Observations on Business Cycle Accounting

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Background

• BCA: A strategy for identifying promising directions for model development

• Fit simple RBC model to data

• Identify ‘wedges’
  – Distortions between marginal rates of substitution in preferences and technology necessary to reconcile model and data.

• Decomposition:
  – Simulate response of model to one wedge, holding other wedges constant.
  – Compare results of simulation to actual business cycle data
Papers on Business Cycle Accounting


CKM’s Conclusion

- Intertemporal wedge not important.
  - accounts for only a small portion of business cycle contractions
  - such wedges cannot be important, because they drive investment and consumption in opposite directions, while both these variables are procyclical in the data.

- Standard models of financial frictions (e.g. Carlstrom-Fuerst (CF) and Bernanke-Gertler-Gilchrist (BGG)) not useful directions for research.

- Results are insensitive to introduction of adjustment costs in investment.
• CKM Finding Potentially of Major Interest

  – Early phases of US Great Depression accompanied by major decline in the stock market and unusually massive decline in investment

  – 2000 recession associated with stock market crash and unusually large drop in investment

  – Researchers Infer from observations like these that financial market imperfections as in CF and BGG are important in business cycles

    • CKM conclude this is a waste of time

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Our Findings:

• BCA may greatly understate the importance in business cycles of financial frictions like those of CF or BGG.

  – Financial frictions likely to generate spillover effects onto other wedges, and these are ignored in BCA.

  – The precise magnitude of spillovers is not identified under BCA, because this requires pinning down the fundamental shocks to the economy. These are not identified under BCA.

• CKM conclusions relative to US and several other countries are not robust to introduction of adjustment costs in investment.

  – A full reconciliation in results with CKM is still being worked on.

  – One factor: CKM adopt a particular measurement error scheme during estimation of their model on US data. We show this scheme is overwhelmingly rejected, and it leads to points in the parameter space where adjustment costs seem not to matter much.
Model

- Preferences:
  \[ E_0 \sum_{t=0}^{\infty} (\beta (1 + g_n))^t \left[ \frac{c_t (1 - l_t)^{\psi}}{1 - \sigma} \right]. \]

- Law of Motion for Capital:
  \[ (1 + g_n) k_{t+1} = (1 - \delta) K_t + x_t - \Phi \left( \frac{x_t}{K_t} \right) K_t. \]

- Household Budget Constraint:
  \[ c_t + (1 + \tau_{x,t}) x_t \leq (1 - \tau_{y}) w_t l_t + \tau_t k_t + T_t. \]

- Resource Constraint:
  \[ c_t + g_t + x_t \leq k_t^\alpha (Z_t l_t)^{1-\alpha} = y_t. \]

- Wedges:
  \[ Z_t, g_t, \tau_{x,t}, \tau_{l,t} \]
Outline

• Distinction between fundamental economic shocks and ‘wedges’
  – Economic shocks originate inside wedges and spill over into other wedges
  – Wedges are correlated

• Illustrate intertemporal wedge.

• Display law of motion of wedges.

• Argument in favor of including investment adjustment costs in an RBC model.

• Explain a priori reasons that adjustment costs might be important in assessing importance of intertemporal wedge.

• Go for the basic results

Correlated Wedges

• The Models We Know of Require that Wedges Be Correlated
  – Example: Suppose there is a Shock Outside the Labor Market and There are Wage-Setting Frictions

\[
labor\ wedge_t = \frac{MRS_t}{MP_{l,t}} = \frac{\frac{w_t}{n_{l,t}}}{f_{n,t}} = \frac{\phi_{ct}}{1-\alpha} \frac{c_t}{1-\alpha y_t} \frac{l_t}{1-\alpha y_t l_t}
\]

  – With Wage-Setting Frictions, Labor Wedge = 1 Each Period.

  – In models of wage-setting frictions,

\[
MP_{l,t} = \frac{W_t}{P_t}, \text{ but } MRS_t \neq \frac{W_t}{P_t},
\]

so labor wedge moves around.
Individual capital producers are competitive and have linear homogeneous technologies. They take prices parametrically. In equilibrium, market price of new capital must equal marginal cost. With more investment, equilibrium price of new capital rises.

\[ y_t = k_t^\alpha (Z_t L_t)^{1-\alpha} = y(k_t, L_t, Z_t). \]

Goods-producing firm first order conditions:
\[ y_{k,t} = r_t, \quad y_{l,t} = w_t. \]

Capital producers’ technology:
\[ k_{t+1} = (1-\delta) k_t + x_t - \Phi \left( \frac{x_t}{k_t} \right) k_t. \]

The Intertemporal Wedge and BGG Financial Frictions

- Competition by capital producers, and optimization leads to first order conditions:

\[ \frac{1}{P_{k,t}} = \frac{1}{1 - \Phi' \left( \frac{x_t}{k_t} \right) \left[ 1 - \delta - \Phi \left( \frac{x_t}{k_t} \right) + \Phi' \left( \frac{x_t}{k_t} \right) \frac{x_t}{k_t} \right]} \]

Individual capital producers are competitive and have linear homogeneous technologies. They take prices parametrically. In equilibrium, market price of new capital must equal marginal cost. With more investment, equilibrium price of new capital rises.

- Rate of return on date \( t \) purchase of capital:

\[ 1 + \Delta r_{t+1} = \frac{r_{t+1} + \frac{P_{k,t+1}}{P_{k,t}}}{P_{k,t}} \]
The Intertemporal Wedge and BGG Financial Frictions ...

- Entrepreneurs own capital goods and rent them out

  - the end of period $t$, entrepreneurs have net worth, $N_{t+1}, N_{t+1} < P_{k_t} k_{t+1}$.
  
  Borrow:
  
  $$b_{t+1} = P_{k_t} k_{t+1} - N_{t+1},$$

  at gross interest rate, $Z_t$.

- entrepreneur experiences shocks, and $k_{t+1}$ becomes $k_{t+1} \omega$, $E \omega = 1$, $\omega$ iid across entrepreneurs and time.

  $$\Pr \omega < x = F(x).$$
The Intertemporal Wedge and BGG Financial Frictions ...

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  - entrepreneur experiences shocks, and $k_{t+1}$ becomes $k_{t+1} \omega$, $E\omega = 1$, $\omega$ iid across entrepreneurs and time.
    \[ \Pr[\omega < x] = F(x). \]

  - the lowest realization of $\omega$ for which it is feasible to repay is $\bar{\omega}_{t+1}$, where
    \[ \bar{\omega}_{t+1} (1 + R_{t+1}^k) P_{k,t}k_{t+1} = Z_{t+1}b_{t+1}. \]

  - for $\omega < \bar{\omega}_{t+1}$ the entrepreneur simply pays all its revenues to the bank:
    \[ (1 + R_{t+1}^k)\omega P_{k,t}k_{t+1}. \]
• Bank

– borrows $b_{t+1}$ from households at $t$. In $t + 1$ the bank pays households

$$(1 + R_{t+1}) b_{t+1}.$$

– sources of funds in $t + 1$:

$$[1 - F(\bar{\omega}_{t+1})] \mathcal{B}_{t+1} b_{t+1} + (1 - \mu) \int_{0}^{\bar{\omega}_{t+1}} \omega dF(\omega) \left(1 + R_{t+1}^b\right) P_{t+1} k_{t+1},$$

or

$$[1 - F(\bar{\omega}_{t+1})] \mathcal{B}_{t+1} \left(1 + R_{t+1}^k\right) P_{k, t+1} k_{t+1} + (1 - \mu) \int_{0}^{\bar{\omega}_{t+1}} \omega dF(\omega) \left(1 + R_{t+1}^k\right) P_{k, t+1} k_{t+1}.$$

The Intertemporal Wedge and BGG Financial Frictions ...

- Bank
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    or
    $$[1 - F(\bar{\omega}_{t+1})] \bar{\omega}_{t+1} (1 + R_{t+1}^b) P_{k,t+1} + (1 - \mu) \int_0^{\bar{\omega}_{t+1}} \omega dF(\omega) \left(1 + R_{t+1}^b\right) P_{k,t+1}$$
  - absence of state-contingent markets for currency in date $t+1$ and zero ex ante profits leads to ex post state by state zero profits:
    $$\frac{P_{k,t+1} (1 + R_{t+1}^b)}{P_{k,t+1} - N_{t+1}} \left([1 - F(\bar{\omega}_{t+1})] \bar{\omega}_{t+1} + (1 - \mu) \int_0^{\bar{\omega}_{t+1}} \omega dF(\omega) k_{t+1}\right) = 1 + R_{t+1}$$

The Intertemporal Wedge and BGG Financial Frictions ...

- Repeat previous
  $$\frac{P_{k,t+1} (1 + R_{t+1}^b)}{P_{k,t+1} - N_{t+1}} \left([1 - F(\bar{\omega}_{t+1})] \bar{\omega}_{t+1} + (1 - \mu) \int_0^{\bar{\omega}_{t+1}} \omega dF(\omega) k_{t+1}\right) = 1 + R_{t+1}$$
- Define
  $$1 + R_{t+1} = \left(1 - \tau_{t+1}^b\right) \left(1 + R_{t+1}^b\right),$$
where
$$1 - \tau_{t+1}^b = \frac{P_{k,t+1} b_{t+1}}{P_{k,t+1} - N_{t+1}} \left([1 - F(\bar{\omega}_{t+1})] \bar{\omega}_{t+1} + (1 - \mu) \int_0^{\bar{\omega}_{t+1}} \omega dF(\omega) k_{t+1}\right)$$
The Intertemporal Wedge and BGG Financial Frictions ...

- Repeat previous:

\[
P_{k,t}k_{t+1} \frac{(1 + R^k_{t+1})}{P_{k,t}k_{t+1} - N_{t+1}} \left[1 - F(\tilde{w}_{t+1})\right] \tilde{w}_{t+1} + (1 - \mu) \int_{0}^{\tilde{w}_{t+1}} \omega dF(\omega) k_{t+1} \right) = 1 + R_t
\]

- Define

\[
1 + R_{t+1} = (1 - \tau^k_{t+1}) (1 + R^k_{t+1}) ,
\]

where

\[
1 - \tau^k_{t+1} = \frac{P_{k,t}k_{t+1}}{P_{k,t}k_{t+1} - N_{t+1}} \left[1 - F(\tilde{w}_{t+1})\right] \tilde{w}_{t+1} + (1 - \mu) \int_{0}^{\tilde{w}_{t+1}} \omega dF(\omega) k_{t+1} \right)
\]

- Household first order condition:

\[
u_{c,t} = \beta E_t u_{c,t+1} (1 - \tau^k_{t+1}) (1 + R^k_{t+1} ),
\]

\[
1 - \tau^k_{t+1} \text{- intertemporal wedge}
\]

The Intertemporal Wedge and BGG Financial Frictions ...

- resource constraint:

\[
\alpha_t + \gamma_t + x_t = k^\alpha_t (Z_t)^{1-\alpha} , \text{ ignore } G_t
\]

- Need other equilibrium conditions to pin down \( N_{t+1}, \tilde{w}_{t+1} \) (optimality condition, law of motion of net worth, conditions to pin down \( R_{t+1} \))

- Impact of Financial frictions:

  - new source of shocks:

\[
\mu, F
\]

  * shocks perturb the intertemporal wedge, plus they ‘spill over’ into other wedges

  - Intertemporal wedge perturbed by shocks that originate in other wedges
• Following is the law of motion for the wedges.

• We follow CKM in allowing virtually unrestricted correlation among wedges.

• This is consistent with the sort of models BCA is designed to shed light on: even though fundamental economic shocks may be independent, wedges will not necessarily be independent

\[
\tilde{Z}_t = \frac{Z_t}{(1 + g_s)^t}, \quad \tilde{g}_t = \frac{g_t}{(1 + g_s)^t}
\]

• Law of Motion:

\[
s_t = P_0 + P_{s_{t-1}} + Q\varepsilon_t, \quad s_t = \begin{pmatrix}
\log \tilde{Z}_t \\
\tau_{t,t} \\
\tau_{x,t} \\
\log \tilde{g}_t
\end{pmatrix}
\]
Law of Motion of Wedges

- Trend:
  \[ \tilde{Z}_t = \frac{Z_t}{(1 + g_t)^t}, \quad \tilde{g}_t = \frac{g_t}{(1 + g_t)^t} \]

- Law of Motion:
  \[ s_t = P_0 + P s_{t-1} + Q \varepsilon_t, \quad s_t = \begin{pmatrix} \log \tilde{Z}_t \\ \tau_{t,t} \\ \tau_{x,t} \\ \log \tilde{g}_t \end{pmatrix} \]

- Here,
  \[ P = \begin{bmatrix} \tilde{P} & 0 \\ 0 & p_{44} \end{bmatrix}, \quad Q = \begin{bmatrix} \tilde{Q} & 0 \\ 0 & q_{44} \end{bmatrix} \]

- Government Spending is Strictly Exogenous
  - Although this Assumption is Arbitrary, We Do Not Find that the Results are Very Sensitive to it.

Law of Motion of Wedges ...

- Wedge Law of Motion, Repeated,
  \[ s_t = P_0 + P s_{t-1} + Q \varepsilon_t, \quad s_t = \begin{pmatrix} \log \tilde{Z}_t \\ \tau_{t,t} \\ \tau_{x,t} \\ \log \tilde{g}_t \end{pmatrix} \]

- where,
  \[ P = \begin{bmatrix} \tilde{P} & 0 \\ 0 & p_{44} \end{bmatrix}, \quad Q = \begin{bmatrix} \tilde{Q} & 0 \\ 0 & q_{44} \end{bmatrix} \]
Law of Motion of Wedges ...

- Wedge Law of Motion, Repeated,

\[ s_t = P_0 + P s_{t-1} + Q \tilde{z}_t, \quad s_t = \begin{pmatrix} \log \tilde{Z}_t \\ T_{l,t} \\ T_{x,t} \\ \log \tilde{y}_t \end{pmatrix} \]

- where,

\[ P = \begin{bmatrix} \tilde{P} & 0 \\ 0 & p_{44} \end{bmatrix}, \quad Q = \begin{bmatrix} \tilde{Q} & 0 \\ 0 & q_{44} \end{bmatrix} \]

- Normalizations:

\[ \tilde{Q} \sim \text{lower triangular} \]
\[ E \tilde{z}_t \tilde{z}_t' = I \]

Law of Motion of Wedges ...

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- Normalizations:

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\[ E \tilde{z}_t \tilde{z}_t' = I \]

- At the Same Time, In the Spirit of the Wedge Analysis:

\[ \bar{P}, \ \tilde{Q} \tilde{Q}' \text{ unrestricted} \]
A Case for Adjustment Costs

• The standard RBC model’s implications for rates of return are strongly counterfactual

• Adjustment costs improve those implications

Investment Adjustment Costs

• Capital Accumulation:

\[(1 + g_n) k_{t+1} = (1 - \delta) K_t + x_t - \Phi \left( \frac{x_t}{K_t} \right) K_t.\]

• Functional Form:

\[\Phi \left( \frac{x_t}{k_t} \right) = \frac{a}{2} \left( \frac{x_t}{k_t} - \frac{x}{k} \right)^2, \text{‘Tobin’s q Elasticity’} \equiv \frac{d \log x_t}{d \log P_{k',t}} = f(a).\]
Investment Adjustment Costs

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- Empirical Estimates of Elasticity:

Abel (1980): 0.50 – 1.14
Eberly (1997): 0.56 – 1.06
Cummins-Hassett-Oliner (1997): 0.42 – 0.55

- Rate of return to capital:

\[1 + R_{t+1}^k = \begin{cases} \frac{MP_{k,t+1} + P_{k,t+1}}{P_{k,t}} & \text{adjustment costs} \\ MP_{k,t+1} + 1 - \delta & \text{no adjustment costs} \end{cases} \]
We go with this elasticity. Could go smaller.

Why Would Adjustment Costs Matter?

- Consider intertemporal Euler equation:

\[ 1 = E_t m_{t+1} (1 - \tau_{t+1}^k) R_{t+1}^k, \]

- Suppose \( \tau_{t+1}^k \) varies very little in the absence of adjustment costs

  - When you add adjustment costs, \( R_{t+1}^k \) fluctuates more and – assuming fluctuations in \( m_{t+1} \) do not change, this requires variance of \( \tau_{t+1}^k \) to increase.
Next:

- Solution of the Model
- Parameter Estimation
- Interesting Property of Solution: VAR Representation

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**Solution of Model**

- Log-linear law of motion for the capital stock:
  \[
  \log \tilde{k}_{t+1} = \lambda \log \tilde{k}_t + \gamma t_s, \quad \tilde{k}_t = k_t / (1 + g_z)^t
  \]

- Data used in the analysis:
  \[
  Y_t = \begin{pmatrix}
  \log \tilde{y}_t \\
  \log \tilde{r}_t \\
  \log \tilde{l}_t \\
  \log \tilde{g}_t
  \end{pmatrix}
  \equiv \begin{pmatrix}
  \log \left[ y_t / (1 + g_z)^t \right] \\
  \log \left[ r_t / (1 + g_z)^t \right] \\
  \log \tilde{l}_t \\
  \log \left[ g_t / (1 + g_z)^t \right]
  \end{pmatrix}
  
- Connection to variables in model:
  \[
  Y_t = \begin{pmatrix} h_0 \\ h_1 \end{pmatrix} 4 \times 1 s_t + \begin{pmatrix} h_0 \\ h_1 \end{pmatrix} 4 \times 1 \log \tilde{k}_t + \upsilon_t,
  \]

  \[
  \upsilon_t \sim \text{measurement error}, \quad E \upsilon_t \upsilon_t' = R = \begin{cases} 
  0.0001 \times I_4 & \text{CKM specification} \\
  0.0 \times I_4 & \text{no measurement error}
  \end{cases}
  \]
Setting up Model in State-Observer Form

- State-Observer Representation
  - state -
  \[ \begin{bmatrix} \xi_t \\ s_t \end{bmatrix} = \begin{bmatrix} \log \hat{k}_t \\ s_t \end{bmatrix}, \]
  
  - State evolution equation:
  \[ \begin{bmatrix} \log \hat{k}_t \\ s_t \end{bmatrix} = \begin{bmatrix} \lambda & \gamma \\ I & P \end{bmatrix} \begin{bmatrix} \log \hat{k}_{t-1} \\ s_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ Q \end{bmatrix} \varepsilon_t, \]
  or
  \[ \xi_t = F\xi_{t-1} + Ds_t + \xi_t. \]
  
  - Observer Equation:
  \[ Y_t = H\xi_t + u_t, \quad E\xi_t u'_t = R_t, \quad \begin{bmatrix} H \\ F \end{bmatrix} = \begin{bmatrix} h_1 \\ h_0 \end{bmatrix}. \]
  
- In this format, estimate parameters \((P_0, P, Q)\) of the model using standard Kalman Filter techniques, and observations, \(Y_t, \ldots, Y_T\) (Hamilton (1994)).

Interesting Rubio-Sargent-Villaverde Result: VAR Representation of the Observed Data

- System:
  \[ \begin{align*}
  \xi_t &= F\xi_{t-1} + Ds_t, \\
  Y_t &= H\xi_{t-1} + HDs_t.
  \end{align*} \]

- Then, since \(HD\) is square (and invertible):
  \[ \varepsilon_t = (HD)^{-1}Y_t - (HD)^{-1}HF\xi_{t-1}. \]

- Put this into the state equation:
  \[ \begin{align*}
  \xi_t &= F\xi_{t-1} + D(HD)^{-1}Y_t - D(HD)^{-1}HF\xi_{t-1} \\
  &= M\xi_{t-1} + D(HD)^{-1}Y_t,
  \end{align*} \]
  where
  \[ M = \left[ I - D(HD)^{-1}H \right] F \]
Identifying the Contribution of Financial Frictions to Business Cycle Dynamics

- Financial Frictions:
  - Source of shocks (e.g., monitoring and risk shocks)
    - operate through two channels:
      - intertemporal wedge
      - Spillovers onto other wedges
  - Source of propagation of other shocks (technology, government spending, etc.)
    - those shocks spill over onto the intertemporal wedge
  - Requires isolating fundamental shocks, but this is impossible under BCA.

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### Interesting Rubio-Sargent-Villaverde Result: VAR Representation of the Observed Data ...

- Repeated substitution (assuming the eigenvalues of \( M \) are less than unity in abs value):
  \[
  \xi_t = D (HD)^{-1} Y_t + MD (HD)^{-1} Y_{t-1} + \ldots + M^j D (HD)^{-1} Y_{t-j} + \ldots
  \]

- Put this together with earlier equation in \( \varepsilon_t \):
  \[
  \varepsilon_t = (HD)^{-1} Y_t - (HD)^{-1} HDF (HD)^{-1} Y_{t-1} - (HD)^{-1} HFM D (HD)^{-1} Y_{t-2} - (HD)^{-1} HFM^2 D (HD)^{-1} Y_{t-3} - \ldots
  \]

- Premultiply by \( HD \) and rearrange to obtain infinite VAR representation:
  \[
  Y_t = B_1 Y_{t-1} + B_2 Y_{t-2} + \ldots + HD \varepsilon_t
  \]
  \[
  B_j = HFM^{j-1} D (HD)^{-1}, \ j \geq 1.
  \]
Identification Problem

- Law of Motion of Wedges:

\[ s_t = P_0 + P s_{t-1} + Q e_t, \quad E c_t e_t' = I, \quad P = \begin{bmatrix} \hat{P} & 0 \\ 0 & p_{44} \end{bmatrix}, \quad Q = \begin{bmatrix} \hat{Q} & 0 \\ 0 & q_{44} \end{bmatrix} \]

- Disturbances in Law of Motion of Wedges:

\[ u_t = s_t - P_0 - P s_{t-1} \]

\[ V = E u_t u_t' = \begin{bmatrix} \hat{Q} & 0 \\ 0 & q_{44}^2 \end{bmatrix}. \]

Identification Problem

- Law of Motion of Wedges:

\[ s_t = P_0 + P s_{t-1} + Q e_t, \quad E c_t e_t' = I, \quad P = \begin{bmatrix} \hat{P} & 0 \\ 0 & p_{44} \end{bmatrix}, \quad Q = \begin{bmatrix} \hat{Q} & 0 \\ 0 & q_{44} \end{bmatrix} \]

- Disturbances in Law of Motion of Wedges:

\[ u_t = s_t - P_0 - P s_{t-1} \]

\[ V = E u_t u_t' = \begin{bmatrix} \hat{Q} & 0 \\ 0 & q_{44}^2 \end{bmatrix}. \]

- Fundamental Economic Shocks, \( e_t \)

\[ u_t = C e_t, \quad E c_t e_t' = I, \quad C C' = V, \quad C = \begin{bmatrix} Q W & 0 \\ 0 & q_{44} \end{bmatrix}, \quad W W' = I. \]
Identification Problem ...

- Problem: Many Admissible \( C \)'s, Each Implies Different \( e_t \):

\[
e_t = C^{-1} u_t = \begin{bmatrix} W' \tilde{Q}^{-1} & 0 \\ 0 & q_{tt}^{-1} \end{bmatrix} u_t
\]

\[
a(\theta_1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta_1) & -\sin(\theta_1) \\ 0 & \sin(\theta_1) & \cos(\theta_1) \end{bmatrix}, \quad b(\theta_2) = \begin{bmatrix} \cos(\theta_2) & 0 & \sin(\theta_2) \\ 0 & 1 & 0 \\ -\sin(\theta_2) & 0 & \cos(\theta_2) \end{bmatrix},
\]

\[
c(\theta_3) = \begin{bmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 \\ \sin(\theta_3) & \cos(\theta_3) & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]

\[
W(\theta) = a(\theta_1) b(\theta_2) c(\theta_3),
\]

\[
\theta \in D \equiv \{ \theta : \theta_i \in [0, 2\pi], i = 1, 2, 3 \}
\]

The identification problem: each value of \( \theta \) gives rise to a different specification of the fundamental shocks, yet the second moment properties of the model are unaffected.
Figure 1: The propagation of economic disturbances through wedges

<table>
<thead>
<tr>
<th>Fundamental Economic Disturbances</th>
<th>Propagation</th>
<th>Wedges, or Shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Other disturbances (tastes, government spending, technology, etc.)</td>
<td>(i)</td>
<td>Other wedges: shock in intratemporal Euler equation, shock in resource constraint</td>
</tr>
<tr>
<td>Financial friction disturbances (monitoring costs, entrepreneurial risk, etc.)</td>
<td>(ii)</td>
<td>Intertemporal wedge: shock that enters intratemporal Euler equation</td>
</tr>
</tbody>
</table>

(*) movements in other wedges due to other disturbances
(i) movements in the intertemporal wedge due to financial disturbances
(ii) movements in the intertemporal wedge due to spillover effects from standard disturbances
(iii) movements in other wedges due to spillovers from financial disturbances

---

**Isolating the Impact of Financial Frictions**

- Law of motion for exogenous shocks (ignoring constant terms):
  \[ s_t = P s_{t-1} + Q \varepsilon_t, \]
  in lag operator form:
  \[ s_t = (I - PL)^{-1} Q \varepsilon_t \equiv F(L) \varepsilon_t \]

  **Original system**
  \[ \begin{bmatrix}
  s_{1t} \\
  s_{2t} \\
  s_{3t} \\
  s_{4t}
  \end{bmatrix} = \begin{bmatrix}
  F_{11}(L) & F_{12}(L) & F_{13}(L) & 0 \\
  F_{21}(L) & F_{22}(L) & F_{23}(L) & 0 \\
  F_{31}(L) & F_{32}(L) & F_{33}(L) & 0 \\
  0 & 0 & 0 & F_{44}(L)
  \end{bmatrix} \begin{bmatrix}
  \varepsilon_{1t} \\
  \varepsilon_{2t} \\
  \varepsilon_{3t} \\
  \varepsilon_{4t}
  \end{bmatrix} \]

  Financial shock

  **Part of system that corresponds to financial frictions**

- Operation of the financial frictions only:
  \[ s_{1t} \]
  \[ \begin{bmatrix}
  s_{1t} \\
  s_{2t} \\
  s_{3t} \\
  s_{4t}
  \end{bmatrix} = \begin{bmatrix}
  0 & 0 & F_{13}(L) & 0 \\
  F_{21}(L) & F_{22}(L) & F_{23}(L) & 0 \\
  0 & 0 & 0 & F_{33}(L)
  \end{bmatrix} \begin{bmatrix}
  \varepsilon_{1t} \\
  \varepsilon_{2t} \\
  \varepsilon_{3t} \\
  \varepsilon_{4t}
  \end{bmatrix} = \tilde{F}(L) \varepsilon_t \]

  **Spillover of other shocks on intertemporal wedge**
  \[ \text{Spillover effects of financial friction shocks} \]

  **Direct effect of financial shock on intertemporal wedge**
Time Series Representations for Wedges

• Full moving average representation of wedges:
  \[ s_t = F(L) \varepsilon_t \]

• Moving average representation of wedges when only effects of financial frictions are allowed to operate
  \[ \tilde{s}_t = \tilde{F}(L) \varepsilon_t \]

Time Series Representations for Observed Data

• Observer equation:
  \[
  Y_t = h_0 s_t + h_1 \log \tilde{k}_t + u_t = \left[ h_0 + h_1 \frac{\gamma L}{1 - \lambda L} \right] s_t + u_t
  \]
  
  \[
  = \left[ h_0 + h_1 \frac{\gamma L}{1 - \lambda L} \right] s_t + u_t
  \]

• Or, in compact notation:
  \[ Y_t = H(L) F(L) \varepsilon_t + u_t, \quad H(L) = h_0 + h_1 \frac{\gamma L}{1 - \lambda L} \]

• Representation of data which isolates financial frictions
  \[ \bar{Y}_t = H(L) \bar{F}(L) \varepsilon_t + u_t. \]
A Measure of the Importance of Financial Frictions

- Statistic:
  \[ f = \frac{\text{var}(H(L)\tilde{F}(L)\varepsilon_t)}{\text{var}(H(L)F(L)\varepsilon_t + \nu_t)} \]

- This object is a function of \( \theta \)
  - Importance of Financial Frictions Not Identified

Identifying the Role of Financial Frictions in the Data

- CKM approach (I'm oversimplifying)
  - Determine recession periods.
  - Feed the measured intratemporal wedge to the model, holding the other wedges fixed at their values at the start of the recession

\[ \log \tilde{k}_{t+1} = \lambda \log \tilde{k}_t + \gamma s_t \quad Y_t = h_0 s_t + h_1 \log \tilde{k}_t + \nu_t \]

- This may understate the role of financial frictions, to the extent that there are spillover effects from financial shocks to other wedges.
Alternative Strategy Which Allows for Spillovers

• Choose $\theta$ to maximize statistic, $f$

• Simulate response of data to financial shock only.
  
  – This underestimates importance of financial frictions to the extent that non-financial shocks move the intertemporal wedge
  – Our way of choosing $\theta$ mitigates this problem.
Percent decline in output at trough of recession, averaged over 5 US recessions, due to intertemporal wedge: adjustment costs make no difference to this quantity which is not huge.

When CKM’s (overwhelmingly rejected) model of measurement error is dropped, adjustment costs are very important though even CKM’s own measure indicates financial frictions are important when there are adjustment costs allowing for spillovers from financial shocks to other wedges has a huge impact on contribution of financial shocks to business cycles.
No adjustment cost case

With no measurement error and no adjustment costs, financial frictions predict booms during recessions.

Strong rejection – against alternative of no measurement error - of CKM model of measurement error for all countries but France and Germany. If the CKM model where ‘true’ the test statistic would be a chi-square with four degrees of freedom.
Conclusion

• Key Conclusion of CKM Analysis: Financial Frictions that Enter Intertemporal Euler Equation Not Important for Understanding Business Cycles.

• With adjustment costs in investment and dropping CKM’s rejected model of measurement error, we find:
  – Financial frictions important in the US, even without allowing for spillovers from financial shocks to other wedges
  – Accounting for spillovers, there is no expectation that financial friction shocks drive consumption and investment (counterfactually) in opposite directions.
  – Allowing for spillovers, the business cycle effects of financial frictions are potentially huge.

• There is nothing in Business Cycle Accounting to warrant abandoning models of financial frictions which distort intertemporal margins (e.g., the CF and BGG models).

Appendix Figures

• Following figures report Figures 1 and 2 for four other US recession episodes.