Expectation Traps and Monetary Policy

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Abstract

Why is it that inflation is persistently high in some periods and persistently low in other periods? We argue that lack of commitment in monetary policy may bear a large part of the blame. We show that, in a standard equilibrium model, absence of commitment leads to multiple equilibria, or *expectation traps,* even in the absence of trigger strategies. In these traps, expectations of high or low inflation lead the public to take defensive actions which then make it optimal for the monetary authority to validate those expectations.

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Many countries have gone through prolonged periods of costly, high inflation, as well as prolonged periods of low inflation. The United States and other industrialized countries went through a high inflation episode during the Great Inflation of the 1970s, and are now in a low inflation episode. Why do high inflation episodes occur? What can be done to prevent them from occurring again? These are two central questions in monetary economics.

One tradition for understanding poor inflation outcomes stems from the time inconsistency literature pioneered by Kydland and Prescott (1978) (KP) and Barro and Gordon (1983) (BG). This literature points to lack of commitment in monetary policy as the main culprit behind high inflation. Static versions of the models in this literature have a unique equilibrium. Inflation rates can fluctuate only if the underlying fundamentals do. In may cases, it is difficult to see what changes in the underlying fundamentals could have generated the episodes of high and low inflation. In infinite horizon versions of the KP and BG models, trigger strategies can be used to produce the observed inflation outcomes. However, such models have embarrassingly many equilibria. It is hard to know what observations would be ruled out by such trigger strategy equilibria.

This paper is squarely within the tradition of the time inconsistency literature in pointing to lack of commitment as the main culprit behind the observed volatility and persistence of inflation. We make three contributions. First, we show how the economic forces in the KP and BG models can be embedded into a standard general equilibrium model. Second, we find that once these forces have been so embedded, inflation rates can be high for prolonged periods and low for prolonged periods, even though we explicitly rule out trigger strategies. Third, we view our models as a promising first step towards developing empirically plausible models of inflation in the United States and other countries.

In the KP and BG models, the key trade-off is between the benefits of higher output from unexpected inflation and the costs of realized inflation. In our general equilibrium model, unexpected inflation raises output because some prices are sticky. This rise in output has benefits because producers have monopoly power and the unexpected inflation reduces the monopoly distortion. In our general equilibrium model, realized inflation is costly because households must use previously accumulated cash to purchase some goods, called *cash goods*. Higher realized inflation forces households to substitute toward other goods, called *credit goods*. This substitution tends to lower welfare. Thus, by design, the general equilibrium model captures the trade-offs between the benefits of unexpected inflation and the costs of realized inflation in the KP and BG framework.

This way of capturing the trade-offs leads to multiple equilibria in our general equilibrium model. Private agents' expectations of high or low inflation can lead these agents to take defensive actions, which then make it optimal for monetary authorities to validate these expectations. The main defensive action we focus on is that sticky price firms set high prices if they expect high inflation and low prices if they expect low inflation. Given these expectations and the associated defensive actions, the monetary authority then chooses policy optimally by equating the marginal benefits of unexpected inflation to the marginal costs of realized inflation. We show analytically that there are at least two sets of policies and allocations at which marginal benefits equal marginal costs. Our analytical procedure only focuses on necessary conditions for monetary authority optimality. In a large class of parameterizations we used numerical methods to identify situations where the necessary conditions are sufficient, and where they are not.

To better understand the multiplicity of equilibria in our model, it is useful to understand how the marginal benefits of unexpected inflation and the marginal costs of realized inflation depend on expected inflation. To a first approximation, the marginal benefit of unexpected inflation is independent of the expected inflation rate. In contrast, the marginal cost of realized inflation as a function of expected inflation has an inverted 'U' shape. That is, it is low at low levels of inflation, high at moderate levels, and low again at high levels of inflation. The shapes of the marginal benefits and costs implies that there are two values of inflation at which marginal benefits and costs are equated.

One way of understanding the inverted 'U' shape of the marginal cost of realized inflation is to draw an analogy with the inflation tax literature. In our model the marginal cost of realized inflation is equal to a measure of the tax revenue from inflation, namely the product of the net interest rate and the stock of real cash balances. In a way, this feature should not be too surprising because in monetary models, inflation acts as a tax on cash balances. The marginal cost of realized inflation has an inverted 'U' shape because, as in the Laffer curve literature, the tax revenue from inflation has an inverted 'U' shape. This tax revenue is 'U' shaped because in our model, money demand is inelastic at low interest rates and elastic at high interest rates.

One feature of the model thus far is that the equilibrium interest rate is independent of shocks to technology and government consumption. Many authors have argued that the response of interest rates and other financial variables to shocks is very different in low and high inflation episodes. We describe a variant of our model with a variable payment technology in which this behavior occurs. This variant provides a related, but different, channel which also leads to multiplicity of equilibria. In this variable payment technology model, households can also take defensive actions to protect themselves against expected high inflation. Specifically, they can choose the fraction of goods purchased with cash and the fraction purchased with credit. The households' choice of payment technology is made before the monetary authority selects the money growth rate. Cash purchases are costly because households forgo interest, while credit purchases require payment of a cost in labor time, which differs depending on the type of good. In our model, as noted above, cash goods must be purchased with previously accumulated cash, so that a monetary expansion, by raising prices, reduces the consumption of cash goods and reduces welfare. These aspects of our model imply that if households expect high inflation and have defensively chosen to purchase few goods with cash, the marginal cost of realized inflation is small. Given the gains of inflation, the monetary authority has an incentive to choose a high level of inflation. If households expect low inflation, however, they do not take defensive actions and choose instead to purchase many goods with cash and the marginal costs of realized inflation are high. Given the gains of inflation, the monetary authority has an incentive to choose a low level of inflation. These considerations reinforce the sources of multiplicity in the fixed payment technology model, so that the variable payment technology model also has multiple equilibria. We also show that with a variable payment technology, interest rates do respond to shocks.

We explore the properties of financial and real variables in a stochastic version of the variable payment technology model. This version also has two equilibria. It turns out that the interest rate response to a technology shock switches sign between the high and low inflation equilibria while the output is increasing in this shock in both equilibria. We show that this sign switch implies that the correlation between output and interest rates is more negative in the high inflation equilibrium than in the low inflation equilibrium. Our model also implies higher volatility of nominal variables in high inflation episodes than in low inflation episodes. In Albanesi, Chari and Christiano (2002a), we examine cross-country data and find support for these implications. While a variety of other models might imply higher volatility, it is hard to see which models would generate the change in the magnitude and sign of the correlation between output and interest rates.

We now briefly compare and contrast the analytic roles played by the two channels we have identified in producing multiplicity of Markov equilibria. In each case, inflation expectations lead agents to take defensive actions, which then make it optimal for the monetary authority to validate the expectations. The two channels focus on the defensive actions of different agents: the sticky price channel focuses on firms' incentives to raise prices when they expect high inflation and the payment technology channel focuses on households' incentive to alter the mix of cash and credit goods when they expect high inflation. The role of the sticky price channel in producing multiple equilibria is clear, because it is the only channel that is operative in our fixed payment technology model. That the payment technology channel also has a potentially important role to play can be seen by considering the analysis in ACC (2002b). There, we describe an environment very similar to the present one, but with a different household utility function. It turns out that in the fixed payment technology version of that model, the equilibrium is unique, so that the sticky price channel does not produce multiple equilibria. We show that in the variable payment technology version of that model, there are multiple equilibria. By displaying an environment in which the payment technology channel is the only channel producing multiple equilibria, that analysis demonstrates the distinct role of that channel. Finally, in our discussion of the sticky price channel we have emphasized the role of a high interest elasticity of money demand at high inflation rates for producing multiplicity. This intuition carries over to the payment technology channel: households' opportunity to alter the mix of cash an credit goods has the effect of increasing their interest elasticity of money demand.

Following Chari, Christiano and Eichenbaum (1998) (CCE), we call the kind of multiplicity identified here an *expectation trap* because the defensive actions induced by changes in expectations in effect 'trap' policy makers into accommodating the expectations. CCE rely on trigger strategies to generate expectation traps. One criticism of trigger strategies is that virtually any inflation outcome can be rationalized as an equilibrium. In this paper, we restrict attention to Markov equilibria that rule out trigger strategies. Furthermore, the Markov equilibrium in CCE is at a corner. One contribution of this paper is that we obtain an interior equilibrium (see also Neiss, 1999).

The notion of an expectation trap may shed light on the continuing debate concerning the interpretation of the successful and, thus far, sustained reduction in inflation since the early 1980s, in the United States and a number of other countries (see Sargent, 1999). Our paper raises the possibility that the inflation of the 1970s was a high inflation expectation trap, and that the inflation may have declined simply because we switched to a low inflation expectation trap. Since the structure of policymaking institutions has not fundamentally changed, the paper raises the possibility that we could once again be caught in a 1970s style high inflation expectation trap.[1]

Our analysis has policy implications. If time inconsistency problems are behind the poor inflation outcomes of many countries, then setting up institutions which promote the ability of central banks to commit to future actions can lead to large gains. Under commitment, the optimal policy in our model has the monetary authority following the Friedman Rule and setting nominal interest rates equal to zero. Without commitment, the economy experiences spells of high inflation and spells of low inflation. Institutional devices which can raise welfare in practice include ways of protecting central bank independence and the design of appropriate incentive contracts for central bankers (as in, for example, Persson and Tabellini, 1993).

The plan of the paper is as follows. Section I describes our model with a fixed payment technology. In Section II, we analyze the equilibria of this model and show that multiplicity is possible. We analyze an economy with a variable payment technology in section III. In Section IV, we discuss the main forces behind the expectation traps we find and in section VI we describe the relationship of our paper to others. The final section concludes.

I A Monetary General Equilibrium Economy

Our economy extends and modifies the Lucas and Stokey (1983) cash-credit goods model in two ways. The first modification is that, in our model, a subset of prices are set in advance by monopolistic firms. The second modification is that, as in Svensson (1985), we require households to use currency accumulated in the previous period to purchase cash goods in the current period. We assume that the monetary authority chooses monetary policy to maximize the welfare of the representative household. Our modifications imply that the trade-off the monetary authority confronts resembles that in the KP and BG models. The sticky price modification implies that an unanticipated monetary expansion tends to raise output and welfare. The cash-in-advance modification implies that the inflation associated with a monetary expansion reduces welfare by reducing the consumption of cash goods relative to credit goods.

Our infinite-horizon economy is composed of a continuum of firms, a representative household and a monetary authority. The sequence of events within a period is as follows. First, the shock to the production technology, θ, and the shock to government consumption, g, are realized. We refer to $s = (\theta, g)$ as the *exogenous state* and assume that s follows a Markov process.[2] Then a fraction, μ, of firms (the *sticky price firms*) set their prices. The average price set by sticky price firms is denoted $P^e(s)$. This price, as well as all other nominal variables, is scaled by the beginning-of-period aggregate stock of money. The remaining fraction, $1 - \mu$, of firms are called *flexible price firms.*

Next, the monetary authority chooses the interest rate, R.[3] We denote the policy rule that the monetary authority is expected to follow by $R(s)$. The state of the economy after the monetary authority makes its decision, the *private sector's state,* is (s, R). Let $X(s, R)$ denote the money growth rate associated with s, R. Households' and firms' production, consumption and employment decisions and the pricing decisions of the flexible price firms depend on the private sector's state.

In what follows, we first describe the problems of households and firms in our economy given s, R and future monetary policy, $R(s)$. We then set up the monetary authority's problem and define a Markov equilibrium. The key part of a Markov equilibrium is that the monetary authority chooses policy optimally. To define the monetary authority's problem, we must specify the private equilibrium allocations as functions of the monetary authority's policy variable, R. We refer to these functions as a private sector equilibrium. A Markov equilibrium is a private sector equilibrium in which policy is set optimally.

A Households

We begin with the household problem. In each period the household consumes a continuum of differentiated goods as in Blanchard and Kiyotaki (1987) and supplies labor. The representative household's preferences are $\sum_{t=0}^{\infty} \beta^t u(c_t, n_t)$, where $0 < \beta < 1$,

$$c_t = \left[\int_0^1 c_t(\omega)^\rho d\omega \right]^{\frac{1}{\rho}}, \; u(c,n) = \frac{\left[c(1-n)^\psi \right]^{1-\sigma}}{1-\sigma},$$

$c_t(\omega)$ denotes consumption of type ω good, l_t denotes labor time, and $0 < \rho < 1$.

Each good in this continuum is one of four types. A fraction μ are produced by sticky price firms and a fraction $1-\mu$ are produced by flexible price firms. A fraction, z, of all goods consists of cash goods, and the fraction, $1-z$, consists of credit goods. The sticky and flexible price firms are randomly distributed over cash and credit goods. Thus, a fraction μz of goods are sticky price goods purchased with cash, a fraction $(1-\mu)z$ are flexible price goods purchased with cash, a fraction $\mu(1-z)$ are sticky price goods purchased with credit, and a fraction $(1-\mu)(1-z)$ are flexible price goods purchased with credit. It turns out that prices for goods within each type are the same. Utility maximization implies that the amounts purchased within each type of good are also the same. Let c_{11} and c_{12} denote quantities of cash goods purchased from sticky and flexible price firms, respectively, and let c_{21} and c_{22} denote the quantities of credit goods purchased from sticky and flexible price goods, respectively. Then we have that

$$c = \left[z\mu c_{11}^\rho + z(1-\mu)c_{12}^\rho + (1-z)\mu c_{21}^\rho + (1-z)(1-\mu)c_{22}^\rho \right]^{\frac{1}{\rho}}. \tag{1}$$

Let A denote the nominal assets of the household, carried over from the previous period. In the asset market, the household trades money, M, and one-period bonds, B, with other households. The asset market constraint is

$$M + B \leq A. \tag{2}$$

Recall that nominal assets, money and bonds are all scaled by the aggregate stock of money. We impose a no-Ponzi constraint of the form $B \leq \bar{B}$, where $\bar{B}$ is a large, finite upper bound.

The household's cash-in-advance constraint is

$$P^e(s) \left[\mu z c_{11} + q(s,R)(1-\mu) z c_{12} \right] \leq M, \tag{3}$$

where $P^e(s)$ denotes the price set by sticky price firms and $q(s,R)P^e(s)$ denotes the price set by flexible price firms. Note that $q(s,R)$ is the relative price of flexible price goods to sticky price goods. Nominal assets evolve over time as follows:

$$
\begin{aligned}
(4) \qquad & zP^e(s)\left[\mu c_{11} + q(s,R)(1-\mu)c_{12}\right] + (1-z)P^e(s)\left[\mu c_{21} + q(s,R)(1-\mu)c_{22}\right] \\
& \qquad + X(s,R)A' \leq W(s,R)n + D(s,R) + (X(s,R)-1) + M + RB.
\end{aligned}
$$

In (4), $W(s,R)$ denotes the nominal wage rate and $D(s,R)$ denotes profits after lump sum taxes. Notice that A' is multiplied by $X(s,R)$. This multiplication reflects that we have scaled all nominal variables by the beginning of period aggregate stock of money and A' is the household's nominal assets scaled by next period's aggregate money stock. Next period's aggregate money stock is simply the current stock multiplied by the growth rate $X(s,R)$.

In is interesting to compare our description of the asset market with that in Svensson (1985). Svensson assumes that each household sees itself as facing a cash in advance constraint in which only previously accumulated cash can be used for cash goods purchases. In our setup, individual households face any such constraint. It is society as a whole that faces the constraint that only previously accumulated cash can be used for cash goods purchases. This constraint manifests itself as an equilibrium condition that $M = 1$. The interest rate adjusts to ensure that the equilibrium condition is satisfied, so that households optimally use only previously accumulated cash for cash goods purchases. The analysis with Svensson's formulation leads to identical results.

Consider the household's asset, goods and labor market decisions. Given that the household expects the monetary authority to choose policy according to $R(s)$ in the future, the household solves the following problem:

$$
(5) \qquad v(A,s,R) = \max_{n,M,A',c_{ij};\ i,j=1,2} u(c,l) + \beta E_{s'}[\max_{z'} v(A',s',R(s'))|s]
$$

subject to (1), (33), (3), (4), and nonnegativity on allocations. Here, we have substituted out for B using (2). The solution to (5) yields decision rules, $d(A, s, R)$, where

$$d(A, s, R) = [n(A, s, R), M(A, s, R), A'(A, s, R), c_{ij}(A, s, R)], \tag{6}$$

$i, j = 1, 2$.

B Firms and Resource Constraint

Each of the differentiated goods is produced by a monopolist using the following production technology

$$y(\omega) = \theta n(\omega),$$

where $y(\omega)$ denotes output and $n(\omega)$ denotes employment for the type ω good. Also, θ is a technology shock that is the same for all goods. The household's problem yields demand curves for each good. The fraction, $1 - \mu$, of firms that are flexible price firms set their price to maximize profits subject to these demand curves. Because the household demand curves have constant elasticity, firms set prices as a fixed markup, $1/\rho$, above marginal cost, W/θ, so that the relative price of flexible to sticky price goods is given by:

$$q(s, R) = \frac{W(s, R)}{P^e(s)\theta\rho}. \tag{7}$$

Sticky price firms set prices at the beginning of the period, after the exogenous shocks are realized. As in Blanchard and Kiyotaki (1987), sticky price firms in our economy must set their price in advance and must produce the amount of goods demanded at that price. These firms, like the flexible price firms, also wish to set their price as a markup, $1/\rho$, over marginal cost, W/θ. In order to do so, they need to forecast the wage rate, W. They do so by taking the wage rate as given by the private sector equilibrium. Thus, the wage they expect to prevail is $W(s, R(s))$. Thus, in equilibrium the price set by sticky price firms satisfies:

$$P^e(s) = \frac{W(s, R(s))}{\theta\rho} \tag{8}$$

Turning to the government, we assume that there is no government debt, government consumption is financed with lump-sum taxes, and government consumption is the same for all goods. As a result, the resource constraint for this economy is

$$\theta n = g + z\left[\mu c_{11} + (1-\mu)c_{12}\right] + (1-z)\left[\mu c_{21} + (1-\mu)c_{22}\right],$$

where g denotes the exogenous level of government consumption. Since there is no government debt, bond market clearing requires $B = 0$, $A = 1$. Also, money market clearing requires $M = 1$.

C Private Sector Equilibrium

We now define an equilibrium for each possible private sector state (s, R) and future monetary policy rule, $R(s)$.

Definition For each s, R, given $R(s)$, a *private sector equilibrium* is a number, $P^e(s)$, and a collection of functions, $q(s,R)$, $W(s,R)$, $X(s,R)$, $v(A,s,R)$, $d(A,s,R)$ such that the following hold:

1. The functions v and d solve (5)
2. Firms maximize profits; that is, $q(s,R)$ satisfies (7) and $P^e(s)$ satisfies (8)
3. The resource constraint is satisfied at $d(1,s,R)$
4. The asset markets clear; i.e., $A'(1,s,R) = M(1,s,R) = 1$.

Notice that a private sector equilibrium is defined for all values of R, not just $R = R(s)$. We define a private sector equilibrium outcome as the allocations and prices that occur when $A = 1$ and actual policy, R, coincides with expectations of policy, $R(s)$:

Definition For each s, a *private sector equilibrium outcome* is a collection of numbers, $P^e(s)$, $q(s,R(s))$, $W(s,R(s))$, $X(s,R(s))$, $v(1,s,R(s))$, $d(1,s,R(s))$.

Combining (7) and (8), we have that in a private sector equilibrium outcome:

$$q(s,R(s)) = 1. \tag{9}$$

D Monetary Authority Problem and Markov Equilibrium

The monetary authority chooses R to maximize the representative household's discounted utility:

$$\max_{R} v(1, s, R), \tag{10}$$

where v is the value function in a private sector equilibrium. Recall that a private sector equilibrium takes as given the evolution of future monetary policy. Thus, in solving (10) the monetary authority implicitly takes as given the evolution of future monetary policy.

We now have the ingredients needed to define a Markov equilibrium.

Definition A *Markov equilibrium* is a private sector equilibrium and a monetary policy rule, $R(s)$, such that $R(s)$ solves (10).

Notice that in a Markov equilibrium, the current money growth rate does not affect discounted utility of the household starting from the next period since it does not affect the next period's state. Therefore, the monetary authority faces the static problem of maximizing current period utility, and we only have to describe how current R affects current allocations. In a parallel fashion to a private sector equilibrium outcome, we define a Markov equilibrium outcome as a Markov equilibrium in which actual policy, R, coincides with expectations of policy, $R(s)$:

Definition For each s, a *Markov equilibrium outcome* is a collection of numbers, $P^e(s)$, $q(s, R(s))$, $W(s, R(s))$, $X(s, R(s))$, $v(1, s, R(s))$, $d(1, s, R(s))$, where $R(s)$ is the monetary policy rule associated with a Markov equilibrium.

II Analysis of Equilibrium

In our analysis, we decompose the first-order condition associated with the monetary authority problem, (10), into benefits and costs of inflation. To obtain these benefits and costs, we begin by characterizing a private sector equilibrium. We then solve the monetary authority's problem. We show that, generically, there are at least two allocations that satisfy the necessary conditions for a Markov equilibrium. We present numerical examples in which these allocations also satisfy the sufficient conditions for a Markov equilibrium.

A Characterizing Private Sector Equilibrium

We first characterize a private sector equilibrium outcome. We use this characterization to construct a private sector equilibrium. Omitting arguments of functions for convenience, the first order necessary conditions for household and firm optimization are:

$$\frac{u_{11}}{u_{12}} = \frac{\mu}{1-\mu}\frac{1}{q}, \tag{11}$$

$$\frac{u_{21}}{u_{22}} = \frac{\mu}{1-\mu}\frac{1}{q}, \tag{12}$$

$$\frac{u_{11}}{u_{21}} = \frac{z}{1-z}R, \tag{13}$$

$$\frac{u_{12}}{u_{22}} = \frac{z}{1-z}R, \tag{14}$$

$$-u_n = \frac{\theta\rho u_{22}}{(1-\mu)(1-z)}, \tag{15}$$

$$\frac{Xu_{21}}{P^e\mu(1-z)} = \beta E_{s'}[v_1(1,s',R(s'))|s], \tag{16}$$

where u_{ij} denotes the partial derivative of u with respect to c_{ij}, and v_1 denotes the partial derivative of v with respect to its first argument. Equations (11) and (12) equate the marginal rate of substitution between sticky and flexible price goods to the relative price of these goods q, and equations (13) and (14) equate the marginal rate of substitution between cash and credit goods to their relative price, the interest rate. Equation (15) is obtained by noting that the marginal rate of substitution between labor and consumption of flexible price credit goods is equated to the ratio of the nominal wage to the price of flexible price goods. This ratio is simply the markup in (7). Finally, (16) is the intertemporal Euler equation for asset accumulation.

The cash-in-advance constraint can be written as

$$\mu z c_{11} + q(1-\mu) z c_{12} \leq \frac{1}{P^e}. \tag{17}$$

A necessary condition for the household problem to be well defined is

$$R \geq 1. \tag{18}$$

It is easy to show that the cash in advance constraint holds with equality if $R > 1$ and that if the cash-in-advance constraint is slack, $R = 1$. These observations imply that the appropriate complementary slackness condition is

$$\left\{ \frac{1}{P^e} - \left[\mu z c_{11} + q(1-\mu) z c_{12}\right] \right\} [R-1] = 0. \tag{19}$$

The resource constraint is

$$g + z\left[\mu c_{11} + (1-\mu) c_{12}\right] + (1-z)\left[\mu c_{21} + (1-\mu) c_{22}\right] = \theta n. \tag{20}$$

We can use the preceding equations to compute a private sector equilibrium outcome. Recall that a private sector equilibrium is conditioned on some given policy rule, $R(s)$. Set $R = R(s)$, $q = 1$ and, for each s, use (9), (11)-(15), (19) and (20) to compute the six numbers $P^e(s)$, $n(1, s, R(s))$, $c_{ij}(1, s, R(s))$, $i, j = 1, 2$. Notice that one of the equations in (11)-(14) is redundant and can be deleted. Thus, we can use these six independent equations to compute the six numbers of interest. The rest of the private sector equilibrium outcome is straightforward to compute. For future use, note that $c(1, s, R(s))$ is obtained from (1) using $c_{ij}(1, s, R(s))$.

Given $P^e(s)$ from a private sector equilibrium outcome, we can compute a private sector equilibrium as follows. For each s and each R, we use (11)-(15), (19) and (20) to compute the functions $n(1, s, R)$, $c_{ij}(1, s, R)$, $i, j = 1, 2$ and $q(s, R)$. As above, note that $c(1, s, R)$ is obtained from (1) using $c_{ij}(1, s, R)$.

B The Monetary Authority's Problem

The monetary authority's problem is static because we focus on Markov equilibria and there are no state variables in our economy. Recall that, in a Markov equilibrium policy makers face dynamic problems only if their decisions affect future state variables. Since there are no state variables in our economy, the monetary authority's problem is simply one of choosing current policy to maximize current period utility. We let

$$U(s,R) = u\left[c(1,s,R), n(1,s,R)\right]$$

denote the utility associated with an interest rate R, where $c(1,s,R)$, $n(1,s,R)$ are the private sector equilibria constructed in the previous subsection. Recall that these functions are defined for some given policy rule, $R(s)$. We suppress this dependence to keep from cluttering the notation. The monetary authority's problem is now

$$\max_R U(s,R), \tag{21}$$

subject to $R \geq 1$.[4] Then, $R(s)$ is the policy rule associated with a Markov equilibrium if it solves (21).

C Characterizing Markov Equilibrium

We can think of constructing a Markov equilibrium in two ways. First, we can treat (21) as defining an operator which maps the space of policy rules into the space of policy rules. The Markov equilibrium policy rule can be constructed by finding a fixed point of this operator. Second, we can think of (9), (11)-(20) and the first order necessary condition associated with (21) as a system of equations which are used simultaneously to solve for a Markov equilibrium. The fist order condition is obtained by differentiating (11)-(20) with respect to R, holding P^e fixed and the derivative is evaluated at a point which solves (11)-(20) with $q = 1$. If the first order condition for the monetary authority is also sufficient, the two approaches are equivalent. We pursue the second approach.

We show that, generically, at least two allocations satisfy the necessary condition associated with (21). In a large class of parameterizations for our economy, we verified numerically that this necessary condition is also sufficient. We also derive a relationship between the payment parameter z and the allocations and prices in a Markov equilibrium. We use this relationship when we discuss a Markov equilibrium with a variable payment technology.

The first-order condition associated with a solution to (21) is

$$U_R(s,R) = u_c c_R + u_n n_R \leq 0, \tag{22}$$

with equality if $R > 1$. In (22) U_R is the derivative of U with respect to R and u_c, u_n are derivatives of the utility function with respect to consumption and employment, respectively. Also, c_R, n_R are the derivatives of the private sector equilibrium functions, $c(1,s,R)$ and $n(1,s,R)$, with respect to R. If $R(s)$ is a Markov equilibrium policy rule, then it satisfies (22).

In what follows, we show that (22) can be decomposed into a part that captures the incentives to increase inflation because of the presence of monopoly power and a part that captures the disincentives arising from the resulting reduction in cash goods consumption. Specifically, we prove the following proposition:

Proposition 1 Suppose $R(s)$ is a Markov equilibrium policy rule. Then, there exists a strictly positive function, $f(c_1,c_2)$, and a pair of functions, $\Psi_{MD}(R,z)$ and $\Psi_{ID}(R)$, given by

$$\psi_{MD}(R,z) = -(1-\rho)R^{\frac{1}{\rho-1}} + \frac{R^{\frac{1}{\rho-1}} + \psi R^{\frac{\rho}{\rho-1}} + \frac{\mu}{1-\mu}\frac{\psi}{\rho}\left(R^{\frac{\rho}{\rho-1}} + \frac{1-z}{z}\right)}{\frac{1+\psi}{1-\rho} + \frac{\psi}{\rho}\left(\frac{z}{1-z}R^{\frac{\rho}{\rho-1}} + 1\right)}, \tag{23}$$

and

$$\psi_{ID}(R) = (R-1)\,R^{\frac{1}{\rho-1}}, \tag{24}$$

such that

$$U_R(s,R(s)) = f(c_1,c_2)\left[-\psi_{ID}(R(s)) + \psi_{MD}(R(s),z)\right],$$

where $c_1 = c_{11}(1, s, R(s)) = c_{12}(1, s, R(s))$ and $c_2 = c_{21}(1, s, R(s)) = c_{22}(1, s, R(s))$. The function, $f(c_1, c_2)$ is provided in the appendix.

Our notation emphasizes the dependence of ψ_{MD} on z because this dependence plays an important role in our discussion of the next section.

Before proving the proposition, we highlight three features. First, in any interior equilibrium, $\psi_{ID}(R(s)) = \psi_{MD}(R(s), z)$, so that determining an equilibrium reduces to finding values of R where the right side of (23) equals the right side of (24). Second, as we show below, the term, $\psi_{MD}(R, z)$ can be interpreted as arising from the distortions induced by monopoly power and the term, $\psi_{ID}(R)$, can be interpreted as the distortion arising from the inflation tax. This interpretation helps us to understand the costs and benefits that the monetary authority weighs in making its policy decision. Third, notice that the shocks, θ and g, do not enter into the functions, ψ_{ID} or ψ_{MD}. Thus, $R(s)$ does not depend on s.

We prove the proposition by proving a lemma. Consider first the term, ψ_{MD}. To obtain this term, note that the efficient allocations in our economy satisfy

$$u_n + \frac{\theta u_{22}}{(1-\mu)(1-z)} = 0. \tag{25}$$

The first term in (25) is the marginal disutility of labor associated with increasing labor input to credit goods production, say, and the second term is the marginal benefit from increased credit goods consumption. In our economy the analog of (25) is (15). Note that because of the presence of monopoly power, the second term in (15) is the same as the second term in (25) multiplied by $\rho < 1$. As a result, the net marginal benefit of increasing labor from its equilibrium value in our economy is positive. This distortion is due to monopoly power and suggests that the left side of (25) is a natural measure of the monopoly distortion in our economy. Add and subtract $\theta u_{22} n_R / [(1-\mu)(1-z)]$ to and from (22), and rearrange terms, to obtain

$$U_R = \left[u_n + \frac{\theta u_{22}}{(1-\mu)(1-z)}\right] n_R + u_c c_R - \frac{\theta u_{22} n_R}{(1-\mu)(1-z)}. \tag{26}$$

The term in square brackets is our measure of the monopoly distortion. In the Appendix, we prove the following lemma regarding the terms in (26).

Lemma 1: In a Markov equilibrium,

$$\left[u_n + \frac{\theta u_{22}}{(1-\mu)(1-z)}\right] n_R = f(c_1, c_2)\psi_{MD}(R, z), \tag{27}$$

and

$$u_c c_R - \frac{\theta u_{22} n_R}{(1-\mu)(1-z)} = -f(c_1, c_2)\psi_{ID}(R), \tag{28}$$

where $\psi_{MD}(R, z)$ and $\psi_{ID}(R)$ are defined in (23) and (24).

Proposition 1 then follows from Lemma 1.

To see that ψ_{ID} is a measure of the inflation distortion, we use a simple consumer surplus type of analysis. In a monetary economy, let $D(r)$ denote the demand for real balances, m, with respect to the net interest rate, $r \equiv R - 1$. Let $g(m)$ be $D^{-1}(r)$. Consumer surplus, S, is the area under the money demand function. A rise in the interest rate acts like a tax and reduces consumer surplus. We are interested in the marginal effects of this tax, namely, the derivative of S with respect to r :

$$\frac{dS}{dr} = \frac{dS}{dm}\frac{dm}{dr} = g(m)D'(r) = rD'(r).$$

In our economy,

$$D(r) = \frac{c}{1 + (1+r)^{\frac{1}{1-\rho}}},$$

where $c = c_1 + c_2$ denotes aggregate consumption. It follows that

$$D'(r)r = -\frac{1}{1-\rho}\frac{c_2}{R}[R^{\frac{1}{\rho-1}}(R-1)] = -\frac{1}{1-\rho}\frac{c_2}{R}\psi_{ID}(R).$$

As we see below, the key features of $\psi_{ID}(R)$ that deliver multiplicity are shared by $D'(r)r$. This result is one motivation for interpreting $\psi_{ID}(R)$ as the inflation distortion.

For another motivation, consider the following. Use $c_2/c_1 = R^{1/1-\rho}$ and the definition of ψ_{ID} to obtain $\psi_{ID}(R) = (R-1)c_1/c_2$. The net interest rate $R - 1$ measures the extent to which cash goods consumption is distorted relative to the efficient level. This distortion is akin to a tax (see Lucas and Stokey (1983)). The base on which this tax is levied is consumption of cash goods. Thus, one way to think of ψ_{ID} is as the product of a tax rate, $R-1$, and the base of taxation, c_1, scaled by a measure of the size of the economy, c_2. This reasoning provides an alternative motivation for using ψ_{ID} to measure the inflation distortion. In the efficient allocations, $R=1$, and $\psi_{ID}(R)=0$.

We now discuss some properties of ψ_{MD} and ψ_{ID}. From (23) the following is clear:

$$\text{(29)} \qquad \psi_{MD}(R,z) \text{ is decreasing in } z \text{ and } \lim_{R\to\infty}\psi_{MD}(R,z)=\frac{\frac{\mu}{1-\mu}\frac{\psi}{\rho}\left(\frac{1-z}{z}\right)}{\frac{1+\psi}{1-\rho}+\frac{\psi}{\rho}}>0.$$

Notice that $\psi_{MD}(R,z)$ does not depend on the shocks θ and g. Next, inspecting (24), we have that $\psi_{ID}\geq 0$ and

$$\text{(30)} \qquad \lim_{R\to\infty}\psi_{ID}(R)=\psi_{ID}(1)=0.$$

That is, there is no inflation distortion when the interest rate is high or low.

A numerical example helps illustrate the results in Proposition 1. We use $\mu=0.1$, $\rho=0.45$, $\psi=1$, $g=0.05$, $\theta=1$. Figure 1 displays the monopoly distortion, ψ_{MD}, and the inflation distortion, ψ_{ID}, for $R\in[1,4.5]$ and for $z=0.13$ and 0.15. The figure shows that the first order necessary condition for monetary authority optimality is satisfied at $R=1.38$ and $R=2.07$ for $z=0.13$ and $R=1.10$ and $R=3.17$ for $z=0.15$. For $z=0.15$ the inflation rate is somewhat below 10 percent in the low inflation equilibrium and just below 217 percent in the high inflation equilibrium.

Using Proposition 1, (22) becomes

$$\text{(31)} \qquad U_R=f(c_1,c_2)\psi(R,z)\leq 0$$

with equality if $R>1$. Here, $\psi(R,z)=\left[-\psi_{ID}(R)+\psi_{MD}(R,z)\right]$. Since $f(c_1,c_2)>0$, a solution to

$$\text{(32)} \qquad \psi(R,z)\leq 0$$

with equality if $R > 1$ satisfies the necessary condition for a Markov equilibrium. If (**??**) is also sufficient, then the interest rate, R, which solves (32) corresponds to a Markov equilibrium policy rule. Given an equilibrium value of the interest rate, we can solve for the allocations and other prices from (11)-(15), (17) with equality, (20) and (9), for each value of θ and g.

We use the properties of the monopoly distortion function, ψ_{MD}, in (29), and the inflation distortion function, ψ_{ID}, in (30), to show that, generically, there are at least two Markov equilibria, if there are any.

Proposition 2 (Generic Multiplicity): Suppose that the monetary authority's first order condition is sufficient for optimality. Then, generically, there are at least two Markov equilibria, or none. Furthermore, the equilibrium interest rate does not depend on θ or g.

Proof: A key property of the function $\psi(R, z)$ is that it is positive for R sufficiently large. This property follows from (29) and (30) which imply

$$\lim_{R\to\infty} \psi(R, z) = \lim_{R\to\infty} [-\psi_{ID}(R) + \psi_{MD}(R, z)] > 0.$$

Suppose first that $\psi(1, z) > 0$. Then, since $\psi(R, z)$ is positive at $R = 1$ and positive for large R, by continuity it follows that if $\psi(R, z)$ is ever zero, it must generically be zero at least twice. A non generic case occurs when the graph of $\psi(R, z)$ against R is tangent to the horizontal axis at a single value of R. Another nongeneric case is when $\psi(1, z) = 0$ and $\psi(R, z) > 0$ for $R > 1$. Both cases are nongeneric because for an arbitrarily larger value of z, one can see that there are multiple equilibria since $\psi(R)$ is strictly decreasing in z. Suppose next that $\psi(1, z) < 0$. Then, $R = 1$ satisfies (32) and corresponds to a Markov equilibrium. In addition, because $\psi(R, z) > 0$ for R sufficiently large, continuity implies that $\psi(R, z)$ must be equal to zero for at least one value of $R > 1$.

>From (24) we have that ψ_{ID} does not depend on θ or g. Since ψ_{MD} does not depend on these variables either, it follows that the equilibrium interest rate, R, does not depend on θ or g. Q.E.D.

We construct two examples to illustrate Proposition 2 and to compare outcomes between the low and high inflation equilibrium. We also constructed a third example to illustrate that the first order condition of the monetary authority may not be sufficient for optimality. In all three examples we use the values of μ, ρ, ψ, g, and θ used in Figure 1. The first example has $z = 0.13$, the second has $z = 0.15$ and the third has $z = 0.152$. In Table 1, we display the candidate private sector equilibrium outcomes which satisfy the first order condition of the monetary authority.

Table 1: Candidate Equilibrium Outcomes in Three Numerical Examples

Inflation	z	c_1	c_2	R	n	P^e
low	0.13	0.17	0.31	1.38	0.339	49.1
high	0.13	0.08	0.32	2.07	0.337	99.2
low	0.15	0.25	0.30	1.10	0.342	26.3
high	0.15	0.04	0.33	3.17	0.336	165.0
low	0.152	0.26	0.30	1.08	0.343	25.4
high	0.152	0.04	0.33	3.27	0.336	171.6

Note from Table 1 that c_1, P and R are quite different in the high and low inflation outcomes. The primary cost of high inflation is that it results in an inefficient level of cash goods consumption. For example when $z = 0.13$, cash goods consumption is over 50 percent lower in the high inflation equilibrium than in the low inflation equilibrium. Note that credit goods consumption changes very little. Employment changes very little because the bulk of labor is allocated to credit goods production.

We found that the first order condition for monetary authority optimality is sufficient in the examples with $z = 0.13$ and $z = 0.15$. We determined sufficiency by examining by examining the the monetary authority's objective function, (21), at each value of P^e corresponding to a private sector equilibrium. When $z = 0.13$, we found in numerical results not reported here, that this objective is concave for a range of values of R up to roughly 4, for both the low and high inflation values of P^e. When $z = 0.15$ or 0.152 this objective function is also concave for the low inflation value of P^e. We illustrate this concavity by graphing the objective function when $z = 0.15$ for the low inflation value of P^e in Figure 2a. In Figure 2b we plot the corresponding objective function for the high inflation value of P^e. This figure shows that the objective function is locally, though not globally, concave. In addition, the figure shows that the high inflation candidate maximizes the monetary authority's objective, and is therefore an equilibrium.

In our third example, it turns out that the low inflation candidate is indeed an equilibrium, but that the high inflation candidate is not an equilibrium. In Figure 2c we plot the monetary authority's objective at the high inflation value of P^e. This figure shows that, although $R = 3.27$ is a local maximum, the global maximum is $R = 1$.[5] This figure illustrates forcefully that it is necessary to check the monetary authority's objective function globally, rather than just locally. Clearly, merely checking second order conditions is not enough.

III An Economy with Variable Payment Technology

In this section, we develop a version of our model with a variable payment technology. For convenience, we refer to the economy of the previous section as the economy with a fixed payment technology. The variable payment version is interesting because it delivers a related but different channel by which monetary policy can be caught in an expectations trap. It is also interesting as a model of financial intermediation in its own right. Finally, we use this model to analyze how equilibrium interest rates fluctuate in response to shocks.

In this version of the model, the fraction of goods purchased with cash, z, is chosen by households at the beginning of the period, before the monetary authority chooses the interest rate, R. This timing assumption turns out to imply that we can characterize the equilibrium with two relationships. The first relationship is between R and z for the fixed payment technology economy. The second relationship is obtained from the optimality condition associated with z.

Consider a version of the fixed payment technology economy in which each consumption good, $c(\omega)$, can be purchased either with cash or with credit, $\omega \in (0,1)$. For goods with $\omega > \bar{z}$ (where $\bar{z}$ is a parameter) the cost of purchasing with credit is zero. Purchasing goods with $\omega \leq \bar{z}$ on credit requires labor time. The household chooses the fraction $z \leq \bar{z}$ such that goods with $\omega < z$ are purchased with cash and goods with $\omega \geq z$ are purchased with credit. This cash-credit decision is made before households know which goods are produced by sticky or flexible price firms, so that the cash-credit good choice is independent of the type of firm.

The labor time required to purchase fraction z of goods with cash is given by $\eta(\bar{z} - z)^{1+\nu}/(1+\nu)$, where $\nu > 0$ is a parameter and $\eta > 0$ is the shock to the payment technology. Since this shock is realized at the beginning of the period, the exogenous state is now given by $s = (\theta, g, \eta)$. The household's labor time, l, is divided between time spent working in the market, n, and time spent on the payment technology as follows:

$$l = n + \frac{\eta(\bar{z} - z)^{1+\nu}}{1+\nu}. \tag{33}$$

Leisure time in the household's utility function is now given by $1 - l_t$, rather than $1 - n_t$.[6]

The decision problem of the household with respect to consumption, employment and asset accumulation described in the previous two sections is unchanged, except that now z is added to the state variables in (5) and (6), and labor is given in (33). The household chooses z to solve the following problem:

$$z(A, s) = \arg\max \ v(A, z, s, R(s)), \tag{34}$$

where v is the analog of the value function in (5). Note that the choice of z depends on the household's expectations of the monetary authority's policy rule, $R(s)$, since z is chosen before R.

A Markov equilibrium, a private sector equilibrium and associated outcomes are defined in the obvious way (see Albanesi, Chari and Christiano (2002a) for these definitions.) We now characterize a Markov equilibrium for the variable payment technology economy. In addition to all the equilibrium conditions for the economy when z is fixed, this equilibrium must satisfy optimality of the choice of z.

A Characterization of Equilibrium

We analyze a Markov equilibrium for this economy by first establishing a relationship between the Markov equilibrium interest rate and the payment technology parameter, z, in the fixed payment technology economy. In Albanesi, Chari and Christiano (2002a), we show that the monetary authority's optimality condition in the variable payment technology economy can be written as $\tilde{f}(c_1, c_2)\psi(R, z) \leq 0$, where $\psi(R, z)$ is given in (31) and $\tilde{f} > 0$. Thus, the equilibrium interest rates in the variable payment technology economy must satisfy the same conditions as in the fixed payment technology economy.

Consider the equilibrium interest rates in the fixed payment technology economy given by the solution to (32). This solution depends on z, as can be seen from (23) and (24). We call this relationship between R and z the *interest rate policy correspondence* (henceforth, *policy correspondence* for short.) The following proposition establishes properties of this correspondence:

Proposition 3 (Interest Rate Policy Correspondence): Suppose that the monetary authority's first-order condition is sufficient for optimality. Suppose also that for some $z < \bar{z}$ a Markov equilibrium exists. Then, there is a critical value of z, say $\hat{z}$, such that for $z < \hat{z}$ there are no Markov equilibria, for $z = \hat{z}$ there is at least one Markov equilibrium, and for $z > \hat{z}$ there at least two Markov equilibria.

Proof: First, we show that when z is sufficiently small, there is no interest rate less than $\bar{R}$ which is an equilibrium, where $\bar{R}$ is arbitrarily large. Notice from (23) that $\psi_{MD}(R, z) \to \infty$ as $z \to 0$ for all $R \in [1, \bar{R}]$, and from (24) that ψ_{ID} is bounded. It follows that there is some value of z, say $\hat{z}_1$, such that for all $z \leq \hat{z}_1$, $\psi(R, z)$ is strictly positive. Thus, there is no equilibrium interest rate less than $\bar{R}$ for z sufficiently small. Second, we show that no interest rate greater than $\bar{R}$ can be an equilibrium. We see from (24) that ψ_{ID} is bounded above by, say, k. Let $\hat{z}_2$ be defined by $\lim_{R\to\infty} \psi_{MD}(R, \hat{z}_2) = 2k$. Such a value of $\hat{z}_2$ exists from (29). Note also that for all $z \leq \hat{z}_2$, $\lim_{R\to\infty} \psi_{MD}(R, z) \geq 2k$. By definition of a limit, some interest rate $\bar{R}$ exists such that for all $R \geq \bar{R}$, $\psi_{MD}(R, \hat{z}_2) \geq 2k - \varepsilon$, where ε is, say, $k/2$. It follows that, for all $R \geq \bar{R}$, $\psi(R, \hat{z}_1) = -\psi_{ID}(R) + \psi_{MD}(R, \hat{z}_1) \geq k/2 > 0$. That is, there is no value of the interest rate greater than $\bar{R}$ which is an equilibrium for $z = \hat{z}_2$. Since $\psi_{MD}(R, z)$ is decreasing in z, there is no value of the interest rate greater than $\bar{R}$ which is an equilibrium for $z \leq \hat{z}_2$. We have established that there is no equilibrium if z is sufficiently small.

Next, $\psi_{MD}(R, z)$ is a continuous function of R and z. As z is increased from some arbitrarily low value, there is some first value of z such that $\psi(R, z) = 0$ for some R. Such a z, call it $\hat{z}$, exists by our assumption that an equilibrium exists for some z. Consider increasing z above $\hat{z}$. Since ψ_{MD} is strictly decreasing, the graph of $\psi(R, z)$ against R must intersect the horizontal axis at at least two points. Thus, for $z > \hat{z}$, there are at least two Markov equilibria. Q.E.D.

Consistent with our theoretical findings, Figure 1 shows that the inflation distortion does not depend on the payment technology parameter, z, while the monopoly distortion is decreasing in this parameter. We graph the policy correspondence in Figure 3.[7] When z is sufficiently small, the monopoly distortion lies above the inflation distortion and there is no equilibrium. As z increases, the monopoly distortion declines. At a critical value of z the economy has a unique equilibrium and for values of z larger than this critical value the economy has two equilibria. Notice that as z increases, the interest rate in the low inflation equilibrium falls and that the interest rate in the high inflation equilibrium rises.

We now develop the second relationship between the equilibrium interest rate, R, and the payment technology parameter z. We obtain this relationship from the first-order condition associated with the household's choice of z:

$$(1 - \frac{1}{\rho})\frac{1 - R^{\frac{\rho}{1-\rho}}}{z + (1-z)R^{\frac{\rho}{1-\rho}}} = \frac{\psi\eta(\bar{z}-z)^{\nu}}{1 - n - \frac{(\bar{z}-z)^{1+\nu}}{\eta/(1+\nu)}}. \tag{35}$$

We can use the equations that define a private sector equilibrium, (11)-(15), (17) with equality, (20) and (9) to substitute for labor, n, in (35). Doing so, we obtain (see Lemma 2 in the Appendix for a derivation):

$$\frac{(\frac{1}{\rho}-1)(1-R^{\frac{\rho}{\rho-1}})}{z\left[(R^{\frac{1}{\rho-1}}-1)+\frac{\psi}{\rho}(R^{\frac{\rho}{\rho-1}}-1)\right]+(1+\frac{\psi}{\rho})} = \frac{\rho\eta(\bar{z}-z)^{\nu}}{1-\frac{(\bar{z}-z)^{1+\nu}\eta}{1+\nu}-\frac{g}{\theta}}. \tag{36}$$

For each z, there is at most one R that solves (36). To see this result, note that the left-hand side is increasing in R, while the right side does not depend on R. Let $R_p(z, g, \theta, \eta)$ denote the value of R that solves (36). We refer to this function as the *payment technology function,* or *payment function*, for short. The set of payment technology parameters z for which this function is defined is developed as follows. As $R \to \infty$, the left side of (36) converges to $(1-\rho)/((\rho+\psi)(1+z))$, which at $z = 0$ becomes $(1-\rho)/(\rho+\psi)$. The right side of (36) at $z = 0$ is $\rho\eta\bar{z}^{\nu}/(1-\bar{z}^{1+\nu}\eta/(1+\nu)-g/\theta)$. If

$$\frac{1-\rho}{\rho+\psi} < \frac{\rho\eta\bar{z}^{\nu}}{1-\bar{z}^{1+\nu}\eta/(1+\nu)-g/\theta},$$

there is some critical value of z, say z^*, at which the function $R_p(z, g, \theta, \eta)$ goes to infinity. Then the function is defined for $(z^*, \bar{z}]$. If not, then the function is defined for $(0, \bar{z}]$. Let the domain of the function be $(\tilde{z}, \bar{z}]$ where $\tilde{z} = z^*$ if the above inequality holds and $\tilde{z} = 0$ otherwise.

It is easy to see from (36) that R_p is decreasing in z, since the left side of (36) is increasing in z, while the right side is decreasing in z. It is also easy to see that R_p is increasing in g/θ and η since an increase in g/θ or η raises the right side of (36) and so increases R for a given value of z.

Each R, z which satisfies the policy correspondence, (31), and the payment function, (36), corresponds to a Markov equilibrium. The other allocations, prices and the monetary authority's policy rule can be obtained by solving (11)-(16), (17) with equality, (20) and (9). Next, we prove a proposition that under certain conditions, there are two Markov equilibria for our economy. We say that the policy correspondence is *horseshoe-shaped* if it satisfies the following conditions: (i) there are two continuous functions, $R_c^1(z)$ and $R_c^2(z)$ which map $[\hat{z}, \bar{z}]$ into the space of interest rates with $R_c^1(z) < R_c^2(z)$, for $z \in (\hat{z}, \bar{z}]$, $R_c^1(\hat{z}) = R_c^2(\hat{z})$, and (ii) for all $z \in [\hat{z}, \bar{z}]$ the solution to (32) is either $R_c^1(z)$ or $R_c^2(z)$, where $\hat{z}$ is defined in Proposition 2.

Proposition 4: Suppose the policy correspondence is horseshoe-shaped. Then, generically, the economy with variable payment technology satisfies the necessary conditions for a Markov equilibrium twice, if at all.

Proof: Suppose to begin with that $\tilde{z} < \hat{z}$. Recalling that $R_p(\bar{z}) = 1$ and $R_c^1(\bar{z}), R_c^1(\bar{z}) \geq 1$, we can divide the proof into two cases. The first case is when $R_p(\bar{z}) < R_c^1(\bar{z})$. The second case is when $R_p(\bar{z}) = R_c^1(\bar{z}) = 1$. Consider the first case, that is, $R_p(\bar{z}) < R_c^1(\bar{z}) \leq R_c^2(\bar{z})$. Now if $R_p(\hat{z}) > R_c^1(\hat{z}) = R_c^2(\hat{z})$, then since R_p is below R_c^1 and R_c^2 at $\bar{z}$ and above R_c^1 and R_c^2 at $\hat{z}$, by continuity, R_p must intersect at least once with each R_c^1 and R_c^2. Each of these intersections corresponds to a Markov equilibrium. If $R_p(\hat{z}) < R_c^1(\hat{z}) = R_c^2(\hat{z})$ then since R_p is below R_c^1 at both $\bar{z}$ and $\hat{z}$, R_p and R_c^1 intersect twice, if at all. The case when $R_p(\hat{z}) > R_c^1(\hat{z}) = R_c^2(\hat{z})$ is clearly non-generic.

Consider the second case, that is, $R_p(\bar{z}) = R_c^1(\bar{z}) = 1$. Then the policies and allocations associated with an interest rate of unity constitute an equilibrium. Generically, there must also be one other equilibrium. To see this, note that, generically, if $R_c^1(\bar{z}) = 1$, some neighborhood of $\bar{z}$ exists such that for all z in this neighborhood, $R_c^1(z) = 1$. Since R_p is strictly decreasing, it follows that for z in this neighborhood, $R_p(z) > 1 = R_c^1(z)$. Suppose that $R_p(\hat{z}) < R_c^1(\hat{z})$. Then, since R_p is above R_c^1 in a neighborhood of $\bar{z}$ and below R_c^1 at $\hat{z}$, by continuity R_p and R_c^1 must intersect at least once. Now suppose that $R_p(\hat{z}) > R_c^1(\hat{z}) = R_c^2(\hat{z})$. Then, since R_p is below R_c^2 at $\bar{z}$ and above R_c^2 at $\hat{z}$, by continuity R_p must intersect at least once with R_c^2. We have established that in this second case, generically, the necessary conditions for equilibrium must be satisfied twice, if at all.

Suppose next that $\tilde{z} > \hat{z}$. Then for z near $\tilde{z}$, R_p is arbitrarily large and must be larger than R_c^2. Exactly the same arguments used above can then be used to conclude that the necessary conditions for a Markov equilibrium must be satisfied twice, if at all. Q.E.D.

The restriction that the policy correspondence be horseshoe-shaped is not severe. In Proposition 2 we have shown that for each $z > \hat{z}$ there are at least two interest rates which belong to the policy correspondence. Using the implicit function theorem, these interest rates can be represented as continuous functions of z. Thus, the assumption that the correspondence is horseshoe-shaped only rules out the possibility that there are three or more interest rates which belong to the correspondence. It is straightforward, but tedious to extend the proof of Proposition 3 to this case. Furthermore, in all the numerical examples we have considered, the correspondence is horseshoe-shaped.

B Properties of Equilibrium

In Figure 4, we plot the interest rate correspondence and the payment function for various realizations of the exogenous shocks in a numerical example. In Figure 4a we plot the interest rate correspondence and the payment function for two realizations of the production technology shock, θ, holding the other shock at its mean value. Figure 4b displays the analogous graph for the payment technology shock, η. These figures display four properties. First, as we have shown in Proposition 1, the policy correspondence does not depend on these shocks. Second, as discussed above, the payment function is decreasing in the interest rate. Third, as also discussed above, the payment function is increasing in η and decreasing in θ. Fourth, there are multiple Markov equilibria. Two of these are easy to see. In one, for every realization of the shocks, the equilibrium is the one associated with the lower intersection of the interest rate correspondence and payment function. We call this the *low inflation equilibrium.* In the other, the equilibrium is the one associated with the higher intersection. We call this the *high inflation equilibrium.*

Figure 4 displays an interesting *sign switch* phenomenon, in the sense that the interest rate response to a shock switches sign between the high and low inflation equilibrium. For example, from Figure 4a, we see that the interest rate is increasing in the technology shock in the low inflation equilibrium and decreasing in this shock in the high inflation equilibrium. We verified, for our numerical example, that in both equilibria output is increasing in the technology shock. If technology shocks were the dominant shocks, the correlation between output and the interest rate would be positive in the low inflation equilibrium and negative in the high inflation equilibrium. From Figure 4b we see the sign switch for the payment shock: the interest rate is decreasing in this shock in the low inflation equilibrium and increasing in this shock in the high inflation equilibrium. In our numerical example, output is increasing in the payment shock in the low inflation equilibrium and decreasing in this shock in the high inflation equilibrium. So, if payment shocks were the dominant shocks the correlation would be negative in both equilibria. It follows that in an economy with both shocks, the correlation of output and the interest rate is negative in the high inflation equilibrium and larger (perhaps even positive) in the low inflation equilibrium. We call this finding the *decreasing correlation implication.*

Our numerical examples also show that the volatility of interest rates in the low inflation equilibrium is substantially smaller. The reason is that the policy correspondence is flatter at the low inflation equilibrium than at the high inflation equilibrium. We call this finding the *increasing volatility implication.*

In Albanesi, Chari and Christiano (2002a), we examine data for high and low inflation episodes in a cross section of countries. We find that some support for the decreasing correlation and increasing volatility implications of the model.

IV Key Features for Generating Expectation Traps

In this section, we ask which features are crucial for generating expectation traps. We focus on six features and find that three of them play essential roles, one plays a convenient role and two play more subsidiary roles. We also briefly discuss extensions of the analysis.

The first essential feature is the assumption that some prices are preset. To see the importance of this assumption, suppose all prices were flexible. Then, the monetary authority cannot reduce the monopoly distortion by making inflation higher than expected because monopolists simply raise their prices in response to expansionary monetary policy, so that the monopoly wedge is invariant to monetary policy. In our model, the only possible benefit of expansionary policy is from reducing the monopoly distortion. At the same time an expansionary policy is costly because it raises the price level, reduces consumption of cash goods and thereby reduces welfare. Indeed, these forces imply that the monetary authority gains by pursuing a contractionary policy, as long as $R > 1$. Thus, when all prices are flexible the unique Markov equilibrium has $R = 1$. Technically, this result can be seen by setting $\mu = 0$ in (23). After some manipulation, we see that $\psi_{MD}(R, z) < 0$ for all R, so that the equilibrium has $R = 1$.

The second essential feature is the assumption that some prices are flexible. To see the importance of this assumption, suppose all prices are fixed. Then, expansionary monetary policy is welfare-enhancing because it reduces the monopoly distortion. Such a policy is not costly because with the price level fixed, cash goods consumption is also fixed. These forces imply that the monetary authority always gains by pursuing an expansionary monetary policy. As a result, no equilibrium exists. Technically, this result can be see from (23), which implies that $\psi_{MD} \to \infty$ as $\mu \to 1$. Since ψ_{ID} is bounded, no equilibrium exists.

The third essential feature is that firms have monopoly power. Again, since the only benefit of expansionary monetary policy is to reduce the monopoly distortion, and since realized inflation is costly, the equilibrium without monopoly power has $R = 1$. Technically, suppose $\rho = 1$ in (23). Then, we see that $\psi_{MD}(R, z) = 0$ for all R. And, the unique equilibrium has $R = 1$.

The fourth feature is our timing assumption under which monetary injections cannot be used to purchase cash goods in the same period. This assumption implies that a monetary expansion, by raising prices, directly reduces consumption of cash goods. This reduction in the consumption of cash goods lowers welfare. An alternative timing assumption is that in Lucas and Stokey (1983). Under this alternative timing, households can use the current monetary injection for current cash goods purchases. Mechanically, Lucas-Stokey timing amounts to adding current money growth to the right side of the cash-in-advance constraint. Since a monetary injection can be used to purchase current cash goods, a greater than expected expansion does not directly change the mix of cash and credit goods consumption. It is possible that induced movements in the interest rate could change this mix, and this possibility would be worth exploring.

The two subsidiary features relate to the shape of the monopoly distortion function and the inflation distortion function. In Albanesi, Chari and Christiano (2002b), we show that if the period utility function is of the following form

$$u(c, n) = \frac{c^{1-\sigma}}{1-\sigma} - an,$$

where a is a parameter, the monetary authority's first order condition can be decomposed into a monopoly distortion and inflation distortion function. In this case, the monopoly distortion at $R = 1$ is negative and the monopoly distortion is positive for R sufficiently large. The inflation distortion function is the same as in this paper. As a result, the fixed payment technology economy has a unique equilibrium. The policy correspondence in Figure 3 becomes a downward-sloping graph. Nevertheless, since the payment function is also downward sloping, there can be multiple intersections and multiple equilibria.

Substituting $c_2/c_1 = R^{1-\rho}$ into (24), we see that

$$\psi_{ID} = (R-1)\frac{c_1}{c_2}.$$

We have already argued that this distortion is akin to the product of a tax, $R - 1$, and the tax base, c_1. When $R = 1$, $\psi_{ID} = 0$. As $R \to \infty$, the behavior of ψ_{ID} depends on the rate at which cash goods consumption falls. In the economy in this paper, c_1 goes to zero faster than R goes to infinity, and thus the product goes to zero. In Albanesi, Chari and Christiano (2002b), we present a model which shares many of the features of the model in this paper, except for the specification of money demand. Interestingly, in that paper the monetary authority's first order condition can be decomposed into terms which are very similar to ψ_{ID} and ψ_{MD}. In that paper, however, ψ_{ID} does not go to zero because c_1 goes to zero at the same rate as R. The fixed payment technology model in that paper has a unique equilibrium. With a variable payment technology, however, multiple equilibria are possible.

In terms of extensions, it would be useful to ask whether these equilibria are stable under various learning schemes. In our numerical examples, including the one associated with Figure 1, the inflation distortion has a single-peaked Laffer curve shape, while the monopoly distortion is relatively flat. This shape is reminiscent of the shape of the monetary Laffer curve in analyses where governments rely on inflation to finance expenditures. (See, for example, Sargent and Wallace (1981).) In this literature there are two steady state levels of inflation. The literature finds that only one of these steady states is stable under a large class of learning schemes. In Albanesi, Chari and Christiano (2002a), we examine the stability properties of the equilibria in our model under a simple learning scheme. We find that both equilibria are stable. Exploring stability under a broader class of learning schemes would be of interest.

It would also be useful to analyze non-stationary equilibria in our model. In this paper, we have focused on Markov equilibria which are stationary in the sense that they cannot depend on time. If we add calendar time as a state variable there are other Markov equilibria as well. For example, one such equilibrium has the economy moving to the low inflation equilibrium on even dates and to the high inflation equilibrium on odd dates. More interesting is the possibility of sunspot driven Markov equilibria in which a sunspot at the beginning of each period coordinates private agents' expectations and induces agents to pick the high or the low inflation equilibrium depending on the realization of the sunspot. Such sunspot equilibria clearly exist and lead to volatility in inflation rates.

V Related Literature

In terms of related literature, Dedola (2002) and Khan, King and Wolman (2002) (KKW) also generate multiple equilibria in a models in which the monetary authority chooses policy without commitment. The mechanism for generating multiplicity in Dedola (2002) is similar to that in this paper. KKW have a finite horizon model in which in every period one-third of firms choose the prices for that they will charge for the next three periods. When firms expect high inflation, they choose high prices. The cost of not validating firms' expectations is that relative prices become distorted and output falls. The staggered setting in KKW plays the same analytic role as the Svensson timing assumption in our paper. Both features have the effect that realized inflation is costly. In some of the literature, firms are allowed effectively to choose different prices for each date (though not allowed to make these prices contingent on shocks). We conjecture that with such a formulation, the equilibrium in KKW would be unique. KKW also simply impose money demand by adding an equation that consumption must equal real balances. to the equilibrium of their model. This additional equation is not the same as a cash-in-advance constraint on households because firms and other households will not accept money for the goods they set in the last period. It would be interesting to ask whether in an infinite horizon version of KKW whether the interest elasticity of money demand would matter for multiplicity.

It is increasingly standard in monetary economics to characterize equilibria without commitment in stochastic economies by studying linear-quadratic approximations around a steady state (see, for example, Clarida, Gali and Gertler, 1999). This literature simply assumes the steady state values of policy variables like inflation. The difficulty is that in determining steady state policy, the policymaker needs to forecast how private agents will respond to alternative policies. That is, an analysis like the one conducted in this paper is necessary to determine steady states before one knows around what point to conduct the approximation. If the linear-quadratic method yields deviations from the state which are independent of the value of the steady state, the method may be a good approximation of equilibria that remain close to steady state. In economies with multiple steady states, like ours, however, the method would entirely miss any equilibria in which the economy switches from one steady state to another.

VI Conclusion

We have shown that discretionary monetary policy can account for prolonged periods of low and high inflation. The model in this paper is a very standard monetary general equilibrium model. Our main theoretical finding is that the model has expectation traps. The main force driving the multiplicity of equilibria is that defensive actions taken by the public to protect itself from high inflation reduce the costs of inflation for a benevolent monetary authority and induce the authority to supply high inflation. This economic force is likely to be present in a large class of monetary models. The main policy implication is that the costs of discretionary monetary policy include not just high average inflation, but volatile and persistent inflation as well. The gains to setting up institutions which increase commitment to future monetary policies are likely to be high.

Notes

[1]We thank Michael Woodford for urging us to emphasize these points.

[2]Notice that we do not include the beginning-of-period aggregate stock of money in our states. In our economy, all equilibria are neutral in the usual sense that if the initial money stock is doubled, an equilibrium exists in which real allocations and the interest rate are unaffected and all nominal variables are doubled. This consideration leads us to focus on equilibria which are invariant with respect to the initial money stock. We are certainly mindful of the possibility of equilibria which depend on the money stock. For example, if multiple equilibria in our sense exist, 'trigger strategy-type' equilibria which are functions of the initial money stock can be constructed. In our analysis we exclude such equilibria.

[3]In Albanesi, Chari and Christiano (2002a), we show that this specification of the monetary authority's choice variable is equivalent to one in which the monetary authority chooses the money growth rate.

[4]Technically, the set of interest rates should also be limited to those where (11)-(15) and (17)-(20) have a solution. Our analysis of the monetary authority's problem uses a first order condition approach which only asks whether small deviations are optimal. One can use the implicit function theorem to show that in some neighborhood of an equilibrium, (11)-(15) and (17)-(20) have a solution. Thus, we will not have to deal with whether the allocation functions are well defined for arbitrary interest rates.

[5]Of course, $R = 1$ is not a Markov equilibrium, because $P^e = 171.6$ and $R = 1$ is not part of a private sector equilibrium outcome.

[6]See Aiyagari, Braun and Eckstein (1998), Cole and Stockman (1992), Dotsey and Ireland (1994), Freeman and Kydland (1994), Ireland (1994), Lacker and Schreft (1996), and Schreft (1992) for payment technology models with similar features.

[7]In all the numerical examples we have studied, the necessary conditions also turned out to be sufficient.

Appendix

To prove Lemma 1 in the text, we use the necessary and sufficient conditions for an interior private sector equilibrium. Using our functional form assumptions, (11)-(15) reduce to

$$c_{12} = c_{11} q^{\frac{-1}{1-\rho}} \tag{37}$$

$$c_{21} = c_{11} R^{\frac{1}{1-\rho}} \tag{38}$$

$$c_{22} = c_{21} q^{\frac{-1}{1-\rho}} \tag{39}$$

$$\frac{\psi}{\rho} c^{\rho} c_{22}^{1-\rho} = \theta(1-n). \tag{40}$$

We have omitted (12) because there are only three linearly independent equations in (11)-(14). These expressions together with (19)-(9) are necessary and sufficient conditions for a private sector equilibrium.

Lemma 1 is established using (37)-(40), (19) with equality, and (20) to construct the functions $c_{ij}(s, P^e, R)$, $q(s, P^e, R)$ and $n(s, P^e, R)$, differentiating these functions with respect to R and evaluating the derivatives at $q = 1$. Mechanically, we first drop n from the system by substituting out for n in (40) using (20). Then, we differentiate (37)-(39) and simplify to obtain one equation in $c_{11,R}$ and q_R. We use (17) to obtain another equation in these variables. We can then evaluate all the other derivatives. We prove the lemma in two parts

Lemma 1a: In a Markov equilibrium ,

$$\frac{(1-\rho)\theta u_{22} n_R}{(1-\mu)(1-z)} = f(c_1, c_2) \psi_{MD}(R, z), \tag{41}$$

where $f(c_1, c_2) > 0$ for $c_1, c_2 > 0$,and $\psi_{MD}(R, z)$ is given in (23).

Proof: Substitute for n from (20) and for c from (1) into (40), to obtain

$$\begin{aligned} & \frac{\psi}{\rho}\left[z\mu c_{11}^{\rho} + z(1-\mu)c_{12}^{\rho} + (1-z)\mu c_{21}^{\rho} + (1-z)(1-\mu)c_{22}^{\rho}\right] c_{22}^{1-\rho} \\ = \; & \theta - g - z\left[\mu c_{11} + (1-\mu)c_{12}\right] + (1-z)\left[\mu c_{21} + (1-\mu)c_{22}\right] - \theta \frac{\eta(\bar{z}-z)^{1+\nu}}{1+\nu}. \end{aligned}$$

Differentiating with respect to R we get

$$
\begin{aligned}
(42)\ & z\left[\mu c_{11,R}+(1-\mu)c_{12,R}\right]+(1-z)\left[\mu c_{21,R}+(1-\mu)c_{22,R}\right] \\
& +\psi\left[z\mu c_1^{\rho-1}c_{11,R}+z(1-\mu)c_1^{\rho-1}c_{12,R}+(1-z)\mu c_2^{\rho-1}c_{21,R}+(1-z)(1-\mu)c_2^{\rho-1}c_{22,R}\right]c_2^{1-\rho} \\
& +\frac{\psi}{\rho}(1-\rho)c^{\rho}c_2^{-\rho}c_{22,R}=0,
\end{aligned}
$$

where all derivatives are evaluated at a value of P^e such that $q=1$. Here, $c_1=c_{11}=c_{12}$ and $c_2=c_{21}=c_{22}$ when $q=1$. Now, differentiate (37)-(39) with respect to R to obtain

$$
\begin{aligned}
c_{12,R} &= c_{11,R}-\frac{c_1}{1-\rho}q_R && (43)\\
c_{21,R} &= c_{11,R}R^{\frac{1}{1-\rho}}+\frac{c_1R^{\frac{\rho}{1-\rho}}}{1-\rho} && (44)\\
c_{22,R} &= c_{21,R}-\frac{c_2}{1-\rho}q_R. && (45)
\end{aligned}
$$

Differentiating (17) with equality and substituting for $c_{12,R}$ from (43), we obtain

$$
\mu zc_{11,R}+(1-\mu)z\left(c_{11,R}-\frac{c_1}{1-\rho}q_R\right)+(1-\mu)zc_1q_R=0.
$$

Simplifying, we obtain

$$
q_R=\frac{1-\rho}{\rho(1-\mu)c_1}c_{11,R}. \tag{46}
$$

>From (43)-(45) and (46), using $(c_2/c_1)^{1-\rho}=R$, we obtain

$$
\mu c_{11,R}+(1-\mu)c_{12,R}=c_{11,R}-\frac{(1-\mu)c_1}{1-\rho}q_R=c_{11,R}(1-1/\rho), \tag{47}
$$

$$
\begin{aligned}
\mu c_{21,R}+(1-\mu)c_{22,R} &= c_{21,R}-\frac{(1-\mu)c_2}{1-\rho}q_R && (48)\\
&= c_{11,R}\left(1-\frac{R^{\frac{1}{1-\rho}}}{\rho(1-\mu)}\right)+\frac{c_1R^{\frac{\rho}{1-\rho}}}{1-\rho} && (49)
\end{aligned}
$$

and

$$
c_{22,R}=c_{11,R}(1-1/\rho)R^{\frac{1}{1-\rho}}+\frac{c_1R^{\frac{\rho}{1-\rho}}}{1-\rho}. \tag{50}
$$

Substitute from (46)-(50) into (42) to obtain

$$
\begin{aligned}
& zc_{11,R}(1-1/\rho)+(1-z)\left[c_{11,R}(1-1/\rho)R^{\frac{1}{1-\rho}}+\frac{c_1R^{\frac{\rho}{1-\rho}}}{1-\rho}\right] \\
& +\psi z c_1^{\rho-1}c_2^{1-\rho}c_{11,R}(1-1/\rho)+\psi(1-z)\left[c_{11,R}(1-1/\rho)R^{\frac{1}{1-\rho}}+\frac{c_1R^{\frac{\rho}{1-\rho}}}{1-\rho}\right] \\
& +\frac{\psi}{\rho}(1-\rho)c^{\rho}c_2^{-\rho}(c_{11,R}(1-\frac{R^{\frac{1}{1-\rho}}}{\rho(1-\mu)})+\frac{c_1R^{\frac{\rho}{1-\rho}}}{1-\rho})=0.
\end{aligned}
$$

Grouping terms, we obtain

$$
\begin{aligned}
& \frac{c_{11,R}}{c_1}\left[z+(1-z)R^{\frac{1}{1-\rho}}+\psi zR+\psi(1-z)R^{\frac{1}{1-\rho}}+\psi\left(\frac{c}{c_2}\right)^{\rho}R^{\frac{1}{1-\rho}}\left(1-\frac{1}{\rho(1-\mu)}\right)\right] \\
= & -\frac{\rho}{\rho-1}\left[(1+\psi)\frac{1-z}{1-\rho}+\frac{\psi}{\rho}\left(\frac{c}{c_2}\right)^{\rho}\right]R^{\frac{\rho}{1-\rho}}.
\end{aligned}
$$

Finally, we obtain the following expression:

(51)

$$
\frac{c_{11,R}}{c_1}=\frac{\frac{\rho}{1-\rho}\left[(1+\psi)\frac{1-z}{1-\rho}+\frac{\psi}{\rho}\left(\frac{c}{c_2}\right)^{\rho}\right]R^{\frac{\rho}{1-\rho}}}{\left(z+(1-z)R^{\frac{1}{1-\rho}}+\psi zR+\psi(1-z)R^{\frac{1}{1-\rho}}\right)+\frac{\psi}{\rho}(1-\rho)\left(\frac{c}{c_2}\right)^{\rho}R^{\frac{1}{1-\rho}}\left(\frac{1}{\rho(1-\mu)}-1\right)}
$$

We use these derivatives to obtain c_R and n_R. Differentiating (1) with respect to R, we obtain

(52)

$$
c_R=c^{1-\rho}\left[z\mu c_1^{\rho-1}c_{11,R}+z(1-\mu)c_1^{\rho-1}c_{12,R}+(1-z)\mu c_2^{\rho-1}c_{21,R}+(1-z)(1-\mu)c_2^{\rho-1}c_{22,R}\right]
$$

Substituting from (47) and (48), we obtain

$$
\frac{c_R}{c^{1-\rho}}=c_1^{\rho-1}zc_{11,R}(1-1/\rho)+(1-z)c_2^{\rho-1}(c_{11,R}(1-1/\rho)R^{\frac{1}{1-\rho}}+\frac{c_1R^{\frac{\rho}{1-\rho}}}{1-\rho}).
$$

Collecting terms:

$$
c_R=c^{1-\rho}c_2^{\rho-1}c_1\left[\frac{c_{11,R}}{c_1}\left(zR+(1-z)R^{\frac{1}{1-\rho}}\right)\left(1-\frac{1}{\rho}\right)+\frac{1-z}{1-\rho}R^{\frac{\rho}{1-\rho}}\right].
$$

Differentiating the resource constraint we obtain n_R :

$$
\theta n_R=z\left[\mu c_{11,R}+(1-\mu)c_{12,R}\right]+(1-z)\left[\mu c_{21,R}+(1-\mu)c_{22,R}\right].
$$

or, after substituting from (47) and (48) and collecting terms:

$$\theta n_R = c_{11,R}(1-\frac{1}{\rho})\left(z+(1-z)R^{\frac{1}{1-\rho}}\right)+(1-z)\frac{c_1}{1-\rho}R^{\frac{\rho}{1-\rho}}. \tag{53}$$

>From (53), using $(c_2/c_1)^{1-\rho}=R$, we obtain

$$\begin{aligned}\theta n_R &= \frac{(1-\frac{1}{\rho})}{1-\rho}c_{11,R}z\left[(1-\rho)\left(1+(\frac{1-z}{z})R^{\frac{1}{1-\rho}}\right)+\frac{(1-z)/z}{(1-\frac{1}{\rho})}\frac{c_1}{c_{11,R}}R^{\frac{\rho}{1-\rho}}\right]\\ &= \frac{c_2}{c_1}\frac{(1-\frac{1}{\rho})}{1-\rho}c_{11,R}z\left[(1-\rho)\left(R^{\frac{1}{\rho-1}}+(\frac{1-z}{z})\right)+\frac{(1-z)/z}{(1-\frac{1}{\rho})}\frac{c_1}{c_{11,R}}R^{-1}\right].\end{aligned}$$

Substituting in (27) and using the result that for our functional forms $u_{22}/(1-\mu)(1-z)=u_c\left(\frac{c}{c_2}\right)^{1-\rho}$, we obtain

$$\begin{aligned}\frac{(1-\rho)\theta u_{22}n_R}{(1-\mu)(1-z)} &= f(c_1,c_2)\left[-(1-\rho)R^{\frac{1}{\rho-1}}-(\frac{1-z}{z})\{(1-\rho)-\frac{\rho}{(1-\rho)}\frac{c_1}{c_{11,R}}R^{-1}\}\right]\\ &= f(c_1,c_2)\psi_{MD}(R,z),\end{aligned}$$

where

$$f(c_1,c_2)=u_c c_2\left(\frac{c}{c_2}\right)^{1-\rho}(\frac{1}{\rho}-1)\frac{c_{11,R}}{c_1}z$$

and

$$\psi_{MD}(R,z)=-(1-\rho)R^{\frac{1}{\rho-1}}+(\frac{1-z}{z})\{\frac{\rho}{(1-\rho)}\frac{c_1}{c_{11,R}}R^{-1}-(1-\rho)\}. \tag{54}$$

Consider the term in parenthesis in (54). When we use (51), this term is

$$\begin{aligned}&\frac{z+(1-z)R^{\frac{1}{1-\rho}}+\psi zR+\psi(1-z)R^{\frac{1}{1-\rho}}+\psi\left(\frac{c}{c_2}\right)^{\rho}R^{\frac{1}{1-\rho}}\left(\frac{1}{\rho(1-\mu)}-1\right)}{\left[(1+\psi)\frac{1-z}{1-\rho}+\frac{\psi}{\rho}\left(\frac{c}{c_2}\right)^{\rho}\right]R^{\frac{\rho}{1-\rho}}}-(1-\rho)\\ =\;&\frac{z+\psi zR+\psi\left(\frac{c}{c_2}\right)^{\rho}R^{\frac{1}{1-\rho}}\left(\frac{1}{\rho(1-\mu)}-1\right)-(1-\rho)\frac{\psi}{\rho}\left(\frac{c}{c_2}\right)^{\rho}R^{\frac{1}{1-\rho}}}{\left[(1+\psi)\frac{1-z}{1-\rho}+\frac{\psi}{\rho}\left(\frac{c}{c_2}\right)^{\rho}\right]R^{\frac{1}{1-\rho}}}\\ =\;&\frac{z(1+\psi R)+\frac{\mu}{1-\mu}\frac{\psi}{\rho}\left(\frac{c}{c_2}\right)^{\rho}R^{\frac{1}{1-\rho}}}{\left[(1+\psi)\frac{1-z}{1-\rho}+\frac{\psi}{\rho}\left(\frac{c}{c_2}\right)^{\rho}\right]R^{\frac{1}{1-\rho}}}.\end{aligned}$$

Substituting for c/c_2 in this expression and then substituting in (54), we obtain:

$$\psi_{MD}(R,z)=-(1-\rho)R^{\frac{1}{\rho-1}}+\left\{\frac{(1-z)\left(R^{\frac{1}{\rho-1}}+\psi R^{\frac{\rho}{\rho-1}}\right)+(\frac{1-z}{z})\frac{\mu}{1-\mu}\frac{\psi}{\rho}\left[zR^{\frac{\rho}{\rho-1}}+1-z\right]}{(1+\psi)\frac{1-z}{1-\rho}+\frac{\psi}{\rho}\left(zR^{\frac{\rho}{\rho-1}}+1-z\right)}\right\}.$$

Dividing the numerator and denominator of the term in braces by $1-z$ and rearranging, we obtain:

$$\psi_{MD}(R,z) = -(1-\rho)R^{\frac{1}{\rho-1}} + \frac{\left(R^{\frac{1}{\rho-1}} + \psi R^{\frac{\rho}{\rho-1}}\right) + \frac{\mu}{1-\mu}\frac{\psi}{\rho}\left(R^{\frac{\rho}{\rho-1}} + \frac{1-z}{z}\right)}{\frac{1+\psi}{1-\rho} + \frac{\psi}{\rho}\left(\frac{z}{1-z}R^{\frac{\rho}{\rho-1}} + 1\right)}.$$

We have proved the first part of Lemma 1. Q.E.D.

Lemma 1b: In a Markov equilibrium, (28) holds, that is,

$$u_c c_R - \frac{\theta u_{22} n_R}{(1-\mu)(1-z)} = -f(c_1,c_2)\,(R-1)\,R^{\frac{1}{\rho-1}}. \tag{55}$$

Proof: Using our functional forms, we obtain

$$u_c c_R - \frac{\theta u_{22} n_R}{(1-\mu)(1-z)} = u_c\left[c_R - \theta\left(\frac{c}{c_2}\right)^{1-\rho} n_R\right]. \tag{56}$$

Substituting for θn_R from (53) and c_R from (52) into (56), we obtain

$$\begin{aligned}
u_c\left[c_R - \theta\left(\frac{c}{c_2}\right)^{1-\rho} n_R\right] &= u_c[\frac{c_{11,R}}{c_1}\left(zR + (1-z)R^{\frac{1}{1-\rho}}\right)\left(1-\frac{1}{\rho}\right) + \frac{1-z}{1-\rho}R^{\frac{\rho}{1-\rho}} \\
&\quad -\frac{c_{11,R}}{c_1}(1-\frac{1}{\rho})\left(z + (1-z)R^{\frac{1}{1-\rho}}\right) - \frac{1-z}{1-\rho}R^{\frac{\rho}{1-\rho}}]c_1\left(\frac{c}{c_2}\right)^{1-\rho} \\
&= u_c\frac{c_{11,R}}{c_1}c_2 z(1-\frac{1}{\rho})\left(\frac{c}{c_2}\right)^{1-\rho}(R-1)\frac{c_1}{c_2} \\
&= -f(c_1,c_2)(R-1)R^{\frac{1}{\rho-1}}
\end{aligned}$$

where

$$f(c_1,c_2) = u_c\frac{c_{11,R}}{c_1}c_2 z(\frac{1}{\rho}-1)\left(\frac{c}{c_2}\right)^{1-\rho}.$$

We have proved the lemma. Q.E.D.

Lemma 2: Equation (35) reduces, in a private sector equilibrium, to (36):

$$\frac{\left(\frac{1}{\rho}-1\right)\left(1-R^{\frac{\rho}{\rho-1}}\right)}{z\left[\left(R^{\frac{1}{\rho-1}}-1\right) + \frac{\psi}{\rho}\left(R^{\frac{\rho}{\rho-1}}-1\right)\right] + (1+\frac{\psi}{\rho})} = \frac{\rho\eta(\bar{z}-z)^{\nu}}{\left(1-\frac{(\bar{z}-z)^{1+\nu}\eta}{1+\nu}\right) - \frac{g}{\theta}}$$

Proof: The necessary and sufficient conditions for a private sector equilibrium are (37)-(39) and the following slightly modified version of (40)

$$\frac{\psi}{\rho} c^{\rho} c_{22}^{1-\rho} = \theta(1 - n - \frac{\eta(\bar{z} - z)^{1+\nu}}{1+\nu}) \tag{57}$$

Using (57) in (35), we obtain:

$$\left(1 - \frac{1}{\rho}\right) \frac{1 - R^{\frac{\rho}{\rho-1}}}{z + (1-z)R^{\frac{\rho}{\rho-1}}} = \frac{\theta\rho\eta(\bar{z} - z)^{\nu}}{(c/c_2)^{\rho}\, c_2}. \tag{58}$$

We use the resource constraint, (20), and (57) to obtain an expression for c_2 in terms of c_1/c_2 and z. Rearranging (57) we obtain:

$$\theta n = \theta \left(1 - \frac{(\bar{z} - z)^{1+\nu}\, \eta}{1+\nu}\right) - \frac{\psi}{\rho} \left(\frac{c}{c_2}\right)^{\rho} c_2.$$

Substituting this equation into the resource constraint, taking into account $c^{\rho} = z c_1^{\rho} + (1 - z) c_2^{\rho}$, and rearranging, we obtain:

$$c_2 = \frac{\theta\left(1 - \frac{(\bar{z}-z)^{1+\nu}\eta}{1+\nu}\right) - g}{z\frac{c_1}{c_2} + \frac{\psi z}{\rho}\left(\frac{c_1}{c_2}\right)^{\rho} + (1-z)(1+\frac{\psi}{\rho})}.$$

Substituting for c_2 in (58), we obtain:

$$(1 - \frac{1}{\rho})\frac{1 - R^{\frac{\rho}{\rho-1}}}{z + (1-z)R^{\frac{\rho}{\rho-1}}} = \frac{\theta\rho\eta(\bar{z} - z)^{\nu}}{z\left(\frac{c_1}{c_2}\right)^{\rho} + 1 - z} \times \frac{z\frac{c_1}{c_2} + \frac{\psi z}{\rho}\left(\frac{c_1}{c_2}\right)^{\rho} + (1-z)(1+\frac{\psi}{\rho})}{\theta\left(1 - \frac{(\bar{z}-z)^{1+\nu}\eta}{1+\nu}\right) - g}.$$

After rearranging and making use of $R = (c_1/c_2)^{\rho-1}$, we obtain (36). Q.E.D.

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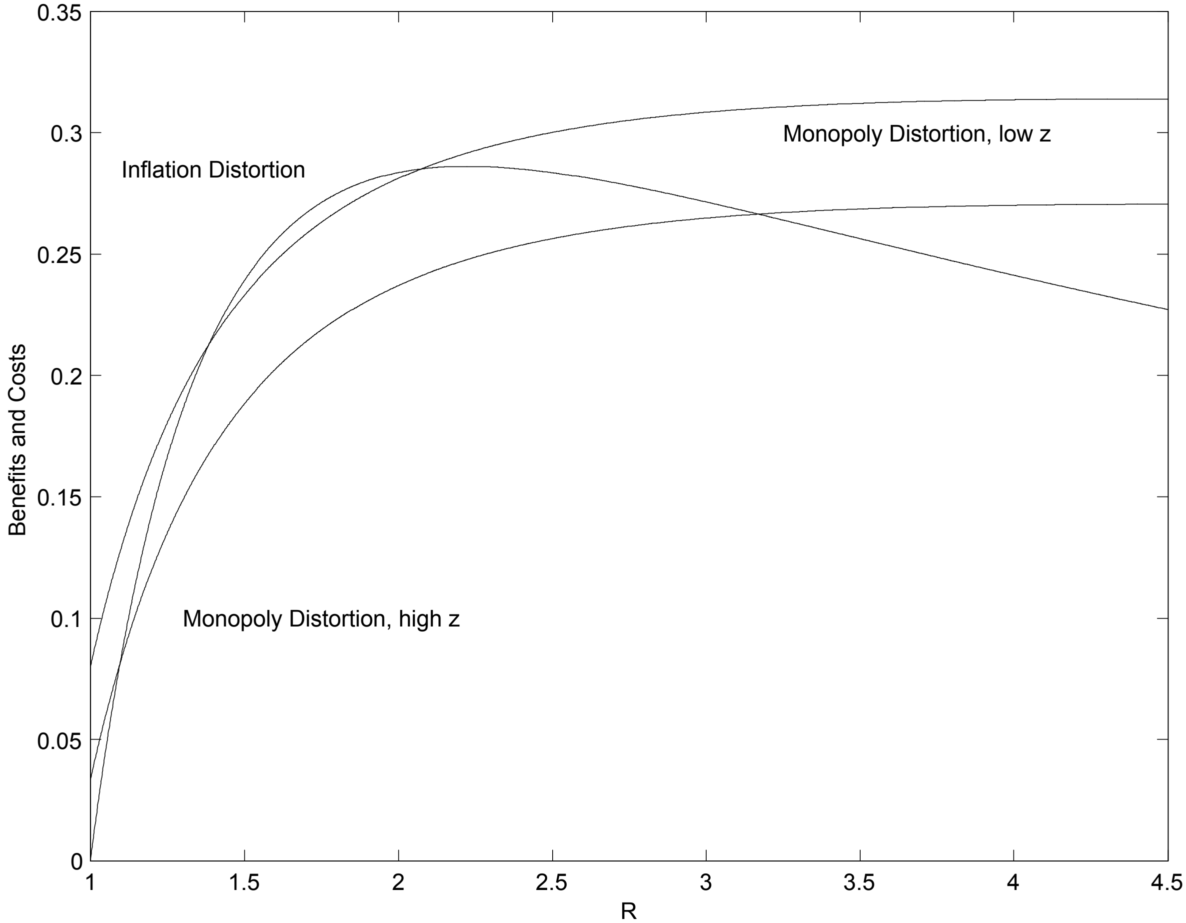
Figure 1: Marginal Benefits and Marginal Costs for Monetary Authority



z = 0.15

Figure 2a: Utility For Deviations From Low Inflation Equilibrium

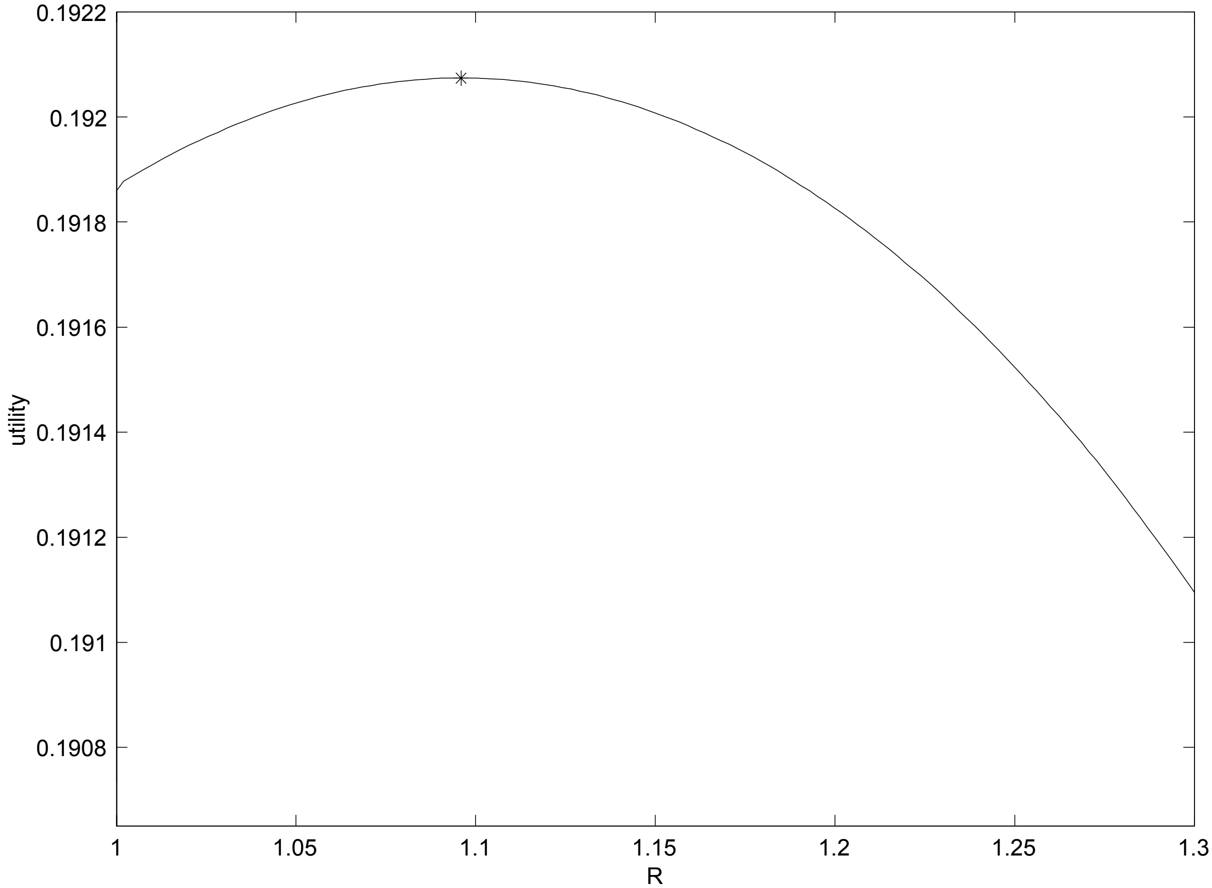


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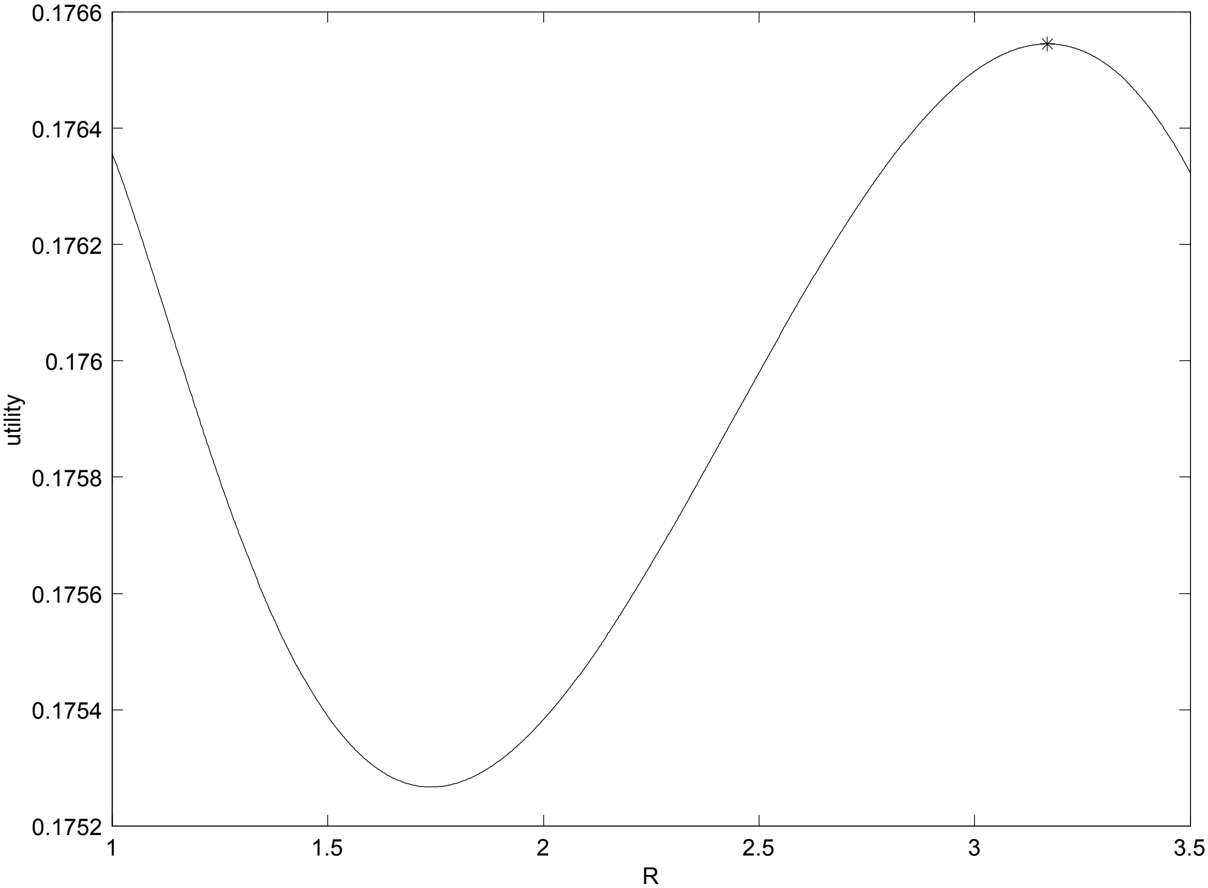
Figure 2b: Utility For Deviations From High Inflation Equilibrium



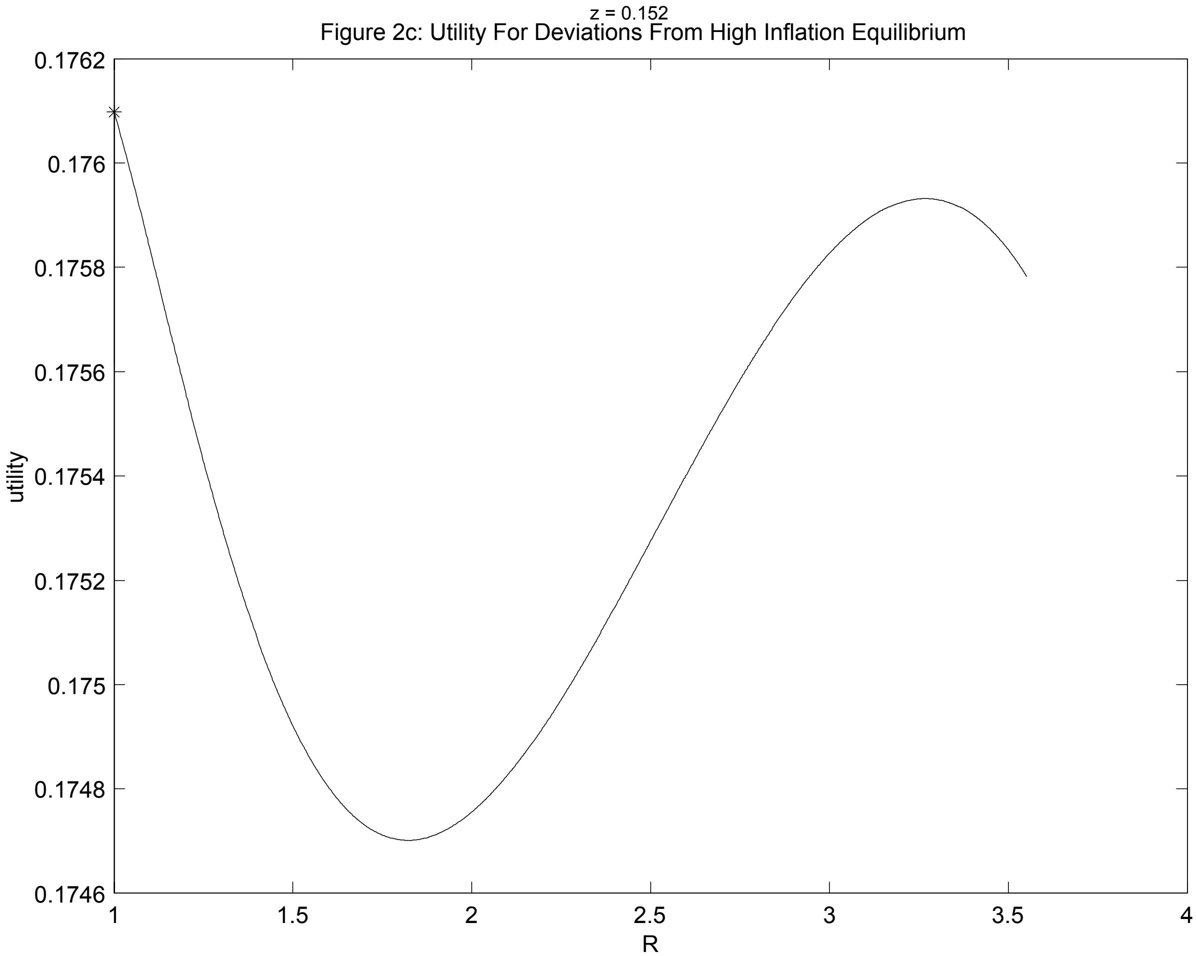
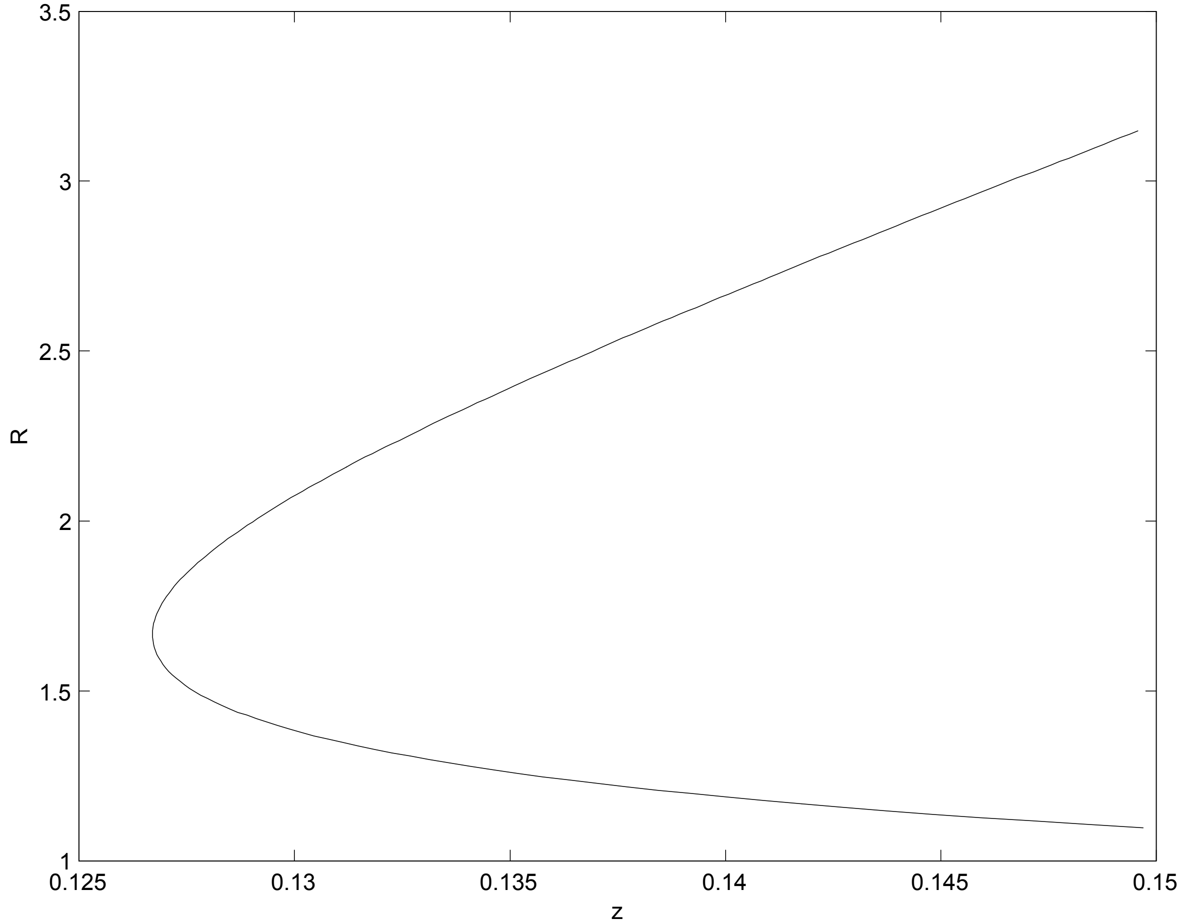

z = 0.152
Figure 2c: Utility For Deviations From High Inflation Equilibrium
0.1762
0.176
0.1758
0.1756
0.1754
0.1752
0.175
0.1748
0.1746
utility
1
1.5
2
2.5
3
3.5
4
R

Figure 3: Interest Rate Policy Correspondence



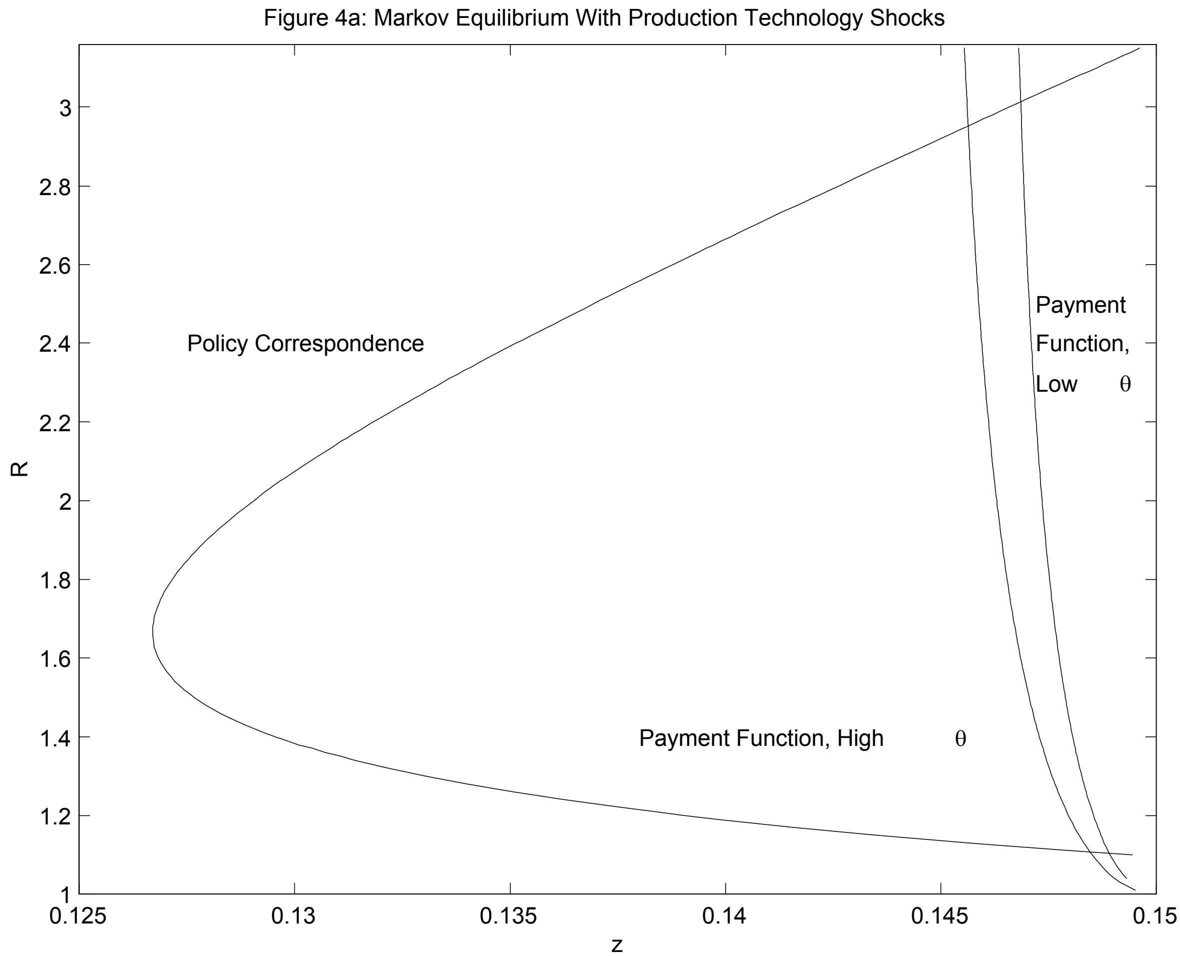


Figure 4a: Markov Equilibrium With Production Technology Shocks
Policy Correspondence
Payment Function, Low θ
Payment Function, High θ
R
z
0.125
0.13
0.135
0.14
0.145
0.15
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
2.8
3

Figure 4b: Markov Equilibrium With Payment Technology Shocks