Chapter 4

The Demand for Money

This chapter seeks to explain one stark fact: the authors used to withdraw \$20 when they went to the ATM, whereas now they tend to withdraw \$300. We are going to make a model to examine this question. In our model, a consumer chooses how often to go to the bank and how much money to withdraw once there.

Let T be the amount of time (in fractions of a year) between a consumer's trips to the bank to get money. If T is $1/3$, then the consumer goes to the bank every 4 months, or three times a year. For arbitrary T, the consumer makes $1/T$ trips to the bank in a year.

Going to the bank is a pain. It takes time and effort, and the bank may charge for each withdrawal. We accumulate all such expenses into some dollar cost γ. We could derive γ by: (i) calculating the consumer's opportunity cost of time; (ii) multiplying that by the amount of time required to go to the bank; and (iii) adding any fees charged by the bank.

The cost per year of this consumer's trips to the bank is just the number of trips times the cost per trip, so the consumer's annual transactions costs are: $(1/T)(\gamma)$. If all the prices in the economy double, then these costs double, since both bank fees and the opportunity cost of the consumer's time double.[1] Accordingly, in order to get the real impact on the consumer of these annual costs, we need to adjust them by the price level P, so the consumers real[2] Here we see that if prices double, then both P and γ double, and those extra factors of two cancel, so real costs do not change, as we require.

[1] If all prices in the economy double, then the prices of anything the consumer produces double. Put differently, the consumer's wage doubles. Either of these implies that the opportunity cost of the consumer's time doubles.

[2] The distinction between "real" and "nominal" values means the same thing here as in Barro's discussion about real versus nominal GDP. (See his Chapter 1.) "Nominal" values are actual dollars. "Real" dollars are scaled so that their purchasing power is constant. In this model, a unit of consumption costs P dollars. This is the observed or "nominal" price. If prices double, each dollar has half the purchasing power, so any nominal amount of dollars goes down in value by a factor of two. In general, we convert from nominal dollar amounts to real dollar amounts by dividing the nominal amounts by the price level P.

Now, going to the bank is costly, but the consumer still does it because the consumer needs to withdraw money in order to buy things. Assume that our consumer spends Pc dollars on consumption each year, where this spending is smooth from day to day. (This gives the consumer c real dollars of consumption each year.) In order to pay for all this consumption, the consumer needs enough money on hand at any given instant to make the purchases.

We can calculate how much money the consumer spends between trips to the bank. (Recall, T measures time between trips, in fractions of a year.) If T is 1, then the consumer spends Pc. If T is $1/2$, then the consumer spends $(Pc)/2$. In general, the consumer spends PcT dollars between trips to the bank. That is the amount the consumer must withdraw on each trip. The consumer can choose to go less often (T bigger), but only if the consumer is willing to withdraw more on each trip.

Barro's Figure 4.1 gives a graphical illustration of how the consumer's money holdings evolve over time. After going to the bank, the consumer's money holdings decline linearly, so the consumer's average money holdings are:

$$\overline{m} = \frac{PcT}{2}.$$

(This uses the fact that the area of a triangle is one half the base times the height.) The consumer's average real money holdings are:

$$\frac{\overline{m}}{P} = \frac{cT}{2}.$$

Notice that the consumer's average money holdings are increasing in the amount of time between bank visits, i.e., the longer between visits, the more money the consumer holds on average.

Since there are transactions costs involved in each trip to the bank, we might wonder why the consumer does not go once, withdraw a ton of money, and get it all over with. All the money that the consumer holds onto between trips to the bank does not earn interest, but money left in the bank does earn interest. This foregone interest is the opportunity cost of holding money. If the annual nominal interest rate is R, then each year the consumer loses out on about:

$$R\overline{m} = \frac{RPcT}{2}$$

dollars of interest. Notice that higher average money holdings result in larger amounts of foregone interest.

We can state this dollar amount of interest in real terms:

$$\text{real interest foregone annually} = \frac{R\overline{m}}{P} = \frac{RcT}{2}.$$

We are now ready to put all this together. The consumer chooses T, the time between bank visits. We have calculated the annual cost of the consumer's bank visits and the annual cost in foregone interest from the consumer's money holdings, both in real terms. Adding these two costs together gives us:

$$\text{total annual real costs} = \frac{\gamma}{PT} + \frac{RcT}{2}. \tag{4.1}$$

This equation is graphed in Barro's Figure 4.2.

We now use calculus to calculate the consumer's optimal behavior. Namely, we want to derive the consumer's cost-minimizing choice of the time T between visits to the bank to withdraw money. We are interested in the minimum costs, so we take the first-order condition of equation (4.1) with respect to T:

$$\frac{\partial}{\partial T}\left(\frac{\gamma}{PT^{\star}} + \frac{RcT^{\star}}{2}\right) = 0, \text{ or :}$$
$$-\frac{\gamma}{P(T^{\star})^2} + \frac{Rc}{2} = 0.$$

Solving this expression for $T^{\star}$ yields:

$$T^{\star} = \sqrt{\frac{2\gamma}{PRc}}.$$

With this answer, we can now write down the algebraic expression for the consumer's average holdings of real money $\overline{m}/P$, which Barro calls $\phi(R, c, \gamma/P)$. The consumer's average money holdings are:

$$\frac{\overline{m}}{P} = \left(\frac{1}{2}\right) cT.$$

When we plug in our expression for $T^{\star}$, we get:

$$\phi(R, c, \gamma/P) = \left(\frac{1}{2}\right) cT^{\star} = \left(\frac{1}{2}\right) c\sqrt{\frac{2\gamma}{PRc}} = \sqrt{\frac{\gamma c}{2PR}}. \tag{4.2}$$

We can do comparative statics to examine how these money holdings are affected by changes in the underlying parameters. See the exercises for examples. The solutions to these exercises provide the answer to question posed at the beginning of this chapter: Why do the authors now withdraw \$300 from the ATM, whereas they used to withdraw only \$20? Well, today they spend more money, the opportunity cost of their time is higher, the transactions costs at the ATM are higher, and interest rates are lower.

Presumably, the consumer that underlies this model of money demand also makes a choice of how much to consume c each year. We now briefly discuss whether it makes sense to have the consumer choose c and T separately.

When a consumer chooses how much to consume c, she considers the price of the goods she would be buying. Higher prices generally mean the consumer chooses to consume less. Now, the costs of a good as rung up at the cash register are not the full costs to the consumer of acquiring the good. The consumer might have to: expend effort to get to the store; spend valuable time waiting in line; or spend time and money to have the cash on hand to make the purchase. All of these things add to the cost that the consumer faces when making a purchase decision. If those additional costs change, then the consumer's consumption will change. This implies that the same things that the consumer considers when choosing T will affect the consumer's optimal choice of c. Since c was one of the things that went into our determination of $T^\star$, it is a shortcoming of our model that we assumed that we could separate these decisions.

Think about the following example. Suppose ATM fees go up temporarily to \$100 per transaction. In our model, this implies that γ increases, so $T^\star$ goes up, since people want to go to the bank less often. Our model assumes that c is fixed, but in reality c will fall because of the new ATM fees, since consumption is now more expensive (especially consumption of goods that have to be purchased with cash). Hence, our solution for $T^\star$ (which assumes a fixed c) is liable differ from that implied by a more sophisticated model. Since c goes down as γ goes up, and $\partial T^\star/\partial c < 0$, T would go up by more in a model that took the relationship between c and T into account.

Variable	Definition
T	Time (in years) between trips to the bank
γ	Opportunity cost of a trip to the bank
P	Price of consumption
c	Consumption per year
$\bar{m}$	Consumer's average money holdings
R	Nominal interest rate
$\phi(\cdot)$	Real money demand

Table 4.1: Notation for Chapter 4

Exercises

Exercise 4.1 (Easy)

1. Determine the effect of an increase in the interest rate R on the consumer's money demand $\phi(\cdot)$, as given by equation (4.2).
2. Determine the effect of an increase in the consumer's consumption c on the consumer's money demand $\phi(\cdot)$, as given by equation (4.2).
3. Determine the effect of an increase in the consumer's real transactions costs γ/P on the consumer's money demand $\phi(\cdot)$, as given by equation (4.2).

(Use calculus for all three parts. The way you do the last one is to replace γ/P with some other variable, say α, and differentiate with respect to the new variable.)