Search with Partially Informed Stopping Decisions

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Abstract

Search models commonly make the simplification that all the relevant information about a sampled object is revealed once it is sampled. In this paper we present and analyze a search model in which searchers have only partial information about the sampled objects at the point of decision. We show that the combination of this imperfection with searchers discretion over which imperfect signal to choose may have a qualitative effect on the equilibrium outcomes. In particular, it generates large multiplicity of equilibria in an environment that would have a unique equilibrium otherwise, and gives rise to equilibria in which stopping decisions might be non-montonic in the strength of the signal that the searcher obtains. These insights owe to the negative search externality that agents exert on each other and which in some form is familiar from other search contexts. But the way in which it plays out here is novel and not immediately obvious.

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1 Introduction

Search models commonly make the simplification that all the relevant information about a sampled object is revealed once it is sampled. In this paper we present and analyze a search model in which searchers have only partial information about the sampled objects at the point of decision. We show that this imperfection may have a qualitative effect on the equilibrium outcomes.

We consider a simple dynamic two-sided search process. On one side, there is a population of searchers; on the other side, a population of objects. Searchers are strategic decision makers who are there to get matched to an object, while objects have no preferences or decisions to make. Upon encountering an object, a searcher observes an imperfect signal of its value. Based on this information, the searcher decides whether to adopt the object and depart or to drop it and continue searching. The equilibrium is a steady state in which each searcher's decisions on adoption are optimal. The steady state means that the outflows of agents and objects who formed a successful match or who "died" (at an exogenous rate) are exactly matched by exogenous inflows of agents and objects.

More specifically, in the main model, an object is characterized by two attributes. Its value for a searcher is the sum of the attributes. The imperfect signal observed by a searcher is the value of one of the attributes. In the benchmark scenario the same attribute (say attribute 1) is observable to all searchers. In this scenario the model has a unique equilibrium and it is of the reservation threshold variety. In a second scenario searchers are free to choose which attribute to observe. In a third scenario searchers again can get information only about a single attribute but they choose at a cost the precision of the signal they observe. In both of these scenarios the model exhibits multiplicity of equilibria and, in the former, some of the equilibria are non-monotonic in the sense that the adoption decision is not everywhere monotonic in the signal observed by the searcher.

In all of these scenarios the equilibrium outcomes are inefficient – they involve too much search or too much information acquisition. This is explained by a negative externality that the searchers exert on each other, which they ignore in their decisions. These search externalities also explain the multiplicity and non-monotonicity mentioned above. Search externalities and their welfare effects are of course familiar from other search models. But the way in which they play out through the endogenous information acquisition decisions in the present model is novel and not immediately obvious.

Our main objective is to expose some possible effects of imperfect information at the decision point in a search environment (rather than to study any particular market). For this reason, we have opted for a lean model that is not guided by the details of a specific application. Nevertheless, since it is useful to have in mind some concrete situation, let us mention that the model captures some of the essential features of competitive search for research and development ideas. The searchers stand for researchers or research units (say, scientists) and the objects are the potential ideas that the researchers sample and examine. The signal observed by a searcher in our model corresponds to preliminary tests conducted prior to starting a research project. The fact that unsuccessful ideas remain available and may be re-examined by different researchers corresponds to the fact that researchers often do not advertise ideas that failed preliminary tests. The "death" and "birth" of objects in our model capture the reality that some potential research topics become obsolete over time, and new ones become relevant. While this model may not fit the research scenario perfectly (e.g., researchers who had been matched to good ideas re-enter or even continue searching), it does capture some of its main features like the one-sidedness, some commonality of values and imperfect observation.

Although our analysis does not apply directly to two-sided with decision makers on both sides[1], it is tempting to speculate that its insights might be relevant in some form for such models as well. This is because the one-sided nature of our model simplifies the calculations but does not seem to play a crucial role in the arguments themselves.

We are not aware of work that bears a close relationship to our analysis. Yet different aspects of the paper are related to different strands of the literature. In terms of its basic model, this paper belongs to the search literature. The basic model (without the information acquisition features) can be thought of as a simple variant of Burdett and Coles (1997). The idea of search with multi-attributes appears in Neeman (1995) and Bar-Isaac, Caruana and Cunat (2011). The relation to our work is more through the general motivation than through the specific analysis. In the former, a searcher faces a sequence of i.i.d. two-attribute objects of which she can observe only one attribute. The main result is that if the distribution of one of the attributes stochastically dominates the distribution of the other in the second order sense, then the optimal search procedure under this limited observability constraint would examine only the dominated attribute. In the latter the focus is on a monopolist's choice of product quality in a world in which consumers are constrained to observe only a subset of the products' attributes.

2 Model

We consider a dynamic two-sided search process. On one side, there is a population of searchers; on the other side, a population of objects. Searchers are strategic decision makers who are there to get matched to an object, while objects have no preferences or decisions to make.

Each object is characterized by two attributes, $(x_1, x_2) \in [0,1]^2$, whose magnitudes are *independent*, and differ across objects. The searcher's payoff from adopting an object characterized by (x_1, x_2) is $u(x_1, x_2) = x_1 + x_2$.

The attributes are not readily observable. Upon encountering an object, the searcher observes a realization of a signal $s = s(x_1, x_2)$. The set of possible signals is denoted S. We will consider two specific forms for S, but for now we will state the model with S in its general form.

The matching of searchers to objects takes place over discrete time. In each period, searchers and objects are matched randomly pairwise. We assume that the maximal number of pairs are formed each

[1] Such as labor markets.

period, so everyone is matched if the populations are equally numerous. If a searcher accepts the object he was matched to, this searcher and object depart; otherwise, the searcher and object return to the pools of the unmatched.

A mass m of new searchers and a mass m of new objects enter the market in the beginning of each period. The quality of entering objects is distributed uniformly on $[0,1]^2$.[2] Searchers and objects stay in the market until they become a party to a successful match or "die". A fraction d of all participants die each period. The payoff to a searcher who "dies" before adopting an object is 0.

A (Markov) *strategy* for a searcher is a choice of a signal $s \in S$ and acceptance rule: $A(s) \in \{\text{accept,reject}\}$.

The *state* of the process is described by masses M of searchers and of objects present in the process, and a distribution F of the quality of objects present in the process.

This configuration describes a *steady state* if the masses M, the distribution F and the distribution of strategies across the searchers' population remain constant over time.

An *equilibrium* is a steady state configuration in which each searcher's behavior is optimal.

3 The benchmark case

Suppose that all searchers observe only x_1. That is, S is a singleton containing only $s(x_1, x_2) = x_1$.

Let $V(A; F)$ denote the expected utility of a searcher who in a steady state situation with distribution F uses the acceptance rule A. The optimal acceptance/rejection decision satisfies

$$A(x_1) = \text{accept} \;\text{ iff }\; x_1 + E(x_2) \geq (1-d)V(A;F), \tag{1}$$

Since $E(x_2)$ is independent of x_1, it follows immediately that the searcher's optimal acceptance rule is of the threshold variety:

$$A(x_1) = \text{accept} \;\text{ iff }\; x_1 \geq \underline{x}.$$

The distribution of the two attributes is independent $F = F_1 \times F_2$ and F_2 is uniform. In a steady state in which all searchers have the same threshold $\underline{x}$, F_1 has two steps:

$$F_1(x) = \begin{cases} \dfrac{x/d}{\underline{x}/d + 1 - \underline{x}} & \text{if} \quad x < \underline{x}; \\ \dfrac{\underline{x}/d + x - \underline{x}}{\underline{x}/d + 1 - \underline{x}} & \text{if} \quad x > \underline{x}. \end{cases}$$

The frequency of objects with quality $x < \underline{x}$ is m/d since they depart only through death, and hence the equality of entry and exit flows in the steady state implies $d \times \text{frequency} = m$. The frequency of objects with quality $x > \underline{x}$ is m, since they are accepted and hence only the newly arrived ones are around.

[2] The uniformity is not required. Everything can be done with an arbitrary distribution. But the greater generality would not contribute anything important to the message of this paper.

The expected utility of a searcher who uses acceptance rule A, $V(A;F)$, satisfies

$$\begin{aligned} V(A;F) &= [1-F(\underline{x})](\frac{1}{2}+\frac{1+\underline{x}}{2})+F(\underline{x})(1-d)V(A;F) \\ &= \frac{(1-\underline{x})}{\underline{x}/d+(1-\underline{x})}(\frac{1}{2}+\frac{1+\underline{x}}{2})+\frac{\underline{x}/d}{\underline{x}/d+(1-\underline{x})}(1-d)V(A;F). \end{aligned}$$

This implies that

$$V(A;F)=(1-\underline{x})\left(1+\frac{\underline{x}}{2}\right).$$

Let $\underline{x}^e$ denote an equilibrium threshold. Since it is optimal to accept an object with x_1 iff $x_1+\frac{1}{2}\geq(1-d)V(A;F)$, the threshold $\underline{x}^e+\frac{1}{2}$ must coincide with $(1-d)V(A;F)$. Using this observation and the above expression for $V(A;F)$, it follows that, for $d\geq\frac{1}{2}$, $\underline{x}^e=0$ and, for $d<\frac{1}{2}$, $\underline{x}^e$ solves the equation

$$\underline{x}^e+\frac{1}{2}=(1-d)(1-\underline{x}^e)(1+\frac{\underline{x}^e}{2}). \tag{2}$$

For $d<\frac{1}{2}$, this quadratic equation has a unique positive solution and, hence, there is a unique equilibrium[3]. It is immediate to observe that in this case $\underline{x}^e>0$.

Welfare will be measured by the searcher's expected payoffs. Since the number of objects equals the number of searchers, welfare is maximized with threshold $x^w=0$. That is, every object that yields positive value should be adopted. Thus, the equilibrium involves excessive search from social point of view. Furthermore, the equilibrium expected utility of a searcher, $(1-\underline{x}^e)(1+\frac{\underline{x}^e}{2})$, is smaller than 1, which is the expected utility that searchers would get if all of them employ threshold x^w. That is, the possibility to examine the object decreases welfare. This is not surprising, since it owes to the *negative externality* imposed by each searcher on the others. A searcher rejects an object and waits to be matched with a better object, which would otherwise be matched with another searcher.

To sum up

Proposition 1 *(i) For $d<\frac{1}{2}$, $\underline{x}^e>0$; for $d\geq\frac{1}{2}$, $\underline{x}^e=0$. (ii) $x^w=0$.*

4 Two attributes case

Consider now the case in which there are two possible signals: $s_i(x_1,x_2)=x_i$, $i=1,2$. That is, each signal exposes an attribute. As we mentioned before, the signals are exclusive. Each searcher has to choose one signal and can observe only the realization of the chosen signal. The assumption that only one attribute

[3] This uniqueness is not an artifact of the uniformity assumption. The counterpart of equation (2) for arbitrary distributions of attributes, H_i,

$$\underline{x}^e+\int x_2dH_2(x_2)=(1-d)\left[\int\max\left\{x_1+\int x_2dH_2(x_2),(1-d)\left(\underline{x}^e+\int x_2dH_2(x_2)\right)\right\}dH_1(x_1)\right],$$

has a unique $\underline{x}^e$ solution since the slope of the LHS in $\underline{x}^e$ is 1, and the slope of the RHS side is bounded away from 1 for any H_i's.

is examined can be viewed as implicitly capturing an underlying cost structure or a time constraint which make the observation of two attributes too costly or impossible.

Recall that a strategy for a searcher is a choice of a signal (attribute) and a function A from $[0,1]$ to $\{$accept,reject$\}$ describing the acceptance decision; an *equilibrium* is a steady state in which each searcher's behavior is optimal.

Let $V_i(A;F)$ denote the expected utility of a searcher who in a steady state situation with distribution F, examines only attribute i and uses the acceptance rule A. Let g_i denote the fraction of the searchers in presence who examine attribute i ($g_2 = 1 - g_1$) and let A_i denote their acceptance strategy.

In equilibrium, the optimality of the searchers' choice of a signal implies

$$\begin{array}{llll} \text{if} & g_i \in (0,1), & \text{then} & V_1(A_1;F) = V_2(A_2;F), \\ \text{if} & g_i = 1, & \text{then} & V_i(A_i;F) \geq V_j(A_j;F), \end{array} \tag{3}$$

and the optimality of searchers' acceptance/rejection implies

$$\begin{array}{lll} \text{if } A_i(x) = \text{accept}, & \text{then} & x + E(x_j \mid x_i = x) \geq (1-d)V_i(A_i;F), \\ \text{if } A_i(x) = \text{reject}, & \text{then} & x + E(x_j \mid x_i = x) \leq (1-d)V_i(A_i;F), \end{array} \tag{4}$$

where the expectation is with respect to the prevailing distribution of attributes F.

The acceptance behavior partitions $[0,1]^2$ into four product sets $RR = \{(x_1,x_2)|A_1(x_1) = A_2(x_2) = \text{reject}\}$, $AR = \{(x_1,x_2)|A_1(x_1) = \text{accept};\ A_2(x_2) = \text{reject}\}$, $RA = \{(x_1,x_2)|A_1(x_1) = \text{reject};\ A_2(x_2) = \text{accept}\}$, and $AA = \{(x_1,x_2)|A_1(x_1) = A_2(x_2) = \text{accept}\}$ with different steady state frequencies of objects (owing to the different rates of exit of objects across these rectangles). Let M_{ij}, $i,j = R,A$, denote the masses of objects in these four sets and let $U(X)$ denote the probability of set X, given the uniform distribution on $[0,1]^2$.

The steady state conditions require equality of the inflows and outflows:

$$\begin{aligned} dM_{RR} &= m \times U(RR);\ [dg_1 + g_2]M_{RA} = m \times U(RA); \\ [g_1 + dg_2]M_{AR} &= m \times U(AR);\ M_{AA} = m \times U(AA). \end{aligned} \tag{5}$$

That is, objects with attributes in RR exit only through death and hence the outflow is dM_{RR}, while objects with attributes in AA are adopted immediately and hence the outflow is M_{AA}.

The expected utility of a searcher who examines attribute 1, $V_1 = V_1(A;F)$, can be expressed in terms of masses M_{ij} as follows:

$$\begin{aligned} V_1 &= \Pr(AA)E(x_1+x_2|(x_1,x_2) \in AA) + \Pr(AR)E(x_1+x_2|(x_1,x_2) \in AR) + \Pr(RR \cup RA)(1-d)V_1 \\ &= \frac{M_{AA}E(x_1+x_2|(x_1,x_2) \in AA) + M_{AR}E(x_1+x_2|(x_1,x_2) \in AR) + (M_{RA}+M_{RR})(1-d)V_1}{M_{AA}+M_{AR}+M_{RA}+M_{RR}} \end{aligned} \tag{6}$$

where, again, the expectations are with respect to the distribution F of attributes in the market. An analogous expression can be obtained for $V_2 = V_2(A;F)$.

Since the distribution F can be expressed in terms of the masses M_{ij}'s (though it would be complicated for arbitrary A_i's), so are the V_i's and the equilibrium conditions (3) and (4). Since by (5) the M_{ij}'s can be expressed in terms of the strategies and the parameters of the model, conditions (4) can be also expressed in terms of the strategies and the parameters alone. This provides the system of inequalities that is necessary and sufficient for strategies to be in equilibrium.

The first question is whether in equilibrium different searchers choose different signals or all choose to employ the same signal. Intuitively, there are arguments in both directions. If all are checking attribute 1, its steady state distribution will be worse than that of attribute 2. This means that it is more prudent to check attribute 1, but at the same time it is more likely to find a good level of attribute 2 if this attribute is being checked. Indeed, as will be shown later, in general both types of pattern might be consistent with equilibrium. However, when the distribution of attributes is *uniform*, as assumed here, the only possible equilibria are such that each attribute is examined by a strict subset of the searchers.

Proposition 2 *: If the two attributes are distributed uniformly, then there is no equilibrium in which all searchers examine the same attribute.*

Proof: Suppose there is. Obviously, the equilibrium acceptance rule for attribute 1 has to be of the threshold variety. Let $\underline{x}$ be that threshold and let V be the searchers' equilibrium expected utility. Observe that

$$V(1-d) = \underline{x} + \frac{1}{2}.$$

The expected utility of a searcher who deviates to following behavior: check attribute 2 once and accept the object if attribute 2 is greater equal than y; otherwise continue with the equilibrium. Consider now this deviation with $y = \underline{x}$. If both attributes of the first sampled object are below $\underline{x}$ or both are above $\underline{x}$, it would be rejected and accepted respectively with or without the deviation. So, the deviation differs from the equilibrium behavior only when the attributes (x_1, x_2) of the first sampled object are such that $x_1 < \underline{x}$ and $x_2 \geq \underline{x}$ or $x_1 \geq \underline{x}$ and $x_2 < \underline{x}$. In the former case the object is rejected under the equilibrium behavior but is accepted under the deviation, while in the latter case the reverse is true. However, the expected value of the object in either of these cases is $\underline{x} + 1/2$ which is exactly equal to $V(1-d)$. Therefore, the searcher's expected utility is not affected by the acceptance/rejection in those regions. So, the deviation yields the same expected utility as the equilibrium behavior. But, the deviation with $y = \underline{x}$ is not the optimal deviation. To verify this observe that the expected utility under the deviation is

$$\frac{(1-y)[\underline{x}m/d\frac{1+\underline{x}+y}{2} + (1-\underline{x})m(1+\frac{\underline{x}+y}{2})]}{\underline{x}m/d + (1-\underline{x})m} + y(1-d)V$$

The derivative with respect to y evaluated at $y = \underline{x}$ is

$$(1-d)V - \frac{3}{2}\underline{x} - \frac{1}{2} + \frac{1}{2}\frac{\underline{x}m/d}{\underline{x}m/d + (1-\underline{x})m}$$

which upon substituting $V(1-d) = \underline{x} + 1/2$ is clearly positive.■

5 Threshold equilibria and payoff discontinuity

The simplest equilibrium in which each of the attributes is being examined by some subset of the searchers is of the threshold variety. A fraction of searchers $g_i \in (0,1)$ examine attribute i and accept the object iff $x_i \geq \underline{x}_i$. The sets AA, AR, etc. are simple rectangles: $RR = [0, \underline{x}_1] \times [0, \underline{x}_2]$, $RA = [0, \underline{x}_1] \times [\underline{x}_2, 1]$, $AR = [\underline{x}_1, 1] \times [0, \underline{x}_2]$ and $AA = [\underline{x}_1, 1] \times [\underline{x}_2, 1]$. Therefore, it follows from (5) that the equilibrium distribution is as shown in Figure 1 below.

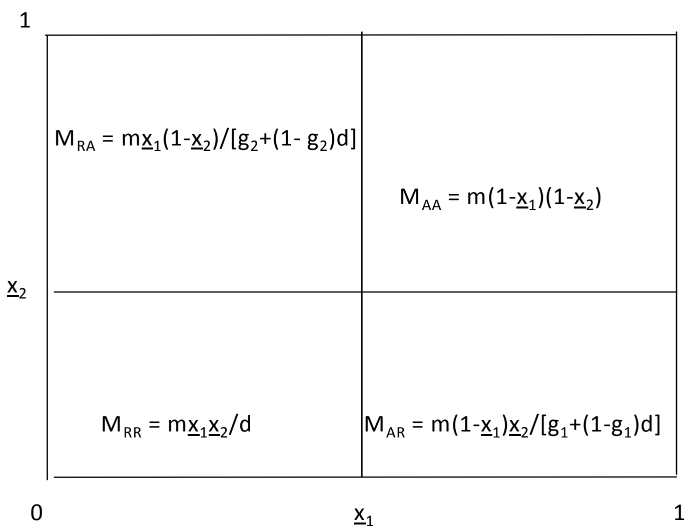

Figure 1: Equilibrium distribution

Therefore (as shown in the proof of Proposition **3**), the equilibrium condition (4) on the optimality of the acceptance/rejection decisions of a searcher who examines attribute 1 can be expressed as[4]

$$\begin{aligned}\underline{x}_1 + E(x_2|x_1 &> \underline{x}_1) \equiv \underline{x}_1 + \frac{M_{AA}\frac{1+\underline{x}_2}{2} + M_{AR}\frac{\underline{x}_2}{2}}{M_{AA}+M_{AR}} \geq (1-d)V_1 \ \text{ if } \underline{x}_1 > 0, \\ \underline{x}_1 + E(x_2|x_1 &< \underline{x}_1) \equiv \underline{x}_1 + \frac{M_{RR}\frac{\underline{x}_2}{2} + M_{RA}\frac{1+\underline{x}_2}{2}}{M_{RA}+M_{RR}} \leq (1-d)V_1 \ \text{ if } \ \underline{x}_1 < 1,\end{aligned} \tag{7}$$

That is, acceptance of x_1 just above $\underline{x}_1$ yields higher payoff than the value of continued search, while acceptance of x_1 just below $\underline{x}_1$ yields lower payoff. Analogous inequalities hold for a searcher who examines attribute 2.

Proposition 3 *A (symmetric) threshold equilibrium exists: (i) for $d < 1/2$, there is a non-degenerate interval $[x^-, x^+]$ such that $\underline{x}$ is a threshold of a symmetric threshold equilibrium iff $\underline{x} \in [x^-, x^+]$; (ii) for $d > 1/2$, the unique symmetric equilibrium threshold is $\underline{x} = 0$.*

[4] More precisely, the first inequality need not hold for $\underline{x}_1 = 0$ ($\underline{x}_2 = 0$, respectively), and the second inequality need not hold for $\underline{x}_1 = 1$ ($\underline{x}_2 = 1$, respectively)

This proof (as well as subsequent proofs that do not appear in the text) is relegated to Appendix A. The multiplicity of equilibria is explained by the externality that searchers exert on each other. Externalities often play an important role in search models. Here, since searchers sample from the same pool and values are common, the externality is the effect of a searcher's decisions on the distribution faced by others[5]. Its special character is in that it involves cross effects from one attribute to another. Observe that the payoff $x_1 + E(x_2 \mid x_1)$ of a searcher who examines attribute 1 and adopts an object with x_1 is in general not continuous at $\underline{x}_1$. This is because

$$\begin{aligned} E(x_2 \quad \mid \quad x_1 < \underline{x}_1) = \frac{M_{RR}\frac{\underline{x}_2}{2} + M_{RA}\frac{1+\underline{x}_2}{2}}{M_{RA} + M_{RR}} < \\ < \quad \frac{M_{AR}\frac{\underline{x}_2}{2} + M_{AA}\frac{1+\underline{x}_2}{2}}{M_{AR} + M_{AA}} = E(x_2 \mid x_1 > \underline{x}_1). \end{aligned}$$

This owes to

$$\frac{M_{RA}}{M_{RR}} = \frac{d(1-\underline{x}_2)}{[d(1-g_2)+g_2]\underline{x}_2} < \frac{[d(1-g_1)+g_1](1-\underline{x}_2)}{\underline{x}_2} = \frac{M_{AA}}{M_{RA}},$$

which in turn follows from $d < [d(1-g_2)+g_2][d(1-g_1)+g_1]$ for any $g_1, g_2 > 0$ such that $g_1 + g_2 = 1$[6]. This implies that the searcher's utility at the equilibrium threshold, $\underline{x}_1 + E(x_2 \mid \underline{x}_1)$, does not coincide with the value of continued search, $(1-d)V_1$. This discontinuity is what gives rise to the multiplicity of equilibria described by the proposition. In contrast, if all searchers were constrained to examine only attribute 1 (which would be equivalent to the benchmark case considered in the very beginning), then $x_1 + E(x_2 \mid x_1)$ would be continuous at the common threshold $\underline{x}_1$, and the searcher's utility at the threshold would coincide with the value of continued search. This difference is explained as follows. When attribute 2 is also examined by some fraction of the searchers, the distribution of x_2 conditional on x_1 depends on whether or not x_1 is accepted by the attribute 1 searchers. Specifically, if the attribute 1 searchers reject x_1' and accept x_1'', then $E(x_2 \mid x_1') < E(x_2 \mid x_1'')$. This is because the objects (x_1', x_2) such that $x_2 < \underline{x}_2$ stay around for longer than objects (x_1'', x_2) such that $x_2 < \underline{x}_2$, and "spoil" the distribution. In other words, the attribute 1 searchers exert externalities on each other which manifest themselves through the distribution of attribute 2.

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[5] In much of the search literature, values are private and the externality is through the meeting probabilities.

[6] It may be interesting to note that $M_{LH}/M_{LL} < M_{HH}/M_{HL}$ also for other, nonuniform distributions of attributes. Indeed, consider independently distributed attributes x_1 and x_2 with cdfs F_i. Then,

$$\frac{M_{LH}}{M_{LL}} = \frac{\frac{m}{d(1-g_2)+g_2}F_1(\underline{x}_1)(1-F_2(\underline{x}_2))}{\frac{m}{d}F_1(\underline{x}_1)F_2(\underline{x}_2)} = \frac{d(1-F_2(\underline{x}_2))}{[d(1-g_2)+g_2]F_2(\underline{x}_2)}$$

and

$$\frac{M_{HH}}{M_{LH}} = \frac{m(1-F_1(\underline{x}_1))(1-F_2(\underline{x}_2))}{\frac{m}{d(1-g_1)+g_1}(1-F_1(\underline{x}_1))F_2(\underline{x}_2)} = \frac{[d(1-g_1)+g_1](1-F_2(\underline{x}_2))}{F_2(\underline{x}_2)}.$$

Therefore $M_{LH}/M_{LL} < M_{HH}/M_{HL}$ follows from the fact that $d < [d(1-g_2)+g_2][d(1-g_1)+g_1]$ for any $g_1, g_2 > 0$ such that $g_1 + g_2 = 1$.

This observation is important, because it will follows that the structure equilibria described in the paper is quite general, as opposed to being specific for uniform, or some slightly larger class of distributions.

Non-symmetric threshold equilibria can be characterized in a similar manner to Proposition 3. The set of possible threshold pairs (one threshold for each attribute) is determined by a set of nonlinear inequalities that are not too instructive and hence are not brought here. Direct computations for different parameter values show that equilibria in which more than half of the searchers examine one of the attributes coexist with equilibria in which more than half examine the other attribute, and with symmetric equilibria in which half of the searchers examine each attribute. Figure 4 in Appendix B depicts the sets of equilibrium thresholds for certain parameter values.

6 Non-monotonicity

Another implication of the discontinuity of the payoff at the equilibrium threshold is **non-monotonic** behavior (which did not arise in the single-attribute case). To see the non-monotonicity, consider a symmetric threshold equilibrium, with threshold $\underline{x}$, for which inequalities (7) hold with strict inequalities and modify the acceptance rule as follows

$$A(x) = \begin{cases} \text{reject} \quad \text{if} \quad x < \underline{x} \text{ or } \underline{x} + \varepsilon \leq x < \underline{x} + 2\varepsilon \\ \text{accept} \qquad\qquad\qquad \text{otherwise} \end{cases}$$

Figure 2 depicts the partition of $[0,1]^2$ induced by this acceptance rule. As before, the different regions exhibit different frequencies of objects. The four corner rectangles have the same frequencies as before. The newly created rectangles also display these same four frequencies as indicated by the arrows pointing to them.

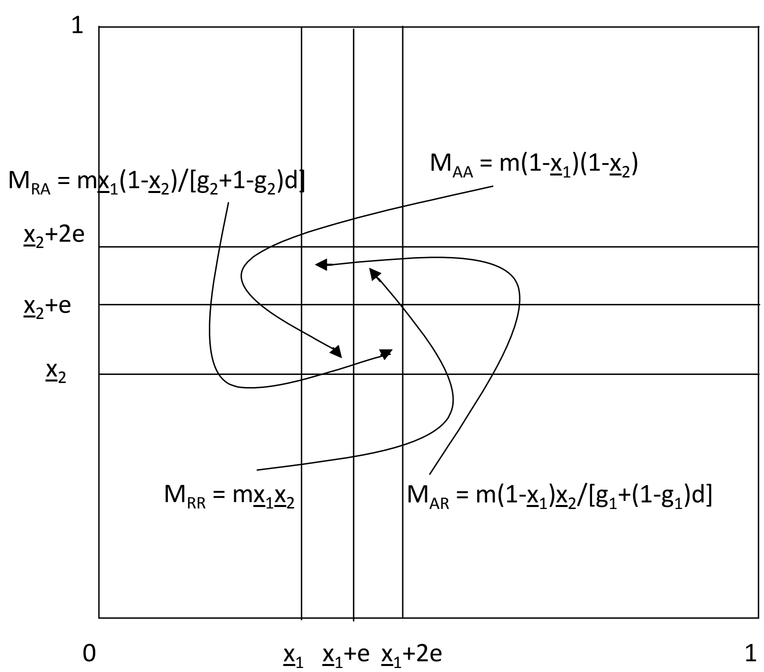

Figure 2: Non-monotonic equilibrium diustribution

Observe that, if the width of these strips, ε, is small enough, this configuration is an equilibrium. First, by symmetry, condition $V_1 = V_2$ holds. Second, since the inequalities in (7) hold strictly in the original equilibrium, their appropriate versions for this case continue to hold. This is because V_i changes only by a small amount while $E[x_2 \mid x_1 < \underline{x}] = E[x_2 \mid x_1 \in [\underline{x} + \varepsilon, \underline{x} + 2\varepsilon)]$ and $E[x_2 \mid x_1 \geq \underline{x} + 2\varepsilon] = E[x_2 \mid x_1 \in [\underline{x}, \underline{x} + \varepsilon)]$ have values that are on the order of ε close to the values that $E[x_2 \mid x_1 < \underline{x}]$ and $E[x_2 \mid x_1 \geq \underline{x}]$, respectively, had in the original equilibrium.

The following proposition summarizes the above observations.

Proposition 4 *For $d < 1/2$, there also exist equilibria that exhibit non-monotonic behavior. Specifically, for any $z \in (x^-, x^+)$, there exists an equilibrium in which some $x < z$ is accepted and some $x > z$ is rejected*[7]*.*

Observe that in the same way we constructed a non-monotonic equilibrium above, we can construct equilibria with an arbitrary finite number of alternating acceptance and rejection intervals. Moreover, the acceptance and rejection regions may be fairly exotic sets rather than just unions of intervals. Simply consider a symmetric equilibrium for which (7) hold with strict inequalities and modify the acceptance rule to

$$A(x) = \begin{cases} \text{reject} \quad if \quad x < \underline{x} \text{ or } x \in C \\ \text{accept} \qquad\qquad \text{otherwise} \end{cases}$$

where C is a positive-measure subset of interval $[\underline{x} + \varepsilon, \underline{x} + 2\varepsilon]$.

Remark: As is evident from the above discussion, what is crucial for the existence of non-monotonic equilibria is not the discontinuity in the value, but rather the dependence of $E(x_2 \mid x_1)$ on the acceptance decisions at x_1 of the searchers who examine attribute 1. Thus, it is possible to perturb the model by introducing some heterogeneity into searchers' preferences that smooths the discontinuity of the value, but still preserves the non-monotonic equilibria. To see this, consider a perturbed version of the model in which the preferences of searchers contain an idiosyncratic component. If a searcher is matched with an object whose attribute is x, the value of this attribute for this particular searcher is $x + \eta$, where the searcher's specific term η is distributed symmetrically around 0 across the population. The threshold equilibria still exist. However, if the distribution of η is continuous, the distribution of attribute 2 contingent on attribute 1 is a continuous function of attribute 1. Indeed, the object with threshold value of attribute 1 is accepted by the searchers whose idiosyncratic preference component η is positive, and rejected by searchers whose idiosyncratic preference component η is negative. That is, a half of the searchers matched with this type of object. While close to the threshold, the object is accepted or rejected by slightly less or more than a

[7] The range over which this proposition holds might extend beyond the interval $[x^-, x^+]$, but its exact characterization might not contribue a great deal to our discussion.

half of searchers matched with this type of object. This implies the continuity of attribute 2 contingent on attribute 1 as a function of attribute 1.

If the idiosyncratic component is sufficiently small, e.g., the support of η is contained in a sufficiently small interval around zero, there will still exist non-monotonic equilibria (i.e., equilibria in which all searchers matched with an object whose attribute 1 is slightly higher than some level x reject it, and all searchers matched with an object whose attribute 1 is slightly lower than x accept it). This follows from an argument analogous to that used to show the existence of non-monotonic equilibria in the original setting.

7 Welfare

As before, welfare is identified with the searcher's expected utility V. We have already observed in the benchmark scenario in which the same attribute was observed by all searchers that the negative externality among searchers gives rise to excessive search in equilibrium relative to the optimal levels. The same holds true for the two-attribute case. Clearly, the equality of the number of objects and searchers and the fact that all objects have positive value imply that welfare is maximized by the no-search scenario in which searchers must take the objects that they are matched with[8]. For the same reason, if one symmetric threshold equilibrium has lower thresholds than another, it would yield higher welfare.

The following propositions report two somewhat more subtle observations about welfare in the two-attribute case. First, the fact that searchers spread themselves across the two attributes lessens the negative externality, and hence improves welfare relative to a situation in which all searchers would examine the same attribute.

Proposition 5 *Any threshold equilibrium of the scenario in which each attribute is examined by a subset of the searchers yields higher welfare than the unique equilibrium of the scenario in which only one attribute can be examined.*

This result is shaped by two opposing considerations. When all searchers examine one attribute, the expected value of the other attribute is unaffected by adverse selection. This induces searchers to be less choosey with respect to the observable attribute which translates to a lower negative externality and hence a positive effect on welfare. But, since all are focusing on the same attribute, the acceptable objects are relatively hard to find which increases the probability of failed search and affects welfare negatively. These two considerations are more moderate when searchers spread themselves across the two attributes: there is more adverse selection with respect to the hidden attribute but it is relatively easier to find an

[8] This is in the absence of an "adoption cost". With such a cost, welfare is not maximized by the no search scenario but still at a lower level of search than the level that prevails at the equilibria.

object whose observable attribute clears the threshold. As the proposition establishes, this trade-off is more conducive to welfare in the latter scenario.

Second, non-monotonic equilibria exhibit further welfare loss in the sense that any symmetric non-monotonic equilibrium yields lower welfare than some threshold equilibrium. This result is not immediately obvious. On the one hand, it is obvious that a symmetric threshold configuration with the same total acceptance region would yield higher welfare, since it differs only in that it rejects inferior objects. But, on the other hand, it is not obvious that for any non-monotonic equilibrium such a threshold configuration can be sustained as an equilibrium, since, in principle, the non-monotonicity might make it possible to sustain greater acceptance region in equilibrium.

Proposition 6 *Any symmetric non-monotonic equilibrium yields lower welfare than some symmetric threshold equilibrium.*

Remark: We conjecture that Proposition 6 may not hold in the case of general, nonuniform distributions. That is, non-monotonic equilibria may not be welfare-dominated by threshold equilibria in the general case. However, Proposition 6 does hold under some, seemingly mild, regularity assumptions on the distribution of attributes. Inspection of the proof (in Appendix A) shows, that it suffices to impose assumptions which would guarantee that $\underline{x} + E^{\underline{x}}(x_2 \mid x_1 \text{ s.t. } x_1 > \underline{x})$ (i.e., the upper upward-sloping line in Figure 3 in Appendix B) is increasing in $\underline{x}$.

8 Choice of signal

It was shown in Section 4 above that there is no equilibrium in which all searchers choose to observe the same attribute. We have already mentioned that this result might not hold in general. The following simple case shows that different patterns may arise in equilibrium. Furthermore, there is a region of the parameter space in which equilibria of all three forms (all checking attribute 1, all checking 2, and each attribute being checked by some) coexist. Suppose that each attribute has two potential values, $x_i \in \{0, 1\}$, and let $p_i = \Pr(x_i = 1)$ at the entry stage. As in the continuous two attributes case, there are two possible signals: $s_i(x_1, x_2) = x_i$, $i = 1, 2$

Proposition 7 *Assume that p_i is bounded away from 0 and 1, i.e., $p_i \in (\varepsilon, 1 - \varepsilon)$, for some constant $\varepsilon > 0$, and that d is small enough (compared to ε). Then: (i) there is an equilibrium in which a fraction $g_i \in (0, 1)$ of the searchers examine each of the attributes. (ii) There also exists an equilibrium in which all searchers examine attribute i iff $p_i(1 + p_j) > \frac{1}{1-d}$ (i.e., if p_1 and p_2 are large enough).*

The steady state equilibrium distribution F is described by the fractions $f_{j\ell} = \Pr(x_1 = j, x_2 = \ell)$, $j, \ell = 0, 1$. The searcher's payoff from examining attribute 1 depends both on the fraction of objects with high attribute 1, $f_{10} + f_{11}$, and on the probability of attribute 2 being high conditional on adopting, $f_{11}/(f_{10} + f_{11})$. Obviously, these magnitudes go in opposite directions across the attributes. That is, if there is a higher fraction of objects with high attribute 1, $f_{10} + f_{11} > f_{01} + f_{11}$, then $f_{11}/(f_{01} + f_{11}) > f_{11}/(f_{10} + f_{11})$, which means that basing the adoption decision on the value of attribute 2 delivers a higher expected value of the hidden attribute 1. Thus, $V_1 = V_2$, which must hold at an equilibrium in which each attribute is examined by some searchers, is achieved either when both attributes exhibit the same distribution and the same conditional distribution of the hidden attribute, or when the two opposing considerations exactly offset each other. The former occurs when $f_{10} = f_{01}$, which is the case analyzed by the proposition. It arises in equilibrium for any parameter configuration in the range under consideration. The other case occurs when $f_{11} = d/(1 - d)$, which may hold for a non-empty subset of (p_1, p_2) values in the range but not for all values in the range.[9]

Observe that there is a region of the parameters for which equilibria of the three possible forms (everybody checking attribute 1, everybody checking attribute 2, and each attribute is examined by a subset) coexist. This multiplicity arises if both p_i, $i = 1, 2$, are sufficiently large to satisfy $p_i(1 + p_j) > 1/(1 - d)$. However, there is also a region (i.e., $p_i(1 + p_j) < 1/(1 - d)$ for $i = 1, 2$) in which only equilibria where each of the attributes is examined by a subset of the searchers exist.

Since all searchers share the same preferences, when there is multiplicity of equilibria, they will be Pareto ranked for the searchers. Thus, when all three equilibria exist, the equilibrium in which all examine attribute 1 is preferred by all searchers to the equilibrium in which all examine attribute 2 if $p_1 > p_2$. But when all examine attribute 2, the steady state distribution of that attribute is sufficiently bad to make it worthwhile for a searcher to focus on it rather than on attribute 1.

9 Costly information acquisition

In the scenarios of the previous sections the searcher could choose between different signals that provide independent information about different aspects of the object. This section introduces costly choice between different signals that represent different amounts of information about a single attribute (as opposed to information about different attributes as above). It is shown that the costly acquisition of information also leads to multiplicity of equilibria (like in the previous scenarios), but it does not give rise to discontinuity of the searcher's value at the threshold levels and to non-monotonicity of the equilibrium behavior.

[9] The precise characterization of the set of (p_1, p_2) for which $f_{11} = d/(1 - d)$ holds is somewhat messy. but it is easy to see that this set is not empty. For example, if $p_1 = p_2 = p$, this condition will be satisfied for $p \in [0.5 + o(d), 0.6 + o(d)]$, where $o(d)$ is a term on the order of d.

Specifically, the value of an object is $x \in [0,1]$ and the distribution of entering objects is uniform[10]. The set of signals is S whose elements $s^\gamma \in S$ are indexed by $\gamma \in [0,1)$

$$s^\gamma(x) = \begin{cases} x & \text{if} \quad x \leq \gamma \\ 1 & \text{if} \quad x > \gamma \end{cases}$$

That is, with signal s^γ, if $x \leq \gamma$, the searcher learns x; if $x > \gamma$, the searcher only learns that $x \in (\gamma, 1]$. Thus, γ is the strength of the signal. It can be thought of as the information acquisition effort or the thoroughness of a test that the searcher uses to learn about x. A test of thoroughness γ is sufficient to completely identify an object whose quality is lower than γ, whereas an object that passes that test is just revealed to be better than γ. The cost of signal s^γ is $C(\gamma)$ assumed increasing and convex with $C(0) = C'(0) = 0$ and $C(\gamma) \to \infty$ as $\gamma \to 1$. The overall payoff of a searcher who ends up adopting an object of quality x after being matched to n objects and making efforts $\gamma_1, ..., \gamma_n$ in examining them is

$$x - \sum_{i=1}^{n} C(\gamma_i).$$

A strategy for searchers specifies how much information to acquire at each match and whether to accept an object given the information acquired. Since information acquisition is costly, an optimizing searcher will choose s^γ with $\gamma > 0$ only if she intends to reject $x < \gamma$ and intends to accept $x \geq \gamma$. We therefore identify the acceptance decision with the signal choice decision, and a searcher's strategy will just specify the signal choice with the understanding that signal s^γ implies acceptance iff $x \geq \gamma$.

Other than the above changes, the model remains as before. In a *steady state* searchers' behavior is constant, and so are the masses M and the distribution F. An (symmetric) *equilibrium* is a steady state configuration in which all searchers use the same Markov strategy and each searcher's behavior maximizes her expected payoff.

Let $V(s; F)$ denote the expected utility of a searcher who uses the strategy s in a steady state situation characterized by F. We will denote by γ a Markov strategy that prescribes the constant choice of signal s^γ in all encounters and by $y \mid \gamma$ a strategy that prescribes the choice of signal s^y once followed by a constant choice of signal s^γ thereafter. Thus,

$$V(y \mid \gamma; F) = -C(y) + [1 - F(y)]E(x \mid x \geq y) + F(y)(1-d)V(\gamma; F),$$

where the expectation is with respect to the prevailing distribution of object quality F. Let F^γ denote the steady state distribution arising when all searchers employ the same Markov strategy γ. The strategy γ constitutes a symmetric equilibrium if

$$V(\gamma; F^\gamma) \geq V(y \mid \gamma; F^\gamma) \text{ for all } y \tag{8}$$

[10] For the purposes of this section we do not need the two-attribute structure. We can retain of course the two-attribute structure, assume that informaton can be obtained only on one and have the other other attribute just hanging around like in Section.

To express this condition explicitly observe that, as before, F^γ has two steps: the frequency of objects with quality below γ is m/d, since they exit only through death and hence in a steady state entry rate $= m = d\times$frequency $=$ exit rate; the frequency of objects with quality above γ is m, since they are accepted immediately and hence only the newly arrived ones are around. Thus,

$$F^\gamma(y) = \begin{cases} \dfrac{y/d}{\gamma/d+1-\gamma} & \text{if} \quad y < \gamma, \\ \dfrac{\gamma/d+y-\gamma}{\gamma/d+1-\gamma} & \text{if} \quad y > \gamma. \end{cases}$$

Therefore,

$$V(y \mid \gamma; F^\gamma) = \frac{1-y}{\gamma/d+1-\gamma}\frac{1+y}{2} + \frac{\gamma/d+y-\gamma}{\gamma/d+1-\gamma}(1-d)V(\gamma;F^\gamma) - C(y)$$

if $y \geq \gamma$, and

$$V(y \mid \gamma; F^\gamma) = \frac{\dfrac{\gamma-y}{d}\dfrac{y+\gamma}{2} + (1-\gamma)\dfrac{1+\gamma}{2}}{\gamma/d+1-\gamma} + \frac{y/d}{\gamma/d+1-\gamma}(1-d)V(\gamma;F^\gamma) - C(y) \tag{9}$$

if $y < \gamma$.

A necessary and sufficient condition for (8) is

$$\frac{\partial V(y \mid \gamma; F^\gamma)}{\partial y}\Big|_{y=\gamma^+} \leq 0 \text{ and } \frac{\partial V(y \mid \gamma; F^\gamma)}{\partial y}\Big|_{y=\gamma-} \geq 0, \tag{10}$$

where $\big|_{y=\gamma^+}$ and $|_{y=\gamma-}$ denote the right and left derivative, respectively. Using (9) to write this condition explicitly, we get that γ^e is a symmetric equilibrium strategy iff it satisfies

$$C'(\gamma)d \leq (1-d)[\frac{1-\gamma}{\gamma/d+1-\gamma}\frac{1+\gamma}{2} - C(\gamma)] - \frac{\gamma}{\gamma/d+1-\gamma} \leq C'(\gamma). \tag{11}$$

The welfare maximizing effort $\gamma^w = \arg\max_\gamma V(\gamma; M, F^\gamma)$ satisfies the condition

$$\frac{dV(\gamma;F^\gamma)}{d\gamma} = -\gamma - C'(\gamma)[\gamma/d+1-\gamma] - C(\gamma)(1-d)/d = 0. \tag{12}$$

The following proposition characterizes the symmetric equilibria of this model. In particular, it points out that the costly decision on information acquisition also produces multiplicity of equilibria.

Proposition 8 *(i) There is an interval* $[\underline{\gamma}^e, \overline{\gamma}^e]$ *of symmetric equilibrium strategies; (ii)* $\gamma^w < \underline{\gamma}^e$*, i.e., every equilibrium exhibits more information acquisition than the welfare maximizing level.*

Proof: (i) The middle part of (11) is decreasing over $[0,1]$, positive at 0 and negative at 1. Therefore, there are $\underline{\gamma}^e < \overline{\gamma}^e$ such that for $\underline{\gamma}^e$ the right inequality holds as equality and for $\overline{\gamma}^e$ the left inequality holds as equality. Therefore, any $\gamma \in [\underline{\gamma}^e, \overline{\gamma}^e]$ satisfies (11) and constitutes an equilibrium.

(ii) Using the first-order condition of welfare maximization (12) to evaluate the middle part of (11) at $\gamma = \gamma^w$, we get

$$\begin{aligned} & (1-d)[\frac{1-\gamma^w}{\gamma^w/d+1-\gamma^w}\frac{1+\gamma^w}{2} - C(\gamma^w)] + C'(\gamma^w) + \frac{C(\gamma^w)(1-d)}{d[\gamma^w/d+1-\gamma^w]} \\ = \;& (1-d)\frac{1-\gamma^w}{\gamma^w/d+1-\gamma^w}\frac{1+\gamma^w}{2} + \frac{(1-d)(1/d-1+\gamma^w)}{[\gamma^w/d+1-\gamma^w]}C(\gamma^w) + C'(\gamma^w) > C'(\gamma^w). \end{aligned}$$

Since the middle part of (11) is decreasing in γ, it follows that $\gamma^w < \underline{\gamma}^e$. ■

Without the costly decision on information acquisition, the model of this section is the same as that of the benchmark model of Section 3 that has a unique equilibrium. Thus, the special effect of the costly acquisition is the emergence of multiplicity of equilibria that did not arise in the benchmark case. The key to this is that the frequency of objects with qualities below the equilibrium joint threshold is different from the frequency of objects with higher quality. The former frequency is m/d while the latter is m. This creates a "kink" in the value of information at the equilibrium level of information acquisition, which explains the multiplicity. When the information is costless, the frequency still changes discontinuously at the equilibrium threshold, but there is no marginal decision like that of information acquisition to be affected by it.

The over-investment in information owes to the externality that searchers confer on each other. Since searchers sample from the same pool, a selective searcher who searches for high quality takes it away from others and imposes on them a longer search. Therefore, an individual searcher who ignores the externality tends to be more selective than is socially optimal and this requires more information acquisition. The difference between the equilibrium and the social optimum is not a consequence of the costly information acquisition, but rather of the above mentioned externality that is already present in the benchmark case in which the information was costlessly observable.

10 Concluding comments

Our objective in this paper has been to expose some effects arising in a common values search environment in which only partial information on the quality of a sampled object is available to searchers at the point of decision. It was shown that, if searchers face an information acquisition decision, be it choice between different signals or a decision on the quantity of information, the model exhibits a large multiplicity of equilibria some of which may be non-monotonic.

We chose to do so with the simplest equilibrium search model that would allow to derive the main insights without dealing with modeling complications that do not seem to be of direct relevance to those points. It would be of course interesting to enrich the basic model and there are several obvious directions. First, a two-sided model search with decision makers on both sides. That is, "objects" also have preferences over searchers and decide whether to accept a match. Second, endogenous decisions of searchers and objects regarding participation in the process. Third, a richer model with heterogeneity across agents and asymmetry across attributes. It would probably make sense to pursue such extensions only in the context of a specific application that would also guide the modeling choices, rather than in the abstract.

11 References

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12 Appendix A

Proposition 3: A (symmetric) threshold equilibrium exists: (i) for $d < 1/2$, there is a non-degenerate interval $[x^-, x^+]$ such that $\underline{x}$ is a threshold of a symmetric threshold equilibrium iff $\underline{x} \in [x^-, x^+]$; (ii) for $d > 1/2$, the unique symmetric equilibrium threshold is $\underline{x} = 0$.

Proof of Proposition : Given the simple rectangular form of the sets AA, AR, etc., the steady state conditions (5) become

$$\begin{aligned} dM_{RR} &= m\underline{x}_1\underline{x}_2;\ [d(1-g_2)+g_2]M_{RA} = m\underline{x}_1(1-\underline{x}_2); \\ [d(1-g_1)+g_1]M_{AR} &= m\underline{x}_2(1-\underline{x}_1);\ M_{AA} = m(1-\underline{x}_1)(1-\underline{x}_2), \end{aligned} \tag{13}$$

$V_1 = V_1(A;F)$ from (6) becomes:

$$\begin{aligned} V_1 &= \Pr([\underline{x}_1,1]\times[\underline{x}_2,1])\frac{2+\underline{x}_1+\underline{x}_2}{2} + \Pr([\underline{x}_1,1]\times[0,\underline{x}_2])\frac{1+\underline{x}_1+\underline{x}_2}{2} \\ &\quad + \Pr([0,\underline{x}_1]\times[0,\underline{x}_2]\cup[0,\underline{x}_1]\times[\underline{x}_2,1])(1-d)V_1 \\ &= \frac{M_{AA}\frac{2+\underline{x}_1+\underline{x}_2}{2} + M_{AR}\frac{1+\underline{x}_1+\underline{x}_2}{2} + (M_{RA}+M_{RR})(1-d)V_1}{M_{AA}+M_{AR}+M_{RA}+M_{RR}}; \end{aligned} \tag{14}$$

and of course $V_2 = V_2(A;F)$ is analogous.

It follows that the equilibrium condition (4) on the optimality of the acceptance/rejection decisions of a searcher who examines attribute 1 can be expressed as

$$\begin{aligned} \underline{x}_1 + \frac{M_{AA}\frac{1+\underline{x}_2}{2} + M_{AR}\frac{\underline{x}_2}{2}}{M_{AA}+M_{AR}} &\geq (1-d)V_1 \ \text{ if } \underline{x}_1 > 0, \\ \underline{x}_1 + \frac{M_{RR}\frac{\underline{x}_2}{2} + M_{RA}\frac{1+\underline{x}_2}{2}}{M_{RA}+M_{RR}} &\leq (1-d)V_1 \ \text{ if } \ \underline{x}_1 < 1, \end{aligned} \tag{15}$$

Using (13), V_i and (15) can be expressed in terms of the thresholds $\underline{x}_i$ and the fractions g_i. At a symmetric configuration with $g_1 = g_2 = 1/2$ and $\underline{x}_1 = \underline{x}_2 = \underline{x}$, both V_1 and (15) can be expressed in terms of $\underline{x}$ alone and thus simplify to

$$V^{\underline{x}} = (1-\underline{x})^2(1+\underline{x}) + \frac{1}{d+1}\underline{x}(1-\underline{x})(2\underline{x}+1) \tag{16}$$

and

$$
\begin{aligned}
\frac{3\underline{x}}{2}+\frac{(1+d)(1-\underline{x})}{2(1+d)(1-\underline{x})+4\underline{x}} &\geq (1-d)V^{\underline{x}} \text{ if } \underline{x}>0, \\
\frac{3\underline{x}}{2}+\frac{d(1-\underline{x})}{(1+d)\underline{x}+2d(1-\underline{x})} &\leq (1-d)V^{\underline{x}} \text{ if } \underline{x}<1.
\end{aligned} \tag{17}
$$

(i) The following argument is summarized in Figure 3 in Appendix B below. In words, $V^{\underline{x}}$ as given by (16) is decreasing in $\underline{x}$, which takes value $V^{\underline{x}}=1$ at $\underline{x}=0$ and value $V^{\underline{x}}=0$ at $\underline{x}=1$. The expressions on the left-hand sides of (17) are equal to $1/2$ at $\underline{x}=0$ and to $3/2$ at $\underline{x}=1$, respectively. In addition, the first of these two expressions is increasing in $\underline{x}$ and exceeds the second for any value of $\underline{x}$; the second expression is convex, so its upward sloping part is monotonic. Therefore, for $d<1/2$, both inequalities in (17) have interior $\underline{x}$ solutions when holding as equations. Let x^{-} and x^{+} be these solutions of the first and second equations, respectively[11]. Thus, any $\underline{x}\in[x^{-},x^{+}]$ is the threshold of some symmetric threshold equilibrium, since by construction of $[x^{-},x^{+}]$ it satisfies the equilibrium conditions. Conversely, in any symmetric threshold equilibrium, the threshold $\underline{x}$ is in $[x^{-},x^{+}]$. (ii) for $d>1/2$ there is no solution for those equations. Condition (17) is satisfied only at $\underline{x}=0$. ■

Proposition5 Any threshold equilibrium of the scenario in which each attribute is examined by a subset of the searchers yields higher welfare than the unique equilibrium of the scenario in which only one attribute can be examined.

Proof: In the scenario in which each attribute is examined by a subset of the searchers, the worst (welfare-wise) threshold equilibrium has the threshold $\underline{x}=x^{+}$ (see Proposition 3 above). Recall that x^{+} and $V^{x^{+}}$ are jointly determined by the following two equations:

$$
\begin{aligned}
V^{x^{+}} &= (1-x^{+})^{2}(1+x^{+})+\frac{1}{d+1}x^{+}(1-x^{+})(2x^{+}+1); \\
\frac{3x^{+}}{2}+\frac{d(1-x^{+})}{(1+d)x^{+}+2d(1-x^{+})} &= (1-d)V^{x^{+}}.
\end{aligned} \tag{18}
$$

Similarly, the threshold and welfare of the unique equilibrium of the scenario in which only one attribute can be examined are jointly determined by

$$
V=\frac{(1-\underline{x})}{\underline{x}/d+(1-\underline{x})}\frac{2+\underline{x}}{2}+\frac{\underline{x}/d}{\underline{x}/d+(1-\underline{x})}(1-d)V,
$$

or equivalently,

$$
V=\frac{(1-\underline{x})(2+\underline{x})}{2} \tag{19}
$$

and

$$
\underline{x}+\frac{1}{2}=(1-d)V.
$$

Direct solution of these polynomial equations yields $V^{x^{+}}>V$ for any $d\in(0,1/2)$. The graphs of these two functions are exhibited in Figure 5 in Appendix B below.■

[11] Figure 2 in the appendix illustrates how x^{-} and x^{+} are determined for $d=0.1$.

Proposition 6 Any symmetric non-monotonic equilibrium yields lower welfare than some symmetric threshold equilibrium.

Proof: Let $A^{nm} = A_1 = A_2$ be the acceptance decision functions of the (symmetric) non-monotonic equilibrium and let

$$\underline{x} = \text{measure of } \{x : A^{nm}(x) = \text{reject}\}.$$

Observe that $\underline{x} = y + z$, where

$$y = \inf\{x : A^{nm}(x) = \text{accept}\},$$

and

$$z = \text{measure of } \{x \geq y : A^{nm}(x) = \text{reject}\}.$$

Consider a symmetric threshold configuration in which objects are accepted iff the examined attribute exceeds $\underline{x}$. If there is a threshold equilibrium whose threshold is smaller than or equal to $\underline{x}$, i.e., when $x^- \leq \underline{x}$, then it obviously yields higher welfare than the original non-monotonic equilibrium. So, the non-monotonic equilibrium can yield higher welfare only when $\underline{x} < x^-$. Consider this case.

By the definition of y,

$$y + E^{nm}(x_2 \mid x_1 \text{ s.t. } A^{nm}(x_1) = \text{accept}) \geq (1-d)V^{nm}, \tag{20}$$

where as above nm indexes magnitudes corresponding to the non-monotonic equilibrium. Since $\underline{x} + E^{\underline{x}}(x_2 \mid x_1 \text{ s.t. } x_1 > \underline{x})$ is increasing in $\underline{x}$ (see Figure 3 in Appendix B), and $x^- + E^{x^-}(x_2 \mid x_1 \text{ s.t. } x_1 > x^-) = (1-d)V^{x^-}$ by the definition of x^-, we have

$$\underline{x} + E^{\underline{x}}(x_2 \mid x_1 \text{ s.t. } x_1 > \underline{x}) \leq (1-d)V^{x^-}. \tag{21}$$

(Recall that we have assumed that $\underline{x} < x^-$.) It follows from (20) and (21) that in order to establish that $V^{nm} \leq V^{x^-}$, it suffices to show that

$$E^{nm}(x_2 \mid x_1 \text{ s.t. } A^{nm}(x_1) = \text{accept}) - E^{\underline{x}}(x_2 \mid x_1 \text{ s.t. } x_1 > \underline{x}) \leq z.$$

Consider a symmetric non-monotonic configuration in which an object is accepted iff the examined attribute belongs to $[y, 1-z]$. Let $E^{[y,1-z]}(x_2 \mid x_1 \text{ s.t. } y \leq x_1 \leq 1-z)$ denote the expectation corresponding to that configuration. Observe that $E^{[y,1-z]}(x_2 \mid x_1 \text{ s.t. } y \leq x_1 \leq 1-z) \geq E^{nm}(x_2 \mid x_1 \text{ s.t. } A^{nm}(x_1) = \text{accept})$. This is because, like in the $nm-$configuration, the rejection region of the $[y, 1-z]-$configuration consists of $[0, y] \cup$ region of measure z. But in the $[y, 1-z]-$configuration, the rejection region of measure z is located at the upper end, which means that the distribution of the unobserved attribute stochastically dominates the corresponding distribution in the $nm-$configuration. Therefore, it suffices to show that

$$E^{[y,1-z]}(x_2 \mid x_1 \text{ s.t. } y \leq x_1 \leq 1-z) - E^{\underline{x}}(x_2 \mid x_1 \text{ s.t. } x_1 > \underline{x}) \leq z.$$

Direct calculations show that indeed

$$E^{[y,1-z]}(x_2 \mid x_1 \text{ s.t. } y \leq x_1 \leq 1-z) - E^{\underline{x}}(x_2 \mid x_1 \text{ s.t. } x_1 > \underline{x}) =$$

$$\frac{\left(\frac{2}{d+1}\right)y\frac{y}{2}+\left(\frac{2}{d+1}\right)z\left(\frac{2-z}{2}\right)+(1-y-z)\left(\frac{1-z+y}{2}\right)}{\left(\frac{2}{d+1}\right)(y+z)+(1-y-z)}-\frac{\left(\frac{2}{d+1}\right)(y+z)\left(\frac{y+z}{2}\right)+(1-y-z)\left(\frac{1+y+z}{2}\right)}{\left(\frac{2}{d+1}\right)(y+z)+(1-y-z)}=$$

$$\frac{\left(\frac{2}{d+1}\right)z-\left(\frac{2}{d+1}\right)z^2-(1-y-z)z-\left(\frac{2}{d+1}\right)yz}{\left(\frac{2}{d+1}\right)(y+z)+(1-y-z)}=z\frac{\frac{1-d}{1+d}(1-y-z)}{\left(\frac{2}{d+1}\right)(y+z)+(1-y-z)}\leq z.$$

where the last inequality follows from $0 \leq y+z \leq 1$. ■

Proposition 7 Assume that p_i is bounded away from 0 and 1, i.e., $p_i \in (\varepsilon, 1-\varepsilon)$, for some constant $\varepsilon > 0$, and that d is small enough (compared to ε). Then: (i) there is an equilibrium in which a fraction $g_i \in (0,1)$ of the searchers examine each of the attributes. (ii) There also exists an equilibrium in which all searchers examine attribute i iff $p_i(1+p_j) > \frac{1}{1-d}$ (i.e., if p_1 and p_2 are large enough).

Proof: The steady state equilibrium distribution F is described by the fractions $f_{j\ell} = \Pr(x_1 = j, x_2 = \ell)$, $j, \ell = 0, 1$. Since the equilibrium acceptance policy always has $A_i(1) =$ accept, we may write $V_i(A;F)$ as $V_i(A_i(0);F)$. We have that

$$V_1(A_1(0);F)=\begin{cases}(f_{10}+f_{11})(1+\frac{f_{11}}{f_{10}+f_{11}})+(f_{00}+f_{01})(1-d)V_i(A_1(0);F) & \text{if} \quad A_1(0)=\text{reject}\\ (f_{10}+f_{11})(1+\frac{f_{11}}{f_{10}+f_{11}})+(f_{00}+f_{01})\frac{f_{01}}{f_{00}+f_{01}} & \text{if} \quad A_1(0)=\text{accept}\end{cases},$$

which yields

$$V_1(\text{reject}; f_1, f_2) = \frac{f_{10}+2f_{11}}{f_{10}+f_{11}+d(f_{00}+f_{01})} \text{ and } V_1(\text{accept}; f_1, f_2) = 1 + f_{11} - f_{00}$$

Since

$$\frac{f_{10}+2f_{11}}{f_{10}+f_{11}} > 1 + f_{11} - f_{00},$$

$V_1(\text{reject}; f_1, f_2) > V_1(\text{accept}; f_1, f_2)$ if d is small. Therefore, in the sequel, we can focus on configurations with $A_i(0) =$ reject, and use V_i as a shorthand for $V_i(\text{reject}; f_1, f_2)$. Observe that $V_1 \gtreqless V_2$ iff

$$\frac{f_{10}+2f_{11}}{f_{10}+f_{11}+d(f_{00}+f_{01})} \gtreqless \frac{f_{01}+2f_{11}}{f_{01}+f_{11}+d(f_{00}+f_{10})}$$

iff

$$[d-(1-d)f_{11}][f_{10}-f_{01}] \gtreqless 0.$$

(i) An equilibrium with $g_i \in (0,1)$, $i = 1,2$, requires $V_1 = V_2$ which in turn requires $f_{10} = f_{01}$ or $f_{11} = d/(1-d)$. That is, if there are $g_i \in (0,1)$, $i = 1,2$, such that either of these equalities holds, then there is an equilibrium in which each attribute is being checked by some fraction of the population. To find out when such $g_i \in (0,1)$ might exist, express $f_{j\ell}$ explicitly in terms of the parameters. Let $M_{j\ell}$, $j, \ell \in \{0,1\}$ denote the mass of objects with attributes 1 and 2 equal to j and ℓ respectively, and recall that m denotes the masses of searchers and objects entering each period. The steady state conditions require equality of the inflows and outflows

$$\begin{aligned} dM_{00} &= m(1-p_1)(1-p_2),\ M_{11} = mp_1p_2, \\ M_{10}g_1 + M_{10}g_2d &= mp_1(1-p_2),\ M_{01}g_2 + M_{01}g_1d = m(1-p_1)p_2. \end{aligned} \tag{22}$$

That is, objects with attributes $(0,0)$ exit only through death and hence the outflow is dM_{00}, while objects with attributes $(1,1)$ are adopted immediately and hence the outflow is M_{11}. Therefore,

$$f_{10} = \frac{M_{10}}{M_{00}+M_{10}+M_{01}+M_{11}} = \frac{m\dfrac{p_1(1-p_2)}{g_1+g_2 d}}{\dfrac{m(1-p_1)(1-p_2)}{d} + m\dfrac{p_1(1-p_2)}{g_1+g_2 d} + m\dfrac{(1-p_1)p_2}{g_2+g_1 d} + mp_1p_2},$$

and f_{00}, $f_{01}$and f_{00} are calculated similarly from (22).

Now, $f_{10} = f_{01}$ is equivalent to

$$\frac{p_1(1-p_2)}{g_1+g_2 d} = \frac{p_2(1-p_1)}{g_2+g_1 d}, \tag{23}$$

and together with $g_2 = 1-g_1$ yields

$$g_i = \frac{p_i(1-p_j)-p_j(1-p_i)d}{[p_i(1-p_j)+p_j(1-p_i)](1-d)}.$$

The assumptions that p_i is bounded away from the boundaries and that d is small imply that $p_i(1-p_j) > dp_j(1-p_i)$. Therefore, $g_i \in (0,1)$, $i=1,2$, and equilibrium of this form in which each attribute is examined by some subset exists for all the parameter configurations in the range we are considering.

(ii) Consider now the possibility of an equilibrium in which all searchers examine the same attribute. If they examine attribute 1, then $g_1 = 1$ and $g_2 = 0$. By the assumption that both p_i, $i=1,2$, are bounded away from 0 and 1 and the assumption that d is small, it follows from (23) that $f_{10} < f_{01}$. Therefore, $V_1 \geq V_2$ requires $f_{11} \geq d/(1-d)$. Since the explicit expression for f_{11} is

$$\frac{p_1p_2}{\dfrac{(1-p_1)(1-p_2)}{d} + \dfrac{p_1(1-p_2)}{g_1+g_2 d} + \dfrac{(1-p_1)p_2}{g_2+g_1 d} + p_1p_2},$$

$f_{11} \geq d/(1-d)$ is equivalent to

$$p_1(1+p_2) > \frac{1}{1-d}$$

Analogously, $V_1 \leq V_2$ is equivalent to $p_2(1+p_1) > \frac{1}{1-d}$. Thus, if $p_i(1+p_j) > \frac{1}{1-d}$, there exists an equilibrium in which all searchers examine attribute i.■

13 Appendix B

This appendix contains three diagrams that complement the derivations in the text.

Figure 3 illustrates how x^- and x^+ (from the proof of Proposition 3) are determined for $d = 0.1$.

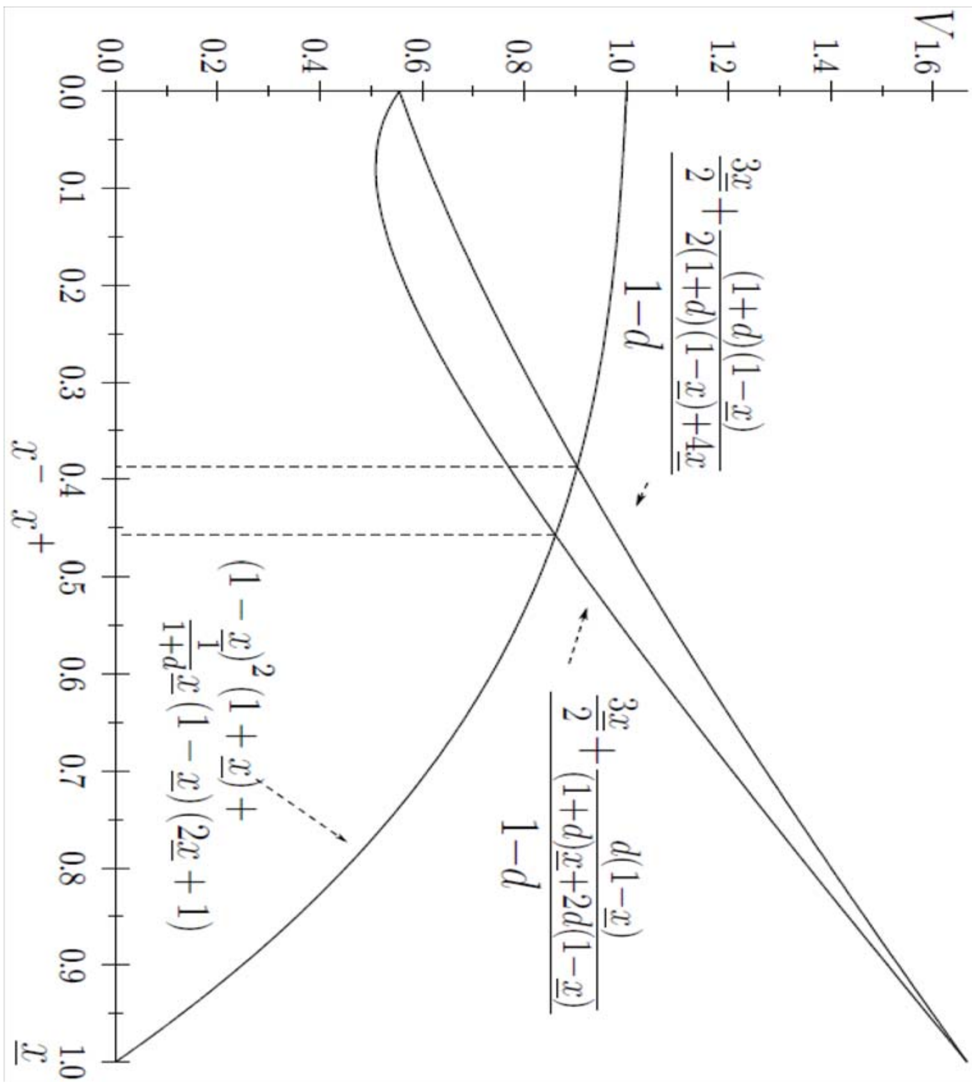

Figure 3

Figure 4 depicts the sets of non-symmetric equilibrium threshold pairs for $d = 0.1$ and $d = 0.3$.

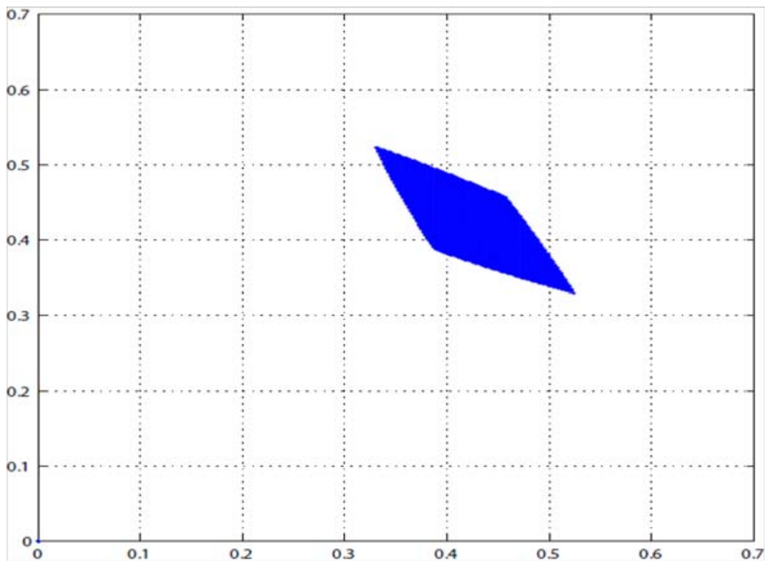

Figure 4(a)

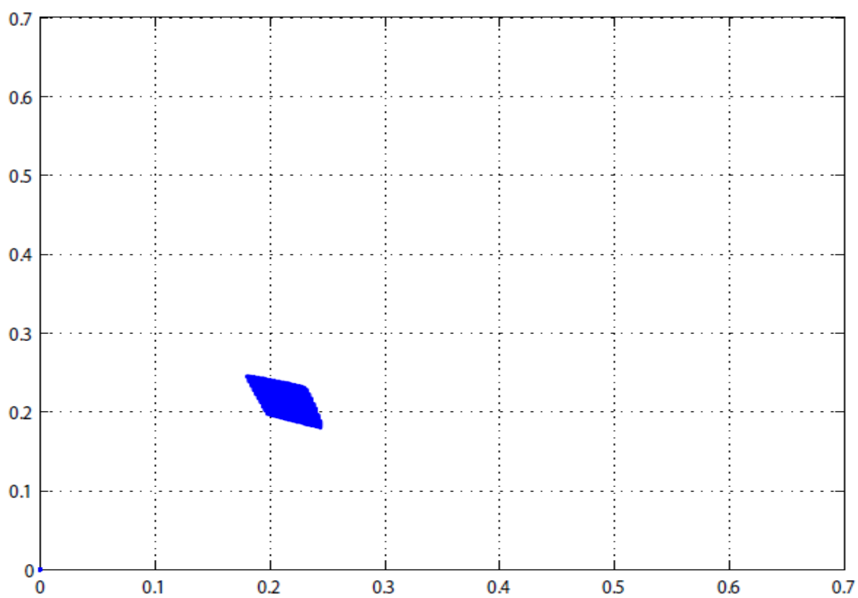

Figure 4(b)

Figure 5 depicts the graphs of the two functions from the end of Proposition 5

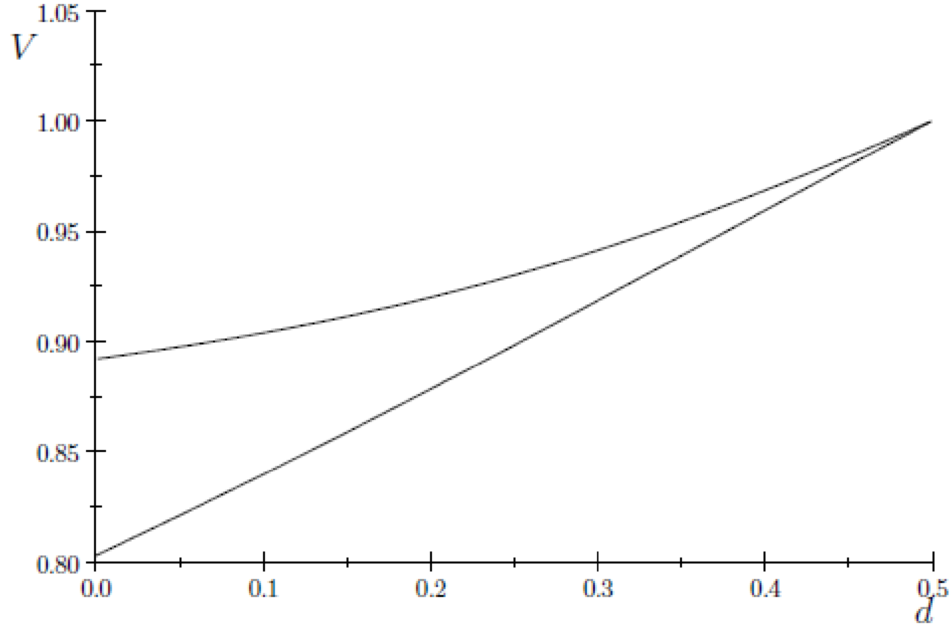

Figure 5