94	21.27	–	28.95	29.35	–
L	leverage ratio	2.000	2.003	–	3.434	3.466	–	3.418	3.452	–
Real variables										
c/y	time 1 consumption	0.679	0.678	0.671	0.609	0.604	0.589	0.612	0.606	0.589
C/y	time 2 consumption	0.665	0.666	0.676	0.559	0.564	0.594	0.552	0.557	0.594
$(B+N)/y$	investment	0.642	0.643	0.650	0.551	0.556	0.571	0.549	0.554	0.571

Note: (i) The columns headed "Panel A: Baseline" correspond to the baseline parameterization reported in the text. The columns headed "Panel B: Drop in N" correspond to the baseline parameterization with replaced by $N = 1/2$. The columns headed "Panel C: Drop in N and rise in σ" correspond to the baseline parameterization with $N = 1/2$ and σ replaced the product of its value in the baseline parameterization and 1.2. (ii) The column headed "$\tau = 0$" reports a private sector equilibrium with zero interest rate subsidy. The column headed "Ramsey" reports the Ramsey equilibrium. The column headed "First best" reports the first best allocation.

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representation as a way to capture casual evidence that there was a general increase in "uncertainty" during the crisis. The results for this case are reported in Panel C. In each panel, the column labeled $\tau = 0$ displays the private sector equilibrium with $\tau = 0$. The column "Ramsey" indicates the equilibrium with the Ramsey-optimal τ. Finally, the column marked first-best indicates the first-best allocations.

Note from Panel A that in normal times the Ramsey interest rate subsidy τ is 0.3 percent. Thus, consistent with proposition 6.2, in normal times the marginal return on investment exceeds the average return. Because equilibrium saving increases in the subsidy, saving and investment are both higher in the Ramsey equilibrium than they are with $\tau = 0$. The increased supply of saving reduces the equilibrium interest rate R, so that τ is effectively a subsidy to banks' cost of funds. The interest rate spread is slightly higher in the Ramsey equilibrium than it is in the private sector equilibrium with $\tau = 0$. This reflects that loans to banks are larger in the Ramsey equilibrium, so that monitoring costs associated with bankruptcies are larger.

We now turn to Panel B. With the drop in N the economy is poorer and so we expect substantial effects, even in the first-best allocations. According to Panel B, the first-best level of consumption in the first and second periods drops. There is a rise in household saving in response to the shock, but the rise is smaller than the fall in N so that investment drops. In terms of the response in the private sector equilibrium with $\tau = 0$, note that there is a substantial jump in the interest rate spread, from 1.23 percent to 7.83 percent. This jump is associated with a very large rise in bankruptcies, from 4 percent to 21 percent. In addition, consumption in both periods and investment all drop by large amounts. In terms of interest rates and quantities, the drop in N generates a response qualitatively similar to what we found in the previous models.

In terms of policy, the optimal interest rate subsidy rises more than fivefold in response to the drop in N. The intervention reduces the cost of funds to banks (R falls an additional nine basis points in the Ramsey equilibrium, compared with the $\tau = 0$ equilibrium). Thus this model shares the implication of the other models in this chapter that indicate that a crisis triggered by a fall in net worth justifies a policy of (increased) interest rate subsidies to banks.

For the results in Panel C, we set $N = 1/2$ and $\sigma = 1.2 \times 0.37$, where 0.37 is the value of σ in the baseline parameterization (see Table 5.1). Of course, this change in our experiment has no impact on the first-best allocations.

The jump in σ causes the interest rate spread to jump by nearly 5 percentage points in both the $\tau = 0$ and Ramsey equilibria. In addition, investment is reduced, though only by a small amount. Note that the increase in σ produces a rise in period 1 consumption. This happens because the increase in σ exacerbates the financial frictions and induces substitution away from activities (investment) that involve finance and toward activities (period 1 consumption) that do not. Christiano et al. (2011), who incorporate the financial frictions described here into a dynamic model, find that fluctuations in σ account for a substantial portion of business cycle fluctuations. This is because other features of their dynamic general equilibrium model reverse our model's prediction that consumption and investment comove negatively in response to disturbances in σ.

Turning to the implications for policy, note in Panel C that the σ shock magnifies the rise in the Ramsey tax subsidy rate. Thus, an increase in uncertainty calls for a greater subsidy to banks' cost of funds and, hence, more leverage.

In sum, the numerical analysis shows that, at a qualitative level, the model of this section rationalizes the view that a drop in net worth reduces investment, raises interest rate spreads, and warrants interest rate subsidies to banks.

6.4.2. Government Loans and Net Worth Transfers to Banks

We consider a government policy that raises a lump-sum tax T on households in the first period. It then lends T to banks using the same technology available to mutual funds and transfers the proceeds to households in the second period in the form of a lump-sum tax rebate. We show that this policy has no impact on equilibrium allocations because it simply displaces, one-for-one, private saving. That is, the Barro-Wallace irrelevance result holds for government purchases of the financial assets of financial businesses. Tax-financed transfers of net worth to banks do help. Indeed, if they are carried out on the right scale, then banks do not require credit and the first-best allocations are supported. This result, which is not surprising, is proved in Appendix C, Section C.2.6.

We suppose that the government and mutual funds offer the same menu of loan contracts to banks. Because banks are all identical, each chooses the same loan contract, regardless of whether they borrow from mutual funds or from the government. That is, each receives the same leverage and interest rate, characterized by (6.9) and (6.10). Denote the fraction of banks that receive their loans from mutual funds by ν, whereas the complementary

fraction receives their loans from the government. Thus, the net worth of banks receiving their loans from mutual funds and the government is νN and $(1 - \nu)N$, respectively.[43]

In the first period, the representative household deposits b with the banking system. The household's first-period budget constraint is

$$c + b + T \leq y. \tag{6.24}$$

Expression (6.24) is also the period 1 resource constraint. The household's second-period budget constraint is

$$C \leq Rb + \text{government lump sum taxes} + \text{banker earnings}.$$

By the zero-profit condition of private mutual funds, household deposits generate the following return in equilibrium:

$$Rb = r^k(\nu N + b) \left[\Gamma(\overline{\omega}) - \mu G(\overline{\omega})\right].$$

The banks financed by mutual funds return the following profits to the households:

$$r^k(\nu N + b) \left[1 - \Gamma(\overline{\omega})\right].$$

Thus, household income from deposits plus the profits generated by the bankers financed by those deposits is

$$Rb + r^k(\nu N + b) \left[1 - \Gamma(\overline{\omega})\right] = r^k(\nu N + b) \left[1 - \mu G(\overline{\omega})\right].$$

Similarly, the households receive a tax rebate from the government plus profits from the bankers financed by the government, in the amount

$$r^k((1 - \nu)N + T)[1 - \mu G(\overline{\omega})].$$

Then, total household income in the second period is

$$r^k \left[(\nu N + b) + ((1 - \nu)N + T)\right] [1 - \mu G(\overline{\omega})]$$
$$= r^k(N + b + T)[1 - \mu G(\overline{\omega})].$$

Combining the household's budget constraint with the equilibrium expressions for banker profits and deposit interest, we obtain the second-period resource constraint:

$$C \leq r^k(N + b + T) \left[1 - \mu G(\overline{\omega})\right]. \tag{6.25}$$

The necessary and sufficient conditions for an (interior) household optimum are that the first- and second-period budget constraints [i.e., (6.24) and (6.25)] hold with equality and that the intertemporal marginal rate of substitution in consumption be equated to the household deposit rate R.

These three conditions coincide with the first three equations in (6.15) if we identify B with $b + T$ and we set $\tau = 0$.

Thus, the equilibrium allocations of the model in this subsection are determined recursively. The five equations in (6.15) with $\tau = 0$ determine the five equilibrium objects in (6.16). Private lending b is then determined by $b = B - T$. If the government increases loans T to bankers, then household loans to bankers via mutual funds are reduced by the same amount. Of course, this assumes (as we do throughout this chapter) that we consider only interior equilibria. For example, if $B < T$ then the nonnegativity constraint $b \geq 0$ would be binding and we expect the tax policy to have real effects.[44] We state this result in the form of a proposition:

Proposition 6.3: *Suppose the government has the same technology for making loans to banks as do mutual funds. Then in an interior equilibrium, tax-financed government loans to bankers have no impact on rates of return, asset prices, consumption, and investment.*

Of course, this proposition is not true if there are differences between the loans provided by the government and those provided by mutual funds. We suspect that interesting asymmetries would involve the government's having a less-efficient technology for making loans than mutual funds have. If so, then we conjecture that social welfare would be reduced with an increase in T. Thus the environment of this section, in contrast to the one in Section 3, appears to provide little rationale for government purchases of securities issued by financial businesses.

6.5. Pecuniary Externalities: A Robustness Check

Recent literature emphasizes the importance of pecuniary externalities that occur when expenditures are constrained by the market value of agents' assets and the market price of assets is endogenous. Our setting incorporates collateral constraints, but our assumptions about technology imply that asset prices are fixed. Here, we report calculations that suggest the conclusions of our analysis are robust to endogenizing asset prices. In particular, we show that the underborrowing property of the model is robust to endogenizing asset prices, as is the result that it is desirable to subsidize the interest rate costs of banks.

We consider a sequence of economies, starting with our baseline specification in which the technology for converting banker resources, $N + B$, into productive capital is linear. In the baseline specification, the price of capital is unity. When there is curvature in the technology for producing

capital, the price of capital becomes endogenous and in principle this intro-
duces a pecuniary externality. We established in proposition 6.2 that in our
baseline linear case the equilibrium is characterized by underborrowing by
banks and mutual funds and undersaving by households. We find that when
curvature in the production of capital is increased the optimal value of the
subsidy τ at first increases, so that underborrowing becomes even more
severe. For yet higher levels of curvature, the optimal value of the subsidy
converges slowly to zero from above, so that the underborrowing result is
attenuated. On net, our numerical results indicate that our underborrowing
result is robust to all but the very highest levels of curvature. And even then,
we never find overborrowing.

To explain our results, we now indicate the key features of the modified
model, which we adapt from BGG. In particular, in the version of the model
with endogenous capital prices, bankers are endowed with a quantity of
capital k at the beginning of the period. As in BGG, bankers sell this capital
to capital producers for a price, P_k. This price defines banker net worth,
$N = P_k k$. Capital producers operate a technology that combines k with
investment goods to produce and sell new capital K at a price P_K. The
technology operated by capital producers is

$$K = k + \left(\frac{I}{k}\right)^{\gamma} k, \quad 0 \leq \gamma \leq 1.$$

Banks go to mutual funds with their net worth and obtain a standard debt
contract with loan amount B. They purchase K by using this loan and their
net worth, subject to

$$P_K K \leq B + N.$$

The banker operates a production technology analogous to the one in (6.1)
and (6.2):

$$\omega K r^k,$$

where ω is distributed as in (6.2) and r^k is a fixed parameter. With our
modification, the prices of old and new capital, P_k and P_K, respectively, are
endogenous. The rate of return on capital R^k is also endogenous, with

$$R^k \equiv \frac{r^k}{P_K}.$$

Now, suppose a mutual fund deviates from the standard debt contract and
makes an additional loan to one particular bank. This has two effects that
may involve pecuniary externalities. First, when the banker uses the loan
to purchase new capital, its price P_K is driven up by a small amount. This
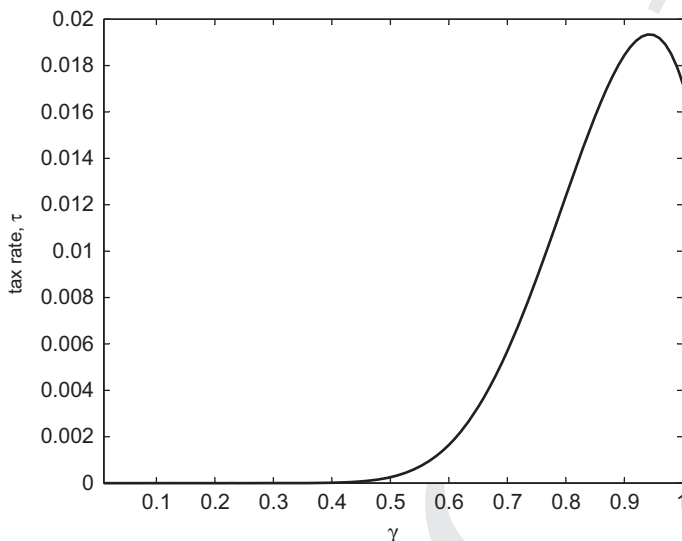
Figure 5.6. Optimal subsidy rate, in presence of pecuniary externality.

encourages capital production and leads to a rise in P_k, thus raising the net worth of other bankers and raising the value of their net worth. As a result, the other bankers are able to borrow more too. The second effect of extending a loan to a banker is that the things the banker buys (in this case K) become more expensive (i.e., P_K increases). Note that the two effects work against each other. Depending on the relative strengths of the two effects, various results could occur in principle. The details of the modified model economy are presented in Appendix C.

We now report our quantitative experiment. We solved for the Ramsey optimal subsidy rate τ for $0.01 \leq \gamma \leq 1$, holding all other parameters fixed at their baseline level (see Table 5.1). Figure 5.6 displays the computed values of τ. Note that as γ decreases, the inefficiency of the economy initially increases because τ increases. However, for γ below roughly 0.85, the inefficiency of equilibrium decreases monotonically with additional reductions in γ. For γ near 0.3, the inefficiency is virtually completely eliminated because the Ramsey optimal τ is close to zero. We interpret the finding $\tau \geq 0$ as indicating the robustness of our underborrowing result and of our result concerning the desirability of subsidizing banks' cost of funds.

7. Concluding Remarks

In the past decade, DSGE models have been constructed that have proved useful for analyzing questions of interest to policy makers.[45] In recent years,

the Federal Reserve has undertaken various actions – a large-scale asset purchase program, reductions in banks' cost of funds – with the objective of correcting dysfunctions in credit markets. The DSGE models developed to place structure on the policy discussions before 2007 are silent on the rationale, design, and appropriate scale of recent policy actions. DSGE models must integrate the right sort of financial frictions to be useful, given the new policy questions. This chapter surveys four candidate models and summarizes some of their implications for recent policy actions.

APPENDIX A: NOTES ON THE UNOBSERVED EFFORT MODEL

A.1. Computational Strategy for Solving the Baseline Model

Computing an equilibrium when the cash constraint, (4.8), is not binding (i.e., $\nu = 0$) is straightforward. Here, we describe an algorithm for computing an equilibrium when the cash constraint is binding, so that $\nu < 0$. There are ten equilibrium conditions. This includes the six conditions associated with the banks, (4.21), the three conditions associated with household optimization, (4.3), (4.2), and (4.16) and the definition of λ, the marginal utility of second-period consumption, (4.17). The ten variables to be solved for are

$$\lambda, c, C, R_g^d, R_b^d, R, e, d, \nu, \eta.$$

When $\nu < 0$, the e and ν equilibrium conditions associated with the banks can be simplified and we do so first.

That $\nu \neq 0$ implies that the ν equation in (4.21) can be replaced by

$$\nu : R_b^d d = R^b(N + d).$$

Note that the μ equation implies that

$$R_g^d - R_b^d = \frac{R - R_b^d}{p(e)}.$$

Use this expression to substitute out for $R_g^d - R_b^d$ in the η equation:

$$e = \lambda b \left[(R^g - R^b)(N + d) - \frac{R - R_b^d}{p(e)} d \right]. \tag{A.1}$$

Note that the d equation implies that

$$R^g - R^b = \frac{R - R^b}{p(e)}.$$

Use this expression to substitute out for $p(e)$ in (A.1) and use the ν equation to substitute out for R_b^d in (A.1) to obtain

$$e = \lambda b \left[(R^g - R^b)(N + d) - (R^g - R^b)\frac{R - R^b \frac{(N+d)}{d}}{R - R^b} d \right].$$

Factor $(R^g - R^b)$, collect terms in d, and rearrange, to obtain the adjustment to the η equation that is possible when $\nu \neq 0$:

$$\eta : e = \lambda b(R^g - R^b)\frac{R}{R - R^b}N. \tag{A.2}$$

The equilibrium conditions associated with the banks, with the appropriate adjustments that are possible when $\nu \neq 0$, are

$$
\begin{aligned}
e &: (\lambda - \nu)b\big(R_g^d - R_b^d\big)d + \eta = 0, \\
d &: R = p(e)R^g + (1 - p(e))R^b, \\
R_g^d &: \nu p(e) = \eta \lambda b, \\
\mu &: R = p(e)R_g^d + (1 - p(e))R_b^d, \\
\eta &: e = \lambda b(R^g - R^b)\frac{R}{R - R^b}N, \\
\nu &: R_b^d d = R^b(N + d).
\end{aligned}
\tag{A.3}
$$

To solve this system, fix $R > R^b$. Solve for λ, c, C, and d by using (4.17) and the household equations, (4.3), (4.2), and (4.16). Compute R_b^d using the ν equation. Compute e using the η equation. Compute R_g^d using the μ equation. Adjust the value of R until the d equation is satisfied. To investigate the possibility of multiple equilibria, we considered values of R on a fine grid over a wide range of values, and it appeared that there is only one value of R that satisfies the d equation. Finally, the e and R_g^d equations can be used to solve linearly for ν and η. For example, with some algebra we find

$$\nu = \frac{b^2\big(R_g^d - R_b^d\big)\lambda^2 d}{b^2\big(R_g^d - R_b^d\big)\lambda d - p(e)}.$$

This completes the discussion of computing the equilibrium.

A.2. Proof of Proposition 4.5

In this appendix, we prove a slightly more precise statement of proposition 4.5 in Subsection 4.5.2. Let

$$B \equiv d + T.$$

Then we obtain the following proposition.

Proposition A.1: *Let T and T' denote two different levels of tax-financed equity finance.*

- *Equilibrium is characterized by the following invariance property. If c, C, e, R, λ, B satisfy the equilibrium conditions under T, then c, C, e, R, λ, B also satisfy the equilibrium conditions under T' as long as $\nu < 0$ in both cases.*
- *In addition, (i) while $\nu < 0$, ν is monotone increasing in T and (ii) there is a T large enough, say T^*, such $T^* < B$ and $\nu = 0$ for $B > T \geq T^*$.*

It is worth stressing that the invariance property applies only to c, C, e, R, λ, and B and not to all the endogenous variables of the model. These include, in addition,

$$\nu, \eta, R_g^d, \text{ and } R_b^d. \tag{A.4}$$

The equilibrium values of the variables in (A.4) do change with T, when $\nu < 0$. We now prove the preceding proposition.

The four equilibrium conditions associated with the household are given by (4.17), (4.27), (4.28), and (4.2):

$$C = R(N + d + T),$$
$$y = c + d + T,$$
$$c^{-\gamma} = \beta R C^{-\gamma},$$
$$\lambda = \beta u'(C).$$

Note that if c, C, R, λ, and B solve these equations for one value of T, then the same c, C, R, λ, and B solve these equations for another value of T. That is, the household equilibrium conditions satisfy the invariance property.

Now consider the equilibrium conditions associated with the banks. Recall from Subsection 4.5.2 that only private deposits d (i.e., not T) enter the bank equilibrium conditions. The equilibrium conditions for the case

$\nu \neq 0$ are stated in (A.3). For convenience, we rewrite the equations here, replacing d with $B - T$:

$$
\begin{aligned}
e : \ & (\lambda - \nu)b\big(R_g^d - R_b^d\big)(B - T) + \eta = 0, \\
d : \ & R = p(e)R^g + (1 - p(e))R^b, \\
R_g^d : \ & \nu p(e) = \eta \lambda b, \\
\mu : \ & R = p(e)R_g^d + (1 - p(e))R_b^d, \\
\eta : \ & e = \lambda b(R^g - R^b)\frac{R}{R - R^b}N, \\
\nu : \ & R_b^d(B - T) = R^b(N + B - T).
\end{aligned}
\tag{A.5}
$$

The d and η equations clearly satisfy the invariance property. It remains to verify that the four equations, e, R_g^d, μ, and ν, do so too. For given B, λ, e, and R, these four equations represent four linear equations in the four unknowns in (A.4). We now verify that these equations have a unique solution. With this result, our invariance property is established.

Using the μ equation to substitute out for $R_g^d - R_b^d$ in the e equation and the R_g^d equation to substitute out for η, the remaining two equations are

$$
\begin{aligned}
e : \ & (\lambda - \nu)b\frac{R - R_b^d}{p(e)}(B - T) + \nu\frac{p(e)}{\lambda b} = 0, \\
\nu : \ & R_b^d(B - T) = R^b(N + B - T).
\end{aligned}
$$

Use the second of these equations to substitute out for R_b^d in the e equation:

$$
e : (\lambda - \nu)b\frac{R(B - T) - (N + B - T)R^b}{p(e)} + \nu\frac{p(e)}{\lambda b} = 0.
$$

Solving for ν, we obtain

$$
-\nu = \frac{\lambda^2}{\dfrac{\left(\dfrac{p(e)}{b}\right)^2}{R(B-T)-(N+B-T)R^b} - \lambda b}.
\tag{A.6}
$$

Given the value of $\nu < 0$ in (A.6), we can now uniquely solve for η, R_g^d, and R_b^d in (A.4). The invariance property is established. Note that the invariance property applies only to the variables in proposition A.1.

We now turn to properties (i) and (ii) in the proposition. According to (A.6), our assumption that the cash constraint is binding in the bad state, $\nu < 0$, implies that

$$R(B - T) - (N + B - T)R^b > 0.$$

Also, note that $R > R^b$ according to the d equation in (A.5). As a result, for fixed B, R, the preceding expression is linear and decreasing in T. Let T^* denote the value of T, where the preceding expression passes through zero. Note that $T^* < B$, so that deposits $B - T^*$ are positive. We can see from (A.6) that as $T \to T^*$, $-\nu \to 0$. That is, the cash constraint is marginally nonbinding for $T = T^*$. For $T > T^*$ the cash constraint is nonbinding and $\nu = 0$. This completes the proof of the proposition.

A.3. Solving the Version of the Model with Bailouts and Leverage

We describe algorithms for solving the versions of the model studied in Subsections 4.5.4 and 4.5.5 in the main text. We begin with the version of the model in Subsection 4.5.4. The first-order conditions associated with the problem in (4.32) are a suitable adjustment on (4.20):

$$
\begin{aligned}
e : \ & \lambda p'(e)\big[\big(R^g - R^b\big)(N + d) - \big(R_g^d - R_b^d\big)d\big] \\
& - e + \mu\, p'(e)\big(R_g^d - (1 + \tau)R_b^d\big)d \\
& + \eta\big(1 - \lambda p''(e)\big[\big(R^g - R^b\big)(N + d) - \big(R_g^d - R_b^d\big)d\big]\big) = 0, \\
d : \ & 0 = \lambda p(e)\big[R^g - R_g^d\big] + \lambda(1 - p(e))\big[R^b - R_b^d\big] \\
& \quad + \mu\big[p(e)R_g^d + (1 - p(e))(1 + \tau)R_b^d - R\big] \\
& \quad - \eta\lambda p'(e)\big[\big(R^g - R^b\big) - \big(R_g^d - R_b^d\big)\big] + \nu\big(R_b^d - R^b\big), \qquad \text{(A.7)} \\
R_g^d : \ & -\lambda p(e) + \mu\, p(e) + \eta\lambda p'(e) = 0, \\
R_b^d : \ & -\lambda(1 - p(e)) + \mu(1 - p(e))(1 + \tau) - \eta\lambda p'(e) + \nu = 0, \\
\mu : \ & R = p(e)R_g^d + (1 - p(e))(1 + \tau)R_b^d, \\
\eta : \ & e = \lambda p'(e)\big[\big(R^g - R^b\big)(N + d) - \big(R_g^d - R_b^d\big)d\big], \\
\nu : \ & R_b^d d - R^b(N + d) = 0.
\end{aligned}
$$

Here, we assume $\nu \neq 0$, so that the cash constraint is binding.

We eliminate the multipliers μ and η and two equations from this system. We add the R_g^d equation to the R_b^d equation and solve for μ:

$$\mu = \frac{\lambda - \nu}{1 + (1 - p(e))\tau}.$$

From the R_g^d equation,

$$\eta \lambda b = \left[\lambda - \frac{\lambda - \nu}{1 + (1 - p(e))\tau}\right] p(e)$$
$$= \tilde{\nu} p(e),$$

where

$$\tilde{\nu} \equiv \frac{\nu + (1 - p(e))\tau \lambda}{1 + (1 - p(e))\tau}. \qquad (A.8)$$

We substitute the expressions for μ and $\eta \lambda b$, as well as the μ equation, into the d equation:

$$d : 0 = \lambda p(e)\left[R^g - R_g^d\right] + \lambda(1 - p(e))\left[R^b - R_b^d\right]$$
$$- \tilde{\nu} p(e)\left[\left(R^g - R^b\right) - \left(R_g^d - R_b^d\right)\right] + \nu\left(R_b^d - R^b\right).$$

or, after rearranging, we obtain

$$d : 0 = (\lambda - \tilde{\nu})\left[p(e)\left(R^g - R_g^d\right) + (1 - p(e))\left(R^b - R_b^d\right)\right]$$
$$+ (\nu - \tilde{\nu})\left(R_b^d - R^b\right).$$

Note that when $\tau = 0$, then $\tilde{\nu} = \nu$, and this equation, together with the μ equation in (4.21), reduces to d in (4.21).[46]

We must adjust the equilibrium conditions of the household to accommodate the taxes required to finance the bank bailouts. The profits π brought home by the bankers in the representative household in period 2 are

$$\pi = p(e)\left[R^g(N + d) - R_g^d d\right] + (1 - p(e))\left[R^b(N + d) - R_b^d d\right].$$

The representative household's second-period budget constraint is

$$C = Rd + \pi - T,$$

where T denotes the lump-sum taxes required to finance τ:

$$T = (1 - p(e))\tau R_b^d d.$$

Substituting out for T and π in the second-period budget constraint, we obtain

$$
\begin{aligned}
C &= Rd + p(e)\big[R^g(N + d) - R_g^d d\big] + (1 - p(e))\big[R^b(N + d) - R_b^d d\big] \\
&\quad -(1 - p(e))\tau R_b^d d \\
&= Rd + p(e)R^g(N + d) + (1 - p(e))R^b(N + d) \\
&\quad -\big[p(e)R_g^d d + (1 - p(e))R_b^d d\big] - (1 - p(e))\tau R_b^d d \\
&= Rd + p(e)R^g(N + d) + (1 - p(e))R^b(N + d) \\
&\quad -\big[p(e)R_g^d d + (1 + \tau)(1 - p(e))R_b^d d - (1 - p(e))\tau R_b^d d\big] \\
&\quad -(1 - p(e))\tau R_b^d d \\
&= Rd + p(e)R^g(N + d) + (1 - p(e))R^b(N + d) - Rd,
\end{aligned}
$$

where the fourth equality makes use of the zero-profit condition of mutual funds. Then,

$$C = \big[p(e)R^g + (1 - p(e))R^b\big](N + d). \tag{A.9}$$

From the household intertemporal Euler equation, $c^{-\gamma} = \beta R C^{-\gamma}$, we have

$$C = c(\beta R)^{\frac{1}{\gamma}}.$$

Substituting d out by using the first- and second-period budget constraints, we obtain

$$c + \frac{C}{p(e)R^g + (1 - p(e))R^b} = N + y.$$

We use the intertemporal first-order condition to substitute out for C, so that the equilibrium conditions for the household are

$$c = \frac{N + y}{1 + \frac{(\beta R)^{\frac{1}{\gamma}}}{p(e)R^g + (1 - p(e))R^b}}, \quad d = y - c.$$

We now collect the equilibrium conditions. We replace the d equation in (A.7), and make use of the expressions for η and μ to obtain the following system:

$$
\begin{aligned}
e:\;& \frac{\lambda - \nu}{1 + (1 - p(e))\tau} b\big(R_g^d - (1 + \tau)R_b^d\big)d + \frac{\tilde{\nu}}{\lambda b}p(e) = 0, \\[4pt]
d:\;& 0 = (\lambda - \tilde{\nu})\big[p(e)\big(R^g - R_g^d\big) + (1 - p(e))\big(R^b - R_b^d\big)\big] \\
& \quad + (\nu - \tilde{\nu})\big(R_b^d - R^b\big), \\[4pt]
\mu:\;& R = p(e)R_g^d + (1 - p(e))(1 + \tau)R_b^d, \\[4pt]
\eta:\;& e = \lambda b\big[(R^g - R^b)(N + d) - \big(R_g^d - R_b^d\big)d\big], \hspace{1.5cm} \text{(A.10)}\\[4pt]
\nu:\;& R_b^d d - R^b(N + d) = 0, \\[4pt]
c =\;& \frac{N + y}{1 + \dfrac{(\beta R)^{\frac{1}{\gamma}}}{p(e)R^g + (1 - p(e))R^b}}, \\[4pt]
d =\;& y - c, \\[4pt]
c^{-\gamma} =\;& \beta R C^{-\gamma},
\end{aligned}
$$

where the last three equations are the equilibrium conditions associated with the household. It is understood that λ is determined according to (4.17) and $\tilde{\nu}$ according to (A.8). In addition, the functional form for $p(e)$ has been used. The expressions in (A.10) represents eight equations in eight unknowns:

$$
\nu,\, e,\, R_g^d,\, R_b^d,\, d,\, c,\, C,\, R.
$$

These equations reduce to (4.21) when $\tau = 0$ in the case $\nu \neq 0$. We solve the preceding equations by solving one equation in R.

We fix a value for R. We now define a mapping from e into itself. We fix a value for e. We use the last three equations in (A.10) to solve for c, C, and d. Then, we use the ν equation to solve for R_b^d:

$$
R_b^d = R^b\left(\frac{N + d}{d}\right).
$$

We use the zero-profit condition for mutual funds to solve for R_g^d:

$$
R_g^d = \frac{R - (1 - p(e))(1 + \tau)R_b^d}{p(e)}.
$$

Finally, we use the η equation to solve for e:

$$\eta : e = \lambda b\big[(R^g - R^b)(N + d) - \big(R^d_g - R^d_b\big)d\big].$$

We adjust the value of e until a fixed point is obtained.

We now have d, c, C, R, R^d_b, R^d_g, and e in hand. It remains to determine a value for v. Substituting out for $\tilde{v}$ in the e equation and multiplying by $1 + (1 - p(e))\tau$, we obtain

$$(\lambda - v)b\big(R^d_g - (1 + \tau)R^d_b\big)d + \frac{1}{\lambda b}\big[v + (1 - p(e))\tau\lambda\big]p(e) = 0.$$

Note that

$$
\begin{aligned}
&(\lambda - v)b\big(R^d_g - (1 + \tau)R^d_b\big)d + \frac{1}{\lambda b}\big[v + (1 - p(e))\tau\lambda\big]p(e) \\
&= \lambda b\big(R^d_g - (1 + \tau)R^d_b\big)d - vb\big(R^d_g - (1 + \tau)R^d_b\big)d \\
&\quad + \frac{1}{\lambda b}vp(e) + \frac{1}{b}(1 - p(e))p(e)\tau \\
&= v\left[\frac{1}{\lambda b}p(e) - b\big(R^d_g - (1 + \tau)R^d_b\big)d\right] \\
&\quad + \lambda b\big(R^d_g - (1 + \tau)R^d_b\big)d + \frac{1}{b}(1 - p(e))p(e)\tau.
\end{aligned}
$$

Then,

$$v = \frac{\lambda b\big(R^d_g - (1 + \tau)R^d_b\big)d + \frac{1}{b}(1 - p(e))p(e)\tau}{b\big(R^d_g - (1 + \tau)R^d_b\big)d - \frac{1}{\lambda b}p(e)}.$$

Finally, we adjust the value of R until the d equation is satisfied.

We now turn to the model considered in Subsection 4.5.5. Relative to the solution to (4.32) that we just analyzed, the solution to (4.33) involves only replacing the zero in the d equation in (A.10) with δ and adding $\overline{LN} = (N + d)$ as an additional equation. We now have nine equations in nine unknowns. We adapt the strategy just described to solve this model. We solve two equations in two unknowns, R and e. We fix values for R and e and solve for d, c, C, R^d_b, v, and R^d_g in the same way as before. These calculations do not involve the d equation. Then, we solve for δ using the modified d equation:

$$
\begin{aligned}
\delta = (\lambda - \tilde{v})&\big[p(e)\big(R^g - R^d_g\big) + (1 - p(e))\big(R^b - R^d_b\big)\big] \\
&+ (v - \tilde{v})\big(R^d_b - R^b\big).
\end{aligned}
$$

Finally, we adjust R and e until the η equation in (A.10) and $\overline{L}N = (N + d)$ are satisfied. In effect, this solution strategy replaces the d equation with the leverage constraint as an equality.

APPENDIX B: PROOF OF PROPOSITION 5.3 FOR
THE ADVERSE-SELECTION MODEL

In this appendix we prove proposition 5.3, which underlies the reason for the assumption in Section 5 that there is an upper bound on the scale of investment projects. Proposition 5.3 addresses what happens if we instead adopt an assumption at the opposite extreme, that investment projects have constant returns to scale without any upper bound. We show that if there is an equilibrium in this version of the model, then it must be that all borrowing is done by the bankers with the lowest value of p and the aggregate profits of those bankers are zero. Thus in this equilibrium the household receives the whole marginal product of his or her saving, and the first-best efficient allocations are supported. For equilibrium to be well defined in this case, we obviously require that there be positive mass on the lower bound of the support for p and that that lower bound be positive. Thus, we suppose that $p \in [\varepsilon, 1]$, where ε is a very small, positive number.

Suppose a banker with a particular $p^* \in [\varepsilon, 1]$ chooses to activate a project; let B_{p^*} denote the amount borrowed by that banker. Such a banker's profits must be no less than what the banker can earn by simply depositing his or her net worth in the bank and not borrowing anything.[47] That is, the analog of (5.5) must hold:

$$(\overline{\theta} - p^* r)B_{p^*} + N(\overline{\theta} - R) \geq 0. \tag{B.1}$$

For each $p \leq p^*$ there exists a B_p such that (B.1) also holds, so that bankers with $p \leq p^*$ also choose to activate their projects. In equilibrium, it must be that

$$\overline{\theta} \leq pr, \quad p \leq p^*, \tag{B.2}$$

for otherwise there is no choice of B_p that optimizes expected banker profits.

For an interior equilibrium in which there is a positive supply of deposits to mutual funds, it must be that $B_p > 0$ for some p. Bankers with lower probabilities of success also borrow positive amounts, and we conclude that

in an interior equilibrium there exists a $p^u \in [\varepsilon, 1]$ such that bankers with $\varepsilon \leq p \leq p^u$ choose $B_p > 0$. For p in this interval, it must be that

$$\overline{\theta} \geq pr, \quad p \leq p^u, \tag{B.3}$$

for otherwise the supposition $B_p > 0$ is contradicted. Let $p^+ \equiv \min(p^u, p^*)$. Combining (B.2) and (B.3), we conclude that

$$\overline{\theta} = pr, \quad \text{for } \varepsilon \leq p \leq p^+.$$

But this expression can hold only if $p^+ = \varepsilon$. We conclude that in an interior equilibrium only the bankers with the lowest probability of success operate their projects. Each of these bankers pays $r = \overline{\theta}/\varepsilon$ in interest and earns zero profits ex ante. Because mutual funds are competitive and so make zero profits, $R = \overline{\theta}$. That is, in equilibrium, households receive the actual social marginal return on loans. As a result, the allocations in equilibrium coincide with the first-best efficient allocations. This establishes proposition 5.3.

APPENDIX C: NOTES ON THE ASYMMETRIC INFORMATION MODEL

C.1. Proof That the Marginal Return Exceeds the Average Return on Loans

For convenience, we repeat the expression for the marginal return on loans, the object to the right of the equality in (6.20), here:

$$R^m \equiv r^k \left\{ 1 - \mu \left[G(\overline{\omega}(B)) + (N + B) G'(\overline{\omega}(B)) \overline{\omega}'(B) \right] \right\}. \tag{C.1}$$

We wish to establish $R^m > R$. Here, R is the equilibrium cost of funds to mutual funds, which we showed is also equal to the average return on loans to bankers [see (6.8)]:

$$R = [\Gamma(\overline{\omega}(B)) - \mu G(\overline{\omega}(B))] \frac{r^k(N + B)}{B}. \tag{C.2}$$

Thus, we establish that (under a regularity condition subsequently stated) the marginal return on loans exceeds the corresponding average return.

In (C.2) and (C.1), $\overline{\omega}(B)$ is defined by (6.18) and (6.19). We reproduce these expressions here [after substituting out for L in (6.18) by using (6.19)] for convenience:

$$\frac{\Gamma'(\overline{\omega}) - \mu G'(\overline{\omega})}{\Gamma(\overline{\omega}) - \mu G(\overline{\omega})} = \frac{N}{B} \frac{\Gamma'(\overline{\omega})}{1 - \Gamma(\overline{\omega})}. \tag{C.3}$$

The mapping $\overline{\omega}(B)$ characterizes how $\overline{\omega}$ changes with a change in B, given the zero-profit condition of mutual funds, (6.9), and the efficiency condition characterizing the solution to the banker contracting problem, (6.10). For our analysis, we require the derivative of $\overline{\omega}(B)$ with respect to B:

$$\overline{\omega}'(B) = \frac{(1-\Gamma)(\Gamma'-\mu G')}{\Gamma'(\Gamma'-\mu G')(N+B) + \Gamma''(\Gamma-\mu G)N - (1-\Gamma)(\Gamma''-\mu G'')B}, \tag{C.4}$$

where we have omitted the argument $\overline{\omega}$ in Γ, G, Γ', and G' for notational simplicity.

The loan contract between banks and mutual funds solves the following optimization problem:

$$\max_{\overline{\omega},B} r^k (N+B)[1-\Gamma(\overline{\omega})], \tag{C.5}$$

subject to

$$r^k(N+B)\left[\Gamma(\overline{\omega}) - \mu G(\overline{\omega})\right] - RB = 0. \tag{C.6}$$

Letting $\lambda \geq 0$ denote the Lagrange multiplier associated with constraint (C.6), we find that the first-order conditions of the problem are

$$r^k[1-\Gamma(\overline{\omega})] + \lambda \left\{ r^k[\Gamma(\overline{\omega}) - \mu G(\overline{\omega})] - R \right\} = 0, \tag{C.7}$$

$$\lambda = \frac{\Gamma'(\overline{\omega})}{\Gamma'(\overline{\omega}) - \mu G'(\overline{\omega})}. \tag{C.8}$$

It is easy to verify that

$$\Gamma'(\overline{\omega}) = 1 - F(\overline{\omega}) > 0, \quad G'(\overline{\omega}) = \overline{\omega} f(\overline{\omega}) > 0, \tag{C.9}$$

where $f(\overline{\omega}) \equiv dF(\overline{\omega})/d\overline{\omega}$. Conditions (C.8), (C.9), and $\lambda \geq 0$ imply[48]

$$\lambda > 1. \tag{C.10}$$

Solving (C.7) for λ and multiplying the numerator and denominator of (C.8) by $r^k(N+B)\overline{\omega}'(B)$, we obtain

$$\lambda = \frac{r^k[1-\Gamma(\overline{\omega})]}{R - r^k[\Gamma(\overline{\omega}) - \mu G(\overline{\omega})]}, \tag{C.11}$$

$$\lambda = \frac{r^k(N+B)\Gamma'(\overline{\omega})\overline{\omega}'(B)}{r^k(N+B)[\Gamma'(\overline{\omega}) - \mu G'(\overline{\omega})]\overline{\omega}'(B)}, \tag{C.12}$$

where $\overline{\omega}'(B)$ is defined in (C.4).

Combining (C.11) and (C.12), we obtain the following expression for λ:

$$\lambda = \frac{r^k[1 - \Gamma] - r^k(N + B)\Gamma'\overline{\omega}'(B)}{R - r^k[\Gamma - \mu G] - r^k(N + B)[\Gamma' - \mu G']\overline{\omega}'(B)}, \quad (C.13)$$

where we suppress the argument $\overline{\omega}$ when doing so does not risk confusion. The numerator of (C.13) consists of the numerator of (C.11) minus the numerator of (C.12). Similarly, the denominator of (C.13) consists of the denominator of (C.11) minus the denominator of (C.12).[49] Now we rewrite the marginal return (C.1), making use of the expression for λ in (C.13):

$$
\begin{aligned}
R^m &= r^k \left[\Gamma(\overline{\omega}) - \mu G(\overline{\omega}) + 1 - \Gamma(\overline{\omega})\right] \\
&\quad - r^k(N + B)[\Gamma'(\overline{\omega}) + \mu G'(\overline{\omega}) - \Gamma'(\overline{\omega})]\overline{\omega}'(B), \\
&= \left\{r^k[1 - \Gamma(\overline{\omega})] - r^k(N + B)\Gamma'(\overline{\omega})\overline{\omega}'(B)\right\} \\
&\quad - \left\{R - r^k\left[\Gamma(\overline{\omega}) - \mu G(\overline{\omega})\right] - r^k(N + B)\right. \\
&\quad \times \left.\left[\Gamma'(\overline{\omega}) - \mu G'(\overline{\omega})\right]\overline{\omega}'(B)\right\} + R, \\
&= R + (\lambda - 1)\left\{R - r^k\left[\Gamma(\overline{\omega}) - \mu G(\overline{\omega})\right] - r^k(N + B)\right. \\
&\quad \times \left.\left[\Gamma'(\overline{\omega}) - \mu G'(\overline{\omega})\right]\overline{\omega}'(B)\right\}, \quad (C.14)
\end{aligned}
$$

where R denotes the average return, (C.2). From (C.10), $\lambda > 1$. Hence, if the object in braces in (C.14) is positive, then $R^m > R$. The object in braces in (C.14) is the denominator of λ in (C.13). Because $\lambda > 0$, we know that the denominator is positive if the numerator of λ is positive. Using the expression for $\overline{\omega}'(B)$, (C.4), we rewrite the numerator as follows:

$$
\begin{aligned}
&r^k[1 - \Gamma] - r^k(N + B)\Gamma'\overline{\omega}'(I) \\
&= r^k[1 - \Gamma] - \frac{r^k(N + B)\Gamma'(\overline{\omega})(1 - \Gamma)(\Gamma' - \mu G')}{\Gamma'(\Gamma' - \mu G')(N + B) + \Gamma''(\Gamma - \mu G)N - (1 - \Gamma)(\Gamma'' - \mu G'')B} \\
&= r^k[1 - \Gamma]\left\{1 - \frac{1}{1 + \frac{\Gamma''(\Gamma - \mu G)N - (1 - \Gamma)(\Gamma'' - \mu G'')B}{(N + B)\Gamma'(\overline{\omega})(\Gamma' - \mu G')}}\right\}.
\end{aligned}
$$

This object is positive if

$$\frac{\Gamma''(\Gamma - \mu G)N - (1 - \Gamma)(\Gamma'' - \mu G'')B}{(N + B)\Gamma'(\Gamma' - \mu G')} \quad (C.15)$$

is positive. A sufficient condition for (C.15) to be positive is that $\overline{\omega} h(\overline{\omega})$ is increasing in $\overline{\omega}$, where $h(\overline{\omega})$ denotes the hazard rate [see (6.22)]. According to BGG, this implies that (i) $\Gamma' - \mu G' > 0$ in equilibrium, and (ii) $\Gamma' G'' - \Gamma'' G' > 0$ for all $\overline{\omega}$.[50] Condition (i) implies that the denominator of (C.15)

is positive. The following result shows that (ii) implies that the numerator of (C.15) is positive:

$$\Gamma''(\Gamma - \mu G)N - (1 - \Gamma)(\Gamma'' - \mu G'')B$$
$$= \Gamma''(\Gamma - \mu G)\frac{(1 - \Gamma)(\Gamma' - \mu G')}{\Gamma'(\Gamma - \mu G)}B - (1 - \Gamma)(\Gamma'' - \mu G'')B$$
$$= \left[\Gamma''(\Gamma' - \mu G') - \Gamma'(\Gamma'' - \mu G'')\right]\frac{1 - \Gamma}{\Gamma'}B$$
$$= \mu\left[\Gamma'G'' - G'\Gamma''\right]\frac{1 - \Gamma}{\Gamma'}B > 0.$$

Here, the first equality uses (C.3) to substitute out for N. This completes the proof of the proposition that the marginal return on B exceeds the corresponding average return. We state this proposition formally as follows:

Proposition C.1: *Suppose that $\overline{\omega}h(\overline{\omega})$ is increasing in $\overline{\omega}$. Then $R^m > R$.*

C.2. Model with Curvature in the Production of Capital

Here, we introduce the modifications to the model that cause the price of capital to be endogenous and possibly be the source of a pecuniary externality. We introduce a representative, competitive firm that produces capital. Rather than building the banker's own capital (as we assume in the main text), the banker uses the net worth and mutual fund loan to purchase capital from a representative capital producer. The following subsection describes the problem of that firm. We then derive all the model equilibrium conditions. The model is virtually identical to the one in the main text. Still, there are some differences in notation, and so we spell out all the details at the risk of some overlap. After that, we describe the algorithm used to compute the equilibrium. This algorithm is the basis for the calculations discussed in the text. Finally, the last subsection describes the proposition that establishes that a sufficiently large net worth transfer to bankers supports the first-best allocations.

C.2.1. Capital Producers

At the start of period 1, each banker is endowed with an equal quantity k of capital goods. Each banker sells his or her capital to capital producers and receives $N = P_k k$, where P_k is the market price of capital in terms of the period 1 numeraire good, consumption. Here, N denotes a banker's net worth after selling his or her capital. In the main text (apart from Section

6.5), $P_k = 1$ always, and so $N = k$ there. The fact that N is a function of the market price P_k creates the potential for a pecuniary externality in this model.

A perfectly competitive, representative capital producer operates the following production function:

$$K = k + f\left(\frac{I}{k}\right)k,$$

where

$$f\left(\frac{I}{k}\right) = \left(\frac{I}{k}\right)^{\gamma}, \quad 0 < \gamma < 1.$$

Here, I denotes a quantity of investment goods, measured in units of the period 1 numeraire good and K denotes the quantity of new capital produced by the capital producer. Profits of the capital producer are given by

$$P_K K - I - P_k k. \tag{C.16}$$

Here, P_K denotes the market price of new capital, in units of the period 1 numeraire good. The representative capital producer takes prices P_K and P_k, as given. In an interior equilibrium, profit maximization leads to the following first-order condition for k:

$$P_K\left[1 + f\left(\frac{I}{k}\right) - f'\left(\frac{I}{k}\right)\frac{I}{k}\right] = P_k.$$

The first-order condition for I is

$$P_K f'\left(\frac{I}{k}\right) = 1.$$

Combining the two first-order conditions with the production function implies the capital producer's profits, (C.16), are zero.

C.2.2. Banks and Mutual Funds

The typical bank takes its net worth N and approaches a mutual fund for a loan B. It combines its net worth and the loan to purchase new capital:

$$P_K K = N + B.$$

In period 2, the banker uses his or her capital K to produce

$$\omega K r^k$$

goods, where r^k is a fixed parameter of technology. In addition, ω is an idiosyncratic productivity shock,

$$\omega \sim F, \int_0^\infty dF(\omega) = 1,$$

where F is the cdf of a log-normal distribution.

A representative mutual fund offers the banker a standard debt contract in period 1, before the realization of ω. Under the contract, the bank pays the mutual fund an amount ZB in period 2 if it is able to do so. Bankers whose ω is too low to pay ZB in full are "bankrupt." Mutual funds verify this by monitoring those bankers, at a cost of

$$\mu\omega K r^k$$

goods, where $\mu > 0$ is a parameter. Bankrupt banks must transfer everything they have to their mutual fund.

The cutoff level of productivity, $\overline{\omega}$, that separates the bankrupt and non-bankrupt banks is defined by

$$\omega K r^k = ZB. \tag{C.17}$$

From the perspective of period 1, an individual bank's expected profits in period 2 are given by

$$\int_{\overline{\omega}}^\infty \left[\omega K r^k - ZB \right] dF(\omega) = \int_{\overline{\omega}}^\infty \left[\omega K r^k - \overline{\omega} K r^k \right] dF(\omega)$$

$$= NR^k L \left(\int_{\overline{\omega}}^\infty [\omega - \overline{\omega}] \, dF(\omega) \right), \tag{C.18}$$

using (C.17). In (C.18), L denotes leverage [see (6.4)] and R^k denotes the rate of return on capital:

$$R^k \equiv \frac{r^k}{P_K}.$$

Expression (C.18) is written in compact notation as follows:

$$NR^k L \int_{\overline{\omega}}^\infty [\omega - \overline{\omega}] \, dF(\omega) = NLR^k (1 - \Gamma(\overline{\omega})), \tag{C.19}$$

where

$$\Gamma(\overline{\omega}) \equiv G(\overline{\omega}) + \overline{\omega} \left[1 - F(\overline{\omega}) \right], \quad G(\overline{\omega}) \equiv \int_0^{\overline{\omega}} \omega \, dF(\omega). \tag{C.20}$$

A bank agrees to maximize (C.19) and remit all its profits to its household in period 2 in exchange for perfect consumption insurance. With bankers maximizing (C.19), the household ensures that his or her bankers as a group maximize the total resources available to the household in period 2. Households can observe everything about their own member bankers (including ω).

We now turn to the mutual funds. There are a large number of competitive mutual funds, each of which makes loans to a diversified group of bankers and takes deposits from households. Because there is no risk on the asset side of the balance sheet, it is feasible for mutual funds to commit in period 1 to paying households a fixed and certain gross rate of interest R on their deposits in period 2. Because mutual funds are competitive, they take R as given. A mutual fund that makes size B loans to each of a large number of banks earns the following per bank:

$$[1 - F(\overline{\omega})]ZB + (1 - \mu) \int_0^{\overline{\omega}} \omega dF(\omega) r^k K.$$

Here, the term before the plus sign indicates the revenues from bankers that are not bankrupt, that is, those with $\omega \geq \overline{\omega}$. The term after the plus sign indicates receipts, net of monitoring costs, from bankers that cannot pay interest, Z. Because the cost of funds is RB, the mutual funds' zero-profit condition is [using (C.17)]

$$[1 - F(\overline{\omega})]\overline{\omega}Kr^k + (1 - \mu) \int_0^{\overline{\omega}} \omega dF(\omega) r^k K = RB,$$

or

$$[\Gamma(\overline{\omega}) - \mu G(\overline{\omega})] \frac{R^k P_K K}{B} = R. \qquad (\text{C.21})$$

As in the main text, the interest rate paid to households is proportional to the average return $R^k P_K K / B$ on loans and not, say, to the marginal return.

Let the combinations of $\overline{\omega}$ and B that satisfy (C.21) define a menu of loan contracts that is available to bankers for given P_K and R^k.[51] It is convenient to express this menu in terms of L and $\overline{\omega}$. Rewriting (C.21), we obtain

$$L = \frac{1}{1 - \frac{R^k}{R} [\Gamma(\overline{\omega}) - \mu G(\overline{\omega})]}. \qquad (\text{C.22})$$

Bankers take N and R^k as given and select the contract $(L, \overline{\omega})$ from this menu, which maximizes expected profits, (C.19). Using (C.22) to substitute

out for L in the banker's objective, the problem reduces to one of choosing $\overline{\omega}$ to maximize

$$NR^k \frac{1 - \Gamma(\overline{\omega})}{1 - \frac{R^k}{R}\left[\Gamma(\overline{\omega}) - \mu G(\overline{\omega})\right]}.$$

The first-order necessary condition for optimization is

$$\frac{1 - F(\overline{\omega})}{1 - \Gamma(\overline{\omega})} = \frac{\frac{R^k}{R}\left[1 - F(\overline{\omega}) - \mu\overline{\omega}F'(\overline{\omega})\right]}{1 - \frac{R^k}{R}\left[\Gamma(\overline{\omega}) - \mu G(\overline{\omega})\right]},$$

which can be solved for $\overline{\omega}$ given R^k and R. Given the solution for $\overline{\omega}$, L, and Z, we solve

$$L = \frac{1}{1 - \frac{R^k}{R}\left[\Gamma(\overline{\omega}) - \mu G(\overline{\omega})\right]}, \quad Z = R^k\overline{\omega}\frac{L}{L - 1},$$

respectively. As in the text, L and Z are independent of net worth N.

C.2.3. Households and Government

In period 1, the household budget constraint is

$$c + B \leq y, \tag{C.23}$$

where c, B, and y denote consumption, bank deposits, and an endowment of output y, respectively. In the second period, deposits generate an after-tax return,

$$(1 + \tau)RB = (1 + \tau)r^k K\left[\Gamma(\overline{\omega}) - \mu G(\overline{\omega})\right],$$

where τ denotes a subsidy on household saving and (C.21) has been used. Total profits brought home by a household's bankers are denoted by π:

$$\pi = Kr^k\left(1 - \Gamma(\overline{\omega})\right).$$

Taking into account that mutual funds have zero profits, the second-period budget constraint is

$$C \leq (1 + \tau)RB + \pi - T, \tag{C.24}$$

where T denotes lump-sum taxes and C denotes second-period consumption. Substituting, we obtain

$$C \leq (1 + \tau)r^k K\left[\Gamma(\overline{\omega}) - \mu G(\overline{\omega})\right] + Kr^k\left(1 - \Gamma(\overline{\omega})\right) - T.$$

The government's budget constraint is

$$T = \tau RB = \tau r^k K\left[\Gamma(\overline{\omega}) - \mu G(\overline{\omega})\right]. \tag{C.25}$$

If we combine the government's budget constraint with the household's second-period budget constraint, we obtain the second-period resource constraint:

$$C \leq r^k K \left[1 - \mu G(\overline{\omega})\right], \tag{C.26}$$

which is Walras's law in our environment. That is, period 2 consumption is no greater than total output, net of the output used up in monitoring by banks.

The representative household maximizes

$$u(c) + \beta u(C),$$

subject to (C.23) and (C.24). The first-order necessary and sufficient conditions corresponding to this problem are

$$\frac{u'(c)}{\beta u'(C)} = (1 + \tau)R, \quad c + \frac{C}{(1+\tau)R} = y + \frac{\pi - T}{(1+\tau)R}.$$

In practice, we assume that

$$u(c) = \frac{c^{1-\alpha}}{1-\alpha}.$$

C.2.4. Equilibrium

We define an equilibrium as follows:

Definition C.2: *A private sector equilibrium is a (C, c, R, K, $\overline{\omega}$, B, P_k, P_K, I, T) such that*

- *(i) the household problem is solved for given τ,*
- *(ii) the problem of the bank is solved,*
- *(iii) mutual fund profits are zero,*
- *(iv) the problem of the capital producer is solved (see Subsection C.2.1),*
- *(v) the government budget constraint is satisfied, and*
- *(vi) the first- and second-period resource constraints are satisfied.*

For convenience, we collect the equations that characterize a private sector equilibrium here. We divide these equilibrium conditions into a household block, and a bank/mutual fund/capital producer block. The first block is

Equation number	Household	Economic description
(1)	$C = c \left(\beta[1 + \tau]R\right)^{\frac{1}{\alpha}}$	household first-order condition
(2)	$C = r^k K \left[1 - \mu G(\overline{\omega})\right]$	period 2 resource constraint
(3)	$c + I = y$	period 1 resource constraint

Expression (3) is the household's first-period budget constraint, with B replaced with I. This replacement is possible for the following reason. Total bank assets $P_K K$ can be written as follows:

$$P_K K = N + B = P_k k + B.$$

Here, the first equality is the banker's expenditure constraint and the second equality uses the definition $N = P_k k$. Zero profits for the capital producers (see Subsection C.2.1) implies that $P_K K = I + P_k k$, and the fact that

$$B = I \qquad\qquad (C.27)$$

follows.

The impact of the household and government budget constraints is completely captured by the period 1 and period 2 resource constraints and expression (1), and so the budget constraints are not included among the equilibrium conditions associated with the household.

The set of equilibrium conditions associated with banks, mutual funds, and capital producers is as follows:

Equation number	Efficiency conditions for firms	Economic description of condition
(4)	$R^k = r^k / P_K$	rate of return on capital
(5)	$\frac{1-F(\overline{\omega})}{1-\Gamma(\overline{\omega})} = \frac{\frac{R^k}{R}\left[1-F(\overline{\omega})-\mu\overline{\omega}F'(\overline{\omega})\right]}{1-\frac{R^k}{R}[\Gamma(\overline{\omega})-\mu G(\overline{\omega})]}$	contract efficiency condition
(6)	$\frac{P_K K}{P_k k} = \frac{1}{1-\frac{R^k}{R}[\Gamma(\overline{\omega})-\mu G(\overline{\omega})]}$	mutual fund zero-profit condition
(7)	$P_k = P_K \left[1 + f\left(\frac{I}{k}\right) - f'\left(\frac{I}{k}\right)\frac{I}{k}\right]$	efficiency condition of capital producers
(8)	$1 = P_K f'\left(\frac{I}{k}\right)$	optimality of I choice by capital producers
(9)	$K = \left[1 + f\left(\frac{I}{k}\right)\right]k$	capital accumulation technology

Equations (1)–(9) represent nine equations in nine private sector equilibrium objects:

$$C, c, R, R^k, K, \overline{\omega}, P_k, P_K, I.$$

The equilibrium value of T can be backed out by imposing either the household or government budget constraint. Note too that the equilibrium rate of interest on banks Z is determined from (6.3) and the facts, (C.27), $N = P_k k$.

It is of interest to show that the equilibrium allocations are first-best when $\mu = 0$. In this case, Eq. (5) can be satisfied only with $R = R^k$ and $\overline{\omega}$ disappears from that equation. Combining Eqs. (6), (7), and (9), we obtain

$$\frac{1}{1 - \Gamma(\overline{\omega})} = \frac{1 + \left(\frac{I}{k}\right)^{\gamma}}{1 + (1 - \gamma)\left(\frac{I}{k}\right)^{\gamma}}.$$

We can think of this equation as defining $\overline{\omega}$ and, hence, Z (i.e., the spread). But this variable does not enter the other equations, and so it plays no role in determining the quantity allocations. The other equations are (1), (2), and (3). Expressing these with $\mu = 0$ and using (8) and (9), we obtain

$(1)'$ $\frac{1}{\beta}\left(\frac{C}{c}\right)^{\alpha} = (1 + \tau)r^k\gamma\left(\frac{I}{k}\right)^{\gamma - 1},$

$(2)'$ $C = r^k k\left[1 + \left(\frac{I}{k}\right)^{\gamma}\right],$

(3) $c + I = y,$

where a prime indicates that the equation has been adjusted using (8) or (9). The preceding system represents three equations in three unknowns, c, C, and I. We substitute out I by using (3):

$(1)''$ $\frac{1}{\beta}\left(\frac{C}{c}\right)^{\alpha} = (1 + \tau)r^k\gamma\left(\frac{y-c}{k}\right)^{\gamma - 1}$

$(2)''$ $C = r^k k\left[1 + \left(\frac{y-c}{k}\right)^{\gamma}\right].$

In sum, the system can be solved as follows. First, solve $(1)''$ and $(2)''$ for C and c. Then, $I = y - c$ and P_K can be computed from (8). Finally, (4) and (5) imply that

$$R = R^k = \frac{r^k}{P_K}.$$

We define the first-best problem as the problem for a planner who observes the bankers' ω realizations. Such a planner obviously does not have to pay monitoring costs. The problem of this planner is

$$\max u(c) + \beta u(C),$$
$$C = r^k k\left[1 + \left(\frac{y - c}{k}\right)^{\gamma}\right]. \tag{C.28}$$

Expressing this in Lagrangian form, we have

$$\max u(c) + \beta u(C) + \lambda\left(r^k k\left[1 + \left(\frac{y - c}{k}\right)^{\gamma}\right] - C\right).$$

The first-order conditions are

$$u'(c) = \lambda r^k \gamma \left(\frac{y - c}{k} \right)^{\gamma - 1},$$

$$\beta u'(C) = \lambda,$$

so that the necessary and sufficient conditions for optimization reduce to

$$\frac{u'(c)}{\beta u'(C)} = \overbrace{r^k \gamma \left(\frac{y - c}{k} \right)^{\gamma - 1}}^{\text{marginal return on investment}},$$

$$C = r^k k \left[1 + \left(\frac{y - c}{k} \right)^{\gamma} \right].$$

Evidently, these equations coincide with (1)$''$ and (2) when $\tau = 0$. We conclude that the equilibrium supports the first-best consumption and investment allocations. We summarize our findings as follows:

Proposition C.3: *Suppose* $0 < \gamma \leq 1$. *When monitoring costs are zero, then the equilibrium consumption and investment allocations coincide with the solution to the first-best problem.*

C.2.5. Computation of Private Sector Equilibrium

Here we describe an algorithm for computing the private sector equilibrium conditional on an arbitrary value of τ. Unlike in the previous subsection, in this subsection we take the model parameters as given and compute values for the nine model endogenous variables that satisfy the nine model equations, (1)–(9) in section C.2.4. Our strategy is to find $\tilde{R} \equiv RP_K$ such that $g(\tilde{R}) = 0$. To define the mapping g from $\tilde{R}$ into the real line, we fix a value for $\tilde{R}$. We solve for $\overline{\omega}$ by using (5) and

$$\frac{R^k}{R} = \frac{r^k}{RP_K} = \frac{r^k}{\tilde{R}}.$$

Next, we compute the object on the right-hand side of (6) and call the result X. Then, (6) is written as

$$\frac{P_K K}{P_k k} = \frac{1 + f \left(\frac{I}{k} \right)}{1 + f \left(\frac{I}{k} \right) - f' \left(\frac{I}{k} \right) \frac{I}{k}} = \frac{1 + \left(\frac{I}{k} \right)^{\gamma}}{1 + (1 - \gamma) \left(\frac{I}{k} \right)^{\gamma}} = X.$$

Rewriting this expression, we obtain

$$\left(\frac{I}{k} \right)^{\gamma} = \frac{X - 1}{1 - X(1 - \gamma)}.$$

and solve for I/k. Then, we solve (9) for K/k, (8) for P_K, and (7) for P_k. Finally,

$$R = \frac{\tilde{R}}{P_K}.$$

We combine (1), (2), and (3):

$$c(\beta[1 + \tau]R)^{\frac{1}{\alpha}} = r^k K[1 - \mu G(\overline{\omega})],$$

$$[y - I](\beta[1 + \tau]R)^{\frac{1}{\alpha}} = r^k K[1 - \mu G(\overline{\omega})],$$

$$\left[\frac{y}{k} - \frac{I}{k}\right](\beta[1 + \tau]R)^{\frac{1}{\alpha}} = r^k \frac{K}{k}[1 - \mu G(\overline{\omega})].$$

We solve the latter for y/k:

$$\frac{y}{k} = \frac{\frac{I}{k}(\beta[1 + \tau]R)^{\frac{1}{\alpha}} + r^k \frac{K}{k}[1 - \mu G(\overline{\omega})]}{(\beta[1 + \tau]R)^{\frac{1}{\alpha}}}.$$

We solve (2) and (3) for c, C. In this way, we define mappings $c(\tilde{R})$ and $C(\tilde{R})$. Then, we let

$$g(\tilde{R}) \equiv C(\tilde{R}) - c(\tilde{R})(\beta[1 + \tau]R)^{\frac{1}{\alpha}}.$$

We adjust $\tilde{R}$ until $g(\tilde{R}) = 0$, that is, (1) is satisfied.

C.2.6. Net Worth Transfers to Bankers

Here, we consider a policy in which the government raises taxes in the first period and then simply gives the banks the proceeds as a subsidy. With this policy, the government in effect can cause the economy to fully circumvent the financial frictions and move to the first-best allocations.

Suppose the government raises $T > 0$ in the first period and transfers the proceeds as a lump-sum gift to banks. Let I^* denote the first-best level of investment [see (C.28)]. The optimal policy is to set $T = I^*$. In this case, the banks have enough net worth that they do not need to borrow from mutual funds. In this way there are no monitoring costs.

It is easy to verify that with one exception the equilibrium conditions of the model considered here coincide with Eqs. (1)–(9) in section equilibrium. The only change is that the object on the left-hand side of the equality in (6) is replaced by

$$\frac{P_K K}{P_k k + T}.$$

This is the leverage ratio of the bankers, and the preceding expression reflects that, with the policy considered here, bankers' net worth consists of the value of the capital they are endowed with, plus the tax transfer received from the government. If we set $T = I$, then leverage is unity by the zero-profit condition on capital producers, (C.16). But if the left-hand side of (6) is unity, the right-hand side must be too. This can be accomplished by $\overline{\omega} = 0$, because in this case $\Gamma(\overline{\omega}) = F(\overline{\omega}) = G(\overline{\omega}) = 0$ [see (6.7)]. Equation (5) then requires $R^k/R = 1$. The remaining equations coincide with the equations of the first-best equilibrium, and so these can be satisfied by setting $T = I^*$. We summarize these results as follows:

Proposition C.4: *A policy of lump-sum tax-financed transfers of net worth to bankers can support the first-best allocations.*

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Notes

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1 We examined monthly data on the interest rate on BAA and AAA rated bonds taken from the St. Louis Federal Reserve bank website. In December 2008 the interest rate spread on BAA over AAA bonds peaked at 3.38 percent, at an annual rate. This is a higher spread than was observed in every month since 1933.

2 The extent to which balance sheets became impaired was hard to assess because there did not exist clear market values for many of the financial assets in the balance sheets of financial institutions.

3 See, for example, Gagnon, Raskin, Remache, and Sack (2010). For a less sanguine perspective on the effectiveness of the Fed's policy, see Krishnamurthy and Vissing-Jorgensen (2010) and Taylor and Williams (2009).

4 There is now a large literature devoted to constructing quantitative dynamic, stochastic general equilibrium models for evaluating the consequences of government asset purchase policies. For a partial list of this work, see Ajello (2010), Bernanke, Gertler, and Gilchrist (1999), Carlstrom and Fuerst (1997), Christiano, Motto, and Rostagno (2003, 2010, 2011), Curdia and Woodford (2009), Del Negro, Eggertsson, Ferrero, and Kiyotaki (2010), Dib (2010), Fisher (1999), Gertler and Karadi (2009), Gertler and Kiyotaki (2011), Hirakata, Sudo, and Ueda (2009a, 2009b, 2010), Liu, Wang, and Zha (2010), Meh and Moran (2010), Nowobilski (2011), Ueda (2009), and Zeng (in press).

5 The Moore and Kiyotaki and Moore ideas are pursued quantitatively in Ajello (2010) and Del Negro et al. (2010).

6 By a "bank" we mean any institution that intermediates between borrowers and lenders.

7 For other analyses in this spirit, see Holmstrom and Tirole (1997) and Meh and Moran (2010).

8 The model (and others in this chapter) assumes that banks cannot increase their net worth. The model offers no explanation for this. The assumption does appear to be roughly consistent with observations. In a private communication, James McAndrews shared the results of his research with Tobias Adrian. That work shows that bond issuance by financial firms declined sharply in the recent crisis, whereas equity issuance hardly rose.

9 In addition, we assume that – unlike the bankers in the model – government employees do not have the opportunity to abscond with tax revenues.

10 There are several analyses of the recent credit crisis that focus on adverse selection in credit markets. See, for example, Chari, Shourideh, and Zetlin-Jones (2010), Fishman and Parker (2010), House (2006), Ikeda (2011a, 2011b), and Kurlat (2010). In addition, see also the argument in Shimer, http://gregmankiw.blogspot.com/2008/09/case-against-paulson-plan.html. Eisfeldt (2004) presents a theoretical analysis that blends adverse-selection and liquidity problems.

11 In particular, equilibrium in the market for credit to banks is a pooling equilibrium. One reason why equilibrium involves pooling is that the environment has the property that the quantity of funds that banks must borrow is fixed.

12 For a prominent example, see BGG. Another example is given by the Christiano et al. (2003, 2010, 2011) analysis of the U.S. Great Depression and the past three decades

of U.S. and Euro Area business cycles. An earlier DSGE model application of the costly state verification and costly monitoring idea can be found in the influential contribution by Carlstrom and Fuerst (1997), as well as in Fuerst (1994). For another contribution of this idea in a DSGE model, see Jonas Fisher's 1994 Northwestern University doctoral dissertation, published in Fisher (1999). Finally, see Williamson (1987).

13 Herein lies a sharp distinction between the model analyzed here and the one in BGG. In BGG, the asymmetric information and monitoring costs lie on the asset side of the bank balance, that is, between the bank and the firm to which it supplies funds. In addition, the bank is perfectly diversified across firms so that in BGG, banks are perfectly safe. Other modifications of the BGG model that introduce risk in banking include, for example, Hirakata et al. (2009a, 2009b, 2010), Nowobilski (2011), Ueda (2009), and Zeng (in press).

14 To our knowledge, the first papers to consider the economic effects of variations in microeconomic uncertainty are Williamson (1987) and Christiano et al. (2003). More recently, this type of shock has also been considered in Arellano, Bai, and Kehoe (2010), Bigio (2011), Bloom (2009), Bloom, Floetotto, and Jaimovich (2010), Christiano et al. (2010, 2011), Ikeda (2011a, 2011b), Jermann and Quadrini (2010), and Kurlat (2010).

15 What we are calling the Barro-Wallace irrelevance proposition is applied to government purchases of long-term debt in Eggertsson and Woodford (2003).

16 The logic in the text may also provide the foundation for a theory of the effectiveness of sterilized interventions in the foreign exchange markets.

17 This section is based on the joint work with Tao Zha.

18 So profits per unit of bank deposits rise when banker net worth is low. However, total bank profits may be low because of the lower net worth of the banks.

19 We assume the environment is such that $c < y$.

20 Whether the "securities" take the form of loans or equity is the same in our model.

21 Given that a bank undertakes a costly search to find a good firm, it would be interesting to explore an alternative model formulation in which the firm and bank that find each other engage in bilateral negotiations.

22 Bankers do not have access to funds other than their own and those provided by depositors.

23 If instead profits were positive, mutual funds would set $d = \infty$, but this exceeds the resources of households who make the deposits. If a positive value of d produced negative profits, then profit-maximizing mutual funds would earn zero profits by setting $d = 0$. But this would be less than the positive amount of deposits supplied by households in the interior equilibria that we study.

24 The d equation in (4.21) is a simplified version of the d equation in (4.20), obtained as follows. Substitute from (4.11) and the R_g^d and μ equations in (4.20) into the d equation in (4.20) to obtain, after some algebra,

$$0 = (\lambda - \nu) \left[p(e) R^g + (1 - p(e)) R^b - R \right].$$

The result follows from the observation that $(\lambda - \nu)$ is strictly positive because $\nu \leq 0$ and $\lambda > 0$ in an interior equilibrium. The e equation in (4.21) is a simplified version of the e equation in (4.20), after making use of (4.11) and the ν equation in (4.20).

25 This requires performing substitutions similar to those in (A.9).

26 See the d and μ equations in (4.21).

27 Although unmodeled, this might reflect that banks must expend a fixed cost to evaluate a firm's project.

28 Our model has the property that the only equilibrium is a pooling equilibrium, that is, one in which banks with different risks receive the same loan contract from mutual funds. For additional discussion of pooling and separating equilibria under adverse selection in credit markets, see Stiglitz and Weiss (1992) and the references they cite.

29 Our model is most closely related to the one in Mankiw (1986).

30 The letter e is not to be confused with effort in the previous section.

31 Our model is an adaptation of the model in Mankiw (1986), especially the example on p. 463.

32 Clearly, an empirically plausible version of our model would require a density function for p that places greater mass on higher p.

33 This phenomenon is captured by a quote from Adam Smith's *Wealth of Nations*, cited in Stiglitz and Weiss (1992). According to Stiglitz and Weiss, Adam Smith wrote that if the interest rate was fixed too high, "... the greater part of the money which was to be lent, would be lent to prodigals and profectors ... Sober people, who will give for the use of money no more than a part of what they are likely to make by the use of it, would not venture into the competition."

34 A sufficient condition is that, in addition to $N < 1$, the parameters satisfy

$$(y + eN)(\overline{\theta}\beta)^{\frac{1}{\gamma}} \leq e\left[(\overline{\theta}\beta)^{\frac{1}{\gamma}} + \overline{\theta} \right].$$

35 For other asymmetric information and monitoring cost models applied specifically to banking, see Hirakata et al. (2009a, 2009b, 2010), Nowobilski (2011), and Zeng (in press).

36 Note that a collection $(\overline{\omega}, B)$ is equivalent to a collection (Z, B) by $\overline{\omega} = ZB/[(N + B)r^k]$ and the fact that N and r^k are exogenous to the banks at the time the contract is undertaken.

37 Here, we have used $\Gamma'(\overline{\omega}) = 1 - F(\overline{\omega})$ and $G'(\overline{\omega}) = \overline{\omega}F'(\overline{\omega})$. See BGG for a formal discussion of the fact that (6.10) uniquely characterizes the solution to the bank optimization problem. A sufficient condition is the regularity condition subsequently defined in (6.23).

38 As noted previously, by the law of large numbers, the expected profits of individual banks, what we defined in (6.6) as π, also corresponds to aggregate profits (in per capita terms) for all the banks in the household.

39 In the numerical examples we have studied, we have found that when there exists a value of $\overline{\omega}$ that solves (6.18) for a given B, that value of $\overline{\omega}$ is unique.

40 We generally ignore imposing the usual nonnegativity constraints such as $B \geq 0$, $y - B \geq 0$, to minimize clutter and in the hope that this does not generate confusion. Throughout, we assume that model parameters are set so that nonnegativity constraints are nonbinding.

41 In his analysis of costly state verification problems, Fisher (1999) also emphasizes that the quantity of lending is determined by the average, not the marginal, return on a loan.

42 See Gertler and Kiyotaki (2011) and Shimer (2010) for other studies that model the shock that triggers the recent financial crisis as a drop in wealth.

43 Recall that all variables are in household per capita terms.
44 We suspect that the environment of this section is not an interesting one for considering equilibria in which the constraint, $b \geq 0$ is binding.
45 See Christiano et al. (2011) for a review.
46 Recall that $\lambda - \nu > 0$ because $\lambda, -\nu > 0$.
47 We implicitly ignore another option for the bank: deposit N in the mutual fund and then borrow from the mutual fund and invest in the project. A property of equilibrium is that $R \leq \bar{\theta}$, so that no bank could increase profits by choosing this option.
48 To see that $\lambda \geq 0$, suppose on the contrary that $\lambda < 0$. Note that $\Gamma'(0) = G(0) = 0$, so that the solution to the Lagrangian representation of the problem solved by the loan contract is $B = \infty$ and $\bar{\omega} = 0$. This does not solve the problem of maximizing (C.5) subject to (C.6) because (C.6) is violated.
49 We have used the following result. Suppose

$$\lambda = \frac{A}{B} = \frac{C}{D}.$$

Then $\lambda = (A + C)/(B + D)$.
50 Condition (i) reflects a combination of the result at the top of p. 1382 in BGG, as well as the observation at the top of p. 1385. Condition (ii) is established on p. 1382 in BGG.
51 Note that a collection $(\bar{\omega}, B)$ is equivalent to a collection (Z, B) by $\bar{\omega} = ZB/[(N + B)R^k]$ and the assumption that banks view N and R^k as parametric.