Firm-Specific Capital, Nominal Rigidities and the Business Cycle

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Objectives

- Contribute Towards Construction of A Dynamic General Equilibrium Model Useable for Policy Analysis

- Resolve Apparent Conflict Between Macro and Micro Data
  - Macro Evidence:
    - Inflation is Inertial
  - Micro Evidence:
    - Prices Change Frequently
Example: Analysis with Calvo-Sticky Prices

• Analysis with Aggregate European and US Data (see Smets-Wouters, Gali-Gertler):
  – Prices Re-optimized Every 6 Quarters

• Micro Evidence:
  – Prices ‘Re-optimized’ Every 1.7 Quarters
Proposed Resolution of Conflict

• Firms Re-optimize Frequently (As in Micro)

• When Firms Re-optimize, They Change Price By a Small Amount
  
  – Firms’ Short Run Marginal Cost Increasing in Own Output
  – Firm-Specific Factors of Production (Capital)
  – Build on Sbordone, Woodford, others
Standard Model

• Capital Is Homogeneous
• Traded in Perfectly Competitive Markets
  – Firm Marginal Cost Independent of Own Output

• Assumptions Unrealistic
  – Made for Computational Simplicity
  – Hope: It Doesn’t Matter
  – In Fact: It Matters A Lot!
Intuition: Rising Marginal Cost and Incentive to Raise Price
More Intuition: Rising Marginal Cost and Incentive to Raise Price

• A Firm Contemplates Raising Price
  – This Implies Output Falls
  – Marginal Cost Falls
  – Incentive to Raise Price Falls

• Effect Quantitatively Important When:
  – Demand Elastic
  – Marginal Cost Steep
Strategy for Evaluating Proposed Resolution of Conflict

• Incorporate Idea Into Otherwise Standard Equilibrium Model

• Estimate Model Parameters Using Macro Data (Elasticity of Demand and Slope of Marginal Cost Particularly Important)

• Ask: Is Model Consistent With
  – Macro Evidence on Inflation Inertia?
  – Micro Evidence on Price Changes?
Key results

• Make Progress On Macro/Micro Conflict
  – Account for Macro Evidence of Inflation Inertia
  – Prices re-optimized on average once every 1.6 quarters.
  – This finding depends on the assumption that capital is firm specific.

• Wage-setting Frictions play Important Role.
  – Wage contracts re-optimized on average once every 3 quarters.

• Monetary Policy Crucial In Transmission of Technology Shocks

• According to our model, in absence of monetary accommodation,
  – Output and hours would fall in the wake of a positive neutral technology shock;
  – Output and hours worked would rise by much less than they actually do after a positive capital embodied technology shock.

• Consistent with findings in Gali, Lopez-Salido and Valles (2002).
Outline

• Model
• Econometric Estimation of Model
  – Fitting Model to Impulse Response Functions
• Model Estimation Results
• Implications for Micro Data on Prices
Model...

• Two Versions of Model
  – Homogeneous Capital
  – Firm-specific Capital

• Describe Model Under Homogeneous Capital Assumption

• What to Change to Obtain Firm-Specific Capital Version
Description of Model

• Timing Assumptions

• Firms

• Households

• Monetary Authority

• Goods Market Clearing and Equilibrium
Timing

• Technology Shocks Realized.
• Agents Make Price/Wage Setting, Consumption, Investment, Capital Utilization Decisions.
• Monetary Policy Shock Realized.
• Household Money Demand Decision Made.
• Production, Employment, Purchases Occur, and Markets Clear.
• Note: Wages, Prices and Output Predetermined Relative to Policy Shock.
Firms

Final Good Firms

- Technology:
  \[ Y_t = \left[ \int_0^1 Y_{it}^{-\frac{1}{\lambda_f}} \, di \right]^{\lambda_f}, \ 1 \leq \lambda_f < \infty \]

- Objective:
  \[ \max P_t Y_t - \int_0^1 P_{it} Y_{it} \, di \]

- Firms and Prices:
  \[ \left( \frac{P_t}{P_{it}} \right)^{\lambda_f} = \frac{Y_{it}}{Y_t}, \ P_t = \left[ \int_0^1 P_{it}^{-\lambda_f} \, di \right]^{(1-\lambda_f)} \]
Intermediate Good Firms -

- Each $Y_{it}$ Produced by a Monopolist, With Demand Curve:
  \[
  \left( \frac{P_t}{P_{it}} \right)^{\frac{\lambda_f}{\lambda_f - 1}} = \frac{Y_{it}}{Y_t}.
  \]

- Technology:
  \[
  Y_{it} = k_{it}^\alpha (z_t L_{it})^{1-\alpha}.
  \]

- Here, $z_t$ is a Technology Shock:
  \[
  \mu_{zt} = \log z_t - \log z_{t-1}, \quad \hat{\mu}_{zt} = \rho \mu_{zt-1} + \varepsilon_{\mu,t}.
  \]
• Calvo Price Setting:
  - With Probability $1 - \xi_p$, $i^{th}$ Firm Sets Price, $P_{it}$, Optimally, to $\tilde{P}_t$.
  - With Probability $\xi_p$, Do Not Optimize Current Price. Instead:

$$P_{i,t} = \pi_{t-1}P_{i,t-1}, \pi_t = \frac{P_t}{P_{t-1}}.$$
• Firms Setting Prices Optimally at $t$ Choose $\tilde{P}_t$ to max:

$$v_t \left[ \tilde{P}_t Y_{i,t} - MC_t Y_{i,t} \right]$$

$$+ \beta \xi_p v_{t+1} \left[ \tilde{P}_t \pi_t Y_{i,t+1} - MC_{t+1} Y_{i,t+1} \right]$$

$$+ (\beta \xi_p)^2 v_{t+2} \left[ \tilde{P}_t \pi_t \pi_{t+1} Y_{i,t+2} - MC_{t+2} Y_{i,t+2} \right]$$

$$+ ...$$

subject to:

$$\left( \frac{P_t}{\tilde{P}_t} \right)^{\frac{y_f}{\lambda_f - 1}} = \frac{Y_{i,t}}{Y_t}.$$  

$v_t$ ~ value of a dividend at $t$

$MC_t$ ~ given
Scaling:

\[ \tilde{p}_t = \frac{\tilde{P}_t}{P_t}, \quad w_t = \frac{W_t}{P_t} \]

\[ r^k_t \text{ rental rate on capital} = \frac{P_t}{P_t} \]

\[ s_t = \frac{MC_t}{P_t}. \]

Real Marginal Cost:

\[ s_t = \left( \frac{1}{1 - \alpha} \right)^{(1-\alpha)} \left( \frac{1}{\alpha} \right)^{\alpha} (r^k_t)^{\alpha} (w_t R_t)^{1-\alpha}  \frac{1}{z_t} \]

Linear approximation:

\[ \hat{x}_t \equiv \frac{x_t - x}{x}. \]
• Price Optimization Leads to:

\[ \hat{p}_t = \hat{s}_t + \sum_{l=1}^{\infty} (\beta \xi_p)^l (\hat{s}_{t+l} - \hat{s}_{t+l-1}) \]

\[ + \sum_{l=1}^{\infty} (\beta \xi_p)^l (\hat{\pi}_{t+l} - \hat{\pi}_{t+l-1}) \]

• Front-Loading:

\[ -\hat{p}_t > \hat{s}_t \text{ if } \hat{s}_{t+l} > \hat{s}_t \text{ and/or } \hat{\pi}_{t+l} > \hat{\pi}_t. \]
• Aggregate Price Level:

\[ P_t = \left[ \int_0^1 P_{it}^{\frac{1}{1-\lambda_f}} \, di \right]^{1-\lambda_f} \]

\[ = \left[ (1 - \xi_p) \tilde{P}_t^{\frac{1}{1-\lambda_f}} + \xi_p \left( \pi_{t-1} P_{t-1} \right)^{\frac{1}{1-\lambda_f}} \right]^{1-\lambda_f} \]

• Scale:

\[ 1 = \left[ (1 - \xi_p) \tilde{P}_t^{\frac{1}{1-\lambda_f}} + \xi_p \left( \frac{\pi_{t-1}}{\pi_t} \right)^{\frac{1}{1-\lambda_f}} \right]^{1-\lambda_f} \]

• Approximately

\[ \tilde{P}_t = \frac{\xi_p}{1 - \xi_p} \left[ \tilde{\pi}_t - \tilde{\pi}_{t-1} \right]. \]
Combining Optimal Price and Aggregate Price Relation:

\[ \Delta \hat{\pi}_t = \beta E_t \Delta \hat{\pi}_{t+1} + \frac{(1 - \beta \xi_p)(1 - \xi_p)}{\xi_p} E_t \hat{s}_t, \]

Under Standard Price-Updating Scheme:

\[ P_{it} = \bar{\pi} P_{i,t-1}. \]

Associated Reduced Form:

\[ \hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{(1 - \beta \xi_p)(1 - \xi_p)}{\xi_p} E_t \hat{s}_t. \]
Households: Sequence of Events

- Technology shock realized.

- Decisions: Consumption, Capital accumulation, Capital Utilization.

- Insurance markets on wage-setting open.

- Wage rate set.

- Monetary policy shock realized.

- Household allocates beginning of period cash between deposits at financial intermediary and cash to be used in consumption transactions.
Households...

- Monopoly supplier of differentiated labor
  - Sets wage subject to Calvo style frictions like firms
- Preferences of \( j^{th} \) household

\[
E_t^j \sum_{i=0}^{\infty} \beta^{l-t} \left[ \log (C_{t+l} - bC_{t+l-1}) - \psi L \frac{h_{j,t+l}^2}{2} \right]
\]

- \( E_t^j \) : expectation operator, conditional on aggregate and household \( j \) idiosyncratic information.
- \( C_t \) : consumption
- \( h_{jt} \) : hours worked.
Habit Persistence and Response of Consumption

- Recall that after an Expansionary Monetary Policy Shock, we see
  - hump-shaped rise in consumption
  - decline in real interest rate.

- Euler Equation in Standard Model:
  \[ \frac{u_{c,t}}{\beta u_{c,t+1}} \approx \frac{c_{t+1}}{\beta c_t} = \frac{g_{t+1}}{\beta} = \frac{R_t}{\pi_{t+1}}, \quad \pi_{t+1} = \frac{P_{t+1}}{P_t}. \]

- Problem: Can’t have \( g_t \) high and \( \frac{R_t}{\pi_{t+1}} \) simultaneously!
• Habit Persistence in Preferences (example):
  \[ u(c_t - b\bar{c}_{t-1}), \quad \bar{c}_{t-1} \sim \text{aggregate consumption} \]

• Euler Equation:
  \[
  \frac{u_{c,t}}{\beta u_{c,t+1}} \approx \frac{c_{t+1} - bc_t}{\beta (c_t - bc_{t-1})} = \frac{g_{t+1} - b}{\beta \left(1 - \frac{b}{g_t}\right)} \\
  \approx \frac{g_{t+1} - bg_t}{\beta (1 - b)}
  \]

• Result:
  – \( g_{t+1} \) and \( g_t \) Can Both be High, as Long as \( g_{t+1} < bg_t \).
  – Consistent with Simultaneous Hump-Shape \( c \) Response and Low Real Rate.

• Habit Persistence Also Helpful for Understanding Asset Prices
Households...

- Asset Evolution Equation:

\[ M_{t+1} = R_t [M_t - Q_t + (x_t - 1)M_t^g] + A_{j,t} + Q_t + W_{j,t} h_{j,t} + P_t r_t^k u_t \bar{K}_t + D_t - P_t \left[ (1 + \eta(V_t)) C_t + \gamma_t^{-1} \left( I_t + a(u_t) \bar{K}_t \right) \right] \]

- \( M_t \): Beginning of Period Base Money; \( Q_t \): Transactions Balances
- \( x_t \): Growth Rate of Base; \( u_t \): Utilization Rate of Capital
  * \( u_t = 1 \) in steady state, \( a(1) = 0, a'(1) > 0, \sigma_a = a''(1)/a'(1) \).
- \( \gamma_t^{-1} \): (Real) Price of investment goods, \( \mu_{\gamma,t} = \gamma_t/\gamma_{t-1} \),

\[ \hat{\mu}_{\gamma,t} = \rho_{\mu,\gamma} \hat{\mu}_{\gamma,t-1} + \varepsilon_{\mu,\gamma,t} \]

- Velocity:

\[ V_t = \frac{P_t C_t}{Q_t}, \]
Money Demand

- Asset Evolution Equation:

\[ M_{t+1} = R_t \left[ M_t - Q_t + (x_t - 1) M_t^a \right] + A_{j,t} + Q_t + W_{j,t} h_{j,t} + P_t r_t^k u_t \bar{K}_t + D_t - P_t \left[ (1 + \eta(V_t)) C_t + \gamma_t^{-1} (I_t + \alpha(u_t) \bar{K}_t) \right] \]

- Increase in \( Q_t \):
  - Marginal Cost of Interest Foregone: \( R_t \)
  - Marginal Benefit:

\[ 1 - P_t \eta'(V_t) C_t \frac{dV_t}{dQ_t} \]

additional cash available at end of period

\[ = \underbrace{1}_{\text{reduction in transactions costs due to extra cash}} + \eta' \left( \frac{P_t C_t}{Q_t} \right) \left( \frac{P_t C_t}{Q_t} \right)^2 \]
Money Demand ...

- Money Demand: Equate Marginal Benefits and Costs of $Q_t$—

$$R_t = 1 + \eta' \left( \frac{P_t C_t}{Q_t} \right) \left( \frac{P_t C_t}{Q_t} \right)^2.$$

- Properties of Money Demand:
  - Unit Consumption Elasticity of Money Demand
    * Increase $C_t$ 1 percent and Hold $R_t$, $P_t$ Fixed $\Rightarrow$ Desired $Q_t$ increases 1 percent
  - $R_t \uparrow$ Implies $Q_t \downarrow$
    * To Induce Households to Hold Additional $Q$, Must Have Lower $R$
    * Money Demand Elasticity is Bigger, the Bigger is $\eta''$
Money Demand ...

- Quantitative Analysis of Money Demand
  - Consider the Following Parametric Function for $\eta$

  $$\eta = AV_t + \frac{B}{V_t} - 2\sqrt{AB}$$

  $$\Rightarrow$$

  $$R = 1 + \eta'(V) \times V^2 = 1 + \left[ A - BV^{-2} \right] V^2 = 1 - B + AV^2$$

- Data:
  * Money - St. Louis Fed’s MZM, 1974-2004
  * Consumption - NIPA Consumption of Services and Nondurables
  * Interest Rate - One Year T-Bills.
  * OLS Regression of $V^2$ on $R \Rightarrow A = 0.0174$ and $B = 0.0187$
Money Demand ...

• Top Graph: Velocity of Money
• Bottom Graph: Actual and Predicted Interest Rate

- Findings: Static Money Demand Equation Fits the Data Well!
Households...

- Capital Evolution:

\[
\bar{K}_{t+1} = (1 - \delta)\bar{K}_t + F(I_t, I_{t-1}),
\]

\[
F(I_t, I_{t-1}) = (1 - S) \left( \frac{I_t}{I_{t-1}} \right) I_t,
\]

\[
S = S'' = 0, \quad S'' > 0 \text{ in steady state}
\]
Wage Decisions

• Households supply differentiated labor.
• Standard Calvo set up as in Erceg, Henderson and Levin and CEE.
Structure of the Labor Market

- Intermediate Good Firms Use Labor Aggregate:

\[ L_t = \left[ \int_0^1 h_{j,t}^{\frac{1}{\lambda_w}} dj \right]^{\lambda_w} . \]

- Price of \( L_t \):

\[ W_t = \left[ \int_0^1 W_{i,t}^{1-\lambda_w} di \right]^{1-\lambda_w} . \]

- Demand for Household Labor Service, \( h_{j,t} \):

\[ h_{j,t} = \left( \frac{W_t}{W_{j,t}} \right)^{\frac{\lambda_m}{\lambda_w-1}} L_t, \quad 1 \leq \lambda_w < \infty. \]

\( W_{j,t} \sim \)wage set by household
\( L_t \sim \)homogeneous aggregate labor
\( W_t \sim \)wage rate of aggregate labor
Household Wage Decision

- Demand for Household’s Specialized Labor:

\[ h = D(w) = w^{-\frac{\lambda_w}{\lambda_w + 1}} \]

- Household Choice of real wage:

\[
\max_{w,c} \; u(c) - z(h) \\
\text{subject to } c \leq wh, \; h = D(w)
\]

- Substitute out for budget constraint and demand curve:

\[
\max_{w,c} \; u(wD(w)) - z(D(w))
\]

- First order condition:

\[ u_c \times [wD'(w) + D(w)] = z' \times D'(w) \]
Household Wage Decision ...

- First order condition:

\[ u_c \times [wD'(w) + D(w)] = z' \times D'(w) \]

or

\[ wu_c \times [wD'(w) + D(w)] = z' \times wD'(w) \]

or

\[ w = \frac{z'}{u_c} \times \frac{wD'(w)}{wD'(w) + D(w)} \]
Household Wage Decision

- First Order Condition:
  \[ w = \frac{z'}{u_c} \times \frac{wD'(w)}{wD'(w) + D(w)} \]

- Note: Household Marginal Cost (consumption value of a unit of leisure):
  \[ \frac{z'}{u_c} = \frac{\frac{d\text{utility}}{d\text{leisure}}}{\frac{d\text{consumption}}{d\text{leisure}}} = \frac{d\text{consumption}}{d\text{leisure}} \]

- Note: With Constant Elasticity Utility Function:
  \[ \frac{wD'(w)}{wD'(w) + D(w)} = \lambda_w \]

- Conclude ‘Wage Equals Markup Times Marginal Cost’:
  \[ w = \lambda_w \frac{z'}{u_c} \]
Calvo-style Wage Setting:

- With Probability $1 - \xi_w$, $i^{th}$ Household Sets Wage, $W_{it}$, Optimally, to $\tilde{W}_t$.
- With Probability $\xi_w$,
  \[
  W_{i,t} = \pi_{t-1}\mu_{z^*} W_{i,t-1},
  \]
  \[\mu_{z^*} \sim \text{steady state growth rate of economy}\]
  \[\mu_{z^*,1} = \frac{z_t^*}{z_{t-1}^*}, \quad z_t^* = \gamma_t^{\frac{\alpha}{1-\alpha}} z_t\]

- First Order Condition:
  \[
  E_{t-1} \sum_{l=0}^{\infty} (\xi_w \beta)^l h_{jt+l} \psi_{t+l} \left[ \frac{\tilde{W}_t X_{t,l}}{P_{t+l}} - \lambda_w \frac{z_{h,t+l}}{\psi_{t+l}} \right] = 0.
  \]

- Households Attempt to Set Price (the wage) as a markup over marginal cost.

  $\psi_{t+l}$ : utility value of consumption (Multiplier on Budget Constraint)
  $z_{h,t+l}$ : Household Marginal utility of Leisure
  $\frac{z_{h,t+l}}{\psi_{t+l}}$ : Marginal Cost (in Consumption Units) of a Unit of Leisure
Monetary and Fiscal Policy

\[ x_t = \frac{M_t}{M_{t-1}} \]

\[ \hat{x}_{M,t} = \rho_M \hat{x}_{M,t-1} + \varepsilon_{M,t} \]
\[ \hat{x}_{z,t} = \rho_{xz} \hat{x}_{z,t-1} + c_z \varepsilon_{z,t} + \chi_{z,t-1} \]
\[ \hat{x}_{\gamma,t} = \rho_{x\gamma} \hat{x}_{\gamma,t-1} + c_{\gamma} \varepsilon_{\gamma,t} + \chi_{\gamma,t-1} \]

- \( \hat{x}_{M,t} \): response of monetary policy to a monetary policy shock, \( \varepsilon_{M,t} \)/
- \( \hat{x}_{z,t} \): response of monetary policy to an innovation in neutral technology, \( \varepsilon_{z,t} \).
- \( \hat{x}_{\gamma,t} \): response of monetary policy to an innovation in capital embodied technology, \( \varepsilon_{\gamma,t} \).
- Government has access to lump sum taxes, pursues a Ricardian fiscal policy.
Loan Market and Final Good Market Clearing Conditions, Equilibrium

- Financial intermediaries receive $M_t - Q_t + (x_t - 1) M_t$ from the household.
  - Lend all of their money to intermediate good firms, which use the funds to pay for $H_t$.
- Loan market clearing
  \[ W_t H_t = x_t M_t - Q_t. \]
- The aggregate resource constraint is
  \[ (1 + \eta(V_t)) C_t + \gamma^{-1}_t [I_t + a(u_t) \bar{K}_t] \leq Y_t. \]
- We adopt a standard sequence-of-markets equilibrium concept.
The Firm - Specific Capital Model

- Firms own their own capital which can’t be adjusted during the period.
  - Can only be increased or decreased over time by varying rate of investment.
- In all other respects, problem of intermediate good firm is same as before.
- Technology for accumulating physical capital by intermediate good firm $i$:
  \[ F(I_t(i), I_{t-1}(i)) = \left(1 - S \left( \frac{I_t(i)}{I_{t-1}(i)} \right) \right) I_t(i), \]

  \[ \bar{K}_{t+1}(i) = (1 - \delta) \bar{K}_t(i) + F(I_t(i), I_{t-1}(i)). \]

- Present discounted value of $i^{th}$ intermediate good’s cash flow:
  \[
  E_t \sum_{j=0}^{\infty} \beta^j \Lambda_{t+j} \left\{ \begin{array}{ll}
  P_{t+j}(i)y_{t+j}(i) - P_{t+j}R_{t+j}w_{t+j}(i)h_t(i) \\
  -P_{t+j} \bar{y}_{t+j}^{-1} \left[ I_{t+j}(i) + a(u_{t+j}(i)) \bar{K}(i)_{t+j} \right]
  \end{array} \right\}. 
  \]
Firm Specific Capital Model...

- Timing:
  - Firms sees technology shock
  - Sets $P_t(i)$, subject to the Calvo frictions.
  - Also decides on $I_t(i)$ and $u_t(i)$.

- Time $t$ monetary policy shock occurs, demand for the firm’s product is realized.
  - Firm must satisfy demand.
Implications for Inflation

- Equations which characterize equilibrium for homogeneous and firm specific capital model are identical:

\[ \Delta \hat{\pi}_t = E [\beta \Delta \hat{\pi}_{t+1} + \gamma \hat{s}_t | \Omega_t] \]

where

\[ \gamma = \frac{(1 - \xi_p)(1 - \beta \xi_p)}{\xi_p} \chi \]

and

\[ \chi = \begin{cases} 
1 & \text{homogeneous capital model} \\
< 1 & \text{firm-specific capital model} 
\end{cases} \]

- In the firm specific capita model, \( \chi \) is a particular non-linear function of the parameters of the model.

- Given \( \gamma \), the two models are observationally equivalent with respect to aggregate prices and quantities.
Implications for Inflation...

\[ \gamma = \frac{(1 - \xi_p)(1 - \beta \xi_p)}{\xi_p} \chi \]

- For example, our estimated of \( \gamma \) is
  \[ \gamma = .035 \]

- Under homogeneous capital model (\( \chi = 1 \)), this implies
  \[ \xi_p = .83 \text{ and } 1/(1 - \xi_p) = 6 \]

- For the firm specific capital model, \( \chi = .031 \), and
  \[ \xi_p = .36 \text{ and } 1/(1 - \xi_p) = 1.6 \]
Econometric Methodology

• Variant of limited information strategy used in CEE (2004).
  – Impose a subset of assumptions made in equilibrium model to estimate impulse response functions of ten key macroeconomic variables to the three shocks in our model.
  – Neutral technology shocks, capital embodied technology shocks and monetary policy shocks.

• Choose values for key parameters of structural model to minimize difference between estimated impulse response functions and analogous objects in model.
Identifying Assumptions

• Shocks to technology
  – Innovations to technology (both neutral and capital embodied) are only shocks that affect level of labor productivity in the long run.
  – Capital embodied technology shocks are the only shocks that affect price of investment goods relative to consumption goods in the long run. (Fisher (2003)).
  – Our equilibrium model is consistent with these assumptions.

• Shocks to monetary policy
  – CEE (2004): there’s exactly one shock - the monetary policy shock - that affects interest rate contemporaneously, over and above shocks that drive aggregate prices and quantities.
  – This assumption is satisfied in our equilibrium.
Identification...

\[
\begin{align*}
Y_t & = \begin{pmatrix}
\Delta \ln(\text{relative price of investment}_t) \\
\Delta \ln(GDP_t/Hours_t) \\
\Delta \ln(\text{GDP deflator}_t) \\
(\text{capacity utilization}_t) \\
\ln(Hours_t) \\
\ln(GDP_t/Hours_t) - \ln(W_t/P_t) \\
\ln(C_t/GDP_t) \\
\ln(I_t/GDP_t) \\
\text{Federal Funds Rate}_t \\
\Delta \ln(\text{GDP deflator}_t) + \ln(GDP_t) - \ln(MZM_t)
\end{pmatrix} \\
& = \begin{pmatrix}
\Delta p_{It} \\
1 \times 1 \quad \Delta a_t \\
1 \times 1 \quad Y_{1t} \\
6 \times 1 \quad R_t \\
1 \times 1 \quad Y_{2t}
\end{pmatrix}
\end{align*}
\]

- Monetary Policy

\[R_t = f(\Omega_t) + \varepsilon_{Mt}.\]

- Function \( f \) is linear, \( \Omega_t \) contains \( Y_{t-1}, Y_{t-2}, Y_{t-3}, Y_{t-4} \) and the only date \( t \) variables in \( \Omega_t \) are \( \{\Delta a_t, \Delta p_{It}, Y_{1t}\} \) and \( \varepsilon_{Mt} \) is orthogonal with \( \Omega_t \).
Estimating Parameters in the Model

- Partition Parameters into Three Groups.
  - Parameters set a priori (e.g., $\beta$, $\delta$, ...)
  - $\zeta_1$: remaining parameters pertaining to the nonstochastic part of the model

$$\zeta_2 = [\xi_w, \gamma, \sigma_a, b, S''', \epsilon]$$

- $\zeta_2$: parameters pertaining to stochastic part of the model
- Number of parameters, $\zeta = (\zeta_1, \zeta_2)$, to be estimated - 18

- Estimation Criterion
  - $\Psi(\zeta)$: mapping from $\zeta$ to model impulse responses
  - $\hat{\Psi}$: 592 impulse responses estimated using VAR
  - Estimation Strategy:
    $$\hat{\zeta} = \arg\min_{\zeta} \left( \hat{\Psi} - \Psi(\zeta) \right)' V^{-1} \left( \hat{\Psi} - \Psi(\zeta) \right).$$
  - $V$: diagonal matrix with sample variances of $\hat{\Psi}$ along the diagonal.
Estimated Parameter Values, $\zeta_1$

<table>
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<tr>
<th>$\lambda_f$</th>
<th>$\xi_w$</th>
<th>$\gamma$</th>
<th>$\sigma_a$</th>
<th>$b$</th>
<th>$\beta''$</th>
<th>$\epsilon$</th>
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<td>.035</td>
<td>2.01</td>
<td>.65</td>
<td>2.22</td>
<td>1.06</td>
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</tbody>
</table>

- $\epsilon$ a little low....
- $b$ similar to other estimates in literature
- $\xi_w$ wages reoptimized on average every 3.6 quarters
- Parameters important in subsequent discussion....
  - $\sigma_a$ costly to vary utilization of capital
  - $\lambda_f$ close to perfect competition
  - $\gamma$ amazingly low! (similar to estimates reported in literature)
Implications for Wage and Price Re-Optimization

• Our benchmark estimates imply that wage decisions are re-optimized on average 3.6 quarters.

• The implication of our estimate of gamma for how frequently firms re-optimize prices depends critically on whether we assume capital is firm specific or homogeneous.
  – If capital is homogeneous, firms re-optimize prices on average once every 6 quarters,
  – If capital is firm specific, firms re-optimize prices once every 1.6 quarters.

• At a broad level, this is consistent with micro evidence from Bils and Klenow, Lucas and Golosov and Klenow and Kryvtsov.

I’ll provide intuition for this in a moment.

Estimated Parameters of Exogenous Shocks, $\zeta_2$

<table>
<thead>
<tr>
<th>$\rho_M$</th>
<th>$\sigma_M$</th>
<th>$\rho_{\mu_z}$</th>
<th>$\sigma_{\mu_z}$</th>
<th>$\rho_{xz}$</th>
<th>$c_z$</th>
<th>$c^p_z$</th>
<th>$\rho_{\mu_T}$</th>
<th>$\sigma_{\mu_T}$</th>
<th>$\rho_{x\gamma}$</th>
<th>$c_\gamma$</th>
<th>$c^p_\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.24</td>
<td>.26</td>
<td>.92</td>
<td>.06</td>
<td>.37</td>
<td>3.36</td>
<td>1.19</td>
<td>.21</td>
<td>.31</td>
<td>.67</td>
<td>.38</td>
<td>.26</td>
</tr>
</tbody>
</table>
Figure 10: Benchmark model – dynamic response to a monetary policy shock
Figure 11: Benchmark model – dynamic response to a neutral technology shock
Figure 12: Benchmark model – dynamic response to an embodied technology shock
Monetary Policy and Technology Shocks

• How would the economy have responded to technology shocks if monetary policy had not been accommodative?
Benchmark model (•) and alternative model (○) - dynamic response to a neutral technology shock
Benchmark model (•) and alternative model (○) - dynamic response to an embodied technology shock
Understanding the Microeconomic Price Implications of the Model

- Reduced Form Parameter, $\gamma$:
  - Model: $\gamma = 0.035$ ‘a 1 percent temporary rise in marginal cost leads to a 0.03% rise in price level’

\[ \hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \gamma \hat{s}_t \]

- Direct Analysis of Macro Data Supports Low Estimate of $\gamma$

- Inflation Inertia:
  - Conventional Def: Sluggish Response of Inflation to (Monetary) Shocks
  - Conventional Solution: Identify Model Features that Slow Rise in Marginal Cost After Shock
  - Does not help with low $\gamma$!
Analysis of relation, $\Delta \pi_t = \gamma s_t + \beta \Delta \pi_{t+1}$
Microeconomic Price Implications of the Model....

- Homogeneous Capital Model:

\[ \gamma = \frac{(1 - \xi_p) \left(1 - \beta \xi_p\right)}{\xi_p} \]

\[ \gamma = 0.035 \rightarrow \xi_p = 0.83, \quad \frac{1}{1 - \xi_p} = 5.8 \]

- Homogeneous Capital Model:
  - Price Seems Not to Respond Much to Marginal Cost - Prices Must be VERY Sticky!
  - Striking Conflict Between Macro (\(\gamma\)) and Micro Evidence
Microeconomic Price Implications of the Model...

- Firm-specific Capital Model:

\[
\gamma = \frac{(1 - \xi_p) (1 - \beta \xi_p)}{\xi_p} \chi(\sigma_a, \lambda_f, \xi_p)
\]

\[
0.031 = \chi(\sigma_a = 2, \lambda_f = 1.01, \xi_p = 0.36)
\]

- Firm-specific capital breaks Micro/Macro by introducing endogenous price stickiness
- How does it do it?
The Experiment

- Begin at steady state and assume there is an expansionary monetary policy shock in period 1.
- Period 1
  - Prices and output is the same for all firms.
- Period 2
  - \((1 - \xi_p)\) firms re-optimize and implement new price, \(\xi_p\) do not.
- Period 3: there are 4 types of firms.
  - \((1 - \xi_p)^2\) re-optimize in period 2 and 3,
  - \(\xi_p^2\) don’t re-optimize in either period 2 or period 3.
  - \((1 - \xi_p)\xi_p\) re-optimized in period 2 but not in period 3.
  - \(\xi_p(1 - \xi_p)\) did not re-optimize in period 2 but did re-optimize in period 3.
- In period \(s\) there are \(2^{s-1}\) different firms.
- For each period \(s\) we calculated the distribution of output and prices across firms.
Features of the Distribution of Output and Prices Across Firms: Homogeneous Capital Model

Share of output and firms in Period 4

Period of most recent optimization

Average relative price in Period 4

Period of most recent optimization

Share of output and firms in Period 8

Period of most recent optimization

Average relative price in Period 8

Period of most recent optimization

Share of output and firms in Period 16

Period of most recent optimization

Average relative price in Period 16

Period of most recent optimization
Features of the Distribution of Output and Prices Across Firms: Firm-specific Capital Model

Share of output and firms in Period 4

Share of output and firms in Period 8

Share of output and firms in Period 16

Average relative price in Period 4

Average relative price in Period 8

Average relative price in Period 16
A Check on the Econometric Procedure

• CKM Have Used an Example to Question Whether Estimated VARs are a Reliable Estimator of Impulse Response Functions to a Shock

• We Did an Experiment to Investigate Whether We Have the Problems They Describe
Basic Idea

• Generate Artificial Data from Economic Model, then Feed it to 10 Variable VAR Program Which Was Applied to Actual Data

• Wait!
  – Economic Model Only Has Three Shocks
  – Can’t Fit 10 Variable VAR to Data From Model

• Solution
  – Empirical Procedure Recognizes We’re Short on Shocks
  – Offers a Natural Solution
Background

- Recall, Structural Form of VAR:

\[ A_0 Y_t = A(L) Y_{t-1} + \epsilon_t \]

- Reduced Form:

\[ Y_t = B(L) Y_{t-1} + C \epsilon_t \]

where

\[ B(L) = A_0^{-1} A(L), \quad C = A_0^{-1}. \]

and

\[ \epsilon_t = \begin{pmatrix} c_{\gamma,t} \oplus c_{z,t} \oplus c_{1t} \oplus c_{Rt} \oplus c_{2t} \end{pmatrix} \]
Background ...

- Can Write:

\[ Ce_t = C_1 \begin{pmatrix} e_{\gamma,t} \\ e_{z,t} \\ e_{R_t} \end{pmatrix} + C_2 \begin{pmatrix} e_{1t} \\ e_{2,t} \end{pmatrix} \]

- So

\[ Y_t = B(L)Y_{t-1} + C_1 \begin{pmatrix} e_{\gamma,t} \\ e_{z,t} \\ e_{R_t} \end{pmatrix} + C_2 \begin{pmatrix} e_{1t} \\ e_{2,t} \end{pmatrix} \]

- Stochastic Process for \( Y_t \) Can Be Decomposed Into Two Orthogonal Pieces:

\[ Y_t = Y_{t}^{Model} + Y_{t}^{Other} \]

\[ Y_{t}^{Model} = B(L)Y_{t-1}^{Model} + C_1 \begin{pmatrix} e_{\gamma,t} \\ e_{z,t} \\ e_{R_t} \end{pmatrix} \]

\[ Y_{t}^{Other} = B(L)Y_{t-1}^{Other} + C_2 \begin{pmatrix} e_{1t} \\ e_{2,t} \end{pmatrix} \]
Background ...

\[ Y_t = Y_t^{Model} + Y_t^{Other} \]

- Piece, \( Y_t^{Model} \), is captured by the equilibrium model.
- Piece, \( Y_t^{Other} \), is left out.

- Implications of the analysis:
  - Data generated from model corresponds to \( Y_t^{Model} \).
  - Data generated from model is missing \( Y_t^{Other} \).

- To run VAR in artificial data:
  - Generate artificial data from economic model, \( Y_t^{Economic\ Model} \).
  - Generate \( Y_t^{Other} \) from
    \[ Y_t^{Other} = B(L)Y_{t-1}^{Other} + C_2 \begin{pmatrix} e_{1t} \\ e_{2,t} \end{pmatrix} \]
  - Construct:
    \[ Y_t = Y_t^{Economic\ Model} + Y_t^{Other} \]
  - Fit VAR to \( Y_t \)
Experiment

• Generate Artificial Data
  – Extremely Long Data Set to Get Plim (20,000 Observations)
  – Many Data Sets of Length 170 Each
• Feed Each Data Set to Same VAR Fit to US Data
• Compute Impulse Response Functions
  – Dotted Lines: Small Sample Means
  – Dashed Lines: Plims
Figure 16: Monetary policy shock

- Output
- M2 Growth
- Inflation
- Fed Funds
- Capacity Util
- Avg Hours
- Real Wage
- Consumption
- Investment
- Velocity
- Price of investment
Figure 18: Neutral technology shock
Figure 17: Investment specific technology shock
Summary

• We constructed a dynamic GE model of cyclical fluctuations.

• Given assumptions satisfied by our model, we identified dynamic response of key US economic aggregates to 3 shocks
  – Monetary Policy Shocks
  – Neutral Technology Shocks
  – Capital Embodied Technology Shocks

• These shocks account for substantial cyclical variation in output.

• Estimated GE model does a good job of accounting for response functions (However, Misses on Inflation Response to Neutral Shock)

• Have Made Progress on Micro/Macro Conflict
  – But, Need to Further Investigate Cross-Sectional Implications of Model
Summary…

• Calvo Sticky Prices and Wages Seems Like Good Reduced Form
  – What is the Underlying Structure?