

Imperfect Information and Optimal Monetary Policy*

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June 2010

Abstract

Most of the analysis of optimal monetary policy is conducted with the Calvo model. This paper studies optimal monetary policy when the slow adjustment of the price level is due to imperfect information by decision-makers in firms. We consider two models: a model with exogenous dispersed information and a rational inattention model. In the model with exogenous dispersed information, complete stabilization of the price level is optimal after aggregate productivity shocks but not after markup shocks. By contrast, in the rational inattention model, complete stabilization of the price level is optimal both after aggregate productivity shocks and after markup shocks. Moreover, in the model with exogenous dispersed information, there is no value from commitment to a future monetary policy. By contrast, in the rational inattention model, there is value from commitment to a future monetary policy because then the private sector can trust the central bank that not paying attention to certain variables is optimal.

JEL: E3, E5, D8.

Keywords: dispersed information, rational inattention, optimal monetary policy

*We thank Fernando Alvarez, Andy Atkeson, Larry Christiano, Jordi Gali, Christian Hellwig, Jonathan Parker, Alessandro Pavan, Pierre-Olivier Weill and seminar participants at CREI, IAE, Maryland, Northwestern and UCLA for helpful comments.

1 Introduction

Most of the analysis of optimal monetary policy is conducted with the Calvo (1983) model. In the Calvo model, prices adjust slowly to shocks because in any given period only an exogenously given fraction of firms can adjust their prices. This paper studies optimal monetary policy in models where prices adjust slowly to shocks because of imperfect information by decision-makers in firms. We consider two models: a model with exogenous dispersed information and a model with rational inattention. In the rational inattention model, decision-makers in firms have limited attention and decide how to allocate their attention.

The motivation for this analysis is threefold. First, conducting monetary policy analysis predominantly with one model may lead to correlated model mistakes. Second, the Calvo (1983) model has difficulties matching the combination of slow and fast adjustment of prices to shocks found in the empirical literature. For example, Christiano, Eichenbaum and Evans (1999), Leeper, Sims and Zha (1996), and Uhlig (2005) find that the price level responds slowly to monetary policy shocks; while Boivin, Giannoni and Mihov (2009) and Maćkowiak, Moench and Wiederholt (2009) find that prices respond very quickly to sector-specific shocks. The Calvo model has difficulties matching this combination of slow and fast adjustment of prices to shocks, whereas certain models with information frictions can match this combination of slow and fast adjustment of prices to shocks. It therefore seems important to understand what these models imply for optimal monetary policy. Third, monetary policy affects the optimal allocation of attention by decision-makers in firms. For example, when due to the central bank's actions the macroeconomic environment becomes more stable, decision-makers in firms will probably pay less attention to the macroeconomy. For these three reasons, it seems useful to study optimal monetary policy in a model with exogenous dispersed information and in a model with rational inattention, and to point out similarities and differences to optimal monetary policy in the Calvo model.

We consider a simple dynamic stochastic general equilibrium (DSGE) model with two types of shocks: aggregate productivity shocks and markup shocks. There is monopolistic competition among firms. Prices may respond slowly to shocks either due to exogenous dispersed information or due to rational inattention by decision-makers in firms. The central bank adjusts the money supply in response to shocks so as to maximize expected utility by the representative household. We study optimal monetary policy under commitment. We obtain the following main results.

In the model with exogenous dispersed information, complete stabilization of the price level is optimal after aggregate productivity shocks but not after markup shocks. By contrast, in the rational inattention model, complete stabilization of the price level is optimal both after aggregate productivity shocks and after markup shocks. Furthermore, in the model with exogenous dispersed information, there is no value from commitment to a future monetary policy. By contrast, in the rational inattention model, there is value from commitment to a future monetary policy because then the private sector can trust the central bank that not paying attention to certain variables is optimal.

This paper is related to four papers that also study optimal monetary policy in models in which price setting firms have imperfect information. First, Ball, Mankiw and Reis (2005) study optimal monetary policy in the sticky information model of Mankiw and Reis (2002). The main difference between their paper and our paper is that in their paper the information structure is exogenous. In particular, in their paper the probability with which firms update their information sets is independent of monetary policy. Second, Adam (2007) studies optimal monetary policy in a model in which firms pay limited attention to aggregate variables. In his model the amount of attention that firms devote to aggregate variables is exogenous; whereas in the rational inattention model presented below the amount of attention that firms devote to aggregate variables is endogenous (and depends on monetary policy). We show that this changes optimal monetary policy in a fundamental way. Third, Lorenzoni (2010) and Angeletos and La’O (2008) study optimal monetary policy in models with dispersed information. In Lorenzoni (2010), price setting firms observe the history of the economy up to the previous period, the sum of aggregate and idiosyncratic productivity, and a noisy public signal about aggregate productivity. There are several differences between his paper and our paper: (i) in his paper the “noise” in the private signal concerning aggregate productivity is idiosyncratic productivity, while in our paper the noise arises from limited attention, (ii) in his paper the information structure is exogenous, while in our paper the information structure is endogenous, and (iii) in his paper the central bank has imperfect information, while we assume that the central bank has perfect information about the state of the economy. We make this assumption to derive the optimal monetary policy response to changes in fundamentals. Afterwards, we study whether the central bank can also implement this optimal monetary policy response with less information. Like in Lorenzoni (2010), agents in Angeletos and La’O (2008) observe the history of the economy

up to the previous period, the sum of aggregate and idiosyncratic productivity, and a noisy public signal about aggregate productivity. In addition, in Angeletos and La'O (2008) agents observe noisy signals concerning endogenous variables with exogenous variance of noise. This creates an informational externality because a stronger response of agents to their private signals makes the signals concerning endogenous variables more informative. Angeletos and La'O (2008) study how this informational externality affects optimal fiscal and monetary policy. In summary, this paper is the first paper that studies optimal monetary policy in a model in which agents choose the attention that they allocate to aggregate variables.

This paper is also related to the literature on the social value of public information, for example, Morris and Shin (2002), Hellwig (2005), and Angeletos and Pavan (2007). In this literature, the main monetary policy question is whether the central bank should provide information about economic fundamentals. We instead ask how the central bank should set the money supply or a nominal interest rate in response to fundamentals. In addition, in the literature on the social value of public information the information structure (i.e., what agents observe) is typically exogenous.

The paper is organized as follows. Section 2 presents the model. Section 3 describes the objective of the central bank. Section 4 states the optimal monetary policy problem under commitment in the model with exogenous dispersed information and in the rational inattention model. Sections 5-7 characterize optimal monetary policy in the two models. Section 5 derives the optimal monetary policy response to aggregate productivity shocks. Section 6 derives the optimal monetary policy response to markup shocks. Section 7 studies the value of commitment. Section 8 concludes.

2 Model

The economy is populated by a representative household, firms and a government.

Household: There is a representative household. The household's preferences in period t over sequences of composite consumption and labor supply $\{C_{t+\tau}, L_{t+\tau}\}_{\tau=0}^{\infty}$ are given by

$$E_t \left[\sum_{\tau=0}^{\infty} \beta^{\tau} \left(\frac{C_{t+\tau}^{1-\gamma} - 1}{1-\gamma} - \frac{L_{t+\tau}^{1+\psi}}{1+\psi} \right) \right], \quad (1)$$

where $C_{t+\tau}$ is composite consumption and $L_{t+\tau}$ is labor supply in period $t + \tau$. The operator E_t is the expectation operator conditioned on the entire history of the economy up to and including period t . The parameter $\beta \in (0, 1)$ is the discount factor. The parameter $\gamma > 0$ is the inverse of

the intertemporal elasticity of substitution and the parameter $\psi \geq 0$ is the inverse of the Frisch elasticity of labor supply. Composite consumption in period t is given by a Dixit-Stiglitz aggregator

$$C_t = \left(\frac{1}{I} \sum_{i=1}^I C_{i,t}^{\frac{1}{1+\Lambda_t}} \right)^{1+\Lambda_t}, \quad (2)$$

where $C_{i,t}$ is consumption of good i in period t . There are I different consumption goods. The elasticity of substitution between consumption goods equals $(1 + 1/\Lambda_t)$ in period t . Since Λ_t will equal the desired markup by firms, we call Λ_t the desired markup. We assume that the log of the desired markup follows a Gaussian first-order autoregressive process

$$\ln(\Lambda_t) = (1 - \rho_\lambda) \ln(\Lambda) + \rho_\lambda \ln(\Lambda_{t-1}) + \nu_t, \quad (3)$$

where the parameter $\Lambda > 0$, the parameter $\rho_\lambda \in [0, 1)$, and the innovation ν_t is *i.i.d.* $N(0, \sigma_\nu^2)$.

The flow budget constraint of the representative household in period t reads

$$M_t + B_t = R_{t-1}B_{t-1} + W_tL_t + D_t - T_t + \left(M_{t-1} - \sum_{i=1}^I P_{i,t-1}C_{i,t-1} \right), \quad (4)$$

where B_{t-1} are the household's holdings of nominal government bonds between period $t-1$ and period t , R_{t-1} is the nominal gross interest rate on those bond holdings, W_t is the nominal wage rate, D_t are nominal aggregate profits, and T_t are nominal lump-sum taxes in period t . The term in brackets on the right-hand side of equation (4) are unspent nominal money balances carried over from period $t-1$ to period t . The household can allocate his pre-consumption wealth in period t (i.e. the right-hand side of equation (4)) between nominal money balances, M_t , and nominal bond holdings, B_t . We assume that the representative household faces the following cash-in-advance constraint in every period

$$\sum_{i=1}^I P_{i,t}C_{i,t} = M_t. \quad (5)$$

Furthermore, the representative household faces a no-Ponzi-scheme condition.

In every period, the representative household chooses a consumption vector, labor supply, nominal money balances, and nominal bond holdings. The representative household takes as given the prices of all consumption goods, the nominal interest rate, the nominal wage rate, nominal aggregate profits, and nominal lump-sum taxes.

Firms: There are I firms. Firm i supplies good i . The technology of firm i is given by

$$Y_{i,t} = A_t L_{i,t}^\alpha, \quad (6)$$

where $Y_{i,t}$ is output and $L_{i,t}$ is labor input of firm i in period t . The parameter $\alpha \in (0, 1]$ is the elasticity of output with respect to labor input. The variable A_t denotes aggregate productivity in period t . The log of aggregate productivity follows a Gaussian first-order autoregressive process

$$\ln(A_t) = \rho_a \ln(A_{t-1}) + \varepsilon_t, \quad (7)$$

where the parameter $\rho_a \in [0, 1)$ and the aggregate technology shock ε_t is *i.i.d.N*($0, \sigma_\varepsilon^2$). The process $\{A_t\}$ is independent of the process $\{\Lambda_t\}$.

Nominal profits of firm i in period t equal

$$(1 + \tau_p) P_{i,t} Y_{i,t} - W_t L_{i,t}, \quad (8)$$

where τ_p is a production subsidy paid by the government.

In every period, each firm sets a price and commits to supply any quantity at that price. The firm takes as given the representative household's composite consumption, the nominal wage rate, and the following price index¹

$$P_t = I^{1+\Lambda_t} \left(\sum_{i=1}^I P_{i,t}^{-\frac{1}{\Lambda_t}} \right)^{-\Lambda_t}. \quad (9)$$

Government: There is a monetary authority and a fiscal authority. The monetary authority commits to set the money supply according to the following rule

$$\ln(M_t^s) = F_t(L) \varepsilon_t + G_t(L) \nu_t, \quad (10)$$

where M_t^s denotes the money supply in period t and $F_t(L)$ and $G_t(L)$ are infinite-order lag polynomials which can depend on t . The last equation simply says that the log of the money supply in period t can be any linear function of the sequence of shocks up to and including period t . We will ask the question which linear function is optimal.

How is money injected into the economy and how does the money market clear? The household can transform any fraction of his pre-consumption wealth in period t into nominal money balances in period t . See equation (4). In equilibrium, the price level and the nominal interest rate will adjust such that the demand for nominal money balances by the representative household will equal the supply of nominal money balances by the monetary authority (i.e. $M_t = M_t^s$).

¹Dixit and Stiglitz (1977), in their original article, also assumed that there is a finite number of goods and that firms take as given the price index. Moreover, it seems to be a good description of the U.S. economy that there is a finite number of physical consumption goods and that firms take the consumer price index as given.

The government budget constraint in period t reads

$$T_t + B_t = R_{t-1}B_{t-1} + \tau_p \left(\sum_{i=1}^I P_{i,t} Y_{i,t} \right). \quad (11)$$

The government has to finance interest on nominal government bonds and the production subsidy. The government can collect lump-sum taxes or issue new nominal government bonds. We assume that the government pursues a Ricardian fiscal policy. In particular, for ease of exposition, we assume that the fiscal authority fixes nominal government bonds at some non-negative level

$$B_t = B \geq 0. \quad (12)$$

We assume that the fiscal authority sets the production subsidy so as to correct, in the non-stochastic steady state, the distortion arising from firms' market power in the goods market. Formally,

$$\tau_p = \Lambda. \quad (13)$$

Alternatively, one could assume that the fiscal authority sets the production subsidy so as to correct fully at each point in time the distortion arising from firms' market power in the goods market. Formally,

$$\tau_{p,t} = \Lambda_t. \quad (14)$$

However, since in the United States fiscal policy has to be approved by Congress while monetary policy decisions by the Federal Reserve are implemented directly, we find it more realistic to assume that the fiscal authority *cannot* adjust the production subsidy quickly while the monetary authority *can* adjust the money supply quickly.

Information: The information set of the price setter of firm i in period t is given by any initial information that the price setter may have as well as the sequence of all signals that the price setter has received up to and including period t

$$\mathcal{I}_{i,t} = \mathcal{I}_{i,-1} \cup \{s_{i,0}, s_{i,1}, \dots, s_{i,t}\}, \quad (15)$$

where $\mathcal{I}_{i,-1}$ is the initial information set of the price setter of firm i in period minus one and $s_{i,t}$ is the signal that the price setter of firm i receives in period t . We assume that, in every period $t \geq 0$, the price setter of firm i receives a two-dimensional signal consisting of noisy signals about

aggregate productivity and the desired markup:

$$s_{i,t} = \begin{pmatrix} \ln(A_t) + \eta_{i,t} \\ \ln(\Lambda_t/\Lambda) + \zeta_{i,t} \end{pmatrix}, \quad (16)$$

where the noise in the signal has the following properties: (i) the stochastic processes $\{\eta_{i,t}\}$ and $\{\zeta_{i,t}\}$ are independent of the stochastic processes $\{A_t\}$ and $\{\Lambda_t\}$, (ii) the stochastic processes $\{\eta_{i,t}\}$ and $\{\zeta_{i,t}\}$ are independent across firms and independent of each other, and (iii) the noise $\eta_{i,t}$ is *i.i.d.N* $(0, \sigma_\eta^2)$ and the noise $\zeta_{i,t}$ is *i.i.d.N* $(0, \sigma_\zeta^2)$. We also assume that the number of firms is sufficiently large so that

$$\frac{1}{I} \sum_{i=1}^I \eta_{i,t} = \frac{1}{I} \sum_{i=1}^I \zeta_{i,t} = 0. \quad (17)$$

Two remarks are in place before we proceed. First, we think of the noise in the signal as being due to limited attention by decision-makers in firms. Therefore, we find it reasonable to assume that the noise in the signal is idiosyncratic and will wash out in the aggregate. Second, the case of noisy signals about aggregate productivity and the desired markup will turn out to be a useful benchmark. Later we will also consider the case of noisy signals about other variables.

We assume that the monetary authority and the representative household have perfect information (i.e. in every period $t \geq 0$, the monetary authority and the representative household know the entire history of the economy up to and including period t). We assume that the monetary authority has perfect information because we are interested in the optimal conduct of monetary policy. We assume that the representative household has perfect information to isolate the implications of imperfect information by price setters for the optimal conduct of monetary policy. Note that the monetary authority, which has perfect information, could announce in every period the entire history of the economy. It is important to point out that this would make no difference so long as we interpret the noise in the signal as arising from limited attention by decision-makers in firms rather than lack of publicly available information.

3 Objective of the central bank

In this section, we state the central bank's objective and we characterize the feasible allocation that maximizes the central bank's objective. We also derive a log-quadratic approximation to the central bank's objective.

We assume that the monetary authority aims to maximize expected utility of the representative household:

$$E \left[\sum_{t=0}^{\infty} \beta^t U(C_t, L_t) \right], \quad (18)$$

where

$$U(C_t, L_t) = \frac{C_t^{1-\gamma} - 1}{1-\gamma} - \frac{L_t^{1+\psi}}{1+\psi}, \quad (19)$$

$$C_t = \left(\frac{1}{I} \sum_{i=1}^I C_{i,t}^{\frac{1}{1+\Lambda_t}} \right)^{1+\Lambda_t}, \quad (20)$$

and

$$L_t = \sum_{i=1}^I \left(\frac{C_{i,t}}{A_t} \right)^{\frac{1}{\alpha}}. \quad (21)$$

Equation (19) is the period utility function, equation (20) is the definition of composite consumption, and equation (21) is the feasibility constraint stating that the representative household has to supply the labor that is required to produce the consumption vector.

By substituting equations (20) and (21) into the period utility function (19), one can express period utility as a function only of the consumption vector in period t and the two exogenous variables A_t and Λ_t . More specifically, it will be convenient to express period utility as a function only of composite consumption in period t , relative consumption of $I - 1$ goods in period t , and A_t and Λ_t . In the following, $\hat{C}_{i,t} = (C_{i,t}/C_t)$ denotes relative consumption of good i in period t . One can write equation (20) as

$$1 = \frac{1}{I} \sum_{i=1}^I \hat{C}_{i,t}^{\frac{1}{1+\Lambda_t}}.$$

Rearranging yields

$$\hat{C}_{I,t} = \left(I - \sum_{i=1}^{I-1} \hat{C}_{i,t}^{\frac{1}{1+\Lambda_t}} \right)^{1+\Lambda_t}. \quad (22)$$

Substituting equations (21) and (22) into the period utility function (19) yields the following expression for expected utility of the representative household:

$$E \left[\sum_{t=0}^{\infty} \beta^t V \left(C_t, \hat{C}_{1,t}, \hat{C}_{2,t}, \dots, \hat{C}_{I-1,t}, A_t, \Lambda_t \right) \right], \quad (23)$$

with

$$\begin{aligned}
& V\left(C_t, \hat{C}_{1,t}, \hat{C}_{2,t}, \dots, \hat{C}_{I-1,t}, A_t, \Lambda_t\right) \\
&= \frac{C_t^{1-\gamma} - 1}{1-\gamma} - \frac{1}{1+\psi} \left(\frac{C_t}{A_t}\right)^{\frac{1}{\alpha}(1+\psi)} \left[\sum_{i=1}^{I-1} \hat{C}_{i,t}^{\frac{1}{\alpha}} + \left(I - \sum_{i=1}^{I-1} \hat{C}_{i,t}^{\frac{1}{1+\Lambda_t}}\right)^{\frac{1}{\alpha}(1+\Lambda_t)} \right]^{1+\psi}. \quad (24)
\end{aligned}$$

Equation (24) gives period utility as a function only of composite consumption in period t , the consumption mix in period t , and the two exogenous variables A_t and Λ_t .

Definition 1 *An efficient allocation in period t is a vector $(C_t, \hat{C}_{1,t}, \hat{C}_{2,t}, \dots, \hat{C}_{I-1,t}) \in \mathbb{R}_{++}^I$ that maximizes expression (24), where $\hat{C}_{i,t} = (C_{i,t}/C_t)$.*

Maximizing expression (24) yields that the unique efficient allocation in period t is

$$C_t^* = \left(\frac{\alpha}{I+1+\psi}\right)^{\frac{1}{\gamma-1+\frac{1}{\alpha}(1+\psi)}} A_t^{\frac{\frac{1}{\alpha}(1+\psi)}{\gamma-1+\frac{1}{\alpha}(1+\psi)}}, \quad (25)$$

and

$$\forall i = 1, 2, \dots, I-1 : \hat{C}_{i,t}^* = 1. \quad (26)$$

The efficient composite consumption in period t is increasing in aggregate productivity in period t . The efficient consumption mix in period t is to consume an equal amount of each good. The efficient allocation in period t does not depend on Λ_t .

In Sections 5-7, we work with a log-quadratic approximation to the central bank's objective (23)-(24). We compute the log-quadratic approximation around the non-stochastic steady state, where a non-stochastic steady state is defined as an equilibrium of the non-stochastic version of the economy with the property that real quantities, relative prices, the nominal interest rate and inflation are constant over time. Let variables without time subscript denote values in the non-stochastic steady state. It is straightforward to show that due to the subsidy (13) we have $C = C^*$ and $\hat{C}_i = \hat{C}_i^*$, that is, in the non-stochastic steady state the equilibrium allocation equals the efficient allocation. Let small variables denote log-deviations from the non-stochastic steady state, that is, $c_t = \ln(C_t/C)$, $\hat{c}_{i,t} = \ln(\hat{C}_{i,t})$, $a_t = \ln(A_t)$ and $\lambda_t = \ln(\Lambda_t/\Lambda)$. Expressing the function V given by equation (24) in terms of log-deviations from the non-stochastic steady state and using

$C = C^*$ and equation (25) yields the following expression for period utility at time t

$$\begin{aligned}
& v(c_t, \hat{c}_{1,t}, \hat{c}_{2,t}, \dots, \hat{c}_{I-1,t}, a_t, \lambda_t) \\
&= \frac{C^{1-\gamma} e^{(1-\gamma)c_t} - 1}{1-\gamma} \\
&\quad - \frac{C^{1-\gamma} e^{\frac{1}{\alpha}(1+\psi)(c_t - a_t)}}{\frac{1}{\alpha}(1+\psi)} \left[\frac{1}{I} \sum_{i=1}^{I-1} e^{\frac{1}{\alpha} \hat{c}_{i,t}} + \frac{1}{I} \left(I - \sum_{i=1}^{I-1} e^{\frac{\hat{c}_{i,t}}{1+\Lambda e^{\lambda_t}}} \right)^{\frac{1}{\alpha}(1+\Lambda e^{\lambda_t})} \right]^{1+\psi}. \quad (27)
\end{aligned}$$

Computing a second-order Taylor approximation to the function v around the origin yields the following proposition.

Proposition 1 (*Objective of the central bank*) *Let v denote the period utility function given by equation (27). Let \tilde{v} denote the second-order Taylor approximation to v at the origin. Let E denote the unconditional expectation operator. Let x_t , z_t and ω_t denote the following vectors*

$$x'_t = \begin{pmatrix} c_t & \hat{c}_{1,t} & \cdots & \hat{c}_{I-1,t} \end{pmatrix}, \quad (28)$$

$$z'_t = \begin{pmatrix} a_t & \lambda_t \end{pmatrix}, \quad (29)$$

$$\omega'_t = \begin{pmatrix} x'_t & z'_t & 1 \end{pmatrix}, \quad (30)$$

and let $\omega_{n,t}$ and $\omega_{k,t}$ denote the n th and k th element of ω_t . Suppose that there exist two constants $\delta < (1/\beta)$ and $\phi \in \mathbb{R}$ such that, for each period $t \geq 0$ and for all n and k ,

$$E |\omega_{n,t} \omega_{k,t}| < \delta^t \phi. \quad (31)$$

Then

$$E \left[\sum_{t=0}^{\infty} \beta^t \tilde{v}(x_t, z_t) \right] - E \left[\sum_{t=0}^{\infty} \beta^t \tilde{v}(x_t^*, z_t) \right] = \sum_{t=0}^{\infty} \beta^t E \left[\frac{1}{2} (x_t - x_t^*)' H (x_t - x_t^*) \right], \quad (32)$$

where the matrix H is given by

$$H = -C^{1-\gamma} \begin{bmatrix} \gamma - 1 + \frac{1}{\alpha}(1+\psi) & 0 & \cdots & \cdots & 0 \\ 0 & 2 \frac{1+\Lambda-\alpha}{I(1+\Lambda)\alpha} & \frac{1+\Lambda-\alpha}{I(1+\Lambda)\alpha} & \cdots & \frac{1+\Lambda-\alpha}{I(1+\Lambda)\alpha} \\ \vdots & \frac{1+\Lambda-\alpha}{I(1+\Lambda)\alpha} & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \frac{1+\Lambda-\alpha}{I(1+\Lambda)\alpha} \\ 0 & \frac{1+\Lambda-\alpha}{I(1+\Lambda)\alpha} & \cdots & \frac{1+\Lambda-\alpha}{I(1+\Lambda)\alpha} & 2 \frac{1+\Lambda-\alpha}{I(1+\Lambda)\alpha} \end{bmatrix}, \quad (33)$$

and the vector x_t^* is given by

$$c_t^* = \frac{\frac{1}{\alpha}(1+\psi)}{\gamma - 1 + \frac{1}{\alpha}(1+\psi)} a_t, \quad (34)$$

and

$$\hat{c}_{i,t}^* = 0. \quad (35)$$

Proof. See Appendix A. ■

After the quadratic approximation to the period utility function (27), the efficient composite consumption in period t is given by equation (34) and the efficient consumption mix in period t is given by equation (35). In addition, the loss in utility in period t in the case of a deviation from the efficient allocation is given by the quadratic form in square brackets on the right-hand side of equation (32). The upper-left element of the matrix H determines the loss in utility in the case of inefficient composite consumption. The lower-right block of the matrix H determines the loss in utility in the case of an inefficient consumption mix. The condition (31) ensures that, in the expression for the expected discounted sum of period utility, after the quadratic approximation to the period utility function (27), one can change the order of integration and summation and the infinite sum converges.

4 Statement of the optimal policy problem

In this section, we state the central bank's optimal policy problem under commitment. We state the problem for the economy described in Section 2 ("exogenous dispersed information") and for an economy that is identical apart from the fact that price setters in firms choose the precision of the signal (16) subject to a cost function ("rational inattention"). In the rational inattention model, we start from the assumption that price setters make their information choice after the central bank has committed to a policy and the policy has become common knowledge.²

4.1 Exogenous dispersed information

When the central bank can commit, the central bank solves

$$\max_{\{F_t(L), G_t(L)\}_{t=0}^{\infty}} E \left[\sum_{t=0}^{\infty} \beta^t V \left(C_t, \hat{C}_{1,t}, \hat{C}_{2,t}, \dots, \hat{C}_{I-1,t}, A_t, \Lambda_t \right) \right], \quad (36)$$

²We think it would be interesting to study the firms' incentive to learn the central bank's policy choice.

subject to (i) the household's optimality conditions

$$P_t C_t = M_t, \quad (37)$$

$$C_{i,t} = \left(\frac{P_{i,t}}{\frac{1}{I} P_t} \right)^{-\left(1 + \frac{1}{\Lambda_t}\right)} C_t, \quad (38)$$

$$P_t = I^{1+\Lambda_t} \left(\sum_{i=1}^I P_{i,t}^{-\frac{1}{\Lambda_t}} \right)^{-\Lambda_t}, \quad (39)$$

$$\frac{W_t}{P_t} = \frac{L_t^\psi}{C_t^{-\gamma}}, \quad (40)$$

(ii) the firms' optimality conditions and information sets

$$E \left[- (1 + \tau_p) \frac{1}{\Lambda_t} P_{i,t}^{-\frac{1}{\Lambda_t}} \left(\frac{1}{I} P_t \right)^{\frac{1+\Lambda_t}{\Lambda_t}} C_t + \frac{1 + \Lambda_t}{\Lambda_t} W_t \frac{1}{\alpha} \left[\frac{\left(\frac{P_{i,t}}{\frac{1}{I} P_t} \right)^{-\frac{1+\Lambda_t}{\Lambda_t}} C_t}{A_t} \right]^{\frac{1}{\alpha}} \middle| \mathcal{I}_{i,t} \right] = 0, \quad (41)$$

$$\mathcal{I}_{i,t} = \mathcal{I}_{i,-1} \cup \{s_{i,0}, s_{i,1}, \dots, s_{i,t}\}, \quad (42)$$

$$s_{i,t} = \begin{pmatrix} \ln(A_t) + \eta_{i,t} \\ \ln(\Lambda_t/\Lambda) + \zeta_{i,t} \end{pmatrix}, \quad (43)$$

(iii) the labor market clearing condition

$$L_t = \sum_{i=1}^I \left(\frac{C_{i,t}}{A_t} \right)^{\frac{1}{\alpha}}, \quad (44)$$

(iv) the laws of motion for aggregate productivity and the desired markup

$$\ln(A_t) = \rho_a \ln(A_{t-1}) + \varepsilon_t, \quad (45)$$

$$\ln(\Lambda_t/\Lambda) = \rho_\lambda \ln(\Lambda_{t-1}/\Lambda) + \nu_t, \quad (46)$$

and (v) the equation for the money supply

$$\ln(M_t) = F_t(L) \varepsilon_t + G_t(L) \nu_t. \quad (47)$$

The function V in objective (36) is given by equation (24), $F_t(L)$ and $G_t(L)$ in equation (47) are infinite-order lag polynomials that can depend on t , and the innovations ε_t , ν_t , $\eta_{i,t}$ and $\zeta_{i,t}$ have the properties described in Section 2.

A log-quadratic approximation of objective (36) around the non-stochastic steady state and a log-linear approximation of the equilibrium conditions (37)-(41) and (44) around the non-stochastic steady state yields the following linear-quadratic optimal policy problem

$$\max_{\{F_t(L), G_t(L)\}_{t=0}^{\infty}} - \sum_{t=0}^{\infty} \beta^t \frac{C^{1-\gamma}}{2} E \left[\begin{array}{l} \left(\gamma - 1 + \frac{1+\psi}{\alpha} \right) (c_t - c_t^*)^2 \\ + \frac{1+\Lambda-\alpha}{I(1+\Lambda)\alpha} \sum_{i=1}^{I-1} \left(\hat{c}_{i,t}^2 + \hat{c}_{i,t} \sum_{k=1}^{I-1} \hat{c}_{k,t} \right) \end{array} \right], \quad (48)$$

subject to

$$p_t + c_t = m_t, \quad (49)$$

$$c_{i,t} = - \left(1 + \frac{1}{\Lambda} \right) (p_{i,t} - p_t) + c_t, \quad (50)$$

$$p_t = \frac{1}{I} \sum_{i=1}^I p_{i,t}, \quad (51)$$

$$w_t - p_t = \psi l_t + \gamma c_t, \quad (52)$$

$$p_{i,t} = E [p_{i,t}^* | \mathcal{I}_{i,t}], \quad (53)$$

$$p_{i,t}^* = p_t + \frac{1}{1 + \frac{1-\alpha}{\alpha} \frac{1+\Lambda}{\Lambda}} (w_t - p_t) + \frac{\frac{1-\alpha}{\alpha}}{1 + \frac{1-\alpha}{\alpha} \frac{1+\Lambda}{\Lambda}} c_t - \frac{\frac{1}{\alpha}}{1 + \frac{1-\alpha}{\alpha} \frac{1+\Lambda}{\Lambda}} a_t + \frac{\frac{\Lambda}{1+\Lambda}}{1 + \frac{1-\alpha}{\alpha} \frac{1+\Lambda}{\Lambda}} \lambda_t, \quad (54)$$

and

$$l_t = \frac{1}{I} \sum_{i=1}^I \frac{1}{\alpha} (c_{i,t} - a_t), \quad (55)$$

where c_t^* is given by equation (34), $\mathcal{I}_{i,t}$ is given by equations (42)-(43), and a_t , λ_t and m_t are given by equations (45)-(47).

The linear-quadratic optimal policy problem (48)-(55) can be stated more concisely. After substituting equations (50)-(51) into objective (48) and after substituting equations (50)-(52) and equation (55) into equation (54), the linear-quadratic optimal policy problem reads

$$\max_{\{F_t(L), G_t(L)\}_{t=0}^{\infty}} - \sum_{t=0}^{\infty} \beta^t \frac{C^{1-\gamma}}{2} E \left[\begin{array}{l} \left(\gamma - 1 + \frac{1+\psi}{\alpha} \right) (c_t - c_t^*)^2 \\ + \frac{1+\Lambda-\alpha}{(1+\Lambda)\alpha} \left(1 + \frac{1}{\Lambda} \right)^2 \frac{1}{I} \sum_{i=1}^I (p_{i,t} - p_t)^2 \end{array} \right], \quad (56)$$

subject to

$$c_t = m_t - p_t, \quad (57)$$

$$p_t = \frac{1}{I} \sum_{i=1}^I p_{i,t}, \quad (58)$$

$$p_{i,t} = E [p_{i,t}^* | \mathcal{I}_{i,t}], \quad (59)$$

$$p_{i,t}^* = p_t + \phi_c c_t - \phi_a a_t + \phi_\lambda \lambda_t, \quad (60)$$

where

$$c_t^* = \frac{\phi_a}{\phi_c} a_t, \quad (61)$$

$$\mathcal{I}_{i,t} = \mathcal{I}_{i,-1} \cup \{s_{i,0}, s_{i,1}, \dots, s_{i,t}\}, \quad (62)$$

$$s_{i,t} = \begin{pmatrix} a_t + \eta_{i,t} \\ \lambda_t + \zeta_{i,t} \end{pmatrix}, \quad (63)$$

$$a_t = \rho_a a_{t-1} + \varepsilon_t, \quad (64)$$

$$\lambda_t = \rho_\lambda \lambda_{t-1} + \nu_t, \quad (65)$$

$$m_t = F_t(L) \varepsilon_t + G_t(L) \nu_t, \quad (66)$$

and

$$\phi_c = \frac{\frac{\psi}{\alpha} + \gamma + \frac{1-\alpha}{\alpha}}{1 + \frac{1-\alpha}{\alpha} \frac{1+\Lambda}{\Lambda}}, \quad (67)$$

$$\phi_a = \frac{\frac{\psi}{\alpha} + \frac{1}{\alpha}}{1 + \frac{1-\alpha}{\alpha} \frac{1+\Lambda}{\Lambda}}, \quad (68)$$

$$\phi_\lambda = \frac{\frac{\Lambda}{1+\Lambda}}{1 + \frac{1-\alpha}{\alpha} \frac{1+\Lambda}{\Lambda}}. \quad (69)$$

In Sections 5-6, we characterize the solution to this problem.

4.2 Rational inattention

In the rational inattention model, the precision of the signals (63) is endogenous. We assume that firms choose the precision of the signals after the central bank has committed to a policy. The attention problem of firm i reads

$$\min_{(\sigma_\eta^2, \sigma_\zeta^2) \in \mathbb{R}_+^2} \left\{ \frac{\omega}{2} E \left[(p_{i,t} - p_{i,t}^*)^2 \right] + \mu \kappa \right\}, \quad (70)$$

subject to

$$p_{i,t}^* = p_t + \phi_c c_t - \phi_a a_t + \phi_\lambda \lambda_t, \quad (71)$$

$$p_{i,t} = E [p_{i,t}^* | \mathcal{I}_{i,t}], \quad (72)$$

$$s_{i,t} = \begin{pmatrix} a_t + \eta_{i,t} \\ \lambda_t + \zeta_{i,t} \end{pmatrix}, \quad (73)$$

and in each period $t \geq 0$,

$$\frac{1}{2} \log_2 \left(\frac{\sigma_{a|s_i}^2}{\sigma_{a|s_i^t}^2} \right) + \frac{1}{2} \log_2 \left(\frac{\sigma_{\lambda|s_i}^2}{\sigma_{\lambda|s_i^t}^2} \right) \leq \kappa. \quad (74)$$

Here the coefficient $\omega > 0$ is a non-linear function of the parameters appearing in the profit function, $\mu \geq 0$ is the per-period marginal cost of attention, and s_i^t is the sequence of signals received by firm i up to and including period t .

5 Optimal policy response to aggregate productivity shocks

In this section, we derive the optimal monetary policy response to aggregate productivity shocks.

5.1 Exogenous dispersed information

To derive the optimal monetary policy response to aggregate productivity shocks, note the following. First, solving for the optimal policy response to aggregate productivity shocks and solving for the optimal policy response to markup shocks are two independent problems. Thus, in this section we can assume without loss of generality that $\lambda_t = 0$ in every period. Second, suppose that price setters have perfect information. Equations (57)-(60) then imply

$$\begin{aligned} c_t &= \frac{\phi_a}{\phi_c} a_t \\ p_{i,t} - p_t &= 0 \\ p_t &= m_t - \frac{\phi_a}{\phi_c} a_t. \end{aligned}$$

Note that when price setters have perfect information, the equilibrium allocation equals the efficient allocation. Equilibrium composite consumption equals efficient composite consumption and the equilibrium consumption mix equals the efficient consumption mix. Furthermore, note that when price setters have perfect information, monetary policy only affects the price level, which has no effect on welfare. Third, suppose that price setters have imperfect information and that the central bank chooses $m_t = \frac{\phi_a}{\phi_c} a_t$. If $m_t = \frac{\phi_a}{\phi_c} a_t$, the profit-maximizing price (60) does not depend on

aggregate productivity and equations (57)-(60) imply

$$\begin{aligned} c_t &= \frac{\phi_a}{\phi_c} a_t \\ p_{i,t} - p_t &= 0 \\ p_t &= 0. \end{aligned}$$

Hence, when price setters have imperfect information, the central bank can replicate one of the equilibria with perfect information: the one with stable prices. Since the allocation associated with this equilibrium is efficient, the central bank can attain the efficient allocation. Fourth, when price setters have imperfect information, no other monetary policy attains the efficient allocation. For any other monetary policy the profit-maximizing price (60) depends on aggregate productivity, implying that firms put some weight on the signal (63), which creates price dispersion. We arrive at the following proposition.

Proposition 2 *Consider the central bank's optimal policy problem (56)-(69). If $\sigma_\eta^2 > 0$, the unique optimal monetary policy response to aggregate productivity shocks is*

$$F_t(L) \varepsilon_t = \frac{\phi_a}{\phi_c} a_t. \tag{75}$$

Under this policy, the price level does not respond to an aggregate productivity shock.

The derivation of this result hopefully also makes clear the limitations and the extensions of this result. If the equilibrium response to aggregate productivity shocks under perfect information was not efficient (e.g., if we had assumed a consumption aggregator with the property that a constant subsidy no longer suffices to correct the distortions due to market power in the goods market), then complete stabilization of the price level in response to an aggregate productivity shock would no longer be optimal. On the other hand, consider any shock with the property that the equilibrium response to this shock under perfect information is efficient. Complete stabilization of the price level in response to this shock is optimal. These examples show that the result stated in Proposition 2 has nothing to do with an aggregate productivity shock being a supply shock rather than a demand shock.

5.2 Rational inattention

The same arguments as in Section 5.1 yield the following proposition.

Proposition 3 Consider the central bank's optimal policy problem (56)-(69) with (70)-(74). If $\mu > 0$, the unique optimal monetary policy response to aggregate productivity shocks is

$$F_t(L) \varepsilon_t = \frac{\phi_a}{\phi_c} a_t. \quad (76)$$

Under this policy, the price level does not respond to an aggregate productivity shock.

6 Optimal policy response to markup shocks

In this section, we study the optimal monetary policy response to markup shocks. We are interested in markup shocks because a markup shock is a simple example of a shock with the property that the response to this shock under perfect information is not efficient. To see this, suppose that price setters have perfect information. Equations (57)-(60) then imply

$$\begin{aligned} c_t &= \frac{\phi_a}{\phi_c} a_t - \frac{\phi_\lambda}{\phi_c} \lambda_t \\ p_{i,t} - p_t &= 0 \\ p_t &= m_t - \left(\frac{\phi_a}{\phi_c} a_t - \frac{\phi_\lambda}{\phi_c} \lambda_t \right). \end{aligned}$$

If $\lambda_t \neq 0$, the equilibrium allocation under perfect information differs from the efficient allocation.

In this section, we assume without loss of generality that $a_t = 0$ in every period.

6.1 Exogenous dispersed information

In this subsection, we study the optimal monetary policy response to markup shocks in the model with exogenous dispersed information. We first solve for the optimal monetary policy analytically in the case of $\rho_\lambda = 0$ and then we solve for the optimal monetary policy numerically in the case of $\rho_\lambda \neq 0$.

Proposition 4 Consider the central bank's optimal monetary policy problem (56)-(69). Suppose $\sigma_\varepsilon^2 = a_{-1} = 0$ and suppose $\rho_\lambda = 0$. Consider policies of the form $G_t(L) \nu_t = g_0 \nu_t$ and equilibria of

the form $p_t = \theta \lambda_t$. The unique equilibrium for given monetary policy is

$$p_t = \frac{\phi_c g_0 + \phi_\lambda}{\phi_c + \frac{\sigma_\zeta^2}{\sigma_\lambda^2}} \lambda_t \quad (77)$$

$$c_t = \frac{\frac{\sigma_\zeta^2}{\sigma_\lambda^2} g_0 - \phi_\lambda}{\phi_c + \frac{\sigma_\zeta^2}{\sigma_\lambda^2}} \lambda_t \quad (78)$$

$$p_{i,t} - p_t = \frac{\phi_c g_0 + \phi_\lambda}{\phi_c + \frac{\sigma_\zeta^2}{\sigma_\lambda^2}} \zeta_{i,t}. \quad (79)$$

If $\sigma_\zeta^2 > 0$, the unique optimal monetary policy is

$$g_0 = \frac{\frac{\gamma-1+\frac{1+\psi}{\alpha}}{(1+\Lambda)\alpha} \left(1+\frac{1}{\Lambda}\right)^2 \phi_\lambda - \phi_c \phi_\lambda}{\frac{\gamma-1+\frac{1+\psi}{\alpha}}{(1+\Lambda)\alpha} \left(1+\frac{1}{\Lambda}\right)^2 \frac{\sigma_\zeta^2}{\sigma_\lambda^2} + \phi_c^2}. \quad (80)$$

Proof. See Appendix B. ■

In the model with exogenous dispersed information, when $\rho_\lambda = 0$, $G_t(L) \nu_t = g_0 \nu_t$ and $p_t = \theta \lambda_t$, complete stabilization of the price level in response to markup shocks is not optimal. We will obtain the opposite result in the rational inattention model.

Next, we solve the central bank's optimal policy problem (56)-(69) numerically in the case of $\rho_\lambda \neq 0$. When we solve the central bank's optimal policy problem (56)-(69) numerically, we turn this infinite-dimensional problem into a finite-dimensional problem by parameterizing the lag polynomial $G_t(L)$ as a lag-polynomial of an ARMA(2,2) process and by restricting $G_t(L)$ to be the same in each period.³ We solve the problem (56)-(69) for the following benchmark parameter values: $\beta = 0.99$, $\gamma = 1$, $\psi = 0$, $\alpha = 2/3$, and $\Lambda = 0.1$.

For comparison, we also solve for the optimal monetary policy response to markup shocks in the Calvo model. In the Calvo model, firms have perfect information, and any given firm can adjust its price in any given period with an exogenous probability equal to δ .⁴

Figure 1 depicts the optimal monetary policy response to a markup shock. The upper panels depict the optimal monetary policy when $\rho_\lambda = 0$. The lower panels depict the optimal monetary policy when $\rho_\lambda = 0.9$. The solid blue lines show the impulse responses of money, output and

³We also worked with the lag-polynomial of an ARMA(3,3) process. We obtained very similar results.

⁴See chapters 6 and 7 in Woodford (2003) for a detailed description of optimal monetary policy in the Calvo model.

inflation at the optimal monetary policy in the imperfect information model. For comparison, the dashed red lines show the impulse responses of money, output and inflation at the optimal monetary policy with commitment in the Calvo model.

We obtain the following results. First, full stabilization of the price level after a markup shock is not the optimal monetary policy in the imperfect information model. Second, whether the optimal monetary policy in the imperfect information model is similar to the optimal monetary policy with commitment in the Calvo model depends on the persistence of the markup shock. For $\rho_\lambda = 0.9$, the two policies appear to be identical, while for $\rho_\lambda = 0$ the two policies are quite different. When $\rho_\lambda = 0$, in the imperfect information model it is optimal for the central bank to respond to the shock only in the period of the shock, whereas in the Calvo model it is optimal for the central bank to commit to respond to the shock also in future periods when the shock no longer affects the desired markup.

6.2 Rational inattention

Proposition 5 *Consider the optimal monetary policy problem (56)-(69) with (70)-(74). Suppose $\sigma_\varepsilon^2 = \rho_a = 0$ and suppose $\sigma_\lambda^2 > 0$ and $\rho_\lambda = 0$. Consider monetary policies of the form $m_t = g_0 \lambda_t$ and equilibria of the form $p_t = \theta \lambda_t$. Assume $\mu > 0$. Define*

$$b \equiv \sqrt{\frac{\omega (\phi_c g_0 + \phi_\lambda)^2 \sigma_\lambda^2 \ln(2)}{\mu}}. \quad (81)$$

First, we characterize the set of equilibria for given monetary policy $g_0 \in \mathbb{R}$. If and only if $b \leq 1$, there exists an equilibrium with

$$\kappa^* = 0. \quad (82)$$

If and only if $\phi_c \in (0, \frac{1}{2}]$ and $b \geq \sqrt{4\phi_c(1-\phi_c)}$ or $\phi_c > \frac{1}{2}$ and $b \geq 1$, there exists an equilibrium with

$$\kappa^* = \log_2 \left(\frac{b + \sqrt{b^2 - 4\phi_c(1-\phi_c)}}{2\phi_c} \right). \quad (83)$$

If and only if $\phi_c \in (0, \frac{1}{2}]$ and $b \in [\sqrt{4\phi_c(1-\phi_c)}, 1]$, there exists an equilibrium with

$$\kappa^* = \log_2 \left(\frac{b - \sqrt{b^2 - 4\phi_c(1-\phi_c)}}{2\phi_c} \right). \quad (84)$$

Furthermore, in equilibrium

$$p_t = \frac{(\phi_c g_0 + \phi_\lambda)(1 - 2^{-2\kappa^*})}{1 - (1 - \phi_c)(1 - 2^{-2\kappa^*})} \lambda_t, \quad (85)$$

$$c_t = \left[g_0 - \frac{(\phi_c g_0 + \phi_\lambda)(1 - 2^{-2\kappa^*})}{1 - (1 - \phi_c)(1 - 2^{-2\kappa^*})} \right] \lambda_t, \quad (86)$$

and

$$E \left[(p_{i,t} - p_t)^2 \right] = \frac{\mu}{\omega \ln(2)} \left(1 - 2^{-2\kappa^*} \right). \quad (87)$$

Second, we characterize optimal monetary policy. If $\phi_c \in [\frac{1}{2}, \infty)$, there exists a unique equilibrium for any policy $g_0 \in \mathbb{R}$ and the unique optimal monetary policy is

$$g_0^* = \begin{cases} 0 & \text{if } \frac{\omega \phi_\lambda^2 \sigma_\lambda^2 \ln(2)}{\mu} \leq 1 \\ -\frac{\phi_\lambda}{\phi_c} + \frac{1}{\phi_c} \sqrt{\frac{\mu}{\omega \sigma_\lambda^2 \ln(2)}} & \text{if } \frac{\omega \phi_\lambda^2 \sigma_\lambda^2 \ln(2)}{\mu} > 1 \end{cases}. \quad (88)$$

At the optimal monetary policy, firms devote no attention to variation in the desired markup, $\kappa^* = 0$, and thus the price level does not respond to markup shocks.

Proof. See Appendix C. ■

Figure 2 depicts an example. This example illustrates how the optimal monetary policy, g_0^* , the price setters' equilibrium attention at the optimal monetary policy, κ^* , and welfare vary with μ/ω , where $\mu \geq 0$ is the price setters' per-period marginal cost of attention and ω appears in the price setters' objective (70). At the optimal monetary policy, price setters devote no attention to markup shocks and therefore prices do not respond to markup shocks. This result holds for any strictly positive cost of attention. The cost of attention can be arbitrarily small so long as it is not zero.

7 The value of commitment

There is value of commitment in the Calvo model when there are markup shocks, $\sigma_\lambda^2 > 0$. By contrast, there is no value of commitment in the model with exogenous dispersed information because in that model future monetary policy simply does not affect today's price setting decisions. Finally, there is value of commitment in the rational inattention model when there are markup shocks, $\sigma_\lambda^2 > 0$, but the value of commitment in the rational inattention model is qualitatively

different from the value of commitment in the Calvo model. In the rational inattention model, there is value of commitment to a future monetary policy because then the private sector can trust the central bank that not paying attention to variation in the desired markup is optimal.

8 Conclusion

This paper studies optimal monetary policy in a model with exogenous dispersed information and in a rational inattention model. In the model with exogenous dispersed information, complete stabilization of the price level is optimal after aggregate productivity shocks but not after markup shocks. By contrast, in the rational inattention model, complete stabilization of the price level is optimal both after aggregate productivity shocks and after markup shocks. Furthermore, in the model with exogenous dispersed information, there is no value from commitment to a future monetary policy, while in the rational inattention model there is value from commitment to a future monetary policy because then the private sector can trust the central bank that not paying attention to certain variables is optimal.

A Proof of Proposition 1

First, we introduce notation. Let x_t denote the vector of all arguments of the function v given by equation (27) that are endogenous variables:

$$x'_t = \begin{pmatrix} c_t & \hat{c}_{1,t} & \cdots & \hat{c}_{I-1,t} \end{pmatrix}. \quad (89)$$

Let z_t denote the vector of all arguments of the function v given by equation (27) that are exogenous variables:

$$z'_t = \begin{pmatrix} a_t & \lambda_t \end{pmatrix}. \quad (90)$$

Second, we compute a log-quadratic approximation to the expression for the expected discounted sum of period utility (23). Let \tilde{v} denote the second-order Taylor approximation to v at the non-stochastic steady state. We have

$$\begin{aligned} & E \left[\sum_{t=0}^{\infty} \beta^t \tilde{v}(x_t, z_t) \right] \\ = & E \left[\sum_{t=0}^{\infty} \beta^t \left(v(0, 0) + h'_x x_t + h'_z z_t + \frac{1}{2} x'_t H_x x_t + x'_t H_{xz} z_t + \frac{1}{2} z'_t H_z z_t \right) \right], \end{aligned} \quad (91)$$

where h_x is the vector of first derivatives of v with respect to x_t evaluated at the non-stochastic steady state, h_z is the vector of first derivatives of v with respect to z_t evaluated at the non-stochastic steady state, H_x is the matrix of second derivatives of v with respect to x_t evaluated at the non-stochastic steady state, H_z is the matrix of second derivatives of v with respect to z_t evaluated at the non-stochastic steady state, and H_{xz} is the matrix of second derivatives of v with respect to x_t and z_t evaluated at the non-stochastic steady state. Third, we rewrite equation (91) using condition (31). Let ω_t denote the following vector

$$\omega'_t = \begin{pmatrix} x'_t & z'_t & 1 \end{pmatrix}. \quad (92)$$

Let $\omega_{n,t}$ and $\omega_{k,t}$ denote the n th and k th element of ω_t . Condition (31) implies that

$$\sum_{t=0}^{\infty} \beta^t E \left| v(0, 0) + h'_x x_t + h'_z z_t + \frac{1}{2} x'_t H_x x_t + x'_t H_{xz} z_t + \frac{1}{2} z'_t H_z z_t \right| < \infty. \quad (93)$$

It follows that one can change the order of integration and summation on the right-hand side of equation (91):

$$\begin{aligned} & E \left[\sum_{t=0}^{\infty} \beta^t \tilde{v}(x_t, z_t) \right] \\ &= \sum_{t=0}^{\infty} \beta^t E \left[v(0, 0) + h'_x x_t + h'_z z_t + \frac{1}{2} x'_t H_x x_t + x'_t H_{xz} z_t + \frac{1}{2} z'_t H_z z_t \right]. \end{aligned} \quad (94)$$

See Rao (1973), p. 111. Condition (31) also implies that the infinite sum on the right-hand side of equation (94) converges to an element in \mathbb{R} . Fourth, we define the vector x_t^* . In each period $t \geq 0$, the vector x_t^* is defined by

$$h_x + H_x x_t^* + H_{xz} z_t = 0. \quad (95)$$

We will show below that H_x is an invertible matrix. Therefore, one can write the last equation as

$$x_t^* = -H_x^{-1} h_x - H_x^{-1} H_{xz} z_t. \quad (96)$$

Hence, x_t^* is uniquely determined and the vector ω_t with $x_t = x_t^*$ satisfies condition (31). Fifth, equation (94) implies that

$$\begin{aligned} & E \left[\sum_{t=0}^{\infty} \beta^t \tilde{v}(x_t, z_t) \right] - E \left[\sum_{t=0}^{\infty} \beta^t \tilde{v}(x_t^*, z_t) \right] \\ &= \sum_{t=0}^{\infty} \beta^t E \left[h'_x (x_t - x_t^*) + \frac{1}{2} x'_t H_x x_t - \frac{1}{2} x_t^{*'} H_x x_t^* + (x_t - x_t^*)' H_{xz} z_t \right]. \end{aligned} \quad (97)$$

Using equation (95) to substitute for $H_{xz} z_t$ in the last equation and rearranging yields

$$\begin{aligned} & E \left[\sum_{t=0}^{\infty} \beta^t \tilde{v}(x_t, z_t) \right] - E \left[\sum_{t=0}^{\infty} \beta^t \tilde{v}(x_t^*, z_t) \right] \\ &= \sum_{t=0}^{\infty} \beta^t E \left[\frac{1}{2} (x_t - x_t^*)' H_x (x_t - x_t^*) \right]. \end{aligned} \quad (98)$$

Sixth, we compute the vector of first derivatives and the matrices of second derivatives appearing in equations (96) and (98). We obtain

$$\begin{aligned} & h_x = 0, \quad (99) \\ & H_x = -C^{1-\gamma} \begin{bmatrix} \gamma - 1 + \frac{1}{\alpha} (1 + \psi) & 0 & \cdots & \cdots & 0 \\ 0 & 2 \frac{1+\Lambda-\alpha}{I(1+\Lambda)\alpha} & \frac{1+\Lambda-\alpha}{I(1+\Lambda)\alpha} & \cdots & \frac{1+\Lambda-\alpha}{I(1+\Lambda)\alpha} \\ \vdots & \frac{1+\Lambda-\alpha}{I(1+\Lambda)\alpha} & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \frac{1+\Lambda-\alpha}{I(1+\Lambda)\alpha} \\ 0 & \frac{1+\Lambda-\alpha}{I(1+\Lambda)\alpha} & \cdots & \frac{1+\Lambda-\alpha}{I(1+\Lambda)\alpha} & 2 \frac{1+\Lambda-\alpha}{I(1+\Lambda)\alpha} \end{bmatrix}, \end{aligned} \quad (100)$$

and

$$H_{xz} = C^{1-\gamma} \begin{bmatrix} \frac{1}{\alpha}(1+\psi) & 0 \\ 0 & 0 \\ \vdots & \vdots \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix}. \quad (101)$$

Seventh, substituting equations (99)-(101) into equation (95) yields the following system of I equations:

$$c_t^* = \frac{\frac{1}{\alpha}(1+\psi)}{\gamma - 1 + \frac{1}{\alpha}(1+\psi)} a_t, \quad (102)$$

and, for all $i = 1, \dots, I - 1$,

$$\hat{c}_{i,t}^* + \sum_{k=1}^{I-1} \hat{c}_{k,t}^* = 0. \quad (103)$$

Finally, we rewrite equation (103). Summing equation (103) over all $i \neq I$ yields

$$\sum_{i=1}^{I-1} \hat{c}_{i,t}^* = 0. \quad (104)$$

Substituting the last equation back into equation (103) yields

$$\hat{c}_{i,t}^* = 0. \quad (105)$$

Collecting equations (98), (100), (102) and (105), we arrive at Proposition 1.

B Proof of Proposition 4

Step 1: Substituting $a_t = 0$, the cash-in-advance constraint $c_t = m_t - p_t$, the monetary policy $m_t = g_0 \lambda_t$, and $p_t = \theta \lambda_t$ into the equation for the profit-maximizing price (60) yields

$$p_{i,t}^* = [(1 - \phi_c) \theta + \phi_c g_0 + \phi_\lambda] \lambda_t.$$

The price set by firm i in period t then equals

$$\begin{aligned} p_{i,t} &= [(1 - \phi_c) \theta + \phi_c g_0 + \phi_\lambda] E[\lambda_t | \mathcal{I}_{i,t}] \\ &= [(1 - \phi_c) \theta + \phi_c g_0 + \phi_\lambda] \frac{\sigma_\lambda^2}{\sigma_\lambda^2 + \sigma_\zeta^2} (\lambda_t + \zeta_{i,t}). \end{aligned}$$

The price level in period t is then given by

$$p_t = [(1 - \phi_c)\theta + \phi_c g_0 + \phi_\lambda] \frac{\sigma_\lambda^2}{\sigma_\lambda^2 + \sigma_\zeta^2} \lambda_t.$$

Thus, the unique rational expectations equilibrium of the form $p_t = \theta \lambda_t$ is given by the solution to the equation

$$\theta = [(1 - \phi_c)\theta + \phi_c g_0 + \phi_\lambda] \frac{\sigma_\lambda^2}{\sigma_\lambda^2 + \sigma_\zeta^2}.$$

Rearranging yields

$$\begin{aligned} \theta &= \frac{(\phi_c g_0 + \phi_\lambda) \frac{\sigma_\lambda^2}{\sigma_\lambda^2 + \sigma_\zeta^2}}{1 - (1 - \phi_c) \frac{\sigma_\lambda^2}{\sigma_\lambda^2 + \sigma_\zeta^2}} \\ &= \frac{\phi_c g_0 + \phi_\lambda}{\phi_c + \frac{\sigma_\zeta^2}{\sigma_\lambda^2}}. \end{aligned}$$

Hence,

$$p_t = \frac{\phi_c g_0 + \phi_\lambda}{\phi_c + \frac{\sigma_\zeta^2}{\sigma_\lambda^2}} \lambda_t \quad (106)$$

$$c_t = \frac{\frac{\sigma_\zeta^2}{\sigma_\lambda^2} g_0 - \phi_\lambda}{\phi_c + \frac{\sigma_\zeta^2}{\sigma_\lambda^2}} \lambda_t \quad (107)$$

$$p_{i,t} - p_t = \frac{\phi_c g_0 + \phi_\lambda}{\phi_c + \frac{\sigma_\zeta^2}{\sigma_\lambda^2}} \zeta_{i,t}. \quad (108)$$

Step 2: Substituting equations (107)-(108), equation (61) and $a_t = 0$ into the central bank's objective (56) yields

$$-\frac{1}{1 - \beta} \frac{C^{1-\gamma}}{2} \left[\begin{aligned} & \left(\gamma - 1 + \frac{1+\psi}{\alpha} \right) \left(\frac{\frac{\sigma_\zeta^2}{\sigma_\lambda^2} g_0 - \phi_\lambda}{\phi_c + \frac{\sigma_\zeta^2}{\sigma_\lambda^2}} \right)^2 \sigma_\lambda^2 \\ & + \frac{1+\Lambda-\alpha}{(1+\Lambda)\alpha} \left(1 + \frac{1}{\Lambda} \right)^2 \left(\frac{\phi_c g_0 + \phi_\lambda}{\phi_c + \frac{\sigma_\zeta^2}{\sigma_\lambda^2}} \right)^2 \sigma_\zeta^2 \end{aligned} \right].$$

If $\sigma_\zeta^2 > 0$, the g_0 that maximizes this expression is

$$g_0 = \frac{\frac{\gamma-1+\frac{1+\psi}{\alpha}}{(1+\Lambda)\alpha} \left(1 + \frac{1}{\Lambda} \right)^2 \phi_\lambda - \phi_c \phi_\lambda}{\frac{\gamma-1+\frac{1+\psi}{\alpha}}{(1+\Lambda)\alpha} \left(1 + \frac{1}{\Lambda} \right)^2 \frac{\sigma_\zeta^2}{\sigma_\lambda^2} + \phi_c^2}. \quad (109)$$

C Proof of Proposition 5

Step 1: Substituting $a_t = 0$, the cash-in-advance constraint $c_t = m_t - p_t$, the monetary policy $m_t = g_0 \lambda_t$, and $p_t = \theta \lambda_t$ into the equation for the profit-maximizing price (60) yields

$$p_{i,t}^* = [(1 - \phi_c) \theta + \phi_c g_0 + \phi_\lambda] \lambda_t.$$

The attention problem of firm i reads

$$\min_{\kappa \in \mathbb{R}_+} \left\{ \frac{\omega}{2} E \left[(p_{i,t} - p_{i,t}^*)^2 \right] + \mu \kappa \right\},$$

subject to

$$p_{i,t}^* = [(1 - \phi_c) \theta + \phi_c g_0 + \phi_\lambda] \lambda_t,$$

$$p_{i,t} = E [p_{i,t}^* | s_{\lambda,i,t}],$$

$$s_{\lambda,i,t} = \lambda_t + \zeta_{i,t},$$

and

$$\frac{1}{2} \log_2 \left(\frac{\sigma_\lambda^2}{\sigma_{\lambda|s_\lambda}^2} \right) \leq \kappa.$$

The solution to this attention problem is

$$\kappa^* = \begin{cases} \frac{1}{2} \log_2 \left(\frac{\omega [(1 - \phi_c) \theta + \phi_c g_0 + \phi_\lambda]^2 \sigma_\lambda^2}{\frac{\mu}{\ln(2)}} \right) & \text{if } \frac{\omega [(1 - \phi_c) \theta + \phi_c g_0 + \phi_\lambda]^2 \sigma_\lambda^2}{\frac{\mu}{\ln(2)}} \geq 1 \\ 0 & \text{otherwise} \end{cases}. \quad (110)$$

The price set by firm i in period t then equals

$$\begin{aligned} p_{i,t} &= [(1 - \phi_c) \theta + \phi_c g_0 + \phi_\lambda] E [\lambda_t | s_{\lambda,i,t}] \\ &= [(1 - \phi_c) \theta + \phi_c g_0 + \phi_\lambda] \frac{\sigma_\lambda^2}{\sigma_\lambda^2 + \sigma_\zeta^2} (\lambda_t + \zeta_{i,t}) \\ &= [(1 - \phi_c) \theta + \phi_c g_0 + \phi_\lambda] \left(1 - 2^{-2\kappa^*} \right) (\lambda_t + \zeta_{i,t}), \end{aligned} \quad (111)$$

where

$$\frac{\sigma_\lambda^2}{\sigma_\zeta^2} = 2^{2\kappa^*} - 1. \quad (112)$$

The price level and composite consumption in period t are then given by

$$p_t = [(1 - \phi_c) \theta + \phi_c g_0 + \phi_\lambda] \left(1 - 2^{-2\kappa^*} \right) \lambda_t, \quad (113)$$

and

$$c_t = \left[g_0 - [(1 - \phi_c)\theta + \phi_c g_0 + \phi_\lambda] (1 - 2^{-2\kappa^*}) \right] \lambda_t. \quad (114)$$

Thus, the set of rational expectations equilibria of the form $p_t = \theta \lambda_t$ is given by the solutions to the following two equations

$$\theta = [(1 - \phi_c)\theta + \phi_c g_0 + \phi_\lambda] (1 - 2^{-2\kappa^*}), \quad (115)$$

and

$$\kappa^* = \begin{cases} \frac{1}{2} \log_2 \left(\frac{\omega [(1 - \phi_c)\theta + \phi_c g_0 + \phi_\lambda]^2 \sigma_\lambda^2}{\frac{\mu}{\ln(2)}} \right) & \text{if } \frac{\omega [(1 - \phi_c)\theta + \phi_c g_0 + \phi_\lambda]^2 \sigma_\lambda^2}{\frac{\mu}{\ln(2)}} \geq 1 \\ 0 & \text{otherwise} \end{cases}. \quad (116)$$

Step 2: Corner solution. We now study under which conditions there exists a solution to equations (115)-(116) with the property $\kappa^* = 0$. We call this a corner solution. It follows from equation (115) that $\kappa^* = 0$ implies $\theta = 0$. Furthermore, it follows from equation (116) that for $\theta = 0$ we have $\kappa^* = 0$ if and only if

$$\frac{\omega (\phi_c g_0 + \phi_\lambda)^2 \sigma_\lambda^2}{\frac{\mu}{\ln(2)}} \leq 1. \quad (117)$$

Thus, there exists a rational expectations equilibrium of the form $p_t = \theta \lambda_t$ with $\kappa^* = 0$ if and only if condition (117) is satisfied.

Step 3: Interior solutions. Next we study under which conditions there exists a solution to equations (115)-(116) with the property

$$\kappa^* = \frac{1}{2} \log_2 \left(\frac{\omega [(1 - \phi_c)\theta + \phi_c g_0 + \phi_\lambda]^2 \sigma_\lambda^2}{\frac{\mu}{\ln(2)}} \right). \quad (118)$$

We call this an interior solution. Solving equation (115) for θ yields

$$\theta = \frac{(\phi_c g_0 + \phi_\lambda) (1 - 2^{-2\kappa^*})}{1 - (1 - \phi_c) (1 - 2^{-2\kappa^*})}. \quad (119)$$

Substituting the last equation into the equation before yields

$$\kappa^* = \frac{1}{2} \log_2 \left(\frac{\omega \left[\frac{1}{1 - (1 - \phi_c) (1 - 2^{-2\kappa^*})} \right]^2 (\phi_c g_0 + \phi_\lambda)^2 \sigma_\lambda^2}{\frac{\mu}{\ln(2)}} \right). \quad (120)$$

Rearranging this equation yields a quadratic equation in 2^{κ^*}

$$\phi_c x^2 - \sqrt{\frac{\omega(\phi_c g_0 + \phi_\lambda)^2 \sigma_\lambda^2}{\frac{\mu}{\ln(2)}}} x + 1 - \phi_c = 0, \quad (121)$$

where

$$x \equiv 2^{\kappa^*}. \quad (122)$$

An interior solution has to satisfy this quadratic equation as well as: $x \in \mathbb{R}$ and $x \geq 1$. Define

$$b \equiv \sqrt{\frac{\omega(\phi_c g_0 + \phi_\lambda)^2 \sigma_\lambda^2}{\frac{\mu}{\ln(2)}}}. \quad (123)$$

The quadratic equation (121) has two solutions:

$$x_1 = \frac{b + \sqrt{b^2 - 4\phi_c(1 - \phi_c)}}{2\phi_c}, \quad (124)$$

and

$$x_2 = \frac{b - \sqrt{b^2 - 4\phi_c(1 - \phi_c)}}{2\phi_c}. \quad (125)$$

We now check whether these two solutions satisfy $x \in \mathbb{R}$ and $x \geq 1$. First, consider the case of $\phi_c \in (0, \frac{1}{2}]$. Then x_1 and x_2 are real if and only if $b \geq \sqrt{4\phi_c(1 - \phi_c)}$. At $b = \sqrt{4\phi_c(1 - \phi_c)}$, we have $x_1 = x_2 = \sqrt{\frac{1}{\phi_c} - 1} \geq 1$. It is straightforward to show that x_1 is increasing in b and thus $x_1 \geq 1$ for all $b \geq \sqrt{4\phi_c(1 - \phi_c)}$, while x_2 is decreasing in b and $x_2 \geq 1$ for all $b \in [\sqrt{4\phi_c(1 - \phi_c)}, 1]$. Hence, if $\phi_c \in (0, \frac{1}{2}]$, then x_1 is an equilibrium so long as $b \geq \sqrt{4\phi_c(1 - \phi_c)}$, while x_2 is an equilibrium so long as $b \in [\sqrt{4\phi_c(1 - \phi_c)}, 1]$. Second, consider the case of $\phi_c \in (\frac{1}{2}, 1]$. Again x_1 and x_2 are real if and only if $b \geq \sqrt{4\phi_c(1 - \phi_c)}$. At $b = \sqrt{4\phi_c(1 - \phi_c)}$, we have $x_1 = x_2 = \sqrt{\frac{1}{\phi_c} - 1} < 1$. It is straightforward to show that x_1 is increasing in b and $x_1 \geq 1$ for all $b \geq 1$, while x_2 is non-increasing in b and thus $x_2 < 1$ for all $b \geq \sqrt{4\phi_c(1 - \phi_c)}$. Hence, if $\phi_c \in (\frac{1}{2}, 1]$, then x_1 is an equilibrium so long as $b \geq 1$, while x_2 is not an equilibrium. Third, consider the case of $\phi_c > 1$. Then x_1 and x_2 are real for all $b \geq 0$. At $b = 0$, we have $x_1 = \sqrt{1 - \frac{1}{\phi_c}} < 1$ and $x_2 = -\sqrt{1 - \frac{1}{\phi_c}} < 0$. It is straightforward to show that x_1 is increasing in b and $x_1 \geq 1$ for all $b \geq 1$, while $x_2 < 0$ for all $b \geq 0$. Hence, if $\phi_c > 1$, then x_1 is an equilibrium so long as $b \geq 1$, while x_2 is not an equilibrium. In summary, if and only if $\phi_c \in (0, \frac{1}{2}]$ and $b \geq \sqrt{4\phi_c(1 - \phi_c)}$ or $\phi_c > \frac{1}{2}$ and $b \geq 1$, then x_1 is an equilibrium. Furthermore, if and only if $\phi_c \in (0, \frac{1}{2}]$ and $b \in [\sqrt{4\phi_c(1 - \phi_c)}, 1]$, then x_2 is an equilibrium.

Step 4: Consumption variance and price dispersion. We now derive expressions for prices, the price level, composite consumption, the variance of composite consumption and price dispersion that will be useful. Solving equation (115) for θ and substituting this equation for θ into equations (111)-(114) yields

$$p_{i,t} = \frac{(\phi_c g_0 + \phi_\lambda) (1 - 2^{-2\kappa^*})}{1 - (1 - \phi_c) (1 - 2^{-2\kappa^*})} (\lambda_t + \zeta_{i,t}), \quad (126)$$

$$p_t = \frac{(\phi_c g_0 + \phi_\lambda) (1 - 2^{-2\kappa^*})}{1 - (1 - \phi_c) (1 - 2^{-2\kappa^*})} \lambda_t, \quad (127)$$

$$c_t = \left[g_0 - \frac{(\phi_c g_0 + \phi_\lambda) (1 - 2^{-2\kappa^*})}{1 - (1 - \phi_c) (1 - 2^{-2\kappa^*})} \right] \lambda_t, \quad (128)$$

and

$$\frac{\sigma_\lambda^2}{\sigma_\zeta^2} = 2^{2\kappa^*} - 1. \quad (129)$$

Equation (128) implies

$$E [c_t^2] = \left[g_0 - \frac{(\phi_c g_0 + \phi_\lambda) (1 - 2^{-2\kappa^*})}{1 - (1 - \phi_c) (1 - 2^{-2\kappa^*})} \right]^2 \sigma_\lambda^2. \quad (130)$$

Equations (126)-(127) imply

$$E [(p_{i,t} - p_t)^2] = \left[\frac{(\phi_c g_0 + \phi_\lambda) (1 - 2^{-2\kappa^*})}{1 - (1 - \phi_c) (1 - 2^{-2\kappa^*})} \right]^2 \sigma_\zeta^2. \quad (131)$$

In the case of an interior solution, equation (120) holds. Substituting equation (120) into equation (131) yields

$$\begin{aligned} E [(p_{i,t} - p_t)^2] &= \frac{\mu}{\omega \sigma_\lambda^2} 2^{2\kappa^*} (1 - 2^{-2\kappa^*})^2 \sigma_\zeta^2 \\ &= \frac{\mu}{\omega \sigma_\lambda^2} \frac{1}{2^{2\kappa^*}} (2^{2\kappa^*} - 1)^2 \sigma_\zeta^2. \end{aligned} \quad (132)$$

Substituting equation (129) into the last equation yields

$$E [(p_{i,t} - p_t)^2] = \frac{\mu}{\omega} (1 - 2^{-2\kappa^*}). \quad (133)$$

Hence, in the case of an interior solution, price dispersion can be expressed as a simple function of (μ/ω) and the equilibrium attention κ^* . Furthermore, note that equation (133) also holds in the case of a corner solution because $E [(p_{i,t} - p_t)^2] = 0$ at a corner solution.

Step 5: Optimal monetary policy has to satisfy $g_0 \geq -\frac{\phi_\lambda}{\phi_c}$. First, at $g_0 = -\frac{\phi_\lambda}{\phi_c}$, we have $b = 0$ and thus the unique equilibrium is a corner solution. It follows that at $g_0 = -\frac{\phi_\lambda}{\phi_c}$ price dispersion equals zero and consumption variance equals $E[c_t^2] = \left(\frac{\phi_\lambda}{\phi_c}\right)^2 \sigma_\lambda^2$. Second, consider $g_0 < -\frac{\phi_\lambda}{\phi_c}$. Consumption variance at $g_0 < -\frac{\phi_\lambda}{\phi_c}$ is strictly larger than consumption variance at $g_0 = -\frac{\phi_\lambda}{\phi_c}$. When the equilibrium at $g_0 < -\frac{\phi_\lambda}{\phi_c}$ is a corner solution, this follows from the fact that at a corner solution consumption variance equals $E[c_t^2] = g_0^2 \sigma_\lambda^2$. When the equilibrium at $g_0 < -\frac{\phi_\lambda}{\phi_c}$ is an interior solution, this follows from the fact that consumption variance is given by equation (130) and, for all $g_0 < -\frac{\phi_\lambda}{\phi_c}$, we have

$$g_0 - \frac{(\phi_c g_0 + \phi_\lambda)(1 - 2^{-2\kappa^*})}{1 - (1 - \phi_c)(1 - 2^{-2\kappa^*})} < -\frac{\phi_\lambda}{\phi_c}. \quad (134)$$

In addition, price dispersion at $g_0 < -\frac{\phi_\lambda}{\phi_c}$ is weakly larger than price dispersion at $g_0 = -\frac{\phi_\lambda}{\phi_c}$. The reason is that price dispersion is always weakly larger than zero. In summary, a policy $g_0 < -\frac{\phi_\lambda}{\phi_c}$ yields strictly larger consumption variance and weakly larger price dispersion than the policy $g_0 = -\frac{\phi_\lambda}{\phi_c}$. Hence, a policy $g_0 < -\frac{\phi_\lambda}{\phi_c}$ cannot be optimal. This result implies that a policy that makes the price level fall after a positive markup shock cannot be optimal. See equation (127).

Step 6: Uniqueness of equilibrium when $\phi_c \geq \frac{1}{2}$. When $\phi_c \geq \frac{1}{2}$, there exists a unique equilibrium for any policy $g_0 \in \mathbb{R}$. In particular, if $b < 1$ then $\kappa^* = 0$ is the unique equilibrium, if $b = 1$ then $\kappa^* = \log_2(x_1) = 0$ is the unique equilibrium, and if $b > 1$ then $\kappa^* = \log_2(x_1)$ is the unique equilibrium. See steps 2 and 3.

Step 7: Optimal monetary policy when $\phi_c \geq \frac{1}{2}$. First, in the derivation of optimal monetary policy we can focus on $g_0 \geq -\frac{\phi_\lambda}{\phi_c}$, or equivalently $\phi_c g_0 + \phi_\lambda \geq 0$. See step 5. Second, consider the case of $\phi_c \geq \frac{1}{2}$ and $\frac{\omega \phi_\lambda^2 \sigma_\lambda^2 \ln(2)}{\mu} \leq 1$. At the policy $g_0 = 0$, we have $b \leq 1$ and thus $\kappa^* = 0$ is the unique equilibrium. Therefore, at the policy $g_0 = 0$, consumption variance equals $E[c_t^2] = 0$ and price dispersion equals $E[(p_{i,t} - p_t)^2] = 0$. Furthermore, at any policy $g_0 \neq 0$, consumption variance equals $E[c_t^2] > 0$ or price dispersion equals $E[(p_{i,t} - p_t)^2] > 0$. In particular, if the equilibrium at the policy $g_0 \neq 0$ is an equilibrium with $\kappa^* = 0$ then $E[c_t^2] > 0$, while if the equilibrium at the policy $g_0 \neq 0$ is an equilibrium with $\kappa^* > 0$ then $E[(p_{i,t} - p_t)^2] > 0$. See equations (130) and (133). It follows from objective (56) that in the case of $\phi_c \geq \frac{1}{2}$ and $\frac{\omega \phi_\lambda^2 \sigma_\lambda^2 \ln(2)}{\mu} \leq 1$ the unique optimal monetary policy is $g_0^* = 0$. Third, consider the case of $\phi_c \geq \frac{1}{2}$

and $\frac{\omega\phi_\lambda^2\sigma_\lambda^2\ln(2)}{\mu} > 1$. Define \bar{g}_0 as the value of $g_0 \in \left[-\frac{\phi_\lambda}{\phi_c}, \infty\right)$ at which $b = 1$. Formally,

$$\bar{g}_0 = -\frac{\phi_\lambda}{\phi_c} + \frac{1}{\phi_c} \sqrt{\frac{\mu}{\omega\sigma_\lambda^2 \ln(2)}}. \quad (135)$$

Note that $\frac{\omega\phi_\lambda^2\sigma_\lambda^2\ln(2)}{\mu} > 1$ implies $\bar{g}_0 < 0$. Thus, to prevent firms from devoting attention to markup shocks, the central bank has to lower the money supply after a positive markup shock. Which policy is the optimal monetary policy among all policies $g_0 \in \left[-\frac{\phi_\lambda}{\phi_c}, \bar{g}_0\right]$? For all $g_0 \in \left[-\frac{\phi_\lambda}{\phi_c}, \bar{g}_0\right]$, we have $b \leq 1$ and thus $\kappa^* = 0$ is the unique equilibrium. Therefore, for all $g_0 \in \left[-\frac{\phi_\lambda}{\phi_c}, \bar{g}_0\right]$, consumption variance equals $E[c_t^2] = g_0^2\sigma_\lambda^2$ and price dispersion equals $E[(p_{i,t} - p_t)^2] = 0$. It follows from objective (56) that the unique optimal monetary policy among all policies $g_0 \in \left[-\frac{\phi_\lambda}{\phi_c}, \bar{g}_0\right]$ is $g_0 = \bar{g}_0$ because this is the policy that causes the smallest consumption variance. Which policy is the optimal monetary policy among all policies $g_0 \geq \bar{g}_0$? For all $g_0 \geq \bar{g}_0$, we have $b \geq 1$ and thus $\kappa^* = \log_2(x_1)$ is the unique equilibrium. We now show that, when $g_0 \geq -\frac{\phi_\lambda}{\phi_c}$ and $\kappa^* = \log_2(x_1)$, composite consumption is given by a simple expression. Equation (128) states that in equilibrium composite consumption equals

$$c_t = \xi \lambda_t, \quad (136)$$

with

$$\xi = g_0 - \frac{(\phi_c g_0 + \phi_\lambda)(1 - 2^{-2\kappa^*})}{1 - (1 - \phi_c)(1 - 2^{-2\kappa^*})}. \quad (137)$$

Furthermore, when $\kappa^* = \log_2(x_1)$, equation (120) holds. Rearranging equation (120) using $\phi_c g_0 + \phi_\lambda \geq 0$ yields

$$\frac{\phi_c g_0 + \phi_\lambda}{1 - (1 - \phi_c)(1 - 2^{-2\kappa^*})} = \sqrt{\frac{\mu}{\omega\sigma_\lambda^2 \ln(2)}} 2^{\kappa^*}. \quad (138)$$

Substituting the last equation into the equation before yields

$$\xi = g_0 - \sqrt{\frac{\mu}{\omega\sigma_\lambda^2 \ln(2)}} (2^{\kappa^*} - 2^{-\kappa^*}). \quad (139)$$

Using again $\kappa^* = \log_2(x_1)$, we arrive at

$$\xi = g_0 - \sqrt{\frac{\mu}{\omega\sigma_\lambda^2 \ln(2)}} \left(x_1 - \frac{1}{x_1}\right), \quad (140)$$

where

$$x_1 = \frac{b + \sqrt{b^2 - 4\phi_c(1 - \phi_c)}}{2\phi_c}, \quad (141)$$

and

$$b = \sqrt{\frac{\omega(\phi_c g_0 + \phi_\lambda)^2 \sigma_\lambda^2}{\frac{\mu}{\ln(2)}}}. \quad (142)$$

In addition, solving equation (142) for g_0 using $\phi_c g_0 + \phi_\lambda \geq 0$ yields

$$g_0 = -\frac{\phi_\lambda}{\phi_c} + \frac{b}{\phi_c} \sqrt{\frac{\mu}{\omega \sigma_\lambda^2 \ln(2)}}. \quad (143)$$

Substituting equations (141) and (143) into equation (140) yields

$$\xi = -\frac{\phi_\lambda}{\phi_c} + \sqrt{\frac{\mu}{\omega \sigma_\lambda^2 \ln(2)}} \left[\frac{b}{\phi_c} - \frac{b + \sqrt{b^2 - 4\phi_c(1-\phi_c)}}{2\phi_c} + \frac{1}{\frac{b + \sqrt{b^2 - 4\phi_c(1-\phi_c)}}{2\phi_c}} \right]. \quad (144)$$

Finally, rearranging the term in square brackets in equation (144) yields

$$\xi = -\frac{\phi_\lambda}{\phi_c} + \sqrt{\frac{\mu}{\omega \sigma_\lambda^2 \ln(2)}} \left[\frac{2}{b + \sqrt{b^2 - 4\phi_c(1-\phi_c)}} \right]. \quad (145)$$

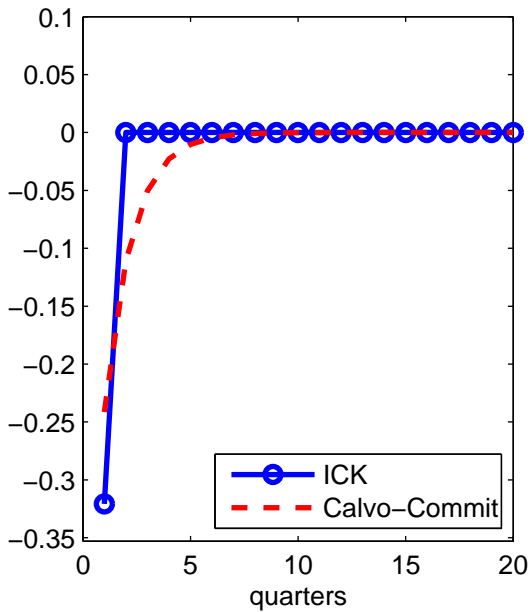
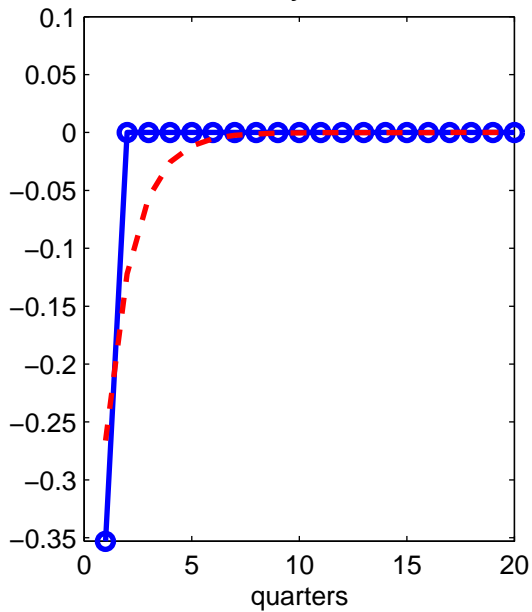
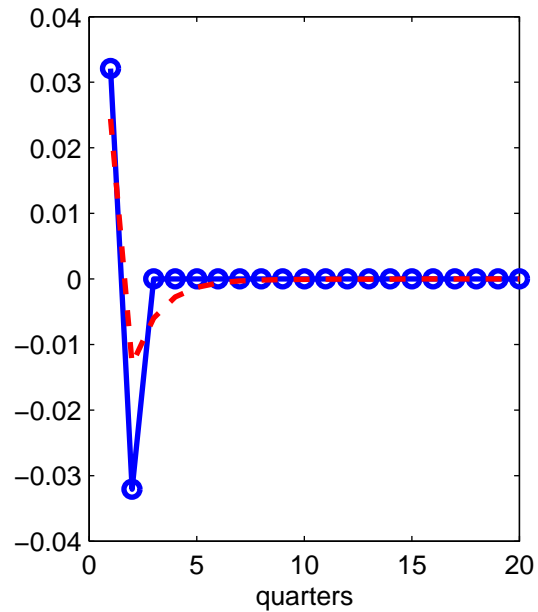
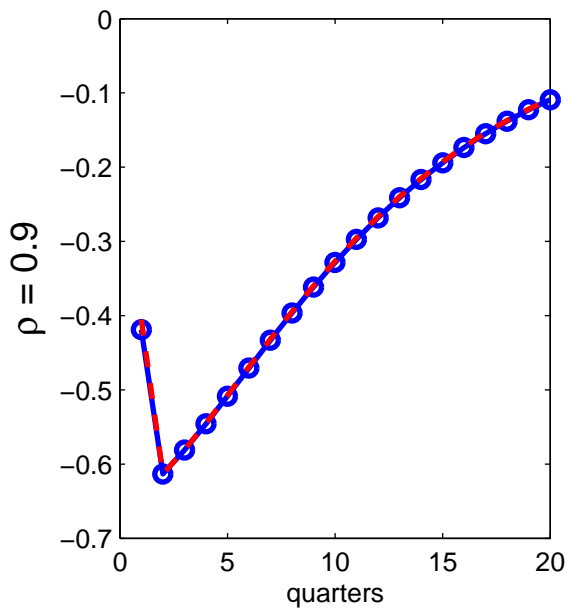
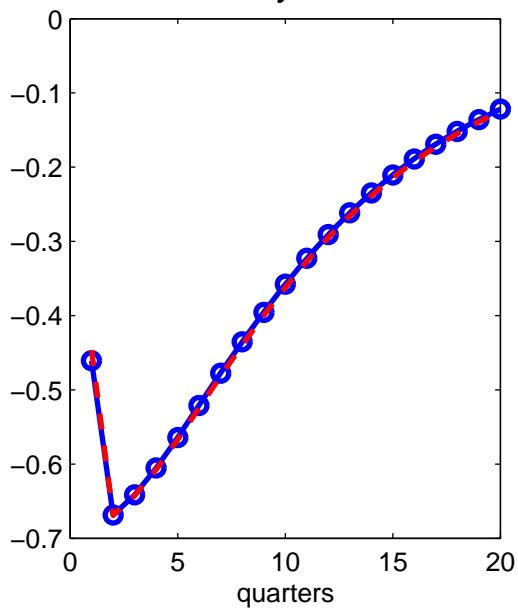
Thus, when $g_0 \geq -\frac{\phi_\lambda}{\phi_c}$ and $\kappa^* = \log_2(x_1)$, composite consumption is given by equations (136) and (145). Note that equation (145) implies: (i) if $\phi_c \geq \frac{1}{2}$ and $\frac{\omega \phi_\lambda^2 \sigma_\lambda^2 \ln(2)}{\mu} > 1$ then $\xi < 0$ at $b = 1$, and (ii) ξ is strictly decreasing in b for all $b \geq 1$. Hence, if $\phi_c \geq \frac{1}{2}$ and $\frac{\omega \phi_\lambda^2 \sigma_\lambda^2 \ln(2)}{\mu} > 1$ then consumption variance $E[c_t^2] = \xi^2 \sigma_\lambda^2$ is strictly increasing in g_0 for all $g_0 \geq \bar{g}_0$. Moreover, equations (133), $\kappa^* = \log_2(x_1)$, (141) and (142) imply that price dispersion $E[(p_{i,t} - p_t)^2]$ is strictly increasing in g_0 for all $g_0 \geq \bar{g}_0$. It follows from objective (56) that in the case of $\phi_c \geq \frac{1}{2}$ and $\frac{\omega \phi_\lambda^2 \sigma_\lambda^2 \ln(2)}{\mu} > 1$, the unique optimal monetary policy among all policies $g_0 \geq \bar{g}_0$ is $g_0 = \bar{g}_0$. Finally, combining results yields that in the case of $\phi_c \geq \frac{1}{2}$ and $\frac{\omega \phi_\lambda^2 \sigma_\lambda^2 \ln(2)}{\mu} > 1$, the unique optimal monetary policy among all policies $g_0 \in \mathbb{R}$ is $g_0 = \bar{g}_0$. In summary, when $\phi_c \geq \frac{1}{2}$, there exists a unique equilibrium for any policy $g_0 \in \mathbb{R}$ and the unique optimal monetary policy is

$$g_0^* = \begin{cases} 0 & \text{if } \frac{\omega \phi_\lambda^2 \sigma_\lambda^2 \ln(2)}{\mu} \leq 1 \\ -\frac{\phi_\lambda}{\phi_c} + \frac{1}{\phi_c} \sqrt{\frac{\mu}{\omega \sigma_\lambda^2 \ln(2)}} & \text{if } \frac{\omega \phi_\lambda^2 \sigma_\lambda^2 \ln(2)}{\mu} > 1 \end{cases}. \quad (146)$$

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