Observations on Business Cycle Accounting

Lawrence J. Christiano
Joshua M. Davis

Background

- BCA: A strategy for identifying promising directions for model development
- Fit simple RBC model to data
- Identify 'wedges'
  - Distortions between marginal rates of substitution in preferences and technology necessary to reconcile model and data.
- Decomposition:
  - Simulate response of model to one wedge, holding other wedges constant.
  - Compare results of simulation to actual business cycle data

Papers on Business Cycle Accounting

- Parkin, Michael, 1988, ‘A Method for Determining Whether Parameters in Aggregative Models are Structural,’ Carnegie-Rochester Conference Series on Public Policy, 29, 215-252.
- Ingram, Beth, Narayana Kocherlakota and N. Savin, 1994, ‘Explaining Business Cycles: A Multiple-Shock Approach,’ Journal of Monetary Economics, 34, 415-428.
- Mulligan, Casey, 2002, ‘A Dual Method of Empirically Evaluating Dynamic Competitive Models with Market Distortions, Applied to the Great Depression and World War II,’ National Bureau of Economic Research Working Paper 8775.
- Chari, V.V., Patrick Kehoe and Ellen McGrattan, 2006, "Business Cycle Accounting," Federal Reserve Bank of Minneapolis Staff Report 328, revised February.

CKM’s Conclusion

- Intertemporal wedge not important.
  - accounts for only a small portion of business cycle contractions
  - such wedges cannot be important, because they drive investment and consumption in opposite directions, while both these variables are procyclical in the data.
- Standard models of financial frictions (e.g. Carlstrom-Fuerst (CF) and Bernanke-Gertler-Gilchrist (BGG)) not useful directions for research.
- Results are insensitive to introduction of adjustment costs in investment.

- CKM Finding Potentially of Major Interest
  - Early phases of US Great Depression accompanied by major decline in the stock market and unusually massive decline in investment
  - 2000 recession associated with stock market crash and unusually large drop in investment
  - Researchers Infer from observations like these that financial market imperfections as in CF and BGG are important in business cycles
    - CKM conclude this is a waste of time

Our Findings:

- BCA may greatly understate the importance in business cycles of financial frictions like those of CF or BGG.
  - Financial frictions likely to generate spillover effects onto other wedges, and these are ignored in BCA.
  - The precise magnitude of spillovers is not identified under BCA, because this requires pinning down the fundamental shocks to the economy. These are not identified under BCA.
- CKM conclusions relative to US and several other countries are not robust to introduction of adjustment costs in investment.
  - A full reconciliation in results with CKM is still being worked on.
  - One factor: CKM adopt a particular measurement error scheme during estimation of their model on US data. We show this scheme is overwhelmingly rejected, and it leads to points in the parameter space where adjustment costs seem not to matter much.

Model

- Preferences:

$$E_0 \sum_{t=0}^{\infty} \left(\beta \left(1+g_n\right)\right)^t \frac{\left[c_t \left(1-l_t\right)^{\psi}\right]^{1-\sigma}}{1-\sigma}.$$

- Law of Motion for Capital:

$$\left(1+g_n\right) k_{t+1} \;=\; \left(1-\delta\right) K_t + x_t - \Phi\left(\frac{x_t}{K_t}\right) K_t.$$

- Household Budget Constraint:

$$c_t + \left(1+\tau_{x,t}\right) x_t \;\leq\; \left(1-\tau_{lt}\right) w_t l_t + r_t k_t + T_t$$

- Resource Constraint:

$$c_t + g_t + x_t \leq k_t^{\alpha} \left(Z_t l_t\right)^{1-\alpha} = y_t.$$

- Wedges:

$$Z_t,\; g_t,\; \tau_{x,t},\; \tau_{l,t}$$

Model

- Preferences:

Intertemporal wedge

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Labor wedge

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Efficiency wedge

Outline

- Distinction between fundamental economic shocks and 'wedges'
  - Economic shocks originate inside wedges and spill over into other wedges
  - Wedges are correlated
- Illustrate intertemporal wedge.
- Display law of motion of wedges.
- Argument in favor of including investment adjustment costs in an RBC model.
- Explain a priori reasons that adjustment costs might be important in assessing importance of intertemporal wedge.
- Go for the basic results

Correlated Wedges

- The Models We Know of Require that Wedges Be Correlated
  - Example: Suppose there is a Shock Outside the Labor Market and There are Wage-Setting Frictions

$$\text{labor wedge}_t = \frac{MRS_t}{MP_{L,t}} = \frac{-\frac{u_{l,t}}{u_{c,t}}}{f_{n,t}} = \frac{\frac{\psi c_t}{1-l_t}}{(1-\alpha)\frac{y_t}{l_t}} = \frac{\psi}{1-\alpha}\frac{c_t}{y_t}\frac{l_t}{1-l_t}$$

  - With Wage-Setting Frictions, Labor Wedge = 1 Each Period.
  - In models of wage-setting frictions,

$$MP_{L,t} = \frac{W_t}{P_t}, \text{ but } MRS_t \neq \frac{W_t}{P_t},$$

    so labor wedge moves around.

The Intertemporal Wedge and BGG Financial Frictions

- Household budget constraint:

$$c_t + B_{t+1} \leq (1 + R_t) B_t + w_t l_t + T_t$$

- Household first order conditions:

$$\begin{aligned} u_{c,t} &= \beta E_t u_{c,t} (1 + R_{t+1}) \\ \frac{-u_{l,t}}{u_{c,t}} &= w_t. \end{aligned}$$

- Technology:

$$y_t = k_t^{\alpha} (Z_t l_t)^{1-\alpha} = y(k_t, l_t, Z_t).$$

- Goods-producing firm first order conditions:

$$y_{k,t} = r_t, \; y_{l,t} = w_t.$$

- Capital producers' technology:

$$k_{t+1} = (1 - \delta) k_t + x_t - \Phi\left(\frac{x_t}{k_t}\right) k_t.$$

The Intertemporal Wedge and BGG Financial Frictions ...

- Competition by capital producers, and optimization leads to first order conditions:

$$P_{k,t} = \frac{1}{1 - \Phi'\left(\frac{x_t}{k_t}\right)} \left[1 - \delta - \Phi\left(\frac{x_t}{k_t}\right) + \Phi'\left(\frac{x_t}{k_t}\right)\frac{x_t}{k_t}\right]$$

$$P_{k',t} = \frac{1 + g_n}{1 - \Phi'\left(\frac{x_t}{k_t}\right)}.$$

Individual capital producers are competitive and have linear homogeneous technologies. They take prices parametrically. In equilibrium, market price of new capital must equal marginal cost. With mo Investment, equilibrium price of new capital rise

- Rate of return on date t purchase of capital:

$$1 + R_{t+1}^k = \frac{r_{t+1} + P_{k,t+1}}{P_{k',t}}$$

The Intertemporal Wedge and BGG Financial Frictions ...

- Entrepreneurs own capital goods and rent them out
  - the end of period t, entrepreneurs have net worth, N_{t+1}, $N_{t+1} < P_{k',t}k_{t+1}$. Borrow:
    $$b_{t+1} = P_{k',t}k_{t+1} - N_{t+1},$$
    at gross interest rate, Z_t.

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    $$\Pr ob\left[\omega < x\right] = F\left(x\right).$$

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  $$\Pr ob\left[\omega < x\right] = F\left(x\right).$$
  - the lowest realization of ω for which it is feasible to repay is $\bar{\omega}_{t+1}$, where
  $$\bar{\omega}_{t+1}\left(1 + R_{t+1}^k\right)P_{k',t}k_{t+1} = Z_{t+1}b_{t+1}.$$
  - for $\omega < \bar{\omega}_{t+1}$ the entrepreneur simply pays all its revenues to the bank:
  $$\left(1 + R_{t+1}^k\right)\omega P_{k',t}k_{t+1}.$$

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  - for $\omega < \bar{\omega}_{t+1}$ the entrepreneur simply pays all its revenues to the bank:
  $$\left(1 + R_{t+1}^k\right)\omega P_{k',t}k_{t+1}.$$
  - Monitoring costs:
  $$\mu\left(1 + R_{t+1}^k\right)\omega P_{k',t}k_{t+1}.$$

The Intertemporal Wedge and BGG Financial Frictions ...

- Bank
  - borrows b_{t+1} from households at t. In $t+1$ the bank pays households

$$(1+R_{t+1})\,b_{t+1}.$$

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- Bank
  - borrows b_{t+1} from households at t. In $t+1$ the bank pays households

$$(1+R_{t+1})\,b_{t+1}.$$

  - sources of funds in $t+1$:

$$[1-F(\bar{\omega}_{t+1})]\,Z_{t+1}b_{t+1}+(1-\mu)\int_0^{\bar{\omega}_{t+1}}\omega dF(\omega)\left(1+R^k_{t+1}\right)P_{k',t}k_{t+1}.$$

or

$$[1-F(\bar{\omega}_{t+1})]\,\bar{\omega}_{t+1}\left(1+R^k_{t+1}\right)P_{k',t}k_{t+1}+(1-\mu)\int_0^{\bar{\omega}_{t+1}}\omega dF(\omega)\left(1+R^k_{t+1}\right)P_{k',t}k_{t+}$$

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or

$$[1-F(\bar{\omega}_{t+1})]\, \bar{\omega}_{t+1}\left(1+R^k_{t+1}\right) P_{k',t} k_{t+1} + (1-\mu)\int_0^{\bar{\omega}_{t+1}} \omega dF(\omega)\left(1+R^k_{t+1}\right) P_{k',t} k_{t+}$$

  - absence of state-contingent markets for currency in date $t+1$ and zero ex ante profits leads to ex post state by state zero profits:

$$\frac{P_{k',t} k_{t+1}\left(1+R^k_{t+1}\right)}{P_{k',t} k_{t+1} - N_{t+1}}\left([1-F(\bar{\omega}_{t+1})]\, \bar{\omega}_{t+1} + (1-\mu)\int_0^{\bar{\omega}_{t+1}} \omega dF(\omega)\, k_{t+1}\right) = 1 + R_{t+}$$

The Intertemporal Wedge and BGG Financial Frictions ...

- Repeat previous..

$$\frac{P_{k',t} k_{t+1}\left(1+R^k_{t+1}\right)}{P_{k',t} k_{t+1} - N_{t+1}}\left([1-F(\bar{\omega}_{t+1})]\, \bar{\omega}_{t+1} + (1-\mu)\int_0^{\bar{\omega}_{t+1}} \omega dF(\omega)\, k_{t+1}\right) = 1 + R_{t+}$$

- Define

$$1+R_{t+1} = \left(1-\tau^k_{t+1}\right)\left(1+R^k_{t+1}\right),$$

where

$$1-\tau^k_{t+1} \equiv \frac{P_{k',t} k_{t+1}}{P_{k',t} k_{t+1} - N_{t+1}}\left([1-F(\bar{\omega}_{t+1})]\, \bar{\omega}_{t+1} + (1-\mu)\int_0^{\bar{\omega}_{t+1}} \omega dF(\omega)\, k_{t+1}\right)$$

The Intertemporal Wedge and BGG Financial Frictions ...

- Repeat previous..

$$\frac{P_{k',t}k_{t+1}\left(1+R_{t+1}^{k}\right)}{P_{k',t}k_{t+1}-N_{t+1}}\left(\left[1-F\left(\bar{\omega}_{t+1}\right)\right]\bar{\omega}_{t+1}+(1-\mu)\int_{0}^{\bar{\omega}_{t+1}}\omega dF\left(\omega\right)k_{t+1}\right)=1+R_{t+}$$

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$$1+R_{t+1}=\left(1-\tau_{t+1}^{k}\right)\left(1+R_{t+1}^{k}\right),$$

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- Household first order condition:

$$u_{c,t}\;=\;\beta E_{t}u_{c,t}\left(1-\tau_{t+1}^{k}\right)\left(1+R_{t+1}^{k}\right),$$

$1-\tau_{t+1}^{k}$ ~ intertemporal wedge

The Intertemporal Wedge and BGG Financial Frictions ...

- resource constraint:

$$c_{t}+G_{t}+x_{t}=k_{t}^{\alpha}\left(Z_{t}l_{t}\right)^{1-\alpha},\text{ ignore } G_{t}$$

- Need other equilibrium conditions to pin down N_{t+1}, $\bar{\omega}_{t+1}$ (optimality condition, law of motion of net worth, conditions to pin down R_{t+1})

- Impact of Financial frictions:
  - new source of shocks:

    $$\mu,\ F$$

    - $*$ shocks perturb the intertemporal wedge, plus they 'spill over' into other wedges
  - Intertemporal wedge perturbed by shocks that originate in other wedges

- Following is the law of motion for the wedges.
- We follow CKM in allowing virtually unrestricted correlation among wedges.
- This is consistent with the sort of models BCA is designed to shed light on: even though fundamental economic shocks may be independent, wedges will not necessarily be independent

Law of Motion of Wedges

- Trend:

$$\tilde{Z}_t = \frac{Z_t}{(1+g_z)^t},\ \tilde{g}_t = \frac{g_t}{(1+g_z)^t}$$

- Law of Motion:

$$s_t = P_0 + P s_{t-1} + Q\varepsilon_t,\ s_t = \begin{pmatrix} \log \tilde{Z}_t \\ \tau_{l,t} \\ \tau_{x,t} \\ \log \tilde{g}_t \end{pmatrix}$$

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- Here,

$$P = \begin{bmatrix} \bar{P} & 0 \\ 0 & p_{44} \end{bmatrix},\ Q = \begin{bmatrix} \bar{Q} & 0 \\ 0 & q_{44} \end{bmatrix}$$

- Government Spending is Strictly Exogenous
  - Although this Assumption is Arbitrary, We Do Not Find that the Results are Very Sensitive to it.

Law of Motion of Wedges ...

- Wedge Law of Motion, Repeated,

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- Normalizations:

$\bar{Q}$ ~ lower triangular
$E\varepsilon_t\varepsilon_t' = I$

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- At the Same Time, In the Spirit of the Wedge Analysis:

$\bar{P},\ \bar{Q}\bar{Q}'$ unrestricted

A Case for Adjustment Costs

- The standard RBC model's implications for rates of return are strongly counterfactual
- Adjustment costs improve those implications

Investment Adjustment Costs

- Capital Accumulation:

$$(1+g_n)\,k_{t+1} = (1-\delta)\,K_t + x_t - \Phi\left(\frac{x_t}{K_t}\right)K_t.$$

- Functional Form:

$$\Phi\left(\frac{x_t}{k_t}\right) = \frac{a}{2}\left(\frac{x_t}{k_t} - \frac{x}{k}\right)^2, \text{ 'Tobin's } q \text{ Elasticity' } \equiv \frac{d\log x_t}{d\log P_{k',t}} = f(a).$$

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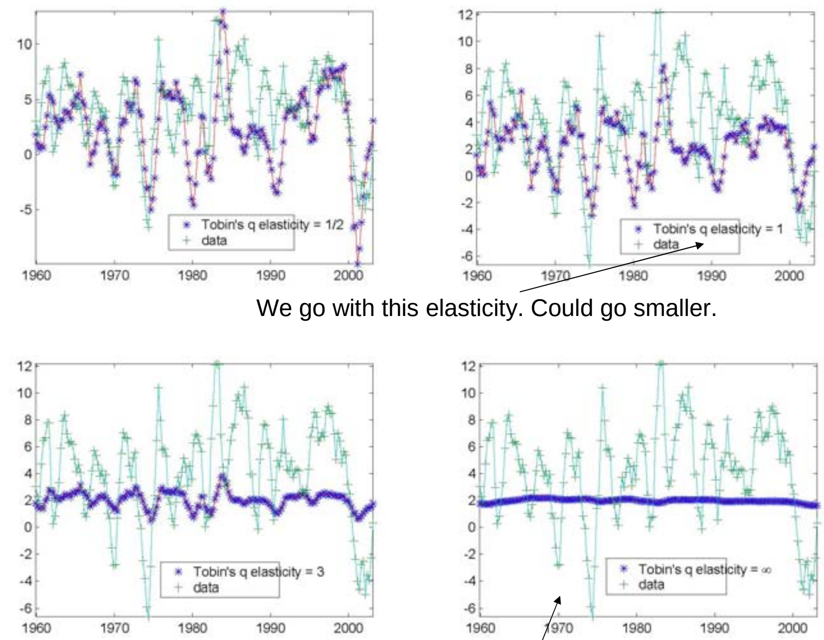
- Empirical Estimates of Elasticity:

Abel (1980):	0.50 – 1.14
Eberly (1997):	0.56 – 1.06
Cummins-Hassett-Oliner(1997) :	0.42 – 0.55

- Rate of return to capital:

$$1 + R_{t+1}^k = \begin{cases} \frac{MP_{k,t+1} + P_{k,t+1}}{P_{k',t}} & \text{adjustment costs} \\ MP_{k,t+1} + 1 - \delta & \text{no adjustment costs} \end{cases}$$

Figure 2: Actual and Model Rates of Return, Different Tobin's q Elasticities



Standard RBC model…

Why Would Adjustment Costs Matter?

- Consider intertemporal Euler equation:

$$1 = E_t m_{t+1}(1-\tau_{t+1}^k)R_{t+1}^k,$$

- Suppose τ_{t+1}^k varies very little in the absence of adjustment costs

  – When you add adjustment costs, R_{t+1}^k fluctuates more and – assuming fluctuations in m_{t+1} do not change, this requires variance of τ_{t+1}^k to increase.

Next:

- Solution of the Model
- Parameter Estimation
- Interesting Property of Solution: VAR Representation

Solution of Model

- Log-linear law of motion for the capital stock:

$$\log \tilde{k}_{t+1} = \lambda \log \tilde{k}_t + \underbrace{\gamma}_{1\times 4} s_t,\ \tilde{k}_t = k_t / (1+g_z)^t$$

- Data used in the analysis:

$$Y_t = \begin{pmatrix} \log \tilde{y}_t \\ \log \tilde{x}_t \\ \log l_t \\ \log \tilde{g}_t \end{pmatrix} \equiv \begin{pmatrix} \log \left[y_t / (1+g_z)^t \right] \\ \log \left[x_t / (1+g_z)^t \right] \\ \log l_t \\ \log \left[g_t / (1+g_z)^t \right] \end{pmatrix}.$$

- Connection to variables in model:

$$Y_t = \underbrace{h_0}_{4\times 4} s_t + \underbrace{h_1}_{4\times 1} \log \tilde{k}_t + \upsilon_t,$$

$$\upsilon_t \sim \text{measurement error},\ E\upsilon_t\upsilon_t' = R = \begin{cases} 0.0001 \times I_4 & \text{CKM specification} \\ 0.0 \times I_4 & \text{no measurement error} \end{cases}$$

Setting up Model in State-Observer Form

- State-Observer Representation
  - state -

$$\underbrace{\xi_t}_{4\times 5} = \begin{pmatrix} \log \tilde{k}_t \\ s_t \end{pmatrix},$$

  - State evolution equation:

$$\begin{pmatrix} \log \tilde{k}_t \\ s_t \end{pmatrix} = \begin{bmatrix} \lambda & \gamma \\ I & P \end{bmatrix} \begin{pmatrix} \log \tilde{k}_{t-1} \\ s_{t-1} \end{pmatrix} + \begin{pmatrix} 0 \\ Q \end{pmatrix} \varepsilon_t,$$

or

$$\xi_t = \underbrace{F}_{5\times 5} \xi_{t-1} + \underbrace{D}_{5\times 4} \varepsilon_t$$

  - Observer Equation:

$$Y_t = H\xi_t + \upsilon_t, \ E\upsilon_t\upsilon_t' = R, \ \underbrace{H}_{4\times 5} = \begin{bmatrix} h_1 & h_0 \end{bmatrix}$$

  - In this format, estimate parameters (P_0, P, Q) of the model using standard Kalman Filter techniques, and observations, $Y_1, ..., Y_T$ (Hamilton (1994)).

Interesting Rubio-Sargent-Villaverde Result: VAR Representation of the Observed Data

- System:

$$\begin{aligned} \xi_t &= F\xi_{t-1} + D\varepsilon_t, \\ Y_t &= HF\xi_{t-1} + HD\varepsilon_t. \end{aligned}$$

- Then, since HD is square (and invertible):

$$\varepsilon_t = (HD)^{-1} Y_t - (HD)^{-1} HF\xi_{t-1}.$$

- Put this into the state equation:

$$\begin{aligned} \xi_t &= F\xi_{t-1} + D(HD)^{-1} Y_t - D(HD)^{-1} HF\xi_{t-1} \\ &= M\xi_{t-1} + D(HD)^{-1} Y_t, \end{aligned}$$

where

$$M = \left[I - D(HD)^{-1} H\right] F$$

Interesting Rubio-Sargent-Villaverde Result: VAR Representation of the Observed Data ...

- Repeated substitution (assuming the eigenvalues of M are less than unity in abs value):

$$\begin{aligned}\xi_t &= D(HD)^{-1}Y_t + MD(HD)^{-1}Y_{t-1}\\ &+M^2D(HD)^{-1}Y_{t-2}\\ &+...\\ &+M^jD(HD)^{-1}Y_{t-j}\\ &+...,\end{aligned}$$

- Put this together with earlier equation in ε_t:

$$\begin{aligned}\varepsilon_t &= (HD)^{-1}Y_t - (HD)^{-1}HFD(HD)^{-1}Y_{t-1}\\ &-(HD)^{-1}HFMD(HD)^{-1}Y_{t-2} - (HD)^{-1}HFM^2D(HD)^{-1}Y_{t-3} - ...\end{aligned}$$

- Premultiply by HD and rearrange to obtain infinite VAR representation:

$$Y_t = B_1Y_{t-1} + B_2Y_{t-2} + ... + HD\varepsilon_t$$

$$B_j = HFM^{j-1}D(HD)^{-1},\ j \geq 1.$$

Identifying the Contribution of Financial Frictions to Business Cycle Dynamics

- Financial Frictions:
  - Source of shocks (e.g., monitoring and risk shocks)
    - operate through two channels:
      - intertemporal wedge
      - Spillovers onto other wedges
  - Source of propagation of other shocks (technology, government spending, etc.)
    - those shocks spill over onto the intertemporal wedge
  - Requires isolating fundamental shocks, but this is impossible under BCA.

Identification Problem

- Law of Motion of Wedges:

$$s_t = P_0 + P s_{t-1} + Q\varepsilon_t,\ E\varepsilon_t\varepsilon_t' = I,\ P = \begin{bmatrix} \bar{P} & 0 \\ 0 & p_{44} \end{bmatrix},\ Q = \begin{bmatrix} \bar{Q} & 0 \\ 0 & q_{44} \end{bmatrix}$$

- Disturbances in Law of Motion of Wedges:

$$u_t = s_t - P_0 - P s_{t-1}$$

$$V = E u_t u_t' = \begin{bmatrix} \bar{Q}\bar{Q}' & 0 \\ 0 & q_{44}^2 \end{bmatrix}.$$

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- Disturbances in Law of Motion of Wedges:

$$u_t = s_t - P_0 - P s_{t-1}$$

$$V = E u_t u_t' = \begin{bmatrix} \bar{Q}\bar{Q}' & 0 \\ 0 & q_{44}^2 \end{bmatrix}.$$

- Fundamental Economic Shocks, e_t

$$u_t = Ce_t,\ Ee_te_t' = I,\ CC' = V,\ C = \begin{bmatrix} \bar{Q}W & 0 \\ 0 & q_{44} \end{bmatrix},\ WW' = I.$$

Identification Problem ...

- Problem: Many Admissible C's, Each Implies Different e_t :

$$e_t = C^{-1}u_t = \begin{bmatrix} W'\bar{Q}^{-1} & 0 \\ 0 & q_{44}^{-1} \end{bmatrix} u_t$$

Identification Problem ...

- Problem: Many Admissible C's, Each Implies Different e_t :

$$e_t = C^{-1}u_t = \begin{bmatrix} W'\bar{Q}^{-1} & 0 \\ 0 & q_{44}^{-1} \end{bmatrix} u_t$$

$$a(\theta_1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta_1) & -\sin(\theta_1) \\ 0 & \sin(\theta_1) & \cos(\theta_1) \end{bmatrix}, \; b(\theta_2) = \begin{bmatrix} \cos(\theta_2) & 0 & \sin(\theta_2) \\ 0 & 1 & 0 \\ -\sin(\theta_2) & 0 & \cos(\theta_2) \end{bmatrix},$$

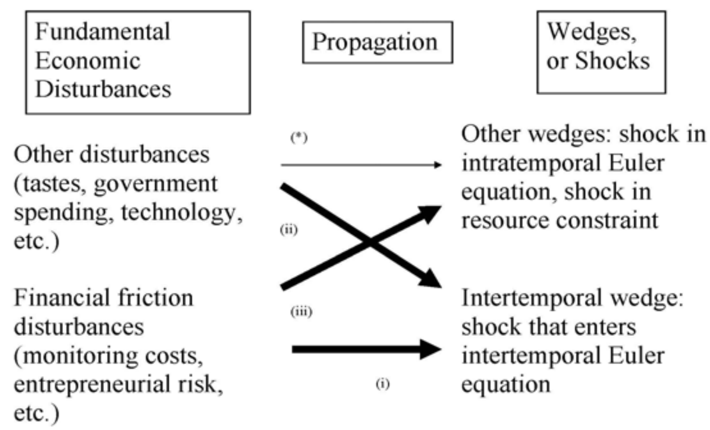
$$c(\theta_3) = \begin{bmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 \\ \sin(\theta_3) & \cos(\theta_3) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The identification problem:
each value of θ gives rise to a different specification of the fundamental shocks, yet the second moment properties of the model are unaffected.

$$W(\theta) = a(\theta_1)\, b(\theta_2)\, c(\theta_3),$$

$$\theta \in D \equiv \{\theta : \theta_i \in [0, 2\pi],\ i = 1, 2, 3\}$$

Figure 1: The propagation of economic disturbances through wedges



(*) movements in other wedges due to other disturbances
(i) movements in the intertemporal wedge due to financial disturbances
(ii) movements in the intertemporal wedge due to spillover effects from standard disturbances
(iii) movements in other wedges due to spillovers from financial disturbances

Isolating the Impact of Financial Frictions

- Law of motion for exogenous shocks (ignoring constant terms):

$$s_t = Ps_{t-1} + Q\varepsilon_t,$$

in lag operator form:

$$s_t = [I - PL]^{-1} Q\varepsilon_t \equiv F(L)\,\varepsilon_t$$

Original system

Intertemporal wedge

$$\begin{pmatrix} s_{1t} \\ s_{2t} \\ s_{3t} \\ s_{4t} \end{pmatrix} = \begin{bmatrix} F_{11}(L) & F_{12}(L) & F_{13}(L) & 0 \\ F_{21}(L) & F_{22}(L) & F_{23}(L) & 0 \\ F_{31}(L) & F_{32}(L) & F_{33}(L) & 0 \\ 0 & 0 & 0 & F_{44}(L) \end{bmatrix} \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t} \end{pmatrix}$$

Financial shock

- Operation of the financial frictions only:

Part of system that corresponds to financial frictions

$$\begin{pmatrix} s_{1t} \\ s_{2t} \\ s_{3t} \\ s_{4t} \end{pmatrix} = \begin{bmatrix} 0 & 0 & F_{13}(L) & 0 \\ 0 & 0 & F_{23}(L) & 0 \\ F_{31}(L) & F_{32}(L) & F_{33}(L) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t} \end{pmatrix} \equiv \tilde{F}(L)\,\varepsilon_t$$

Spillover of other shocks on Intertemporal wedge

Direct effect of financial shock on intertemporal wedge

Spillover effects of financial friction shocks

Time Series Representations for Wedges

- Full moving average representation of wedges:

$$s_t = F(L)\varepsilon_t$$

- Moving average representation of wedges when only effects of financial frictions are allowed to operate

$$\tilde{s}_t = \tilde{F}(L)\varepsilon_t$$

Time Series Representations for Observed Data

- Observer equation:

$$Y_t = h_0 s_t + h_1 \log \tilde{k}_t + \upsilon_t = \left[h_0 + h_1 \frac{\gamma L}{1-\lambda L}\right] s_t + \upsilon_t$$

$$= \left[h_0 + h_1 \frac{\gamma L}{1-\lambda L}\right] s_t + \upsilon_t$$

- Or, in compact notation:

$$Y_t = H(L)F(L)\varepsilon_t + \upsilon_t,\ H(L) = h_0 + h_1 \frac{\gamma L}{1-\lambda L}$$

- Representation of data which isolates financial frictions

$$\tilde{Y}_t = H(L)\tilde{F}(L)\varepsilon_t + \upsilon_t.$$

A Measure of the Importance of Financial Frictions

- Statistic:

$$f = \frac{var(H(L)\tilde{F}(L)\varepsilon_t)}{var(H(L)F(L)\varepsilon_t + \upsilon_t)}$$

- This object is a function of θ
  - Importance of Financial Frictions Not Identified

Identifying the Role of Financial Frictions in the Data

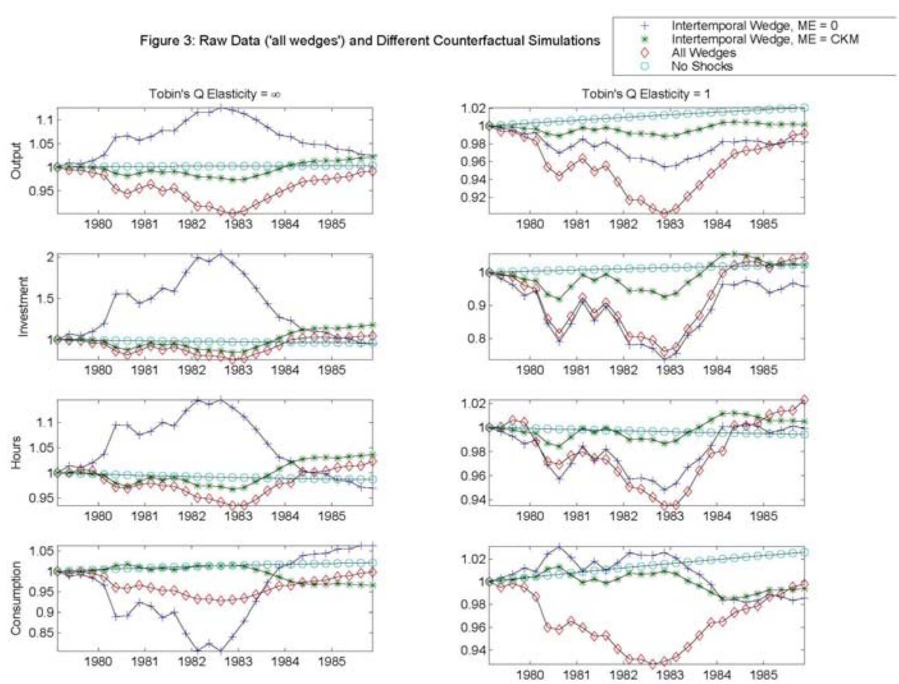
- CKM approach (I'm oversimplifying)
  - Determine recession periods.
  - Feed the measured intratemporal wedge to the model, holding the other wedges fixed at their values at the start of the recession

$$\log\tilde{k}_{t+1} = \lambda\log\tilde{k}_t + \gamma s_t \qquad Y_t = h_0 s_t + h_1\log\tilde{k}_t + \upsilon_t$$

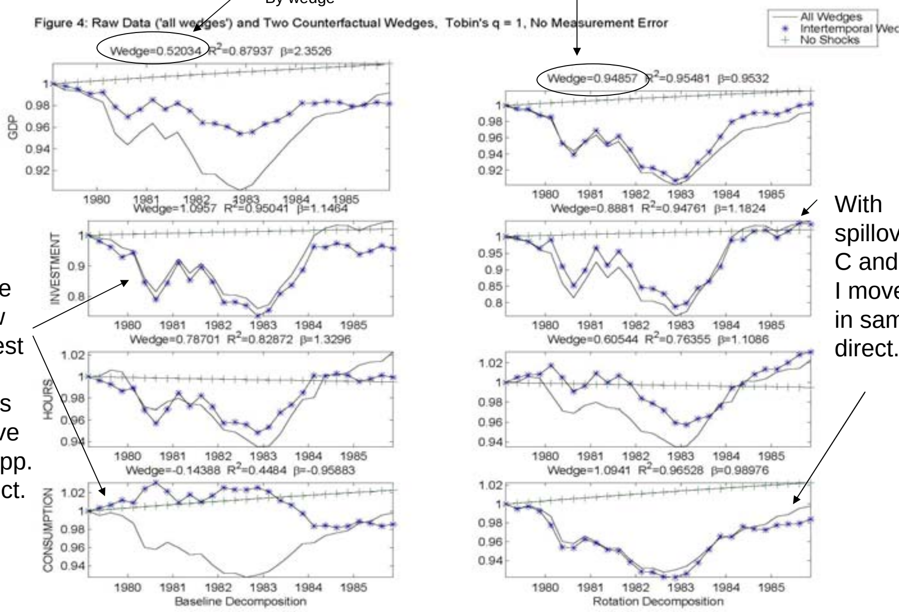
- This may understate the role of financial frictions, to the extent that there are spillover effects from financial shocks to other wedges.

Alternative Strategy Which Allows for Spillovers

- Choose θ to maximize statistic, f
- Simulate response of data to financial shock only.
  - This understates importance of financial frictions to the extent that non-financial shocks move the intertemporal wedge
  - Our way of choosing θ mitigates this problem.



Fraction of drop in output at trough accounted for
By wedge



Note how invest and cons move in opp. direct.

With spillover C and I move in same direct.

essage: when the (statistically rejected) model of measurement error is dropped, anda conservative amount of adjustment costs are
sed, CKM measure of importance of intertemporal wedge is big (let column). With spillovers, financial frictions could be EVERYTHING

Percent decline in output at trough of recession, averaged over 5 US recessions, due to intertemporal wedge: adjustment costs make no difference to this quantity which is not huge.

When CKM's (overwhelmingly rejected) model of measurement error is dropped, adjustment costs are very important though even CKM's own measure indicates financial frictions are important when there are adjustment costs

Table 1: Evidence from the post-war US

Likelihood ratio test of CKM measurement error	CKM measurement error: baseline	CKM measurement error: rotation	No measurement error: baseline	No measurement error: rotation
	Tobin's q elasticity $= \infty$			
493	0.28	0.91	-1.02	-0.11
	Tobin's q elasticity $= 1$			
454	0.28	0.93	0.53	0.89

Notes: (1) Likelihood ratio statistic - twice difference between the log likelihood under estimation with CKM measurement error specification, and under estimation with measurement error set to zero; (2) CKM measurement error: results based on estimation with CKM specification of R in (??), (3) No measurement error: based on estimation subject to $R = 0$, (4) baseline decomposition - see text, (5) rotation - rotation of shocks which maximizes importance of financial frictions, as defined in text

Allowing for spillovers from financial shocks to other wedges has a huge impact on contribution of financial shocks to business cycles

No adjustment cost case

With no measurement error and no adjustment costs, financial frictions predict booms during Recessions.

Table 2: BGG Wedge, Tobin's q Elasticity

Country	Likelihood Ratio test of CKM Meas Error	CKM Measurement Error		No Measurement Error	
		Baseline	Rotation	Baseline	Rotation
United States	492.73	0.28	0.91	-1.02	-0.11
Belgium	109.63	0.60	0.83	-1.12	0.24
Canada	186.41	0.21	1.11	-0.50	0.95
Denmark	31.57	0.39	1.15	0.18	1.11
Finland	43.81	1.18	1.57	0.23	1.31
France	-225.74	0.18	1.60	-3.29	4.88
Germany	-394.38	0.44	1.10	-1.85	-3.25
Italy	327.92	-4.84	1.86	-0.19	1.38
Japan	39.25	0.12	1.02	0.36	1.59
Mexico	60.76	0.01	1.06	-0.06	1.05
Netherlands	153.39	2.50	3.03	-0.01	1.25
Norway	63.67	1.27	-0.24	-0.55	0.79
Spain	287.91	1.48	1.49	0.04	1.64
Switzerland	274.95	-0.10	0.95	-0.20	0.89
England	132.62	0.25	1.10	0.04	1.19

Strong rejection – against alternative of no measurement error - of CKM model of measurement error for all countries but France and Germany. If the CKM model where 'true' the test statistic would be a chi-square with four degrees of freedom.

Table 3: BGG Wedge, Tobin's q Elasticity = 1

Country	Likelihood Ratio test of CKM Meas Error	CKM Measurement Error		No Measurement Error	
		Baseline	Rotation	Baseline	Rotation
United States	454.20	0.28	0.93	0.53	0.89
Belgium	39.95	0.14	0.88	-0.38	1.20
Canada	201.43	0.40	1.23	0.77	1.75
Denmark	2.44	0.12	1.05	-0.18	0.95
Finland	312.63	-4.22	-3.61	-0.56	0.50
France	202.06	1.41	1.39	1.16	1.05
Germany	10.29	0.41	1.07	-0.05	0.90
Italy	183.11	0.65	1.30	-0.54	3.77
Japan	-266.11	0.80	1.03	0.64	1.50
Mexico	42.84	-0.04	0.92	0.40	1.04
Netherlands	197.29	0.77	1.25	0.01	1.54
Norway	-1.34	0.11	1.08	-0.23	1.50
Spain	142.28	1.69	1.21	0.72	1.25
Switzerland	259.61	0.43	1.09	0.14	1.29
England	80.24	0.02	1.06	-0.02	1.24

Conclusion

- Key Conclusion of CKM Analysis: Financial Frictions that Enter Intertemporal Euler Equation Not Important for Understanding Business Cycles.
- With adjustment costs in investment and dropping CKM's rejected model of measurement error, we find:
  - Financial frictions important in the US, even without allowing for spillovers from financial shocks to other wedges
  - Accounting for spillovers, there is no expectation that financial friction shocks drive consumption and investment (counterfactually) in opposite directions.
  - Allowing for spillovers, the business cycle effects of financial frictions are potentially huge.
- There is nothing in Business Cycle Accounting to warrant abandoning models of financial frictions which distort intertemporal margins (e.g., the CF and BGG models).

Appendix Figures

- Following figures report Figures 1 and 2 for four other US recession episodes.

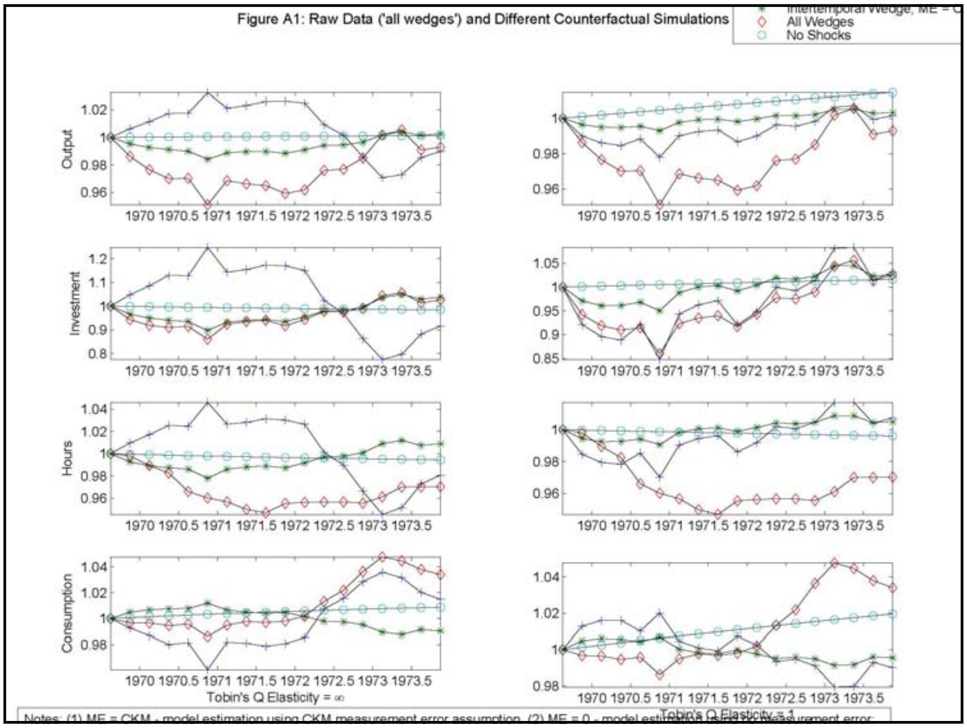


Figure A1: Raw Data ('all wedges') and Different Counterfactual Simulations
All Wedges
No Shocks
Output
Investment
Hours
Consumption
Tobin's Q Elasticity = ∞
Notes: (1) ME = CKM - model estimation using CKM measurement error assumption. (2) ME = 0 - model estimation

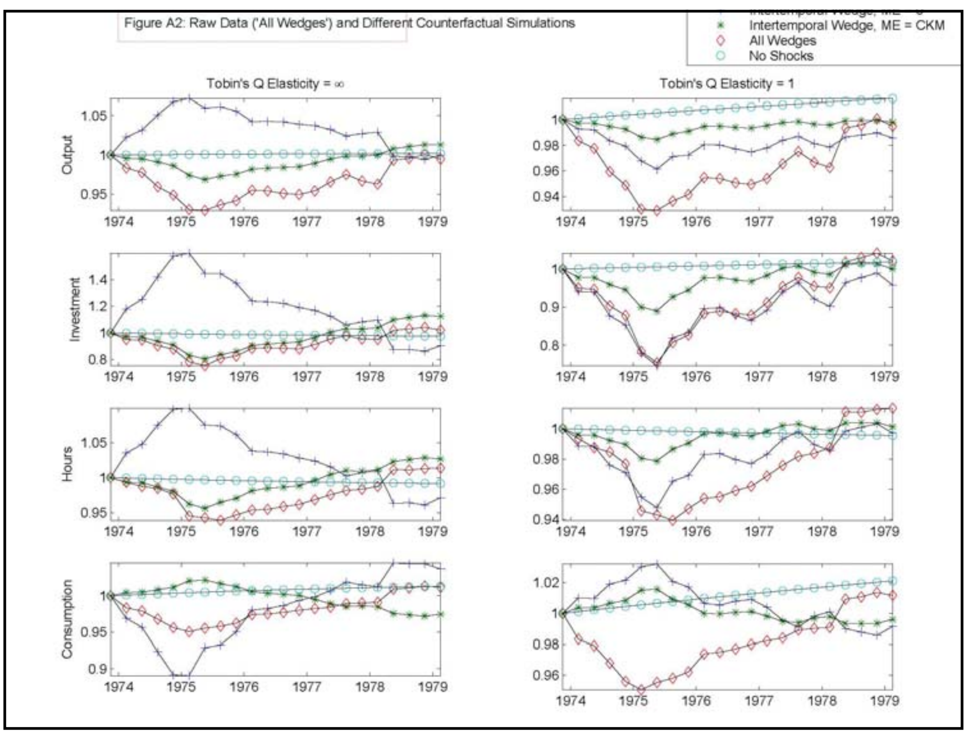


Figure A2: Raw Data ('All Wedges') and Different Counterfactual Simulations
Intertemporal Wedge, ME = CKM
All Wedges
No Shocks
Tobin's Q Elasticity = ∞
Tobin's Q Elasticity = 1
Output
Investment
Hours
Consumption

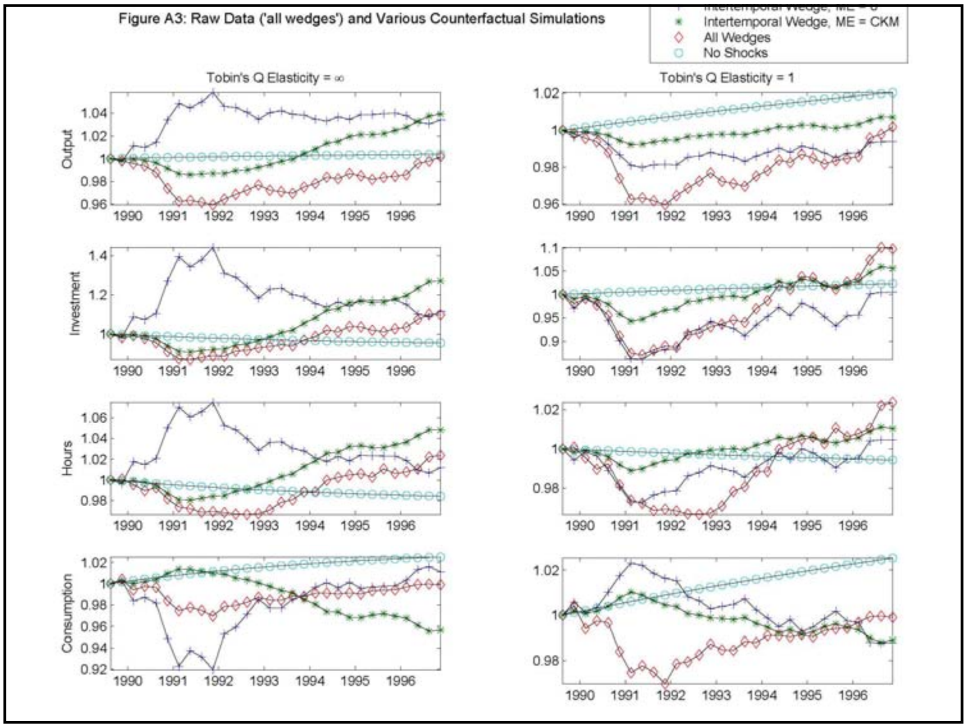


Figure A3: Raw Data ('all wedges') and Various Counterfactual Simulations

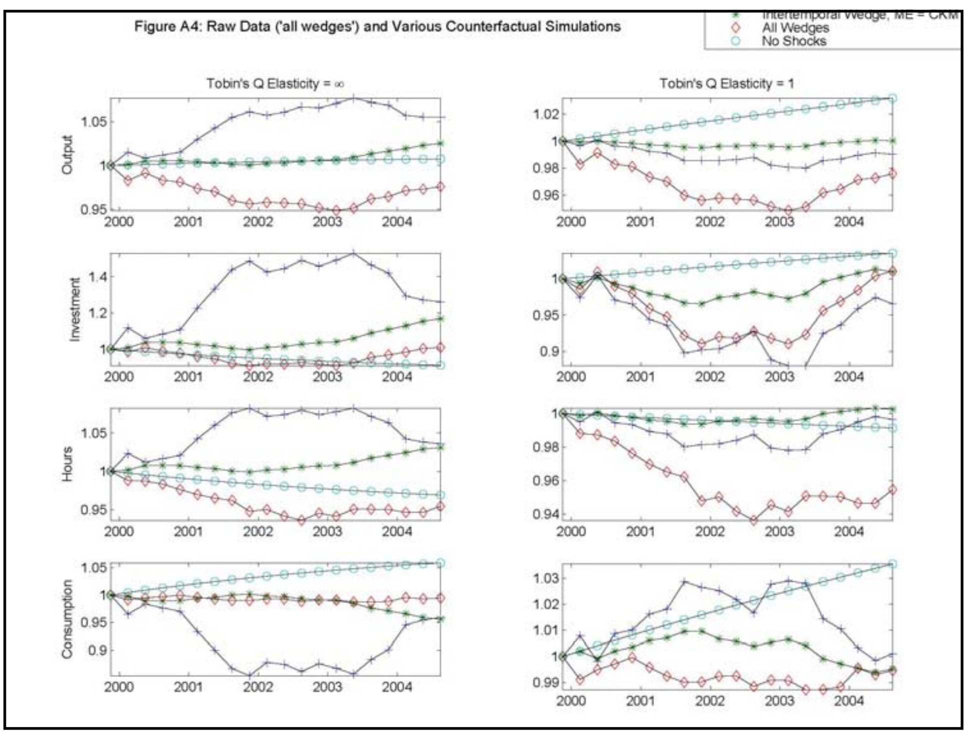


Figure A4: Raw Data ('all wedges') and Various Counterfactual Simulations

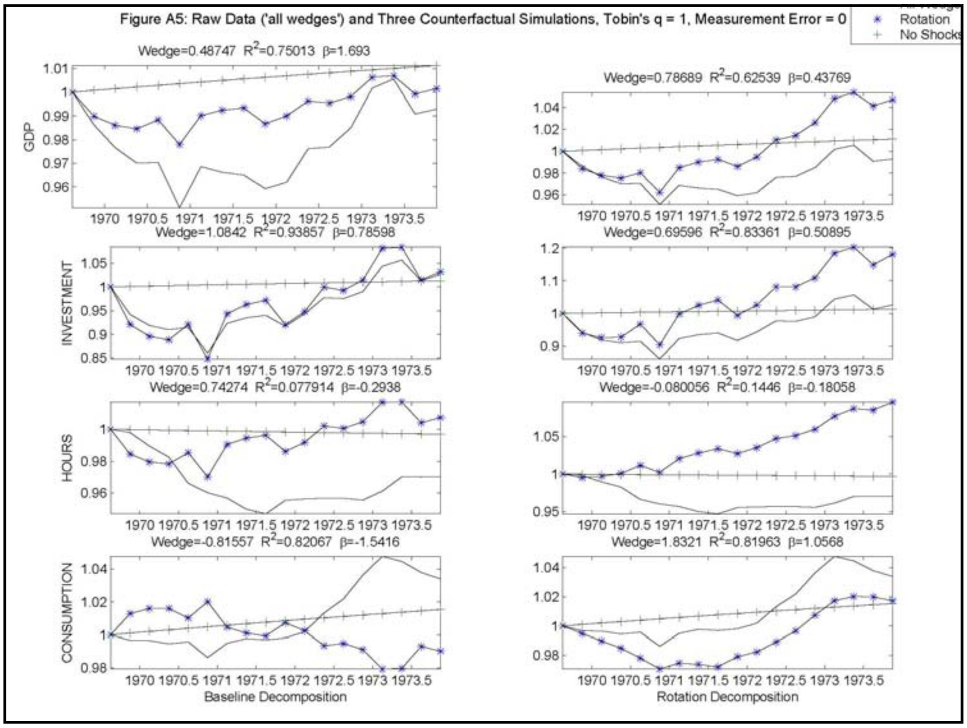


Figure A5: Raw Data ('all wedges') and Three Counterfactual Simulations, Tobin's q = 1, Measurement Error = 0

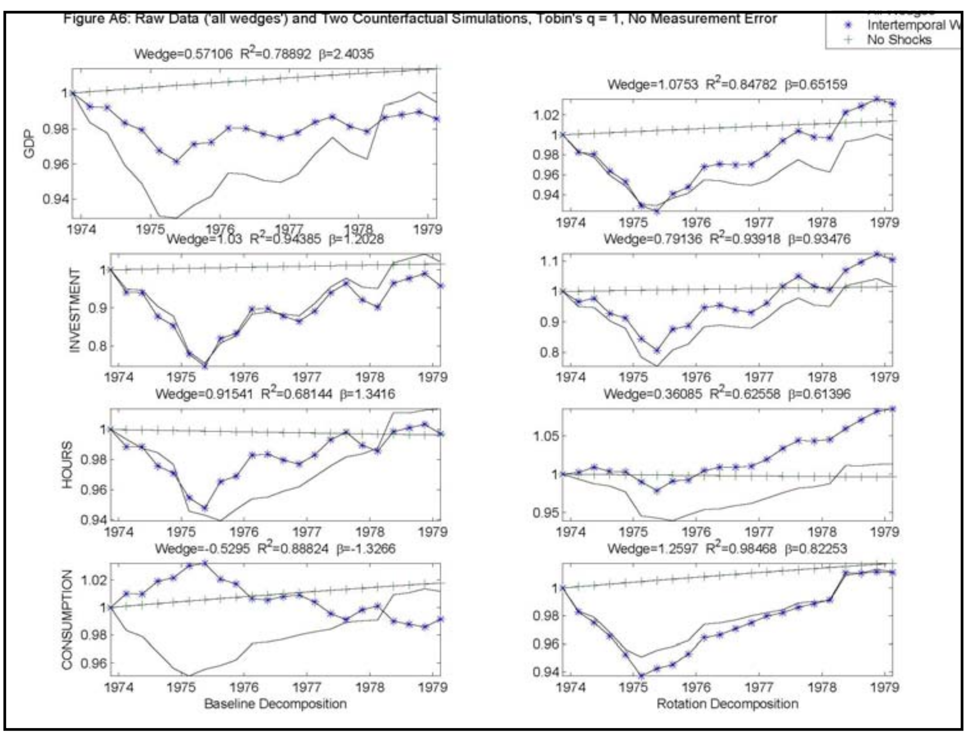


Figure A6: Raw Data ('all wedges') and Two Counterfactual Simulations, Tobin's q = 1, No Measurement Error

Figure A7: Raw Data ('all wedges') and Two Counterfactual Simulations, Tobin's q = 1, Measurement Error=0

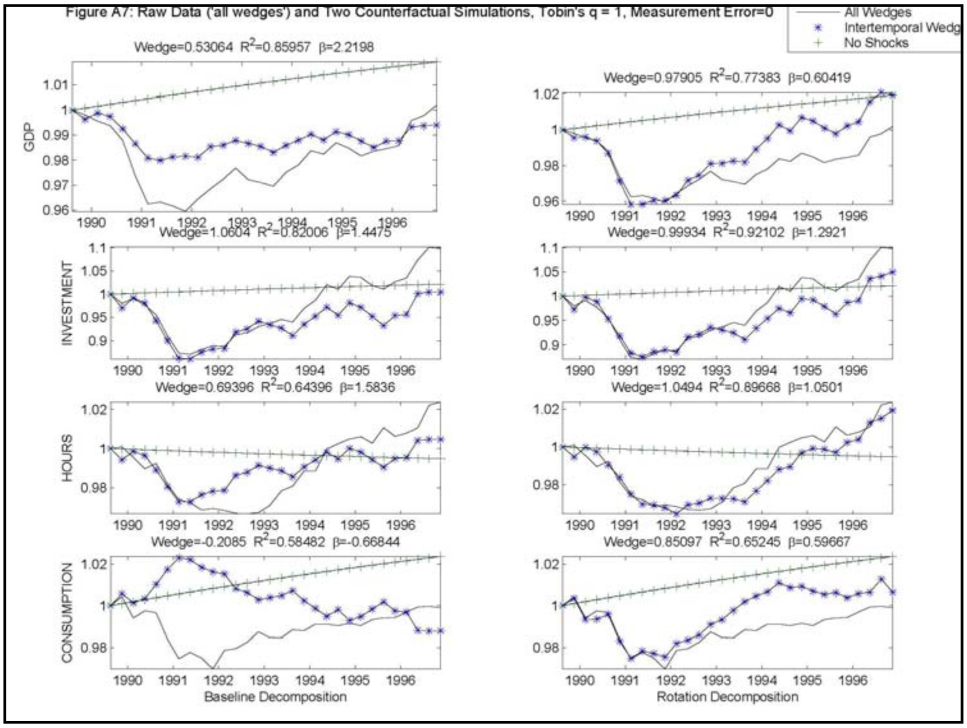


Figure A8: Raw Data ('all wedges') and Two Counterfactual Simulations, Tobin's q = 1, no measurement error