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ABSTRACT

Why is it that inflation is persistently high in some periods and persistently low in other periods? We argue that lack of commitment in monetary policy may bear a large part of the blame. We show that, in a standard equilibrium model, absence of commitment leads to multiple equilibria, or expectation traps. In these traps, expectations of high or low inflation lead the public to take defensive actions which then make it optimal for the monetary authority to validate those expectations. We find support in cross-country evidence for key implications of the model.

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Many countries have gone through prolonged periods of costly, high inflation, as well as prolonged periods of low inflation. Why do high inflation episodes occur? What can be done to prevent them from occurring again? These are two central questions in monetary economics.

One tradition for understanding poor inflation outcomes stems from the time inconsistency literature pioneered by Kydland and Prescott (1978) and Barro and Gordon (1983). This literature points to lack of commitment in monetary policy as the main culprit behind high inflation. Static versions of the models in this literature have a unique equilibrium. Inflation rates can fluctuate only if the underlying fundamentals do. In may cases, it is difficult to see what changes in the underlying fundamentals could have generated the episodes of high and low inflation. In infinite horizon versions of the Kydland-Prescott and Barro-Gordon models, trigger strategies can be used to produce the observed inflation outcomes. However, such models have embarrassingly many equilibria. It is hard to know what observations would be ruled out by such trigger strategy equilibria.

This paper is squarely within the tradition of the time inconsistency literature in pointing to lack of commitment as the main culprit behind the observed volatility and persistence of inflation. We make two contributions. First, we show how the economic forces in the Kydland-Prescott and Barro-Gordon models can be embedded into a standard general equilibrium model. Second, we find that once these forces have been embedded into a standard model, inflation rates can be high for prolonged periods and low for prolonged periods, even though we explicitly rule out trigger strategies. We find some support in cross-country data for key implications of the model.

In the Kydland-Prescott and Barro-Gordon models, the key trade-off is between the benefits of higher output from unexpected inflation and the costs of realized inflation. In our general equilibrium model, unexpected inflation raises output because some prices are sticky. This rise in output has benefits for households because producers have monopoly power and the unexpected inflation reduces the monopoly distortion. In our general equilibrium model, realized inflation is costly because households must use previously accumulated cash to purchase some goods, called *cash goods*. The realized inflation forces households to substitute toward other goods, called *credit goods*. This substitution tends to lower welfare. Thus, by design, the general equilibrium model captures the trade-offs between the benefits of unexpected inflation and the costs of realized inflation in the Kydland-Prescott and Barro-Gordon framework.

Interestingly, this way of capturing the trade-offs leads to multiple equilibria in our general equilibrium model. Specifically, private agents' expectations of high or low inflation can lead these agents to take defensive actions, which then make it optimal for monetary authorities to validate these expectations. We focus on two kinds of defensive actions. The first is that sticky price firms set high prices if they expect high inflation and low prices if they expect low inflation. The second is that households change the nature of payment technologies depending on their expectations of inflation. To explain these defensive actions we briefly describe key features of our model.

In our model, goods are produced in monopolistically competitive markets. The monopoly power of firms causes output to be inefficiently low. A subset of monopolists set their prices before the monetary authority selects the money growth rate, while the rest of the monopolists set prices afterward. Because of the preset, or *sticky*, prices, a greater than expected monetary expansion can raise output. Such a monetary expansion tends to raise welfare because output is inefficiently low. If sticky price firms expect inflation to be high, they take appropriate defensive actions and set their prices correspondingly high. If the monetary authority fails to validate the expectations of firms, output will be low. A benevolent monetary authority may find it optimal to validate firms' expectations. Indeed, in our general equilibrium model, we show that this kind of logic holds and plays a role in leading to multiple equilibria.

In our model, households can also take defensive actions to protect themselves against expected high inflation. Specifically, they can choose the fraction of goods purchased with cash and the fraction purchased with credit. This choice is made before the monetary authority selects the money growth rate. Cash purchases are costly because households forgo interest, while credit purchases require payment of a cost in labor time, which differs depending on the type of good. In our model, as noted above, cash goods must be purchased with previously accumulated cash, so that a monetary expansion, by raising prices, reduces the consumption of cash goods and reduces welfare. These aspects of our model imply that if households expect high inflation and have chosen to purchase few goods with cash, the marginal cost of unanticipated inflation is small. The monetary authority has a strong incentive to inflate. If households expect low inflation, however, they choose to purchase most goods with cash and the marginal costs of unexpected inflation are high. The monetary authority then does not have a strong incentive to inflate. These arguments suggest that multiple equilibria are possible in our model.

This multiplicity is the reason we can account for persistent and variable inflation. We think this multiplicity is likely to be robust across a wide range of economic models because the underlying economics is so compelling. As noted above, existing models in the Kydland-Prescott and Barro-Gordon literature have unique equilibria. This uniqueness reflects assumptions that best response functions are linear. We have found that the best response functions in general equilibrium models are inherently non-linear and that multiplicity occurs naturally.

Following Chari, Christiano and Eichenbaum (1998), we call this kind of multiplicity an *expectation trap* because changes in private decisions induced by changes in expectations trap policy makers into having to accommodate the expectations. Chari, Christiano and Eichenbaum (1998) show that expectation traps can occur in conventional general equilibrium monetary models. They rely, however, on trigger strategies on the part of the monetary authority to support such outcomes. One criticism of trigger strategies is that for folk-theorem-like reasons, virtually any inflation outcome can be rationalized as an equilibrium. In this paper, we restrict attention to Markov equilibria that rule out trigger strategies. A key finding is that expectation traps occur even in the absence of trigger strategies. We show that, generically, the economy has at least two equilibria or none at all. In our numerical example, we find there are two equilibria. We label these the high-inflation and low inflation equilibrium.

The expectation traps have novel implications for the properties of financial and real variables across the high and low inflation equilibria in a stochastic version of the model with shocks to technology and to the payment system. The interest rate response to a shock switches sign between the high and low inflation equilibria. For example, the interest rate is increasing in the technology shock in the low inflation equilibrium and decreasing in this shock in the high inflation equilibrium. Output is increasing in this shock in both equilibria. When other shocks are present, we show that this sign switch implies that the correlation between output and interest rates is more negative in the high inflation equilibrium than in the low inflation equilibrium. We examine cross-country data and find that within high inflation countries, the correlation between output and interest rates is quite negative when these countries experience high inflation episodes and is essentially zero when these countries experience low inflation episodes. We also find that this correlation is typically positive in low inflation economies and typically negative in high inflation countries. Our model also implies higher volatility of nominal variables in high inflation episodes than in low inflation episodes. This last finding is also present in the data. While a variety of other models might imply higher volatility, it is hard to see which models would generate the change in the magnitude and sign of the correlation between output and interest rates.

If time inconsistency problems are behind the poor inflation outcomes of many countries, the policy implications are that setting up institutions which promote the ability of central banks to commit to future actions can lead to large gains. Under commitment, the optimal policy in our model has the monetary authority following the Friedman Rule and setting nominal interest rates equal to zero. Without commitment, the economy experiences spells of high inflation and spells of low inflation. Institutional devices which can raise welfare in practice include ways of protecting central bank independence and the design of appropriate incentive contracts for central bankers (as in, for example, Persson and Tabellini, 1993).

The plan of the paper is as follows. Section I describes our model. Section II analyzes a restricted version of the model, in which the payment technology is exogenously fixed. The endogenously determined payment technology case is analyzed in section III. In Section IV we discuss cross-country evidence for key implications of the model. In Section V, we discuss the main forces behind the expectation traps we find. The final section concludes.

I A Monetary General Equilibrium Economy

Our economy extends and modifies the Lucas and Stokey (1983) cash-credit goods model. Two of our modifications are intended to capture the benefits and costs emphasized in the literature following Kydland-Prescott and Barro-Gordon. This literature points to gains of unanticipated monetary expansion from higher output and direct costs of realized inflation. In our model, a subset of prices are set in advance by monopolistic firms. This feature implies that an unanticipated monetary expansion tends to raise output and welfare. We adopt the timing assumption in Svensson (1985) by requiring that households use currency accumulated in the previous period to purchase cash goods. This timing assumption implies that a realization of high inflation reduces the consumption of cash goods relative to credit goods and thereby tends to reduce welfare. Our third modification is intended to capture the idea that when people expect high inflation, they take defensive actions to protect themselves. Specifically, in our model each good can be paid for either with cash or with credit. To purchase any good with credit requires a payment of an intermediation cost, which varies across goods. For each good, households trade off the forgone interest from using cash against the intermediation cost.[1]

Our infinite-horizon economy is composed of a continuum of firms, a representative household and a monetary authority. The sequence of events within a period is as follows. First, the shocks to the production technology, θ, and to the payment technology, η are realized. We refer to $s = (\theta, \eta)$ as the *exogenous state*, and we assume that s follows a Markov process. Then households choose the fraction z of goods to purchase with cash, and a fraction μ of firms (the *sticky price firms*) set their prices. These decisions depend on the exogenous state. Let $Z(s)$ denote the economy wide average value of z and $P^e(s)$ denote the average price set by sticky price firms. Here, and in what follows, we scale all nominal variables by the beginning-of-period aggregate stock of money.

Next, the monetary authority makes its policy decision. We denote the actual money growth rate by x and the policy rule that the monetary authority is expected to follow by $X(s)$. The state of the economy after the monetary authority makes its decision, the *private sector's state,* is (s, x). Households' and firms' production, consumption and employment decisions depend on the private sector's state.

Notice that we do not include the beginning-of-period aggregate stock of money in our states. In our economy, all equilibria are neutral in the usual sense that if the initial money stock is doubled, an equilibrium exists in which real allocations and the interest rate are unaffected and all nominal variables are doubled. This consideration leads us to focus on equilibria which are invariant with respect to the initial money stock. We are certainly mindful of the possibility of equilibria which depend on the money stock. For example, if multiple equilibria in our sense exist, 'trigger strategy-type' equilibria which are functions of the initial money stock can be constructed. In our analysis we exclude such equilibria and we normalize the aggregate stock of money at the beginning of each period to unity.

As is customary in defining a Markov equilibrium, we begin with the decisions at the end of the period and work our way back to the beginning of the period. Accordingly, we first describe the end-of-period problem of households and flexible price firms given (s, x) and future monetary policy $X(s)$. We then describe the problem of sticky price firms and the household's choice of z. These problems and market clearing allow us to define a private sector equilibrium for arbitrary x. We then describe the monetary authority's problem and define a Markov equilibrium.

A Private Sector at the End of the Period

Here we discuss the decision problems of households and firms at the end of the period.

We begin with the household problem. In each period the household consumes a continuum of differentiated goods as in Blanchard and Kiyotaki (1987) and supplies labor. The representative household's preferences are $\sum_{t=0}^{\infty} \beta^t u(c_t, n_t)$, where $0 < \beta < 1$,

$$c_t = \left[\int_0^1 c_t(\omega)^\rho d\omega \right]^{\frac{1}{\rho}}, \; u(c,l) = \frac{\left[c(1-l)^\psi \right]^{1-\sigma}}{1-\sigma},$$

$c_t(\omega)$ denotes consumption of type ω good, l_t denotes labor time, and $0 < \rho < 1$.

Each good in this continuum is one of four types. A fraction μ are produced by sticky price firms and a fraction $1-\mu$ are produced by flexible price firms. The sticky and flexible price firms are randomly distributed over the goods. In addition, each good can be purchased with cash or with credit. Let z denote the fraction of goods the household chooses to purchase with cash. This cash-credit decision is made before households know which goods are produced by sticky or flexible price firms, so that the cash-credit good choice is independent of the type of firm. Thus, a fraction μz of goods are sticky price goods purchased with cash, a fraction $(1-\mu)z$ are flexible price goods purchased with cash, a fraction $\mu(1-z)$ are sticky price goods purchased with credit, and a fraction $(1-\mu)(1-z)$ are flexible price goods purchased with credit. It turns out that prices for goods within each type are the same. Utility maximization implies that the amounts purchased of each type of good are the same. Let c_{11} and c_{12} denote quantities of cash goods purchased from sticky and flexible price firms, respectively, and let c_{21} and c_{22} denote the quantities of credit goods purchased from sticky and flexible price goods, respectively. Then we have that

$$c = \left[z\mu c_{11}^\rho + z(1-\mu)c_{12}^\rho + (1-z)\mu c_{21}^\rho + (1-z)(1-\mu)c_{22}^\rho \right]^{\frac{1}{\rho}}. \tag{1}$$

The household divides its labor time, l, into time supplied to goods-producing firms, n, and time allocated to the payment technology according to

$$l = n + \frac{\eta(\bar{z} - z)^{1+\nu}}{1+\nu}. \tag{2}$$

We discuss the determination of z below.

Let A denote the nominal assets of the household, carried over from the previous period. In the asset market, the household trades money, M, and one-period bonds, B, with other households. The asset market constraint is

$$M + B \leq A. \tag{3}$$

Recall that nominal assets, money and bonds are all scaled by the aggregate stock of money. We impose a no-Ponzi constraint of the form $B \leq \bar{B}$, where $\bar{B}$ is a large, finite upper bound.

The household's cash-in-advance constraint is

$$M - \left[P^e(s)\mu z c_{11} + \hat{P}(s,x)(1-\mu)z c_{12}\right] \geq 0, \tag{4}$$

where $P^e(s)$ denotes the price set by sticky price firms and $\hat{P}(s,x)$ denotes the price set by flexible price firms. Nominal assets evolve over time as follows:

$$\begin{aligned} (5)0 \;\leq\;& W(s,x)n + (1 - R(s,x))M - z\left[P^e(s)\mu c_{11} + \hat{P}(s,x)(1-\mu)c_{12}\right] \\ & -(1-z)\left[P^e(s)\mu c_{21} + \hat{P}(s,x)(1-\mu)c_{22}\right] + R(s,x)A + (x-1) + D(s,x) - xA'. \end{aligned}$$

In (5), $W(s,x)$ denotes the nominal wage rate, $R(s,x)$ denotes the gross nominal rate of return on bonds, and $D(s,x)$ denotes profits after lump sum taxes. Finally, B has been substituted out in the asset equation using (3). Notice that A' is multiplied by x. This multiplication reflects that we have scaled all nominal variables by the beginning of period aggregate stock of money and A' is the household's nominal assets scaled by next period's aggregate money stock. Next period's aggregate money stock is simply the current stock multiplied by the growth rate x.

Consider the household's asset, goods and labor market decisions for a given value of z. Given that the household expects the monetary authority to choose policy according to

$X(s)$ in the future, the household solves the following problem:

(6) $$v(A,z,s,x) = \max_{n,M,A',c_{ij};\, i,j=1,2} u(c,l) + \beta E_{s'}[\max_{z'} v(A',z',s',X(s'))|s]$$

subject to (1), (2), (3), (4), (5), and nonnegativity on allocations. The solution to (6) yields decision rules, $d(A,z,s,x)$, where

(7) $$d(A,z,s,x) = [n(A,z,s,x), M(A,z,s,x), A'(A,z,s,x), c_{ij}(A,z,s,x)],$$

$i,j = 1,2$.

We turn now to the decision problems of firms at the end of the period. Each of the differentiated goods is produced by a monopolist using the following production technology

$$y(\omega) = \theta n(\omega),$$

where $y(\omega)$ denotes output and $n(\omega)$ denotes employment for the type ω good. Also, θ is a technology shock that is the same for all goods. The household's problem yields demand curves for each good. The fraction, $1-\mu$, of firms that are flexible price firms set their price, $\hat{P}(s,x)$, to maximize profits subject to these demand curves. Because the household demand curves have constant elasticity, firms set prices as a fixed markup, $1/\rho$, above marginal cost, W/θ, so that

(8) $$\hat{P}(s,x) = \frac{W(s,x)}{\theta\rho}.$$

Turning to the government, we assume that there is no government debt, government consumption is financed with lump-sum taxes, and government consumption is the same for all goods. As a result, the resource constraint for this economy is

$$\theta n = g + z\left[\mu c_{11} + (1-\mu)c_{12}\right] + (1-z)\left[\mu c_{21} + (1-\mu)c_{22}\right],$$

where g denotes an exogenous fixed level of government consumption. Since there is no government debt, bond market clearing requires $B = 0$, $A = 1$. Also, money market clearing requires $M = 1$.

B Private Sector at the Beginning of the Period

At the beginning of the period, after the exogenous shocks are realized, sticky price firms set prices and households make their payment technology decision, z.

As in Blanchard and Kiyotaki (1987), sticky price firms in our economy must set their price in advance and must produce the amount of goods demanded at that price. These firms, like the flexible price firms, also wish to set their price as a markup, $1/\rho$, over marginal cost, W/θ. In order to do so, they need to forecast the wage rate, W. They do so by taking the wage rate as given by the private sector equilibrium. Thus, the wage they expect to prevail is $W(s, X(s))$. Thus, in equilibrium the price set by sticky price firms is given by

$$P^e(s) = \frac{W(s, X(s))}{\theta\rho} \tag{9}$$

We now discuss the household's payment technology decision. As noted above, each consumption good can be purchased either with cash or with credit. For goods with $\omega > \bar{z}$ (where $\bar{z}$ is a parameter between zero and one) the cost of purchasing with credit is zero. Purchasing goods with $\omega \leq \bar{z}$ on credit requires labor time. The household chooses a fraction $z \leq \bar{z}$ such that goods with $\omega < z$ are purchased with cash and goods with $\omega > z$ are purchased with credit. The labor time required to purchase fraction z of goods with cash is given by $\eta(\bar{z} - z)^{1+\nu}/(1+\nu)$, where $\nu > 0$ is a parameter and $\eta > 0$ is the shock to the payment technology. The household's labor time, including time spent working in the market, n, is given in (2). The household chooses z to solve the following problem:

$$z(A, s) = \arg\max \; v(A, z, s, X(s)). \tag{10}$$

We now define an equilibrium for each possible private sector state (s, x) and future monetary policy rule, $X(s)$.

Definition For each s and each x, given $X(s)$ a private sector equilibrium is a collection of functions $P^e(s)$, $Z(s)$, $\hat{P}(s, x)$, $W(s, x)$, $R(s, x)$, $v(A, z, s, x)$, $d(A, z, s, x)$, $z(A, s)$ such that the following are true:

1. The functions v and d solve (6)
2. The function $z(A, s)$ solves (10) and $z(1, s) = Z(s)$
3. Firms maximize profits; that is, $\hat{P}(s, x)$ satisfies (8) and $P^e(s)$ satisfies (9)
4. The resource constraint is satisfied at $d(1, Z(s), s, x)$
5. The asset markets clear; i.e., $A'(1, s, x) = M(1, s, x) = 1$.

We find it convenient to define another private sector equilibrium concept. A *private sector equilibrium with a fixed payment technology* is a private sector equilibrium with the restriction that z is fixed and is not a choice variable.

C Monetary Authority

The monetary authority chooses x to maximize the representative household's discounted utility:

$$\max_x v(1, Z(s), s, x), \tag{11}$$

where v is the value function in a private sector equilibrium. Recall that a private sector equilibrium takes as given the evolution of future monetary policy. Thus, in solving (11) the monetary authority implicitly takes as given the evolution of future monetary policy.

D Markov Equilibrium

We now have the ingredients needed to define a Markov equilibrium.

Definition A *Markov equilibrium* is a private sector equilibrium and a monetary policy rule, $X(s)$, such that $X(s)$ solves (11).

Two properties of a Markov equilibrium deserve emphasis. First, the current money growth rate does not affect discounted utility of the household starting from the next period since it does not affect the next period's state. Therefore, the monetary authority faces the static problem of maximizing current period utility, and we only have to describe how current money growth affects current allocations. Second, inspection of (8) and (9) shows that $\hat{P}(s, X(s)) = P^e(s)$ in a Markov equilibrium. We use these properties below.

In our analysis of a Markov equilibrium, we find it convenient to define another Markov equilibrium concept. The *Markov equilibrium with a fixed payment technology* is a Markov equilibrium in which z is exogenously fixed and beyond the control of the households.

II Analysis with Fixed Payment Technology

In this section we discuss a version of our model in which the payment technology is fixed, in the sense that households cannot alter the value of z. We do this for two reasons. First, this version of the model is a building block for the analysis of the model with a variable payment technology. Second, the model with a fixed payment technology is of interest in its own right because it is the simplest adaptation of a standard monetary model designed to capture the frictions emphasized in Kydland-Prescott and Barro-Gordon.

In our analysis, we decompose the first-order condition associated with the monetary authority problem, (11), into benefits and costs of inflation. Unexpected inflation has benefits because some prices are sticky and there is a monopoly distortion. With sticky prices, higher inflation tends to raise output, while the monopoly distortion makes higher output desirable. These are the reasons the monetary authority in our model has a temptation to stimulate the economy. Inflation is costly because it leads to a reduction in the relative consumption of cash goods.

To analyze a Markov equilibrium, we first characterize a private sector equilibrium. We then solve the monetary authority's problem. We then show that, generically, there are at least two Markov equilibria for the economy with a fixed payment technology.

A Characterizing Private Sector Equilibrium

We now develop a set of necessary and sufficient conditions for a private sector equilibrium. We find it convenient to adopt a change of variables. Let the relative prices of flexible and sticky price goods $q = \hat{P}/P^e$. Omitting arguments of functions for convenience, the first order necessary conditions for household and firm optimization are:

$$\frac{u_{11}}{u_{12}} = \frac{\mu}{1-\mu}\frac{1}{q}, \tag{12}$$

$$\frac{u_{21}}{u_{22}} = \frac{\mu}{1-\mu}\frac{1}{q}, \tag{13}$$

$$\frac{u_{11}}{u_{21}} = \frac{z}{1-z}R, \tag{14}$$

$$\frac{u_{12}}{u_{22}} = \frac{z}{1-z}R, \tag{15}$$

$$-u_n = \frac{\theta\rho u_{22}}{(1-\mu)(1-z)}, \tag{16}$$

$$\frac{xu_{21}}{P^e\mu(1-z)} = \beta E_{s'}[v_1(1, z, s', X(s'))|s], \tag{17}$$

where z is fixed. Here, u_{ij} denotes the partial derivative of u with respect to c_{ij}, and v_1 denotes the partial derivative of v with respect to its first argument. Equations (12) and (13) equate the marginal rate of substitution between sticky and flexible price goods to the relative price of these goods q, and equations (14) and (15) equate the marginal rate of substitution between cash and credit goods to the interest rate R which is their relative price R. Equation (16) is obtained by noting that the marginal rate of substitution between labor and consumption of flexible price credit goods is equated to the ratio of the nominal wage to the price of flexible price goods. This ratio is simply the markup in (8).

The cash-in-advance constraint can be written as

$$\mu z c_{11} + q(1-\mu)zc_{12} \leq \frac{1}{P^e}. \tag{18}$$

A necessary condition for the household problem to be well defined is

$$R \geq 1. \tag{19}$$

It is easy to show that the cash in advance constraint holds with equality if $R > 1$ and that if the cash-in-advance constraint is slack, $R = 1$. These observations imply that the appropriate complementary slackness condition is

$$\left\{\frac{1}{P^e} - [\mu z c_{11} + q(1-\mu) z c_{12}]\right\}[R-1] = 0. \tag{20}$$

The resource constraint is

$$g + z\left[\mu c_{11} + (1-\mu)c_{12}\right] + (1-z)\left[\mu c_{21} + (1-\mu)c_{22}\right] = \theta n. \tag{21}$$

Combining (8) and (9), we have that

$$q(s, X(s)) \equiv \frac{\hat{P}(s, X(s))}{P^e(s)} = 1. \tag{22}$$

In equation (22) we reintroduce the dependence of variables on s and x to emphasize that P^e coincides with $\hat{P}$ only when $x = X(s)$. The conditions (12)-(22) are necessary and sufficient for a private sector equilibrium. That is, these conditions can be used to construct the allocation and pricing functions stated in the definition of the private sector equilibrium above, namely, $P^e(s)$, $\hat{P}(s,x)$, $R(s,x)$, $d(1,z,s,x)$ with $W(s,x) = \theta\rho\hat{P}(s,x)$. The value function is also straightforward to construct.

B The Monetary Authority's Problem

The monetary authority's problem is static in our economy for two reasons. First, we focus on Markov equilibria. In such equilibria, policy makers face dynamic problems only if their decisions affect future state variables. Second, there are no state variables in our economy. Thus, the monetary authority's problem is simply one of choosing current money growth to maximize current period utility.

We find it convenient to set up the monetary authority's problem as one of choosing the interest rate R rather than the money growth rate x. This change in instruments makes the analysis of the variable payment technology economy much easier. As long as the cash-in-advance constraint holds with equality, the two instruments are equivalent. The equivalence argument is as follows. With x as the instrument, (12)-(21) define allocation and pricing functions $(c_{ij}(s,x,P^e), n(s,x,P^e), R(s,x,P^e), q(s,x,P^e))$. These functions evaluated at $P^e(s)$ are the allocation and pricing functions stated in the definition of a private sector equilibrium. Under our functional form assumptions, it is tedious but straightforward to verify that a unique set of allocations and prices solves (12)- (21) for each x and each P^e and that a unique x is associated with each allocation. With the interest rate as the monetary authority's instrument, we use (12)-(16) and (18)- (21) to define allocations and prices as functions of the interest rate,

$$c_{ij}(s,P^e,R),\ i,j=1,2,\ q(s,P^e,R),\ n(s,P^e,R) \tag{23}$$

and let x be simply defined by (17). If the cash-in-advance constraint holds with equality, under our functional form assumptions, a unique set of allocations and prices solves these equations for each R and a unique R exists for each allocation and relative price q. Thus the two formulations are equivalent if the cash-in-advance constraint holds with equality. If the cash-in-advance constraint holds with inequality it is easy to see that there are many allocations which solve (12)-(16) and (18)-(21) for given $R=1$. Each of these allocations is associated with a different value of x. In the Appendix, we prove the following lemma, which allows us to set up the monetary authority's problem as one of choosing the interest rate R rather than the money growth rate x.

Lemma 1: In a Markov equilibrium, the cash-in-advance constraint (18) holds with equality.

We now set up the monetary authority's (static) problem. Substituting from (23) into the utility function, we let $U(s,P^e,R)=u\left[c(s,P^e,R),n(s,P^e,R)\right]$ denote the utility associated with an interest rate R, where c is defined in (1). The monetary authority's problem is now

$$\max_R U(s,P^e,R), \tag{24}$$

subject to $R \geq 1$.[2] Let $R(s,P^e)$ denote the solution to this problem.

C Markov Equilibria

Here we derive a relationship between the payment parameter z and the allocations and prices in a Markov equilibrium with a fixed payment technology. We also show that, for given z generically at least two allocations satisfy the necessary conditions for a Markov equilibrium. In a large class of parameterizations for our economy, we verified numerically that the necessary conditions are sufficient for a Markov equilibrium.

The first-order condition associated with a solution to (24) is

$$U_R(s, P^e, R) = u_c c_R + u_n n_R \leq 0, \tag{25}$$

with equality if $R > 1$. In (25) U_R is the derivative of U with respect to R and u_c, u_n are derivatives of the utility function with respect to c and n, respectively, and c_R, n_R are the derivatives of c and n with respect to R evaluated at the allocations which satisfy the conditions of a private sector equilibrium. In addition to conditions (12)-(21), a private sector equilibrium must satisfy the analog of (22), namely, $q(s, P^e(s), R(s, P^e(s))) = 1$. Therefore, in (25) the derivatives are evaluated at a value of P^e such that $q(s, P^e(s), R(s, P^e(s))) = 1$. From here on we suppress the arguments of functions, and evaluate all functions at their equilibrium values.

In what follows, we show that (25) can be decomposed into a part that captures the incentives to increase inflation because of the presence of monopoly power and a part that captures the disincentives arising from the resulting reduction in cash goods consumption. Consider the role of monopoly power. The efficient allocations with respect to the labor-leisure choice in our economy satisfy

$$u_n + \frac{\theta u_{22}}{(1-\mu)(1-z)} = 0. \tag{26}$$

The first term in (26) is the marginal disutility of labor associated with increasing labor input to credit goods production, say, and the second term is the marginal benefit from increased credit goods consumption. In our economy the analog of (26) is (16). Note that because of the presence of monopoly power, the second term in (16) is the same as the second term in (26) multiplied by $\rho < 1$. As a result, the net marginal benefit of increasing labor from its equilibrium value in our economy is positive. This distortion is due to monopoly power and suggests that the left side of (26) is a natural measure of the monopoly distortion in our economy. Add and subtract $\theta u_{22} n_R / [(1-\mu)(1-z)]$ to and from (25) to obtain

$$U_R = u_c c_R - \frac{\theta u_{22} n_R}{(1-\mu)(1-z)} + \left[u_n + \frac{\theta u_{22}}{(1-\mu)(1-z)} \right] n_R \leq 0. \tag{27}$$

The term in square brackets is our measure of the monopoly distortion. Substituting from (16) into (27), we obtain

$$U_R = u_c c_R - \frac{\theta u_{22} n_R}{(1-\mu)(1-z)} + \frac{(1-\rho)\theta u_{22} n_R}{(1-\mu)(1-z)} \leq 0. \tag{28}$$

In the Appendix, we prove the following lemma regarding the last term in (28).

Lemma 2: In a Markov equilibrium with a fixed payment technology,

$$\frac{(1-\rho)\theta u_{22} n_R}{(1-\mu)(1-z)} = f(c_1, c_2)\psi_{MD}(R, z), \tag{29}$$

where

$$f(c_1, c_2) > 0 \text{ for } c_1, c_2 > 0, \tag{30}$$

and

$$\psi_{MD}(R, z) = -(1-\rho) R^{\frac{1}{\rho-1}} + \frac{R^{\frac{1}{\rho-1}} + \psi R^{\frac{\rho}{\rho-1}} + \frac{\mu}{1-\mu}\frac{\psi}{\rho}\left(R^{\frac{\rho}{\rho-1}} + \frac{1-z}{z}\right)}{\frac{1+\psi}{1-\rho} + \frac{\psi}{\rho}\left(\frac{z}{1-z}R^{\frac{\rho}{\rho-1}} + 1\right)}. \tag{31}$$

>From (31) it is clear that $\psi_{MD}(R, z)$ satisfies the following properties:

$$\psi_{MD}(R, z) \text{ is decreasing in } z \text{ and } \lim_{R \to \infty} \psi_{MD}(R, z) = \frac{\frac{\mu}{1-\mu}\frac{\psi}{\rho}\left(\frac{1-z}{z}\right)}{\frac{1+\psi}{1-\rho} + \frac{\psi}{\rho}} > 0. \tag{32}$$

Notice that $\psi_{MD}(R, z)$ does not depend on the shocks θ and η.

Now consider the disincentives to increase inflation. In the Appendix, we prove the following lemma.

Lemma 3: The first two terms to the right of the equality in (28) can be written as

$$u_c c_R - \frac{\theta u_{22} n_R}{(1-\mu)(1-z)} = -f(c_1, c_2)\,(R-1)\,R^{\frac{1}{\rho-1}}. \tag{33}$$

Let

$$\psi_{ID}\,(R) = (R-1)\,R^{\frac{1}{\rho-1}}. \tag{34}$$

Using $c_2/c_1 = R^{1/1-\rho}$, we have that $\psi_{ID}\,(R) = (R-1)c_1/c_2$. The net interest rate $R-1$ measures the extent to which cash goods consumption is distorted relative to the efficient level. This distortion is akin to a tax (as Lucas and Stokey (1983) have argued). The base on which this tax is levied is consumption of cash goods. Thus, one way to think of ψ_{ID} is as the product of a tax rate, $R-1$, and the base of taxation, c_1, scaled by a measure of the size of the economy, c_2. In this sense, ψ_{ID} measures the inflation distortion. In the efficient allocations, $R=1$, and the term on the right side of (33) is zero. Inspecting (34), we have that $\psi_{ID} \geq 0$ and

$$\lim_{R\to\infty} \psi_{ID}(R) = \psi_{ID}(1) = 0. \tag{35}$$

That is, there is no inflation distortion when the interest rate is high or low.

Substituting (29), (33) and (34) into (28), we obtain

$$U_R = f(c_1, c_2)\left[-\psi_{ID}\,(R) + \psi_{MD}\,(R, z)\right] \leq 0 \tag{36}$$

with equality if $R > 1$. Let $\psi(R, z) = -\psi_{ID}\,(R) + \psi_{MD}\,(R, z)$. Then a solution to

$$\psi(R, z) \leq 0 \tag{37}$$

with equality if $R > 1$ satisfies the necessary condition for monetary authority optimality. If (36) is also sufficient, then the interest rate, R, which solves (37) corresponds to a Markov equilibrium with fixed payment technology. Given an equilibrium value of the interest rate, we can solve for the allocations and other prices from (12)-(16), (18) with equality, (21) and

(22), for each value of θ, η and z. We can then obtain the monetary authority's policy rule from (17).

We use the properties of the monopoly distortion function, ψ_{MD}, in (32), and the inflation distortion function, ψ_{ID}, in (35), to show that, generically, there are at least two Markov equilibria, if there are any.

Proposition 1 (Generic Multiplicity): Consider the version of our economy with a fixed payment technology. Suppose that the monetary authority's first order condition is sufficient for optimality. Then, except for a set of z of Lebesgue measure zero, there are at least two Markov equilibria, or none. Furthermore, the equilibrium interest rate does not depend on θ or η.

Proof: A key property of the function $\psi(R, z)$ is that it is positive for R sufficiently large. This property follows from (32) and (35) which imply

$$\lim_{R \to \infty} \psi(R, z) = \lim_{R \to \infty} [-\psi_{ID}(R) + \psi_{MD}(R, z)] > 0.$$

Suppose first that $\psi(1, z) > 0$. Then, since $\psi(R, z)$ is positive at $R = 1$ and positive for large R, by continuity it follows that if $\psi(R, z)$ is ever zero, it must generically be zero at least twice. A non generic case occurs when the graph of $\psi(R, z)$ against R is tangent to the horizontal axis at a single value of R. Another nongeneric case is when $\psi(1, z) = 0$ and $\psi(R, z) > 0$ for $R > 1$. Both cases are nongeneric because for an arbitrarily larger value of z, one can see that there are multiple equilibria since $\psi(R, z)$ is strictly decreasing in z. Suppose next that $\psi(1, z) < 0$. Then, $R = 1$ satisfies (37) and corresponds to a Markov equilibrium. In addition, because $\psi(R, z) > 0$ for R sufficiently large, continuity implies that $\psi(R, z)$ must be equal to zero for at least one value of $R > 1$.

>From (34) we have that ψ_{ID} does not depend on θ or η. Since ψ_{MD} does not depend on these variables either, it follows that the equilibrium interest rate, R, does not depend on θ or η. Q.E.D.

An example helps illustrate the results in Proposition 1. Figure 1 displays the monopoly distortion, ψ_{MD}, and the inflation distortion, ψ_{ID}, for $R \in [1, 4.5]$ and for $z = 0.13$ and 0.15.[3] The figure shows that the first order necessary condition for monetary authority optimality is satisfied at $R = 1.38$ and $R = 2.07$ for $z = 0.13$ and $R = 1.10$ and $R = 3.17$ for $z = 0.15$. Thus, for $z = 0.15$ the inflation rate is somewhat below 10 percent in the low inflation equilibrium and just below 217 percent in the high inflation equilibrium. To verify that the first order condition for monetary authority optimality is also sufficient, in Figure 2a we graph the monetary authority's objective for $z = .15$, (24), as a function of R for the value of P^e corresponding to the low inflation candidate equilibrium, and in Figure 2b we graph the corresponding objective for the high inflation candidate equilibrium. (The values of P^e are 26.3 and 165.0 for the low and high inflation equilibria, respectively.) These figures show that the first-order conditions are indeed sufficient. They also show that the utility function is not necessarily concave. This is why it is necessary to check monetary authority's utility level globally, rather than just locally.

In the numerical example, the inflation distortion has a single-peaked Laffer curve shape, while the monopoly distortion is relatively flat. We found these properties to hold across a range of parameterizations of the economy. The shape of the inflation distortion is reminiscent of the shape of the monetary Laffer curve in analyses where governments rely on inflation to finance expenditures. (See, for example, Sargent and Wallace (1981).) Below we explore the relationship between our analysis and the analysis in the monetary Laffer curve literature.

The set of interest rates, R, and payment technology, z, which solves (37) plays a key role in our analysis of the equilibrium with variable payment technology. We call the graph of R against z which solves (37) the *interest rate policy correspondence* (henceforth, *policy correspondence* for short.) The following proposition establishes properties of this correspondence:

Proposition 2 (Interest Rate Policy Correspondence): Suppose that the monetary authority's first-order condition is sufficient for optimality. Suppose also that for some $z < \bar{z}$

a Markov equilibrium exists. Then, there is a critical value of z, say $\hat{z}$, such that for $z < \hat{z}$ there are no Markov equilibria, for $z = \hat{z}$ there is at least one Markov equilibrium, and for $z > \hat{z}$ there at least two Markov equilibria.

Proof: First, we show that there is no interest rate less than $\bar{R}$ which is an equilibrium, where $\bar{R}$ is arbitrarily large. Notice from (31) that $\psi_{MD}(R, z) \to \infty$ as $z \to 0$ for all $R \in [1, \bar{R}]$, and from (34) that ψ_{ID} is bounded. It follows that there is some value of z, say $\hat{z}_1$, such that for all $z \leq \hat{z}_1$, $\psi(R, z)$ is strictly positive. Thus, there is no equilibrium interest rate less than $\bar{R}$ for z sufficiently small. Second, we show that no interest rate greater than $\bar{R}$ can be an equilibrium. We see from (34) that ψ_{ID} is bounded above by, say, k. Let $\hat{z}_2$ be defined by $\lim_{R\to\infty} \psi_{MD}(R, \hat{z}_2) = 2k$. Such a value of $\hat{z}_2$ exists from (32). Note also that for all $z \leq \hat{z}_2$, $\lim_{R\to\infty} \psi_{MD}(R, z) \geq 2k$. By definition of a limit, some interest rate $\bar{R}$ exists such that for all $R \geq \bar{R}$, $\psi_{MD}(R, \hat{z}_2) \geq 2k - \varepsilon$, where ε is, say, $k/2$. It follows that, for all $R \geq \bar{R}$, $\psi(R, \hat{z}_1) = -\psi_{ID}(R) + \psi_{MD}(R, \hat{z}_1) \geq k/2 > 0$. That is, there is no value of the interest rate greater than $\bar{R}$ which is an equilibrium for $z = \hat{z}_2$. Since $\psi_{MD}(R, z)$ is decreasing in z, there is no value of the interest rate greater than $\bar{R}$ which is an equilibrium for $z \leq \hat{z}_2$. We have established that there is no equilibrium if z is sufficiently small.

Next, $\psi_{MD}(R, z)$ is a continuous function of R and z. As z is increased from some arbitrarily low value, there is some first value of z such that $\psi(R, z) = 0$ for some R. Such a z, call it $\hat{z}$, exists by our assumption that an equilibrium exists for some z. Consider increasing z above $\hat{z}$. Since ψ_{MD} is strictly decreasing, the graph of $\psi(R, z)$ against R must intersect the horizontal axis at at least two points. Thus, for $z > \hat{z}$, there are at least two Markov equilibria. Q.E.D.

Consistent with our theoretical findings, Figure 1 shows that the inflation distortion does not depend on the payment technology parameter, z, while the monopoly distortion is decreasing in this parameter. We graph the policy correspondence in Figure 3. When z is sufficiently small, the monopoly distortion lies above the inflation distortion and there is no equilibrium. As z increases, the monopoly distortion declines. At a critical value of z

the economy has a unique equilibrium and for values of z larger than this critical value the economy has two equilibria. Notice that as z increases, the interest rate in the low inflation equilibrium falls and that the interest rate in the high inflation equilibrium rises.

III Analysis with Variable Payment Technology

We now characterize a Markov equilibrium in the full-blown version of our economy in which the payment technology is variable. This equilibrium must satisfy all the conditions of a Markov equilibrium with a fixed payment technology. It must in addition satisfy the condition that the payment technology parameter z is chosen optimally. We have already shown that a Markov equilibrium with fixed payment technology is characterized by the relationship between R and z defined given by (37). Here, we show that the first order condition for the optimal choice of z yields a second relation between R and z. The necessary conditions for an equilibrium are completely characterized by values of R and z which satisfy both relationships.[4] In effect, we collapse the set of equilibrium necessary conditions to just two. This simplification makes key properties of the equilibrium transparent. A simple argument establishes that, generically, there are multiple Markov equilibria. In addition, we are able to use the two equations to analyze the effects of exogenous shocks on equilibrium allocations and prices.

The first-order condition associated with the household's choice of z is

$$(1-\frac{1}{\rho})\frac{1-R^{\frac{\rho}{1-\rho}}}{z+(1-z)R^{\frac{\rho}{1-\rho}}}=\frac{\psi\eta(\bar{z}-z)^{\nu}}{1-n-\frac{(\bar{z}-z)^{1+\nu}}{\eta/(1+\nu)}}. \tag{38}$$

We can use the equations that define a private sector equilibrium, (12)-(16), (18) with equality, (21) and (22) to substitute for labor, n, in (38). Doing so, we obtain (see Lemma 4 in the Appendix for a derivation):

$$\frac{(\frac{1}{\rho}-1)(1-R^{\frac{\rho}{\rho-1}})}{z\left[(R^{\frac{1}{\rho-1}}-1)+\frac{\psi}{\rho}(R^{\frac{\rho}{\rho-1}}-1)\right]+(1+\frac{\psi}{\rho})}=\frac{\rho\eta(\bar{z}-z)^{\nu}}{1-\frac{(\bar{z}-z)^{1+\nu}\eta}{1+\nu}-\frac{g}{\theta}}. \tag{39}$$

For each z, there is at most one R that solves (39). To see this, note that the left-hand side is increasing in R, while the right side does not depend on R. Let $R_p(z, g, \theta, \eta)$ denote the value of R that solves (39). We refer to this function as the *payment technology function,* or *payment function*, for short. The set of payment technology parameters z for which this function is defined is developed as follows. As $R \to \infty$, the left side of (39) converges to $(1-\rho)/((\rho+\psi)(1+z))$, which at $z=0$ becomes $(1-\rho)/(\rho+\psi)$. The right side of (39) at $z=0$ is $\rho\eta\bar{z}^{\nu}/(1-\bar{z}^{1+\nu}\eta/(1+\nu)-g/\theta)$. If

$$(1-\rho)/(\rho+\psi) < \rho\eta\bar{z}^{\nu}/(1-\bar{z}^{1+\nu}\eta/(1+\nu)-g/\theta),$$

there is some critical value of z, say z^*, at which the function $R_p(z, g, \theta, \eta)$ goes to infinity. Then the function is defined for $(z^*, \bar{z}]$. If not, then the function is defined for $(0, \bar{z}]$. Let the domain of the function be $(\tilde{z}, \bar{z}]$ where $\tilde{z} = z^*$ if the above inequality holds and $\tilde{z} = 0$ otherwise.

It is easy to see from (39) that R_p is decreasing in z, since the left side of (39) is increasing in z, while the right side is decreasing in z. It is also easy to see that R_p is increasing in g/θ and η since an increase in g/θ or η raises the right side of (39) and so increases R for a given value of z.

Each R, z which satisfies the policy correspondence, (36), and the payment function, (39), corresponds to a Markov equilibrium. The other allocations, prices and the monetary authority's policy rule can be obtained by solving (12)-(17), (18) with equality, (21) and (22). Next, we prove a proposition that under certain conditions, there are two Markov equilibria for our economy. We say that the policy correspondence is *horseshoe-shaped* if it satisfies the following conditions: (i) there are two continuous functions, $R_c^1(z)$ and $R_c^2(z)$ which map $[\hat{z}, \bar{z}]$ into the space of interest rates with $R_c^1(z) < R_c^2(z)$, for $z \in (\hat{z}, \bar{z}]$, $R_c^1(\hat{z}) = R_c^2(\hat{z})$, and (ii) for all $z \in [\hat{z}, \bar{z}]$ the solution to (37) is either $R_c^1(z)$ or $R_c^2(z)$, where $\hat{z}$ is defined in Proposition 2.

Proposition 3: Suppose the policy correspondence is horseshoe-shaped. Then, generically, the economy with variable payment technology has two Markov equilibria, if any.

Proof: Suppose to begin with that $\tilde{z} < \hat{z}$. Recalling that $R_p(\bar{z}) = 1$ and $R_c^1(\bar{z})$, $R_c^1(\bar{z}) \geq 1$, we can divide the proof into two cases. The first case is when $R_p(\bar{z}) < R_c^1(\bar{z})$. The second case is when $R_p(\bar{z}) = R_c^1(\bar{z}) = 1$. Consider the first case, that is, $R_p(\bar{z}) < R_c^1(\bar{z}) \leq R_c^2(\bar{z})$. Now if $R_p(\hat{z}) > R_c^1(\hat{z}) = R_c^2(\hat{z})$, then since R_p is below R_c^1 and R_c^2 at $\bar{z}$ and above R_c^1 and R_c^2 at $\hat{z}$, by continuity, R_p must intersect at least once with each R_c^1 and R_c^2. Each of these intersections corresponds to a Markov equilibrium. If $R_p(\hat{z}) < R_c^1(\hat{z}) = R_c^2(\hat{z})$ then since R_p is below R_c^1 at both $\bar{z}$ and $\hat{z}$, R_p and R_c^1 intersect twice, if at all. The case when $R_p(\hat{z}) > R_c^1(\hat{z}) = R_c^2(\hat{z})$ is clearly non-generic.

Consider the second case, that is, $R_p(\bar{z}) = R_c^1(\bar{z}) = 1$. Then the Ramsey policy and allocations constitute an equilibrium. Generically, there must also be one other equilibrium. To see this, note that, generically, if $R_c^1(\bar{z}) = 1$, some neighborhood of $\bar{z}$ exists such that for all z in this neighborhood, $R_c^1(z) = 1$. Since R_p is strictly decreasing, it follows that for z in this neighborhood, $R_p(z) > 1 = R_c^1(z)$. Suppose that $R_p(\hat{z}) < R_c^1(\hat{z})$. Then, since R_p is above R_c^1 in a neighborhood of $\bar{z}$ and below R_c^1 at $\hat{z}$, by continuity R_p and R_c^1 must intersect at least once. Now suppose that $R_p(\hat{z}) > R_c^1(\hat{z}) = R_c^2(\hat{z})$. Then, since R_p is below R_c^2 at $\bar{z}$ and above R_c^2 at $\hat{z}$, by continuity R_p must intersect at least once with R_c^2. We have established that in this second case, generically, there must be at least two equilibria.

Suppose next that $\tilde{z} > \hat{z}$. Then for z near $\tilde{z}$, R_p is arbitrarily large and must be larger than R_c^2. Exactly the same arguments used above can then be used to conclude that there must be two Markov equilibria. Q.E.D.

The restriction that the policy correspondence be horseshoe-shaped is not severe. In Proposition 2 we have shown that for each $z > \hat{z}$ there are at least two interest rates which belong to the policy correspondence. Using the implicit function theorem, these interest rates can be represented as continuous functions of z. Thus, the assumption that the correspondence is horseshoe-shaped only rules out the possibility that there are three or more interest rates which belong to the correspondence. It is straightforward, but tedious to extend the proof of Proposition 3 to this case. Furthermore, in all the numerical examples we have considered, the correspondence is horseshoe-shaped.

In Figure 4, we plot the interest rate correspondence and the payment function for various realizations of the exogenous shocks in our numerical example. In Figure 4a we plot the interest rate correspondence and the payment function for two realizations of the production technology shock, θ, holding the other shock at its mean value. Figure 4b displays the analogous graph for the payment technology shock, η. These figures display four properties. First, as we have shown in Proposition 1, the policy correspondence does not depend on these shocks. Second, as discussed above, the payment function is decreasing in the interest rate. Third, as also discussed above, the payment function is increasing in η and decreasing in θ. Fourth, there are multiple Markov equilibria. Two of these are easy to see. In one, for every realization of the shocks, the equilibrium is the one associated with the lower intersection of the interest rate correspondence and payment function. We call this the *low inflation equilibrium.* In the other, the equilibrium is the one associated with the higher intersection. We call this the *high inflation equilibrium*.

Figure 4 displays an interesting *sign switch* phenomenon, in the sense that the interest rate response to a shock switches sign between the high and low inflation equilibrium. For example, from Figure 4a, we see that the interest rate is increasing in the technology shock in the low inflation equilibrium and decreasing in this shock in the high inflation equilibrium. We verified, for our numerical example, that in both equilibria output is increasing in the technology shock. If technology shocks were the dominant shocks, the correlation between output and the interest rate would be positive in the low inflation equilibrium and negative in the high inflation equilibrium. From Figure 4b we see the sign switch for the payment shock: the interest rate is decreasing in this shock in the low inflation equilibrium and increasing in this shock in the high inflation equilibrium. In our numerical example, output is increasing in the payment shock in the low inflation equilibrium and decreasing in this shock in the high inflation equilibrium. So, if payment shocks were the dominant shocks the correlation would be negative in both equilibria. It follows that in an economy with both shocks, the correlation of output and the interest rate is negative in the high inflation equilibrium and larger (perhaps even positive) in the low inflation equilibrium. We call this finding the *decreasing correlation implication.*

Our numerical examples also show that the volatility of interest rates in the low inflation equilibrium is substantially smaller. The reason is that the policy correspondence is flatter at the low inflation equilibrium than at the high inflation equilibrium. We call this finding the *increasing volatility implication.*

Thus far we have focused on Markov equilibria which are stationary in the sense that they cannot depend on time. We should point out that if we add calendar time as a state variable there are other Markov equilibria as well. For example, one such equilibrium has the economy moving to the low inflation equilibrium on even dates and to the high inflation equilibrium on odd dates. More interesting is the possibility of sunspot driven Markov equilibria in which a sunspot at the beginning of each period coordinates private agents' expectations and induces agents to pick the high or the low inflation equilibrium depending on the realization of the sunspot. Such sunspot equilibria clearly exist and lead to volatility in inflation rates.

IV Interest Rates and Output in Cross-Country Data

The model's decreasing correlation and increasing volatility implications receive support from within-country data and cross-country data. We analyzed data from the International Financial Statistics (2000) on output and interest rates for a number of countries. We obtained annual data from high and low inflation countries. We defined a high inflation country as one for which output and interest rate data are available and in which interest rates exceed 100 percent in at least one year. Our low inflation countries are the developed countries of Western Europe, the United States, Canada, Japan, Australia and New Zealand.

The list of high inflation countries is in Table 1, and the list of all countries appears in Table 2. In all cases, the tables show the relevant sample periods. The correlations reported in the tables are based on logged, Hodrick-Prescott filtered output and Hodrick-Prescott filtered interest rates.[5]

We begin with the within-country data analysis. Typically, the high inflation countries in our sample experience episodes of high inflation and episodes of relatively low inflation. One interpretation is that the high inflation episodes correspond to our high inflation equilibrium and the low inflation episodes to our low inflation equilibrium. Under this interpretation, the model suggests that the correlation between output and interest rates should be negative in the high inflation episodes and larger in the low inflation episodes. We define episodes of high inflation to be periods when the nominal interest rate exceeds 50 percent per year, while we define low inflation episodes to be all other periods. Fortunately, these episodes turned out - with minor exceptions - to be contiguous. As can be seen from Table 1, there are seven high inflation countries. Five of these countries have had episodes of both high and low inflation. With one exception, the correlation between output and interest rates is higher in the low inflation episodes than in the high inflation episodes. Table 1 also reports the mean value of the correlation between output and the interest rate for all countries in low inflation episodes and in high inflation episodes. The correlation is -0.08 in low inflation episodes and -0.45 in high inflation episodes. Table 1 also provides evidence for the increasing volatility implication. In the low inflation episodes, the standard deviation of the interest rate is 3.57, and in the high inflation episodes, this standard deviation is 350190. For comparison purposes note that the percentage standard deviation of output is 2.34 in the low inflation episodes and 4.57 in the high inflation episodes.

Table 2 provides cross-country evidence for the decreasing correlation and increasing volatility implications. Table 2a shows that the mean value of the correlation between output and interest rates is $-.33$ for the high inflation counties and Table 2b shows that this average is .20 for the low inflation countries. This table also provides evidence for the increasing volatility implication. The standard deviation of the interest rate is 283324 for the high inflation countries and 1.84 for the low inflation countries. For comparison purposes note that the percentage standard deviation of output is 4.43 in the high inflation countries and 2.26 in the low inflation countries.

We also simulated our model and computed the correlation between output and interest rates and the standard deviations of both output and the interest rate. The parameter values are the same as those used in Figure 4. The autocorrelations of both shocks are 0.9, the shocks are uncorrelated, and the standard deviations of θ and η are 0.04 and 9735.1, respectively. We took 500 observations from our model and filtered the simulated data from the model in the same way that the cross-country data were filtered. We found $\sigma_R = 1.90$ and $\sigma_R = 0.12$ in the low and high inflation equilibria, respectively. and the standard deviation of output is essentially the same in both equilibria. The model obviously fails to match the level of volatility in these variables in the data. However, it is interesting that the model predicts the interest rate is an order of magnitude more volatile in the high inflation equilibrium, while output volatility is essentially the same. We also computed the correlation between logged and filtered output and the filtered interest rate. That correlation is 0.013 in the low inflation equilibrium and -0.019 in the high inflation equilibrium. These statistics from the model are qualitatively similar to the corresponding statistics in the data.

V Key Features for Generating Expectation Traps

In this section, we ask which features are crucial for generating expectation traps. We focus on four features and find that two of them play essential roles and two play more subsidiary roles. We also ask whether introducing learning, staggered price setting, or capital accumulation is likely to alter the results significantly.

The two essential features are the ex post benefits of higher than expected inflation and the costs of realized inflation. The benefits of higher than expected inflation come from our assumption that some prices are preset and the presence of monopoly power. The importance of the assumption that some prices are preset can be seen by considering the case when none are, that is, when $\mu = 0$. Setting $\mu = 0$ in (31), after some manipulation, we see that $\psi_{MD}(R, z) < 0$ for all R. Thus, if $\mu = 0$, the unique equilibrium with both a fixed and a variable payment technology has $R = 1$. To see the importance of monopoly power, note that if $\rho = 1$, the markup of prices over marginal cost is 0. Setting $\rho = 1$ in (31), we see that $\psi_{MD}(R, z) = 0$ for all R. Thus, if $\rho = 1$, the unique equilibrium with both a fixed and a variable payment technology has $R = 1$.

The cost of realized inflation in our model comes from the timing assumption under which households must use previously accumulated currency to purchase cash goods. To see the importance of this timing assumption, suppose instead we had adopted the timing assumption in Lucas and Stokey (1983). Under Lucas-Stokey timing, open market operations are conducted in the securities market at the beginning of the period. Households can use the current monetary injection for current cash goods purchases. Mechanically, this amounts to adding current money growth to the right side of the cash-in-advance constraint. A greater than expected monetary expansion, therefore does not in and of itself change the mix of cash and credit goods expansion. We should emphasize that anticipations of high inflation in the future do change the relative amounts of cash and credit goods consumption. Basically, under Lucas-Stokey timing, with flexible prices a one-time, unanticipated change in the quantity of money is neutral, prices change by the quantity of the money change and all real variables are unaffected. With sticky prices, there is a one-time increase in output and all future real variables are unaffected. With Lucas-Stokey timing, there is no Markov equilibrium in our model. The argument is by contradiction. Suppose there were such an equilibrium. A monetary expansion raises output and therefore tends to raise welfare. The only cost is the distortion in the relative prices of sticky and flexible prices. But, in any equilibrium this relative price is one and thus changes in this relative price have a second order effect on welfare. The monetary authority always has an incentive to raise the inflation rate. Thus, there is no equilibrium.

The two subsidiary features relate to the shape of the inflation distortion function and the monopoly distortion function. Substituting $c_2/c_1 = R^{1-\rho}$ into (34), we see that

$$\psi_{ID} = (R-1)\frac{c_1}{c_2}.$$

We have already argued that this distortion is akin to the product of a tax, $R-1$, and the tax base, c_1. When $R = 1$, $\psi_{ID} = 0$. As $R \to \infty$, the behavior of ψ_{ID} depends on the rate at which cash goods consumption falls. In the economy in this paper, c_1 goes to zero faster than R goes to infinity, and thus the product goes to zero. In Albanesi, Chari and Christiano (2002), we present a model in which ψ_{ID} does not go to zero because c_1 goes to zero at the same rate as R. The fixed payment technology model in that paper has a unique equilibrium. With a variable payment technology, however, multiple equilibria are possible.

In our model the monopoly distortion is positive for R sufficiently large. This result implies that there are two equilibria with a fixed payment technology so that the policy correspondence is horseshoe-shaped. In Albanesi, Chari and Christiano (2002), we show that if the period utility function is of the following form

$$u(c,n) = \frac{c^{1-\sigma}}{1-\sigma} - an,$$

where a is a parameter, the fixed payment technology economy has a unique equilibrium. The policy correspondence in Figure 3 becomes a downward-sloping graph. Nevertheless, since the payment function is also downward sloping, there can be multiple intersections and multiple equilibria.

We also ask whether these equilibria are stable under a simple learning scheme. One reason to do so is that if one of these equilibria is unstable, it might regarded implausible. We will show that under one widely used learning scheme, the low inflation equilibrium is always locally stable and the high inflation equilibrium is also stable if the payment function is sufficiently steep. When households determine the period t payment technology, z_t, they have to form expectations of monetary policy. We assume that they expect the monetary authority to set the interest rate equal to its last period value, R_{t-1}. So the value of z_t is given by the payment function with R_{t-1} substituted for the interest rate. We assume that the monetary authority solves the same problem as before so that, for example, it neglects the impact of its current policy action on agents' expectations next period. Thus, the monetary policymaker's correspondence is unaffected. Suppose that in the neighborhood of an equilibrium the payment function and the policy correspondence are given approximately by

$$z_t = a + bR_{t-1}$$

$$R_t = d + ez_t$$

where a, b, d, e are parameters. Substituting for z_t we obtain

$$R_t = d + ea + ebR_{t-1}.$$

Thus, the local stability of the dynamical system governing the interest rate depends on whether the absolute value of be is greater than or less than one. The slope of the payment function is $1/b$, and the slope of the policy correspondence is e. At the low inflation equilibrium, the payment function is steeper than the policy correspondence, or $-1/b > -e$. It follows that the value of be is positive and less than 1, and the system is dynamically stable. At the high inflation equilibrium, if $-1/b > -e$, the same argument applies and the system is locally stable. Inspection of Figures 4a and 4b reveals that the high and low inflation equilibria in our numerical example are locally stable under learning in the sense discussed here. Thus, stability under learning does not provide a device for selecting equilibria in this example.

In this paper prices are preset for one period. In principle, it is straightforward to allow for Taylor- or Calvo- style staggered price setting so that cohorts of firms set their prices for many periods at a time. We could also allow for capital accumulation. A particularly interesting source of dynamics is to consider models in which it takes time or resources to change z. We conjecture that in any of these extensions, outcomes similar to those described in this paper would emerge as steady states.

VI Conclusion

We have shown that discretionary monetary policy can account for prolonged periods of low and high inflation. The model in this paper is a very standard monetary general equilibrium model. Our main theoretical finding is that the model always has expectation traps. The data provide some support for the decreasing correlation and the increasing volatility implications.

The main force driving the multiplicity of equilibria is that defensive actions taken by the public to protect itself from high inflation reduce the costs of inflation for a benevolent monetary authority and induce the authority to supply high inflation. This economic force is likely to be present in a large class of monetary models. The main policy implication is that the costs of discretionary monetary policy include not just high average inflation, but volatile and persistent inflation as well. The gains to setting up institutions which increase commitment to future monetary policies are likely to be high.

Notes

[1]See Aiyagari, Braun and Eckstein (1998), Cole and Stockman (1992), Dotsey and Ireland (1994), Freeman and Kydland (1994), Ireland (1994), Lacker and Schreft (1996), and Schreft (1992) for payment technology models with similar features.

[2]Technically, the set of interest rates should also be limited to those where (12)-(16) and (18)-(21) have a solution. Our analysis of the monetary authority's problem uses a first order condition approach which only asks whether small deviations are optimal. One can use the implicit function theorem to show that in some neighborhood of an equilibrium, (12)-(16) and (18)-(21) have a solution. Thus, we will not have to deal with whether the allocation functions are well defined for arbitrary interest rates.

[3]In the numerical example used throughout this paper, $\mu = 0.1$, $\rho = 0.45$, $\psi = 1$, $\nu = 2$, $\bar{z} = 0.15$, $\eta = 28000$.

[4]In all the numerical examples we have studied, the necessary conditions also turned out to be sufficient.

[5]The smoothness parameter in the Hodrick-Prescott filter was set to 100. Each country and sample period was treated as a distinct series for constructing the Hodrick-Prescott filter.

Appendix

To prove the lemmas in the text, we use the necessary and sufficient conditions for an interior private sector equilibrium. Using our functional form assumptions, (12)-(16) reduce to

$$c_{12} = c_{11} q^{\frac{-1}{1-\rho}} \tag{40}$$

$$c_{21} = c_{11} R^{\frac{1}{1-\rho}} \tag{41}$$

$$c_{22} = c_{21} q^{\frac{-1}{1-\rho}} \tag{42}$$

$$\frac{\psi}{\rho} c^{\rho} c_{22}^{1-\rho} = \theta\left(1 - n - \frac{\eta(\bar{z} - z)^{1+\nu}}{1+\nu}\right). \tag{43}$$

We have omitted (13) because there are only three linearly independent equations in (12)-(15). These expressions together with (20)-(22) are necessary and sufficient conditions for a private sector equilibrium.

Lemma 1: In a Markov equilibrium, the cash-in-advance constraint (18) holds with equality.

Proof: Suppose that the cash-in-advance constraint holds as a strict inequality in a Markov equilibrium. We will show that there is some small deviation in the money growth rate x from its supposed equilibrium value which raises the utility of the representative household. Note from (20) that $R = 1$ for all x in some neighborhood of x. In that small neighborhood of the equilibrium value of x, expressions (40)-(43) and (21) with $R = 1$ implicitly define functions $c_{ij}(s, x, P^e)$, $c(s, x, P^e), q(s, x, P^e)$, and $n(s, x, P^e)$. Since (40)-(43) and (17)-(22) are necessary and sufficient conditions for a private sector equilibrium, these functions evaluated at the equilibrium value of $P^e = P^e(s)$ are the allocation rules and prices in a Markov equilibrium. The monetary authority's problem is $\max_x u(c(s, x, P^e), n(s, x, P^e))$ and the first-order condition for this problem is

$$u_c c_x + u_n n_x = 0 \tag{44}$$

where c_x and n_x denote derivatives with respect to x and where all functions are evaluated at their supposed equilibrium values. Since $R = 1$ and $q = 1$ in equilibrium, we have that $c_{ij} = c$ for all i, j. Using our functional form assumptions and (43) in (44), we have

$$u_c(c_x - \theta\rho n_x) = 0. \tag{45}$$

Differentiating (1) with respect to x and evaluating at $c_{ij} = c$, we obtain

$$c_x = z\mu c_{11,x} + z(1-\mu)c_{12,x} + (1-z)\mu c_{21,x} + (1-z)(1-\mu)c_{22,x}$$

where $c_{ij,x}$ is the derivative of c_{ij} with respect to x. Differentiating the resource constraint with respect to x, we obtain

$$\theta n_x = z\mu c_{11,x} + z(1-\mu)c_{12,x} + (1-z)\mu c_{21,x} + (1-z)(1-\mu)c_{22,x}.$$

Substituting for c_x and θn_x into (45), we have a contradiction. Q.E.D.

Lemmas 2 and 3 are established using (40)-(43), (20) with equality, and (21) to construct the functions $c_{ij}(s, P^e, R)$, $q(s, P^e, R)$ and $n(s, P^e, R)$, differentiating these functions with respect to R and evaluating the derivatives at $q = 1$. Mechanically, we first drop n from the system by substituting out for n in (43) using (21). Then, we differentiate (40)-(42) and simplify to obtain one equation in $c_{11,R}$ and q_R. We use (18) to obtain another equation in these variables. We can then evaluate all the other derivatives.

Substitute for n from (21) and for c from (1) into (43), to obtain

$$\begin{aligned}
& \frac{\psi}{\rho}\left[z\mu c_{11}^{\rho} + z(1-\mu)c_{12}^{\rho} + (1-z)\mu c_{21}^{\rho} + (1-z)(1-\mu)c_{22}^{\rho}\right] c_{22}^{1-\rho} \\
= \; & \theta - g - z\left[\mu c_{11} + (1-\mu)c_{12}\right] + (1-z)\left[\mu c_{21} + (1-\mu)c_{22}\right] - \theta\frac{\eta(\bar{z}-z)^{1+\nu}}{1+\nu}.
\end{aligned}$$

Differentiating with respect to R we get

$$\begin{aligned}
& z\left[\mu c_{11,R} + (1-\mu)c_{12,R}\right] + (1-z)\left[\mu c_{21,R} + (1-\mu)c_{22,R}\right] \\
& +\psi\left[z\mu c_1^{\rho-1}c_{11,R} + z(1-\mu)c_1^{\rho-1}c_{12,R} + (1-z)\mu c_2^{\rho-1}c_{21,R} + (1-z)(1-\mu)c_2^{\rho-1}c_{22,R}\right] c_2^{1-\rho} \\
& +\frac{\psi}{\rho}(1-\rho)c^{\rho}c_2^{-\rho}c_{22,R} = 0,
\end{aligned} \tag{46}$$

where all derivatives are evaluated at a value of P^e such that $q = 1$. Here, $c_1 = c_{11} = c_{12}$ and $c_2 = c_{21} = c_{22}$ when $q = 1$. Now, differentiate (40)-(42) with respect to R to obtain

$$c_{12,R} = c_{11,R} - \frac{c_1}{1-\rho} q_R \tag{47}$$

$$c_{21,R} = c_{11,R} R^{\frac{1}{1-\rho}} + \frac{c_1 R^{\frac{\rho}{1-\rho}}}{1-\rho} \tag{48}$$

$$c_{22,R} = c_{21,R} - \frac{c_2}{1-\rho} q_R. \tag{49}$$

Differentiating (18) with equality and substituting for $c_{12,R}$ from (47), we obtain

$$\mu z c_{11,R} + (1-\mu) z \left(c_{11,R} - \frac{c_1}{1-\rho} q_R \right) + (1-\mu) z c_1 q_R = 0.$$

Simplifying, we obtain

$$q_R = \frac{1-\rho}{\rho(1-\mu)c_1} c_{11,R}. \tag{50}$$

>From (47)-(49) and (50), using $(c_2/c_1)^{1-\rho} = R$, we obtain

$$\mu c_{11,R} + (1-\mu) c_{12,R} = c_{11,R} - \frac{(1-\mu)c_1}{1-\rho} q_R = c_{11,R}(1 - 1/\rho), \tag{51}$$

$$\mu c_{21,R} + (1-\mu) c_{22,R} = c_{21,R} - \frac{(1-\mu)c_2}{1-\rho} q_R \tag{52}$$

$$= c_{11,R}\Big(1 - \frac{R^{\frac{1}{1-\rho}}}{\rho(1-\mu)}\Big) + \frac{c_1 R^{\frac{\rho}{1-\rho}}}{1-\rho} \tag{53}$$

and

$$c_{22,R} = c_{11,R}(1 - 1/\rho) R^{\frac{1}{1-\rho}} + \frac{c_1 R^{\frac{\rho}{1-\rho}}}{1-\rho}. \tag{54}$$

Substitute from (50)-(54) into (46) to obtain

$$\begin{aligned} & z c_{11,R}(1 - 1/\rho) + (1-z) \left[c_{11,R}(1 - 1/\rho) R^{\frac{1}{1-\rho}} + \frac{c_1 R^{\frac{\rho}{1-\rho}}}{1-\rho} \right] \\ & + \psi z c_1^{\rho-1} c_2^{1-\rho} c_{11,R}(1 - 1/\rho) + \psi(1-z) \left[c_{11,R}(1 - 1/\rho) R^{\frac{1}{1-\rho}} + \frac{c_1 R^{\frac{\rho}{1-\rho}}}{1-\rho} \right] \\ & + \frac{\psi}{\rho}(1-\rho) c^{\rho} c_2^{-\rho} \Big(c_{11,R}\Big(1 - \frac{R^{\frac{1}{1-\rho}}}{\rho(1-\mu)}\Big) + \frac{c_1 R^{\frac{\rho}{1-\rho}}}{1-\rho}\Big) = 0. \end{aligned}$$

Grouping terms, we obtain

$$\begin{aligned} & \frac{c_{11,R}}{c_1}\left[z+(1-z)R^{\frac{1}{1-\rho}}+\psi zR+\psi(1-z)R^{\frac{1}{1-\rho}}+\psi\left(\frac{c}{c_2}\right)^{\rho}R^{\frac{1}{1-\rho}}\left(1-\frac{1}{\rho(1-\mu)}\right)\right] \\ = & -\frac{\rho}{\rho-1}\left[(1+\psi)\frac{1-z}{1-\rho}+\frac{\psi}{\rho}\left(\frac{c}{c_2}\right)^{\rho}\right]R^{\frac{\rho}{1-\rho}}. \end{aligned}$$

Finally, we obtain the following expression:

(55)

$$\frac{c_{11,R}}{c_1}=\frac{\frac{\rho}{1-\rho}\left[(1+\psi)\frac{1-z}{1-\rho}+\frac{\psi}{\rho}\left(\frac{c}{c_2}\right)^{\rho}\right]R^{\frac{\rho}{1-\rho}}}{\left(z+(1-z)R^{\frac{1}{1-\rho}}+\psi zR+\psi(1-z)R^{\frac{1}{1-\rho}}\right)+\frac{\psi}{\rho}(1-\rho)\left(\frac{c}{c_2}\right)^{\rho}R^{\frac{1}{1-\rho}}\left(\frac{1}{\rho(1-\mu)}-1\right)}$$

We use these derivatives to obtain c_R and n_R. Differentiating (1) with respect to R, we obtain

(56)

$$c_R=c^{1-\rho}\left[z\mu c_1^{\rho-1}c_{11,R}+z(1-\mu)c_1^{\rho-1}c_{12,R}+(1-z)\mu c_2^{\rho-1}c_{21,R}+(1-z)(1-\mu)c_2^{\rho-1}c_{22,R}\right]$$

Substituting from (51) and (52), we obtain

$$\frac{c_R}{c^{1-\rho}}=c_1^{\rho-1}zc_{11,R}(1-1/\rho)+(1-z)c_2^{\rho-1}(c_{11,R}(1-1/\rho)R^{\frac{1}{1-\rho}}+\frac{c_1R^{\frac{\rho}{1-\rho}}}{1-\rho}).$$

Collecting terms:

$$c_R=c^{1-\rho}c_2^{\rho-1}c_1\left[\frac{c_{11,R}}{c_1}\left(zR+(1-z)R^{\frac{1}{1-\rho}}\right)\left(1-\frac{1}{\rho}\right)+\frac{1-z}{1-\rho}R^{\frac{\rho}{1-\rho}}\right].$$

Differentiating the resource constraint we obtain n_R :

$$\theta n_R=z\left[\mu c_{11,R}+(1-\mu)c_{12,R}\right]+(1-z)\left[\mu c_{21,R}+(1-\mu)c_{22,R}\right].$$

or, after substituting from (51) and (52) and collecting terms:

(57) $$\theta n_R=c_{11,R}(1-\frac{1}{\rho})\left(z+(1-z)R^{\frac{1}{1-\rho}}\right)+(1-z)\frac{c_1}{1-\rho}R^{\frac{\rho}{1-\rho}}.$$

We now prove Lemma 2.

Lemma 2: In a Markov equilibrium with a fixed payment technology,

$$\frac{(1-\rho)\theta u_{22} n_R}{(1-\mu)(1-z)} = f(c_1,c_2)\psi_{MD}(R,z), \tag{58}$$

where $f(c_1,c_2) > 0$ for $c_1, c_2 > 0$,and $\psi_{MD}(R,z)$ is given in (31).

Proof: From (57), using $(c_2/c_1)^{1-\rho} = R$, we obtain

$$\begin{aligned}
\theta n_R &= \frac{(1-\frac{1}{\rho})}{1-\rho} c_{11,R} z \left[(1-\rho)\left(1+(\frac{1-z}{z})R^{\frac{1}{1-\rho}}\right) + \frac{(1-z)/z}{(1-\frac{1}{\rho})}\frac{c_1}{c_{11,R}} R^{\frac{\rho}{1-\rho}}\right] \\
&= \frac{c_2}{c_1}\frac{(1-\frac{1}{\rho})}{1-\rho} c_{11,R} z \left[(1-\rho)\left(R^{\frac{1}{\rho-1}} + (\frac{1-z}{z})\right) + \frac{(1-z)/z}{(1-\frac{1}{\rho})}\frac{c_1}{c_{11,R}} R^{-1}\right].
\end{aligned}$$

Substituting in (29) and using the result that for our functional forms $u_{22}/(1-\mu)(1-z) = u_c\left(\frac{c}{c_2}\right)^{1-\rho}$, we obtain

$$\begin{aligned}
&\frac{(1-\rho)\theta u_{22} n_R}{(1-\mu)(1-z)} = f(c_1,c_2)\left[-(1-\rho)R^{\frac{1}{\rho-1}} - (\frac{1-z}{z})\{(1-\rho) - \frac{\rho}{(1-\rho)}\frac{c_1}{c_{11,R}}R^{-1}\}\right] \\
= \;& f(c_1,c_2)\psi_{MD}(R,z),
\end{aligned}$$

where

$$f(c_1,c_2) = u_c c_2 \left(\frac{c}{c_2}\right)^{1-\rho}(\frac{1}{\rho}-1)\frac{c_{11,R}}{c_1} z$$

and

$$\psi_{MD}(R,z) = -(1-\rho)R^{\frac{1}{\rho-1}} + (\frac{1-z}{z})\{\frac{\rho}{(1-\rho)}\frac{c_1}{c_{11,R}}R^{-1} - (1-\rho)\}. \tag{59}$$

Consider the term in parenthesis in (59). When we use (55), this term is

$$\begin{aligned}
& \frac{z + (1-z)R^{\frac{1}{1-\rho}} + \psi z R + \psi(1-z)R^{\frac{1}{1-\rho}} + \psi\left(\frac{c}{c_2}\right)^{\rho} R^{\frac{1}{1-\rho}}\left(\frac{1}{\rho(1-\mu)} - 1\right)}{\left[(1+\psi)\frac{1-z}{1-\rho} + \frac{\psi}{\rho}\left(\frac{c}{c_2}\right)^{\rho}\right] R^{\frac{\rho}{1-\rho}}} - (1-\rho) \\
= \;& \frac{z + \psi z R + \psi\left(\frac{c}{c_2}\right)^{\rho} R^{\frac{1}{1-\rho}}\left(\frac{1}{\rho(1-\mu)} - 1\right) - (1-\rho)\frac{\psi}{\rho}\left(\frac{c}{c_2}\right)^{\rho} R^{\frac{1}{1-\rho}}}{\left[(1+\psi)\frac{1-z}{1-\rho} + \frac{\psi}{\rho}\left(\frac{c}{c_2}\right)^{\rho}\right] R^{\frac{1}{1-\rho}}} \\
= \;& \frac{z(1+\psi R) + \frac{\mu}{1-\mu}\frac{\psi}{\rho}\left(\frac{c}{c_2}\right)^{\rho} R^{\frac{1}{1-\rho}}}{\left[(1+\psi)\frac{1-z}{1-\rho} + \frac{\psi}{\rho}\left(\frac{c}{c_2}\right)^{\rho}\right] R^{\frac{1}{1-\rho}}}.
\end{aligned}$$

Substituting for c/c_2 in this expression and then substituting in (59), we obtain:

$$\psi_{MD}(R,z) = -(1-\rho)R^{\frac{1}{\rho-1}} + \left\{ \frac{(1-z)\left(R^{\frac{1}{\rho-1}} + \psi R^{\frac{\rho}{\rho-1}}\right) + (\frac{1-z}{z})\frac{\mu}{1-\mu}\frac{\psi}{\rho}\left[zR^{\frac{\rho}{\rho-1}} + 1 - z\right]}{(1+\psi)\frac{1-z}{1-\rho} + \frac{\psi}{\rho}\left(zR^{\frac{\rho}{\rho-1}} + 1 - z\right)} \right\}.$$

Dividing the numerator and denominator of the term in braces by $1-z$ and rearranging, we obtain:

$$\psi_{MD}(R,z) = -(1-\rho)R^{\frac{1}{\rho-1}} + \frac{\left(R^{\frac{1}{\rho-1}} + \psi R^{\frac{\rho}{\rho-1}}\right) + \frac{\mu}{1-\mu}\frac{\psi}{\rho}\left(R^{\frac{\rho}{\rho-1}} + \frac{1-z}{z}\right)}{\frac{1+\psi}{1-\rho} + \frac{\psi}{\rho}\left(\frac{z}{1-z}R^{\frac{\rho}{\rho-1}} + 1\right)}.$$

We have proved the lemma. Q.E.D.

Lemma 3: The first two terms to the right of the equality in (28) can be written as

$$u_c c_R - \frac{\theta u_{22} n_R}{(1-\mu)(1-z)} = -f(c_1, c_2)(R-1)R^{\frac{1}{\rho-1}}. \tag{60}$$

Proof: Using our functional forms, we obtain

$$u_c c_R - \frac{\theta u_{22} n_R}{(1-\mu)(1-z)} = u_c \left[c_R - \theta\left(\frac{c}{c_2}\right)^{1-\rho} n_R\right]. \tag{61}$$

Substituting for θn_R from (57) and c_R from (56) into (61), we obtain

$$\begin{aligned} u_c \left[c_R - \theta\left(\frac{c}{c_2}\right)^{1-\rho} n_R\right] &= u_c[\frac{c_{11,R}}{c_1}\left(zR + (1-z)R^{\frac{1}{1-\rho}}\right)\left(1 - \frac{1}{\rho}\right) + \frac{1-z}{1-\rho}R^{\frac{\rho}{1-\rho}} \\ &\quad -\frac{c_{11,R}}{c_1}(1-\frac{1}{\rho})\left(z + (1-z)R^{\frac{1}{1-\rho}}\right) - \frac{1-z}{1-\rho}R^{\frac{\rho}{1-\rho}}]c_1\left(\frac{c}{c_2}\right)^{1-\rho} \\ &= u_c\frac{c_{11,R}}{c_1}c_2 z(1-\frac{1}{\rho})\left(\frac{c}{c_2}\right)^{1-\rho}(R-1)\frac{c_1}{c_2} \\ &= -f(c_1,c_2)(R-1)R^{\frac{1}{\rho-1}} \end{aligned}$$

where

$$f(c_1, c_2) = u_c\frac{c_{11,R}}{c_1}c_2 z(\frac{1}{\rho} - 1)\left(\frac{c}{c_2}\right)^{1-\rho}.$$

We have proved the lemma. Q.E.D.

Lemma 4: Equation (38) reduces, in a private sector equilibrium, to (39):

$$\frac{\left(\frac{1}{\rho}-1\right)\left(1-R^{\frac{\rho}{\rho-1}}\right)}{z\left[\left(R^{\frac{1}{\rho-1}}-1\right)+\frac{\psi}{\rho}\left(R^{\frac{\rho}{\rho-1}}-1\right)\right]+\left(1+\frac{\psi}{\rho}\right)}=\frac{\rho\eta(\bar{z}-z)^{\nu}}{\left(1-\frac{(\bar{z}-z)^{1+\nu}\eta}{1+\nu}\right)-\frac{g}{\theta}}$$

Proof: Using (43) in (38), we obtain:

$$\left(1-\frac{1}{\rho}\right)\frac{1-R^{\frac{\rho}{\rho-1}}}{z+(1-z)R^{\frac{\rho}{\rho-1}}}=\frac{\theta\rho\eta(\bar{z}-z)^{\nu}}{(c/c_2)^{\rho}\,c_2}. \tag{62}$$

We use the resource constraint, (21), and (43) to obtain an expression for c_2 in terms of c_1/c_2 and z. Rearranging (43) we obtain:

$$\theta n=\theta\left(1-\frac{(\bar{z}-z)^{1+\nu}\eta}{1+\nu}\right)-\frac{\psi}{\rho}\left(\frac{c}{c_2}\right)^{\rho}c_2.$$

Substituting this into the resource constraint, taking into account $c^{\rho}=zc_1^{\rho}+(1-z)c_2^{\rho}$, and rearranging, we obtain:

$$c_2=\frac{\theta\left(1-\frac{(\bar{z}-z)^{1+\nu}\eta}{1+\nu}\right)-g}{z\frac{c_1}{c_2}+\frac{\psi z}{\rho}\left(\frac{c_1}{c_2}\right)^{\rho}+(1-z)(1+\frac{\psi}{\rho})}.$$

Substituting for c_2 in (62), we obtain:

$$(1-\frac{1}{\rho})\frac{1-R^{\frac{\rho}{\rho-1}}}{z+(1-z)R^{\frac{\rho}{\rho-1}}}=\frac{\theta\rho\eta(\bar{z}-z)^{\nu}}{z\left(\frac{c_1}{c_2}\right)^{\rho}+1-z}\times\frac{z\frac{c_1}{c_2}+\frac{\psi z}{\rho}\left(\frac{c_1}{c_2}\right)^{\rho}+(1-z)(1+\frac{\psi}{\rho})}{\theta\left(1-\frac{(\bar{z}-z)^{1+\nu}\eta}{1+\nu}\right)-g}.$$

After rearranging and making use of $R=(c_1/c_2)^{\rho-1}$, we obtain (39). Q.E.D.

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Table 1: Evidence from High Inflation Economies

Country	Low Inflation				High Inflation				Low Inflation	High Inflation
	$\rho(y,R)$	mean R	σ_y	σ_R	$\rho(y,R)$	mean R	σ_y	σ_R	Period	Period
Argentina	−0.49	8.15	3.28	2.34	−0.57	$9.24{*}10^5$	3.95	$24.45{*}10^5$	1992 - 2000	1980 - 1991
Brazil	−0.31	21.39	3.28	4.06	0.03	2362.59	3.08	3713.38	1963 - 1980	1981 - 1995
Brazil	−0.80	25.16	1.11	3.46	NA	NA	NA	NA	1996 - 2000	NA
Chile	−0.11	25.85	3.03	6.37	−0.65	73.89	7.83	26.04	1984 - 2000	1977 - 1983
Israel	0.28	21.93	1.70	3.47	−0.68	245.70	1.30	241.92	1972 - 2000	1979 - 1987
Peru	0.46	28.78	1.65	1.72	−0.58	846.35	7.77	1483.92	1995 - 2000	1986 - 1994
Turkey	NA	NA	NA	NA	−0.41	68.15	3.13	18.98	NA	1987 - 2000
Uruguay	NA	NA	NA	NA	−0.30	88.10	4.92	16.31	NA	1976 - 2000
Column mean	−0.08	21.88	2.34	3.57	−0.45	132516	4.57	350190	NA	NA

References

[1] Notes: In this table y denotes the logged, Hodrick-Prescott filtered level of output, R denotes the Hodrick-Prescott filtered interest rate, $\rho(y,R)$ is the correlation between y and R, σ_y is the standard deviation of y mutiplied by 100 and σ_R is the standard deviation of the interest rate. All data are from the *International Financial Statistics*.

Table 2a: Full Sample Evidence from High Inflation Economies					
Country	$\rho(y,R)$	mean R	σ_y	σ_R	Period
Argentina	-0.59	$5.28*10^5$	4.78	$1.98*10^6$	1980 - 2000
Brazil	0.03	946.05	3.51	2304.49	1963 - 2000
Chile	-0.36	39.86	5.53	15.17	1977 - 2000
Israel	-0.24	113.47	1.66	154.95	1979 - 2000
Peru	-0.46	519.32	7.48	1128.99	1979 - 1993
Turkey	-0.41	68.15	3.13	18.98	1987 - 2000
Uruguay	-0.30	88.10	4.92	16.31	1976 - 2000
Mean, High Inflation	-0.33 (-0.29)	75677	4.43	283324	NA

Table 2b: Full Sample Evidence from Low Inflation Economies					
Country	$\rho(y, R)$	mean R	σ_y	σ_R	Period
Australia	0.54	8.89	1.74	1.92	1970 - 2000
Austria	0.48	6.09	1.78	1.55	1967 - 1998
Belgium	0.32	5.22	1.79	1.48	1953 - 1998
Canada	0.40	8.36	2.48	2.45	1975 - 2000
Denmark	-0.31	9.81	1.82	2.06	1972 - 2000
Finland	0.18	9.68	4.03	1.94	1978 - 2000
France	0.09	6.96	1.48	1.60	1950 - 1998
Germany	0.54	5.38	2.58	1.91	1960 - 2000
Netherlands	-0.04	5.79	4.07	1.75	1960 - 1998
Ireland	0.15	10.65	2.51	2.27	1971 - 1999
Italy	0.09	11.28	1.53	2.08	1969 - 2000
Japan	0.24	6.21	3.06	1.69	1957 - 2000
New Zealand	0.48	11.11	2.61	2.10	1985 - 2000
Norway	-0.25	9.44	1.87	1.48	1972 - 2000
Spain	0.33	11.60	2.18	2.40	1974 - 2000
Sweden	0.01	8.75	2.16	2.20	1966 - 2000
Switzerland	0.43	3.40	2.64	1.59	1969 - 2000
United Kingdom	0.03	7.78	2.28	2.20	1969 - 2000
United States	0.20	6.15	2.15	1.78	1955 - 2000
Mean, Low Inflation	0.20 (0.20)	8.03	2.26	1.84	NA

Notes: In this table y denotes the logged, Hodrick-Prescott filtered level of output, R denotes the Hodrick-Prescott filtered interest rate, $\rho(y, R)$ is the correlation between y and R, σ_y is the standard deviation of y mutiplied by 100 and σ_R is the standard deviation of the interest rate. All data are from the *International Financial Statistics*.

Figure 1: Marginal Benefits and Marginal Costs for Monetary Authority

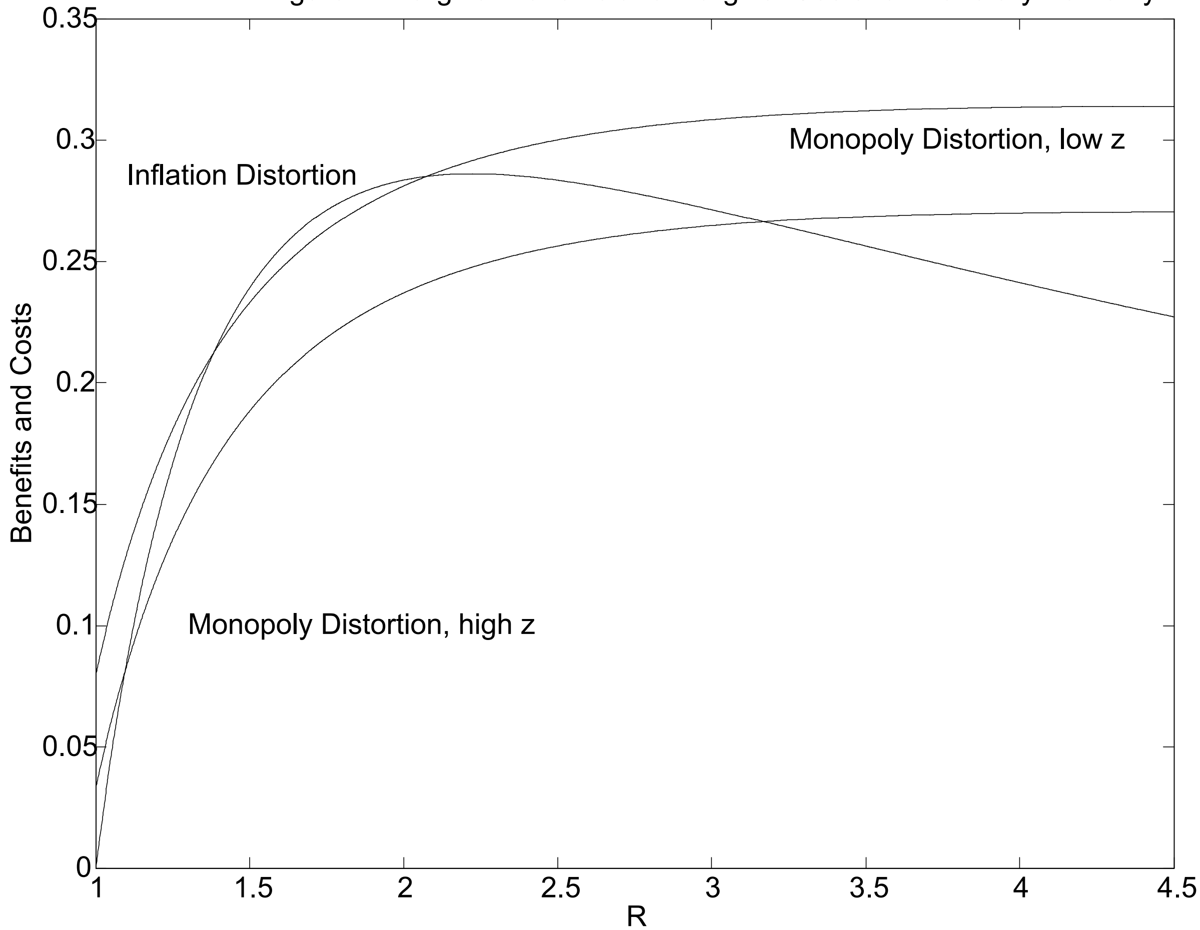


Figure 2a: Utility for Deviations from Low Inflation Equilibrium

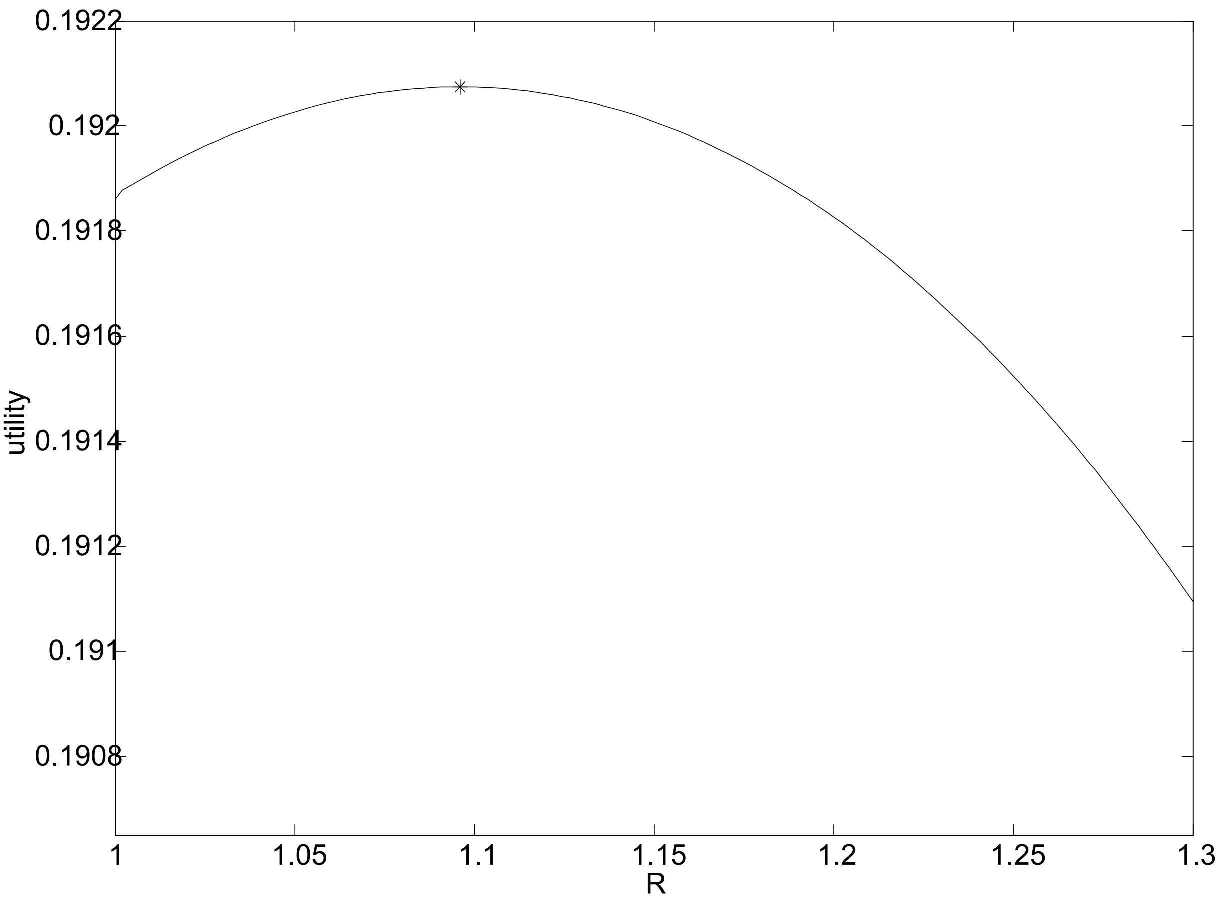
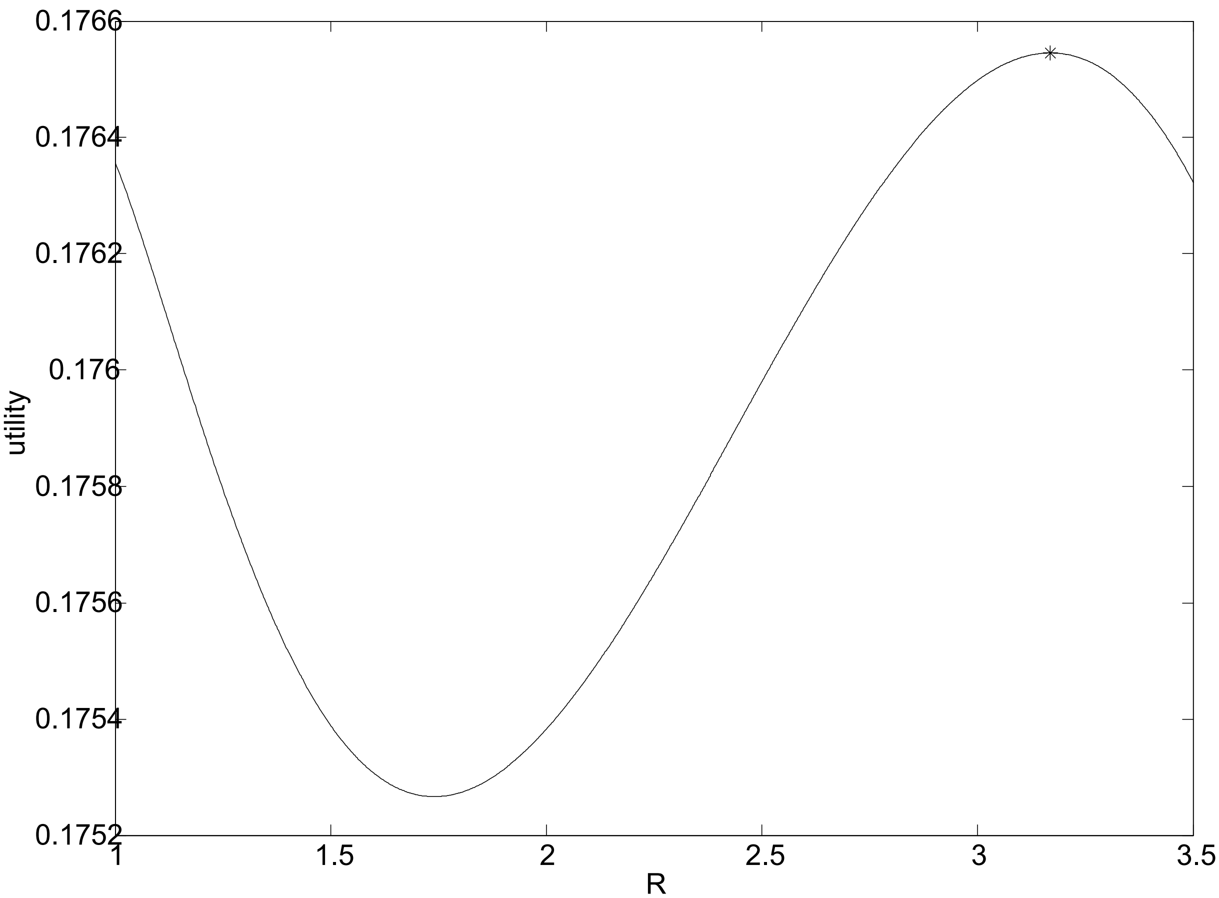


Figure 2b: Utility for Deviations from High Inflation Equilibrium



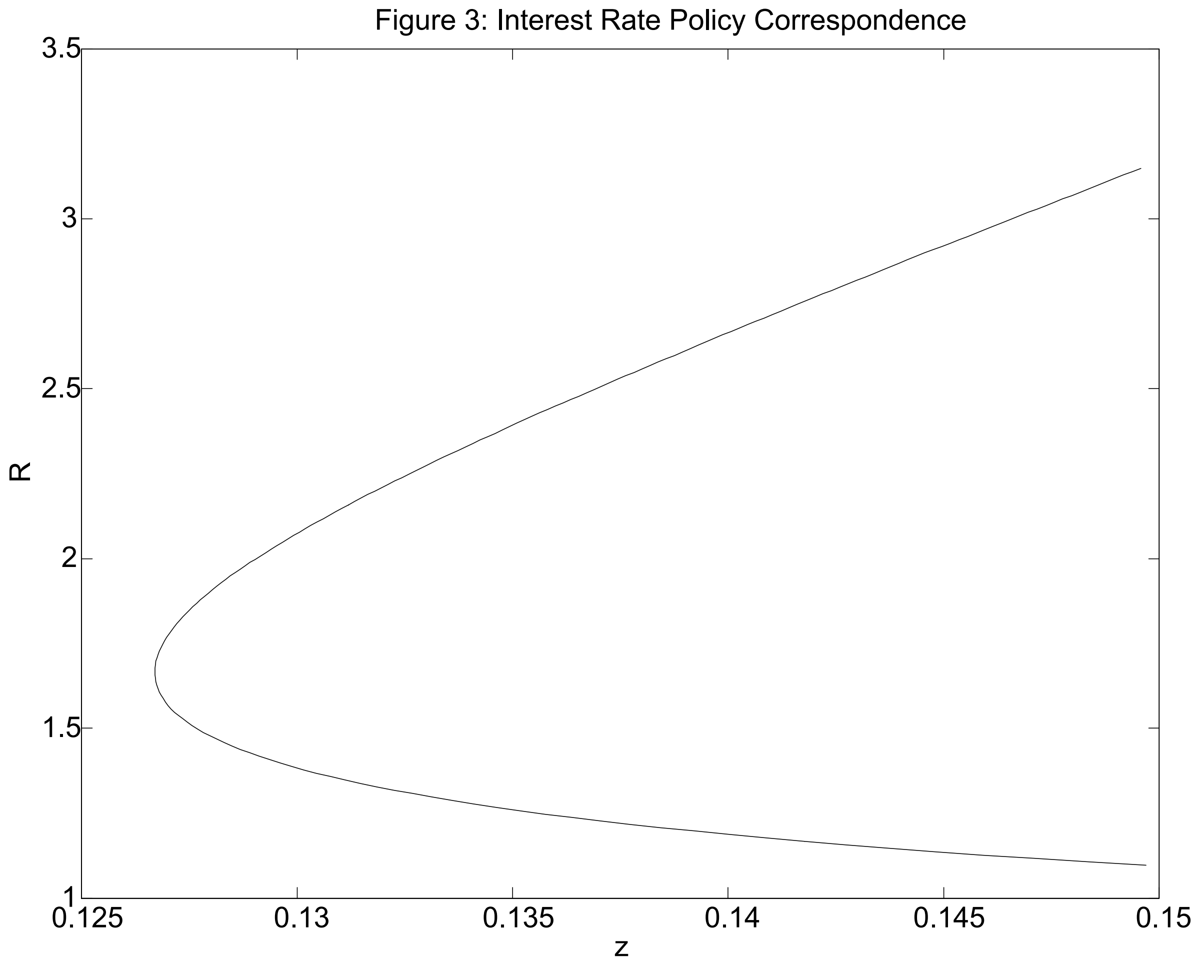


Figure 3: Interest Rate Policy Correspondence
3.5
3
2.5
2
1.5
1
R
0.125
0.13
0.135
0.14
0.145
0.15
z

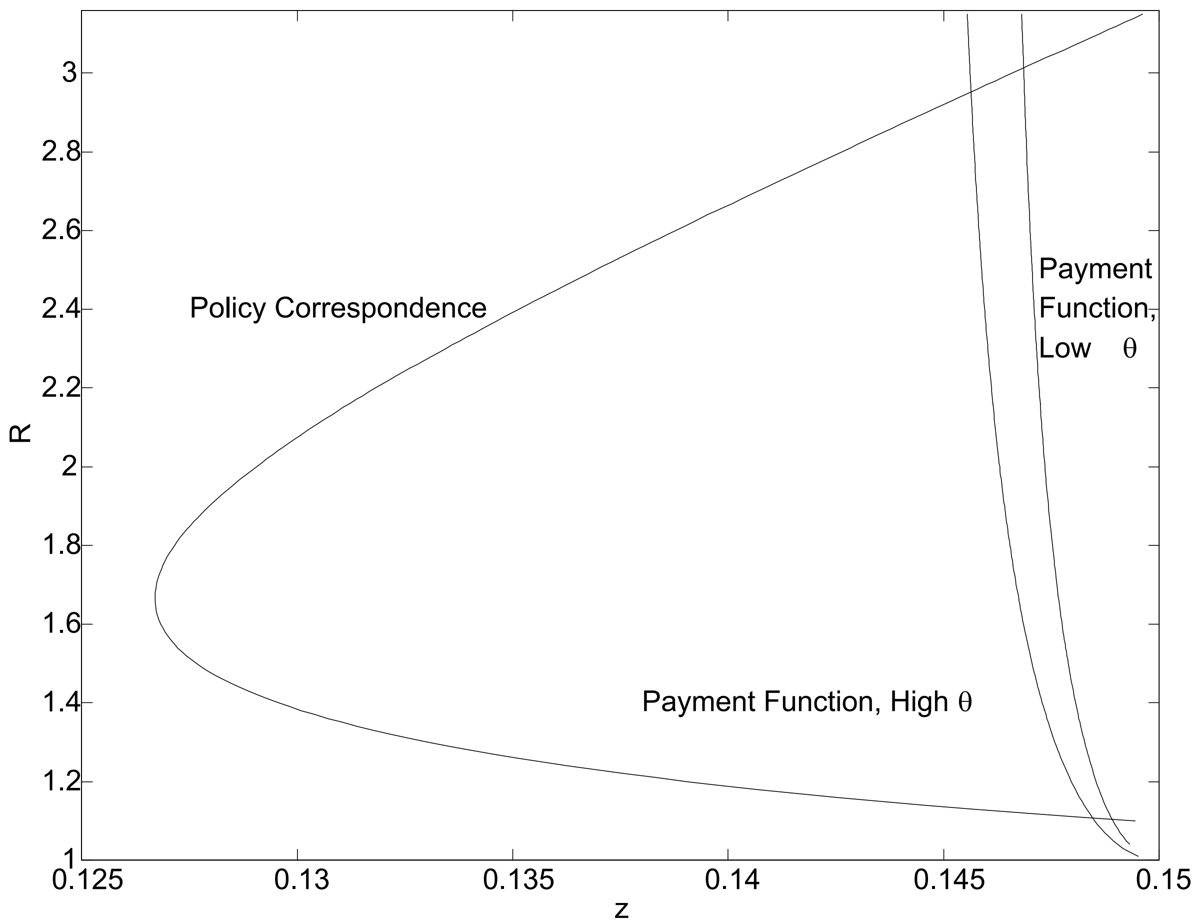

Policy Correspondence
Payment Function, Low θ
Payment Function, High θ
R
z
0.125
0.13
0.135
0.14
0.145
0.15
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
2.8
3

Figure 4b: Markov Equilibrium With Payment Technology Shocks