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On the Relationship Between Historic Cost, Forward Looking Cost and Long Run Marginal Cost

William P. Rogerson

Abstract

This paper considers a simple model where a regulated firm must make sunk investments in long-lived assets in order to produce output, assets exhibit a known but arbitrary pattern of depreciation, there are constant returns to scale within each period, and the replacement cost of assets is weakly falling over time due to technological progress. It is shown that a simple formula can be used to calculate the long run marginal cost of production each period and that the firm breaks even if prices are set equal to long run marginal cost. Furthermore, the formula for calculating long run marginal cost can be interpreted as a formula for calculating forward looking cost (where the current cost of using assets is based on the current replacement cost of assets). However, through appropriate choice of the accounting depreciation rule, it can also be interpreted as a formula for calculating historic cost (where the current cost of using assets is based on the historic purchase cost of assets). In particular, the results derived in the simple benchmark model of this paper contradict the commonly expressed view that measures of forward looking cost are superior to measures of historic cost in environments with declining asset prices.

KEYWORDS: historic cost, forward looking cost, long run marginal cost, cost allocation, depreciation

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1. INTRODUCTION

Under traditional rate of return regulation, the per period cost of using long-lived assets to produce goods and services is calculated by allocating the original purchase cost of each asset across all of the periods that the asset will be used. Since the cost of using long-lived assets in any given period will therefore depend on the original purchase costs of all of the assets being used in that period, such costs are often referred to as "historic costs." In the telecommunications industry, the replacement cost of many of the long-lived assets that firms use to produce telecommunications services has been dropping dramatically over time due to technological progress. Motivated by the intuition that historical costing methods may overstate the current long run marginal cost of production in this case, (because *current* long run marginal cost should depend on the *current* cost of replacing assets which is lower than the *historic* cost of purchasing assets), regulators in many countries, including the United States and most countries in Western Europe, have recently begun to base prices on costs calculated using the current replacement cost of assets instead of their original purchase cost. Costs calculated under such a methodology are often referred to as "forward looking costs."[1] Under a forward looking methodology, the regulator estimates the total cost of replacing the existing assets of the firm with functionally equivalent new assets and then allocates a share of the estimated total replacement cost to the current period.

Note that either method of calculating costs requires the regulator to make a decision on how to allocate costs. In the case of historic cost, the regulator must choose what share of the historic purchase price of an asset to allocate to each period of the asset's lifetime. This will be called the "historic allocation rule." In the case of forward looking cost, the regulator must choose what share of the estimated total replacement cost to allocate to the current period. This will be called the "forward looking allocation share." There has been considerable controversy and confusion over the related issues of how investment costs should be allocated for purposes of calculating either type of cost, which costing method is superior for purposes of setting cost-based prices, and how the answer to these questions depends on factors such as the rate of technological progress and the depreciation pattern of the underlying assets. This paper provides a theory which addresses these questions.

In particular, two major results are proven. First, it is shown that a very simple formula can be used to calculate the long run marginal cost of production in

[1] In the United States this cost concept is often referred to as Total Element Long Run Incremental Cost (TELRIC) and in Western Europe, New Zealand, and Australia it is often referred to as Total Service Long Run Incremental Cost (TSLRIC). See Salinger (1998), Hausman (2000), Falch (2002), Rosston and Noll (2002), Tardiff (2002) and Federal Communications Commission (2003) as well as other references cited below for further discussion and institutional background.

each period, and that the firm breaks even if price each period is set equal to long run marginal cost. Second, it is shown that the formula for calculating long run marginal cost can be interpreted either as a formula for calculating forward looking cost or as a formula for calculating historic cost so long as the correct method for allocating costs is used in either case. For the case of historic cost, the correct historic cost allocation rule is a relatively simple and natural rule which is called the relative replacement cost (RRC) rule. For the case of forward looking cost, the correct forward looking allocation share is the unique forward looking allocation share that allows the firm to break even.

Note that the results of this paper show that *either* method of calculating costs can be used to set optimal prices *regardless* of the rate of technological progress.[2] In particular, the intuition that basing prices on forward looking cost somehow becomes more appropriate as the rate of technological progress increases is shown to be false in the simple benchmark model of this paper. This result potentially has important public policy implications. It can be argued that one advantage that historically based costing rules have over forward looking rules in real-world applications is that they are based on more objective data and thus reduce the cost of regulatory proceedings and also allow regulators to make more binding commitments. If forward looking rules do not offer some compensating advantage, then it is not clear why they should be used.

In the formal model of this paper it is assumed that assets have a known but arbitrary depreciation pattern and that the purchase price of new assets decreases at a known constant rate over time. The RRC allocation rule is defined to be the unique allocation rule that satisfies the two properties that (i) the share of cost allocated to each period is proportional to the cost of replacing the surviving amount of the asset with new assets and (ii) the present discounted value of the cost allocations calculated using the firm's cost of capital is equal to the original purchase price of the asset. Property (i) can be interpreted as a method of matching the time pattern of cost allocations for an asset to the time pattern of benefits created by the asset. Property (ii) simply requires that the firm be fully reimbursed for its investments. While the RRC allocation rule is somewhat different than the sorts of allocation rules traditionally used in rate-of-return regulation, it is intuitively reasonable, simple, and easy to calculate, and would therefore be extremely suitable for use in real regulatory proceedings.

The welfare maximization problem of a social planner (choosing prices to maximize discounted total surplus) is very similar in structure to the profit

[2] Of course, as will be seen, in either case the correct method of cost allocation depends on the rate of technological progress.

maximization problem of an unregulated firm (choosing prices to maximize discounted cash flows) studied in the optimal investment literature. Therefore the results of this paper do not require the development of new analytic techniques, but instead follow fairly directly from existing results in the optimal investment literature that show how the concept of "user cost" can be employed to dramatically simplify the analysis of investment problems.[3] For this paper's purposes, a significant limitation of most of this literature is that it restricts itself to considering the case of exponential depreciation, where a constant fraction of the capital stock is assumed to depreciate each period. This assumption dramatically simplifies the analysis because the age profile of the existing capital stock can be ignored. However, for the purposes of this paper's study of cost allocation rules, it is important to allow for general patterns of depreciation, because one of the most interesting questions to investigate regarding cost allocation rules is how the nature of the appropriate cost allocation rule should change as the depreciation pattern of the underlying assets changes. Obviously the pattern of depreciation must be a factor which can be exogenously varied in order to investigate this question. Furthermore, the case of exponential depreciation is not a particularly natural case to consider for applications, since most regulators assume that depreciation occurs according to the so-called one-hoss shay pattern where assets have a fixed lifetime and remain equally useful over their lifetime. In an early paper, Arrow (1964) showed that the user cost approach could actually be generalized to apply to the case of general patterns of depreciation. Recently, Rogerson (2008) has extended these results by deriving a simpler formula for user cost and showing that the RRC allocation rule can be used to calculate user cost.

This paper's results follow from the results in Arrow (1964) and Rogerson (2008). In the context of this paper's model, the Arrow/Rogerson results provide a simple formula for calculating a vector of "user costs" for any pattern of depreciation such that the present discounted cost of producing any vector of outputs can be calculated by assuming that the firm has a constant marginal cost of production each period equal to that period's user cost.[4] Thus, the seemingly complex dynamic optimal investment problem actually collapses into a series of additively separable single period problems where the firm has a constant marginal cost of production each period. Obviously, setting price each period equal to user cost generates efficient consumption and allows the firm to break even. It turns out that user cost in any period is equal to a constant multiplied by the total cost of purchasing a unit

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[3] See Jorgensen (1963) for an early treatment and Abel (1990) for an extensive literature review.

[4] Note, however, that this paper presents a different and more direct derivation of the vector of user costs than in presented in Rogerson (2008). See footnote 13 for details.

of the asset in that period. By definition, this formula can therefore be interpreted as a formula for calculating forward looking cost where the constant is the forward looking allocation share.[5] Rogerson's (2008) result, that the RRC allocation rule can be used to calculate user cost, directly implies that the RRC allocation rule can also be used to calculate efficient prices.

For purposes of describing the incremental contributions of this paper to the literature on dynamic regulatory pricing, this paper can be viewed as showing that three different results hold true for any exogenously specified asset depreciation pattern:

(i) A simple formula exists to calculate rental rates for capital which can be interpreted as being hypothetical perfectly competitive rental rates in the sense that they allow the firm to break even on an investment made in any period.

(ii) The hypothetical perfectly competitive rental rate in any period is actually equal to the long run marginal cost of using capital in that period. Therefore the efficient price for output in any period is simply equal to the hypothetical perfectly competitive cost of renting sufficient capital to produce one unit of output.

(iii) If historic cost is calculated using a simple and natural allocation rule called the RRC allocation rule, the unit accounting cost of production in each period is equal to the long run marginal cost of production in that period. Thus cost-based pricing results in efficient prices when the RRC allocation rule is used to calculate costs.

The existing literature has established result (i) for the case of any pattern of depreciation and result (ii) for the case of exponential depreciation. Therefore the incremental contribution of this paper is to establish that result (ii) holds true for the case of depreciation patterns other than the exponential pattern and to establish result (iii). More specifically, Biglaiser and Riordan (2000), Laffont and Tirole (2000, Box 4.2, page 152), and Hausman (1997) consider the case of exponential depreciation and establish results (i) and (ii) for this case. Salinger (1998) derives the formula for perfectly competitive prices for the case of general patterns of depreciation and Mandy and Sharkey (2003) derive the formula for the special case of one-hoss shay depreciation but neither paper explicitly investigates the welfare problem (or equivalently, the problem of how to calculate the long run marginal cost of

[5] This is true when one unit of the asset is defined to be the amount of the asset necessary to produce one unit of output.

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production in any given period).[6] Regarding result (iii), it is well understood that, as an accounting identity, any pricing rule that allows the firm to break even can be thought of as being a cost-based rule for some method of depreciating assets over time.[7] Therefore result (ii) immediately implies that there must be some method of allocating investment costs with the property that prices will be efficient if they are set equal to historic costs calculated using this allocation method. The contribution of this paper is to show that the allocation rule takes a remarkably simple and natural form that would be suitable for use in practice.

The main formal model in the economics literature that analyzes the welfare effects of inter-temporal cost allocation rules in the context of cost-based regulation is due to Baumol (1971).[8] Baumol suppresses the issue of long run efficiency by simply assuming that the firm has already made a single fixed exogenous expenditure on long-lived assets and no further investment of any sort is possible. It is assumed that the firm can vary its output from period to period only by varying the amount of non-capital inputs it uses each period. In this analysis, it would be efficient to set price each period equal to short run marginal cost and, in general, setting prices at this level would not allow the firm to recover its investment cost. Baumol solves for a second best price path that maximizes total surplus subject to the constraint that the firm must be allowed to recover its investment cost. Therefore in Baumol's model, allocating the cost of long-lived investments across time is a sort of "necessary evil" that has to be endured in order that the firm be reimbursed for its investment expenses. This paper shows that a dramatically different sort of result can occur in a model where it is assumed that investment occurs every period and the issue of long run efficiency is considered. Namely, allocating the cost of long-lived investments across time can help play a role in making consumption decisions more efficient by ensuring that prices reflect long run marginal cost.

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[6] Also, see Mandy (2002) for an estimation of the extent to which prices set under existing regulatory practices diverge from hypothetical perfectly competitive prices.

[7] The relevant depreciation method is simply the "Hotelling" or "economic" depreciation associated with the stream of revenues generated by the pricing rule. Given any fixed stream of revenues, the Hotelling or economic depreciation in any period is defined to be the change in the present discounted value of the remaining revenue stream. See Hotelling (1925) for the original treatment of this concept of depreciation. See Schmalensee (1989) for a clear treatment of the role of this concept in cost-based pricing. Salinger (1998) and Biglaiser and Riordan (2000) both note that the pricing rules they identify could in principle be implemented by a cost based pricing rule.

[8] Also see Crew and Kleindorfer (1992) who consider the issue that it may be necessary to front-load the reimbursement of a firm's investment if future entry of competitors is expected. Rogerson (1992) analyzes the effect of various depreciation rules on a regulated firm's incentives to choose an efficient level of investment in the presence of regulatory lag.

In a recent paper, Guthrie, Small and Wright (2006)compare the performance of access pricing rules based on forward looking vs. historic cost measures and conclude that historic access prices generally create superior investment incentives to forward looking access prices. Guthrie, Small and Wright focus on the effects of uncertainty and the option value created by uncertainty and abstract away from issues relating to ongoing investment by assuming that there is a one-time investment. This paper, in contrast, focuses on issues created by ongoing investment and abstracts away from issues related to uncertainty by considering a model with no uncertainty. The two papers thus provide complementary analyses of different economic factors that affect the relative performance of forward looking vs. historic cost measures.

The paper is organized as follows. Section 2 describes the basic model. Section 3 describes the relevant user cost result from the optimal investment literature and its implications for the efficient pricing rule in the model of this paper. Section 4 briefly discusses the effect of changes in the rate of technological progress on long run marginal cost. Section 5 considers forward looking pricing rules. Section 6 considers historic pricing rules. Section 7 considers policy implications for the choice between forward looking vs. historic pricing rules. Section 8 draws a brief conclusion. More technical proofs are contained in an appendix.

2. THE MODEL

Let $q_t \in [0, \infty)$ denote the firm's output in period $t \in \{1, 2, \ldots\}$ and let $\mathbf{q} = (q_1, q_2, \ldots)$ denote the vector of outputs. Similarly, let $K_t \in [0, \infty)$ denote the firm's capital stock in period $t \in \{1, 2, \ldots\}$ and let $\mathbf{K} = (K_1, K_2, \ldots)$ denote the vector of capital stocks. Assume that a capital stock of K_t in period t allows the firm to produce up to K_t units of output in period t. For simplicity, assume that no other inputs are required to produce output and the firm has no assets at the beginning of period 0.

Let $I_t \in [0, \infty)$ denote the number of units of capital that the firm purchases in period $t \in \{0, 1, \ldots\}$ and let $\mathbf{I} = (I_0, I_1, \ldots)$ denote the entire vector of investments. Assume that a unit of capital becomes available for use one period after it is purchased and then gradually wears out or depreciates over time. It will be convenient to use notation that directly defines the share of the asset that survives, and is thus available for use in each period, rather than the share that depreciates. Let s_t denote the share of an asset that survives until at least the t^{th} period of the asset's lifetime and let $\mathbf{s} = (s_1, s_2, \ldots)$ denote the entire vector of survival shares. Then the vector of capital stocks generated by any vector of investments is given by

$$(1) \qquad K_t = \sum_{i=1}^{t} s_i I_{t-i}$$

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Assume that $s_t \in [0,1]$ for every t, $s_1 = 1$, and that s_t is weakly decreasing in t. Two natural and simple examples of depreciation patterns are the cases of exponential depreciation given by

$$(2) \qquad s_t = \beta^{t-1}$$

for some $\beta \in (0,1)$ and one-hoss shay depreciation given by

$$(3) \qquad s_t = \begin{cases} 1, & t \in \{1, 2, \ldots, T\} \\ 0, & \text{otherwise} \end{cases}$$

where T is a positive integer.

Let $\delta \in (0,1)$ denote the discount factor. Let $z_t \in [0, \infty)$ denote the price of purchasing a new unit of the asset in period t. Assume that asset prices are weakly decreasing and change at a constant rate over time. Formally, assume that asset prices are given by

$$(4) \qquad z_t = z_0 \alpha^t$$

for some $\alpha \in (0, 1]$ and $z_0 \in (0, \infty)$.[9] Thus, lower values of α correspond to higher rates of technological progress. Let $E(\mathbf{I})$ denote the present discounted value of the expenditures required to create the vector of investments **I**, given by

$$(5) \qquad E(\mathbf{I}) = z_0 \sum_{t=0}^{\infty} I_t (\delta\alpha)^t$$

A vector of investments **I** will be said to be efficient for a vector of outputs **q** if it minimizes the expected discounted cost of investment subject to the constraint that sufficient capital is available every period to allow production of **q** and subject to the constraint that investment every period must be non-negative. The

[9] The assumption that asset prices change at a constant rate is necessary for some but not all of the conclusions of this analysis. The basic user cost result can still be derived in the general case where it is only assumed that asset purchase prices are weakly decreasing over time. It is still true that it is efficient to set price each period equal to that period's user cost and that the formula for calculating efficient prices can be interpreted as being either a formula for calculating forward looking cost or historic cost. However the formulas are much more complicated and have no simple interpretation. Thus, although the basic conclusion that the efficient pricing rule can be interpreted as being a formula for calculating either forward-looking or historic cost remains correct, the formulas are no longer simple enough to be obviously suitable for applied use. See Rogerson (2008) for details.

assumptions that $\alpha \leq 1$ and $\delta < 1$ imply that it will never be efficient for the firm to stockpile assets ahead of time. Therefore, for any vector of outputs, the unique efficient vector of investments can be calculated sequentially beginning with period 0. In each period the firm purchases the minimum number of assets required produce next period's output. (If next period's capital stock will already be greater than the required level without any investment, then no assets are purchased.) Let $\phi_t(q_1, \ldots, q_{t+1})$ denote the efficient choice of I_t and let $\phi(\mathbf{q}) = (\phi_0(q_1), \phi_1(q_1, q_2), \ldots)$ denote the entire vector of efficient investment choices. Let C(q) denote the minimum cost of producing the vector of outputs q. This will be called the firm's cost function and is defined by

$$(6) \qquad C(\mathbf{q}) \quad = \quad E(\phi(\mathbf{q})) \quad = \quad z_0 \sum_{t=0}^{\infty} \phi_t(q_1, \ldots, q_{t+1})\, (\delta\alpha)^t$$

A vector of outputs will be said to satisfy the fully utilized investment (FUI) property if there is never any excess capacity when the vector of outputs is produced efficiently. Formally, **q** satisfies the FUI property if

$$(7) \qquad q_t \quad = \quad \sum_{i=1}^{t} s_i \phi_{t-i}(q_1, \ldots, q_{t-i+1})$$

For future reference, note that a sufficient condition for a vector of outputs to satisfy the FUI property is that output be weakly increasing over time.[10]

Let $p_t \in [0, \infty)$ denote the price of output in period t and let $P_t: (0, \infty) \to [0, \infty)$ denote the period t inverse demand function. Assume that $P_t(q_t)$ is greater than or equal to zero, strictly decreasing and differentiable where it is strictly positive, that $P_t(q_t)$ converges to 0 as q_t converges to ∞, and that $P_t(q_t)$ converges to ∞ as q_t converges to 0.[11] Let $D_t(p_t)$ denote the period t demand function. One assumption with real economic content will need to be made about demand. This is that demand is weakly increasing over time. Formally, it will be assumed that

[10] The proof is by induction. The firm obviously operates with no excess capacity in period 1. It is also easy to see that if output is weakly increasing over time and the firm operates with no excess capacity in period t, then it will operate with no excess capacity in period t+1.

[11] The last assumption is made simply to avoid the extra notational burden of describing corner solutions at $q_t = 0$.

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(8) $$D_{t+1}(p) \geq D_t(p) \qquad \text{for every } p \in [0, \infty) \text{ and } t \in \{1, 2, \ldots\}.$$

Let $B_t(q_t)$ denote the consumer benefit of output q_t in period t, given by

(9) $$B_t(q_t) = \int_{x=0}^{q_t} P_t(x)dx.$$

Let B(**q**) denote the discounted social consumer benefit of the vector of outputs **q** given by

(10) $$B(\mathbf{q}) = \sum_{t=1}^{\infty} B_t(q_t)\,\delta^t$$

Finally let W(**q**) denote the discounted welfare of the vector of outputs **q** given by

(11) $$W(\mathbf{q}) = B(\mathbf{q}) - C(\mathbf{q})$$

A vector of quantities will be said to be efficient if it maximizes W(**q**). A vector of prices will be said to be efficient if it induces consumers to demand an efficient vector of quantities.

It will be useful to introduce one additional piece of notation. Let π(**p**) denote the present discounted value of the firm's cash flows if it charges prices according to **p** and invests efficiently to supply all demand at these prices. It is given by

(12) $$\pi(\mathbf{p}) = \sum_{t=1}^{\infty} D_t(p_t)p_t\delta^t - C(D(\mathbf{p}))$$

It will be said that the firm earns zero (postive; negative) profit at **p** if π(**p**) = (>; <) 0.

Finally, it is worth drawing attention to two assumptions implicitly introduced in the description of the model which likely play an important role in generating the specific results of this paper. The first assumption is that the cost of acquiring capital in any given period is linear and that capital can be acquired in infinitely divisible quantities. If there were economies or diseconomies of scale in acquiring capital in any given period, or if capital was "lumpy," it would be necessary to look multiple periods ahead to determine the correct level of investment in any given period and the basic "user cost" result from the optimal investment literature would no longer hold. The second assumption is that there is no uncertainty. The optimal investment problem changes dramatically in a number of ways once uncertainty is introduced. Investigating the effects of nonlinear capital acquisition costs and uncertainty on efficient pricing rules and the extent to which they can be achieved by forward looking vs. historic cost measures is an interesting and important subject for future research.[12]

3. THE USER COST RESULT AND EFFICIENT PRICES

Suppose for a moment that, instead of having to purchase long-lived assets, the firm could rent assets on a period-by-period basis and the cost of renting one unit of the asset in period t was equal to c_t. In this case, the welfare problem would collapse into a series of simple additively separable single period problems where the firm has a constant marginal cost of production in period t equal to c_t. Obviously the efficient solution would be for the firm to charge a price of c_t in period t and the firm would break even at this solution. The essential result of the user cost approach is that a very simple formula exists to calculate a vector of hypothetical perfectly competitive rental prices or user costs and that, over the relevant range of output, the firm's true cost function, given that it must purchase assets, is actually equal to the hypothetical cost function it would have if it could rent assets at these rates. In particular, over the relevant range of output the firm has a constant marginal cost of production in each period equal to the hypothetical perfectly competitive rental rate of capital or user cost in that period. The efficient solution is therefore for the firm to charge a price for output in each period equal to that period's hypothetical perfectly competitive rental rate or user cost and the firm breaks even at this solution.

To derive the formula for the hypothetical perfectly competitive rental rates, consider a hypothetical situation where there is a rental market for assets and a supplier of rental services can enter the market in any period by purchasing one unit of the asset and then renting out the available capital stock over the asset's life. Let c_t denote the price of renting one unit of capital stock in period t and let $\mathbf{c} = (c_1, c_2, \ldots)$ denote the entire vector of rental prices. Assume that suppliers incur no extra costs besides the cost of purchasing the asset, that they can rent the full remaining amount of the asset every period and that their discount factor is equal to δ. Then the zero profit condition that must be satisfied by a perfectly competitive equilibrium is

[12] As mentioned in the introduction, Guthrie, Small and Wright (2006)compare the performance of access pricing rules based on forward looking vs. historic cost measures in a model which focuses on the effects of uncertainty.

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(13) $$z_t = \sum_{i=1}^{\infty} c_{t+i}\, s_i \delta^i \qquad \text{for every } t \in \{0, 1, 2, \ldots\}$$

After substituting equation (4) into (13), it is straightforward to verify that the vector of rental rates **c*** = (c_1*, c_2*, . . .) as defined below by equations (14)-(15) satisfies equation (13).

(14) $$c_t^* = k^* z_t$$

(15) $$k^* = 1/ \sum_{i=1}^{\infty} s_i (\alpha\delta)^i$$

The vector of rental rates **c*** will be called the vector of hypothetical perfectly competitive rental rates or the vector of user costs. Note that the rental rate in period t is equal to the positive constant k* multiplied by the cost of purchasing a unit of capital in that period. Therefore, all of the rental rates are strictly positive and they decline at the same rate that the purchase price of assets declines at.

Let H(**q**) denote the cost function that the firm would have if, instead of having to purchase assets, it was able to rent assets at the hypothetical perfectly competitive rental rates.

(16) $$H(\mathbf{q}) = \sum_{t=1}^{\infty} c_t^* q_t \delta^t$$

This will often be referred to simply as the "hypothetical" cost function. Recall that C(q) denotes the firm's cost function given that it is unable to rent assets but instead must purchase them. This will sometime be referred to as the firms "true" cost function to distinguish it from the hypothetical cost function. Let **p*** and **q*** denote the unique vectors of prices and quantities that would be efficient in the hypothetical case where the firm could rent assets at the hypothetical perfectly competitive rental rates. These are obviously determined by

(17) $$p_t^* = c_t^*$$

(18) $$q_t^* = D_t(c_t^*)$$

Furthermore, it is also obvious that the firm would break even at this solution.

If it could be shown that the firm's true cost function was always equal to the hypothetical cost function, it would therefore follow immediately that **p*** and **q*** were also the unique vectors of efficient prices and quantities for the true case of interest and that the firm breaks even at these prices. It turns out that only a somewhat weaker relationship between the two cost functions can be established.

Lemma 1:

(19) $C(\mathbf{q}) = H(\mathbf{q})$ for every $\mathbf{q}$ satisfying the FUI property

(20) $C(\mathbf{q}) \geq H(\mathbf{q})$ for every $\mathbf{q} \in [0, \infty)^{\infty}$

Proof:
See Appendix. QED

Equation (19) states the firm's true cost function is equal to the hypothetical cost function for every vector of outputs satisfying the FUI property. Equation (20) states that the firm's true cost function is always greater than or equal to the hypothetical cost function. Both results are very intuitive. The zero profit condition (13) can be interpreted as stating that the cost of purchasing any single asset must be equal to the hypothetical cost of renting assets to produce the vector of outputs that would be produced if the asset was fully utilized. Since the rental rates are all positive this also means that the cost of purchasing any single asset must be greater than or equal to the hypothetical cost of renting assets to produce any vector of outputs that this asset was able to produce. Obviously, the same conditions must hold for the present discounted value of any sequence of assets purchased over time, which is what is stated in equations (19) and (20). (The proof of Proposition 1 in the appendix is simply a formalization of this reasoning.)

It turns out that Lemma 1 is sufficient to establish the result of interest. The reason for this is that the assumption that demand is weakly increasing implies that the solution to the hypothetical welfare problem, **q*,** is also weakly increasing and therefore satisfies the FUI property. It follows immediately from this and Lemma 1 that **q*** must therefore also be the solution to the true welfare problem.

Proposition 1:
The unique vectors of efficient prices and quantities are **p*** and **q***. The firm breaks even at this solution, i.e, $\pi(\mathbf{p}^*) = 0$.

Proof:
See Appendix. QED

In summary, the user cost approach essentially shows that the firm's cost function is linear and additively separable in each period's output over the relevant range of outputs. Therefore the seemingly complex multi-period problem actually collapses into a series of additively separable single period problems. Welfare is maximized by setting price each period equal to the marginal cost of production and the firm breaks even at these prices. Furthermore, there is a simple formula to calculate the marginal cost of production in any period. [13]

The result that the firm's cost function is linear and additively separable over a broad range of outputs might initially seem somewhat surprising in light of the fact that each asset represents a joint cost of production across multiple periods. A widely accepted general principle in both the economics and managerial accounting literatures that study cost allocation is that there is generally no economically meaningful way assign a joint cost to individual products.[14] Thus we might expect that the cost of producing a vector of joint products would inherently not be additively separable in each product. Yet this is precisely what happens in the model of this paper. The resolution to this apparent conflict lies in the fact that there are "multiple overlapping" joint costs in the model of this paper instead of a single joint cost. When there is a single joint cost for all products, the only way to increase the output of a single product is to increase investment in the joint cost and this results in increased output of all products. Thus, increasing the production of one good necessarily results in increases in the production of all goods. However, in the model

[13] It is possible to directly calculate the vector of user costs without using the zero profit condition (13) by inverting the linear function in equation (1) to directly calculate the coefficients of the linear function ϕ and substitute these into (4) to directly calculate $C(\mathbf{q})$ as a linear function of $\mathbf{q}$. This is the approach originally used by Arrow (1964) and yields a formula for user cost that depends on the coefficients of ϕ. While the coefficients of ϕ have a natural interpretation (they are the series of marginal changes to investment that the firm would have to make in order to increase the stock of capital in one period while holding the stock of capital in all other periods fixed) they are difficult to calculate because they are determined by an infinite series of recursively defined equations. Rogerson (2008) presents Arrow's derivation and then directly shows that the more simple formula for user cost in (14)-(15) is equivalent to Arrow's more complex formula. As seen above, this paper takes a different approach which completely avoids calculating the coefficients of ϕ and avoids deriving the more complex formula as an intermediate stage. It instead directly observes that the vector of rental rates defined by (14)-(15) satisfies the zero profit condition (13) and then uses (13) to directly show that these rental rates are equal to the marginal cost of production over the relevant rage of output. While the approach presented in this paper is much simpler, Arrow's original approach provides some extra intuition because it shows that the marginal cost of producing one more unit of output in any period is equal to the present discounted value of the series of marginal changes to investment that would produce one more unit of capital in that period while holding the level of capital fixed in all other periods. See Rogerson (2008) for more details.

[14] See, for example, Demski (1981), Thomas (1978), and Young (1985).

of this paper, where there are multiple overlapping joint costs, this is not necessarily true.

An illuminating way to see this point is to directly calculate marginal cost for a simple example by directly determining the adjustments in asset purchases that are necessary to produce an additional unit of output in a given period while holding output in all other periods fixed. The present discounted value of these adjustments is, of course, by definition the marginal cost of production. Consider, for example, the case of the one-hoss shay pattern of asset decay given by equation (3), where an asset lasts with undiminished productivity for T years. Suppose that the firm has made investment plans to produce a particular vector of outputs over time and is engaging in at least one unit of investment in each period. Then the firm can increase output in period 1 by one unit while holding output in all other periods constant by implementing the following series of adjustments to its investment plans. The firm must purchase an additional unit of the asset in period 0 to increase production by one unit in period 1. However, it will now be able to reduce its asset purchases by one unit in period 1. Now when period T arrives, the extra asset that the firm purchased in period 0 will no longer be available the next period, so the firm will have to purchase an extra unit of the asset in that period to maintain its level of production at the previously planned level in period T+1. However, as before, it will now be able to reduce its asset purchases by one unit in period T+1. This process continues indefinitely. That is, the firm can produce exactly one more unit of output in the period 1 and hold output in all other periods fixed by shifting the purchase of one unit of the asset forward in time from period 1 to 0, T+1 to T, 2T+1 to 2T, etc. The present discounted value calculated in period 1 of the cost of these adjustments is, by definition, the marginal cost of increasing output by one unit in period 1. It is straightforward to directly calculate this value and show that it is equal to k^*z_1.

Thus, even though each asset can be viewed as a joint cost of production over multiple periods, it is still possible to increase production in one period while holding output in all other periods constant by adjusting the entire vector of overlapping joint costs. The result is that the cost function is linear and additively separable even though there are joint costs of production.

4. THE EFFECT OF THE RATE OF TECHNOLOGICAL PROGRESS ON MARGINAL COST

The effect of changing the rate of technological progress on the vector of marginal costs and thus on the vector of efficient prices can now be investigated. Recall that lower values of α correspond to higher rates of technological progress in the sense that asset prices fall more rapidly. Since a higher rate of technological progress strictly reduces the purchase price of assets in all periods subsequent to period 0, it

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is clear that an increase in the rate of technological progress must always reduce the present discounted cost of producing any vector of outputs whose production requires any investment after period 0. Based on this observation, one might suspect that an increase in the rate of technological progress would therefore reduce the marginal cost of production in every period. This turns out *not* to be true. Instead, it is always the case that an increase in the rate of technological progress *increases* the marginal cost of production in periods immediately following the change and only reduces the marginal cost of production in later periods.

To see this result, rewrite equation (14) so that the user cost in any period is expressed as a function of the purchase price of assets in the previous period. This yields

$$c_t^* = k^{**} z_{t-1} \tag{21}$$

$$k^{**} = k^*\alpha = 1/\sum_{i=1}^{\infty} s_i \alpha^{i-1}\delta^i \tag{22}$$

It is easy to see that k** increases in the rate of technological progress (i.e., k** decreases in α). Obviously, the price of assets in the current period, z_0, does not change with the rate of technological progress. However, the prices of assets in all subsequent periods decrease in the rate of technological progress (i.e. they increase in α). Therefore an increase in the rate of technological progress will unambiguously increase the marginal cost of production for period 1. However, its effect on marginal costs in subsequent periods will be ambiguous because the increase in k** will be counteracted by a decrease in asset prices. The decrease in asset prices will grow more significant over time as the increased rate of technological progress operates over more periods. Therefore, we would expect the second effect to eventually dominate for periods far enough into the future. That is, we would expect an increase in the rate of technological progress to raise marginal cost in early periods but to decrease marginal costs in later periods. Proposition 2 formally states this result.

Proposition 2:
Let $c_t^*(\alpha)$ denote the user cost in period t given α. Then there exists a value $\tau(\alpha) \in (1, \infty)$ defined by equation (49) in the appendix such that

$$c_t^{*\prime}(\alpha) \begin{matrix} > \\ = \\ < \end{matrix} 0 \quad \Leftrightarrow \quad t \begin{matrix} > \\ = \\ < \end{matrix} \tau(\alpha) \tag{23}$$

Proof:
See Appendix. QED

Thus, even though an increase in the rate of technological progress will reduce the total discounted cost of producing any given vector of outputs, it will actually increase the marginal cost of production in early periods. The explanation for this result is that the firm produces more output in the current period by shifting purchases of assets from the future to the present. When there is a higher rate of technological progress, asset prices decrease more rapidly over time, and the opportunity cost of shifting asset purchases ahead in time is therefore higher. In the first period of production this is the only effect and marginal cost therefore rises. A second effect that becomes more important over time is that technological progress will reduce marginal cost by reducing the future purchase price of assets. This second effect eventually dominates and causes the marginal cost of production to fall in periods far enough in the future.

5. FORWARD LOOKING PRICES

Recall that forward looking cost in a given period is determined by first determining the total cost that the firm would incur to purchase sufficient new assets to produce the desired level of output and then allocating a share of this cost to the given period. The share of the total hypothetical cost allocated to the current period is called the forward looking allocation share. In the simple model of this paper where one unit of the asset is required to produce one unit of output, a forward looking pricing method must therefore be a rule of the form

$$p_t = k_t z_t \tag{24}$$

where k_t is the forward looking allocation share in period t. That is, a forward looking pricing rule is simply a rule specifying a vector of constants $\mathbf{k} = (k_1, k_2, \ldots)$ where the regulated price in period t is set equal to the share k_t of the price of purchasing assets, and k_t is the forward looking allocation share in period t. Define a stationary forward looking rule to be a rule that uses the same forward looking allocation share in each period.

A comparison of (24) and (14) shows that the efficient vector of prices is produced if and only if k_t is set equal to the constant k^* in every period. That is, there is unique forward looking allocation rule that generates efficient prices and it is the stationary rule where the forward looking allocation share is set equal to k^* in every period. Now consider any stationary forward looking rule that uses the allocation share k in every period. Obviously prices set equal to forward looking cost

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calculated using the forward looking allocation share k will be strictly greater than (equal to, strictly less than) the efficient vector of prices if and only if k is strictly greater than (equal to, strictly less than) k*. It has already been observed that the firm earns zero profit if k is set equal to k* and it is easy to see that the firm earns strictly positive (strictly negative) profit if k is strictly greater than (strictly less than) k*.[15] It therefore follows that setting k=k* yields the unique stationary forward looking allocation rule that causes the firm to earn zero profit and it will earn higher (lower) profit if k is set higher (lower) than k*. Proposition 3 summarizes these conclusions.

Proposition 3:
Suppose that a regulator sets prices equal to forward looking cost calculated using the forward looking allocation share k. Then

(i) The resulting prices will be efficient if and only if k_t is set equal to k* defined by (15)

(ii) Consider a stationary forward looking allocation rule where the forward looking allocation share is set equal to k in each period. Setting k=k* yields the unique stationary forward looking allocation rule that causes the firm to earn zero profit and it will earn higher (lower) profit if k is set higher (lower) than k*.

Proof:
As above. QED

6. Historic Cost

6.1. Allocation and Depreciation Rules And Historic Cost

Define a depreciation rule to be a vector **d** = $(d_1, d_2, \ldots)$ such that $d_i \geq 0$ for every i and

$$\sum_{i=1}^{\infty} d_i = 1 \tag{25}$$

where d_i is interpreted as the share of depreciation allocated to the i^{th} period of the

[15] Forward looking prices change at the same rate as asset prices for any value of k. This means that the resulting vector of quantities satisfies the FUI property. The conclusion follows immediately from this.

asset's life. Define an allocation rule to be a vector $\mathbf{a} = (a_1, a_2, \ldots)$ that satisfies $a_i \geq 0$ for every i and

$$(26) \qquad \sum_{i=1}^{\infty} a_i \gamma^i = 1$$

for some discount factor $\gamma \in (0,1)$. Let $\Gamma(\mathbf{a})$ denote the value of γ such that (26) is satisfied. The allocation rule **a** will be said to be complete with respect to the discount factor $\Gamma(\mathbf{a})$.

Regulators generally think of themselves as directly choosing a depreciation rule and a discount factor instead of directly choosing an allocation rule. The cost allocated to each period is then calculated as the sum of the depreciation allocated to that period plus imputed interest on the remaining (non-depreciated) book value of the asset. Formally, for any depreciation rule **d** and discount factor γ, the corresponding allocation rule is given by

$$(27) \qquad a_i = d_i + \{(1-\gamma)/\gamma\} \sum_{j=i}^{\infty} d_j .$$

It is straightforward to verify that the resulting allocation rule determined by (27) is complete with respect to γ. It is also straightforward to verify that for any allocation rule, **a** , there is a unique $(\mathbf{d}, \gamma)$ such that (27) maps $(\mathbf{d}, \gamma)$ into **a**. It is defined by $\gamma = \Gamma(\mathbf{a})$ and

$$(28) \qquad d_i = \sum_{j=i+1}^{\infty} \gamma^{j-i} a_j - \sum_{j=i+2}^{\infty} \gamma^{j-i-1} a_j$$

Therefore one can equivalently think of the regulator as choosing either a depreciation rule and discount factor or as choosing an allocation rule. For the purposes of this paper, it will be more convenient to view the firm as directly choosing an allocation rule.

Let $A_t(I_0, \ldots, I_{t-1}, \mathbf{a})$ denote the total accounting cost assigned to period t given the vector of investments $(I_0, \ldots, I_{t-1})$ and the allocation rule **a**. It is defined by

$$(29) \qquad A_t(I_0, \ldots, I_{t-1}, \mathbf{a}) = \sum_{i=1}^{t} I_{t-i} z_{t-i} a_i$$

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6.2. The RRC Allocation Rule

An allocation rule **a** = (a_1, a_2, . . .) can be said to allocate costs in proportion to the cost of replacing the surviving amount of the asset with new assets if it satisfies

$$(30) \qquad a_i = k s_i \alpha^i$$

for some positive real number k. It is easy to verify that an allocation rule of the form in (30) is complete with respect to δ if and only if the constant k is equal to the value k* defined by (15). Let **a*** denote the allocation rule determined by setting k equal to k*, i.e.,

$$(31) \qquad a_i^* = k^* s_i \alpha^i$$

This will be called the relative replacement cost (RRC) allocation rule. It is the unique allocation rule that satisfies the two properties that: (i) it allocates costs in proportion to replacing the surviving amount of the asset with new assets, and (ii) it is complete with respect to δ. Property (i) can be interpreted as a version of the "matching principle" from accrual accounting that suggests that costs should be allocated across the periods of an asset's lifetime in proportion to the benefits that the asset creates in each period where the benefit that an asset creates in a period is interpreted to be the avoided cost of purchasing new assets. Property (ii) is simply the requirement that the firm break even taking the time value of money into account.

For applied purposes, note that the RRC allocation rule takes the following simple form for the case where assets follow the one-hoss shay depreciation pattern defined by equation (3).

$$(32) \qquad a_i^* = \begin{cases} k^*\alpha^i, & i \in \{1, \ldots, T\} \\ 0, & i \in \{T+1, \ldots\} \end{cases}$$

That is, for the case of one-hoss shay depreciation, the cost of purchasing an asset is allocated across the periods of an asset's lifetime to satisfy the requirements that (i) the cost allocations decrease at the same rate that the purchase price of assets is decreasing at and (ii) the present discounted value of the cost allocations is equal to the initial purchase price of the asset.

While the RRC allocation rule is simple and intuitive, it is somewhat different than the sorts of allocation rules actually used in practice. As explained above, regulators generally view themselves as directly choosing a depreciation rule and then calculating the total cost allocated to any period as the sum of that period's depreciation plus interest on the non-depreciated book value. Perhaps for this reason, they have tended to focus directly on the time pattern of depreciation shares instead of the time pattern of allocation shares. In contrast, the approach suggested by this paper would require regulators to focus directly on the time pattern of allocation shares that a depreciation rule induces. While the RRC rule is therefore somewhat different than the sorts of rules traditionally used in practice, it is very simple, intuitively reasonable, and easy to calculate, and would therefore be very suitable for use in real regulatory proceedings.

Lemma 2 now describes a key property of the RRC allocation rule

Lemma 2:

(33) $$a_i{*}z_{t-i} = c_t{*}s_i \quad \text{for every } t \in \{1, 2, \ldots\} \text{ and } i \in \{1, 2, \ldots, t\}.$$

Proof:
Substitute equation (4) into equation (31) and reorganize. QED

To interpret equation (33), consider any period $t \in \{1, 2, \ldots\}$ and suppose that the firm purchases one unit of capital i periods earlier in period t-i for any $i \in \{1, \ldots, t\}$. The LHS of (33) is the accounting cost allocated to period t if the firm uses the RRC allocation rule. The RHS of equation (33) is the user cost in period t multiplied by the surviving share of the asset. Therefore equation (33) states that the RRC allocation rule has the property that the cost allocated to any period of an asset's lifetime is equal to that period's user cost multiplied by the surviving amount of the asset. That is, under the RRC allocation rule, the per unit accounting cost of capital in a given period is equal to that period's user cost regardless of when the capital was purchased! It follows immediately from this that, under the RRC allocation rule, the accounting cost in any period is simply equal to that period's user cost multiplied by that period's capital stock. This result is stated as Lemma 3.

Lemma 3:
Let **I** denote any vector of investments and let **K** denote the vector of capital stocks generated by **I** according to equation (1). Then

(34) $$A_t(I_0, \ldots, I_{t-1}, \mathbf{a}) = c_t{*}\,K_t \quad \text{for every } t \in \{1, 2, \ldots\}$$

Proof:
Substitute equation (31) into equation (29) and reorganize. QED

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6.3. Regulatory Equilibrium

An ordered pair of vectors of prices and outputs (**p, q**) will be defined to be a regulatory equilibrium given the allocation rule **a** if it satisfies the following two requirements.

(35) $$D_t(p_t) = q_t \qquad \text{for every } t \in \{1, 2, \ldots\}$$

(36) $$p_t q_t - A_t(q_1, \ldots, q_t, \mathbf{a}) \leq 0 \qquad \text{for every } t \in \{1, 2, \ldots\}$$

Equation (35) simply requires that the firm supply all demand at the prices it is charging. Equation (36) requires that the firm's revenue in any period is always less than or equal to its accounting cost for that period.[16]

Lemma 3 can be interpreted as stating that the RRC allocation rule has the property that the historic accounting cost per unit of *capital* in any period is equal to that period's user cost. It therefore immediate that the RRC accounting rule also has the property that the historic accounting cost per unit of *output* in any period must be equal to that period's user cost so long as the vector of outputs satisfies the FUI property. Since the vector of efficient outputs **q*** has already been shown to satisfy the FUI property, it therefore follows that (**p***, **q***) is a regulatory equilibrium under the RRC allocation rule. It has already been observed that the firm earns zero profit when at the price **p***. It is straightforward to show that the constraints in (36) imply that the firm's profit can never be greater than zero. This means that there can be no other regulatory equilibrium that the firm strictly prefers to (**p*, q***). This result is stated as Proposition 4.

Proposition 4:
The efficient vector of prices and quantities (**p***, **q***) is an regulatory equilibrium under the RRC allocation rule. The firm earns zero profit in this equilibrium and there is no regulatory equilibrium where the firm earns strictly positive profit.

Proof:
As above. QED

[16] The LHS of equation (36) is often referred to as the residual income of the firm in period t. Thus equation (36) simply requires that the firm's residual income be less than or equal to zero in every period.

6.4. The Effect of the Rate of Technological Progress on the RRC Allocation Rule

Substitution of (4) and (14) into (31) yields

(37) $$a_i^* = s_i c_i^*/z_0.$$

Therefore a_i^* is strictly increasing (constant, strictly decreasing) in α if and only if c_i^* is strictly increasing (constant, strictly decreasing) in α. That is, changes in the rate of technological progress have the same qualitative effect on the time pattern of allocation shares under the RRC allocation rule as they have on the vector of user costs. In particular then, Proposition 2 implies that increases in the rate of technological progress will always increase the cost allocated to early periods and decrease the cost allocated to later periods.

7. FORWARD LOOKING VS. HISTORIC PRICING RULES

Proposition 4 shows that regulators' intuition that the rapid pace of technological progress in the telecommunications industry required them to switch from basing prices on historical costs to basing prices on forward looking costs is not correct in the simple benchmark model of this paper. In theory, either method can be used to calculate efficient prices when technological progress is causing the replacement cost of assets to fall over time. Under a historical pricing method, increases in the rate of technological progress simply require the regulator to use a more accelerated allocation rule to correctly reflect the impact of technological progress on the marginal cost of production in each period.

Furthermore, it can be argued that basing prices on forward looking cost is likely to create a whole host of extra problems that do not arise when prices are based on historic cost because of a factor not captured in the formal model. Namely, in reality, calculations of historic cost are likely to be based on much more objective data that are less subject to manipulation than are calculations of forward looking cost.[17] Historic cost is based on the amount of money that was actually spent to purchase an asset. However, forward looking cost is based on the amount of money that the regulator *estimates* that it would cost to purchase *functionally equivalent* assets. In the formal model of this paper these problems are glossed over because it is assumed that the asset is a simple homogenous commodity that does not change over time that is sold at some easily measured market price. The reality of the

[17] See Tardiff (2002), section 3, for a detailed discussion of this issue illustrated with many examples from real cases.

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situation is, of course, likely to be quite different. This creates two related problems. First, at a minimum, it is very expensive to conduct the sort of investigations required to determine what the current replacement cost of assets is. It is widely recognized that the regulatory proceedings in the United States used to determine forward looking cost have become highly adversarial and very expensive. Second, to the extent that forward looking cost is manipulable, this allows regulators the opportunity to essentially attempt to reneg *ex post* on their commitment to reimburse the firm for its investments in sunk assets.[18] To the extent that some sort of *ex ante* commitment is necessary and desirable in order to alleviate the hold-up problem, the fact that a forward pricing rule weakens this commitment ability may be undesirable.

Of course the policy implications suggested by any theoretical model are only relevant to the extent that the model has captured all of the important economic factors relevant to evaluating the effects of the policy. It is possible that future research will show that there is some important factor not taken into account by the simple benchmark model of this paper that can provide some formal justification for the common view that forward looking pricing rules become more appropriate than historic pricing rules as the rate of technological progress increases. One particular possibility for such a factor might be the effect of uncertainty. One difference between a historic rule and a forward looking rule is that a historic rule determines the amount of cost that will be allocated to a particular period years before the period occurs. A forward looking rule, on the other hand, waits until a period arrives before determining the cost allocated to that period. The efficient price in any period should depend upon the amount of technical progress that has occurred up until that point in time. This means that, so long as the rate of technical progress is uncertain, there would generally be an advantage in waiting to determine the price charged in any given period until the actual amount of technical progress that has occurred up until that period is known. Therefore it seems possible that a forward looking pricing rule might have an advantage over a historic pricing rule in a world where technical progress is uncertain, because the forward pricing rule could base the price in any period on the actual amount of technical progress that had occurred up until that point. To put this another way, one might interpret the results of this paper as showing that a historic pricing rule can be just as effective as a forward looking pricing rule at taking the effects of fully anticipated technical progress into account. However, it might still be the case that a forward pricing rule could have an advantage in taking the effects of unanticipated technical progress into account. Of course even if this was an advantage of forward pricing rules, it would have to be weighed against the disadvantage that forward pricing rules are likely to be based on less objective data.

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[18] See Ergas (2009) for a recent discussion of this issue and empirical documentation of its importance.

While building a model to formally investigate generalizations of this sort is beyond the scope of this paper, hopefully it has made a contribution to the longer term project of fully investigating the comparative advantages of historic vs. forward looking pricing rules by showing that the rules are equally effective in a simple benchmark model.

8. CONCLUSION

This paper considers a simple model where a regulated firm must make sunk investments in long-lived assets in order to produce output, assets exhibit a known but arbitrary pattern of depreciation, there are constant returns to scale within each period and the replacement cost of assets is weakly falling over time due to technological progress. It is shown that a simple formula can be used to calculate the long run marginal cost of production each period and that the firm breaks even if prices are set equal to long run marginal cost. Furthermore, the formula for calculating long run marginal cost can be interpreted either as a formula for calculating forward looking cost or as a formula for calculating historic cost so long as the correct method for allocating costs is used in either case. For the case of forward looking cost, the correct forward looking allocation share is the unique forward looking allocation share that allows the firm to break even. For the case of historic cost, the correct historic cost allocation rule is a relatively simple and natural rule called the relative replacement cost (RRC) rule.

To some extent, regulators have introduced the practice of basing prices on forward looking cost because of the intuition that historic costing methods become less appropriate as the rate of technological progress grows higher. However, the switch to basing prices on forward looking costs has created a whole host of problems associated with the fact that, in the real world, estimating the cost of replacing the existing assets of the firm with functionally equivalent new assets is a considerably more complicated and difficult exercise than simply determining the actual cost that the firm incurred to purchase the assets that it actually owns. Thus the results of this paper at least raise the question of whether or not this switch of pricing methods was a wise idea. Additional research to determine whether the results of this paper continue to hold in more general models will be necessary to provide a more definitive answer to this question.

More generally, the results of this paper provide regulators with a theory that explains how to optimally allocate investment costs over time for any given rate of technological progress and any asset depreciation pattern.

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9. APPENDIX - PROOFS OF PROPOSITIONS AND LEMMAS

Lemma 1:
Consider any vector of outputs $\mathbf{q} = (q_1, q_2, \ldots)$. Let $\mathbf{I} = (I_1, I_2, \ldots) = \phi(\mathbf{q})$ denote the efficient vector of investments for $\mathbf{q}$ and let $\mathbf{q}' = (q_1', q_2', \ldots)$ defined by

$$(38) \qquad q_t' = \sum_{i=1}^{t} s_i I_{t-i}$$

denote the vector of outputs that could be produced if the efficient capital stock for $\mathbf{q}$ was used at full capacity every period. It is obvious that

$$(39) \qquad \mathbf{q} \leq \mathbf{q}'$$

(40) $\mathbf{q} = \mathbf{q}'$ if and only if $\mathbf{q}$ satisfies the FUI property

The fact that $\mathbf{c^*}$ satisfies equation (13) implies that

$$(41) \qquad z_t I_t = I_t \sum_{i=1}^{\infty} c_{t+i}^* s_i \delta^i \qquad \text{for every } t \in \{0, 1, \ldots\}$$

Therefore

$$(42) \qquad \sum_{t=0}^{\infty} z_t I_t \delta^t = \sum_{t=0}^{\infty} \sum_{i=1}^{\infty} I_t c_{t+i}^* s_i \delta^{i+t}$$

Reorganize the summation on the RHS of (42) by using the index j = t+i to yield

$$(43) \qquad \sum_{t=0}^{\infty} z_t I_t \delta^t = \sum_{j=1}^{\infty} \sum_{i=1}^{j} I_{j-i} c_j^* s_i \delta^j$$

The LHS of (43) is by definition $C(\mathbf{q})$. Substitute (38) into the RHS of (43) to yield

$$(44) \qquad C(\mathbf{q}) = \sum_{j=1}^{\infty} q_j' c_j^* \delta^j$$

The results now follow from (39), (40), (44) and the fact that $c_j^* > 0$ for every j.
QED

Proposition #1:

Let $W^H(\mathbf{q}) = B(\mathbf{q}) - H(\mathbf{q})$ denote the welfare function for the hypothetical problem where the firm's cost function is $H(\mathbf{q})$. From Lemma 1 it follows that

(45) $$W(\mathbf{q}) = W^H(\mathbf{q}) \qquad \text{for every } \mathbf{q} \text{ satisfying the FUI property}$$

(46) $$W(\mathbf{q}) \leq W^H(\mathbf{q}) \qquad \text{for every } \mathbf{q} \in [0, \infty)^\infty$$

Recall that $\mathbf{q}^*$ is the unique vector of outputs that maximizes $W^H(\mathbf{q})$. It follows immediately from (45) and (46) that if $\mathbf{q}^*$ satisfies the FUI property then it must also be the unique vector of outputs that maximizes $W(\mathbf{q})$. Equation (14) implies that c_t^* is weakly decreasing in t. Therefore equation (18) and the assumption that demand is weakly increasing over time (equation (8)) imply that q_t^* is weakly increasing in t. It has already been noted (see footnote 10 and the associated text) that this implies that $\mathbf{q}^*$ satisfies the FUI property. QED

Proposition 2:
Define $c_t^*(\alpha)$ by

(47) $$c_t^*(\alpha) = k^{**}(\alpha) z_0 \alpha^{t-1}$$

where $k^{**}(\alpha)$ is defined by equation (22). Then

(48) $$c_t^{*\prime}(\alpha)/c_t^*(\alpha) = [k^{**\prime}(\alpha)/k^{**}(\alpha)] + (t-1)/\alpha$$

Obviously $c_t^{*\prime}(\alpha)$ will be positive (zero, negative) if and only if $c_t^{*\prime}(\alpha)/c_t(\alpha)$ is positive (zero, negative). The result then follows where τ is defined by

(49) $$\tau(\alpha) = 1 - [k^{**\prime}(\alpha)\alpha/k^{**}(\alpha)]$$

It is easy to see that the term in square brackets is negative, which implies that $\tau(\alpha) \in (1, \infty)$. QED

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