Money as a Unit of Account
Matthias Doepke and Martin Schneider
Question

- Explain emergence of a common unit of account for future payments.
  - Why coordinate on a common unit of account?
  - What should be the unit of account?
Examples

Treasury Debt, 2002: U.S. Dollars
Mühlenerbzins, 1794: Meissnische Gulden, bushels of bran
Unit of account often different from medium of exchange.

Accounting currencies:
- Distinct from any existing medium of exchange.
- Livre tournois in France, ECU in Europe.

Common unit of account in areas with intensive trade:
- Many currencies used for payment, contracts mostly in one.
- Vereinsthaler in Northern Germany before unification.
- Use of dollar denominated contracts in world trade.

Government-issued fiat money as unit of account:
- More common recently as governments borrow more . . .
- . . . but not when value too uncertain (dollarization).
Why Coordinate?

- Candidates for unit of account:
  - Goods or assets with quoted prices.

- Three features lead to dominant unit of account:
     - Demand for unit of account that hedges relative-price risk.
  2. Trade along credit chains.
     - Demand for common unit of account in chains.
     - Demand for dominant unit of account in entire economy.
What Should Be the Unit of Account?

- Properties of dominant unit of account:
  - Stable in value relative to revenue of borrowers in many transactions.
  - If government is large, government debt works well . . .
  - . . . but only if value of debt is stable.
  - In areas with a lot of trade, common unit of account is useful: “currency areas.”
Literature

- Hedging through denomination of (bilateral) contracts:

- Credit chains:
  - Kiyotaki-Moore (2001), ...

- Coordination on indexation:

- Medium of exchange and unit of account:

- Matching and currency areas:
  - Matsuyama-Kiyotaki-Matsui (1993), Trejos-Wright (2001), Rey (2003) ...

- Redistribution effects of inflation:
  - Bohn (1990), Doepke-Schneider (2006), Auclert (2006), Doepke et al. (2017) ...
Outline

- General setup.
- Large default cost and divisible projects:
  - Noncontingent contracts, no default, inefficient production.
  - Unit of account maximizes scale of production.
  - Application to government IOUs.
  - Application to optimal currency areas.
- Small default cost and indivisible projects (not today):
  - Contingent contracts, costly default, efficient production.
  - Unit of account minimizes default costs.
Model: Agents, Dates, Goods

- Continuum of agents: Farmers and artisans.
  - Meet and write contracts at date 0.
  - Work at date 1.
  - Exchange goods and consume at date 2.

- Goods:
  - Farm goods: Traded in spot markets at date 2.
  - Artisanal goods: Tailored to matched customer.
  - Labor.
Utility function:

\[ u_i(c, x, h) = u(c) + (1 + \lambda)x - h. \]

\( u(c) \): Homogeneous utility derived from vector of farm goods \( c \).
\( x \): Customized artisanal good.
\( h \leq 1 \): labor supply.
Farmer type $i \in \{A, B\}$ with mass 0.5 each.
- Farmer of type $i$ endowed with one unit of farm good $i$ at date 2.
- Farm good $i$ trades in spot market at date 2 at price $p_i$.
- **Price risk**: Price of farm good $i$ is random.
- Vector of farm-good prices $\mathbf{p} \in \mathbf{P}$ is only source of aggregate risk.
- Prices and units of measurement normalized such that utility is linear in wealth and $E(p_i) = 1$. 
Model: Artisan Technology

- Mass one each of artisans at location $i \in [1, 2, \ldots, N]$ along highway.
- One unit of labor at date 1 makes one unit of customized artisanal good at date 2.
- Artisans of type 1 produce for farmers, artisans of type $i + 1$ produce for artisans of type $i$.
- Artisanal good valuable only for matched customer.
- Artisanal goods do not trade in spot market and do not have a quoted market price.
Model: Matching Process

- Farmers and artisans linked in chains along the highway:

  Farmer ← 1 ← 2 ← 3 ← 4 ← ... ← N.
Model: Matching Process

- Farmers and artisans linked in chains along the highway:
  Farmer $\leftarrow 1 \leftarrow 2 \leftarrow 3 \leftarrow 4 \leftarrow \ldots \leftarrow N$.
- Chains created at date 0 by random matching:
Model: Matching Process

- Farmers and artisans linked in chains along the highway:
  
  Farmer 1 ←− 2 ←− 3 ←− 4 ←− ... ←− N.

- Chains created at date 0 by random matching:
  - **Morning**: Odd $i$ artisans travel east and contract with supplier.
Model: Matching Process

- Farmers and artisans linked in chains along the highway:
  
  Farmer $\rightarrow\leftarrow 1 \rightarrow\leftarrow 2 \rightarrow\leftarrow 3 \rightarrow\leftarrow 4 \rightarrow\leftarrow \ldots \rightarrow\leftarrow N$.

- Chains created at date 0 by random matching:
  - **Morning**: Odd $i$ artisans travel east and contract with supplier.
  - **Night**: Odd $i$ artisans travel west and contract with customer.
Model: Matching Process

- Farmers and artisans linked in chains along the highway:
  
  Farmer $\leftrightarrow 1 \leftrightarrow 2 \leftrightarrow 3 \leftrightarrow 4 \leftrightarrow \ldots \leftrightarrow N$.

- Chains created at date 0 by random matching:
  
  - **Morning**: Odd $i$ artisans travel east and contract with supplier.
  - **Night**: Odd $i$ artisans travel west and contract with customer.
  - **Matching risk**: Identity of farmer in chain unknown in morning.
In every meeting, customer and supplier can enter into contract specifying:

1. Quantity $x = h$ to be produced by supplier in period 1 and delivered in period 2.
2. Payment from customer to supplier in spot market in period 2.

Introduce friction that favors simple (non-contingent) payment promise:

- Contract consists of both non-contingent promise and (possibly lower) contingent actual payment.
- Settling cost if actual payment is lower than promise.
- Today: Settling cost is infinite: non-contingent promise.
Promise of payment $\pi_{i,j}$:
- Fixed, non-contingent vector of farm goods.

Unit of account: Denomination of the promise.

\[
\pi_{i,j} = q_{i,j} \begin{pmatrix} u_{i,j} \\ 1 - u_{i,j} \end{pmatrix}.
\]
Planning Approach

- To define equilibrium would need to pin down:
  - Bargaining process.
  - Expectations over contracts in other matches.
- Instead, adopt planning approach:
  - Find system of contracts that maximizes total welfare.
  - Planner chooses (among other things) unit(s) of account for promises.
  - Social optimum is an equilibrium for a specific distribution of bargaining power.
Planning Problem

- Maximizing equally weighted welfare is equivalent to maximizing production of artisanal goods.
- Maximization subject to payment feasibility of payments:
  - If $i$ is artisan with costumer $g$ and supplier $j$, for any $p$:
    $$ p' \pi_{g,i} \geq p' \pi_{i,j}. $$
  - If $i$ is farmer with supplier $j$, for any $p$:
    $$ p_i \geq p' \pi_{i,j}. $$
- Maximization also subject to participation constraints.
Examples for Setup with Large Default Cost

- Assumption on farm good prices:
  - Symmetric price distribution.
  - Lower bound of relative price $\underline{p} = \min \{p_i/p_{-i}\} < 1$ independent of $i$.
  - Upper bound of relative price $\overline{p} = 1/p > 1$ independent of $i$. 
One type of farmer and one type of artisan:

\[ A \leftarrow 1. \]

One stage of matching. Price risk only.

Decide on \( x_A = h_1 \) and \( \pi_{A,1} = q_{A,1}(u_A, 1 - u_A)' \).

Constraints:

- Payment feasibility: for all \( p \in \mathbf{P} \),
  \[
p_a \geq p' \pi_{A,1}
  \]

- Participation constraints:
  \[
  1 - q_{A,1} + (1 + \lambda)x_A \geq 1, \\
  q_{A,1} - x_A \geq 0.
  \]
Can achieve first-best production:

- Set artisanal production to $x_A = 1$.
- Make promise in terms of the farmer's good: $u_{A,1} = 1$.
- Scale $q_A$ of payment then has to satisfy:

\[
p_A \geq p_A q_{A,1},
\]
\[
1 - q_{A,1} + 1 + \lambda \geq 1,
\]
\[
q_{A,1} - 1 \geq 0.
\]

- Hence, $q_{A,1} = 1$.

Could not get first-best production with other unit of account.
One Farmer, Two Artisans: Unit of Account Passed On

- One type of farmer and two types of artisans:
  \[ A \leftarrow 1 \leftarrow 2. \]

- Two stages of matching. Price risk only.

- Can still achieve first best:
  - Set \( x_A = x_1 = 1. \)
  - Set \( u_{A,1} = u_{1,2} = 1. \)
  - Scales of payments need to satisfy:
    \[ q_{A,1} = q_{1,2} = 1. \]
Two Farmers, Two Artisans: Dominant Unit of Account

- Highway with two types of farmer and two types of artisan:

\[
\begin{pmatrix}
A \\
B
\end{pmatrix} \leftarrow 1 \leftarrow 2.
\]

- Two stages of matching. Both price and matching risk.
- Problem: In morning matches of 1 and 2, always possible that night partner of 1 (A or B) will not correspond to the chosen unit of account.
- Scale of production needs to be lowered to avoid default.
Consider optimal choice of unit of account $u$, where:

$$\pi_{1,2} = q_{1,2} \begin{pmatrix} u \\ 1 - u \end{pmatrix}.$$

The optimal $u$ solves:

$$u = \arg\max_u \left\{ \min_p \left\{ \frac{p_i}{p_A u + p_B (1 - u)} \right\} \right\}.$$

Under symmetric price distribution have:

$$\min_p \left\{ \frac{p_i}{p_A u + p_B (1 - u)} \right\} = \frac{p}{\max\{u, 1 - u\} + p \min\{u, 1 - u\}},$$

Thus, optimal unit is $u = 0.5$: Equally weighted bundle of farm goods.
Highway with two types of farmer and four types of artisan:

\[
\begin{pmatrix}
A \\
B
\end{pmatrix} \leftarrow 1 \leftarrow 2 \leftarrow 3 \leftarrow 4.
\]

Optimal to use equally weighted bundle \((u = 0.5)\) in 3-4 morning matches as well.

Without coordination on dominant unit of account, additional sources of mismatch, resulting in lower scale of production.
Extensions

- Income risk for farmers: place more weight on good with higher income risk.
- Price distribution not symmetric: farm goods with less volatile prices are better unit of account.
- Small default costs: use unit of account to minimize probability of default.
- Optimal allocation can be decentralized with Nash bargaining at each stage.
- Unit of account is independent of bargaining weights.
- Bargaining weights matter for distribution of surplus across farmers and artisans.
Government Debt and the Optimal Unit of Account

- Model shows that dominant unit of account is optimal.
- In reality, why is money often used, as opposed to a commodity bundle?
- Introduce government that issues IOUs.
- Will private contracts be denominated in government IOUs?
In period 0, government buys fraction $g$ of farmers’ output in exchange for $g$ units of government IOUs. IOU is claim on tax revenue $T$. Tax revenue is realized at end of date 2, after spot market closes, but before consumption takes place.

At start of period 2, news about $T$ arrives. IOUs trade in spot market at price:

$$p_{IOU} = E_2(T).$$
Assume symmetric distribution for $p_A$ and $p_B$.

- $p_{IOU}$ symmetric with respect to $p_A$ and $p_B$.
- At extremes of the relative price distribution, $\frac{p_{IOU}}{\max\{p_A, p_B\}} \in [p_{IOU}, \bar{p}_{IOU}]$, $p_{IOU} < \frac{p + 1}{2}$.

- IOUs can serve as unit of account:

\[
\pi_{i,j} = \begin{pmatrix} \pi_{i,j}^{IOU} \\ \pi_{i,j}^A \\ \pi_{i,j}^B \\ \pi_{i,j} \\ u_{i,j} \\ u_{i,j}^A \\ u_{i,j}^B \end{pmatrix} = q_{i,j} \begin{pmatrix} u_{i,j}^{IOU} \\ u_{i,j}^A \\ u_{i,j}^B \end{pmatrix}
\]
Optimal unit of account:

- If $\overline{p}_{IOU} < \frac{p+1}{2}$, choose IOUs: $u^{IOU} = 1$.
- Else, choose:

$$u^{IOU} = \frac{g}{g + (1 - g)\frac{2p}{p+1}},$$

$$u_{i,j}^A = u_{i,j}^B = \frac{1 - u^{IOU}_{i,j}}{2}.$$ 

Interpretation: “dollarization” when inflation becomes too volatile.
Optimal Currency Areas

- Consider model in which there are two locations/countries:

  Country A:  \( A \leftarrow 1 \leftarrow 2 \leftarrow 3 \leftarrow 4 \)
  Country B:  \( B \leftarrow 1 \leftarrow 2 \leftarrow 3 \leftarrow 4 \)

- At each stage of matching, probability \( x < 0.5 \) of meeting someone from the other country.
- If matched in “wrong” country, can pay cost to rematch.
- Should a common unit of account be adopted?
Optimal Currency Areas

- Separate units (A for A, B for B):
  - Maximizes production conditional on matching within one country.
  - But requires paying rematch cost to avoid possibility of default.

- Common unit of account:
  - Some ex-post risk due to meeting partners from either country.
  - But no need to pay rematch cost.

- Common unit optimal when $x$ sufficiently large.
- Common unit more attractive when chains of credit are longer.
Summary

Three features lead to common unit of account:

2. Trade along credit chains.

Properties of optimal unit of account:

- Stable in value relative to revenue of borrowers in many transactions.
- Government debt works well if large and not too volatile.
- Common “currency areas” optimal if lots of trade.
Next Steps

- Explain history of units of accounts and currency areas.
- Examine role of financial intermediaries.
- Examine costs of monetary instability.
Setup with Small Default Cost

- Discrete labor supply $h \in \{0, 1\}$.
- Small default costs: $\kappa < \lambda$.
- Everyone works under optimal allocation.
- Maximize surplus by minimizing probability of default.
- Do this by coordinating on a dominant unit of account.
- Intuition as in large-default-cost case, but rather than extremes of price distribution, probability of default matters.
Optimal Contract

- All agents work: $h_i = 1$ for all $i$.
- Farmers promise and pay their entire harvest.
- Choose promise $\pi$ in matches between artisans to maximize:
  \[
  E \left[ \Pr \left[ p_h (1 + \lambda) \geq p' \pi \right] \right]
  \]
  subject to:
  \[
  E \left[ \min \{ p_h (1 + \lambda), p' \pi \} \right] \geq 1.
  \]
- Actual payment by artisan $i$ in chain headed by farmer $h$:
  \[
  v_{i,j}(\mathcal{N}, \mathbf{p}) = \min \{ p' \pi, p_h (1 + \lambda) \}.
  \]