

# An Expected-Efficient Status Quo Allows Efficient Bargaining\*

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## Abstract

In a general model of asymmetric-information bargaining with independent private values and quasilinear utilities, efficient bargaining is possible when the status-quo allocation (the distribution of property rights) is the expected efficient allocation. The only assumption needed for this is that the total surplus is convex in the allocation, which holds as long as randomized allocations are allowed.

## 1 Introduction

Myerson and Satterthwaite (1983) first demonstrated, in a bilateral trading setting, that private information and property rights may make it impossible for parties to bargain towards fully efficient trade. Specifically, they showed that when privately-known valuations are statistically independent, under very weak conditions no efficient, Bayesian incentive-compatible, budget-balanced, and interim individually rational mechanism exists when a seller owns a good that a buyer may wish to purchase.<sup>1</sup>

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<sup>1</sup>Mailath and Postlewaite (1990) derive a similar result for public good provision.

Subsequently, the literature has asked whether efficiency can be attained for *some* status quo allocations (that is, allocations of ex ante “property rights”). Cramton et al. (1987) first showed that in the problem of allocating a divisible good to the highest-value agent, efficiency can be achieved with symmetric agents when the status-quo allocation of the good is close enough to equal shares. Schmitz (2002), Che (2006), and Figueroa and Skreta (2007) demonstrated other examples in which efficiency is possible, while Neeman (1999) considered a pollution model in which the existence of an efficiency-permitting status-quo allocation depended on some parameters. Schweizer (2006, Proposition 2) established the existence of an efficiency-permitting status-quo allocation in a model that generalized the preceding ones.

This note contributes to the literature by describing a status-quo allocation that allows efficient bargaining in a very general model, which strictly generalizes all the preceding models for which existence has been shown. In contrast to Schweizer’s result, we explicitly describe a natural status-quo allocation that permits efficient bargaining – it is the expectation of the ex post efficient allocation. The only assumption needed (in addition to the quasilinearity of payoffs and independent private values) is that the total surplus is convex in the allocation. Moreover, the proof of the result is surprisingly simple.

## 2 The Model and Result

Consider  $I$  expected-utility maximizing agents whose utilities are quasilinear in money. The set of non-monetary allocations  $X$  is a subset of a measurable topological vector space. The agents’ privately observed types are independently distributed random variables  $\tilde{\theta}_1, \dots, \tilde{\theta}_I$  with values in measurable spaces  $\Theta_1, \dots, \Theta_I$  respectively. The state space is thus  $\Theta = \Theta_1 \times \dots \times \Theta_I$ . The utility of each agent  $i$  is given by  $u_i(x, \theta_i) + t_i$ , where  $t_i \in \mathbb{R}$  is the payment to the agent,  $x \in X$  is the nonmonetary allocation, and  $\theta_i \in \Theta_i$  is the agent’s type. The functions  $u_i : X \times \Theta_i \rightarrow \mathbb{R}$  are measurable and uniformly bounded (i.e.,  $\sup_{i,x \in X, \theta_i \in \Theta_i} |u_i(x, \theta_i)| < \infty$ ).<sup>2</sup> The total surplus given allocation  $x$  and state  $\theta$  is  $s(x, \theta) \equiv \sum_i u_i(x, \theta_i)$ .

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<sup>2</sup>The last two assumptions ensure that expectations exist and that the inf in the proof of the Proposition is finite.

**Definition 1**  $\chi^* : \Theta \rightarrow X$  is an *efficient allocation rule* if

$$s(\chi^*(\theta), \theta) = \max_{x \in X} s(x, \theta) \equiv S(\theta) \text{ for all } \theta \in \Theta.$$

Throughout we shall assume that an efficient allocation rule  $\chi^*(\cdot)$  exists.

**Definition 2** Status quo  $\hat{x} \in X$  *permits efficient bargaining* if there exists a Bayesian incentive-compatible and interim individually rational mechanism implementing an efficient allocation rule in which the total payment to the agents is zero in all states.

By the Revelation Principle we focus on direct revelation mechanisms  $\langle \chi^*(\cdot), \tau(\cdot) \rangle$  implementing an efficient allocation rule  $\chi^*(\cdot)$  with a payment rule  $\tau : \Theta \rightarrow \mathbb{R}^I$ . The Bayesian Incentive Compatibility (IC) and interim individual rationality (IR) constraints for each agent  $i$  in this mechanism can be written as

$$\mathbb{E}_{\tilde{\theta}_{-i}} \left[ u_i(\chi^*(\theta_i, \tilde{\theta}_{-i}), \theta_i) - \tau_i(\theta_i, \tilde{\theta}_{-i}) \right] \geq \mathbb{E}_{\tilde{\theta}_{-i}} \left[ u_i(\chi^*(\theta'_i, \tilde{\theta}_{-i}), \theta_i) - \tau_i(\theta'_i, \tilde{\theta}_{-i}) \right] \\ \text{for all } \theta_i, \theta'_i \in \Theta, \quad (\text{IC})$$

$$\mathbb{E}_{\tilde{\theta}_{-i}} \left[ u_i(\chi^*(\theta_i, \tilde{\theta}_{-i}), \theta_i) - \tau_i(\theta_i, \tilde{\theta}_{-i}) \right] \geq u_i(\hat{x}, \theta_i) \text{ for all } \theta_i \in \Theta. \quad (\text{IR})$$

In particular, consider the direct mechanism  $\langle \chi^*(\cdot), \bar{\tau}(\cdot|\hat{x}) \rangle$  with payments to each agent  $i$  in state  $\theta$  of

$$\bar{\tau}_i(\theta|\hat{x}) = \sum_{j \neq i} u_j(\chi^*(\theta), \theta_j) - \inf_{\hat{\theta}_i \in \Theta_i} \left( \mathbb{E}_{\tilde{\theta}_{-i}} \left[ S(\hat{\theta}_i, \tilde{\theta}_{-i}) \right] - u_i(\hat{x}, \hat{\theta}_i) \right). \quad (1)$$

The expected profit for an intermediary running this mechanism is

$$\bar{\pi}(\hat{x}) \equiv - \sum_i \mathbb{E} \left[ \bar{\tau}_i(\tilde{\theta}|\hat{x}) \right] \\ = - (I - 1) \mathbb{E} \left[ S(\tilde{\theta}) \right] + \inf_{\hat{\theta} \in \Theta} \left( \sum_i \mathbb{E}_{\tilde{\theta}_{-i}} \left[ S(\hat{\theta}_i, \tilde{\theta}_{-i}) \right] - s(\hat{x}, \hat{\theta}) \right). \quad (2)$$

The following lemma, which has a number of (more restrictive) precedents in the literature, provides a useful sufficient condition for efficient bargaining to be possible:<sup>3</sup>

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<sup>3</sup>Precedents can be found, e.g., in Makowski and Mezzetti (1994), Krishna and Perry (1998), Neeman (1999), Williams (1999), Schweizer (2006), Che (2007), and Figueroa and Skreta (2008). However, these precedents make restrictive assumptions on type spaces and utility functions, in order to show that the condition is necessary as well as sufficient for permitting efficient bargaining. (We state such necessity result in Lemma 2 below.)

**Lemma 1** *Any status quo  $\hat{x} \in X$  such that  $\bar{\pi}(\hat{x}) \geq 0$  permits efficient bargaining.*

**Proof.** First note that  $\langle \chi^*(\cdot), \bar{\tau}(\cdot|\hat{x}) \rangle$  is a VCG mechanism, hence it is dominant-strategy incentive compatible and therefore satisfies IC. This mechanism also satisfies IR, since the slack in the IR constraint of agent  $i$ 's type  $\theta_i$  (his interim net expected utility over the status quo) is

$$\mathbb{E}_{\tilde{\theta}_{-i}} \left[ S(\theta_i, \tilde{\theta}_{-i}) \right] - u_i(\hat{x}, \theta_i) - \inf_{\hat{\theta}_i \in \Theta_i} \left( \mathbb{E}_{\tilde{\theta}_{-i}} \left[ S(\hat{\theta}_i, \tilde{\theta}_{-i}) \right] - u_i(\hat{x}, \hat{\theta}_i) \right) \geq 0.$$

We now construct a mechanism satisfying IC and IR in which the profit is zero in every state. The payment to each agent  $i$  in the new mechanism is

$$\psi_i(\theta) \equiv \mathbb{E}_{\tilde{\theta}_{-i}} \left[ \bar{\tau}_i(\theta_i, \tilde{\theta}_{-i}|\hat{x}) \right] + \frac{1}{I-1} \sum_{j \neq i} \left( \mathbb{E} \left[ \bar{\tau}_j(\tilde{\theta}|\hat{x}) \right] - \mathbb{E}_{\tilde{\theta}_{-j}} \left[ \bar{\tau}_j(\theta_j, \tilde{\theta}_{-j}|\hat{x}) \right] \right) + \frac{1}{I} \bar{\pi}(\hat{x}).$$

By construction,  $\sum_i \psi_i(\theta) = 0$  for all  $\theta \in \Theta$ . Also by construction, for all  $i$ ,  $\theta_i \in \Theta_i$ ,

$$\mathbb{E}_{\tilde{\theta}_{-i}} \left[ \psi_i(\theta_i, \tilde{\theta}_{-i}) \right] - \mathbb{E}_{\tilde{\theta}_{-i}} \left[ \bar{\tau}_i(\theta_i, \tilde{\theta}_{-i}) \right] = \frac{1}{I} \bar{\pi}(\hat{x}) \geq 0,$$

hence mechanism  $\langle \chi^*(\cdot), \psi(\cdot) \rangle$  inherits both IC and IR from mechanism  $\langle \chi^*(\cdot), \bar{\tau}(\cdot|\hat{x}) \rangle$ .

■

We now establish our main result:

**Proposition 1** *Suppose that the total surplus  $s(x, \theta)$  is convex in  $x$  for all  $\theta \in \Theta$ , that  $\chi^* : \Theta \rightarrow X$  is an efficient allocation rule, and that the status quo is the expected efficient allocation  $\mathbb{E}[\chi^*(\tilde{\theta})] \in X$ . Then this status quo permits efficient bargaining.*

**Proof.** Given any status quo  $\hat{x}$ , the expected profit  $\bar{\pi}(\hat{x})$  is bounded below as follows:

$$\begin{aligned} \bar{\pi}(\hat{x}) &= \inf_{\hat{\theta} \in \Theta} \left[ - (I-1) \mathbb{E} \left[ S(\tilde{\theta}) \right] + \sum_i \mathbb{E} \left[ S(\hat{\theta}_i, \tilde{\theta}_{-i}) \right] - s(\hat{x}, \hat{\theta}) \right] \\ &\geq \inf_{\hat{\theta} \in \Theta} \mathbb{E} \left[ - (I-1) \mathbb{E} \left[ S(\tilde{\theta}) \right] + \sum_i s \left( \chi^*(\tilde{\theta}), (\hat{\theta}_i, \tilde{\theta}_{-i}) \right) + s(\hat{x}, \hat{\theta}) \right] \\ &= \inf_{\hat{\theta} \in \Theta} \mathbb{E} \left[ s(\chi^*(\tilde{\theta}), \hat{\theta}) - s(\hat{x}, \hat{\theta}) \right], \end{aligned} \tag{3}$$

where the inequality holds by the definition of  $S(\widehat{\theta}_i, \tilde{\theta}_{-i})$ .

Condition (3) and Jensen's inequality yields

$$\bar{\pi}(\mathbb{E}[\chi^*(\tilde{\theta})]) \geq \inf_{\hat{\theta} \in \Theta} \mathbb{E} \left[ s(\chi^*(\tilde{\theta}), \hat{\theta}) - s(\mathbb{E}[\chi^*(\tilde{\theta})], \hat{\theta}) \right] \geq 0, \quad (4)$$

so the result follows from Lemma 1. ■

### 3 Remarks

#### 3.1 Role of Convexity

When the allocation space  $X$  is not convex, it may not contain an expected-efficient allocation  $\mathbb{E}[\chi^*(\tilde{\theta})]$ , in which case Proposition 1 does not apply and there may not exist a feasible status quo permitting efficient bargaining:

**Example 1 (Bilateral Trade)** *Suppose that  $I = 2$ , each  $\tilde{\theta}_i$  is distributed on  $\Theta_i = [0, 1]$  according to a continuous strictly increasing c.d.f.  $F_i$ ,  $X = \{(1, 0), (0, 1)\}$ , and  $u_i(x, \theta_i) = \theta_i x_i$ . In this setting, the efficient allocation rule has  $\chi_i^*(\theta) = 1$  whenever  $\theta_i > \theta_{-i}$ . By the Myerson-Satterthwaite Theorem [Myerson and Satterthwaite (1983)], neither possible status-quo allocation allows efficient bargaining.*

Even with a convex allocation space  $X$ , the assumption of convexity of the total surplus  $s(x, \theta)$  in the allocation  $x$  is also crucial. Neeman (1999, p.685) offers an example of pollution in which the convexity assumption fails and there does not exist an efficiency-permitting status-quo allocation.

On the other hand, both convexity assumptions are trivially ensured by allowing randomized allocations, in which case we take  $X$  to be the space of probability distributions over a primitive set of pure allocations, and the agents' expected utilities are linear in  $x$ . Allowing randomizations does not affect the efficiency of a deterministic allocation rule, but a randomized status quo allowing efficient bargaining will exist by Proposition 1. In Example 1, the space of randomized allocations is the simplex  $X = \{(x_1, x_2) \in \mathbb{R}_+^2 : x_1 + x_2 = 1\}$ , and the utilities are linear in  $x$ , hence Proposition 1 applies.

### 3.2 Role of Asymmetric-Information Bargaining

The fact that an expected efficient status quo maximizes surplus depends on the assumption of optimal bargaining under asymmetric information. This conclusion need not follow for different bargaining procedures or different sources of bargaining failure. For example, suppose that there is no bargaining at all. Then, if  $X$  is a compact convex subset of a Euclidean space and  $s(x, \theta)$  is convex in the allocation  $x$ , the expected surplus  $\mathbb{E}[s(x, \theta)]$  is maximized by an extreme point of  $X$  (this follows from the Krein-Milman Theorem), and generically only by such a point. For, instance, in Example 1, if one agent has a higher value of  $\mathbb{E}[\theta_i]$  than the other, then expected surplus is uniquely maximized by giving the entire asset to one agent. The same is true if there is an exogenous probability that bargaining fails, with perfect Coasian bargaining taking place with the complementary probability. Intuitively, when bargaining inefficiencies are instead endogenous and caused by asymmetric information, they can be ameliorated by making the status quo intermediate rather than extreme.

### 3.3 Comparison to Schweizer's (2006) Proposition 2

Schweizer's approach may be described as follows: Consider an intermediary who offers a mechanism with an efficient allocation rule  $\chi^*(\cdot)$  and payments to each agent  $i$  of the form

$$\hat{\tau}_i(\theta|\hat{x}, \hat{\theta}) = \sum_{j \neq i} u_j(\chi^*(\theta), \theta_j) - S(\hat{\theta}_i, \tilde{\theta}_{-i}) + u_i(\hat{x}, \hat{\theta}_i) \quad (5)$$

for some  $\hat{x} \in X$  and  $\hat{\theta} = (\hat{\theta}_1, \dots, \hat{\theta}_I) \in \Theta$ . Note that this is a VCG mechanism and that it makes the IR constraints of "critical types"  $\hat{\theta}_1, \dots, \hat{\theta}_I$  hold with equality given the status-quo allocation  $\hat{x}$ . The intermediary's expected profit in this mechanism is

$$\pi(\hat{x}, \hat{\theta}) \equiv -\mathbb{E} \left[ \sum_i \hat{\tau}_i(\tilde{\theta}|\hat{x}, \hat{\theta}) \right] = -(I-1) \mathbb{E} [S(\tilde{\theta})] + \sum_i \mathbb{E} [S(\hat{\theta}_i, \tilde{\theta}_{-i})] - s(\hat{x}, \hat{\theta}). \quad (6)$$

Observe that the interim expectations of payments (1) in the mechanism considered earlier are  $\mathbb{E}_{\theta_{-i}}[\hat{\tau}_i(\theta|\hat{x})] = \sup_{\hat{\theta}_i \in \Theta_i} \mathbb{E}_{\theta_{-i}}[\hat{\tau}_i(\theta|\hat{x}, \hat{\theta})]$  for all  $i$  and  $\theta$ , which are the smallest interim expected payments ensuring participation of all types, and the profit (2) in that mechanism is  $\bar{\pi}(\hat{x}) = \inf_{\hat{\theta} \in \Theta} \pi(\hat{x}, \hat{\theta})$ .

Schweizer imposes sufficient conditions to ensure the existence of a saddle point of the function  $\pi(\hat{x}, \hat{\theta})$ , a pair  $(\hat{x}^\circ, \hat{\theta}^\circ)$  such that

$$\min_{\hat{\theta} \in \Theta} \pi(\hat{x}^\circ, \hat{\theta}) = \pi(\hat{x}^\circ, \hat{\theta}^\circ) = \max_{\hat{x} \in X} \pi(\hat{x}, \hat{\theta}^\circ), \quad (7)$$

which corresponds to a Nash equilibrium of the zero-sum game in which the intermediary chooses the status-quo allocation  $\hat{x}$  to maximize  $\pi(\cdot)$  and an adversary chooses the critical types  $\hat{\theta}$  to minimize it. Using (6), the two saddle point conditions in (7) can be rewritten respectively as

$$\hat{\theta}_i^\circ \in \arg \min_{\hat{\theta}_i \in \Theta_i} \mathbb{E}_{\hat{\theta}_{-i}} \left[ S(\hat{\theta}_i, \hat{\theta}_{-i}) \right] - u_i(\hat{x}^\circ, \hat{\theta}_i) \text{ for all } i, \quad (8)$$

$$\hat{x}^\circ \in \arg \min_{\hat{x} \in X} s(\hat{x}, \hat{\theta}^\circ). \quad (9)$$

Condition (8) says that each agent  $i$ 's critical type  $\hat{\theta}_i^\circ$  is his worst-off type in the mechanism given the status-quo allocation  $\hat{x}$ . Condition (9) says that the status-quo allocation  $\hat{x}$  minimizes the total surplus in the state in which the agents have their critical types.

Schweizer then shows that the status quo allocation  $\hat{x}^\circ$  permits efficient bargaining. To derive Schweizer's result, one way to proceed is to observe that, by an argument parallel to the one leading to (3), for any  $\hat{x}$  and  $\hat{\theta}$  we have

$$\pi(\hat{x}, \hat{\theta}) \geq \mathbb{E}[s(\chi^*(\hat{\theta}), \hat{\theta}) - s(\hat{x}, \hat{\theta})]. \quad (10)$$

Combining (7) and (10) yields

$$\bar{\pi}(\hat{x}) = \min_{\hat{\theta} \in \Theta} \pi(\hat{x}^\circ, \hat{\theta}) = \max_{\hat{x} \in X} \pi(\hat{x}, \hat{\theta}^\circ) \geq \mathbb{E} \left[ s(\chi^*(\hat{\theta}^\circ), \hat{\theta}^\circ) - \min_{\hat{x} \in X} s(\hat{x}, \hat{\theta}^\circ) \right] \geq 0.$$

So by Lemma 1, status quo  $\hat{x}^\circ$  permits efficient bargaining.

To ensure existence of a saddle point, Schweizer assumes that the sets of allocations  $X$  and of types  $\Theta_1, \dots, \Theta_I$  are convex and compact subsets of Euclidean spaces, that the total surplus  $s(x, \theta)$  is linear in types  $\theta$ , convex in allocations  $x$ , and continuous in  $(x, \theta)$ .<sup>4</sup> One advantage of our approach is that it allows us to dispense with these assumptions — in particular,

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<sup>4</sup>Schweizer also assumes in footnotes 1 and 2 that the payoffs are continuously differentiable in  $(x, \theta)$  and the efficient allocation rule  $\chi^*$  is continuous a.e., but these assumptions do not seem to play any role in his Proposition 2.

we allow for infinite-dimensional allocations or types, non-convex or non-compact types spaces, and utility functions being nonlinear or discontinuous. The only indispensable assumptions prove to be convexity of the total surplus in the allocation and the feasibility of the expected efficient allocation. A second and perhaps more important advantage is our explicit description of a natural status-quo allocation that permits efficiency.<sup>5</sup>

It is interesting to note that, even when a saddle-point status-quo allocation does exist, it generally differs from an expected-efficient allocation (i.e., the expected-efficient status-quo allocation need not maximize the intermediary's expected profits):

**Example 2** Consider the bilateral trade setting of Example 1, with the randomized allocation space  $X = \{(x_1, x_2) \in \mathbb{R}_+^2 : x_1 + x_2 = 1\}$ . This setting has a unique saddle point, which is found as follows: Given a status-quo allocation  $\hat{x}^\circ = (\hat{x}_1^\circ, \hat{x}_2^\circ)$ , condition (8) is satisfied only by the types  $\hat{\theta}_i^\circ$  whose interim expected consumption equals  $\hat{x}_i^\circ$ , i.e.,  $\mathbb{E}_{\hat{\theta}_{-i}} [\chi_i^*(\hat{\theta}_i, \hat{\theta}_{-i})] = F_{-i}(\hat{\theta}_i^\circ) = \hat{x}_i^\circ$  (any other type can obtain a greater interim net expected utility than type  $\hat{\theta}_i^\circ$  by announcing  $\hat{\theta}_i^\circ$ ). To satisfy condition (9), we must have  $\hat{\theta}_1^\circ = \hat{\theta}_2^\circ$  (unless  $\hat{x}_i^\circ = 1$  for some  $i$  and  $\hat{\theta}_i^\circ < \hat{\theta}_{-i}^\circ \leq 1$ , but this contradicts the previous condition  $F_{-i}(\hat{\theta}_i^\circ) = \hat{x}_i^\circ = 1$ ). The equation  $F_1(\hat{\theta}^\circ) + F_2(\hat{\theta}^\circ) = 1$  then uniquely defines the saddle-point types  $\hat{\theta}_1^\circ = \hat{\theta}_2^\circ = \hat{\theta}^\circ$  and the status-quo allocation  $\hat{x}^\circ = (F_2(\hat{\theta}^\circ), F_1(\hat{\theta}^\circ))$ .<sup>6</sup> Note that this status-quo allocation does not change if we perturb  $F_1$  and  $F_2$  in ways that keep  $F_1(\hat{\theta}^\circ)$  and  $F_2(\hat{\theta}^\circ)$  fixed, but such perturbations generally alter the expected-efficient allocation  $\mathbb{E}[\chi^*(\hat{\theta})]$ .

### 3.4 Efficiency with Non-convex Surplus

In some cases, an expected efficient status quo permits efficient bargaining even though randomization is infeasible and the total surplus  $s(x, \theta)$  is not

<sup>5</sup>Schweizer's saddle-point status quo  $\hat{x}^\circ$  allows one advantage: one can show that  $\sum_i \hat{\tau}_i(\theta | \hat{x}^\circ, \hat{\theta}^\circ) \leq 0$  for all  $\theta \in \Theta$ , so the mechanism  $\langle \chi^*(\cdot), \hat{\tau}(\cdot | \hat{x}^\circ, \hat{\theta}^\circ) \rangle$  never requires a monetary infusion. [The argument uses a bound like in (10), but that holds for each state  $\theta$  rather than in expectation.] Unlike the mechanism  $\langle \chi^*(\cdot), \psi(\cdot) \rangle$  in the proof of our Proposition, which also requires no monetary infusion, this mechanism is dominant-strategy incentive compatible rather than just Bayesian incentive compatible. Note, however, that participation is not a dominant strategy.

<sup>6</sup>This allocation has been considered by Schmitz (2002, proof of Proposition 3), Che (2006), and Figueroa and Skreta (2008).

convex in  $x$ .

**Example 3** Suppose that  $X$  and each  $\Theta_i$  are bounded subsets of  $\mathbb{R}^K$ ,  $\mathbb{E}[\theta_i] = 0$ , and  $u_i(x, \theta_i) = \theta_i \cdot x - \frac{1}{2I} x \cdot x$ . Assume, in addition, that  $\sum_i \theta_i \in X$  for all  $\theta \in \Theta$ . Then  $\chi^*(\theta) = \sum_i \theta_i$  and  $S(\theta) = \frac{1}{2} (\sum_i \theta_i) \cdot (\sum_i \theta_i)$ . Given a status quo  $\hat{x}$ , the types that achieve the infimum in (2) are  $\hat{\theta}_i(\hat{x}) = \hat{x}$  for each  $i$ , and the resulting expected profit can be calculated as (using the statistical independence of agents' types):

$$\bar{\pi}(\hat{x}) = \pi(\hat{x}, \hat{\theta}(\hat{x})) = - \left( \frac{I-1}{2} \right) \hat{x} \cdot \hat{x}.$$

This function is strictly concave in  $\hat{x}$  and is maximized at  $\hat{x} = 0$ , the expected efficient allocation, which yields  $\bar{\pi}(0) = 0$ . Thus, by Lemma 1, the expected efficient status quo  $\hat{x} = 0$  permits efficient bargaining. (Moreover, by Lemma 2 below, it is the unique status quo that permits efficient bargaining in this example whenever each  $\Theta_i$  is a smoothly connected set.)<sup>7</sup>

However, as the discussion at the end of the previous subsection indicates, an expected efficient status quo generally differs from the status quo that maximizes the intermediary's expected profit. As a result, with a concave surplus efficiency may be possible for some status quo allocation, but not with an expected efficient status quo. To provide an example, we first note that for certain classical settings, a converse to Lemma 1 also holds, and so we have:

**Lemma 2** Suppose that for each agent  $i$ ,  $\Theta_i$  is a smoothly connected subset of a Euclidean space, and  $u_i(x, \theta_i)$  is differentiable in  $\theta_i$ , with a bounded gradient on  $X \times \Theta$ . Then status quo  $\hat{x} \in X$  permits efficient bargaining if only if  $\bar{\pi}(\hat{x}) \geq 0$ .

**Proof.** The “if” statement is by Lemma 1. For the “only if” statement, by the Revenue Equivalence Theorem (which follows from Corollary 1 in Milgrom and Segal (2002)), the expected profit in any IC mechanism implementing an efficient allocation rule  $\chi^*$  in which the IR of each agent  $i$ 's type  $\hat{\theta}_i$  holds is bounded above by (6). Thus, the expected profit in any mechanism that satisfies IC and IR is bounded above by  $\inf_{\hat{\theta} \in \Theta} \pi(\hat{x}, \hat{\theta}) = \bar{\pi}(\hat{x})$ . ■

<sup>7</sup>The same result holds in any setting in which the agents' utilities are linear-quadratic in  $(\theta, x)$  and the total surplus  $s(x, \theta)$  is strictly concave in  $x$ . Indeed, any such setting can be transformed into the model in the example using affine transformations of  $x$  and  $\theta$  and adding baseline decision-dependent transfers.

**Example 4** Suppose that  $I = 2$ , that  $\Theta_i = X = [0, 1]$  for  $i = 1, 2$ . Suppose also that  $u_1(x, \theta_1) = u_1(x, \theta_1) = -\theta_1^{1-a}x^a/a$  and  $u_2(x, \theta_2) = \theta_2^{1-1/a}x$ , with  $a > 1$ . (Agent 1 is the “seller,” agent 2 the “buyer,” and  $x$  is the buyer’s purchase.) Then  $\chi^*(\theta) = \theta_1\theta_2^{1/a}$  and  $S(\theta) = \left(\frac{a-1}{a}\right)\theta_1\theta_2$ . Given status quo  $\hat{x}$ , the types that achieve the infimum in (2) are  $\hat{\theta}_1(\hat{x}) = \mathbb{E}[\theta_2]^{-1/a}\hat{x}$  and  $\hat{\theta}_2(\hat{x}) = \mathbb{E}[\theta_1]^{-a}\hat{x}^a$ , and the resulting expected profit is

$$\bar{\pi}(\hat{x}) = -\frac{1}{a}\mathbb{E}[\theta_1]^{1-a}\hat{x}^a + \mathbb{E}[\theta_2]^{1-1/a}\hat{x} - \left(\frac{a-1}{a}\right)\mathbb{E}[\theta_2]\mathbb{E}[\theta_1].$$

This function is strictly concave in  $\hat{x}$  and is maximized at  $\hat{x}^\circ = \mathbb{E}[\theta_1]\mathbb{E}[\theta_2]^{1/a}$ , which yields  $\bar{\pi}(\hat{x}^\circ) = 0$ . Thus, by Lemma 2, this is the only status quo that permits efficient bargaining. In particular, efficient bargaining is impossible with the expected efficient status quo  $\mathbb{E}[\chi^*(\theta)] = \mathbb{E}[\theta_1]\mathbb{E}[(\theta_2)^{1/a}] < \hat{x}^\circ$  (by Jensen’s inequality).

### 3.5 Relation to Hold-Up Models

Suppose agents first choose unobservable ex ante investments, with each agent  $i$ ’s private investment choice  $a_i \in A_i$  determining the distribution of his type  $\theta_i$ . (Any investment costs can be subsumed into utilities.) Let  $a^* = (a_1^*, \dots, a_I^*)$  be an efficient investment profile, i.e., maximizing the expected total surplus  $\mathbb{E}[S(\tilde{\theta})|a]$ . Then if following the realizations of their types the agents participate in the budget-balanced mechanism  $\langle \chi^*, \psi \rangle$  constructed in the proof of Proposition 1 where all expectations are taken conditional on the efficient investment profile  $a^*$ , this profile will form a Nash equilibrium of the ex ante investment game. This is analogous to Rogerson’s (1992) Proposition 2, since mechanism  $\langle \chi^*, \psi \rangle$  is an expected-externality mechanism, but unlike Rogerson’s result, we also ensure interim participation of all the agents, both on and off the equilibrium path. Indeed, with our expected-efficient status quo (or any status quo satisfying the condition in Lemma 1 conditional on efficient investments), the mechanism also satisfies each agent  $i$ ’s IR and IC constraints for any realization of his type  $\theta_i$ , provided that the other agents have made efficient investments  $a_{-i}^*$ . Thus, the strategies in which each agent  $i$  invests  $a_i^*$  and then participates in the mechanism and reports truthfully for each realized  $\theta_i$  form a Perfect Bayesian Equilibrium of the mechanism.

The use of an expected-efficient allocation as the status quo is reminiscent of the result of Edlin and Reichelstein (1996), who show that in a two-agent hold-up model in which observable but not verifiable investments in private values are followed by symmetric-information Nash bargaining, such a status quo sustains efficient investments in equilibrium, provided that payoffs satisfy a separability condition. Our result, however, is formally quite distinct: e.g., the investments here are unobservable, our bargaining mechanism is quite different from Nash bargaining, and the assumptions of convexity and separability are non-nested.

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