Dynamic Decision-Making under Ambiguity: Recent developments

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Ambiguity and Dynamic Choice: Overview

The plan:

- Choice under ambiguity: "executive sumary"
- Updating ambiguous beliefs
- The key issue: dynamic (in)consistency
- Solution 1: tree consistency
- Solution 2: sophisticated choice
- Solution 3: non-consequentialist choice

Focus on multiple priors (MEU; Gilboa-Schmeidler, 1989)

Most ideas generalize to "fancier" models

The Ellsberg Paradox, three-color urn edition

90 balls: 30 red, 60 green or blue.

One ball will be drawn; bets on its color.

	r	g	b
f_r	1	0	0
f_g	0	1	0
f_{rb}	1	0	1
f_{gb}	0	1	1

Modal preferences: $f_r \succ f_g$, $f_{rb} \prec f_{gb}$: **ambiguity aversion**

Inconsistent with probabilistic reasoning: prefs indicate

$$P(r) > P(g), \quad P(r) + P(b) < P(g) + P(b)!$$

Notation and Setup

Basic setup:

- Ω: state space, with sigma-algebra Σ.
- charges $ba_1(\Sigma)$, or $\Delta(\Sigma)$ if finite.
- X: set of consequences
  Anscombe-Aumann: X cvx subset of lin space: e.g. $\Delta(Z)$
- Acts are Σ-measurable functions $f : \Omega \to X$.
  - L_c or X: constant acts
  - L_0: simple acts, i.e. f such that $f^{-1}(\Omega)$ discrete
- Mixtures on L_0 taken pointwise : $\alpha f(\omega) + (1-\alpha)g(\omega)$.
- Preferences on L_0: $\succcurlyeq$ and (for $E \in \Sigma$ "not null") $\succcurlyeq_E$

Representing functionals:

- $V : L_0 \to \mathbb{R}$ such that $f \succcurlyeq g$ iff $V(f) \geq V(g)$.
- EU: $V(h) = \mathrm{E}_p[u \circ h]$, $p \in ba_1(\Sigma)$
- Maxmin EU: $V(h) = \min_{p \in C} \mathrm{E}_p[u \circ h]$, $C \subset ba_1(\Sigma)$.

Conditional representing functional: $V_E : L_0 \to \mathbb{R}$, $E \in \Sigma$.

Basics

Compound acts:

Definition

For $f, g \in L_0$ and $E \in \Sigma$, fEg is the act with $fEg(s) = f(s)$ for $s \in E$ and $fEg(s) = g(s)$ for $s \in \Omega \setminus E$.

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"Zero-probability" events:

Definition (Null event)

$E \in \Sigma$ is **null for** $\hat{\succsim}$ iff, for some $x, y \in X$ with $x \succ y$, $xEy \sim y$.

As in EU, condition on non-null events.

If condition on E, only outcomes in states $s \in E$ matter:

NC **Null Complement**: For all non-$\succsim$-null $E \in \Sigma$, $\Omega \setminus E$ is $\succsim_E$–null

Classical Updating Rules for MEU

$$V(h) = \min_{q \in C} \mathrm{E}_q[u \circ f], \qquad u : X \to \mathbb{R}, \quad C \subset ba_1(\Sigma).$$

Non-null means that $\min_{q \in C} q(E) > 0$.

Conditional MEU preference : $V_E = \min_{q \in C_E} \mathrm{E}_q[u \circ f]$

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Two "natural" updating rules:

- Prior-by-prior : $C_E = \{q(\cdot|E) \,:\, q \in C,\ q(E) > 0\}$
- Maximum-likelihood :
  $C_E = \{q(\cdot|E) \,:\, q \in C,\ q \in \arg\max_{q' \in C} q'(E)\}$

Rich history: Walley (1990), Gilboa-Schmeidler (1993), Jaffray (1994), Pires (2001)...

MEU: Prior-by-prior updating and the fixpoint axiom

Jaffray (1994), Pires (2001).

EU: for all $f \in L_0$, E non-null, $x \sim_E f$ iff $fEx \sim x$:

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& u(x) = \mathrm{E}[u \circ f | E] \\
\Leftrightarrow & P(E)u(x) + P(\Omega \setminus E)u(x) = P(E)\mathrm{E}[u \circ f | E] + P(\Omega \setminus E)u(X) \\
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Use as axiom:

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Intuition: local mean

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Proposition (Jaffray, Pires)

If $\succcurlyeq$, $\succcurlyeq_E$ are MEU with same u, then NC and FP iff
$C_E = \{q(\cdot|E) : q \in C\}$.

MEU: Maximum-likelihood updating and pessimism axiom

Gilboa and Schmeidler (1993): actually MEU $\cap$ CEU

Assume there exist best, worst prize x^*, x_*.

PE **Pessimism** For all $f \in L_0$: $f \succcurlyeq_E g$ iff $fEx^* \succcurlyeq gEx^*$.

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Updating: A summary

- More rules: Horie (2007), Eichberger, Grant, Kelsey (2009)...
- Good: Attitudes toward updating!
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Two main problems (or sides of the same coin):

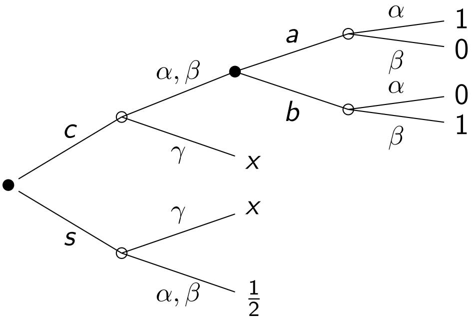
- Dynamic Inconsistency
- Is $\succcurlyeq_E$ observable or counterfactual?

Ambiguity and Dynamic Inconsistency

$\Omega = \{\alpha, \beta, \gamma\}$, $E = \{\alpha, \beta\}$. MEU prefs, $C = \{q : q(\alpha) = \frac{1}{3}\}$.

Prior-by-prior updating: $C_E = \{q : q(\alpha) \geq \frac{1}{3},\ q(\gamma) = 0\}$.

For $x = 0, 1$, let $f_x =$



Actions: a, b, s. Effectively acts (in this example).

Plans: ca_0, ca_1, cb_0, cb_1. Reduce to acts: e.g. $ca_1(\gamma) = 1$, etc.

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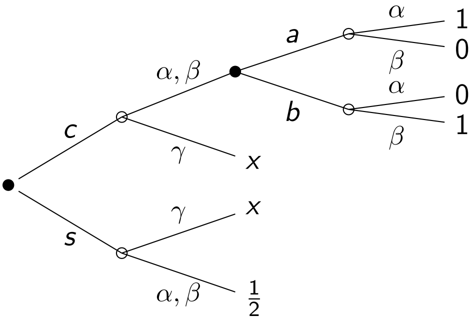

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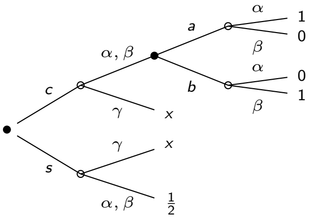
$a \succ_E b$, but $cb_1 \succ ca_1$: **dynamic inconsistency** when $x = 1$.

How to fix this?



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Three possible fixes (in order of popularity, log scale):

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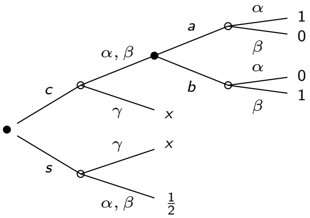

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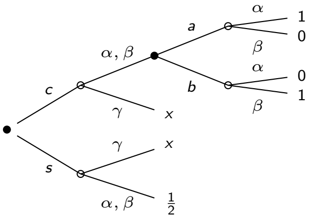

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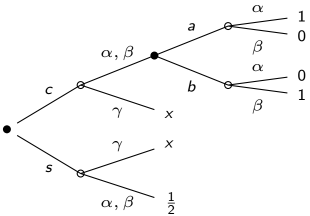

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  - In f_1, DM knows that c leads to a, so f_1 "as if" ca_1.
- Non-consequentialist choice: $\succcurlyeq_E$'s can depend on the tree
  - Hanany and Klibanoff (2007/9), based on Machina (1989).
  - DM prefers a in f_0 but b in f_1.

Dynamic Consistency and the Sure-Thing Principle (1)

Start with classic, or "Full" Dynamic Consistency axiom:

FDC For all $f, g \in L_0$ and non-null $E \in \Sigma$: if $f \succcurlyeq_E g$ (resp. $f \succ_E g$) and $f \succcurlyeq_{\Omega \setminus E} g$, then $f \succcurlyeq g$ (resp. $f \succ g$)

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Also recall the Sure-Thing Principle

STP For all $f, g, h, k \in L_0$ and $E \in \Sigma$ not $\succcurlyeq$-null: $fEh \succcurlyeq gEh$ implies $fEk \succcurlyeq gEk$

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Modal behavior in the Ellsberg paradox violates the STP:

	r	g	b
f_r	1	0	0
f_g	0	1	0
f_{rb}	1	0	1
f_{gb}	0	1	1

$f_r(\omega) = f_{rb}(\omega)$ for $\omega = r, g$, etc. Yet $f_r \succ f_g$, $f_{rb} \prec f_{gb}$

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One advantage of STP: can define/elicit conditional prefs!

SavU For all $f, g \in L_0$ and $E \in \Sigma$ not $\succcurlyeq$-null: $f \succcurlyeq_E g$ iff $fEh \succcurlyeq gEh$ for some $h \in L_0$.

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Theorem ("Folk theorem" of dynamic choice)

For $\succcurlyeq, \succcurlyeq_E$ weak orders, the following are equivalent:
(1) $\succcurlyeq$ satisfies STP and $\succcurlyeq_E$ is obtained via SavU
(2) $\succcurlyeq$, $\succcurlyeq_E$ jointly satisfy NC and FDC

Furthermore, if $\succcurlyeq, \succcurlyeq_E$ are EU, Bayesian updating.

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Foundation for standard approach to dynamic choice:

- Reduce (continuation) plans to acts
- Update prefs (e.g. via Bayes' Rule if EU)
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Then, in standard approach, ambiguity and FDC are inconsistent

Dynamic Consistency in a tree

Epstein and Schneider, *JET* 2003. Many followers!

- Fix a filtration $\mathcal{F} = (\mathcal{F}_0, \ldots, \mathcal{F}_T)$ in Σ
- $\mathcal{F}$-adapted prefs $\succcurlyeq_{t,\omega}$ over $\mathcal{F}$–adapted consumption plans:

$$h = (h_t)_{t=0,\ldots,T}, \quad h_t : \Omega \to \mathbb{R}\ \mathcal{F}_t\text{-meas.}$$

- Impose DC on every $E \in \mathcal{F}_t$, $t = 0, \ldots, T$
- Consider MEU conditional prefs. Literature considers other models: variational, smooth.
- Recursive Multiple Priors:

$$V_t(h)(\omega) = u(h_t(\omega)) + \beta \min_{p \in C_t(\omega)} \mathrm{E}_p[V_{t+1}(h)]$$

V_t, C_t $\mathcal{F}_t$-meas., $V_{T+1}(h) = u \circ h$, $p(\mathcal{F}_t(\omega)) = 1\ \forall p \in C_t(\omega)$

- Key: characterizing sets C_t

Rectangularity: the case $T = 1$

To understand key issues, assume:

- $T = 1$, $\mathcal{F}_0 = \{\Omega\}$, $\mathcal{F}_1 = \{E_1, \ldots, E_N\}$.
- Consumption at $t = T + 1 = 2$ only

Can identify consumption plans with acts

Write $\succcurlyeq_{1,n}$ and $C_{1,n}$ for $\succcurlyeq_{t,\omega}$, $C_1(\omega)$ with $\omega \in E_n$.

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$\mathcal{F}_1$–NC **Null Complement**: For all n, $\Omega \setminus E_n$ is $\succsim_{1,n}$–null

$\mathcal{F}_1$–DC For all $f, g \in L_0$: if $f \succsim_{1,n} g$ for each n (resp and $f \succ_{1,m} g$ for some m) then $f \succsim_0 g$ (resp. $f \succ_0 g$)

$\mathcal{F}_1$–STP For all $f, g, h, k \in L_0$, all n and $E_n \in \mathcal{F}_1$: $fE_nh \succsim_0 gE_nh$ implies $fE_nk \succsim_0 gE_nk$

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Proposition

Let $\succcurlyeq_0, \succcurlyeq_{1,1}, \ldots \succcurlyeq_{1,N}$ be weak orders. Assume each $E \in \mathcal{F}_1$ is not $\succcurlyeq_0$–null. The following are equivalent:

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Furthermore, if $\succcurlyeq_0$ is MEU with priors C_0, then (1)-(2) hold iff

(3) For every n, Prior-by-Prior Updating: $\succcurlyeq_{1,n}$ is MEU with beliefs

$$C_{1,n} = \{q(\cdot|E_n) \, : \, q \in C_0\};$$

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$$C_{1,n} = \{q(\cdot|E_n) \,:\, q \in C_0\}\,;$$

morevorer, $\mathcal{F}_1$–Rectangularity:

$$C_0 = \left\{\sum_{n=1}^{N} q_0(E_n) q_n : q_0 \in C_0, q_n \in C_{1,n} n = 1, \ldots, N\right\}.$$

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Key idea: can choose

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Implies, indeed equivalent to

$$\min_{q \in C_0} \mathrm{E}_q[u \circ h] = V_0(h) = \min_{q_0 \in C_0} \sum_{n=1}^{N} q_0(E_n) \min_{q_n \in C_{1,n}} \mathrm{E}_{q_n}[u \circ h]$$

i.e. recursion.

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Then, for every $n = 1, \ldots, N$, by $\mathcal{F}_1$–NC:

- $h_{n-1} \sim_{1,n} f \succcurlyeq_{1,n} g \sim_{1,n} h_n$.

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- Thus $fE_n k \succcurlyeq_0 gE_n k$ by $\mathcal{F}_1$–SavU

Hence $\mathcal{F}_1$–STP.

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Assume $\succcurlyeq_0$ is MEU. (3) $\Rightarrow$ (2) not hard; focus on (2) $\Rightarrow$ (3)

To show $\succcurlyeq_{1,n}$ is MEU with Full Bayesian Updating, fix $f \in L_0$.

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- Hence $\bar{C}_0 \equiv \{\sum_n q_0(E_n) q_n : q_0 \in C_0, q_n \in C_{1,n}\}$ represents $\succcurlyeq_0$; by uniqueness of priors, $C_0 = \bar{C}_0$. Q.E.D

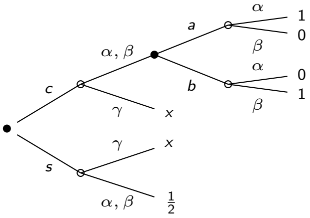
The general case and extensions

- With arbitrary horizon, add discounting
- Rectangularity extends naturally: Def. 3.1
- Recursive approach equivalent to ex-ante MEU:

$$V_0(h)(\omega) = \min_{q \in C_0} \mathrm{E}_q \left[\sum_{t=0}^{T+1} \beta^t u \circ h_t \right].$$

- Extensions/adaptations:
  - variational/multiplier: Maccheroni, Marinacci and Rustichini ECMA 2006, JET 2006
  - smooth ambiguity: Klibanoff, Marinacci and Mukerji ECMA 2005, JET 2009
  - vector expected utility: yours truly ECMA 2009, in progress 2010

The price of rectangularity

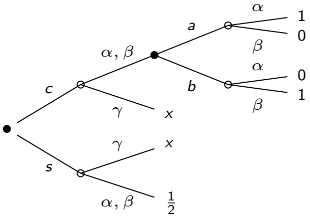


Back to our example:

- Must take $\mathcal{F}_1 = \{\{\alpha, \beta\}, \{\gamma\}\} \equiv \{E, \Omega \setminus E\}$.
- $\mathcal{F}_1$–DC implies $\mathcal{F}_1$–STP
- But then $ca_0 \succcurlyeq_0 cb_0$ iff $ca_1 \succcurlyeq_0 cb_1$: **no Ellsberg!**
- Indeed $C = \{q \in \Delta(\{\alpha, \beta, \gamma\}) : q(\alpha) = \frac{1}{3}\}$ is not rectangular: take $q_0, q_{1,1}, q_{1,2}$ s.t. $q_0(\{\alpha, \beta\}) = 1$, $q_{1,1}(\beta) = 0$; then $q \equiv \sum_n q_0(E_n) q_{1,n} \notin C$ as $q(\alpha) = 1$.

Sophistication and Consistent Planning

Strotz (1956); for ambiguity yours truly (mimeo, 2009)



MEU prefs, $C = \{q : q(\alpha) = \frac{1}{3}\}$, $C_{\alpha,\beta} = \{q : q(\alpha) \geq \frac{1}{3}, q(\gamma) = 0\}$.

- In tree with $x = 1$, $a \succ_E b$.
- Sophistication: DM should anticipate a at $t = 0$.
- Hence DM realizes c is same as $ca_1 \prec_0 s$
- So, even though $cb_1 \succ_0 s$, DM will choose s if $x = 1$.
- (for completeness, c then a if $x = 0$)

Consistent Planning and its challenges

Consistent Planning (CP) generalizes/strengthens this idea:

> *[The DM should choose] the best plan among those that he will actually follow (Strotz, 1956, p. 173)*

- Multi-period algorithm / procedure
- Like backward induction with specific tie-breaking rule

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Essential to adopt preferences over **trees** as primitive

Formalizing Sophistication

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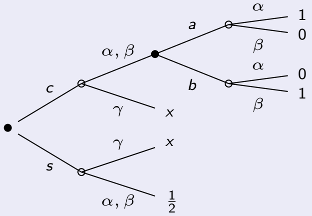
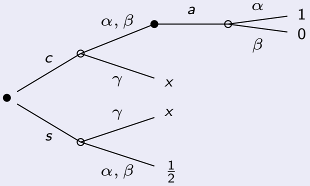
Axiom (Sophistication in the tree f_x)

If $a \succ_E b$ then



$\sim_0$



Note: to state this axiom, $\sim_0$ must be defined on trees.

Sophistication: a caveat

Note use of strict preference in premise: $a \succ_E b$.

What if we allowed for $\succcurlyeq_E$? Gives Strong Sophistication

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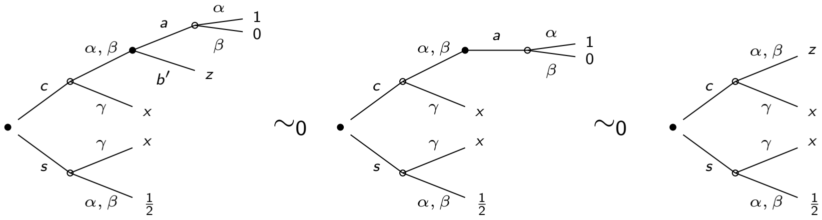

Strong Soph $\Rightarrow$ can replace a with CE

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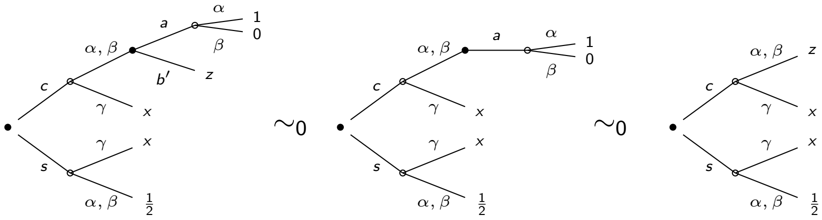

Strong Soph $\Rightarrow$ can replace a with CE $\Rightarrow$ **recursion**

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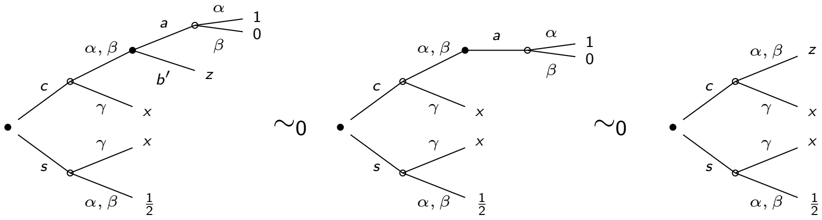
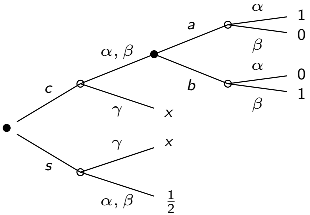


Strong Soph ⇒ can replace a with CE ⇒ **recursion** ⇒ $\mathcal{F}$–**DC**!

This is a general result. See paper.

Tie-breaking in CP



Different MEU prefs:
$C = \{q : \frac{1}{90} \leq q(\alpha) \leq \frac{30}{90}, \frac{2}{90} \leq q(\beta) \leq \frac{15}{90}\}$,
$C_{\alpha,\beta} = \{q : q(\alpha), q(\beta) \geq \frac{1}{16}, q(\gamma) = 0\}$.

- In tree with $x = 1$, $a \sim_E b$.
- However, $ca_1 \succ_0 cb_1$.
- Now Sophistication has no bite
- Should DM be able to "commit" to a? Strotz says "yes"!
- Must formalize this tie-breaking assumption in CP.

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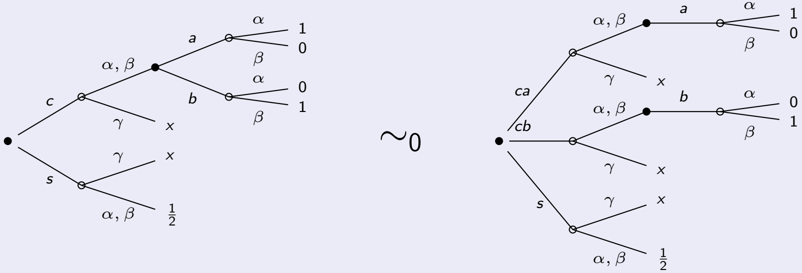
What does it mean for DM to be able to "commit to a"?

- If $a \sim_E b$, then no reason at $t = 1$ to overrule time-0 choice
- Tree f_1 (i.e. $x = 1$) same as tree with immediate ($t = 0$) commitment to a vs. b

Axiom (Weak Commitment in the tree f_x)

If $a \sim_E b$ then



Note: to formalize, need precise notation for **tree surgery**.

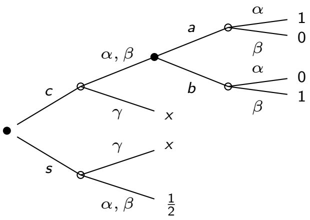
If you really want to know more

Rest of the paper:

- Make CP precise, formal characterization result
- Eliciting conditional preferences
- Application to value of information
- Application to Raiffa's critique
- Related literature, esp. Kreps, DLR, Gul-Pesendorfer.

Non-Consequentialist choice

Machina (1989); ambiguity Hanany-Klibanoff (2007/9)



Prior MEU prefs: $C = \{q : q(\alpha) = \frac{1}{3}\}$.

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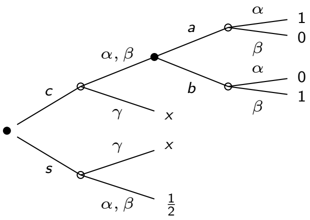

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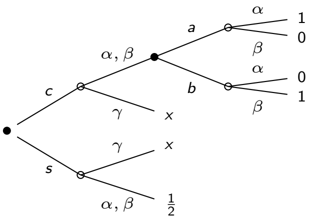

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- Turn problem on its head: *what conditional preferences guarantee WeakDC*?
- Here: $b \succcurlyeq_{E,x=1} a$ if $x = 1$, $a \succcurlyeq_{E,x=0} b$ if $x = 0$.

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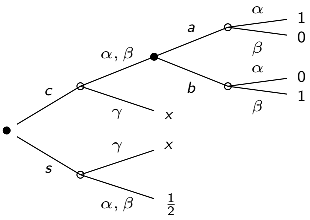

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- Conditional preferences may depend on **context**
- Machina: "experiencing, not realizing, possibility of x" may influence conditional preferences.

Dynamically Consistent MEU rules

Given:

- Conditioning event $E \in \Sigma$ (non-null)
- Feasible acts $B \subset L_0$ (restrictions — think plans in tree)
- Ex-ante optimal act $g \in B$ (possibly one of many)

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Hanany-Klibanoff propose "two-step procedure": given C,

1. $Q^{E,g,B} = \{q \in C : \mathrm{E}_q[u \circ g] \geq \mathrm{E}_q[u \circ f] \; \forall f \in B$ s.t. $f(\omega) = g(\omega) \; \forall \omega \notin E\}$
2. $C_{E,g,B} \subset \{q(\cdot|E) : q \in C\}$ such that, for some $q^* \in Q^{E,g,B}$, $q^*(\cdot|E) \in \arg\min_{q \in C_{E,g,B}} \mathrm{E}_q[u \circ g]$.

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Our example: $C = \{q : q(\alpha) = \frac{1}{3}\}$, $E = \{\alpha, \beta\}$

- $B_x = \{s, ca_x, cb_x\}$ (identify plans with acts)
- $x = 0$: $g = ca_0$
  - $Q^{E,g,B} = \{q : q(\beta) \leq \frac{1}{3} = q(\alpha)\}$
  - $C_{E,g,B} \subset \{q : q(\gamma) = 0, \frac{1}{2} \leq q(\alpha) \leq 1$
- $x = 1$: $g = cb_1$
  - $Q^{E,g,B} = \{q : q(\beta) \geq \frac{1}{3} = q(\alpha)\}$
  - $C_{E,g,B} \subset \{q : q(\gamma) = 0, \frac{1}{3} \leq q(\alpha) \leq \frac{1}{2} \leq q(\beta)\}$

Hanany-Klibanoff 2007

What's in the paper:

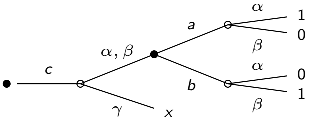
- Update rule: from $\succcurlyeq_0$, $B \subset L_0$, $g \in B$ ($\succcurlyeq_0$–optimal in B) and $E \in \Sigma$ to $\succcurlyeq_{E,g,B}$.
- An update rule is WeakDC iff

$$g \succcurlyeq_0 f \ \forall f \in B \quad \Rightarrow \quad g \succcurlyeq_{E,g,B} f \ \forall f \in B$$

  for all E, g, B.
- Characterize WeakDC update rules for MEU and UAP
- Characterize Maximal-Ambiguity update rule for MEU
- Compare with stronger forms of DC (impossibilities)

(Non-)Consequential choice under ambiguity

One reason for concern.



Prior MEU prefs: $C = \{q : q(\alpha) = \frac{1}{3}, q(\beta) \leq \frac{1}{3}\}$. Uniform q_u.

- $x = 0$: ca_0 optimal. Max ambiguity rule: update all of C.
- $x = 1$: cb_1 optimal. Must update only q_u!
- Hence after E, EU if $x = 1$ and MEU if $x = 0$.
- Conditional perception of ambiguity can depend on x!
- Runs counter to usual interpretation of ambiguity.