Efficient Sorting in a Dynamic Adverse-Selection Model*

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This version, May 2004.

Abstract

We discuss a class of markets for durable goods where efficiency (or approximate efficiency) is obtained despite the presence of information asymmetries. In the model, the number of times a good has changed hands (the vintage of the good) is an accurate signal of its quality, each consumer self-selects into obtaining the vintage that the social planner would have assigned to her, and consumers' equilibrium trading behavior in secondary markets is not subject to adverse selection. We show that producers have the incentive to choose contracts that lead to the efficient allocation, and to supply the efficient output. We also provide a contrast between leasing contracts, resale contracts, and different kinds of rental contracts. Resale contracts do not lead to the efficient allocation. A specific kind of rental contract provides the appropriate incentives to consumers.

1 Introduction

Since Akerlof's (1970) seminal paper, adverse selection has been recognized to be a potential source of inefficiency in durable-goods markets. This paper presents a model of a durable-goods market in which full information payoffs and allocations can be achieved in a competitive equilibrium even under asymmetric information. Our interpretation of this result is that inefficiency in standard adverse selection models of durables is not due solely to asymmetric information, but to a combination of other restrictions of trading possibilities.[1]

Some recent literature has pointed out that the extent of inefficiency in the classic model depends on restrictions of trading opportunities. Two features of these restrictions of the classic model have been examined separately by different strands of the literature: *exogenous ownership* and *restricted secondary markets*. The first strand of the literature (Hendel and Lizzeri 1999, 2002, and Johnson and Waldman 2003) departs from the *exogenous ownership* assumption. In markets for

*We thank Patrick Bolton, Melvyn Coles, Alessandro Gavazza, and Steve Tadelis for helpful discussion and suggestions. We also thank the editor, Mark Armstrong, and two anonymous referees for useful comments.

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[1]While some of these restrictions are realistic, we argue that it is useful to understand the exact sources of inefficiency both for theoretical reasons and to better evaluate policy recommendations.

durable goods such as cars, the identity of the owners of used goods is endogenous: high-valuation consumers self-select into the new-goods market, and lower valuation consumers self-select into the used-goods market. This self-selection can reduce the inefficiencies, but some distortions remain because multiple qualities of used goods are traded in a single market.[2] The second strand of the literature (Janssen and Roy 2001, 2002) deals with the *restricted secondary markets* assumption but retains the exogenous ownership assumption. Janssen and Roy show that, when used-goods markets are open at every date, more welfare-enhancing trades are possible. Prices and traded qualities increase over time, and the time to trade acts as a sorting device because owners of high quality cars are more willing to wait to obtain the high prices. However, a delay in efficient sorting is necessary to induce incentive compatible trade, and this leads to a welfare loss. This welfare loss does not disappear as the discount factor goes to one.

We show that removing both restrictions of Akerlof's model at the same time can lead to the first-best outcome, if the appropriate kind of rental contracts is offered, and if consumers observe the number of times a good has been traded. Thus, the combination of multiple secondary markets and endogenous assignment of new goods can completely eliminate the inefficiencies caused by asymmetric information. Because the identity of new car consumers is endogenously determined, the highest-value cars are allocated to the right consumers; the existence of multiple secondary markets leads to the allocation of used units to the right used-car consumers. Because efficient sorting requires that each consumer type always holds goods of a single quality, special (but simple) contracts are needed to induce this holding pattern.

The basic intuition for our results emerges most starkly in an asymmetric-information environment we call the *simple depreciation model.* We assume that all *new* units of the durable good have the same, known quality. However, goods depreciate stochastically: if the good is of quality q_n at date t, there is a positive probability that the good depreciates to quality q_{n+1} by date $t+1$. This implies that a good produced some time in the past may be of several distinct qualities, numbered 0 (highest) through N (lowest). In steady state, there is a distribution of qualities, with newly produced goods replacing units that have depreciated. Smoothly functioning secondary markets play two key roles in this environment: (1) they must allow high-valuation consumers whose units have depreciated to replace them with new goods, transferring the used good to lower-valuation consumers; (2) they must allocate used units efficiently among low-valuation consumers. If the quality of the good is publicly observable, trade in secondary markets achieves these two goals and leads to the efficient allocation. If, on the other hand, a prospective buyer cannot observe the quality of the good prior to purchase, adverse selection could in principle preclude efficiency.

However, we show that, even in the presence of these information asymmetries, there is a competitive equilibrium in which a menu of rental contracts induces precisely the same allocation that would prevail if quality were observable; furthermore, per-period rental rates are the same as under observable quality. In this equilibrium, all that consumers need to observe is the *vintage* of a unit, i.e. the number of distinct consumers who have used it in the past. Thus, in this model,

[2] An exception that is explored by Hendel and Lizzeri and Johnson and Waldman is the case in which there are only two types of consumers. In this case, only one type consumes used goods, so sorting in secondary markets is not an issue. We will see below that another exception is the case of two qualities.

as long as this limited amount of information about the trading history of a good is available, asymmetric information about quality is completely harmless.[3,4]

The equilibrium may be briefly described as follows. Every vintage (including vintage 0, which corresponds to new goods) is traded at a different (rental) price. In each period, a high-valuation consumer rents a new (vintage-0) unit, and stops renting that unit when it depreciates for the first time. At that point the vintage of the unit increases from zero to one. Consumers with somewhat lower valuation rent a vintage-1 unit and keep renting it until the unit depreciates again, at which point its vintage increases to 2, and is passed on to consumers with yet lower valuation. The process continues until the good "falls apart and dies". Consumers who stop renting a unit of vintage n obtain another unit of the same vintage. Therefore, in equilibrium, the vintage of a unit is a perfect signal of its quality: a good that has had n previous consumers is of quality q_n.

Rental contracts provide the "right" incentives to consumers; in particular, consumers have no reason to keep renting a unit once that unit has depreciated: better units are available at the same rental price. Thus, consumers of inferior vintages have no reason to worry about adverse selection. In contrast, a system of resale markets generates the "wrong" incentives: some consumers find it profitable to keep a unit after it depreciates, so that its vintage is no longer a perfect signal of quality. The intuition behind the difference between the two alternative mechanisms (resale vs rental) is the following. In a system of resale markets, consumers suffer a capital loss when the good changes hands (and hence its vintage increases). This loss is instead borne by the producer of the good when the good is rented. Thus, with rental contracts, the consumer has no incentive to retain depreciated units.

Given that the desirable efficiency properties of rental contracts depend on transferring the capital loss generated by a change in vintage from consumers to producers, it is important to understand whether producers are willing to bear such losses. We show that there is a competitive equilibrium in which the efficient amount of output is produced and the efficient menu of rental contracts is chosen by each firm. Thus, firms are indeed willing to bear the capital losses associated with changing vintages. It should however be emphasized that this result does require that consumers be able to observe the contracts offered by individual firms. This is a strong, albeit standard, requirement which is consistent with the spirit of competitive equilibrium analysis.

In the simple depreciation model, vintage is an effective signal of quality because there is no uncertainty about the quality of the new good (it is known to be q_0), and depreciation occurs only one step at a time (e.g. from q_n to q_{n+1}, but not to q_{n+k}, for $k > 1$). In order to examine the robustness of the intuition just described, we analyze a more general model that allows both for initial uncertainty about the quality of newly produced goods, and for more general depreciation processes. Again, efficiency involves assortative matching of qualities to consumers. The quality of a unit is only observed by a consumer who has tried it in the past. In contrast with the

[3] Note that this information is commonly available to used car consumers in the US through Carfax.

[4] Consider the following alternative assumptions: cars follow the depreciation process specified in the text, but a used unit can only be traded once, t periods after it is produced. This yields a model that is very similar to the ones studied by Hendel and Lizzeri and Johnson and Waldman: from the point of view of prospective consumers, the quality of a unit offered on the used-car market is a random variable with a distribution determined by the depreciation rates. Under this interpretation, the only difference between our simple depreciation model and the environments studied in the papers cited above is the number of active secondary markets for a given unit.

simple depreciation model, in this more general model, in order to find a good of the right quality, consumers must *experiment* with different units.

We show that it is still possible to employ the vintage of the good to signal its quality and to achieve sorting; however, in contrast with the simple depreciation model, sorting occurs with delay because experimentation is required. Each unit changes hands until it finds its right match, and the unit increases in vintage each time it changes hands. In turn, a consumer continues experimenting with a particular vintage until she gets the top quality of that vintage, and then keeps the unit until it depreciates. If the car depreciated only one step, the next consumer to obtain that unit will hold on to it until the next time it depreciates. If, on the other hand, the car depreciated $k > 1$ steps, the next consumer to obtain that unit will immediately return it to try another car of the same vintage, while the car is traded at least k times, increasing in vintage with every trade (exactly k times if the car does not depreciate while this process continues). Thus, as in the previous model, units "trickle down" from consumers with high valuation to consumers with lower valuations.

As in the simple depreciation model, a menu of rental contracts induces consumers to follow an experimentation policy leading to the efficient matching of goods to consumers.[5] Moreover, the utility cost of the delay connected with experimentation becomes negligible if retrading can happen quickly. Furthermore, the rental prices that induce consumers to follow this experimentation policy converge to the observable-quality rental prices when the time between transaction converges to zero. We show that, as a consequence, producers' incentives are approximately in line with efficiency.

Our analysis of the general depreciation model highlights a role for secondary markets to facilitate experimentation. Since experimentation is only necessary when quality is not publicly observable, secondary markets are more active when quality is unobservable.

An implication of our model is that, if transactions involve rental contracts, observability of the vintage of the good is welfare-improving. The role of the observability of trading histories in the case of resale markets is less obvious. Indeed, House and Leahy (2001) provide a model in which observability of trade histories can create additional distortions. In their setup, there are two car qualities; consumers are homogeneous in their valuations for quality, but the match value of a consumer/car pair deteriorates stochastically over time. Thus, while the good should change hands as often as possible sorting does not matter. Adverse selection creates a distortion, since owners of good-quality units may refrain from trading and observability of trade histories may delay trade because delay is a signal that a car is high quality (as in Janssen and Roy).

In contrast, extensive numerical analysis of a three-quality version of our simple depreciation model indicates that, even in the case of resale markets, the allocation when the vintage of the good is observable is more efficient than in the case in which it is unobservable. This contrast may be explained by noting that gains from trade stem from different sources in the two models. In our model, consumers are heterogeneous in their valuation for quality; as a consequence, efficiency does depend upon the identity of the agents buying a used unit: it is efficient to match high-quality used units to quality-sensitive consumers. Since trade history is a signal of the quality of traded units, observability can enhance the buyer-car match.[6]

[5] As in the simple depreciation model, resale markets generate the wrong incentives.

[6] Stolyarov (2002) develops a model of trade in secondary markets with transaction costs. He shows that the

We should note that the goal of the paper is not to construct a realistic model of a durable-goods market. Rather, our model is designed to enable us to evaluate the distortions caused by adverse selection in the absence of any other friction in secondary markets. A more realistic model would acknowledge the importance of frictions such as transaction costs. However, by assuming away additional complications, we are able to isolate the role of informational asymmetries as a barrier to the efficient operation of secondary markets.

2 Preliminaries

2.1 Model

We consider a discrete-time, infinite-horizon economy. Time is measured in some specified *unit* (e.g. days, months, years), and every time *period* lasts for $\Delta \in (0, 1]$ units. There is a unit mass of infinitely lived consumers who differ in their valuation for quality, characterized by a "type" $\theta \in [\underline{\theta}, \overline{\theta}] \subset \mathbf{R}_+$, distributed according to the c.d.f. F; the latter is assumed to have a strictly positive density. The total mass of cars equals $Y < 1$; at any time, the quality of a car may take up one of finitely many values, denoted $q_0 > q_1 > \ldots > q_N \geq 0$.[7]

Consumers discount utility streams at the instantaneous rate ρ; thus, u utils at time $t > 0$ are worth $e^{-\rho t}u$ utils at time 0. Car qualities and consumer valuations determine instantaneous flow utility from consumption, as follows: if a type-θ consumer drives a quality-q car for τ units of calendar time, she receives utility equal to

$$\int_0^\tau e^{-\rho t} q\theta \; dt = \frac{1 - e^{-\rho\tau}}{\rho} q\theta.$$

Associating quality levels with *instantaneous* (as opposed to per-period) utility from consumption simplifies the comparison of consumption streams in economies characterized by periods of different length Δ. Finally, utility is quasi-linear in "money". Specifically, for every Lebesgue-measurable function $q : \mathbb{R}_+ \to \{q_0, \ldots, q_N\}$ and every pair of sequences $\{P_k\}_{k\geq 0}, \{t_k\}_{k\geq 0}$ in $\mathbb{R}_+$, the utility of a type-θ consumer who, at each time $t \in \mathbb{R}_+$, drives a car of quality $q(t)$ and effects a (lump-sum) payment of P_k at time t_k for every $k \geq 0$, is

$$\int_0^\infty e^{-\rho t} q(t)\theta \; dt - \sum_{k\geq 0} e^{-\rho t_k} P_k.$$

Each period consumers receive an endowment e of 'money'. We assume that e is finite and 'large', namely e is large enough that consumers can potentially afford any quality they wish to

probability of trade is non-monotonic in the age of the good. In his model inefficiencies arise because consumers may keep goods too long due to transaction costs. Potential transaction costs in our setting are endogenous and due to asymmetric information. Tadelis (1999, 2002) develops an adverse-selection model where the name of a firm summarizes its reputation.

[7] $Y < 1$ will hold in equilibrium as long as costs of production are not too small. The discrete distribution of qualities is convenient but does not play an important role.

consume.[8]

In any *period*, a car may depreciate (i.e. its quality may deteriorate), or it may break down, i.e. "die", in which case it exits the economy and is replaced by a newly-produced car. We assume throughout that the "death" of a car is publicly observable, regardless of whether or not quality is.

We analyze two models of the depreciation process. In the **simple depreciation** model, the quality of a newly produced car is known to be q_0. For $n = 0, \ldots, N-1$, at the end of each period, a car currently of quality q_n depreciates to q_{n+1} with probability $\gamma_n \Delta$; a quality-q_N car does not depreciate, but may die with probability $\gamma_N \Delta$ at the end of each period. It turns out that the analysis is independent of the length Δ of time periods.

In the **general depreciation** model, for $n = 0, \ldots N$, a newly produced car has quality q_n with probability $\chi_n \geq 0$, where $\sum_{n=0}^{N} \chi_n = 1$. Moreover, for $n = 0, \ldots, N$ and $m = n+1, \ldots, N+1$, a car of quality q_n depreciates to q_m (if $n < m \leq N$) or dies (if $m = N+1$) with probability $\gamma_{n,m}\Delta$ in every time period. The simple depreciation case corresponds to $\chi_0 = 1$ and $\gamma_{n,m} = 0$ for $m > n+1$. Thus, there are two generalization relative to the simple depreciation model: (1) initial quality is uncertain; (2) depreciation can occur in more than one step.

It can be shown that, as $\Delta \to 0$, the general depreciation process has a well-defined continuous-time limit, which may be (somewhat loosely) described as follows: if, at time t, car quality equals q_n, then (i) the time until the next depreciation event is exponentially distributed, with parameter $\sum_{m=n+1}^{N+1} \gamma_{n,m}$; and (ii) conditional upon a depreciation event, the quality of the car becomes q_m with probability $\left(\sum_{m=n+1}^{N+1} \gamma_{n,m}\right)^{-1} \gamma_{n,m}$.

To avoid redundancies, we assume that the general depreciation process generates a positive mass of cars of each quality level. Formally, for every $n = 0, \ldots, N$, there is at least one sequence $m_0 < \ldots < m_M = n$, with $M \geq 0$, such that $\chi_{m_0} > 0$ and $\gamma_{m_\ell, m_{\ell+1}} > 0$ for all $\ell = 0, \ldots, M-1$. To clarify, for $n = 0$, this requires that newly produced cars attain the highest quality level q_0 with positive probability; for $n = 1$, it requires that either newly produced cars attain quality level q_1 with positive probability, or that they attain quality level q_0 with positive probability, and that quality-q_0 cars depreciate to q_1 with positive probability; and so on.

2.2 Efficiency

We now define our reference notion of efficiency. At each time $t \geq 0$, positive assortative matching of consumer types to cars must obtain; thus, we need to determine cutoff types $\theta_0^*, \ldots, \theta_N^* \in [\underline{\theta}, \overline{\theta}]$ such that types $\theta \in [\theta_0^*, \overline{\theta}]$ hold a quality-q_0 car, types $\theta_1 \in [\theta_1^*, \theta_0^*)$ hold a quality-q_1 car, and so on; types $\theta < \theta_N^*$ will not hold any car (recall that the mass Y of cars is less than 1, the mass of consumers).

In order to determine these cutoff types, we must first derive the steady-state masses of cars of each quality, as determined by the depreciation process. Let v_n^* denote the steady-state mass of cars of quality q_n. We consider the general specification of the depreciation process, as it entails

[8]The assumption that the endowment is large allows us to rule out equilibria with price 'bubbles', where resale prices of a good escalate because consumers expect them to rise in the future. The assumption of a large endowment is made to avoid keeping track of the wealth of each consumer. Such a problem would complicate the analysis, and seems tangential to the issue studied here.

only a slight penalty in terms of analytical complexity. Recall that Δ denotes the length of a period in terms of the chosen units of calendar time.

It is convenient to introduce the following notation. First, for all $n = 0, \ldots, N$ and $m = n+1, \ldots, N+1$, let $G_{n,m} = \sum_{\ell=m}^{N+1} \gamma_{n,\ell}$, so $G_{n,m}\Delta$ is the probability that a quality-q_n car depreciates at all (or dies) in a period; we assume that $G_{n,n+1} \leq 1$ for all $n = 0, \ldots, N$. Next, denote by $\gamma_{n,n}(\Delta)$ the probability that a car of quality q_n does *not* depreciate in a single period: that is, $\gamma_{n,n}(\Delta) = 1 - G_{n,n+1}\Delta$.

For $n = 0, \ldots, N$, the steady-state mass v_n^* must satisfy the following system of equations:

$$v_0^* = \gamma_{0,0}(\Delta)v_0^* + \chi_0 y^* \tag{1}$$

$$v_n^* = \gamma_{n,n}(\Delta)v_n^* + \chi_n y^* + \sum_{k=0}^{n-1} \gamma_{k,n}\Delta v_k^* \tag{2}$$

$$y^* = \sum_{n=0}^{N} \gamma_{n,N+1}\Delta v_n^* \tag{3}$$

$$Y = \sum_{n=0}^{N} v_n^*. \tag{4}$$

That is: Y is the total mass of cars (equation 4) and y^* is the mass of cars that die in each time period, and are replaced by newly produced cars (equation 3). The mass of cars of quality q_0 consists of quality-q_0 cars that have not depreciated in the previous period, and of newly-produced quality-q_0 cars (equation 1). Finally, the mass of quality-q_n cars, for $n > 0$, is given by undepreciated quality-q_n cars, newly-produced quality-q_n cars, and cars previously of higher quality that just depreciated to q_n (equation 2).

Straightforward manipulations[9] show that the above system of equations admits a unique solution, which is independent of Δ (so our benchmark is unaffected by the duration of a period). Furthermore, a simple induction argument shows that, under the assumption on the depreciation process stated at the end of the preceding section, $v_n^* > 0$ for all $n = 0, \ldots, N$.

The *ex-post* efficient steady-state allocation of cars to consumers ("efficient sorting" hereafter) can then be described as follows. First, let $\theta_{-1} := \overline{\theta}$; next, proceeding iteratively for $n = 0, \ldots, N$, assuming that θ_{n-1}^* has been defined, choose θ_n^* such that

$$\forall n = 0, \ldots, N, \quad F(\theta_{n-1}^*) - F(\theta_n^*) = v_n^*; \tag{5}$$

observe that $\theta_{-1} > \theta_0^* > \ldots > \theta_N^*$ by construction; also, $\theta_N^* > \underline{\theta}$, because $Y < 1$.

Thus, for every $n = 0, \ldots, N$, the mass of consumers with types $\theta \in [\theta_n^*, \theta_{n-1}^*]$ is equal to the mass of cars of quality q_n. As noted above, we then assign all cars of quality q_n to consumer types $\theta \in [\theta_n^*, \theta_{n-1}^*]$.

[9] For any $y^* \geq 0$, Eqs. (1) and (2) determine $v_0^*, \ldots, v_N^*$; furthermore, adding up Eqs. (1) and (2) and solving for y^* shows that Eq. (3) is automatically satisfied. If $y^* = 0$ then $v_n^* = 0$ for all n, and since $\chi_0 > 0$, there exists y^* such that Eq. (4) holds. Substituting for y^* in Eqs (1), it becomes apparent that the solution is independent of Δ.

We are interested in analyzing the incentives of consumers and producers. For expository reasons, it is convenient to focus on consumers' incentives first, assuming that production is exogenous, and then extend the analysis to the supply side of the economy. Correspondingly, we distinguish between **consumer equilibrium**, which assumes exogenous production, and **market equilibrium**, which encompasses firms' optimal choice of output and contracts.

3 Simple Depreciation Model

3.1 Observable-Quality Benchmark and Trickle-Down

We begin by briefly analyzing consumer equilibrium in the simple depreciation model, under the assumption that quality is observable. This will serve as a benchmark, and also illustrate our notation.

Regardless of whether cars are sold or rented, the following strategies constitute a consumer equilibrium. Whenever consumers of type $\theta \in [\theta_0^*, \overline{\theta}]$ do not have a car, they obtain a car of quality q_0 and keep it as long as the car remains of quality q_0. As soon as the car depreciates to q_1 they get rid of the car and obtain a new car of quality q_0. Consumers of type $\theta \in [\theta_n^*, \theta_{n-1}^*]$ behave in an analogous fashion with cars of quality q_n. Clearly, the resulting equilibrium allocation is ex-post efficient; moreover, this equilibrium allocation is essentially[10] unique.

Under *rental*, a consumer who rents a car of quality q_n pays a fee $\frac{1-e^{-\rho\Delta}}{\rho} r_n$ at the beginning of the period (hence, if she keeps renting the same quality, she pays this fee at times $0, \Delta, 2\Delta, \ldots$). This can be seen as the discounted value of a constant, *instantaneous rental price* r_n to be paid throughout the period. Consequently, the per-period utility for a type-θ consumer who rents a quality-q_n car can be written as $\int_0^\Delta e^{-\rho t}[q_n\theta - r_n]\, dt = \frac{1-e^{-\rho\Delta}}{\rho}[q_n\theta - r_n]$. The $N+1$ rental prices that sustain this equilibrium allocation can then be defined exactly in the same way as the sorting prices in a static model:

$$\theta_n^* q_n - r_n^* = \theta_n^* q_{n+1} - r_{n+1}^*, \quad n = 0, 1, ...N \tag{6}$$

where, by convention, $r_{N+1}^* = q_{N+1} = 0$. These instantaneous rental prices are defined by the indifference of marginal type θ_n^* between renting a good of quality q_n and renting a good of quality q_{n+1}. Also notice that the prices defined in equation (6) are independent of Δ.

Under *selling*, the prices that sustain this consumer equilibrium are defined by the expected present value of rental prices (where the expectation is necessary since the time until the good depreciates and is sold is stochastic). It is easy to verify that these prices are defined by the conditions

$$p_n^* - \sum_{k=1}^{\infty}(1-\gamma_n\Delta)^{k-1}\gamma_n\Delta e^{-\rho\Delta k} p_{n+1}^* = \sum_{k=0}^{\infty}(1-\gamma_n\Delta)^k \gamma_n\Delta \frac{1-e^{-\rho\Delta k}}{\rho} r_n^*, \quad n = 0, 1, ...N \tag{7}$$

where, by convention, $p_{N+1}^* = 0$, and r_n^* is defined in equation (6). The right-hand side of this expression is the expected present value of the rental payment for a unit of a good of quality q_n,

[10] If the car she is currently renting does not depreciate, a consumer is indifferent between keeping the same unit and renting another unit of the same quality.

given that the consumer will stop renting it once the unit depreciates. In the left-hand side, the expected present value of revenues from resale is subtracted from the price of the quality-q_n good.

We use the term ***trickle-down*** to denote the process by which these goods are allocated in equilibrium: the good trickles down from the high-valuation consumers to the low-valuation ones as it declines in quality.

The equilibrium just described has the following key feature: the quality of any given unit of the good offered on the market can be exactly inferred from its *vintage*, i.e. the number of times the unit has changed hands. A unit of vintage n is of quality q_n.

Therefore, the equilibrium strategies described above can equivalently be formulated as follows: for each $n = 0, ..., N$, consumer types $\theta \in [\theta_n^*, \theta_{n-1}^*]$ rent or buy vintage-n cars, and keep the same unit until it depreciates. Notice that, in order to implement these strategies, only the vintage of a unit must be observed, not its quality or its age. Yet, if all consumers follow these strategies, the ex-post efficient allocation will ensue, and qualities will be fully revealed. For this reason, we deem these strategies *revealing*.[11]

3.2 Failure of Resale Markets under Asymmetric Information

We now turn to the analysis of the simple depreciation model with unobservable quality. Specifically, assume that consumers cannot observe the quality of a specific car without using it. Moreover, a consumer who is using a specific car at time t observes the time-t realization of the depreciation process that determines the quality of the car at time $t + 1$. It is notationally convenient (but without loss of generality) to assume that realizations of the depreciation process occur at the end of each period.

We continue to assume that the trading history of each unit is observable. Thus, the revealing strategies described in the previous section are still well-defined, and it is natural to ask whether they still constitute a consumer equilibrium.

Consider resale markets first. We show that, if there are more than two qualities, under asymmetric information, revealing strategies do not constitute a consumer equilibrium with resale. Moreover, we show that there is no consumer equilibrium with resale that achieves the efficient allocation.

By analogy with the complete-information case, we begin by analyzing trading environments consisting of $N + 1$ resale markets; a vintage-n unit is sold at a price p_n, for $n = 0, ..., N$.

Theorem 1 *(i) If there are more than two qualities, in a system of resale markets, there is no set of $N + 1$ vintage-dependent prices that supports the revealing strategy profile.*

(ii) If there are only two qualities, then there exists an ex-post efficient consumer equilibrium.

Observe that both statements are true for *any* value of Δ.

Proof. (i) Assume (by contradiction) that a vintage-n car is indeed of quality q_n. Denote by $V_n(\theta)$ the ex-ante value of purchasing a vintage-n good, and then behaving as prescribed by

[11] In this simple setting, whether or not the age of a car is observed is irrelevant. This is not necessarily the case if consumers may keep more than one quality (see the discussion in section 3.6).

the vintage$-n$ revealing strategy: keep the car until it depreciates to q_{n+1} and then buy another vintage-n car. Denote by $W_n(\theta, q_n)$ the value of already owning a car of quality q_n for a consumer who is a vintage-n consumer and who follows the policy just described. We have

$$V_n(\theta) = -p_n + \frac{1-e^{-\rho\Delta}}{\rho} q_n\theta + e^{-\rho\Delta}[\gamma_n\Delta(V_n(\theta)+p_{n+1}) + (1-\gamma_n\Delta)W_n(\theta,q_n)]$$

$$W_n(\theta, q_n) = V_n(\theta) + p_n \tag{8}$$

Thus,

$$V_n(\theta) = \frac{\frac{1-e^{-\rho\Delta}}{\rho} q_n\theta - p_n(1-e^{-\rho\Delta}(1-\gamma_n\Delta)) + e^{-\rho\Delta}\gamma_n\Delta p_{n+1}}{1-e^{-\rho\Delta}} \tag{9}$$

In order to achieve the efficient allocation, the following condition inducing efficient ex-ante sorting must be satisfied:

$$V_N(\theta_N^*) = 0, \text{ and } V_n(\theta_n^*) = V_{n+1}(\theta_n^*), \text{ for } n = 0, ...N-1. \tag{10}$$

These $N+1$ equations determine prices $p_0, p_1, ..., p_N$. To obtain the efficient allocation, these prices must induce the right ex-post keeping behavior: no consumer in $[\theta_{n-1}^*, \theta_n^*]$ should want to keep any good of quality lower than q_n. We will show that, if equations (10) are satisfied, then the consumer of type θ_n^* strictly prefers keeping quality q_{n+1}. The value of selling the good is:

$$\begin{aligned} p_{n+1} + V_n(\theta_n^*) &= p_{n+1} + V_{n+1}(\theta_n^*) = W_{n+1}\left(\theta_n^*, q_{n+1}\right) = \\ &= \frac{\frac{1-e^{-\rho\Delta}}{\rho} q_{n+1}\theta + e^{-\rho\Delta}\gamma_{n+1}\Delta p_{n+2}}{1-e^{-\rho\Delta}\left(1-\gamma_{n+1}\Delta\right)} + \frac{e^{-\rho\Delta}\gamma_{n+1}\Delta V_{n+1}(\theta_n^*)}{1-e^{-\rho\Delta}\left(1-\gamma_{n+1}\Delta\right)} \end{aligned} \tag{11}$$

where the first equality uses equation (10), the second equality uses equation (8) and the last equality is just a solution for $W_{n+1}\left(\theta_n^*\right)$. Consider the alternative strategy for the consumer of holding on to the good until it depreciates to q_{n+2} and then following the policy of buying vintage $n+1$. Denote by W^K the value of this strategy.

$$W^K(\theta_n^*) = \frac{1-e^{-\rho\Delta}}{\rho} q_{n+1}\theta + e^{-\rho\Delta}\left(\gamma_{n+1}\Delta\left(V_{n+1}(\theta_n^*) + p_{n+1}\right) + \left(1-\gamma_{n+1}\Delta\right)W^K(\theta_n^*)\right)$$

Thus,

$$W^K(\theta_n^*) = \frac{\frac{1-e^{-\rho\Delta}}{\rho} q_{n+1}\theta + e^{-\rho\Delta}\gamma_{n+1}\Delta p_{n+1}}{1-e^{-\rho\Delta}\left(1-\gamma_{n+1}\Delta\right)} + \frac{e^{-\rho\Delta}\gamma_{n+1}\Delta V_{n+1}(\theta_n^*)}{1-e^{-\rho\Delta}\left(1-\gamma_{n+1}\Delta\right)}.$$

Subtract the right-hand side of this equation from the expression for $W_{n+1}\left(\theta_n^*, q_{n+1}\right)$ in equation (11) to obtain

$$W_{n+1}\left(\theta_n^*, q_{n+1}\right) - W^K(\theta_n^*) = -\left(\frac{e^{-\rho\Delta}\gamma_{n+1}\Delta}{1-e^{-\rho\Delta}\left(1-\gamma_{n+1}\Delta\right)}\right)(p_{n+1} - p_{n+2}) \tag{12}$$

If we can show that $p_n > p_{n+1}$ for every n, we can conclude that keeping quality q_{n+1} is better than selling it for type θ_n^* (and, given the continuity of payoffs with respect to θ, for all types sufficiently close to θ_n^*). We now claim that equation (10) implies that $p_n > p_{n+1}$, $n = 0, 1, \ldots N-1$. We prove this by induction: First, note that for $n = N-1$, we can rewrite equation (10) as:

$$\frac{1-e^{-\rho\Delta}}{\rho}\theta_{N-1}^*(q_{N-1}-q_N) + e^{-\rho\Delta}\gamma_N\Delta p_N = (p_{N-1}-p_N)(1-e^{-\rho\Delta}(1-\gamma_{N-1}\Delta))$$

since $p_{N+1} = 0$ because $q_{N+1} = 0$. Thus, $p_{N-1} > p_N$ and the claim is true for $n = N-1$. Assume the claim is true for $n+1$ (i.e., $p_{n+1} > p_{n+2}$). We can rewrite equation (10) as:

$$\frac{1-e^{-\rho\Delta}}{\rho}\theta(q_n - q_{n+1}) + e^{-\rho\Delta}\gamma_{n+1}(p_{n+1}-p_{n+2}) = (p_n - p_{n+1})(1-e^{-\rho\Delta}(1-\gamma_n\Delta))$$

so that $p_n > p_{n+1}$, which proves the inductive step.

(ii) If $N = 1$ (i.e., there are only two qualities), existence of an efficient equilibrium is established if we can show that the prices defined by equations (10) induce the correct keeping behavior. For consumers of vintage 1, this is trivial. For consumers of vintage 0, the right keeping behavior requires that all type $\theta \geq \theta_0^*$ be willing to sell the car the moment it depreciates to q_1. This requires that for all such θ's,

$$p_1 + V_0(\theta) \geq \frac{1-e^{-\rho\Delta}}{\rho}\theta q_1 + e^{-\rho\Delta}\left(1-\gamma_1\Delta\right)p_1 + e^{-\rho\Delta}V_0(\theta). \tag{13}$$

Rewrite equation (13) as

$$V_0\left(\theta\right) \geq \frac{\frac{1-e^{-\rho\Delta}}{\rho}\theta q_1 - \left(1-e^{-\rho\Delta}\left(1-\gamma_1\Delta\right)\right)p_1}{1-e^{-\rho\Delta}}$$

Note that the right hand side of this equation is equal to $V_1\left(\theta\right)$ (since $p_2 = 0$). Thus, the ex-post keeping condition is equivalent to the ex-ante condition that $V_0\left(\theta\right) \geq V_1\left(\theta\right)$, implying that all types who buy vintage 0 will never want to keep quality q_1. ■

The intuition for this result is the following. For revealing strategies to be an equilibrium with resale markets, type θ_n^* must be just willing to be a vintage-n consumer ex-ante, i.e., $V_n(\theta_n^*) = V_{n+1}(\theta_n^*)$. Furthermore, he should be willing to sell the good as soon as it becomes of quality q_{n+1}. These two conditions together imply that he should be willing to sell a vintage-n good that just depreciated to quality q_{n+1} and then buy a vintage-$(n+1)$ good whose quality (in equilibrium) is q_{n+1}. However, this cannot be optimal. The reason is that by keeping the vintage n car that is of quality q_{n+1} until it depreciates again, the consumer enjoys a quality q_{n+1} good which he can then sell for p_{n+1}. In contrast, if he buys a vintage $n+1$ car, he would still enjoy a quality q_{n+1} unit, but would only be able to sell it for p_{n+2}. Thus, it is less costly to consume q_{n+1} if one happens to have a vintage-n good than it is to consume the same quality with a vintage $n+1$ good: the resale price is higher in the first case. Note that this logic fails for $n = N-1$ because when a good of quality $N-1$ depreciates twice it dies, and we have assumed that this event is observable. This implies that efficiency is possible when $N = 1$.

Remark 1 *Inefficiency does not vanish in the limit as $\Delta \to 0$.*

Proof. In the proof of part (i) of Theorem 1 we showed that in a system of resale markets buyers have an incentive to keep the wrong qualities. Thus, it is enough to show that these incentives do not disappear in the limit as $\Delta \to 0$. To do this, recall that the right-hand side of equation (12) expresses the (negative) payoff from keeping quality q_{n+1} for the marginal type who should instead be keeping only q_n. We now show that this expression is bounded away from zero. Observe that

$$\lim_{\Delta\to 0}\left(\frac{e^{-\rho\Delta}\gamma_{n+1}\Delta}{1-e^{-\rho\Delta}\left(1-\gamma_{n+1}\Delta\right)}\right)=\frac{\gamma_{n+1}}{\gamma_{n+1}+\rho}>0.$$

Now suppose by contradiction that the limit allocation is efficient. It is easy to show that then prices must converge to the observable-quality prices implicitly described in equation (7); consequently, $(p_{n+1}-p_{n+2})$ must converge to a finite positive quantity. But this implies that the payoff to type θ_n^* from keeping the lower quality q_{n+1} is bounded away from zero as $\Delta \to 0$. Furthermore, since payoffs are continuous in θ, the mass of types who benefit from keeping the lower quality is bounded away from zero as $\Delta \to 0$. This contradicts the assumption that the limit allocation is efficient, and proves the claim. ■

We now show that the only candidate for an efficient consumer equilibrium under resale is the revealing strategy profile. This result, jointly with Theorem 1, implies that *there is no efficient consumer equilibrium under stochastic depreciation and resale.* By Remark 1, this is true even in the limit as $\Delta \to 0$.

We consider $K+1$ markets, numbered $0,...,K \leq \infty$. New cars are traded in market 0. For all $k = 1,...,K$, equilibrium determines the price p_k that clears market k, as well as the (average) quality of cars traded in that market. We do not distinguish between markets where the same qualities are traded and the same prices prevail.

Proposition 1 *If there exists a system of $K+1$ resale markets and an equilibrium strategy profile that yields the efficient allocation, then $K = N$ and the consumer equilibrium consists of the revealing strategy profile.*

Proof. Note first that, if in some market k, positive masses of goods of two or more different qualities are sold, there is a positive mass of consumers who obtain cars of the wrong quality. But this is ruled out by efficiency; hence, a single quality is traded in every market. Now consider the markets where quality q_N is traded; the price in all such markets must be equal to $\frac{1-e^{-\rho\Delta}}{\rho}\theta_N q_N$.[12] Hence, in effect, there is a unique market for quality-q_N cars. Now suppose that qualities q_m, for $m = n,...,N$, are traded each in a unique market. Then, all cars of quality q_{n-1} must have the same resale value, and by a similar argument to the one for quality-q_N cars, it follows that the market for quality-q_{n-1} cars is also unique. Hence, there are exactly $N+1$ markets, one for each quality. Furthermore, efficiency also implies that consumers of type $\theta \in [\theta_n^*, \theta_{n-1}^*]$ are only active in market n. ■

[12] This requires that endowments be finite; otherwise there could be sequences of increasing prices sustained by expectations of ever-increasing resale values.

3.3 Rental and Efficiency under Asymmetric Information

We now show that the set of instantaneous rental prices $\{r_n^*\}_{n=0}^N$ that prevail under observable quality (cf. equation (6)) lead to the efficient allocation even under asymmetric information. Specifically, we consider rental contracts that specify an instantaneous rental price r_n the consumer pays for renting vintage n. Moreover, the consumer can keep renting the same unit as long as she wishes, and stop paying the rental fee the moment she wishes to return the unit (without any cancellation fees to the manufacturer). Consumers are not allowed to rent units they returned in the past. This prevents the consumers from strategically returning cars to lower their rental payments.

Consider the ex-ante value $V_n(\theta)$ of pursuing the ex-post efficient policy of renting vintage n and keeping until (and only until) the good depreciates to q_{n+1}. It is easy to see that

$$V_n(\theta) = \sum_{k=0}^{\infty} e^{-\rho k\Delta} \frac{1 - e^{-\rho\Delta}}{\rho}[q_n\theta - r_n^*] = \frac{q_n\theta - r_n^*}{\rho}. \tag{14}$$

We now verify that, given the observable-quality rental prices (equation 6), consumers optimally follow efficient policies. This requires: (i) the ex-ante sorting condition that types $\theta \in [\theta_n^*, \theta_{n-1}^*]$ be willing to rent a vintage n car; (ii) the "ex-post keeping" condition that consumer types $\theta \in [\theta_n^*, \theta_{n-1}^*]$ not be willing to keep any quality below q_n.

Condition (i) is satisfied if for every n, $V_n(\theta_n^*) = V_{n+1}(\theta_n^*)$. Given equation (14), this condition is equivalent to $\theta_n^* q_n - r_n^* = \theta_n^* q_{n+1} - r_{n+1}^*$ which is clearly satisfied given that $\{r_n^*\}_{n=0}^N$ are defined by equation (6). That is, observable-quality rental prices are such that the self-selection conditions are satisfied even if quality is not observable.

In contrast to the case of resale, ex-post keeping (condition (ii)) is automatically satisfied under rental. No consumer has an incentive to keep any quality below the highest quality of any vintage. Suppose that a consumer of type θ is renting vintage n and he is currently consuming a quality-m good. Consuming this good for one period and then resuming tomorrow the efficient policy of only keeping quality q_n yields a payoff of $\frac{1-e^{-\rho\Delta}}{\rho}[q_m\theta - r_n^*] + e^{-\rho\Delta}V_n(\theta)$. If instead the consumer returns quality q_m immediately to start the efficient policy today, he obtains a payoff of $V_n(\theta) = \frac{1-e^{-\rho\Delta}}{\rho}[\theta q_n - r_n^*] + e^{-\rho\Delta}V_n(\theta)$. We can therefore conclude that, under rental, incentives to keep are always guaranteed to hold. Thus, the instantaneous rental prices $\{r_n^*\}_{n=0}^N$ guarantee that both ex-ante sorting and ex-post keeping incentives are consistent with the ex-post efficient allocation that obtains under observable quality. As above, note that the conclusion holds for *any* positive value of Δ.

Theorem 2 *If the goods are rented, there is a consumer equilibrium under asymmetric information that features the same allocation, strategies, and instantaneous rental prices as under observable quality.*

An important feature of the rental contracts that implement the efficient allocation is that they have *indeterminate duration*. A rental contract that required that the good be returned after some fixed number of periods would not lead to the efficient allocation. To see this, recall that the key feature of the mechanism is that the vintage of the good is a perfect signal of quality. Thus, the trading behavior of consumers must signal the depreciation history. If a consumer were required to return the good after a fixed number of periods, then the fact that the good is returned would not convey any information regarding its quality—the good may or may not have depreciated by the time the good is returned.

A menu of *leasing contracts* would not lead to an efficient allocation either. A leasing contract consists of two prices: a rental price that the consumer pays for a pre-specified length of time, and a purchase price that the consumer pays at the end of the period if he chooses to purchase the unit. Thus, a leasing contract suffers from a combination of the shortcomings of resale markets and of the type of fixed-duration rental contracts just discussed. To see this, we need to consider two cases: (1) either the consumer returns the good at the termination of the lease for all depreciation histories, or, (2) for some depreciation histories, the consumer purchases the good at the termination of the lease. In case (1), it is not possible to infer the quality of the good from the behavior of the consumer. Therefore, at some point in the future, potential consumers of the unit are uncertain about the quality of the good. This means that, with positive probability, the good is allocated inefficiently for at least one period. In case (2), efficiency requires that the consumer who purchases the good sell it once it depreciates. However, the consumer now faces incentives that are similar to those she faces in a system of resale markets, and we have seen that efficiency cannot be obtained in that case either.

The inefficiency of leasing contracts is in contrast with the result obtained by Johnson and Waldman (2001), who show that leasing contracts can lead to efficiency. The difference is due to the fact that in Johnson and Waldman (2001) there are only two consumer types and, as in Hendel and Lizzeri (2002), the timing of depreciation is deterministic.

3.4 Supply Side

We now consider the incentives of car producers. We will show that, if producers are competitive, there is a market equilibrium where firms maximize profits by renting the goods at the observable-quality rental prices. Thus, efficient sorting is achieved in equilibrium. Furthermore, the efficient amount of output is supplied. Thus, the equilibrium we characterize leads to the first-best allocation in spite of the presence of asymmetric information in secondary markets.

Assume that there is a unit measure of producers, each of whom has an opportunity to produce a single unit of the good at a cost c in every period.[13] Firms have the same instantaneous discount factor ρ as consumers.

Denote by $R(y)$ the per-unit expected present value of revenue as a function of the total industry output y. If this output is offered according to the efficient rental contracts, the value of this revenue can be obtained recursively as follows. Define $R_n(y)$ as the expected present value of revenue conditional on quality being q_n in the current period:

$$R_n(y) = \frac{1 - e^{-\rho\Delta}}{\rho} r_n^*(y) + e^{-\rho\Delta}((1 - \gamma_n\Delta)R_n(y) + \gamma_n\Delta R_{n+1}(y)), \quad n = 0, ...N$$

[13]The assumption that a producer can produce only one unit every period ensures that each producer is 'small' and simplifies the analysis considerably.

where $R_{N+1} = 0$ by convention. It can be verified that

$$R_N(y) = \frac{\frac{1-e^{-\rho\Delta}}{\rho} r_N^*(y)}{1 - e^{-\rho\Delta}(1-\gamma_N\Delta)}, \quad R_n(y) = \sum_{k=0}^{N-n} \frac{\frac{1-e^{-\rho\Delta}}{\rho} r_{n+k}(y) e^{-\rho k\Delta} \prod_{j=0}^{k-1} \gamma_{n+j}\Delta}{\prod_{j=0}^{k}(1 - e^{-\rho\Delta}(1-\gamma_{n+j}\Delta))}, \; n = 0, \ldots, N-1.$$

Because every new unit is born with quality q_0, it must be the case that $R(y) = R_0(y)$. It is easy to see that $R(y)$ is decreasing in y since each $r_n(y)$ is decreasing in y for every y. We assume that $R(1) < c < R(0)$.[14]

Let y^* be the output defined by the solution of $R(y^*) = c$. This is the output that leads to zero profits for all firms in the industry. Since $R(y)$ is decreasing and continuous, and $R(1) < c < R(0)$, y^* exists and is unique.

We now construct a market equilibrium with the following features: a fraction y^* of firms produce each period. Active firms offer rental contracts at instantaneous prices $\{r_n^*(y^*)\}_{n=0}^N$. The remaining $1-y^*$ firms are inactive. Thus, the equilibrium under asymmetric information is identical to the equilibrium that would obtain under observable quality.

To formalize the market equilibrium concept, we need to describe the class of contracts that firms can offer. Each firm can offer a sequence of mechanisms, one for every consumer who enters into a relationship with the firm during the lifetime of the car (recall that each firm produces a single car). For instance, the rental contracts described in the previous subsection can be viewed as a sequence of $N + 1$ mechanisms, each corresponding to a vintage; the consumer is induced to return the car as soon as it depreciates, and a returned car previously offered under the "vintage-n" mechanism is offered under the "vintage-$(n + 1)$" mechanism in the subsequent time period. However, we can allow for more general mechanisms. Informally, each mechanism features the following ingredients (see the Appendix for a formal description and analysis):

(i) the deviating firm partially or fully reveals information it has gathered concerning the car's history to date;

(ii) upon entering the mechanism, and prior to receiving the car, the consumer pays a price r_0 and sends a message m_0;

(iii) at the end of each period k, the consumer sends a message m_k and pays a price r_k, which may depend on all messages sent up to and including time k (where time is indexed relative to the inception of the relationship between the consumer and the firm);

(iv) finally, the message m_k indicates (possibly among other things) whether or not the consumer desires to continue the relationship with the firm; similarly, after receiving the message m_k, the firm can indicate that it intends to terminate the mechanism. In the latter case, an additional terminal transfer $\bar{r}_k$ will be effected.

The role of messages is twofold: first, they allow the firm to design type-dependent payments and consumption histories; second, they allow the firm to extract information about qualities. Of course, the rental contracts in the previous subsection do not use any of this additional structure:

[14] Assuming $R(0) > c$ implies that some production is viable. The assumption that $R(1) < c$ guarantees that a unit mass of firms is sufficient to exhaust industry profits. We also want to avoid dealing with the case where costs are so low that, under observable quality some qualities would be available at zero price. In this case, all qualities below some level would not be purchased by anybody.

they are very simple contracts. By allowing for this richness, our assumptions impose almost no restrictions on the possible mechanisms.

In order to focus on the distortions caused by adverse selection, we abstract from issues related to lack of commitment; that is, we assume that each firm is bound to the menu of mechanisms it announces. We also emphasize that we make the strong informational assumption that consumers observe the entire contract terms offered by a firm. Note however that, it would be easy to enrich the model to show that, given that other firms publicize their contract terms, it is optimal for a firm to also do so, because hiding prior contract terms would be construed as a bad signal about the history of the unit that the firm rents.

Finally, we assume that firms expect consumers to best-respond to the mechanisms they offer, both on and off the equilibrium path.

Theorem 3 *The following constitutes a market equilibrium for any $\Delta > 0$:*

(i) firms produce the first-best output y^*, and offer $N+1$ vintage-dependent rental contracts at the instantaneous rental prices $r_0, \ldots, r_N$ determined by equation (6);

(ii) for every $n = 0, \ldots, N$, consumer types $\theta \in [\theta_n^*, \theta_{n-1}^*]$ rent vintage-n cars and only keep cars of quality q_n, where the cutoffs $\theta_0^*, \ldots, \theta_N^*$ are determined by equation (5).

The proof of this result is in the Appendix; we now provide a brief sketch of the argument. Individual rationality implies that a consumer of type $\theta \in [\theta_n, \theta_{n-1}]$ will agree to transact with a deviating firm under some mechanism M only if the value of participating in mechanism M, then reverting to renting vintage-n cars is at least as large as her payoff if she rents vintage-n cars forever. This provides an upper bound on the revenues that a deviating firm may obtain from mechanism M by transacting with type θ. We employ this bound to show that any deviation is dominated by a menu of one-period rental contracts, each targeted to a specific consumer type. In this menu, the deviator fully reveals the quality history of the car, and chooses each rental price so as to leave the target type indifferent between (i) renting the deviator's car in the current period, then continuing with her designated putative equilibrium rental contract, and (ii) employing her designated putative equilibrium contract in the current as well as in all subsequent periods. We then show that individual rationality implies that the rental prices charged by the deviator for each quality cannot exceed the rental prices for vintages defined by equation (6); hence, there can be no profitable deviation even if the deviator had full knowledge of the quality history of the car. Furthermore, since industry output equals y^*, which is determined by the zero-profit condition, no new entry can occur.

3.5 Discussion

We now discuss the robustness of the results of this section to possible modifications of the model.

The depreciation process Theorems 1 and 2 continue to hold even if we allowed for a much more general depreciation process where the probability that a car depreciates is a function of the age of the unit. A menu of rental contracts would still lead to efficient sorting since consumers' incentives are not affected by the time structure of the probabilities of depreciation. The logic of the failure of resale markets is also unaffected. Finally, while the structure of the proof of Theorem 3 relies on stationarity, we conjecture that the result would still hold when depreciation is time-dependent.

Note also that Theorem 2 does not rely on the depreciation rate being observable. Consumers just need to know that, by renting vintage n, they can obtain a quality-q_n good. Indeed, none of the calculations concerning consumer incentives in the discussion preceding (and proving) Theorem 2 depend on the probability of depreciation. Asymmetric information of depreciation rates would probably increase the size of the distortions in the case of resale markets.

On the other hand, the exact efficiency result of this section does rely in an important way on the fact that depreciation occurs one step at a time, since this implies that, in equilibrium, the quality of the good is known. If at any point in time the quality of the good can decrease by one or more than one step, then it is not possible to obtain first-best efficiency through rental contracts. Section 4 deals with this case.

Noise traders Suppose that there is a fraction of consumers who have to trade for exogenous reasons (e.g., moving to another country). This phenomenon has been explored by Greenwald (1986) in a modification of Akerlof's adverse selection model. In his model, the presence of such 'noise traders' has a positive welfare effect because it increases the volume of trade. Greenwald showed that noise traders generate a multiplier effect because the they cause a price increase which induces some non-noise traders to sell generating more beneficial trades. In contrast, in our model, noise traders would have a negative welfare effect because they would make the vintage a less precise signal of quality. A car of vintage n may be returned by a noise trader prior to depreciation, when its quality is still q_n; this car would now be of vintage $n+1$, and it would therefore end up in the hands of a consumer with lower valuation who should be consuming quality q_{n+1} instead. Note however, that this distortion may not be too large. First, the good would go to some consumer in the interval $[\theta_{n+1}, \theta_n]$ instead of some consumer in $[\theta_n, \theta_{n-1}]$; these intervals are "close" if there are many qualities. Second, the good is misallocated only until it depreciates, because at that point the vintage-$(n+1)$ consumer will keep the good, which is now the right match for him. Rental prices would have to be adjusted to reflect these misallocations, but the adjustment is minor as long as the fraction of noise traders is not too large.

3.6 Quantifying the Distortions Under Selling: observable vs unobservable trading histories

Theorem 1 shows that, with resale markets and more than two qualities, there is no efficient equilibrium. In order to analyze the nature of the distortions generated by asymmetric information, we now fully characterize equilibrium in a setting with three qualities. Furthermore, to clarify the role of observability of trading histories under resale markets, we compare our framework with observable vintages to a scenario in which consumers can only distinguish new and used goods, but do not observe the number of times a good has been traded.

It is clear from our Theorem 2 that, under rental, observability of trading histories is beneficial. If consumers could not observe the vintage of a car, matching would, by necessity, be much coarser.

However, under resale markets, the equilibrium allocation is inefficient, so it is not immediately clear that observability is similarly beneficial. For example, as mentioned in the Introduction, House and Leahy (2001), in a different setup, show that observing trading histories can have a negative welfare impact.

As we have seen in Theorem 1 three is the minimal number of qualities such that asymmetric information introduces distortions in resale markets. Going beyond three qualities is conceptually simple but tedious and adds no new insight. Furthermore, the efficiency gain from making vintages observable is likely to increase with the number of different qualities. This is because, in equilibrium, each vintage can have distinct average quality which could lead to finer sorting. Thus, the efficiency gain that we calculate here is likely to be a lower bound.

The proofs of the two Propositions provided here can be found in a "Web Appendix" available from the authors' Web sites.

In the preceding analysis, behavior and prices do not depend on the age of the car despite the fact that age is observable. In the equilibrium under selling however, if consumers observe the age of the car, there may not be equilibria in which prices are independent of age. This is because, under selling, consumers of a given vintage may behave differently: lower types may keep more qualities. In this case, older cars are likely to be of lower quality, even conditional on vintage. Hence, in these circumstances, older cars would sell at lower prices. Unfortunately, while a qualitative characterization of equilibrium with age-dependent prices would not be difficult, computing equilibria becomes extremely hard: because cars have indefinite lifespans, we would need to compute infinitely many prices. For simplicity and to highlight the beneficial role of vintages, we will thus proceed by assuming that the age of the car is unobservable.

Observable Vintages In a steady-state equilibrium, the set of consumer types is partitioned into four intervals: (1) types in $[\underline{\theta}, \theta_2]$ never buy any car; (2) types in $[\theta_2, \theta_1]$ buy vintage 2; (3) types in $[\theta_1, \theta_{01}]$ buy vintage 1; and types in $\left[\theta_{01}, \overline{\theta}\right]$ buy vintage 0. Denote by q_n^e the average quality of cars that were vintage $n-1$ the previous date and just became vintage n; in other words, q_n^e is the average quality of cars that were just traded. Clearly, $q_0^e = q_0$. The following proposition characterizes equilibrium.

Proposition 2 *(i) There exists a consumer equilibrium under resale with observable vintages. In this equilibrium:*
(ii) $q_2^e = q_2 \leq q_1^e < q_1$.
*(iii) Buyers of vintage-2 cars keep their cars until they die. Buyers of vintage-*1 *cars keep quality-*q_1 *cars and sell quality-*q_2 *cars. Finally, there exists* $\theta_0 \in (\theta_{01}, \overline{\theta}]$ *such that types* $\theta \in [\theta_{0,1}, \theta_0]$ *buy vintage 0, keep qualities* q_0 *and* q_1*, and sell* q_2*, whereas types* $\theta \in \left[\theta_0, \overline{\theta}\right]$ *buy vintage 0, keep* q_0 *only, and sell all other qualities.*

Note first that only quality-q_2 cars are sold as vintage 2 goods; for this reason, $q_2^e = q_2$. On the other hand, since $q_2 \leq q_1^e < q_1$, a positive mass of cars that are offered on the market as vintage-1 goods must be of quality q_2. Part (iii) implies that no quality-q_0 car is ever offered for resale.

Note also that, while $\theta_0 > \theta_{01}$, there are parameter values for which $\theta_0 = \overline{\theta}$. That is, it is always the case that some high types buy vintage-0 cars and keep both qualities q_0 and q_1; however, it may be the case that *all* high types adopt this policy. In such cases, no cars of quality q_1 are offered for resale, and the equilibrium effectively features two vintages.

Unobservable Vintages Now assume that consumers cannot distinguish goods that have been sold only once from goods that have been sold more than once. Now consumers will be partitioned into three intervals (1) types in $[\underline{\theta}, \theta_2]$ who never buy any car; (2) types in $[\theta_2, \theta_u]$ who buy used cars; (3) types in $\left[\theta_u, \overline{\theta}\right]$ who buy vintage 0. Denote by q_u the average quality of used cars. The following proposition characterizes equilibrium.

Proposition 3 *(i) There exists a consumer equilibrium under resale with unobservable vintages. In this equilibrium:*
(ii) $q_2 \leq q_u < q_1$.
(iii) Buyers of used cars keep quality-q_1 *cars and sell quality-*q_2 *cars. There exists* $\theta_0 \in (\theta_u, \bar{\theta}]$ *such that types* $\theta \in [\theta_u, \theta_0]$ *buy new cars, keep qualities* q_0 *and* q_1*, and sell* q_2*, whereas types* $\theta \in \left[\theta_0, \overline{\theta}\right]$ *buy new cars, only keep* q_0*, and sell all other qualities.*

Thus, a positive mass of used cars is of quality q_2. However, no quality-q_0 cars are offered for resale.

The behavior of new good consumers is qualitatively the same as that of vintage-zero consumers in the case of observable quality. The behavior of used good consumers reflects a form of arbitrage: specifically, a consumer who owns a used good of quality q_2 can sell it at a price p^u and buy another used good for the same price. If $q_u > q_2$, which is the case whenever $\theta_0 < \bar{\theta}$, all used goods consumers are better off selling quality-q_2 used goods and only keeping quality-q_1 cars.

Finally, as above, it is always the case that some high types keep both qualities q_0 and q_1 (i.e. $\theta_0 > \theta_u$); furthermore, for certain parameter values, *all* high types adopt this policy (i.e. $\theta_0 = \bar{\theta}$).

Comparison of the two scenarios and the role of observable vintages The differences in equilibrium outcomes between the two scenarios[15] may be understood by focusing on two forces. Suppose first that the resale behavior of new goods consumers is the same in the two scenarios. Then, multiple secondary markets allow better sorting of used units. Loosely speaking, the ability to distinguish between two used-car vintages effectively unbundles goods sold by new goods consumers; within at most two periods, quality-q_1 goods are allocated to vintage-1 consumers, and quality-q_2 goods are allocated to vintage-2 consumers. Thus, higher-quality used cars are assigned to higher-valuation consumers. If instead vintages are unobservable, such unbundling is impossible and all consumers who are not new goods consumers end up consuming the same average quality over their lifetimes.

Second, in the case of unobservable vintages, cars that are sold by new goods consumers are pooled with quality-q_2 cars that are sold by used goods consumers.[16] Such pooling will reduce the

[15]This discussion assumes that $\theta_0 \in (\theta_{01}, \bar{\theta})$ in the observable-vintages case, and $\theta_0 \in (\theta_u, \bar{\theta})$ when vintages are unobservable. When the interval $\left[\theta_0, \overline{\theta}\right]$ is empty in both scenarios, the allocations are the same.

[16]These include consumers who owned quality q_1 units that depreciated, and those who purchased a used good the previous period and found its quality to be q_2.

resale value of new goods, and this will induce a higher fraction of new goods consumers to keep quality-q_1 cars.

A direct comparison of the equilibrium allocations in the two scenarios cannot be provided because the overall equilibrium cannot be solved for in closed form. We therefore proceeded numerically. In our extensive numerical analysis, *we always found social surplus to be higher when vintages are observable.* We now briefly describe the computations we carried out.[17]

We compute two measures of the efficiency gain resulting from vintage observability. Let S_v and S_{nv} denote per-period surplus under observable and unobservable vintages respectively; also let S_{eff} denote per-period surplus under the efficient allocation. Finally, let S_1 denote per-period surplus in the absence of secondary markets, computed as follows: let $\theta_2 = F^{-1}(1-Y)$ and

$$S_1 = \frac{1-e^{-\rho\Delta}}{\rho} \int_{\theta_2}^{\bar{\theta}} E(q) \cdot \theta \; d\theta.$$

S_1 is the maximum surplus achieved by opening only the market for new goods: the $1-Y$ low-valuation consumers are excluded, but cars are randomly assigned to higher-valuation consumers.

One possible measure of relative efficiency is then $\frac{S_v}{S_{\mathit{eff}}} - \frac{S_{nv}}{S_{\mathit{eff}}}$, the difference between the fraction of the efficient social surplus realized with and without vintage observability. Alternatively, we can measure realized efficiency as a fraction of $S_{\mathit{eff}} - S_1$, the *maximum possible efficiency gain* relative to a single-market environment; the quantities $\frac{S_v - S_1}{S_{\mathit{eff}} - S_1}$ and $\frac{S_{nv} - S_1}{S_{\mathit{eff}} - S_1}$ are the fractions of this gain actually realized under observable and unobservable vintages respectively, so another measure of relative efficiency is their difference $\frac{S_v - S_{nv}}{S_{eff} - S_1}$.

For the purposes of comparing social surplus, the minimum and maximum quality levels can be chosen arbitrarily; we set $q_0 = 1$ and $q_2 = 0$. Also, recall that, under simple depreciation, the length of a period is immaterial; we set $\Delta = 1$. Hence, a full parameterization of both models consists of the intermediate quality $q_1 \in [q_2, q_0]$, the depreciation probabilities $\gamma_0, \gamma_1, \gamma_2$, the instantaneous discount factor ρ, the total mass of cars Y, and the distribution F of consumer types.

We assume that types are uniformly distributed on $[0,1]$,[18] and set $e^{-\rho} = 0.9$ and $Y = 0.8$. We consider qualities $q_1 \in (q_2, q_0)$ on a grid of size 20. Finally, we analyze two choices of minimum and maximum depreciation probabilities $\gamma^{\min}, \gamma^{\max}$ ("high" and "low"), and consider values of $\gamma_0, \gamma_1, \gamma_2 \in [\gamma^{\min}, \gamma^{\max}]$, each on a grid of size 20. Table 1 summarizes some of our findings.

In rows 4 and 5, for each of the two measures described above, we report the maximum and minimum efficiency gains due to vintage observability, calculated over the $20^4 = 160,000$ different parameterizations generated by each choice of $\gamma^{\min}$ and $\gamma^{\max}$. As noted above, vintage observability is always beneficial; however, for certain parameter values, the equilibrium is the same regardless of whether vintages are observable or not (i.e no quality-q_1 cars are traded in either setting); in these cases the efficiency gain is zero. Overall, these figures suggest that the efficiency gain from vintage observability, especially when measured relative to a single-market environment, can be substantial, and is larger when depreciations are less frequent.

[17] The matlab code is available upon request.

[18] We also experimented with Beta-distributed types. Again, social surplus was higher with observable vintages.

	High Depreciation		Low Depreciation	
	$\gamma^{\min}=0.1$, $\gamma^{\max}=0.9$		$\gamma^{\min}=0.01$, $\gamma^{\max}=0.1$	
	Max	Min	Max	Min
$\frac{S_v-S_{nv}}{S_{\mathit{eff}}}$	5.49%	0	8.31%	0
$\frac{S_v-S_{nv}}{S_{\mathit{eff}}-S_1}$	18.6%	0	41.87%	0
$\frac{S_v}{S_{\mathit{eff}}}$	–	89.87%	–	92.81%
$\frac{S_v-S_1}{S_{\mathit{eff}}-S_1}$	–	38.14%	–	42.53%
$\frac{S_1}{S_{\mathit{eff}}}$	95.90%	62.48%	96.20%	62.26%

Table 1: Efficiency Gains

Rows 6 and 7 report the *minimum* realized efficiency gain $\frac{S_v}{S_{\mathit{eff}}}$ under resale markets and observable vintages, and the minimum realized gain $\frac{S_v-S_1}{S_{\mathit{eff}}-S_1}$ relative to a single-market environment.[19] Since rental contracts yield the efficient allocation, these quantities measure the potential inefficiency associated with resale.

We wish to emphasize that the numbers in row 6 correspond to "worst-case scenarios", i.e. parameter values for which resale markets perform particularly poorly. For different parameter values, the gains from using rental contracts rather than selling contracts are not very large. This may help explain why rental contracts are not commonly observed in the car market; perhaps the gains are not large enough to justify the larger administrative costs, and the potential problems of moral hazard in maintenance that are likely to be associated with rental contracts. Furthermore, we note that leasing contracts (which constitute more than a third of transactions in the new car market) share some of the advantages of rental contracts—although, as pointed out in Section 3.3, the two are not perfect substitutes.

Finally, the last row reports the minimum and maximum surplus achievable in a single-market environment; these figures can be useful as a reference.

4 General Depreciation Model

We now consider the general environment described in Section 2.1. In contrast with the model discussed in Section 3, the initial quality of the new good is uncertain, and the good may depreciate by an arbitrary number of quality steps (or die) in any time period. As before, efficiency requires assortative matching of qualities to consumers. It is easy to verify that the system of instantaneous rental prices defined by equation (6) still sustain the efficient allocation if quality is observable.

We now assume that the quality of the good is not observable before purchase: it becomes observable only to the current user at the end of the first period of consumption. The key distinction

[19] By Theorem 1, resale contracts achieve efficiency if there are only two qualities; thus, the *maximum* efficiency gain can be made to approach 1 by choosing q_1 close to q_0 or q_2. For this reason, this figure is not shown in Table 1.

between the present model and the simple one-step depreciation model presented in Section 3 is the fact that efficient sorting now requires *experimentation*: for instance, highest-valuation consumers need to try several units before finding one of quality q_0. Consequently, in the environment under consideration, it is impossible to obtain the first-best allocation: the first consumer of the good consumes the 'wrong' quality with positive probability. However, we will show that approximate payoff efficiency is possible when the length of the periods Δ shrinks to zero (i.e. trading becomes more and more frequent).

The goal of this section is to show that the efficiency properties of vintage-dependent rental contracts of indeterminate duration are robust: approximate efficiency can be obtained in a much more general setting than the simple depreciation model. We do **not** claim that these rental contracts are the *best* mechanism in this more complex environment. In particular, menus of contracts that depend more finely on the trading history may well be superior. However, our results imply that any gains from more complex mechanisms would be negligible in a world where retrading costs are small.

4.1 Experimentation and the Trickle-Down Algorithm

As in the simpler setting of Section 3, we assume that the *vintage* of each car is observable.

For $n = 0, \dots, N$ and $m = n, \dots, N$, denote by v_m^n the mass of cars of vintage n and quality m. These quantities must satisfy the following equations:

$$\begin{aligned} v_0^0 &= \gamma_{0,0}(\Delta)v_0^0 + \chi_0 y \\ v_m^0 &= \chi_m y \\ v_n^n &= \gamma_{n,n}(\Delta)v_n^n + \gamma_{n-1,n}\Delta v_{n-1}^{n-1} + \gamma_{n,n}(\Delta)v_n^{n-1} \\ v_m^n &= \sum_{\ell=n-1}^{m-1} \gamma_{\ell,m}\Delta v_\ell^{n-1} + \gamma_{m,m}(\Delta)v_m^{n-1}. \end{aligned} \tag{15}$$

That is: vintage-0 cars consists of quality-q_0 cars that have not depreciated, and newly-produced cars. Since vintage-0 cars worse than q_0 are immediately retraded, the stock of quality-q_m cars of vintage 0, for $m > 0$, consists solely of newly-produced cars. Vintage-n cars of quality q_n come from three sources: vintage-n, quality-q_n cars that have not depreciated, vintage-$(n-1)$ quality-q_{n-1} cars that have depreciated to q_n, and vintage-$(n-1)$ quality-q_n cars that have not depreciated. Vintage-n cars of quality worse than q_n all come from the stock of vintage-$(n-1)$ cars, and are immediately retraded.

Observe that the masses v_m^n, for $n \neq m$, measure the efficiency loss due to the fact that consumers need to experiment in order to receive a car of their designated quality.

Furthermore, the mass of new cars must correspond to the mass of cars dying in each period:

$$y = \sum_{\ell=0}^{N} \sum_{k=\ell}^{N} \gamma_{k,N+1}\Delta v_k^\ell. \tag{16}$$

Finally, there is a total of Y cars on the market at each given time:

$$\sum_{\ell=0}^{N}\sum_{k=\ell}^{N} v_k^\ell = Y. \tag{17}$$

We can now define cutoff types to identify consumers of each vintage. Let $\theta_{-1} = \overline{\theta}$ for notational convenience; then, for $n = 0, \ldots, N$, let θ_n be defined by the condition

$$F(\theta_{n-1}) - F(\theta_n) = \sum_{m=n}^{N} v_m^n. \tag{18}$$

4.2 Rental and Experimentation under Asymmetric Information

We now turn to the analysis of consumer incentives. Each consumer is facing a stationary, infinite-horizon dynamic programming problem, with states $\emptyset$ (corresponding to the event that the car just died), $q_0, \ldots, q_N$. The possible actions (controls) are "rent your current car for an additional period" (not available in state $\emptyset$) and "return your current car, if any, and rent a vintage-n car", for $n = 0, \ldots, N$. The transition probabilities are determined by actions and depreciation probabilities in the obvious way. As in the simpler setting of Section 3, we consider contracts that specify vintage-dependent rental fees represented by instantaneous rental prices $r_0, \ldots, r_N$. Other aspects of the trading history of a specific unit (e.g., age and timing of past trades) do not affect the terms under which it is offered.[20]

We assume that consumers are not allowed to exercise choice among units of the same vintage, but possibly different trading histories; equivalently, one could assume that consumers are informed of the full trading history of a unit only *after* they decide to rent it.[21] This assumption allows for a more straightforward comparison with the analysis of section 3, and is of no consequence as far as the interpretation of our results is concerned: as mentioned in the beginning of this Section, we do not claim that the rental mechanism we examine is the *best* in the current, general setting; rather, our results show that any other mechanism can only lead to ϵ-improvements in either welfare from the social planner's point of view, or profits from the point of view of the firms. This is the case regardless of how finely alternative mechanisms depend upon or disclose past trading history.

[20] For instance, suppose Car A is first rented at time $t-1$ and first returned at time t, whereas Car B is first rented at time $t-15$ and first returned at time t. Then, at the end of time t, both Cars A and B are deemed to be of vintage 1, and hence are offered at rental price r_1.

[21] Note that while in section 3 the vintage of a car was a sufficient statistic for quality, this is not necessarily true if one allows for both uncertainty about the quality of new cars, and multi-step depreciation. However, this is of no consequence when $\Delta \to 0$. Loosely speaking, to allow for more complicated history dependent rental prices, we would simply need to keep track of a larger set of consumer intervals. When $\Delta \to 0$, a simple extension of Lemma 2 implies that the rental prices of all cars with the same vintage would converge to the same limit, regardless of the actual trading histories. This in turn implies that, in the limit, all consumers and all firms are approximately indifferent among all rental contracts for cars of the same vintage.

In order to describe the value functions, it is useful to introduce additional notation. The probability that a newly rented vintage-n car is of quality q_m equals

$$\lambda_m^0 = \chi_m \quad \text{and} \quad \lambda_m^n = \frac{\sum_{\ell=n-1}^{m-1} \gamma_{\ell,m} \Delta v_\ell^{n-1} + \gamma_{m,m}(\Delta) v_m^{n-1}}{\sum_{k=n}^{N} \left(\sum_{\ell=n-1}^{k-1} \gamma_{\ell,k} \Delta v_\ell^{n-1} + \gamma_{k,k}(\Delta) v_k^{n-1} \right)}, \tag{19}$$

for $m = n, \ldots, N$. These expressions are derived by looking at the supply of vintage-n cars of each quality, as it appears in Eqs. (15); for vintage 0, the supply consists solely of newly produced cars, whose quality is distributed according to the proportions $\chi_0, \ldots, \chi_N$. For vintages $n > 0$, the supply consists of vintage-$(n-1)$ cars, and we keep track of the various ways a quality-q_m car might be offered in the vintage-n market according to the trickle-down algorithm.

We denote the expectation operator corresponding to the distribution $\lambda_n^n, \ldots, \lambda_N^n$ by E^n; for simplicity, we also define $L_m^n = \sum_{\ell=m+1}^{N} \lambda_\ell^n$. Note that, although the notation does not emphasize this fact, both λ_m^n and E^n are also a function of Δ.

By standard arguments, there exists a stationary policy that is optimal for the consumer. We now describe the set of stationary policies. If the current state is $\emptyset$ (no car), the policy must specify which vintage to rent: thus, this portion of the policy can be represented by an integer $n \in \{0, \ldots, N\}$. If the current state is instead q_n, for $n = 0, \ldots, N$, the policy must specify whether to keep the current car (i.e. rent the currently rented unit for another time period), or return it and rent another car. A consumer who chooses to return her current car faces the same problem as a consumer whose car has just died: hence, it is without loss of generality to restrict attention to policies that prescribe that the *same* vintage be rented if the current car dies, or if it is returned. Such policies are thus fully specified by a pair (n, M), where $n \in \{0, \ldots, N\}$ is the vintage the consumer rents in state $\emptyset$, and $M \subset \{n, \ldots, N\}$ is a (possibly empty) collection of quality indices corresponding to qualities the consumer keeps.[22]

Consider one such stationary policy (n, M) with $M \neq \emptyset$ (see Lemma 3 for the case $M = \emptyset$). The value function for consumer type θ in state $\emptyset$ can be written as follows:

$$\begin{aligned}
V_M^n(\theta, \emptyset) &= \frac{1 - e^{-\rho\Delta}}{\rho} \left[E^n(q | q \le q_n)\theta - r_n \right] + \qquad (20) \\
&+ e^{-\rho\Delta} \Bigg\{ \Bigg[\sum_{m \ge n \,:\, m \notin M} \lambda_m^n \Big(\gamma_{m,m}(\Delta) + \sum_{\ell > m \,:\, \ell \notin M} \gamma_{m,\ell} \Delta \Big) + \sum_{m \ge n \,:\, m \in M} \lambda_m^n \sum_{\ell > m \,:\, \ell \notin M} \gamma_{m,\ell} \Delta \Bigg] V_M^n(\theta, \emptyset) + \\
&+ \sum_{m \in M} \lambda_m^n \Bigg[\gamma_{m,m}(\Delta) V_M^n(\theta, q_m) + \sum_{\ell > n \,:\, \ell \in M \setminus \{m\}} \gamma_{m,\ell} \Delta V_M^n(\theta, q_\ell) \Bigg] + \sum_{m \ge n \,:\, m \notin M} \lambda_m^n \Bigg(\sum_{\ell > m \,:\, \ell \in M} \gamma_{m,\ell} \Delta V_M^n(\theta, q_\ell) \Bigg) \Bigg\};
\end{aligned}$$

The first term on the right-hand side of equation (20) is the flow payoff from experimenting with a vintage–n good which has an uncertain quality $E^n(q | q \le q_n)$. The remaining terms refer to the

[22] In general, one cannot guarantee *a priori* (without fixing rental prices and solving for the optimal policy) that restricting attention to cutoff policies—i.e. $M = \{n, \ldots, m\}$ for some $m \ge n$—will be w.l.o.g.. Intuitively, without restrictions on the depreciation probabilities $\gamma_{n,m}$, it may be the case that a car of *current* quality q_{n+1} yields a higher expected discounted utility than a car of current quality q_n (e.g. if quality q_{n+1} depreciates slowly, whereas q_n does not depreciate but dies with high probability).

possible events that the consumer faces the first period after experimentation. In particular, the consumer returns the car (so her state switches to $\emptyset$) if its quality is outside the "keeping set" M at the beginning of the period, and either (i) there is no depreciation, or (ii) the car depreciates to another quality not in M; the consumer also returns the car if its quality is initially in the keeping set M, but depreciation yields a quality not in M. These possibilities are captured in the second line of equation (20). Symmetrically, the consumer keeps her car if its quality is initially in the keeping set M, and either (i) there is no depreciation, or (ii) the car depreciates to another quality in M; she also keeps the car if its quality was initially outside M, but depreciation yields a quality level in the keeping set M. The third line of equation (20) corresponds to these considerations.

If instead the consumer is currently renting a quality-q_ℓ car, with $\ell \in M$,

$$\begin{aligned} V_M^n(\theta, q_\ell) \quad &= \quad \frac{1-e^{-\rho\Delta}}{\rho}\left[q_\ell\theta - r_n\right] + e^{-\rho\Delta}\left[\left(\sum_{k>\ell\,:\,k\notin M}\gamma_{\ell,k}\Delta\right)V_M^n(\theta,\emptyset) + \right. \\ &\left. + \left(\gamma_{\ell,\ell}(\Delta)V_M^n(\theta,q_\ell) + \sum_{k>\ell\,:\,k\in M}\gamma_{\ell,k}\Delta V_M^n(\theta,q_k)\right)\right]. \end{aligned} \tag{21}$$

We now define the instantaneous rental prices $r_0, \ldots r_N$ so as to ensure that

$$V_{\{N\}}^N(\theta_N,\emptyset) = 0, \qquad V_{\{n\}}^n(\theta_n,\emptyset) = V_{\{n+1\}}^{n+1}(\theta_n,\emptyset),\ n = 0,\ldots,N-1. \tag{22}$$

The main result of this paper can now be stated.

Theorem 4 *There exists $\Delta^* > 0$ such that, for all $\Delta \in (0,\Delta^*)$, there is a consumer equilibrium wherein cutoff types and instantaneous rental prices are determined by equations (18) and (22) respectively, and for every $n = 0,\ldots,N$, consumer types $\theta \in [\theta_n,\theta_{n-1}]$ rent vintage-n cars and only keep cars of quality q_n.*

Furthermore, as $\Delta \to 0$, cutoff types and instantaneous rental prices converge to their observable-quality counterparts: $\theta_n \to \theta_n^*$ and $r_n \to r_n^*$ for all $n = 0,\ldots,N$.

We prove this result via several lemmas. The first, Lemma 1, shows that the Eqs. (15), (16) and (17) uniquely determine masses in each stage of the trickle-down algorithm, and show that these masses converge to the appropriate efficient quantities as the length of each time period shrinks. The proof of this Lemma is available in a "Web Appendix" on the authors' Web sites.

Lemma 1 *There is a unique solution to Eqs. (15), (16) and (17). Furthermore, as $\Delta \to 0$, for every $n = 0,\ldots N$, m $v_n^n \to v_n^*$ and $v_m^n \to 0$ for $m = n+1,\ldots N$. Consequently, $\theta_n \to \theta_n^*$ as $\Delta \to 0$.*

Turn now to consumer incentives. The following Lemma provides the key step in the proof of Theorem 4: it shows that the value functions $V_{\{n\}}^n(\theta,\emptyset)$ can be written as a weighted average of a "long-run" component, which corresponds to the net payoff from renting a car of known quality q_n in each period, and an "experimentation" component; furthermore, the weight on the latter vanishes as $\Delta \to 0$. This is then shown to imply that rental prices, as defined above, converge to their observable-quality counterparts as $\Delta \to 0$.

Lemma 2 *For every $n = 0, \ldots, N$,*

$$V^n_{\{n\}}(\theta, \emptyset) = (1 - w_n)\frac{E^n(q|q \leq q_n)\theta - r_n}{\rho} + w_n \frac{q_n\theta - r_n}{\rho}$$

where $w_n = \frac{e^{-\rho\Delta}\lambda^n_n(1-G_{n,n+1}\Delta)}{1-e^{-\rho\Delta}(1-G_{n,n+1}\Delta)+e^{-\rho\Delta}\lambda^n_n(1-G_{n,n+1}\Delta)} \in (0,1)$, and $w_n \to 1$ as $\Delta \to 0$. Therefore, the rental prices defined in equation (22) satisfy $r_n \to r^*_n$ as $\Delta \to 0$.

Proof. See Appendix. ■

We now employ the decomposition of payoffs provided in Lemma 2 to show that experimentation policies of the form (n, M) with $M \neq \{n\}$ cannot be optimal for *any* type, and "pure-consumption" policies ($M = \emptyset$) can be disregarded w.l.o.g. .

Lemma 3 *There exists $\Delta^* > 0$ such that, for $\Delta \in (0, \Delta^*)$, and for all θ, the policies (n, M) with $M \neq \{n\}$ are suboptimal. Furthermore, for every $n = 0, \ldots, N$, every Δ, and every θ, in each state $\emptyset, q_0, \ldots, q_N$, the policy $(n, \emptyset)$ is not strictly better than the policy $(n, \{n\})$.*

Proof. Consider type $\underline{\theta}$ and policy (n, M), and let $m = \max\ M$; by Lemma 2, there exists $\Delta_{n,m} > 0$ such that, for $\Delta \in (0, \Delta_{n,m})$, $\frac{q_m\underline{\theta}-r_n}{\rho} < (1 - w_n)\frac{E^n(q|q\leq q_n)\underline{\theta}-r_n}{\rho} + w_n\frac{q_n\underline{\theta}-r_n}{\rho} = V^n_{\{n\}}(\underline{\theta}, \emptyset)$ and $q_m \leq w_n q_n \leq (1 - w_n)E^n(q|q \leq q_n) + w_n q_n = \frac{\partial V^n_{\{n\}}(\theta,\emptyset)}{\partial\theta}(1 - e^{-\rho\Delta})$. This implies that, for $\Delta \in (0, \Delta_{n,m})$, $\frac{q_m\theta-r_n}{\rho} < V^n_{\{n\}}(\theta, \emptyset)$ for all types θ. Now consider one such Δ, and suppose that (n, M) is optimal for some type θ. Then, in particular, $V^n_M(\theta, \emptyset) \geq V^n_{\{n\}}(\theta, \emptyset) > \frac{q_m\theta-r_n}{\rho}$. Moreover, by equation (21), since $m = \max\ M$ (so $k > m$ implies $k \notin M$) and $\gamma_{m,m}(\Delta) = 1 - G_{m,m+1}\Delta$,

$$\begin{aligned} V^n_M(\theta, q_m) &= \frac{1 - e^{-\rho\Delta}}{\rho}[q_m\theta - r_n] + e^{-\rho\Delta}\left[G_{m,m+1}\Delta V^n_M(\theta, \emptyset) + \gamma_{m,m}(\Delta)V^n_M(\theta, q_m)\right] = \\ &= \frac{1 - e^{-\rho\Delta}}{1 - e^{-\rho\Delta}(1 - G_{m,m+1}\Delta)}\frac{q_m\theta - r_n}{\rho} + \frac{e^{-\rho\Delta}G_{m,m+1}\Delta}{1 - e^{-\rho\Delta}(1 - G_{m,m+1}\Delta)}V^n_M(\theta, \emptyset); \end{aligned}$$

it follows that $V^n_M(\theta, q_m) < V^n_M(\theta, \emptyset)$: that is, renting another car of vintage n and then reverting to (n, M) is more profitable for type θ than following the policy (n, M) if her current car is of quality q_m, i.e. the lowest quality the consumer is supposed to keep under policy (n, M).

Choosing $\Delta^* = \min_{n,m} \Delta_{n,m}$ completes the proof of the first claim. As for the second, for any type θ and any $n = 0, \ldots, N$, the policy $(n, \emptyset)$ yields $\frac{E^n(q|q\leq q_n)\theta-r_n}{\rho}$ in any state; by Lemma 2, $V^n_{\{n\}}(\theta, \emptyset) = (1 - w_n)\frac{E^n(q|q\leq q_n)\theta-r_n}{\rho} + w_n\frac{q_n\theta-r_n}{\rho}$, with $w_n \in [0, 1]$. Since $E^n(q|q \leq q_n) \leq q_n$ for all $n = 0, \ldots, N$, it follows that, in any state $\emptyset, q_0, \ldots, q_N$, the consumer is at least as well off returning the current car and switching to the policy $(n, \{n\})$. ■

The argument can now be concluded. An optimal policy of the form (n, M) exists for every $\theta \in [\theta_N, \bar{\theta}]$; by Lemma 3, for $\Delta \in (0, \Delta^*)$, we can restrict attention to policies of this class with $M = \{n\}$. But rental prices are defined so as to ensure that, for all types $\theta \in [\theta_n, \theta_{n-1}]$, it is optimal to adopt policy $(n, \{n\})$ in state $\emptyset$ and adhere to it in the continuation; hence, $(n, \{n\})$ must be an optimal policy for these types.

4.3 Supply Side in the General Depreciation Model

The analysis of producers' incentives is more delicate than in the simple depreciation environment. Since initial quality is uncertain and depreciation by more than one quality level is possible, experimentation is necessary in the trickle-down mechanism for consumers to obtain the 'right' car quality. This implies that some delay is inevitable before the right match between cars and consumers is achieved. When the time between periods Δ is small, this delay is short; however, the presence of this delay raises the possibility that producers may choose to deviate from the menu of rental contracts to offer a mechanism that *accelerates experimentation*. For instance, a firm might require consumers to report the current quality of the car when they return it, then offer it to the 'right' consumer type in the following period. Even in the absence of such reports, it can be shown that profitable deviations from the menu of rental contracts defined above are possible.

For instance, learning the current quality of the car from its previous consumer is beneficial to a firm for two reasons: (i) one or more steps in the trickle-down mechanism may be bypassed, and (ii) the next consumer will not need to experiment in order to find the right quality for her; therefore, she will be willing to pay a higher per-period rental price to the deviating firm. However, if the time between periods is small, *the gain from such deviations is also small*: if Δ is small, (i) allocating a car via the trickle-down mechanism only imposes a short delay, and (ii) the cost of experimentation is small.

These intuitive observations can be formalized and developed in two directions. In the setting of Section 3.4, it is possible to adapt the proof of Theorem 3 to establish the following approximate equilibrium result.

Theorem 5 *For every $\varepsilon > 0$, there exists $\Delta_\varepsilon > 0$ such that, for all $\Delta \in (0, \Delta_\varepsilon)$, the following constitutes a market ε-equilibrium:*

(i) firms produce the first-best output, and offer $N+1$ vintage-dependent rental contracts at the instantaneous rental prices $r_0, \ldots, r_N$ determined by equation (22);

(ii) for every $n = 0, \ldots, N$, consumer types $\theta \in [\theta_n, \theta_{n-1}]$ rent vintage-n cars and only keep cars of quality q_n, where the cutoffs $\theta_0, \ldots, \theta_N$ are determined by equation (18).

Furthermore, as $\Delta \to 0$, for every $n = 0, \ldots, N$, $\theta_n \to \theta_n^*$ and $r_n \to r_n^*$.

The proof of Theorem 5 can be found in the Appendix; here we provide a brief sketch. As explained in Section 3.4, each deviation can be shown to be dominated by a menu of one-period rental contracts; the argument is independent of the specific features of the depreciation process. Prices in the dominating menu of rental contracts are set so as to ensure that each target consumer is indifferent between renting a car from the deviating firm, then reverting to experimentation with her designated vintage, and experimenting with that vintage forever.

As suggested by the above intuitive discussion, under general depreciation, the rental prices charged in each period by the deviating firm for a car of quality q_n can be larger than $\frac{1-e^{-\rho\Delta}}{\rho} r_n$, because the latter is determined taking into account the cost of experimentation borne by consumers. We also noted above that the gain from such a one-period deviation is "small" if Δ is not too large; however, we must quantify gains from deviations *per unit of calendar time*, because as Δ becomes smaller, the expected number of periods until the car dies grows larger. We thus show that gains per time unit vanish as $\Delta \to 0$, which completes the proof.

Theorem 5 may be interpreted as stating that, if Δ is small, then there is an *approximate* equilibrium wherein *all* firms offer the rental contracts described in Section 4.2. The previous version of this paper (Hendel, Lizzeri and Siniscalchi, 2002) considered a model characterized by initial uncertainty about quality, no depreciation, and a positive probability that the car dies in each period; these assumptions correspond to $\gamma_{n,m} = 0$ for $m = n+1, \ldots, N$ and $\gamma_{n,N+1} > 0$. In this environment, we established a complementary result: when Δ is small, there is an *exact* equilibrium wherein *almost all* firms offer the rental contracts described above, but a small mass of firms offer other types of contracts.

5 Concluding Remarks

We now outline some possible extensions of our analysis.

Throughout this paper, we have assumed that the consumer learns the quality of a unit as soon as she uses it for the first time. This leads to rather strong efficiency results. If instead one were to assume that the quality discovery process may take a minimal amount of time, and that such learning may be imperfect, then new effects would arise out of the interaction between asymmetric information and slow learning. In particular, we conjecture that in such a model, a consumer may get rid of a high quality unit if he is sufficiently convinced that it is low quality. Once the high quality good is in the hands of a low valuation consumer, it may become impossible, under asymmetric information, to get it back in the hands of high valuation consumers. Thus, some degree of misallocation may be inescapable.

It would be interesting to extend the model to study matching under asymmetric information in the labor market, so as to understand the relation between job mobility and wage growth. To this end, two additional key features should be incorporated in the model. First, both sides in the labor market can take actions after learning the quality of a match: both employers and workers can in principle choose to dissolve a match, whereas in our model the car cannot decide to get rid of the consumer. Second, idiosyncratic components are likely to be a more important feature of match quality in labor markets than in markets for durable goods.

Finally, it may be instructive to contrast our results with those from the literature on the Coase conjecture. This literature deals with a monopolistic producer of a durable good of *known* quality. In that setting, a monopolist may prefer a rental contract over a sale contract, because the former avoids the commitment problem.[23] In the context of a durable-goods monopoly, if consumers are patient, the stationary subgame-perfect equilibrium outcome under selling is approximately efficient.[24] In contrast, under rental, the equilibrium outcome involves the monopolist producing too little output. Thus, the consequences for efficiency of these alternative contractual arrangements are the opposite of those we find in our model.

This paper's broader contribution is that it provides an efficiency benchmark that is a polar opposite of the classic model. Having these two opposites - extreme efficiency and extreme inefficiency creates a scale that makes it easier to identify the sources and the extent of inefficiencies arising in the asymmetric information environments with durables.

[23]See Bulow (1982) for this argument.

[24]See Gul, Sonneschein, and Wilson (1986).

6 Appendix

6.1 Proof of Lemma 2

Begin with a preliminary result; for the proof see the Webappendix.

Lemma 4 *For all* $n = 0, \ldots, N$, $\liminf_{\Delta \to 0} \lambda_n^n > 0$.

Turn now to the proof of Lemma 2. Note that, for $M = \{n\}$, the functions $V^n_{\{n\}}$ can be rewritten in the following simpler form:

$$V^n_{\{n\}}(\theta, \emptyset) = \frac{1 - e^{-\rho\Delta}}{\rho} \left[E^n(q|q \leq q_n)\theta - r_n \right] + \tag{23}$$
$$+ e^{-\rho\Delta} \left\{ \left(L^n_{n+1} + \lambda^n_n G_{n,n+1}\Delta \right) V^n_{\{n\}}(\theta, \emptyset) + \lambda^n_n \gamma_{n,n}(\Delta) V^n_{\{n\}}(\theta, q_n) \right\},$$

$$\begin{aligned} V^n_{\{n\}}(\theta, q_n) &= \frac{1 - e^{-\rho\Delta}}{\rho} \left[q_n\theta - r_n \right] + e^{-\rho\Delta} \left\{ G_{n,n+1}\Delta V^n_{\{n\}}(\theta, \emptyset) + \gamma_{n,n}(\Delta) V^n_{\{n\}}(\theta, q_n) \right\} = \\ &= \frac{\left(1 - e^{-\rho\Delta}\right) \frac{q_n\theta - r_n}{\rho} + e^{-\rho\Delta} G_{n,n+1}\Delta V^n_{\{n\}}(\theta, \emptyset)}{1 - e^{-\rho\Delta}(1 - G_{n,n+1}\Delta)}. \end{aligned} \tag{24}$$

Plugging back into equation (23) yields

$$\begin{aligned} V^n_{\{n\}}(\theta, \emptyset) &= \frac{1 - e^{-\rho\Delta}}{\rho} \left[E^n(q|q \leq q_n)\theta - r_n \right] + e^{-\rho\Delta} \left(L^n_{n+1} + \lambda^n_n G_{n,n+1}\Delta \right) V^n_{\{n\}}(\theta, \emptyset) + \\ &+ e^{-\rho\Delta} \lambda^n_n \gamma_{n,n}(\Delta) \frac{\left(1 - e^{-\rho\Delta}\right) \frac{q_n\theta - r_n}{\rho} + e^{-\rho\Delta} G_{n,n+1}\Delta V^n_{\{n\}}(\theta, \emptyset)}{1 - e^{-\rho\Delta}(1 - G_{n,n+1}\Delta)} = \\ &= \frac{\left(1 - e^{-\rho\Delta}\right) \frac{E^n(q|q \leq q_n)\theta - r_n}{\rho} + e^{-\rho\Delta} \lambda^n_n (1 - G_{n,n+1}\Delta) \left(1 - e^{-\rho\Delta}\right) \frac{q_n\theta - r_n}{\rho}}{1 - e^{-\rho\Delta} \left[L^n_{n+1} + \lambda^n_n G_{n,n+1}\Delta + \lambda^n_n (1 - G_{n,n+1}\Delta) \frac{e^{-\rho\Delta} G_{n,n+1}\Delta}{1 - e^{-\rho\Delta}(1 - G_{n,n+1}\Delta)} \right]}. \end{aligned}$$

Rewrite the denominator as follows:

$$\begin{aligned} &1 - e^{-\rho\Delta} \left[1 - \lambda^n_n + \lambda^n_n G_{n,n+1}\Delta + \lambda^n_n (1 - G_{n,n+1}\Delta) \frac{e^{-\rho\Delta} G_{n,n+1}\Delta}{1 - e^{-\rho\Delta}(1 - G_{n,n+1}\Delta)} \right] = \\ &= 1 - e^{-\rho\Delta} \left[1 - \lambda^n_n (1 - G_{n,n+1}\Delta) + \lambda^n_n (1 - G_{n,n+1}\Delta) \frac{e^{-\rho\Delta} G_{n,n+1}\Delta}{1 - e^{-\rho\Delta}(1 - G_{n,n+1}\Delta)} \right] = \\ &= 1 - e^{-\rho\Delta} \left[1 - \lambda^n_n (1 - G_{n,n+1}\Delta) \left(1 - \frac{e^{-\rho\Delta} G_{n,n+1}\Delta}{1 - e^{-\rho\Delta}(1 - G_{n,n+1}\Delta)} \right) \right] = \\ &= 1 - e^{-\rho\Delta} \left[1 - \lambda^n_n (1 - G_{n,n+1}\Delta) \frac{1 - e^{-\rho\Delta}}{1 - e^{-\rho\Delta}(1 - G_{n,n+1}\Delta)} \right] = \\ &= (1 - e^{-\rho\Delta}) + \frac{e^{-\rho\Delta} \lambda^n_n (1 - G_{n,n+1}\Delta)(1 - e^{-\rho\Delta})}{1 - e^{-\rho\Delta}(1 - G_{n,n+1}\Delta)} = (1 - e^{-\rho\Delta}) \left\{ 1 + \frac{e^{-\rho\Delta} \lambda^n_n (1 - G_{n,n+1}\Delta)}{1 - e^{-\rho\Delta}(1 - G_{n,n+1}\Delta)} \right\} \end{aligned}$$

Therefore, we can write

$$\begin{aligned} V^n_{\{n\}}(\theta,\emptyset) &= (1-w_n)\frac{E^n(q|q\le q_n)\theta-r_n}{\rho}+w_n\frac{q_n\theta-r_n}{\rho}= \\ &= \frac{\left[(1-w_n)E^n(q|q\le q_n)+w_nq_n\right]\theta-r_n}{\rho}, \end{aligned}$$

where

$$\begin{aligned} w_n &= \frac{\frac{e^{-\rho\Delta}\lambda^n_n(1-G_{n,n+1}\Delta)}{1-e^{-\rho\Delta}(1-G_{n,n+1}\Delta)}}{1+\frac{e^{-\rho\Delta}\lambda^n_n(1-G_{n,n+1}\Delta)}{1-e^{-\rho\Delta}(1-G_{n,n+1}\Delta)}}= \\ &= \frac{e^{-\rho\Delta}\lambda^n_n(1-G_{n,n+1}\Delta)}{1-e^{-\rho\Delta}(1-G_{n,n+1}\Delta)+e^{-\rho\Delta}\lambda^n_n(1-G_{n,n+1}\Delta)}. \end{aligned}$$

By Lemma 4, $\liminf_{\Delta\to 0}\lambda^n_n\equiv\Lambda_n>0$, so

$$w_n\ge\frac{e^{-\rho\Delta}\Lambda_n(1-G_{n,n+1}\Delta)}{1-e^{-\rho\Delta}(1-G_{n,n+1}\Delta)+e^{-\rho\Delta}\Lambda_n(1-G_{n,n+1}\Delta)}\to\frac{\Lambda_n}{0+\Lambda_n}=1,$$

i.e. $w_n\to 1$. Now consider rental prices. Clearly, $r_N=r^*_N$; thus, assume that $r_{n+1}\to r^*_{n+1}$ for $n<N$; then $V^n_{\{n\}}(\theta_n,\emptyset)=V^{n+1}_{\{n+1\}}(\theta_n,\emptyset)$ iff

$$\begin{aligned} r_n &= r_{n+1}+\left[(1-w_n)E^n(q|q\le q_n)+w_nq_n-(1-w_{n+1})E^{n+1}(q|q\le q_{n+1})+w_{n+1}q_{n+1}\right]\theta_n\to \\ &\to r^*_{n+1}+(q_n-q_{n+1})\theta^*_n=r^*_n, \end{aligned}$$

because $w_n\to 1$ and $\theta_n\to\theta^*_n$.

6.2 Supply Side under Simple and General Depreciation

The proofs of Theorems 3 and 5 are very similar, except that an additional step is required for the latter. It is thus convenient to present them together.

We begin by describing the realizations of the quality process, or *quality histories.* Recall that a car of quality q_n that depreciates becomes a car of quality q_m with probability $\gamma_{n,m}\Delta$. Also, when a car of quality q_N depreciates, it disappears ("dies"). Thus, we are led to consider quality histories of the form $(q_0,\ldots,q_0,q_1,...,q_1,q_2,...,q_n,...,q_N,...,q_N,0)$, where 0 denotes that the car has died. Formally, let $\mathcal{Q}$ be the set of all finite sequences $\{q^0,...,q^J\}$ such that (i) if $q^0=q_n$, then $\chi_n>0$, and (ii) $q^j\ge q^{j+1}$ for all $j=0,...,J-1$; also, let $\bar{\mathcal{Q}}$ the set of all complete quality histories: that is, $(q^0,...,q^J)\in\bar{\mathcal{Q}}$ iff $(q^0,...,q^J)\in\mathcal{Q}$ and $q^J=0$.

Let Q denote the set of all qualities: that is, $Q=\{q_0,...,q_N,0\}$; then, for all integers m, Q^m denotes the Cartesian product of m copies of Q (in particular, $Q^0=\emptyset$).

Recall that depreciation events occur *at the end of the period.* Therefore, the history $(q^0,...,q^J)$ should be interpreted as follows: q^0 is the initial quality of the car; then, for $j>0$, q^j is the quality of the car in period j, which is determined by the realization of the depreciation process at the end of period $j-1$.

We now describe deviations from the putative equilibrium rental contracts. A deviation consists of a collection of mechanisms, each targeted to a specific type of consumer. We begin by analyzing single mechanisms.

It turns out that, in order to assess whether a deviation is profitable, only certain elements of a mechanism need to be explicitly described. In particular, below we derive upper bounds on the revenues of a deviating firm. These bounds are determined solely by individual rationality considerations, taking into account the fact that equilibrium contracts offered by other firms are always available to consumers. Therefore, we only need a representation of a mechanism that allows us to compute consumers' utility and payment flows.

Moreover, it is technically convenient to analyze a larger set of deviations than would be feasible for a firm operating in the environment described in the main text; in particular, we assume that (i) the deviating firm knows the initial quality of its newly-produced car, and (ii) the firm can ascertain the type of any consumer it transacts with. Since the firm can commit to the contracts it offers, having access to such information can only have a positive effect on revenues; therefore, our upper bound will be valid a fortiori when all informational constraints are taken into account.

We first specify under what circumstances a car may be offered via the mechanism; we do so by indicating a set of *initial quality histories*. The interpretation is that the consumer who enters the mechanism does not necessarily know the previous history of the car, but knows that it belongs to the specified initial set. It is up to the deviating firm to decide how much to reveal to consumers.

Second, we must be able to establish when the car exits the mechanism—either because it dies, or because it is returned to the firm. A specific mechanism will prescribe that certain actions be taken (e.g. the consumer is supposed to keep the car for 3 periods, then return it if the car has depreciated at least once, and otherwise keep it for 2 more periods). These prescriptions and actions determine a set of *final quality histories*; our minimalistic description of a deviation only requires the specification of the latter.

Third, we define revenues. Again, a specific mechanism will prescribe that certain transfers be effected, possibly contingent on the actions taken by the consumer (e.g. the consumer pays a price p upon entering the mechanism; then, if she keeps the car for more than 3 periods, she pays a rental price r for each additional period.) And, again, such specifics are irrelevant for our purposes; we only define a *revenue function* that indicates, for every continuation history that is consistent with some initial history and leads to a final history, the transfer effected by the consumer to the firm.

Finally, we specify a set of *target consumer types* that are allowed to enter the mechanism. We need not describe the specifics of the mechanism that result in only certain types entering the mechanism; as noted above, for the purposes of the present analysis, we simply assume that firms can decide whether or not to transact with her.

Definition 1 *A* ***(reduced-form) mechanism*** *is a tuple* $M = (I, F, R, \Theta)$*, where:*

- $I, F \subset \mathcal{Q}$ *and both sets are nonempty.*
- *If* $(q^0, ..., q^J) \in F$*, then there exists* $j_0 < J$ *such that* $(q^0, ..., q^{j_0}) \in I$*;*

- *If* $(q^0, ..., q^{j_0}) \in I$*, then there exist* $J > j_0$ *and* $\{q^{j_0+1}, ..., q^J\} \in Q^{J-(j_0+1)}$ *such that* $(q^0, ..., q^{j_0}, q^{j_0+1}, ..., q^J) \in F$*;*

- *If* $(q^0, ..., q^{j_0}) \in I$*, then there does not exist* $j_1 > 0$ *and* $\{q^{j_0+1}, ..., q^{j_1}\} \in Q^{j_1-(j_0+1)}$ *such that* $(q^0, ..., q^{j_0}, q^{j_0+1}, ..., q^{j_1}) \in I$*.*

Now define the set of continuation histories

$$H = \left\{(q^{j_0}, ..., q^J) \in Q^{J-j_0+1} : (q^0, ..., q^{j_0}, q^{j_0+1}, ..., q^J) \in F \text{ for some } (q^0, ..., q^{j_0}) \in I\right\}$$

- *If* $(q^{j_0}, ..., q^J) \in H$*, then there is no* $\{q^{J+1}, ..., q^K\} \in Q^{K-J}$ *such that* $(q^{j_0}, ..., q^J, q^{J+1}, ..., q^K) \in H$*.*

- $R : H \to \mathbb{R}$

- $\Theta \subset [\underline{\theta}, \overline{\theta}]$*.*

Suppose that, in period j_0, a consumer enters the mechanism and receives a car characterized by the initial quality history $(q^0, ..., q^{j_0}) \in I$. The consumer does not observe the entire history; however, as soon as she receives the car, she learns q^{j_0}. She then keeps the car until its realized partial history is one of the elements of the set F—say, $(q^0, ..., q^{j_0}, q^{j_0+1}, ..., q^J)$. The consumer then returns the car at the end of period J, and her total payments to the firm from period j_0 through time J are given by $e^{\rho\Delta(J-j_0)}R(q^{j_0}, ..., q^J)$.[25] That is, $R(q^{j_0}, ..., q^J)$ is the discounted value of the transfer at the beginning of period j_0.

The last restriction on initial histories rules out the possibility that both a history and one of its subhistories be elements of I. For instance, the set $\{(0, 0), (0, 0, 1)\}$ violates this restriction. The intuition is that, if $(0, 0)$ is an initial history, then the consumer enters the mechanism in period 2, so $(0, 0, 1)$ could not also be an initial history.

The restriction on continuation histories is a definiteness requirement: the consumer must be able to tell whether a final history has obtained based on what she observes. If $(q^{j_0}, ..., q^J)$ and $(q^{j_0}, ..., q^J, q^{J+1}, ..., q^K)$ were both possible continuation histories, the consumer would not be able to decide whether or not to exit in period J. Thus, we eliminate this possibility.

For example, under *simple* depreciation, the equilibrium mechanism for vintage-1 cars is defined as follows: I consists of all partial histories $(q^0, ..., q^{j_0})$ such that $q^{j_0} = q_1$ and $q^j = q_0$ for all $j < j_0$; F contains all histories $(q^0, ..., q^J)$ such that $q^J = q_2$ and $q^j > q_2$ for $j < J$; H is a set containing all partial histories of the form $(q_1, ..., q_1, q_2)$ (any number of repetitions of q_1) and $R(q^{j_0}, ..., q^J)$ equals $\sum_{j=j_0}^{J-1} \frac{1-e^{-\rho\Delta}}{\rho} r_1 e^{-\rho\Delta(j-j_0)} = \frac{1-e^{-\rho\Delta[(J-1)-j_0]}}{\rho} r_1^*$.

[25] If $(q^{j_0}, ..., q^J)$ is a feasible intermediate partial history, then in particular q^{j_0} is one of the possible initial qualities of the car, i.e. $(q^0, ..., q^{j_0}) \in I$. In other words, the very first observation the consumer makes is the initial quality of the car (which was realized in period $j_0 - 1$).

Also note that, as a consequence of the definition, intermediate partial histories have length at least 2: they contain the initial quality of the car, and the quality resulting from the realization of the depreciation process at the end of the first period of the mechanism. Thus, initial histories can never be complete histories.

We emphasize that this is *not* a complete description of a mechanism and/or of the consumer's optimizing behavior conditional upon entering the mechanism; it is merely a reduced-form representation of those elements that are essential to the analysis.

The initial quality distribution $(q_n, \chi_n : n = 0, \ldots, N+1)$ and depreciation probabilities $\gamma_{n,m}\Delta$ determine a probability distribution $\Pr[\cdot]$ over the set of complete histories $\bar{\mathcal{Q}}$. Certain derived probabilities will now be obtained. First, for any partial history $(q^0, ..., q^j)$,

$$\Pr[(q^0, ..., q^j)] = \Pr[\{(\bar{q}^0, ..., \bar{q}^J) \in \bar{\mathcal{Q}} : \forall j' = 0, ..., j, \; \bar{q}^{j'} = q^{j'}\}].$$

It also makes sense to define conditional probabilities of the following type:

$$\Pr[(q^{j_0+1}, ..., q^j)|(q^0, ..., q^{j_0})] = \frac{\Pr[(q^0, ..., q^{j_0}, q^{j_0+1}, ..., q^j)]}{\Pr[(q^0, ..., q^j)]}.$$

Finally, fix a mechanism $M = (I, F, R, \Theta)$. We are interested in the conditional probability of reaching a mechanism by way of a specific initial history $(q^0, ..., q^{j_0}) \in I$, given that the mechanism is reached in period j_0. Assuming throughout there are histories of such length in I, this probability can be computed as follows:

$$\Pr[(q^0, ..., q^{j_0})|I, j_0] = \frac{\Pr[(q^0, ..., q^{j_0})]}{\Pr[\{(\hat{q}^0, ..., \hat{q}^{j_0}) \in I\}]}$$

A collection of mechanisms that constitute a deviation must be internally consistent. To motivate, consider the following two mechanisms: M_0 is such that $I_0 = \{(q_0)\}$, and the consumer is supposed to keep the car for exactly 3 periods, then return it to the firm regardless of the realization of the depreciation process; M_1 is such that $I_1 = \{(q_0, q_1, q_2)\}$, and the consumer keeps the car until it dies. The pair (M_0, M_1) is not a well-defined deviation, because it does not specify what to do if the car does not depreciate each period. This motivates the following definition.

Definition 2 *A* ***menu*** *is a collection $\mathcal{M}$ of (reduced-form) mechanisms such that, for every partial history $(q^0, \ldots, q^J)$, there is a unique mechanism $M = (I, F, R, \Theta) \in \mathcal{M}$ and period $j_0 \leq J$ such that*

1. *$(q^0, \ldots, q^{j_0}) \in I$;*
2. *for some $J' > J$ and $(q^{J+1}, \ldots, q^{J'}) \in \mathcal{Q}^{J'-(J+1)}$, $(q^0, \ldots, q^J, q^{J+1}, \ldots, q^{J'}) \in F$.*

That is: at any point in the course of the car's quality history, it is always clear which mechanism must be used (or is being used). As noted above, we allow for menus that distinguish between different initial qualities of the deviator's car (more precisely, between histories that differ in their period-zero component, which corresponds to the initial quality of a new car).

Throughout the remainder of this proof, let $V^e(\theta)$ denote the expected payoff to type θ if she uses the putative equilibrium rental contracts and follows the appropriate policy for her; under simple depreciation, for $\theta \in [\theta_n^*, \theta_{n-1}^*]$, $V^e(\theta) = \frac{q_n\theta - r_n^*}{\rho}$; under general depreciation, $\theta \in [\theta_n, \theta_{n-1}]$, $V^e(\theta) = V_{n,\{n\}}(\theta, \emptyset)$. For the sake of notational uniformity, we denote cutoff types for the simple depreciation model by θ_n, etc, suppressing stars.

Suppose that the deviating firm offers a menu $\mathcal{M}$, and consider an arbitrary mechanism $M = (I, F, R, \Theta) \in \mathcal{M}$; the firm's expected revenues from M, if a consumer enters it in period j_0, can be expressed as follows:

$$\begin{aligned} R_{M,j_0} &= \sum_{(q^0,...,q^{j_0})\in I} \Pr[(q^0, ..., q^{j_0})|I, j_0] \\ &\quad \times \sum_{\substack{(q^{j_0+1},...,q^j): \\ (q^0,...,q^{j_0},q^{j_0+1},...,q^j)\in F}} \Pr[(q^{j_0+1}, ..., q^j)|(q^0, ..., q^{j_0})] R(q^{j_0}, ..., q^j). \end{aligned}$$

Suppose that a consumer $\theta \in [\theta_n, \theta_{n-1}]$ is targeted by mechanism M, so $\theta \in \Theta$. Individual rationality then determines an upper bound on her willingness to pay for M in period j_0. Specifically, let $V(\theta)$ denote the value from type θ's best strategy not involving participation in M when she does not have a car; note that this strategy may prescribe participating (at a later date) in some other mechanism offered by the deviator—i.e. it is not necessarily confined to the putative equilibrium rental contracts. Hence, in general, $V(\theta) \geq V^e(\theta)$. In any case, in period j_0, type θ only accepts to participate in the mechanism M if her expected payment does not exceed the difference between (i) the consumption value of entering M in period j_0, then following her best continuation policy when the mechanism terminates, and (ii) the value of adopting her best continuation policy at j_0. This determines the following upper bound on expected revenues from type θ to the deviator:

$$\begin{aligned} \bar{R}_{M,j_0}(\theta) &= -V(\theta) + \sum_{(q^0,...,q^{j_0})\in I} \Pr[(q^0, ..., q^{j_0})|I, j_0] \sum_{\substack{(q^{j_0+1},...,q^j): \\ (q^0,...,q^{j_0},q^{j_0+1},...,q^j)\in F}} \Pr[(q^{j_0+1}, ..., q^j)|(q^0, ..., q^{j_0})] \\ &\quad \times \left(\sum_{k=j_0}^{j-1} e^{-\rho\Delta(k-j_0)} \frac{1-e^{-\rho\Delta}}{\rho} q^k\theta + e^{-\rho\Delta j} V(\theta) \right) \\ &= \sum_{(q^0,...,q^{j_0})\in I} \Pr[(q^0, ..., q^{j_0})|I, j_0] \sum_{\substack{(q^{j_0+1},...,q^j): \\ (q^0,...,q^{j_0},q^{j_0+1},...,q^j)\in F}} \Pr[(q^{j_0+1}, ..., q^j)|(q^0, ..., q^{j_0})] \\ &\quad \times \left(\sum_{k=j_0}^{j-1} e^{-\rho\Delta(k-j_0)} \frac{1-e^{-\rho\Delta}}{\rho} q^k\theta - (1-e^{-\rho\Delta j}) V(\theta) \right). \end{aligned} \tag{25}$$

We now construct a new menu $\mathcal{M}'$ that still satisfies each target consumer's individual rationality constraint, and yields at least as much revenues as $\mathcal{M}$ to the deviating firm. The new menu consists of one-period "rental" contracts, targeted to a single consumer type, wherein the firm fully discloses the history of the car up to the current period; each mechanism in the original menu is replaced by a collection of such one-period rental contracts, and payments are defined so as to leave the target consumer indifferent between taking up the contract (for one period), then reverting to her designated putative equilibrium rental contract, and choosing the latter right away. This implies that any policy involving her designated putative equilibrium contract, as well as any contract made available to her by the deviator, yields exactly the same expected payoff, so the new menu consists of individually rational mechanisms.

Formally, consider an arbitrary $M = (I, F, R, \Theta) \in \mathcal{M}$; for every $\theta \in \Theta$ and partial history $(q^0, \dots, q^{j_0}, \dots, q^j)$ such that (i) $(q^0, \dots, q^{j_0}) \in I$ and (ii) for some $(q^{j+1}, \dots, q^J) \in \mathcal{Q}^{J-(j+1)}$, $(q^0, \dots, q^j, \dots, q^J) \in F$, define a mechanism $M(\theta, q^0, \dots, q^j)$ with $(q^0, \dots, q^j)$ as unique initial history,

$$\{(q^0, \dots, q^j, q^{j+1}) \ : \ \Pr[(q^0, \dots, q^j, q^{j+1}) | (q^0, \dots, q^j)] > 0\}$$

as set of final histories, θ as unique target type, and

$$R_\theta(q^j, q^{j+1}) = \frac{1 - e^{-\rho\Delta}}{\rho} q^j \theta - (1 - e^{-\rho\Delta}) V^e(\theta) \tag{26}$$

as revenue function. It is clear that the collection of mechanisms thus obtained is a menu; note that, in particular, this menu prescribes different contracts for a newly produced car, depending on its initial quality.

To verify individual rationality, observe that, by construction, if a consumer of type θ enters the mechanism $M(\theta, q^0, \dots, q^{j_0})$ [which can happen only in period j_0, following the partial history $(q^0, \dots, q^{j_0})$ of the car offered by the deviator], her per-period payoff is $(1 - e^{-\rho\Delta}) V^e(\theta)$. Hence, her per-period payoff from *any* mechanism offered by the deviator is the same, and of course it coincides with the expected per-period payoff from her designated putative equilibrium rental contracts.

We now verify that the deviator does not lose by offering the menu $\mathcal{M}'$ in lieu of $\mathcal{M}$. Consider any mechanism $M = (I, F, R, \Theta) \in \mathcal{M}$, any initial history $(q^0, \dots, q^{j_0}) \in I$, any final history $(q^0, \dots, q^{j_0}, q^{j_0+1}, \dots, q^j) \in F$ consistent with $(q^0, \dots, q^{j_0})$, and any type θ. Under the menu $\mathcal{M}'$, the firm receives

$$\begin{aligned}
& \sum_{k=j_0}^{j-1} e^{-\rho\Delta(k-j_0)} R_\theta(q^k, q^{k+1}) \\
= \; & \sum_{k=j_0}^{j-1} e^{-\rho\Delta(k-j_0)} \left[\frac{1 - e^{-\rho\Delta}}{\rho} q^k \theta - (1 - e^{-\rho\Delta}) V^e(\theta) \right] \\
= \; & \sum_{k=j_0}^{j-1} e^{-\rho\Delta(k-j_0)} \frac{1 - e^{-\rho\Delta}}{\rho} q^k \theta - (1 - e^{-\rho\Delta}) V^e(\theta) \sum_{k=j_0}^{j-1} e^{-\rho\Delta(k-j_0)} \\
= \; & \sum_{k=j_0}^{j-1} e^{-\rho\Delta(k-j_0)} \frac{1 - e^{-\rho\Delta}}{\rho} q^k \theta - (1 - e^{-\rho\Delta(j-j_0)}) V^e(\theta)
\end{aligned}$$

Notice that this quantity appears in the last line of equation (25); hence, taking conditional expectations over all histories $(q^0, \dots, q^{j_0}, q^{j_0+1}, \dots, q^j) \in F$ consistent with $(q^0, \dots, q^{j_0})$, and then over all $(q^0, \dots, q^{j_0}) \in I$, yields precisely the upper bound $\bar{R}_{M,j_0}(\theta)$ on revenues accruing to the deviating firm from mechanism M if it transacts with type θ beginning in period j_0. Since this is true for all target types and all mechanisms, the new menu $\mathcal{M}'$ yields at least as much revenues as the initial one.

Recall that, in the case of simple depreciation, for $\theta \in [\theta_n, \theta_{n-1}]$, $V^e(\theta) = \frac{q_n\theta - r_n}{\rho}$. In the general depreciation case, Lemma 2 shows that, for every $n = 0, \ldots, N$, $V^e(\theta) = V_{n,\{n\}}(\theta, \emptyset) = (1 - w_n)\frac{E^n(q|q\leq q_n)\theta - r_n}{\rho} + w_n \frac{q_n\theta - r_n}{\rho}$; write this as $\frac{\tilde{q}_n\theta - r_n}{\rho}$, where

$$\tilde{q}_n = (1 - w_n)E^n(q|q \leq q_n) + w_n q_n, \quad n = 0, \ldots, N.$$

It is then possible to rewrite equation (22), which determines the putative equilibrium rental prices $r_0, \ldots, r_N$, as follows: $\tilde{q}_N\theta_N - r_N = 0$, $\tilde{q}_n\theta_n - r_n = \tilde{q}_{n+1}\theta_n - r_{n+1}$ for $n = 0, \ldots, N - 1$. Notice that this is analogous to equation (6), except that the "experimentation-corrected" quantities $\tilde{q}_n$ are used in lieu of the actual ones (note however that $\tilde{q}_N = q_N$). In other words, the simple depreciation case corresponds to setting $w_n = 1$ independently of Δ.

For $n < N$, $r_n = r_{n+1} + \theta_n(\tilde{q}_n - \tilde{q}_{n+1})$, and hence

$$r_n = \tilde{q}_N\theta_N + \sum_{m=n}^{N-1} \theta_m(\tilde{q}_m - \tilde{q}_{m+1}). \tag{27}$$

Considering $\theta \in [\theta_n, \theta_{n-1}]$ and substituting for $V^e(\theta)$ in equation (26) then yields

$$R_\theta(q^j, q^{j+1}) = \frac{1 - e^{-\rho\Delta}}{\rho} q^j\theta - (1 - e^{-\rho\Delta})\frac{\tilde{q}_n\theta - r_n}{\rho} = \frac{1 - e^{-\rho\Delta}}{\rho}\left[(q^j - \tilde{q}_n)\theta + r_n\right].$$

Assume for concreteness that $q^j = q_\ell$, and rewrite the above as

$$R_\theta(q^j, q^{j+1}) = \frac{1 - e^{-\rho\Delta}}{\rho}\left[(\tilde{q}_\ell - \tilde{q}_n)\theta + r_n\right] + \frac{1 - e^{-\rho\Delta}}{\rho}(q_\ell - \tilde{q}_\ell)\theta.$$

We claim first that $(\tilde{q}_\ell - \tilde{q}_n)\theta + r_n \leq r_\ell$. Suppose first that $\ell \leq n$: from equation (27),

$$r_\ell - r_n = \sum_{m=\ell}^{n-1} \theta_m(\tilde{q}_m - \tilde{q}_{m+1}) \geq \theta \sum_{m=\ell}^{n-1} (\tilde{q}_m - \tilde{q}_{m+1}) = (\tilde{q}_\ell - \tilde{q}_n)\theta,$$

because, for $m = \ell, \ldots, n - 1$, $\theta_m \geq \theta_{n-1} \geq \theta \in [\theta_n, \theta_{n-1}]$. If instead $\ell > n$,

$$r_n - r_\ell = \sum_{m=n}^{\ell-1} \theta_m(\tilde{q}_m - \tilde{q}_{m+1}) \leq \theta \sum_{m=n}^{\ell-1} (\tilde{q}_m - \tilde{q}_{m+1}) = (\tilde{q}_n - \tilde{q}_\ell)\theta,$$

because, for $m = n, \ldots, \ell - 1$, $\theta_m \leq \theta_n \leq \theta \in [\theta_n, \theta_{n-1}]$. Therefore, if $q^j = q_\ell$,

$$R_\theta(q^j, q^{j+1}) \leq \frac{1 - e^{-\rho\Delta}}{\rho} r_\ell + \frac{1 - e^{-\rho\Delta}}{\rho}(q_\ell - \tilde{q}_\ell)\theta \leq \frac{1 - e^{-\rho\Delta}}{\rho} r_\ell + \frac{1 - e^{-\rho\Delta}}{\rho}(q_\ell - \tilde{q}_\ell)\bar{\theta}.$$

Therefore, the menu $\mathcal{M}'$ (hence, the original menu $\mathcal{M}$) cannot improve upon the menu consisting of the putative equilibrium rental contracts by more than $\frac{1-e^{-\rho\Delta}}{\rho}\max_n(q_n - \tilde{q}_n)\bar{\theta}$ per period. Under simple depreciation, $\tilde{q}_n = q_n$ for all n, which concludes the proof of Theorem 3.

To complete the proof of Theorem 5, note that the gains from deviating from the putative equilibrium rental contracts cannot exceed the maximum per-period gain times the expected lifetime of the car (i.e. the expected number of periods until the car dies). For every $n = 0, \ldots, N$, the number of periods until a car of quality q_n depreciates is a geometric random variable with parameter $G_{n,n+1}\Delta$, so the expected number of periods until depreciation is $\frac{1}{G_{n,n+1}\Delta}$ (recall that a car depreciates at the end of the period, so if depreciation occurs in the first time period, this means that the car has remained at quality level q_n for one period). We can then argue inductively as follows. Let L_n be the expected lifetime (in periods) of a car of quality q_n, where $n = N+1$ signifies death. Then $L_{N+1} = 0$ and

$$L_n = \frac{1}{G_{n,n+1}\Delta} + \sum_{m=n+1}^{N+1} \frac{\gamma_{n,m}}{G_{n,n+1}} L_m.$$

Thus, if the deviating firm has a car of quality q_n, she cannot improve upon the putative equilibrium menu by more than $\frac{1-e^{-\rho\Delta}}{\rho}\max_\ell(q_\ell - \tilde{q}_\ell)\bar{\theta} \cdot L_n$, where again we let $n = N+1$ signify that the car has already died. We argue that, for all n, $\frac{1-e^{-\rho\Delta}}{\rho}\max_\ell(q_\ell - \tilde{q}_\ell)\bar{\theta} \cdot L_n \to 0$ as $\Delta \to 0$. This is trivially true for $n = N+1$. Thus, assume it is true for $n+1$. Then

$$\begin{aligned}
& \lim_{\Delta\to 0} \frac{1-e^{-\rho\Delta}}{\rho} \max_\ell (q_\ell - \tilde{q}_\ell)\bar{\theta} \cdot L_n \\
= \; & \lim_{\Delta\to 0} \left[\frac{1-e^{-\rho\Delta}}{\rho} \max_\ell (q_\ell - \tilde{q}_\ell)\bar{\theta} \cdot \frac{1}{G_{n,n+1}\Delta} + \sum_{m=n+1}^{N+1} \frac{\gamma_{n,m}}{G_{n,n+1}} \frac{1-e^{-\rho\Delta}}{\rho} \max_\ell (q_\ell - \tilde{q}_\ell)\bar{\theta} \cdot L_m \right] \\
= \; & \lim_{\Delta\to 0} \frac{1-e^{-\rho\Delta}}{\rho G_{n,n+1}\Delta} \max_\ell (q_\ell - \tilde{q}_\ell)\bar{\theta} \\
= \; & \lim_{\Delta\to 0} \frac{1-e^{-\rho\Delta}}{\rho G_{n,n+1}\Delta} \cdot \lim_{\Delta\to 0} \max_\ell (q_\ell - \tilde{q}_\ell)\bar{\theta} = \frac{1}{G_{n,n+1}} \cdot \lim_{\Delta\to 0} \max_\ell (q_\ell - \tilde{q}_\ell)\bar{\theta} = 0,
\end{aligned}$$

because $\tilde{q}_\ell \to q_\ell$ for all ℓ as $\Delta \to 0$. This proves Theorem 5.

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