Observations on Business Cycle Accounting*

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Abstract

Chari, Kehoe and McGrattan (2005) advocate the use of business cycle accounting to identify directions for improvement in equilibrium business cycle models. This procedure computes the ‘wedges’ in a first-generation RBC model that are necessary for the model to reproduce major macroeconomic time series. As a demonstration of the power of the approach, CKM argue that it can be used to rule out a prominent class of explanations of the Great Depression. In particular, they conclude that models of financial frictions which create wedges in the intertemporal Euler equation are not promising avenues for understanding the dynamics of the Great Depression.

We have two main findings. First, when we modify the RBC model to include investment adjustment costs, we find that the CKM results are overturned: shocks in the intertemporal Euler equation account for a significant fraction (25-40 percent) of the fall in output in the Great Depression. Second, we show that the use of business cycle accounting to determine the importance of intertemporal Euler equation shocks in output dynamics depends on identification assumptions. But, there are many observationally equivalent assumptions that one can make, and each has different implications for the importance of intertemporal Euler equation errors.

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1. Introduction

Chari, Kehoe and McGrattan (2005) (CKM) argue that a procedure they call Business Cycle Accounting (BCA) is useful for identifying promising directions for model development.\(^1\) The strategy works with the first-generation real business cycle (RBC) model as a benchmark. It isolates the wedges between marginal rates of substitution in preferences and marginal rates of substitution in technology necessary for the model to be able to reproduce key macroeconomic aggregates. Under BCA, the parts of the model that deserve further development are the ones with the most important wedges. After applying their methodology to the US Great Depression, CKM reach a surprising conclusion. Financial frictions of the sort that manifest themselves primarily in the intertemporal wedge (the one between the intertemporal marginal rate of substitution in consumption and the rate of return on capital) are not useful for thinking about the US Great Depression. This appears to rule out models of financial frictions like those analyzed by Carlstrom and Fuerst (1997) (CF) and Bernanke, Gertler and Gilchrist (1999) (BGG).

We make two observations. First, we focus on the assumption in CKM's benchmark RBC model, that the technology for producing new capital is linear in investment and old capital. We find that when we introduce curvature ('adjustment costs') into this relationship, the CKM conclusion is overturned.\(^2\) Second, we show that the use of BCA to identify the most important wedges is hampered by a fundamental identification problem. Determining the importance of a wedge in model dynamics requires making identification assumptions, but the framework offers no way to evaluate alternative assumptions.

We now discuss our two findings in greater detail. We document the importance of curvature by introducing adjustment costs into CKM's benchmark model and redoing their wedge calculations. We choose the degree of adjustment costs by calibrating the model's implication for the Tobin's \(q\) elasticity, that is, the elasticity of the investment to capital ratio with respect to the price of capital. To be conservative, we adopt the smallest adjustment cost consistent with the empirical evidence on Tobin's \(q\) reported in Abel (1980) and Eberly (1997). When we apply BCA using the modified RBC model, we find that financial frictions which enter primarily through the intertemporal wedge account for 25-40 percent of the fall of output during the Great Depression. The reason CKM’s findings are not robust to the presence of adjustment costs is that adjustment costs and the intertemporal wedge in effect cancel each other, when viewed from the perspective of the first-generation RBC model.\(^3\) Leave out adjustment costs and there is no apparent need for an intertemporal wedge. The intuition for this is simple.

The rate of return on capital - measured in the way that is consistent with the standard RBC model - falls at roughly the same rate as does consumption growth during the first

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\(^1\)This strategy is closely related to that advocated in Parkin (1988), Ingram, Kocherlakota and Savin (1994), Hall (1997), and Mulligan (2002).

\(^2\)In considering robustness to perturbations in their analysis, CKM also consider adjustment costs in investment. In contrast to our findings, they report that their results are robust to the introduction of adjustment costs. Their paper does not provide enough information for us to assess the reason for the different results. For example, CKM do not report the numerical value of the parameter of their adjustment cost function.

\(^3\)We are grateful to Mark Gertler, who conjectured this result in a private communication.
years of the Great Depression. Equality of the intertemporal marginal rate of substitution in consumption, \( MRS \), and rate of return on capital, \( 1 + R^k \), occurs without the need of a wedge between the two. In particular,

\[
MRS \approx 1 + R^k,
\]
during the first few years of the Great Depression, when \( R^k \) is measured as in the first-generation RBC model.\(^4\) When adjustment costs are introduced, a capital gain term appears in the rate of return on capital. The modified model associates the collapse in output and investment at the start of the Great Depression with a low value of the price of capital. Because of the stationarity properties of the RBC model, the price of capital is expected to rise back up to its steady state value. This anticipated capital gain implies that \( R^k \) actually rises in the first few years of the Great Depression. Now a wedge, \( 1 - \tau^k \), is required to bring \( MRS \) and \( 1 + R^k \) into line:

\[
MRS = (1 - \tau^k) (1 + R^k). \tag{1}
\]
The intertemporal wedge, \( 1 - \tau^k \), must fall with the rise in \( R^k \).\(^5\)

We now briefly describe our point about lack of identification in BCA. In modeling the wedges, a flexible time series representation is adopted in which the dynamic and contemporaneous interactions between wedges is left unrestricted. This is very much in the spirit of the analysis, because most extensions of the first generation RBC models do imply that the wedges are correlated. A shock that originates inside one wedge moves all other wedges.\(^6\)

Our identification point can be understood by contemplating closely the sort of question that BCA is designed to answer: ‘what is the contribution to aggregate fluctuations of the sort of financial frictions that manifest themselves in the form of intertemporal wedges?’ This question can be interpreted in at least two ways: (i) ‘what is the role of financial frictions, as a source of macroeconomic impulses?’ For example, when the financial frictions studied by CF or BGG are introduced, one can contemplate new shocks, such as shocks to monitoring costs, or to the riskiness of entrepreneurs.\(^7\) Another interpretation is (ii), ‘what is the role of financial frictions as a propagation mechanism for traditional macroeconomic impulses?’ Answering (ii) requires comparing the actual effects of non-financial friction shocks with what their effects are when financial frictions are shut down. But, given the lack of structure in BCA, there is no hope of addressing this question. We take it for granted that BCA cannot address question (ii). We argue that underidentification also mars the ability to address (i).

\(^4\)In this case, \( 1 + R^k = MP_k + (1 - \delta) \), where \( MP_k \) is the marginal physical product of capital.

\(^5\)Primiceri, Schaumburg and Tambalotti (2005) make a related argument. They note that if one measures \( R^k \) using a market return, then one finds that \( 1 - \tau^k \) varies substantially in post-war data. Market based measures of \( R^k \) exhibit orders of magnitude greater fluctuations than does \( R^k \) in the first-generation RBC model. This argument is related to the rejections of intertemporal Euler equations reported by Hansen and Singleton (1982, 1983) and others.

\(^6\)For example, with sticky wages and/or sticky prices the marginal rate of substitution between consumption and leisure is only equal to the marginal product of labor on average. A shock that enters the intertemporal Euler equation will in general have a different impact on the marginal rate of substitution than on the marginal product of labor. That is, it will move what CKM call the leisure wedge. This effect is expected to occur contemporaneously, and over time.

\(^7\)Shocks like these are explored in the context of the Great Depression by Christiano, Rostagno and Motto (2004) and in the context of recent business cycles in the Euro Area and the US by Christiano, Rostagno and Motto (2005).
The implicit identification assumption underlying CKM’s implementation of BCA is that the shocks originating in the intertemporal wedge are exclusively financial friction shocks, and that those shocks do not spill over into the other wedges. Identification is a substantial issue when, under CKM’s implicit identification assumptions, one finds that intertemporal wedges matter. This is the relevant case, because, as noted above, we find that the intertemporal wedge matters when adjustment costs are included in the RBC model. The problem is that one can interpret the estimated covariation among wedges as implying that much of the movement in the intertemporal wedge actually reflects shocks originating in other wedges. In this case, the 25-40 percent number we reported above is an overestimate of the importance of shocks originating in the intertemporal wedge. Alternatively, one can interpret the covariation among wedges as reflecting that shocks originating in the intertemporal wedge spill over into other wedges. In this case, our 25-40 percent estimate understates the importance of shocks originating in the intertemporal wedge. To make this point concrete, we find an identification which has the implication that shocks originating in the intertemporal wedge account for 100% of the behavior of output, employment and investment during the Great Depression. There is no way to select between the alternative identification assumptions that support these different conclusions about the importance of the shocks originating in the intertemporal wedge. This is because each identification assumption is observationally equivalent.

In the following section, we describe the model used in the analysis. In section 3, we discuss our model solution and estimation strategy. In section 4 we spell out the identification problem discussed above. In section 5 we discuss how the importance of a particular wedge is assessed, given a set of identification assumptions. Section 6 displays our results. Concluding remarks appear in section 7.

2. Model

According CKM’s first-generation RBC model, households maximize:

\[
E \sum_{t=0}^{\infty} \left( \beta (1 + g_n) \right)^t \left[ \log c_t + \psi \log (1 - l_t) \right], \quad 0 < \beta < 1,
\]

where \( c_t \) and \( l_t \) denote per capita consumption and employment, respectively. The household budget constraint is

\[
c_t + (1 + \tau_{x,t}) x_t \leq r_t k_t + (1 - \tau_{l,t}) w_t l_t + T_t,
\]

where \( T_t \) denotes lump sum taxes, \( x_t \) denotes investment and \( \tau_{l,t} \) denotes the labor wedge. Also, \( k_t \) denotes the beginning-of-period \( t \) stock of capital divided by the period \( t \) population. As discussed in the introduction the intertemporal wedge, \( \tau_{x,t} \), is motivated using the CF model. In addition, the household satisfies the following accumulation equation for investment:

\[
(1 + g_n) k_{t+1} = (1 - \delta) k_t + x_t.
\]

(2.1)

The household maximizes utility by choice of \( k_{t+1}, x_t, c_t \) and \( l_t \), subject to its budget constraint, the capital evolution equation and the usual inequality constraints and no-Ponzi scheme condition.
The resource constraint is:
\[ c_t + G_t + x_t \leq y (k_t, l_t, Z_t) = k_t^\alpha (Z_t l_t)^{1-\alpha}, \]
(2.2)
where
\[ Z_t = \tilde{Z}_t (1 + g_z)^t, \]
and \( \tilde{Z}_t \), the efficiency wedge, is an exogenous stationary stochastic process. In the resource constraint, \( G_t \) denotes government purchases of goods and services, which have the following representation:
\[ g_t = \tilde{g}_t (1 + g_z)^t, \]
where \( g_t \) is a stationary, exogenous stochastic process and \( g_z \geq 0 \).

Combining firm and household first order necessary conditions for optimization,
\[ u_{c,t} = (1 - \delta) y_{l,t}, \]
(2.3)
\[ u_{c,t} = \beta E_{t, t+1} y_{l,t+1} + (1 + \tau_{x,t}) (1 - \delta) \]
(2.4)
where \( u_{c,t} \) and \(-u_{l,t}\) are the derivatives of period utility with respect to consumption and leisure, respectively. In addition, \( y_{l,t} \) and \( y_{k,t} \) are the marginal products of labor and capital, respectively. The solution to the model is obtained by solving (2.1)-(2.4) subject to the transversality condition and the law of motion for the exogenous shocks. This is given by:
\[ s_t = P_0 + P s_{t-1} + Q \varepsilon_t, \quad s_t = \begin{pmatrix} \log \tilde{Z}_t \\ \tau_{l,t} \\ \tau_{x,t} \\ \log \tilde{g}_t \end{pmatrix}, \quad E \varepsilon_t \varepsilon'_t = I, \]
(2.5)
where
\[ P = \begin{bmatrix} \bar{P} & 0 \\ 0 & \bar{p}_{44} \end{bmatrix}, \quad Q = \begin{bmatrix} \bar{Q} & 0 \\ 0 & \bar{q}_{44} \end{bmatrix}. \]
Here, \( \bar{P} \) is unrestricted and \( \bar{Q} \) is normalized to be lower triangular with non-negative diagonal terms. The shocks, \( \varepsilon_t \), have no particular economic significance. The assumption about their variance and cross-correlations, as well as the lower triangularity of \( \bar{Q} \) are simply normalizations. In effect, the dynamic interaction between the first three wedges (i.e., elements of \( s_t \)) is left unrestricted. Their statistical relationship to \( \tilde{g}_t \) is sharply restricted. However, we did not find that the results in this note are very sensitive to this assumption, and so we do not explore that more here.

CKM also consider the case of adjustment costs in investment, \( a > 0 \). We consider the model of adjustment costs used in CKM:
\[ (1 + g_a) k_{t+1} = (1 - \delta) k_t + x_t - \Phi \left( \frac{x_t}{k_t} \right) k_t, \]
(2.6)
where
\[ \Phi \left( \frac{x_t}{k_t} \right) = \frac{a}{2} \left( \frac{x_t}{k_t} - b \right)^2. \]
(2.7)
Here, $b$ is the steady state investment to capital ratio.

In Appendix A, we derive the equilibrium conditions for the CF model when the technology for producing new capital is given by (2.6)-(2.7) with $a > 0$. We establish a proposition displaying the wedges that must be added to the RBC economy to ensure that the equilibrium allocations of the RBC economy coincide with those of the CF economy with adjustment costs. We show that the intertemporal wedge implied by the CF model has the following form:

$$u_{c,t} = \beta E_t u_{c,t+1} \left(1 - \tau_{t+1}^k\right) \frac{y_{k,t+1} + P_{k,t+1}}{P_{k',t}} + u_{c,t+1}, \quad (2.8)$$

where,

$$P_{k',t} = \frac{1}{1 - \Phi' \left(\frac{\bar{x}_t}{k_t}\right)} \quad (2.9)$$

$$P_{k,t} = \frac{1 - \delta - \Phi \left(\frac{\bar{x}_t}{k_t}\right) + \Phi' \left(\frac{\bar{x}_t}{k_t}\right) \bar{x}_t}{1 - \Phi' \left(\frac{\bar{x}_t}{k_t}\right)} \quad (2.10)$$

Here, $P_{k,t}$ denotes the market price of period $t$ used capital, $k_t$. In Appendix A we show that the CF model with adjustment costs implies $\tau_{t+1}^k$ is a function of uncertainty realized at date $t$, but not at date $t+1$. That appendix provides a careful derivation of our result, because our finding for the way the intertemporal wedge enters (2.8) differs from CKM. We follow CKM in presuming that all wedges implied by the CF financial frictions, but the intertemporal wedge, $1 - \tau_{t+1}^k$, are quantitatively small and can be ignored. So, the equilibrium conditions for the RBC model with wedges, in the adjustment cost case, are (2.2)-(2.3), (2.6), (2.8), (2.9) and (2.10).

In Appendix B we derive the intertemporal wedge associated with the BGG model. That model also implies that the intertemporal wedge enters as $1 - \tau_{t+1}^k$ in (2.8). The only difference is that under BGG, $\tau_{t+1}^k$ is a function of the period $t + 1$ realization of uncertainty. In the BGG model, $P_{k,t}$ is also the market price of period $t$ used capital, $k_t$.

### 3. Model Solution and Estimation

Here, we describe how we assigned values to the model parameters. A subset of the parameters were not estimated. These were set as in CKM:

$$\beta = 0.9722, \quad \theta = 0.35, \quad \delta = 0.0464, \quad \psi = 2.24, \quad (3.1)$$

$$g_n = 0.015, \quad g_z = 0.016.$$

CKM estimate the nonstochastic steady state values of the shocks, $\tau_t$, $\tau_x$ and $\zeta$ (i.e., the elements of $P_0$) as well as the elements of $P$ and $Q$. Annual data covering the period, 1901-1940 were used.\(^8\) The elements of the matrices, $P$ and $Q$ are estimated subject to the zero

\(^8\)The data were obtained from Ellen McGrattan’s website.
restrictions described in section 2, and to the restriction that the maximal eigenvalue of $P$ not exceed 0.995.

The first step of estimation is to set up the model’s solution in state space - observer form:

$$
Y_t = H(\xi_t, v_t; \gamma) \tag{3.2}
$$

$$
\xi_t = F(\xi_{t-1}, \epsilon_t; \gamma) \tag{3.3}
$$

$$
\gamma = (P, P_0, V, a) .
$$

Here, $\xi_t$ is the state of the system:

$$
\xi_t = \left( \begin{array}{c}
\log \tilde{k}_t \\
\log s_t 
\end{array} \right),
$$

where $\tilde{k}_t = k_t / (1 + g_z)^t$. Also, $Y_t$ is the observation vector:

$$
Y_t = \left( \begin{array}{c}
\log \tilde{y}_t \\
\log \tilde{x}_t \\
\log \tilde{l}_t \\
\log \tilde{g}_t 
\end{array} \right),
$$

where $\tilde{x}_t = x_t / (1 + g_z)^t$. Also, $v_t$ is a $4 \times 1$ vector of measurement errors, with

$$
E v_t v_t' = 0.0001 \times I_4,
$$

where $I_4$ is the four-dimensional identity matrix.

We compute two representations of (3.2) and (3.3). The first is based on a standard log-linear approximation of the model solution (see, e.g., Christiano (2002)). The second is based on a second order perturbation, which we compute using the algorithm in Kim, Kim, Schaumburg, and Sims, (2005). When we use the log-linear approximation, we use our state space - observer system to construct the Gaussian likelihood function using the procedure described in Hamilton (1994). When we use the second order perturbation solution, we use the state space - observer representation to approximate the Gaussian density using the particle filter described in Wan and van der Merwe (2001).

When we analyze the version of the model with adjustment costs, we need to assign a value to the adjustment cost parameter, $a$. Our calibration of $a$ is based on our interpretation of the variable, $P_{k',t}$. On this dimension, the CF and BGG models differ slightly (for details, see the Appendices). Both agree that $P_{k',t}$ is the marginal cost, in units of consumption goods, of producing new capital when only (2.6) is considered. However, in the CF model, financial frictions introduce a wedge between the market price of capital and $P_{k',t}$. In that model, there is a conflict between the producers of capital and their lenders, and this also adds to the marginal cost of $k_{t+1}$. This wedge reflects that the production of new capital necessarily involves some destruction of capital as a by-product of the monitoring that banks must do of capital producers. Still, in practice the discrepancy between $P_{k',t}$ and the market price of

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9It is easy to verify that $P_{k',t}$ in (2.9) corresponds to the price of investment goods (i.e., unity) divided by the marginal product of investment goods in producing end of period capital.
new capital in the CF model with adjustment costs may be quantitatively small. To see this, it is instructive to consider the response of the variables in the CF model (where \( P_{k,t} = 1 \) always) to a technology shock. According to CF (see Figure 2 in CF), the contemporaneous response of the market price of capital is only one-tenth the contemporaneous response of investment. That simulation suggests that the distinction between \( P_{k,t} \) and the market price of capital may not be large in the CF model.

In the BGG model, financial frictions occur in the relationship between the managers of capital and banks, and they do not enter into the production of capital. As a consequence, \( P_{k,t} \) corresponds to the market price of capital in that model. Under the interpretation of \( P_{k,t} \) as the market price of capital, we can calibrate \( a \) based on empirical estimates of the elasticity of investment with respect to the price of capital (i.e., Tobin’s \( q \)). According to estimates reported in Abel (1980) and Eberly (1997), Tobin’s \( q \) lies in the range of 0.5 to 1. Interestingly, if we just consider the period of largest fall in the Dow Jones Industrial average during the Great Depression, 1929Q4 to 1932Q4, the ratio of the percent change in investment to the percent change in the Dow is 0.68.10 This is an estimate of Tobin’s \( q \) under the assumption that the movement in the Dow reflects primarily the price of capital (tangible and intangible), and not its quantity. This estimate lies in the middle of the Abel-Eberly range of estimates. To be conservative, when we introduce adjustment costs in capital, we pick the smallest value of \( a \) consistent with the Abel-Eberly interval. Thus, we set \( a \) so that Tobin’s \( q \) is unity.

4. Identification and the Contribution of a Particular Wedge to Economic Dynamics

In this section we make precise the observations about identification made in the introduction. Let

\[
    u_t \equiv s_t - P_0 - Ps_{t-1},
\]

be the disturbances in the time series representation of the wedges. Then,

\[
    V = E u_t u_t' = \begin{bmatrix} \bar{Q} \bar{Q}^t & 0 \\ 0 & q_{44}^t \end{bmatrix},
\]

where \( \bar{Q} \) is the unique lower triangular Choleski decomposition of the upper 3 \times 3 block of \( V \).

Let the fundamental economic shocks be denoted \( e_t \). We suppose, for the purpose of our illustration, that there is exactly one shock that originates inside each wedge, and that the third element in \( e_t \) corresponds to that shock.11 We suppose that \( u_t \) and \( e_t \) are related as follows12:

\[
    u_t = C e_t, \quad E e_t e_t' = I, \quad CC' = V.
\]

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10 This is the ratio of the log difference in investment to the log difference in the Dow, over the period indicated. Both variables were in nominal terms.

11 In an agency cost model, these shocks could be perturbations to the variance of idiosyncratic disturbances affecting entrepreneurs, or to the survival rate of entrepreneurs. See Christiano, Motto and Rostagno (2004, 2005) for examples.

12 Implicitly, we rule out the identification problems discussed by Fernandez-Villaverde, Rubio-Ramirez and Sargent (2005). In doing so, we put the CKM analysis in the best possible light.
There are many $C$’s which satisfy this condition. In particular, write

$$ C = \begin{bmatrix} QW & 0 & 0 \\ 0 & 0 & q_{44} \end{bmatrix}, $$

where $W$ is any orthonormal matrix. Each $C$ in the above class is observationally equivalent, in that it implies the same likelihood value. However, each $C$ implies a different $e_t$. To see this, note that for any sequence of fitted disturbances, $u_t$, one can recover a time series of $e_t$ using

$$ e_t = C^{-1}u_t = \begin{bmatrix} W^{-1}Q^{-1} & 0 \\ 0 & q_{44}^{-1} \end{bmatrix}. \quad (4.3) $$

To see how many $e_t$’s there are, for given $V$ and sequence $u_t$, let

$$ a(\theta_1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta_1) & -\sin(\theta_1) \\ 0 & \sin(\theta_1) & \cos(\theta_1) \end{bmatrix}, $$

$$ b(\theta_2) = \begin{bmatrix} \cos(\theta_2) & 0 & \sin(\theta_2) \\ 0 & 1 & 0 \\ -\sin(\theta_2) & 0 & \cos(\theta_2) \end{bmatrix}, $$

$$ c(\theta_3) = \begin{bmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 \\ \sin(\theta_3) & \cos(\theta_3) & 0 \\ 0 & 0 & 1 \end{bmatrix}, $$

$$ W(\theta) = a(\theta_1)b(\theta_2)c(\theta_3), $$

where $\theta = (\theta_1, \theta_2, \theta_3)$ and

$$ \theta \in D \equiv \{ \theta : \theta_i \in [0, 2\pi], i = 1, 2, 3 \}. $$

Note that since $a$, $b$, and $c$ are orthonormal, it follows that $W(\theta)$ is too, for each $\theta \in D$. For a fixed set of observed $u_t$, $t = 1, ..., T$, there is a different sequence, $e_t$, $t = 1, ..., T$, associated with each $\theta \in D$. The likelihood of the data is invariant to $\theta \in D$.

We adopt the normalization that the third element in $e_t$ is the innovation in the shock originating in the financial wedge. Each $\theta \in D$ adopts a different interpretation of the cross-shock correlations in the upper left block of $V$. For example, when $\theta = (0, 0, 0)$, then $e_t = \varepsilon_t$. In this case, the third element of $u_t$ is entirely due to the innovation in the shock in the financial wedge, and all the correlation between the innovations in the wedges is attributed to causation going from the shock in the financial wedge to the other wedges.

We will show that, because the data suggest that the off-diagonal terms in the upper $3 \times 3$ block of $V$ are non-trivial, there are many different answers to question (i) in the introduction: ‘what is the importance of the financial frictions shock?’ Each is equally likely, according to the likelihood function of the data.

5. Computing the Importance of the Intertemporal Wedge

In this section, we describe three procedures for computing the role of the intertemporal wedge in the dynamics of output, investment and employment in the 1930s. Each procedure
corresponds to a different way to isolate what would have happened if only the intertemporal wedge had been operative and none other. The first procedure (the ‘rotation decomposition’) is designed to illustrate our observations about identification in the previous section. The second two procedures (the ‘impulse decomposition’ and the ‘expectation decomposition’) are alternative representations of the approach taken in CKM. Since the latter two produce roughly the same results, we only report results based on the impulse decomposition in the next section.

Our rotation procedure is as follows. We compute $u_t, t = 1929, ..., 1939$, using (4.1). Then, we fix $W(\theta)$ for some $\theta \in D$ and compute the implied sequence, $e_t$, for $t = 1929, ..., 1939$ using (4.3). Then, to identify the importance of the shocks originating in the financial wedge, we set to zero all elements in $e_t$ but the third one. After that, we compute the implied sequence of disturbances, $u_t^W, t = 1929, ..., 1939$ using (4.2). Here, the superscript $W$ appears in order to highlight dependence on the orthonormal matrix, $W$.

For input into our state space - observer system, (3.3)-(3.2), we require $\varepsilon_t$. We compute a sequence, $\varepsilon_t^W, t = 1929, ..., 1939$ using $\varepsilon_t^W = Q^{-1}u_t^W$. Thus, the mapping from the disturbances, $u_t$, and $\theta \in D$, to $\varepsilon_t^W$ is given by:

$$\varepsilon_t^W = \begin{bmatrix} Q^{-1} & 0 & 0 \\ 0 & q_{44}^{-1} & QW(\theta) \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} W(\theta)^T \bar{Q}^{-1} & 0 \\ 0 & q_{44}^{-1} \end{bmatrix} u_t,$$

for $t = 1929, ..., 1939$. For each sequence, $\varepsilon_t^W, t = 1929, ..., 1939$, we simulate (3.3)-(3.2) to obtain a time series on what investment, output and employment would have been had only the shocks originating in the intertemporal wedge (i.e., the third element of $e_t$) been active. We then compute the sum of squares of the discrepancy between the simulated investment, output and employment data, and choose $\theta \in D$ to minimize that discrepancy.13

The two versions of the CKM procedure are different ways to simulate the system in response to the realized intertemporal wedge, holding the other wedges fixed at their 1929 values. We interpret this as reflecting the following two identification assumptions. The first is that the whole of the movement in the intertemporal wedge reflects shocks originating in the financial friction sector and the second is the notion that financial friction shocks do not induce movements in the other wedges.

For the impulse decomposition, we find the sequence, $\varepsilon_t, t = 1929, ..., 1930$ which has the property that when this is input into (2.5), the third element of the simulated $s_t, t = 1929, ..., 1930$, is set to its estimated values and the other elements of $s_t$ are fixed at their 1929 value. For the expectation decomposition, we adopt a different time series representation for $s_t$. We suppose that only the third element of $s_t$ is stochastic, while the other wedges are simply constants at their 1929 values. The stochastic process that we adopt for the intertemporal wedge is the actual one obtained during the estimation. It is (2.5) itself, where the elements of $s_t$ other than the third are simply treated as information variables.

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13Our strategy of selecting a rotation based on the implied dynamic properties of a model resembles identification using sign restrictions, applied in, for example, Uhlig (2002).
6. Results

The results are presented in Figures 1-3 and Tables 1-3. The results in each of these figures is based on the log-linear approximation to model solutions. The information in the tables indicates that our results are robust to working with a second-order approximation.

Consider Figure 1 first, which is based on the absence of investment adjustment costs, \( a = 0 \). The first column reproduces the CKM results when \( a = 0 \). In the first, second and third rows of graphs, the solid lines indicate the actual output, investment and hours worked data, after removal of a log-linear trend. The lines with a ‘+’ indicate the movements in each variable that is attributed to the intertemporal wedge. Note that the movement in output due to the intertemporal wedge, identified as CKM do, is trivial. The movement in investment and hours worked due to the intertemporal wedge are slightly greater, but still not very substantial. This corresponds to results reported in CKM.\(^{14}\)

The second column of Figure 1 displays results based on our rotation decomposition. The results there show that it is possible to identify a rotation such that 100% of the movement in output, investment and hours throughout the 1930s is due to the intertemporal wedge. This illustrates our observations about identification. In particular, there is correlation among the wedges and without further structure, one is free to interpret this as causation passing from shocks originating in the intertemporal wedge, and inducing movements in the other wedges. However, it is not clear that this finding poses a problem for the CKM conclusion. It is perhaps fair to interpret the results in the first and second columns of Figure 1 as indicating that if there are financial frictions, they manifest themselves primarily in non-intertemporal wedges. CKM emphasize that financial frictions of this kind are consistent with the conclusions of their analysis. Of course, in practice the analyst would be left guessing about whether this is a fair conclusion.

Figure 2 repeats the calculations in Figure 1, with adjustment costs. Note that the results are now very different. The results in the first column indicate that by the CKM metric, the drop in output due to the intertemporal wedge is about 12 percent, or about 1/3 of the total fall in output. Under the CKM identification, this suggests the intertemporal wedge is very important. The wedge is even more important when we consider the fall in hours worked and investment.

Column 2 provides illustrates the identification problem. The drop in output, investment and hours worked in the first column of Figure 2 overstate the importance of the intertemporal wedge to the extent that the movement in the intertemporal wedge reflects induced effects from shocks originating in other wedges. At the other extreme, the results in the first column of Figure 2 understate the importance of shocks emanating from the intertemporal wedge to the extent that those shocks also induce movements in the other wedges. To see how important this second possibility can be, consider the second column of Figure 2. There we report the results based on a rotation in which shocks originating in the intertemporal wedge account for 100 percent of the movement in output, investment and hours worked throughout the 1930s. The fact is that we cannot, using this framework, tell whether the intertemporal wedge accounts for 100 percent of the fall in output, 50 percent, or zero. A

\(^{14}\)Actually, CKM report a slight rise in output, when the results are based on a non-linear solution of the model. When we solve the model by a second order perturbation method, we find the same result. This will be evident in the discussion of Table 1, below.
rotation can be found to yield either conclusion, and given the lack of structure in BCA, it is not possible to make a meaningful choice between them.

To gain intuition into why it is that with $a > 0$ the importance of the intertemporal wedge increases, consider Figure 3. The bottom row applies to the case, $a = 0$. The left column shows our estimate of the anticipated rate of return on capital, $E_t (1 + R^k_{t+1})$, for the decade of the 1930s. Note how this quantity falls during this period. Note too, from the right column, that there is very little movement in the intertemporal wedge, $\tau^k_t$. Now consider the first row of graphs. With $a > 0$, $E_t (1 + R^k_{t+1})$ actually rises as the economy falls into Depression. Note that now there is a sharp rise in the intertemporal wedge. These findings support the intuition described in the introduction.

Tables 1-3 summarize our findings in tabular form. In addition, they provide evidence on the robustness of our results to using a second-order approximation to our model solution. Each entry in these tables is:

$$\frac{1 - \text{data(wedge)}}{1 - \text{data}} \times 100.$$ 

Here, ‘data(wedge)’ denotes an estimate of what the (linearly detrended) data would have been if only the intertemporal wedge had been operative, normalized to unity in 1929. Also, ‘data’ denotes the actual data, normalized by its 1929 value.

Consider Table 1, which pertains to GNP only. The first two columns pertain to the no adjustment cost case, in which case Tobin’s $q$ is infinite. The results that pertain to the linear approximation are transformations on the numbers displayed in Figure 1. Note how the percent of the decline in output accounted for by the intertemporal wedge is relatively small. When we approximate the model solution with a second order approximation, the fall is even less. Indeed, with the CKM identifying assumptions we conclude that the intertemporal wedge alone acted to raise output. Our findings confirm the results in CKM, who show that when a nonlinear approximation is used to solve the model, the drop in output due to the wedge is smaller and actually becomes a rise.

Note the very sharp contrast in the results when we move to the last two columns in Table 1. When adjustment costs are incorporated into the analysis, and we adopt the CKM identifying assumptions, we find that the intertemporal wedge accounts for 25-40 percent of the drop in output. Both linear and nonlinear approximations generate similar results. The results in Table 2 show that with adjustment costs, under the CKM identification assumptions the intertemporal wedge accounts for a huge portion of the decline in investment. According to Table 3, the fraction of the drop in hours worked due to the intertemporal wedge is roughly similar to what we found for output.

In sum, we find that with adjustment costs and with the CKM identification assumptions, the intertemporal wedge has a substantial impact on the major macroeconomic variables during the Great Depression. However, these results are dependent on the CKM identification assumptions. Other identification assumptions are observationally equivalent. There are some that support the view that the intertemporal wedge had virtually nothing to do with the Great Depression. Others support the view that the only wedge that mattered was the intertemporal wedge.

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15We estimated this variable by including it in the state evolution of our Kalman filter system, and then using the two-sided Kalman smoother.
7. Conclusion

Chari, Kehoe and McGrattan (2005) advocate the use of ‘wedges’ to identify directions for improvement in equilibrium models. As a demonstration of the power of the approach they argue that it can be used to rule out a prominent class of explanations for the Great Depression. In particular, they conclude that models of financial frictions that create wedges in the intertemporal Euler equation are not promising avenues for understanding the dynamics of the Great Depression.

We show that the conclusions are not robust to a small change in CKM’s model environment. In particular, when a modest amount of investment adjustment costs are introduced, we find that the CKM findings about financial frictions are reversed. We also show that business cycle accounting suffers from a significant identification problem: determining the importance of a wedge requires identifying assumptions, and there is no way to choose between the alternative possibilities. The problem is similar to the well known problem of identification in a vector autoregression (VAR). Fundamental economic shocks are not identified in VARs. Additional restrictions must be incorporated from economic theory if questions about the importance of particular economic shocks, such as financial frictions shocks, are to be answered.

Fortunately, there are other methods to identify promising avenues for model development. Computational methods, software and econometric methods have now reached a stage where it is straightforward to specify and evaluate alternative model specifications quickly and efficiently. There is no reason to guess what these explorations are likely to produce using a method like business cycle accounting.16

16 This strategy for determining the importance of financial frictions is pursued by Christiano, Motto and Rostagno (2004, 2005). They use likelihood methods to investigate the role of financial frictions as sources of shocks and propagation in the US Great Depression and in recent business cycles in the Euro Area and the US.
Table 1: Movements in GDP Accounted for by Intertemporal Wedge

<table>
<thead>
<tr>
<th>Year</th>
<th>$q$ Elasticity $= \infty$</th>
<th>$q$ Elasticity $= \infty$</th>
<th>$q$ Elasticity $= 1$</th>
<th>$q$ Elasticity $= 1$</th>
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</tr>
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</tr>
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<tr>
<td>1935</td>
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<tr>
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Table 2: Movements in Investment Accounted for by the Intertemporal Wedge

<table>
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<td>52.07</td>
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Table 3: Movements in Hours Worked Accounted for by the Intertemporal Wedge

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<td>1939</td>
<td>0.96</td>
<td>-5.04</td>
<td>16.99</td>
<td>15.72</td>
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8. Appendix A: The Carlstrom-Fuerst Financial Friction Wedge

This section considers a version of the CF model, modified to include the adjustment costs in capital considered in CKM. We identify the version of the RBC model with wedges whose equilibrium coincides with that of the CF model with adjustment costs. We state the result as a proposition. For the RBC wedge economy to have literally the same equilibrium as the CF economy with adjustment costs requires several wedges and other adjustments. We then describe the parameter settings required in the original CF model to ensure that the adjustments primarily take the form of a wedge in the intertemporal Euler equation, and nowhere else. In this respect we follow the approach taken in CKM. To simplify the notation, we set the population growth rate to zero throughout the discussion in the appendix.

8.1. RBC Model With Adjustment Costs

To establish a baseline, we describe the version of the RBC model with adjustment costs. Preferences are:

\[ E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t). \]

The resource constraint and the capital accumulation technology are, respectively,

\[ c_t + x_t \leq k_t^\alpha (Z_t l_t)^{1-\alpha} \]  
(8.1)

and

\[ k_{t+1} = (1 - \delta) k_t + x_t - \Phi \left( \frac{x_t}{k_t} \right) k_t. \]  
(8.2)

The first order necessary conditions for optimization are:

\[ \frac{-u_{c,t}}{u_{c,t}} = (1 - \alpha) \left( \frac{k_t}{l_t} \right)^\alpha Z_t^{1-\alpha} \]  
(8.3)

\[ 1 = \beta E_t \frac{u_{c,t+1}}{u_{c,t}} (1 + R_{t+1}^k), \]  
(8.4)

where the gross rate of return on capital is:

\[ 1 + R_{t+1}^k = \alpha \left( \frac{k_{t+1}}{Z_{t+1} l_{t+1}} \right)^{\alpha-1} + P_{k,t+1}, \]

where \( P_{k,t} \) and \( P_{k,t+1} \) are given in (2.9) and (2.10).

In the following two subsections, we argue that the CF financial frictions act like a tax on the gross return on capital, \( 1 + R_{t+1}^k \), in (8.4). In particular, \( 1 + R_{t+1}^k \) is replaced by

\[ (1 + R_{t+1}^k) \left( 1 - \tau_t^k \right). \]

This statement is actually only true as an approximation. Below we state, as a proposition, what the exact RBC model with wedges is, which corresponds to the CF model. We then explain the sense in which the wedge equilibrium just described is an approximation.
8.2. The CF Model With Adjustment Costs

Here, we develop the version of the CF model in which there are adjustment costs in the production of new capital. The economy is composed of firms, an \( \eta \) mass of entrepreneurs and a mass, \( 1 - \eta \), of households. The sequence of events through the period proceeds as follows. First, the period \( t \) shocks are observed. Then, households and entrepreneurs supply labor and capital to competitive factor markets. Because firm production functions are homogeneous, all output is distributed in the form of factor income. Households and entrepreneurs then sell their used capital on a capital market. The total net worth of households and entrepreneurs at this point consists of their earnings of factor incomes, plus the proceeds of the sale of capital. Households divide this net worth into a part allocated to current consumption, and a part that is deposited in the bank. Entrepreneurs apply their entire net worth to a technology for producing new capital. They produce an amount of capital that requires more resources than they can afford with only their own net worth. They borrow the rest from banks. At this point the entrepreneur experiences an idiosyncratic shock which is observed to him, while the bank can only see it by paying a monitoring cost. This creates a conflict between the entrepreneur and the bank which is mitigated by the bank extending the entrepreneur a standard debt contract. After capital production occurs, entrepreneurs sell the new capital, and pay off their bank loan. Households receive a return on their deposits at the bank, and use the proceeds to purchase new capital. Entrepreneurs use their income after paying off the banks to buy consumption goods and new capital. All the newly produced capital is purchased by households and entrepreneurs, and all the economy’s consumption goods are consumed. The next period, everything starts all over.

We now provide a formal description of the economy. The household problem is

\[
\max_{\{c_t, k_{c,t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t, l_t),
\]

subject to:

\[
c_t + q_t k_{c,t+1} \leq w^e_t l_t + [r_t + P_{k,t}] k_{c,t}
\]  

(8.5)

where \( c_t \) and \( k_{c,t} \) denote household consumption and the household stock of capital, respectively. In addition, and \( l_t \) denotes household employment, \( w^e_t \) denotes the household’s competitive wage rate, \( P_{k,t} \) denotes the price of used capital and \( q_t \) denotes the price of capital available for production in the next period (the reason for not denoting this price by \( P_{k',t} \) will be clear momentarily). After receiving their period \( t \) income, households allocate their net worth (the right side of (8.5)) to \( c_t \) and the rest, \( w^e_t l_t + [r_t + P_{k,t}] k_{c,t} - c_t \), is deposited in a bank. These deposits earn a rate of return of zero. This is because markets are competitive and the opportunity cost to the household of the output they lend to the bank is zero. Later in the period, when the deposit matures, the households use the principal to purchase \( k_{c,t+1} \) units of capital. The first order conditions of the household are (8.5) with the equality strict and:

\[
1 = \sum_{t} E_t \beta^t u_{c,t+1} \left[ \frac{r_{t+1} + P_{k,t+1}}{q_t} \right] \]

(8.6)

\[
\frac{-u_{l,t}}{u_{c,t}} = w^e_t.
\]

(8.7)
where \( u_{c,t} \) and \(-u_{l,t}\) denote the time \( t \) marginal utilities of consumption and leisure, respectively.

The \( \eta \) entrepreneurs’ present discounted value of utility is:

\[
E_0 \sum_{t=0}^{\infty} (\beta \gamma)^t c_{et}.
\]

After the period \( t \) shocks are realized, the net worth of entrepreneurs, \( a_t \), is:

\[
a_t = w_t^e + \left[ r_t + P_{k,t} \right] k_{et},
\]

where \( w_t^e \) is the wage rate earned by the entrepreneur, who inelastically supplies his one unit of labor. The entrepreneur uses the \( a_t \) consumption goods, together with a loan from the bank to purchase the inputs into the production of capital goods. Entrepreneurs have access to the technology for producing capital, (2.6) and (2.7). The technology proceeds in two stages. In the first stage, the entrepreneur produces an intermediate input, \( i_t \). In the second stage, that input results in \( \omega_i \) units of capital, which has a price, in consumption goods, \( q_t \).

The random variable, \( \omega \), is independently distributed across entrepreneurs, has mean unity, and cumulative density function,

\[
\Psi(z) \equiv \text{prob}[\omega \leq z].
\]

The entrepreneur who wishes to produce \( i_t \) units of the capital input faces the following cost function:

\[
C(i_t; P_{k,t}) = \min_{\varphi_{k,t}, \varphi_{x,t}} P_{k,t} \varphi_{k,t} + \varphi_{x,t} + \lambda_t \left[ i_t - (1 - \delta) \varphi_{k,t} - \varphi_{x,t} + \Phi \left( \frac{\varphi_{x,t}}{\varphi_{k,t}} \right) \varphi_{k,t} \right],
\]

where the constraint is that the object in square brackets is no less than zero. In addition, \( \varphi_{k,t} \) and \( \varphi_{x,t} \) denote the quantity of old capital and investment goods, respectively, purchased by the entrepreneur. The first order conditions for \( \varphi_{k,t} \) and \( \varphi_{x,t} \) are:

\[
P_{k,t} = \lambda_t \left[ 1 - \delta - \Phi \left( \frac{\varphi_{x,t}}{\varphi_{k,t}} \right) + \Phi' \left( \frac{\varphi_{x,t}}{\varphi_{k,t}} \right) \varphi_{x,t} \right], \quad (8.8)
\]

\[
1 = \lambda_t \left[ 1 - \Phi' \left( \frac{\varphi_{x,t}}{\varphi_{k,t}} \right) \right], \quad (8.9)
\]

respectively. The reason for denoting the time \( t \) price of old capital by \( P_{k,t} \) is now apparent. Substituting out for \( \lambda_t \) in (8.8) from (8.9), we see that the formula for \( P_{k,t} \) here coincides with with the one implied by (2.9)-(2.10). The reason for not denoting the price of new capital by \( P_{k,t} \) is also apparent. Comparing (8.9) with (2.9)-(2.10) we see that the formula for \( \lambda_t \) coincides with the formula for \( P_{k,t} \) implied by (2.9)-(2.10). However, the equilibrium value of \( q_t \) will not coincide with \( \lambda_t \) here. This is because \( \lambda_t \) does not capture all the costs of producing new capital. It measures the marginal costs implied by the production technology. However, it is missing the marginal cost that arises from the conflict between entrepreneurs and banks, which has the consequence that the production of capital necessarily involves some monitoring, and therefore also involves some destruction of capital.
Solving (8.9) for $x_t/k_t$ in terms of $\lambda_t$, and using the result to substitute out for $\varphi_{x,t}/\varphi_{k,t}$ in (8.8):

$$P_{k,t} = \lambda_t \left[ (1 - \delta) - \frac{a}{2} \left( \frac{1}{\lambda_t} - 1 \right)^2 + a \left( \frac{1}{\lambda_t} - 1 \right) \left( \frac{1}{\lambda_t} + b \right) \right]$$

Solving this for $\lambda_t$, provides the marginal cost function for producing $i_t$:

$$\lambda_t = \lambda(P_{k,t}) \quad (8.10)$$

Because all entrepreneurs face the same $P_{k,t}$, they will all choose the same ratio, $\varphi_{x,t}/\varphi_{k,t}$, regardless of the scale of production, $i_t$. Moreover, that ratio must be equal to the ratio of aggregate investment to the aggregate stock of capital.

The constant returns to scale feature of the production function implies that the total cost of producing $i_t$ is:

$$C(i_t; P_{k,t}) = \frac{1}{2} \lambda(P_{k,t}) i_t a > 0$$

Consider an entrepreneur who has $a_t$ units of goods and wishes to produce $i_t \geq a_t$, so that the entrepreneur must borrow $\lambda(P_{k,t}) i_t - a_t$ from the bank. Following CF, we suppose that the entrepreneur receives a standard debt contract. This specifies a loan amount and an interest rate, $R^a_t$, in consumption units. If the revenues of the entrepreneur turn out to be too low for him to repay the loan, then the entrepreneur is ‘bankrupt’ and he simply provides everything he has to the bank. In this case, the bank pays a monitoring cost which is proportional to the scale of the entrepreneur’s project, $\mu i_t^a$. We now work out the equilibrium value of the parameters of the standard debt contract.

The value of $\omega$ such that entrepreneurs with smaller values of $\omega$ are bankrupt, is $\bar{\omega}_t^a$, where

$$[\lambda(P_{k,t}) i_t - a_t] R^a_t = \bar{\omega}_t^a i_t q_t.$$

Using this we find that the average, across all entrepreneurs with asset level $a_t$, of revenues is:

$$i_t q_t \int_0^\infty \omega dF(\omega) - \int_0^{\bar{\omega}_t^a} R^a_t (\lambda(P_{k,t}) i_t^a - a_t) dF(\omega) - i_t q_t \int_0^{\bar{\omega}_t^a} \omega dF(\omega)$$

$$= i_t q_t f(\bar{\omega}_t^a),$$

where

$$f(\bar{\omega}_t^a) = \int_{\bar{\omega}_t^a}^\infty \omega d\Phi(\omega) - \bar{\omega}_t^a (1 - \Phi(\bar{\omega}_t^a)).$$

The average receipts to banks, net of monitoring costs, across loans to all entrepreneurs with assets $a_t$ is:

$$q_t i_t^a \left[ \int_0^{\bar{\omega}_t^a} \omega d\Phi(\omega) - \mu \Phi(\bar{\omega}_t^a) \right] + [\lambda(P_{k,t}) i_t - a_t] R^a_t [1 - \Phi(\bar{\omega}_t^a)]$$

$$= q_t i_t^a g(\bar{\omega}_t^a),$$
where

\[ g(\omega_t^a) = \int_0^\omega_t^a \omega d\Phi(\omega) - \mu \Phi(\omega_t^a) + \omega_t^a [1 - \Phi(\omega_t^a)]. \]

The contract with entrepreneurs with asset levels, \( a_t \), that is assumed to occur in equilibrium is the one that maximizes the expected state of the entrepreneur at the end of the contract, subject to the requirement that the bank be able to pay the household a gross rate of interest of unity. The interval during which the entrepreneur produces capital is one in which there is no alternative use for the output good. So, the condition that must be satisfied for the bank to participate in the loan contract is:

\[ q_t i_t g(\omega_t^a) \geq \lambda(P_{k,t}) i_t - a_t. \]

The contract solves the following Lagrangian problem:

\[ \max_{\omega_t^a, i_t} i_t q_t f(\omega_t^a) + \mu [q_t i_t g(\omega_t^a) - \lambda(P_{k,t}) i_t + a_t]. \]

The first order conditions for \( \omega_t^a \) and \( i_t \) are, after solving out for \( \mu \) and rearranging:

\[ q_t f(\omega_t^a) = \frac{f'(\omega_t^a)}{g'(\omega_t^a)} [q_t g(\omega_t^a) - \lambda(P_{k,t})] \quad (8.11) \]

\[ i_t = \frac{a_t}{\lambda(P_{k,t}) - q_t g(\omega_t^a)} \quad (8.12) \]

From (8.11), we see that \( \omega_t^a = \bar{\omega}_t \) for all \( a_t \), so that \( R_t^a = R_t \) for all \( a_t \). It then follows from (8.12) that the loan amount is proportional to \( a_t \). As in the no adjustment case in CF, these two properties imply that in studying aggregates, we can ignore the distribution of assets across entrepreneurs.

The expected net revenues of the entrepreneurs, expressed in terms of \( a_t \), are, after making use of (8.12):

\[ i_t q_t f(\omega_t) = \frac{q_t f(\bar{\omega}_t)}{\lambda(P_{k,t}) - q_t g(\omega_t)} a_t. \quad (8.13) \]

At the end of the period, after the debt contract with the bank is paid off, the entrepreneurs who do not go bankrupt in the process of producing capital have income that can be used to buy consumption goods and new capital goods:

\[ c_{et} + q_t k_{et+1} = \begin{cases} 
  i_t q_t \omega - R_t (\lambda(P_{k,t}) i_t - a_t) & \omega \geq \bar{\omega}_t \\
  0 & \omega < \bar{\omega}_t 
\end{cases}. \quad (8.14) \]

An entrepreneur who is bankrupted in period \( t \) must set \( c_{et} = 0 \) and \( k_{et+1} = 0 \). In period \( t + 1 \), these entrepreneurs start with net worth \( a_{t+1} = w_{t+1} \). Entrepreneurs who are not bankrupted in period \( t \) can purchase positive amounts of \( c_{et} \) and \( k_{e,t+1} \) (except in the non-generic case, \( \omega = \bar{\omega}_t \)). For these entrepreneurs, the marginal cost of purchasing \( k_{e,t+1} \) is \( q_t \) units of consumption. The time \( t \) expected marginal payoff from \( k_{e,t+1} \) at the beginning of period \( t + 1 \) is \( E_t [r_{t+1} + P_{k,t+1}] \). In each aggregate state in period \( t + 1 \), the entrepreneur expands his net worth by the value of \( [r_{t+1} + P_{k,t+1}] \) in that state. This extra net worth can be leveraged into additional bank loans, which in turn permit an expansion in the
entrepreneur’s payoﬀ by investing in the capital production technology. The expected value of this additional payoﬀ (relative to date \( t + 1 \)) idiosyncratic uncertainty) corresponds to the coefﬁcient on \( a_t \) in (8.13). So, the expected rate of return available to entrepreneurs who are not bankrupt in period \( t \) is:

\[
E_t \left[ \frac{r_{t+1} + P_{k,t+1}}{q_t} \times \zeta_{t+1} \right],
\]

which they equate to \( 1/(\beta \gamma) \). Here,

\[
\zeta_{t+1} = \max \left[ \frac{q_{t+1} f (\tilde{\omega}_{t+1})}{\lambda (P_{k,t+1}) - q_{t+1} g (\tilde{\omega}_{t+1})}, 1 \right].
\]

The expression to the left of ‘\( \times \)’ in (8.15) is the rate of return enjoyed by ordinary households. The reason that \( \zeta_{t+1} \) cannot be less than unity is that an entrepreneur can always obtain unity, simply by consuming his net worth in the following period and not producing any capital. Averaging over all budget constraints in (8.14):

\[
c_{et} + q_t k_{et+1} = \frac{q_t f (\tilde{\omega}_t)}{\lambda (P_{k,t}) - q_t g (\tilde{\omega}_t)} a_t.
\]

Here, \( c_{et} \) and \( k_{et+1} \) refer to averages across all entrepreneurs.

Output is produced by goods-producers using a linear homogeneous technology,

\[
y (k_t, l_t, \eta, Z_t) = k_t^\alpha ((1 - \eta) Z l_t)^{1 - \alpha - \zeta} \eta^\zeta,
\]

where \( k_t \) is the sum of the capital owned by households and the average capital held by entrepreneurs:

\[
k_t = (1 - \eta) k_{et} + \eta k_{e,t}.
\]

The argument, \( \eta \), in \( y \) is understood to apply to the second occurrence of \( \eta \). The arguments in the production function reﬂect our assumption that the entrepreneur supplies one unit of labor, and households supply \( l_t \) units of labor. Profit maximization implies:

\[
y_{k,t} = r_{t}, \: y_{lt} = w^c_{it}, \: y_{3,t} = w^e_{it}.
\]

We now collect the equilibrium conditions for the economy. The production of new capital goods by the average entrepreneurs is:

\[
i_t \int_{0}^{\infty} \omega dF (\omega) - \mu i_t \int_{0}^{\tilde{\omega}_t} dF (\omega)
\]

\[
= i_t [1 - \mu F (\tilde{\omega}_t)].
\]

Since there are \( \eta \) entrepreneurs, the total new capital produced is \( k_{t+1} = \eta i_t \left[ 1 - \mu F (\tilde{\omega}_t) \right] \), so that

\[
k_{t+1} = \left[ (1 - \delta) k_t + x_t - \Phi \left( \frac{x_t}{k_t} \right) k_t \right] [1 - \mu F (\tilde{\omega}_t)].
\]

The resource constraint is:

\[
(1 - \eta) c_t + \eta c^e_t + x_t = k_t^\alpha ((1 - \eta) Z_t l_t)^{1 - \alpha - \zeta} \eta^\zeta.
\]
Substituting (8.17) into (8.6) and (8.7):

\[
1 = \beta E_t \frac{u_{c,t+1} y_{k,t+1} + P_{k,t+1}}{q_t} \tag{8.20}
\]

\[
\frac{-u_{l,t}}{u_{c,t}} = y_{l,t}. \tag{8.21}
\]

The budget constraint of the entrepreneur is (after multiplying by \(\eta\)):

\[
c_{e,t} + q_t k_{e,t+1} = \lambda (P_{k,t}) \frac{k_{t+1}}{\eta [1 - \mu F(\bar{\omega}_t)]} q_t f(\bar{\omega}_t) \tag{8.22}
\]

The efficiency conditions associated with the contract are:

\[
q_t f'(\bar{\omega}_t) = \frac{f'(ar{\omega}_t)}{g'(ar{\omega}_t)} [q_t g(\bar{\omega}_t) - \lambda (P_{k,t})] \tag{8.23}
\]

\[
\frac{k_{t+1}}{\eta [1 - \mu F(\bar{\omega}_t)]]} = \frac{a_t}{\lambda (P_{k,t}) - q_t g(\bar{\omega}_t)} \tag{8.24}
\]

\[
a_t = y_{3,t} + y_{k,t} k_{e,t} + P_{k,t} k_{e,t} \tag{8.25}
\]

The intertemporal efficiency condition for the entrepreneur is (assuming the condition, \(\zeta_{t+1} \geq 1\), is not binding):

\[
E_t \left[ F_{k,t+1} + P_{k,t+1} \frac{q_{t+1} f(\bar{\omega}_{t+1})}{\lambda (P_{k,t+1}) - q_{t+1} g(\bar{\omega}_{t+1})} \right] = \frac{1}{\gamma \beta} \tag{8.26}
\]

Taking the ratio of (8.8) to (8.9), we obtain:

\[
P_{k,t} = \frac{1 - \delta - \Phi \left( \frac{\bar{\omega}_t}{k_t} \right) + \Phi' \left( \frac{\bar{\omega}_t}{k_t} \right) \frac{\bar{\omega}_t}{k_t}}{1 - \Phi' \left( \frac{\bar{\omega}_t}{k_t} \right)} \tag{8.27}
\]

The 10 variables to be determined with the 10 equations, (8.18)-(8.27) are: \(c_t, c_{e,t}, x_t, k_t, k_{e,t}, l_t, P_{k,t}, q_t, \bar{\omega}_t, a_t\).

It is convenient to define a sequence of markets equilibrium formally. Let \(s^t\) denote a history of realizations of shocks. Then,

**Definition 8.1.** An equilibrium of the CF economy with adjustment costs is a sequence of prices, \(\{P_k(s^t), q(s^t), w^e(s^t), w^c(s^t), r(s^t)\}\), quantities, \(\{c(s^t), c_e(s^t), x(s^t), k(s^t), k_e(s^t), l(s^t), a(s^t)\}\), and \(\{\bar{\omega}(s^t)\}\) such that:  

(i) Households optimize (see (8.20), (8.21))  
(ii) Entrepreneurs optimize (see (8.22), (8.25), (8.26), (8.27))  
(iii) Firms optimize (see (8.17))  
(iv) Conditions related to the standard debt contract are satisfied (see (8.23), (8.24))  
(v) The resource constraint and capital accumulations equations are satisfied (see (8.18), (8.19))
8.3. The CF Model as an RBC Model with Wedges

We now construct a set of wedges for the RBC economy, such that the equilibrium for that economy coincides with the equilibrium for the CF economy. We begin by constructing the following state-contingent sequences:

\[ \psi (s^t) = 1 - \mu F (\bar{\omega} (s^t)) , \quad (8.28) \]
\[ \tau_x (s^t) = \frac{\psi (s^t) \tilde{q} (s^t)}{\lambda \left( \tilde{P}_{k,t} (s^t) \right)} - 1 , \]
\[ \theta (s^t) = \frac{\tilde{P}_k (s^t) \tau_x (s^t)}{\tilde{r} (s^t)} , \]
\[ G (s^t) = \eta (\bar{c}^e (s^t) - \bar{c} (s^t)) , \]
\[ T (s^t) = G (s^t) - \tau_x (s^t) \bar{x} (s^t) - \theta (s^t) \tilde{r} (s^t) \tilde{k} (s^{t-1}) , \]
\[ D (s^t) = \tilde{w}^e (s^t) \eta , \]

where \( \tilde{q}, \bar{c}^e, \bar{c}, \tilde{w}^e, \tilde{r}, \tilde{k}, \tilde{x}, \bar{\omega} \) and \( \tilde{P}_k \) correspond to the objects without ‘\( \tilde{\cdot} \)’ in a CF equilibrium. Also, \( \lambda \) is the function defined in (8.10). In this subsection, we treat \( D, \psi, \theta, \tau_x, G \) and \( T \) as given exogenous stochastic processes. Here, \( D, G, \) and \( T \) represent exogenous sequences of profits, government spending and lump sum taxes. Also, \( \theta \) and \( \tau_x \) are tax rates on capital rental income and investment good purchases. Finally, \( \psi \) is a technology shock in the production of physical capital.

Consider the following budget constraint for the household:

\[ c (s^t) + (1 + \tau_x (s^t)) x (s^t) \leq (1 - \theta (s^t)) r (s^t) k (s^{t-1}) + w (s^t) l (s^t) - T (s^t) + D (s^t) . \quad (8.29) \]

Here, \( r \) is the rental rate on capital, \( w \) is the wage rate, and \( l \) measures the work effort of the household. Each of these variables is a function of \( s^t \) and is determined in an RBC wedge economy. At time 0 the household takes prices, taxes and \( k (s^{-1}) \) as given and chooses \( c, k \) and \( l \) to maximize utility:

\[ \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi (s^t) u (c (s^t), l (s^t)) \]

subject to the budget constraint and non-negativity constraints. Here, \( \pi (s^t) \) is the probability of history, \( s^t \).

Households operate the following backyard technology to produce new capital:

\[ k (s^t) = \left[ (1 - \delta) k (s^t) + x (s^t) - \Phi \left( \frac{x (s^t)}{k (s^{t-1})} \right) k (s^{t-1}) \right] \psi (s^t) . \quad (8.30) \]
The first order necessary conditions for household optimization are:
\[
\begin{align*}
  u_t(s') + u_c(s') w(s') &= 0, \quad (8.31) \\
  1 + \tau_x(s') P_k'(s') &= \sum_{s'+1|s'} \beta \pi(s'+1|s') \frac{u_c(s'+1)}{u_c(s')} \left[ r(s'+1)(1 - \theta(s'+1)) + (1 + \tau_x(s'+1)) P_k(s'+1) \right], \quad (8.32)
\end{align*}
\]
where
\[
   P_k(s') \equiv \frac{1 - \delta - \Phi\left(\frac{x(s')}{k(s')}\right) + \Phi'\left(\frac{x(s')}{k(s')}\right) x(s')}{1 - \Phi'\left(\frac{x(s')}{k(s')}\right)}. \quad (8.33)
\]
Equation (8.31) is the first order condition associated with the optimal choice of \(l(s')\). Equation (8.32) combines the first order order conditions associated with the optimal choice of \(x(s')\) and \(k(s')\). Also,
\[
   P_{k'}(s') \equiv \frac{1}{1 - \Phi'\left(\frac{x(s')}{k(s')}\right)}, \quad (8.34)
\]
is the pre-tax marginal cost of producing new capital, in units of the consumption good. In addition, \(\pi(s'+1|s') \equiv \pi(s'+1)/\pi(s')\) is the conditional probability of history \(s'+1\) given \(s'\).

The technology for firms is taken from (8.16):
\[
   y(k, l, \eta, Z) = k^\alpha \left( (1 - \eta) Zl \right)^{1 - \alpha - \zeta} \eta^\zeta,
\]
where, as before, the third argument in \(y\) refers only to the second occurrence of \(\eta\). There are three inputs: physical capital, household labor and another factor whose aggregate supply is fixed at \(\eta\). Profit maximization leads to:
\[
   r(s') = y_k(s'), \quad w(s') = y_l(s'), \quad w^e(s') = y_\eta(s'). \quad (8.35)
\]
The household is assumed to own the representative firm, and it receives the earnings of \(\eta\) in the form of lump-sum profits, \(D(s')\). We do allow allow trade in claims on firms, a restriction that is non-binding on allocations because the households are identical.

We now state the equilibrium for the RBC wedge economy:

**Definition 8.2.** An RBC wedge equilibrium is a set of quantities, \(\{c(s'), l(s'), k(s'), x(s')\}\), and prices \(\{p_k(s'), p_{k'}(s'), r(s'), w(s')\}\), and a set of taxes, profits and government spending, \(\{G(s'), \tau_x(s'), \theta(s'), T(s')\}\), technology shocks, \(\{\delta(s'), \psi(s')\}\), such that

(i) The quantities solve the household problem given the prices, taxes, profits, government spending and the shock to the backyard investment technology

(ii) Firm optimization is satisfied

(iii) Relations (8.28) is satisfied, for given state-contingent sequences, \(\tilde{q}, \tilde{c}, \tilde{c}, \tilde{w}, \tilde{r}, \tilde{k}, \tilde{x}, \tilde{\omega}\) and \(\tilde{P}_k\).
The variables to be determined in an RBC wedge equilibrium are \(c, l, k, x, P_k, P_{k'}, r\) and \(w\). The 8 equations that can be used to determine these are (8.29)-(8.35). It is easily verified that \(c, l, k, x, P_k, r\) and \(w\) coincide with the corresponding objects in a CF equilibrium. In addition, \(P_{k'}\) coincides with \(\lambda(P_k)\) in a CF equilibrium. To see this, one verifies that the equilibrium condition in the RBC wedge economy coincides with the equilibrium conditions in the CF economy. First, (8.30) coincides with (8.18). After using (8.35), we see that (8.31) coincides with (8.21). Consider the household budget equation evaluated at equality. Substituting out for lump sum transfers:

\[
c (s^t) + (1 + \tau_x (s^t)) x (s^t) = (1 - \theta (s^t)) r (s^t) k (s^{t-1}) + w (s^t) l (s^t) + \tau_x (s^t) x (s^t) + \theta (s^t) r (s^t) k (s^{t-1}) + w^e (s^t) \eta + G (s^t),
\]

or,

\[
(1 - \eta) c (s^t) + x (s^t) + \eta c^e (s^t)
\]

Equation (8.36) coincides with (8.19). Substitute out for \(\theta\) and \(\tau_x\) from (8.28) into (8.32), and rearranging, we obtain:

\[
1 = E_t \frac{u_c (s^{t+1})}{u_c (s^t)} \left[ \frac{r (s^{t+1}) + P_k (s^{t+1})}{\frac{1 + \tau_x (s^t)}{\psi (s^t)} P_{k'} (s^t)} \right].
\]

Note that by definition of \(1 + \tau_x (s^t)\) in (8.28),

\[
\frac{1 + \tau_x (s^t)}{\psi (s^t)} P_{k'} (s^t) = \frac{P_{k'} (s^t) q (s^t)}{\lambda(P_{k',t} (s^t))}.
\]

Combining (8.10) and (8.9), we find that \(\lambda(P_{k,t} (s^t)) = P_{k'} (s^t)\), so that the household’s intertemporal Euler equation reduces to (8.20), or (after making use of (8.35)):

\[
E_t \frac{u_c (s^{t+1})}{u_c (s^t)} \left[ \frac{y k (s^{t+1}) + P_k (s^{t+1})}{P_{k'} (s^t)} (1 - \tau^k (s^t)) \right] = 1,
\]

where

\[
1 - \tau^k (s^t) = \frac{\psi (s^t)}{1 + \tau_x (s^t)}.
\]

We conclude that conditions (8.18)-(8.21) in the CF economy are satisfied. The remaining equilibrium conditions are satisfied, given (8.28). We state this result as a proposition:

**Proposition 8.3.** Consider a CF equilibrium, and a set of taxes, technology shocks and transfers computed in (8.28). The objects, \(\{c (s^t), l (s^t), k (s^t), x (s^t)\}, \{P_k (s^t), r (s^t), w (s^t)\}\) and \(P_{k'} (s^t) = \lambda(P_k (s^t))\) in the CF equilibrium correspond to an RBC wedge equilibrium.

For \(\eta\) and \(\zeta\) close to zero and \(\psi\) close to unity, the RBC wedge equilibrium converges to the equilibrium conditions of the RBC model with adjustment costs in section 8.1 with a wedge, \(1 - \tau^k\), in the intertemporal Euler equation, (8.4).
9. Appendix B: The Bernanke-Gertler-Gilchrist Financial Friction Wedge

In this section we briefly review the BGG model and derive the RBC wedge model to which it corresponds. In the model there are households, capital producers, entrepreneurs and banks. At the beginning of the period, households supply labor to factor markets, and entrepreneurs supply capital. Output is then produced and an equal amount of income is distributed among households and entrepreneurs. Households then make a deposit with banks, who lend the funds on to entrepreneurs. Entrepreneurs have a special expertise in the ownership and management of capital. They have their own net worth with which to acquire capital. However, it is profitable for them to leverage this net worth into loans from banks, and acquiring more capital than they can afford with their own resources. The source of friction is a particular conflict between the entrepreneur and the bank. In the management of capital, idiosyncratic things happen, which either make the management process more profitable than expected, or less so. The problem is that the things that happen in this process are observed only by the entrepreneur. The bank can observe what happens inside the management of capital, but only at a cost. As a result, the entrepreneur has an incentive to underreport the results to the bank, and thereby attempt to keep a greater share of the proceeds for himself. To mitigate this conflict, it is assumed that entrepreneurs receive a standard debt contract from the bank.

The capital that entrepreneurs purchase at the end of the period is sold to them by capital producers. The latter use the old capital used within the period, as well as investment goods, to produce the new capital that is sold to the entrepreneurs. Capital producers have no external financing need. They finance the purchase of used capital and investment goods using the revenues earned from the sale of new capital.

The budget constraint of households is:

\[ c_t + B_{t+1} \leq (1 + R_t) B_t + w_t l_t + T_t, \]

where \( R_t \) denotes the interest earned on deposits with the bank, \( b_t \) denotes the beginning-of-period stock of those deposits, \( w_t \) denotes the wage rate, \( l_t \) denotes employment and \( T_t \) denotes lump sum transfers. Subject to this budget constraint and a no-Ponzi condition, households seek to maximize utility:

\[ E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t). \]

Households’ first order conditions, in addition to the transversality condition, are:

\[
\frac{u_{c,t}}{u_{c,t}} = \beta E_t u_{c,t} (1 + R_{t+1})
\]

\[
\frac{-u_{l,t}}{u_{c,t}} = w_t.
\]

Firms have the following production function:

\[ y_t = k_t^\alpha \left( Z_t l_t \right)^{1-\alpha} = y(k_t, l_t, Z_t). \]
They rent capital and hire labor in perfectly competitive markets at rental rate, \( r_t \), and wage rate, \( w_t \), respectively. Optimization implies:

\[
y_{k,t} = r_t, \quad y_{l,t} = w_t.
\]

Capital producers purchase investment goods, \( x_t \), and old capital, \( k_t \), to produce new capital, \( k_{t+1} \), using the following linear homogeneous technology:

\[
k_{t+1} = (1 - \delta) k_t + x_t - \Phi \left( \frac{x_t}{k_t} \right) k_t.
\]

The competitive market prices of \( k_t \) and \( k_{t+1} \) are \( P_{k,t} \) and \( P_{k',t} \), respectively. Capital producer optimization leads to the following conditions:

\[
P_{k,t} = \frac{1}{1 - \Phi' \left( \frac{x_t}{k_t} \right)} \left[ 1 - \delta - \Phi \left( \frac{x_t}{k_t} \right) + \Phi' \left( \frac{x_t}{k_t} \right) \frac{x_t}{k_t} \right]
\]

\[
P_{k',t} = \frac{1 + g_n}{1 - \Phi' \left( \frac{x_t}{k_t} \right)}.
\]

At the end of period \( t \), entrepreneurs have net worth, \( N_{t+1} \), and it is assumed that \( N_{t+1} < P_{k',t} k_{t+1} \). As a result, in an equilibrium in which the entire stock of capital is to be owned and operate, entrepreneurs must borrow:

\[
b_{t+1} = P_{k',t} k_{t+1} - N_{t+1}. \quad (9.1)
\]

As soon as an individual entrepreneur purchases \( k_{t+1} \), he experiences a shock, and \( k_{t+1} \) becomes \( k_{t+1} \omega \). Here, \( \omega \) is a random variable that is iid across entrepreneurs and has mean unity. The realization of \( \omega \) is unknown before the loan is made and it is known only to the entrepreneur after it is realized. The bank which extends the loan to the entrepreneur must pay a monitoring cost in order to observe the realization of \( \omega \). The cumulative distribution function of \( \omega \) is \( F \), where

\[
\Pr \{ \omega < x \} = F(x).
\]

Entrepreneurs receive a standard debt contract from their bank, which specifies a loan amount, \( b_{t+1} \), and a gross rate of return, \( Z_{t+1} \), in the event that it is feasible for the entrepreneur to repay. The lowest realization of \( \omega \) for which it is feasible to repay is \( \bar{\omega}_{t+1} \), where

\[
\bar{\omega}_{t+1} \left( 1 + R_{t+1}^k \right) P_{k',t} k_{t+1} = Z_{t+1} b_{t+1}. \quad (9.2)
\]

For \( \omega < \bar{\omega}_{t+1} \) the entrepreneur simply pays all its revenues to the bank:

\[
\left( 1 + R_{t+1}^k \right) \omega P_{k',t} k_{t+1}.
\]

In this case, the bank monitors the entrepreneur. Following BGG, we assume that monitoring costs are a fraction, \( \mu \), of the total earnings of the entrepreneur:

\[
\mu \left( 1 + R_{t+1}^k \right) \omega P_{k',t} k_{t+1}.
\]
The bank’s source of funds is the deposits of households, and so for each amount the bank lends, it must pay a gross rate of return, \(1 + R_{t+1}\). Competition and non-negativity of profits implies that banks make zero profits in each date and state. The zero profit condition is:

\[
[1 - F(\tilde{\omega}_{t+1})] Z_{t+1} b_{t+1} + (1 - \mu) \int_{0}^{\tilde{\omega}_{t+1}} \omega dF(\omega) (1 + R^k_{t+1}) P_{k',t} k_{t+1} = (1 + R^e_{t+1}) b_{t+1}.
\]

Substituting from (9.2) for \(Z_{t+1} b_{t+1}\) and dividing by \((1 + R^e_{t+1}) P_{k',t} k_{t+1}\):

\[
[1 - F(\tilde{\omega}_{t+1})] \omega_{t+1} + (1 - \mu) \int_{0}^{\tilde{\omega}_{t+1}} \omega dF(\omega) = \left( \frac{1 + R^e_{t+1}}{1 + R^k_{t+1}} \right) \frac{b_{t+1}}{P_{k',t} k_{t+1}}.
\]

We conclude that the gross return on capital can be expressed:

\[
1 + R_{t+1} = (1 - \tau^k_{t+1}) \left( 1 + R^k_{t+1} \right),
\]

where the ‘wedge’, \(1 - \tau^k_{t+1}\), satisfies:

\[
1 - \tau^k_{t+1} = \frac{P_{k',t} k_{t+1} - N_{t+1}}{P_{k',t} k_{t+1}} \left( [1 - F(\tilde{\omega}_{t+1})] \omega_{t+1} + (1 - \mu) \int_{0}^{\tilde{\omega}_{t+1}} \omega dF(\omega) \right).
\]

The wedge, \(\tau^k_{t}\), contains two additional endogenous variables, \(N_{t+1}\) and \(\tilde{\omega}_{t+1}\). These are determined in general equilibrium by the introduction of two additional equations: the condition associated with the fact that the standard debt contract maximizes the utility of the entrepreneur, as well as the law of motion for entrepreneurial net worth.

The resource constraint for this economy is:

\[
c_t + G_t + x_t = k_t^\alpha (Z_t l_t)^{1-\alpha},
\]

where \(G_t\) includes any consumption of entrepreneurs, as well as monitoring costs incurred by banks. As long as these latter can be ignored, then the BGG financial frictions is to, in effect, introduce a tax on the rate of return on capital in, \(1 + R^k_{t+1}\), in (8.4). In particular, \(1 + R^k_{t+1}\) is replaced by

\[
(1 + R^k_{t+1}) (1 - \tau^k_{t+1}).
\]

Note there is a slight difference with CF financial frictions in that the latter imply the tax rate is not a function of period \(t + 1\) uncertainty, while the BGG frictions imply that in general it is a function of this uncertainty.
References


Figure 1: Wedges With No Adjustment Costs

- **Impulse Decomposition: GDP**
- **Rotation Decomposition: GDP**
- **Efficiency Wedge**
- **Impulse Decomposition: INV**
- **Rotation Decomposition: INV**
- **Intertemporal Wedge**
- **Impulse Decomposition: HOURS**
- **Rotation Decomposition: HOURS**
- **Labor Wedge

Data: CKM
Figure 2: Wedges with Tobin’s q Elasticity Set to Unity (a=12.88)

Impulse Decomposition: GDP
Rotation Decomposition: GDP
Efficiency Wedge

Impulse Decomposition: INV
Rotation Decomposition: INV
Intertemporal Wedge

Impulse Decomposition: HOURS
Rotation Decomposition: HOURS
Labor Wedge

Data
CKM
BGG
Figure 3: Expected Returns and Intertemporal Wedge with Adjustment Costs (top row), and Without Adjustment Costs (bottom row)

E_t(1 + R^k_{t+1}), (a=10)

\tau^k_t, (a=10)

E_t(1 + R^k_{t+1}), (a = 0)

\tau^k_t, (a=0)