When is the Government Spending Multiplier Large?

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Abstract

This paper argues that if the zero bound on nominal interest rates is binding, then the government spending multiplier is large.

1. Introduction

A classic question in macroeconomics is: what is the size of the government spending multiplier? There is a large empirical literature that grapples with this question. Authors such as Barro (1981) argue that the multiplier is around 0.8 while authors such as Ramey (2008) estimate the multiplier to be closer to 1.2. These differences primarily reflect the use of alternative identifying assumptions to isolate exogenous movements in government spending.

There is also a large literature that uses general equilibrium models to study the size of the government spending multiplier. For example,in their study of new-Keynesian models, Monacelli and Perotti (2008) report that the government spending multiplier can be somewhat above or below one depending on the exact specification of agent's preferences. An increase in government spending also increases output in frictionless real business cycle models. In these models the multiplier effect of temporary increases in government spending is typically less than one (see e.g. Aiyagari, Christiano, and Eichenbaum (1992), Baxter and King (1993), Burnside, Eichenbaum and Fisher (2004), Ramey and Shapiro (1998), and Ramey (2008)).

Much of the existing empirical literature is drawn from data in which the zero bound on nominal interest rates is arguably not binding. In addition, the general equilibrium models that are used to calculate the government spending multiplier typically abstract from the zero bound on nominal interest rates. In this paper we build on insights in Eggertsson and Woodford (2003) and Christiano (2004) to argue that, whenever the zero bound on nominal interest rates is binding, the government spending multiplier is much bigger than one. Consequently, our model implies that it can be socially optimal to substantially raise government spending in response to the shocks that make the zero bound on the nominal interest rate binding. We articulate this argument in a model in which the government spending multiplier is quite modest when the nominal interest rate is not zero.

Our analysis proceeds in two steps. First, we consider an economy with Calvo-style pricing frictions, no capital, and a monetary authority that follows a standard Taylor rule. We study the effects of two types of shocks. The first is a temporary, unanticipated rise in the discount factor of the representative agent. The second is a fear of deflation shock, i.e. self-fulfilling expected deflation induced by a sunspot.

Other things equal, the first shock increases desired savings. Absent capital, aggregate savings must be zero in equilibrium. When the shock is small enough, the real interest rate falls and there is a modest decline in output. However, when the shock is large enough, the zero bound becomes binding before the real interest rate falls by enough to make aggregate savings zero. The only force that can induce the fall in saving required to re-establish equilibrium is a very large transitory fall in output.

Why is the fall in output so large when the economy hits the zero bound? For a given fall in output, marginal cost falls, and there is expected deflation. Consequently, the real interest rate rises. This rise leads to an increase in desired savings which partially undoes the effect of a given fall in output. As a consequence, the total fall in output required to reduce desired savings to zero is very large. This scenario captures the paradox of thrift originally emphasized by Keynes (1936) and recently analyzed by Krugman (1998), Eggertsson and Woodford (2003), and Christiano (2004). The government spending multiplier is large when the zero bound is binding because an increase in government spending lowers desired national savings and shortcuts the meltdown created by the paradox of thrift.

Consider now a fear of deflation shock. Suppose that for non-fundamental reasons agents expect a large fall in prices in the next period. For a given nominal interest rate, the higher is the expected rate of deflation the higher is the real interest rate and the higher is desired savings. Recall that savings must be zero in equilibrium. So, as above, the nominal interest rate could hit zero before the real interest rate falls by enough to make savings equal to zero. When the zero bound is binding the equilibrium is established by a large, transitory fall in output. The fall in output is associated with a fall in marginal cost and expected deflation, thus validating agents' initial expectation of a fall in prices. The government spending multiplier is large in these circumstances because an increase in government purchases counteracts the rise in desired private savings induced by the fear of deflation. In our simple model self-fulfilling fear of deflation equilibria always exist. These equilibria capture the negative consequences of deflation that some policy makers are concerned about.

The exact value of the multiplier depends on how long the zero bound is expected bind. In our simple benchmark model without capital the value of the multiplier goes from 1.3, when the zero bound is expected to last for one quarter, to 3.7 when the zero bound is expected to last for five quarters. The corresponding value of the multiplier when the zero bound is not binding is 1.05.

In the second step of our analysis we incorporate capital accumulation into the model. In addition to the discount factor shock we also allow for a neutral technology shock and an investment-specific shock. For computational reasons we consider temporary shocks that make the zero bound binding for a deterministic number of periods. Again, we find that the government spending multiplier is larger when the zero bound is binding. Allowing for capital accumulation has two effects. First, for a given size shock it reduces the likelihood that the zero bound becomes binding. Second, when the zero bound binds, the presence of capital accumulation tends to increase the size of the government spending multiplier.

The intuition for this result is that, in our model, investment is a decreasing function of the real interest rate. When the zero bound binds, the real interest rate generally rises. So, other things equal, savings and investment diverge as the real interest rate rises, thus exacerbating the meltdown associated with the zero bound. As a result, the fall in output necessary to bring savings and investment into alignment is larger than in the model without capital.

One practical objection to using fiscal policy when the zero bound binds is that there are long lags in implementing an increase in government spending. Motivated by this consideration, we study the size of the government spending multiplier in the presence of implementation lags. We find that the key determinant of the size of the multiplier is the state of the world in which new government spending actually comes on line. If it comes on line in future periods when the nominal interest rate is zero there is a large effect on current output. If it comes on line in future periods where the nominal interest rate is positive the current effect on government spending is smaller.

According to our analysis the response of aggregate demand to shocks plays a key role in making the zero-bound constraint binding. In general the more responsive aggregate demand is to shocks the more likely the zero bound binds after various shocks. In practice researchers such as Christiano, Eichenbaum and Evans (2005) (henceforth CEE) and Smets and Wouters (2005, 2007) find that it is important to include sources of inertia in aggregate demand in order for DSGE models to fit the data. Consequently, we investigate the robustness of our results to incorporating the sources of inertia that these researchers stress. First, we adopt the investment adjustment cost specification proposed by CEE. Second, we use the model developed in Altig, Christiano, Eichenbaum, and Lindé (2005), which allows for habit formation in consumption as well as the CEE specification for investment adjustment costs. We find that our basic results are robust in the following sense. Incorporating sources of inertia in aggregate demand makes it less likely that the zero bound binds after a shock. However, conditional on the zero bound binding the government spending multiplier is larger than it would be under normal circumstances.

As emphasized by Eggertsson and Woodford (2003), an alternative way to escape the negative consequences of a shock that makes the zero bound binding is for the central bank to commit to future inflation. We abstract from this possibility in this paper. We do so for a number of reasons. First, this theoretical possibility is well understood. Second, we do not think that it is easy in practice for the central bank to credibly commit to future high inflation. Third, the optimal trade-off between higher government purchases and anticipated inflation depends sensitively on how agents value government purchases and the costs of anticipated inflation. Studying this issue is an important topic for future research.

Our analysis is related to several recent papers on the zero bound. Eggertson (2009) focuses on the effects of transitory tax cuts when the zero bound on nominal interest rates binds. Bodenstein, Erceg, and Guerrieri (2009) analyze the effects of shocks to open economies when the zero bound binds. Braun and Waki (2006) use a model in which the zero bound binds to account for Japan's experience in the 1990s. Their results for fiscal policy are broadly consistent with our results. Braun and Waki (2006) and Coenen and Wieland (2003) investigate whether alternative monetary policy rules could have avoided the zero bound and led to more desirable economic outcomes.

Our paper is organized as follows. In section 2 we analyze the size of the government spending multiplier when the zero bound does not bind in a standard new-Keynesian model without capital. In section 3 we modify the analysis to incorporate a binding zero-bound constraint on the nominal interest rate. In section 4 we extend the model to incorporate capital. In section 5 we discuss the robustness of our results to allowing for various sources of inertia in aggregate demand. Section 6 concludes.

2. The standard multiplier in a model without capital

In this section we present a simple new-Keynesian model and analyze its implications for the size of the "standard multiplier," by which we mean the size of the government spending multiplier when the zero bound is not binding.

Households The economy is populated by a representative household, whose life-time utility, U, is given by:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\left[C_t^{\gamma} (1 - N_t)^{1-\gamma} \right]^{1-\sigma} - 1}{1-\sigma} + v\left(G_t\right) \right]. \tag{2.1}$$

Here E_0 is the conditional expectation operator, and C_t, G_t, and N_t denote time-t consumption, government consumption, and hours worked, respectively. We assume that $\sigma > 0$, $\gamma \in (0, 1)$, and that $v(.)$ is a concave function.

The household budget constraint is given by:

$$P_t C_t + B_{t+1} = B_t \left(1 + R_t\right) + W_t N_t + T_t, \tag{2.2}$$

where T_t denotes firms' profits net of lump-sum taxes paid to the government. The variable B_{t+1} denotes the quantity of one-period bonds purchased by the household at time t. Also, P_t denotes the price level and W_t denotes the nominal wage rate. Finally, R_t denotes the one-period nominal rate of interest that pays off in period t. The household's problem is to maximize utility given by equation (2.1) subject to the budget constraint given by equation (2.2) and the condition $E_0 \lim_{t \to \infty} B_{t+1}/[(1 + R_0)(1 + R_1)...(1 + R_t)] \geq 0$.

Firms The final good is produced by competitive firms using the technology,

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \ \varepsilon > 1, \tag{2.3}$$

where $Y_t(i)$, $i \in [0,1]$ denotes intermediate good i.

Profit maximization implies the following first-order condition for $Y_t(i)$:

$$P_t(i) = P_t \left(\frac{Y_t}{Y_t(i)} \right)^{\frac{1}{\varepsilon}}, \tag{2.4}$$

where $P_t(i)$ denotes the price of intermediate good i and P_t is the price of the homogeneous final good.

The intermediate good, $Y_t(i)$, is produced by a monopolist using the following technology:

$$Y_t(i) = N_t(i),$$

where $N_t(i)$ denotes employment by the i^{th} monopolist. We assume there is no entry or exit into the production of the i^{th} intermediate good. The monopolist is subject to Calvo-style price-setting frictions and can optimize its price, $P_t(i)$, with probability $1-\theta$. With probability θ the firm sets:

$$P_t(i) = P_{t-1}(i).$$

The discounted profits of the i^{th} intermediate good firm are:

$$E_t \sum_{j=0}^{\infty} \beta^{t+j} \upsilon_{t+j} \left[P_{t+j}(i) Y_{t+j}(i) - (1-\nu) W_{t+j} N_{t+j}(i) \right], \tag{2.5}$$

where $\nu = 1/\varepsilon$ denotes an employment subsidy which corrects, in steady state, the inefficiency created by the presence of monopoly power. The variable υ_{t+j} is the multiplier on the household budget constraint in the Lagrangian representation of the household problem. The variable W_{t+j} denotes the nominal wage rate.

Firm i maximizes its discounted profits, given by equation (2.5), subject to the Calvo price-setting friction, the production function, and the demand function for $Y_t(i)$, given by equation (2.4).

Monetary policy We assume that monetary policy follows the rule:

$$R_{t+1} = \max(Z_{t+1}, 0), \tag{2.6}$$

where

$$Z_{t+1} = (1/\beta)(1+\pi_t)^{\phi_1}(Y_t/Y)^{\phi_2}(R_t/R)^{\rho_R} - 1.$$

The variable Y denotes the steady-state level of output. The variable π_t denotes the time-t rate of inflation. We assume that $\phi_1 > 1$ and $\phi_2 \in (0,1)$.

According to equation (2.6) the monetary authority follows a Taylor rule as long as the implied nominal interest rate is non-negative. Whenever the Taylor rule implies a negative nominal interest rate, the monetary authority simply sets the nominal interest rate to zero. For convenience we assume that steady-state inflation is zero. This assumption implies that the steady-state net nominal interest rate is $1/\beta - 1$.

Fiscal policy As long as the zero bound on the nominal interest rate is not binding, government spending evolves according to:

$$G_{t+1} = (1-\rho)G + \rho G_t + \varepsilon_{t+1}. \tag{2.7}$$

Here G is the level of government spending in the non-stochastic steady state and ε_{t+1} is an i.i.d. shock with zero mean. To simplify our analysis, we assume that government spending and the employment subsidy are financed with lump-sump taxes. The exact timing of these taxes is irrelevant because Ricardian equivalence holds under our assumptions.

Equilibrium The economy's resource constraint is:

$$C_t + G_t = Y_t. \tag{2.8}$$

A 'monetary equilibrium' is a collection of stochastic processes,

$$\{C_t, N_t, W_t, P_t, Y_t, R_t, P_t(i), Y_t(i), N_t(i), \upsilon_t, B_{t+1}, \pi_t\},$$

such that for given $\{G_t\}$ the household and firm problems are satisfied, the monetary and fiscal policy rules are satisfied, markets clear, and the aggregate resource constraint is satisfied.

To solve for the equilibrium we use a linear approximation around the non-stochastic steady state of the economy. Throughout, $\hat{Z}_t$ denotes the percentage deviation of Z_t from its non-stochastic steady state value, Z. The equilibrium is characterized by the following set of equations.

The Phillips curve for this economy is given by:

$$\pi_t = E_t\left(\beta\pi_{t+1} + \kappa\widehat{MC}_t\right), \tag{2.9}$$

where $\kappa = (1-\theta)(1-\beta\theta)/\theta$. In addition, MC_t denotes real marginal cost which, under our assumptions, is equal to the real wage rate. Absent labor market frictions, the percent deviation of real marginal cost from its steady state value is given by:

$$\widehat{MC}_t = \hat{C}_t + \frac{N}{1-N}\hat{N}_t. \tag{2.10}$$

The linearized intertemporal Euler equation for consumption is:

$$\begin{aligned} & \frac{\gamma(\sigma-1)}{1-g}\hat{N}_t - [\gamma(\sigma-1)+1]\hat{C}_t \qquad (2.11) \\ = \; & E_t\left\{\frac{\gamma(\sigma-1)}{1-g}\hat{N}_{t+1} - [\gamma(\sigma-1)+1]\hat{C}_{t+1} + \beta(R_{t+1}-R) - \pi_{t+1}\right\}. \end{aligned}$$

The linearized aggregate resource constraint is:

$$\hat{Y}_t = (1-g)\hat{C}_t + g\hat{G}_t, \tag{2.12}$$

where $g = G/Y$.

Combining equations (2.9) and (2.10) and using the fact that $\hat{N}_t = \hat{Y}_t$ we obtain:

$$\pi_t = \beta E_t(\pi_{t+1}) + \kappa\left[\left(\frac{1}{1-g} + \frac{N}{1-N}\right)\hat{Y}_t - \frac{g}{1-g}\hat{G}_t\right]. \tag{2.13}$$

Similarly, combining equations (2.11) and (2.12) and using the fact that $\hat{N}_t = \hat{Y}_t$ we obtain:

$$\begin{aligned} &\hat{Y}_t - g\left[\gamma(\sigma-1)+1\right]\hat{G}_t = \\ &E_t\left\{-(1-g)\left[\beta(R_{t+1}-R)-\pi_{t+1}\right] + \hat{Y}_{t+1} - g\left[\gamma(\sigma-1)+1\right]\hat{G}_{t+1}\right\}. \end{aligned} \tag{2.14}$$

As long as the zero bound on the nominal interest rate does not bind, the linearized monetary policy rule is given by:

$$R_{t+1} - R = \rho_R(R_t - R) + \frac{1-\rho_R}{\beta}\left(\phi_1\pi_t + \phi_2\hat{Y}_t\right).$$

Whenever the zero bound binds, $R_{t+1} = 0$.

We solve for the equilibrium using the method of undetermined coefficients. For simplicity, we begin by considering the case in which $\rho_R = 0$. Under the assumption that $\phi_1 > 1$, there is a unique linear equilibrium in which π_t and $\hat{Y}_t$ are given by:

$$\pi_t = A_\pi \hat{G}_t, \tag{2.15}$$

$$\hat{Y}_t = A_Y \hat{G}_t. \tag{2.16}$$

The coefficients A_π and A_Y are given by:

$$A_\pi = \frac{\kappa}{1-\beta\rho}\left[\left(\frac{1}{1-g} + \frac{N}{1-N}\right)A_Y - \frac{g}{1-g}\right], \tag{2.17}$$

$$A_Y = g\frac{(\rho-\phi_1)\kappa - \left[\gamma(\sigma-1)+1\right](1-\rho)(1-\beta\rho)}{(1-\beta\rho)\left[\rho - 1 - (1-g)\phi_2\right] + (1-g)(\rho-\phi_1)\kappa\left(\frac{1}{1-g}+\frac{N}{1-N}\right)}. \tag{2.18}$$

The effect of an increase in government spending Using equation (2.12) we can write the government spending multiplier as:

$$\frac{dY_t}{dG_t} = \frac{1}{g}\frac{\hat{Y}_t}{\hat{G}_t} = 1 + \frac{1-g}{g}\frac{\hat{C}_t}{\hat{G}_t}. \tag{2.19}$$

This equation implies that the multiplier is less than one whenever consumption falls in response to an increase in government spending. Equation (2.16) implies that the government spending multiplier is given by:

$$\frac{dY_t}{dG_t} = \frac{A_Y}{g}. \tag{2.20}$$

From this equation we see that the multiplier is constant over time. To analyze the magnitude of the multiplier outside of the zero bound we consider the following baseline parameter values:

$$\theta = 0.85,\ \beta = 0.99,\ \phi_1 = 1.5,\ \phi_2 = 0,\ \gamma = 0.29,\ g = 0.2,\ \sigma = 2,\ \rho_R = 0,\ \rho = 0.8. \tag{2.21}$$

These parameter values imply that $\kappa = 0.03$ and $N = 1/3$. Our baseline parameter values imply that the government spending multiplier is 1.05. Figure 1 displays the impulse response of output, inflation, and the nominal interest rate to a government spending shock.

In our model Ricardian equivalence holds. From the perspective of the representative household the increase in the present value of taxes equals the increase in the present value of government purchases. In a typical version of the standard neoclassical model we would expect some rise in output driven by the negative wealth effect on leisure of the tax increase. But in that model the multiplier is generally less than one because the wealth effect reduces private consumption. From this perspective it is perhaps surprising that the multiplier in our baseline model is greater than one. This perspective neglects two key features of our model, the frictions in price setting and the complementarity between consumption and leisure in preferences. When government purchases increase, total demand, $C_t + G_t$, increases. The presence of sticky prices has the consequence that, in the wake of a rise in demand, price over marginal cost falls. As emphasized in the literature on the role of monopoly power in business cycles, the fall in the markup induces an outward shift in the labor demand curve. This shift amplifies the rise in employment following the rise in demand. Given our specification of preferences, $\sigma > 1$ implies that the marginal utility of consumption rises with the increase in employment. As long as this increase in marginal utility is large enough, it is possible for private consumption to actually rise in response to an increase in government purchases. Indeed, consumption does rise in our benchmark scenario which is why the multiplier is larger than one.

To assess the importance of our preference specification we redid our calculations using the basic specification for the momentary utility function commonly used in the new-Keynesian DSGE literature:

$$u = \left(C_t^{1-\varsigma} - 1\right) / \left(1 - \varsigma\right) - \eta N_t^{1+\vartheta} / \left(1 + \vartheta\right), \tag{2.22}$$

where, ς, η, and ϑ are positive. The key feature of this specification is that the marginal utility of consumption is independent of hours worked. Consistent with the intuition discussed above, we found that, across a wide set of parameter values, dY/dG is always less than one with this preference specification.[1]

To provide additional intuition for the determinants of the multiplier, Figure 2 displays dY/dG for various parameter configurations. In each case we perturb one parameter at a time relative to the benchmark parameter values. The $(1, 1)$ element of Figure 2 shows that a rise in σ is associated with an increase in the multiplier. This result is consistent with the intuition above which builds on the

[1] See Monacelli and Perotti (2008) for a discussion of the impact of preferences on the size of the government spending multiplier in models with Calvo-style frictions when the zero bound is not binding.

observation that the marginal utility of consumption is increasing in hours worked. This dependence is stronger the higher is σ. Note that multiplier can be bigger than unity even for σ slightly less than unity. This result presumably reflects the positive wealth effects associated with the increased competitiveness of the economy associated with the reduction in the markup.

The $(1,2)$ element of Figure 2 shows that the multiplier is a decreasing function of κ. In other words, the multiplier is larger the higher is the degree of price stickiness. The result reflects the fall in the markup when aggregate demand and marginal cost rise. This effect is stronger the stickier are prices. The multiplier exceeds one for all $\kappa < 0.13$. In the limiting case when prices are perfectly sticky $(\kappa = 0)$ the multiplier is given by:

$$\frac{dY_t}{dG_t} = \frac{\left[\gamma\left(\sigma - 1\right) + 1\right]\left(1 - \rho\right)}{1 - \rho + \left(1 - g\right)\phi_2} > 0.$$

Note that when $\phi_2 = 0$ the multiplier is greater than one as long as σ is greater than one.

When prices are perfectly flexible $(\kappa = \infty)$ the markup is constant. In this case the multiplier is given by:

$$\frac{dY_t}{dG_t} = \frac{1}{1 + \left(1 - g\right)\frac{N}{1-N}} < 1.$$

Note that the multiplier is less than one. This result reflects the fact that with flexible prices an increase in government spending has no impact on the markup. As a result, the demand for labor does not rise as much as in the case in which prices are sticky.

The $(1,3)$ element of Figure 2 shows that as ϕ_1 increases, the multiplier falls. The intuition for this effect is that the expansion in output increases marginal cost which in turn induces a rise in inflation. According to equation (2.6) the monetary authority increases the interest rate in response to a rise in inflation. The rise in the interest rate is an increasing function of ϕ_1. In general higher values of ϕ_1 lead to higher values of the real interest rate which are associated with lower levels of consumption. So, higher values of ϕ_1 lead to lower values of the multiplier.

The $(2,1)$ element of Figure 2 shows that as ϕ_2 increases, the multiplier falls. The intuition underlying this effect is similar to that associated with ϕ_1. When ϕ_2 is large there is a substantial increase in the real interest rate in response to a rise in output. The contractionary effects of the rise in the real interest rate on consumption reduce the size of the multiplier.

The $(2,2)$ element of Figure 2 shows that as ρ_R increases the multiplier rises. The intuition for this result is as follows. The higher is ρ_R the less rapidly the monetary authority increases the interest rate in response to the rise in marginal cost and inflation that occur in the wake of an increase in government purchases. This result is consistent with the traditional view that the government spending multiplier is greater in the presence of accommodative monetary policy. By accommodative we mean that the monetary authority keeps interest rates low in the presence of a fiscal expansion.

The $(2,3)$ element of Figure 2 shows that the multiplier is a decreasing function of the parameter governing the persistence of government purchases, ρ. The intuition for this result is that the present value of taxes associated with a given innovation in government purchases is an increasing function of ρ. So the negative wealth effect on consumption is an increasing function of ρ.

We conclude this subsection by noting that we redid Figure 2 using a forward-looking Taylor rule in which the interest rate responds to the one-period-ahead expected inflation and output gap. The results that we obtained were very similar to the ones discussed above.

Viewed overall, our results indicate that, from the perspective a simple new-Keynesian model, it is quite plausible that the multiplier is above one. However, it is difficult to obtain multipliers above 1.2 for plausible parameter values.

3. The zero-bound multiplier in a model without capital

In this section we analyze the government spending multiplier in our simple new-Keynesian model when the zero bound on nominal interest rates becomes binding. We first assume, as in Eggertsson and Woodford (2003) and Christiano (2004), that the shock that makes the zero bound binding is an increase in the discount factor. We think of this shock as representing a temporary rise in agent's propensity to save. We then consider a self-fulfilling fear of deflation shock. Finally, we analyze the impact of the timing of government spending increases on the magnitude of the multiplier.

A discount factor shock We modify agent's preferences, given by (2.1), to allow for a stochastic discount factor,

$$U = E_0 \sum_{t=0}^{\infty} d_t \left[\frac{\left[C_t^{\gamma}(1-N_t)^{1-\gamma}\right]^{1-\sigma} - 1}{1-\sigma} + v\left(G_t\right) \right]. \tag{3.1}$$

The cumulative discount factor, d_t, is given by:

$$d_t = \begin{cases} \frac{1}{1+r_1}\frac{1}{1+r_2}\cdots\frac{1}{1+r_t}, & t \geq 1 \\ 1 & t = 0 \end{cases}. \tag{3.2}$$

The time-t discount factor, r_t, can take on two values: r and r^l, where $r^l < 0$. The stochastic process for r_t is given by:

$$\Pr\left[r_{t+1} = r^l | r_t = r^l\right] = p,\ \Pr\left[r_{t+1} = r | r_t = r^l\right] = 1-p,\ \Pr\left[r_{t+1} = r^l | r_t = r\right] = 0. \tag{3.3}$$

The value of r_{t+1} is realized at time t.

We consider the following experiment. The economy is initially in the steady state, so $r_t = r$. At time zero r_1 takes on the value r^l. Thereafter r_t follows the process described by equation (3.3). The discount factor remains high with probability p and returns permanently to its normal value, r, with probability $1-p$. In what follows we assume that r^l is sufficiently high that the zero-bound constraint on nominal interest rates is binding. We assume that $\hat{G}_t = \hat{G}^l \geq 0$ in the lower bound and $\hat{G}_t = 0$ otherwise.

To solve the model we suppose (and then verify) that the equilibrium is characterized by two values for each variable: one value for when the zero bound is binding and one value for when it is not. We denote the values of inflation and output in the zero bound by π^l and $\hat{Y}^l$, respectively. For simplicity we assume that $\rho_R = 0$, so there is no interest rate smoothing in the Taylor rule, (2.6). Since there are no state variables and $\hat{G}_t = 0$ outside of the zero bound, as soon as the zero bound is not binding the economy jumps to the steady state.

We can solve for $\hat{Y}^l$ using equation (2.13) and the following version of equation (2.14), which takes into account the discount factor shock:

$$\begin{aligned} &\hat{Y}_t - g\left[\gamma\left(\sigma-1\right)+1\right]\hat{G}_t = \qquad (3.4) \\ &E_t\left\{-\beta\left(1-g\right)\left(R_{t+1}-r^l\right)+\hat{Y}_{t+1}-g\left[\gamma\left(\sigma-1\right)+1\right]\hat{G}_{t+1}+(1-g)\pi_{t+1}\right\}. \end{aligned}$$

Equations (2.13) and (3.4) can be re-written as:

$$\hat{Y}^l = g\left[\gamma\left(\sigma-1\right)+1\right]\hat{G}^l + \frac{1-g}{1-p}\left(\beta r^l + p\pi^l\right), \qquad (3.5)$$

$$\pi^l = \beta p \pi^l + \kappa\left(\frac{1}{1-g}+\frac{N}{1-N}\right)\hat{Y}^l - \frac{g}{1-g}\kappa\hat{G}^l. \qquad (3.6)$$

Equations (3.5) and (3.6) imply that π^l is given by:

$$\begin{aligned} \pi^l \;=\; & \frac{\kappa\left(\frac{1}{1-g}+\frac{N}{1-N}\right)\beta r^l}{\left(1-\beta p\right)\left(1-p\right)/(1-g) - p\kappa\left(\frac{1}{1-g}+\frac{N}{1-N}\right)} \qquad (3.7) \\ & + g\kappa\frac{\left(\frac{1}{1-g}+\frac{N}{1-N}\right)\gamma\left(\sigma-1\right)+\frac{N}{1-N}}{1-\beta p-\kappa\left(\frac{1}{1-g}+\frac{N}{1-N}\right)p\frac{1-g}{1-p}}\hat{G}^l. \end{aligned}$$

The government spending multiplier is given by:

$$\frac{dY^l}{dG^l} = [\gamma(\sigma - 1) + 1] + \kappa \frac{\left[1 + \frac{N}{1-N}(1-g)\right][\gamma(\sigma - 1) + 1] - 1}{(1 - \beta p)(1 - p)/p - \kappa\left[1 + \frac{N}{1-N}(1-g)\right]}. \qquad (3.8)$$

Empirical estimates of ϕ_2 based on U.S. data are small and insignificantly different from zero. For convenience, in what follows we set ϕ_2 to zero. In addition we suppose that $\sigma \geq 1$. A necessary condition for the zero bound to be binding is that the denominator in equation (3.5) must be positive:

$$(1 - \beta p)(1 - p)/(1 - g) - p\kappa\left(\frac{1}{1-g} + \frac{N}{1-N}\right) > 0$$

When this condition holds the multiplier dY^l/dG^l is positive.

In analyzing the size of this multiplier we assume that the discount factor shock is sufficiently large to make the zero bound binding.[2] Conditional on this bound being binding, the size of the multiplier does not depend on the size of the shock. In our discussion of the standard multiplier we assume that the first-order serial correlation of government spending shocks is 0.8. To make the experiment in this section comparable we choose $p = 0.8$. This choice implies that the first-order serial correlation of government spending in the zero bound is also 0.8. All other parameter values are given by the baseline specification in (2.21). We only consider values of κ for which the zero bound is binding, so we display results for $0.013 \leq \kappa \leq 0.038$.

Figure 3 displays the government-spending multiplier and the response of output to the discount rate shock in the absence of a change in government spending as a function of the parameter κ. The '$*$' indicates results for our benchmark value of κ. Three key features of this figure are worth noting. First, the multiplier is very large. For our benchmark specification it is 3.7, which is roughly three

[2] For example, a rise in the discount factor from its steady state value of four percent (APR) to -1 percent (APR) would make the zero bound binding

times larger than the standard multiplier. Second, absent a change in government spending, the decline in output is increasing in the degree of price flexibility, i.e. it is increasing in κ. Finally the government spending multiplier is also an increasing function of κ.

A natural question is how sensitive the multiplier is to the duration of the discount factor shock. It turns out that the multiplier is an increasing, highly nonlinear function of p. Suppose, for example, that p is equal to 0.75, then the multiplier is 'only' 1.83.[3] On the other hand, for $p = 0.5$ the multiplier falls to 1.3.[4] Further declines in p have very little impact on the multiplier. Regardless of the value of p the multiplier is larger than the analogue 'standard' multiplier when the zero bound is not binding.

To provide intuition for this result it is useful to focus on why the drop in output is so large absent a change in government spending. The basic shock to the economy is an increase in agent's desire to save. In this economy savings must be zero in equilibrium. With completely flexible prices the real interest rate would simply fall to discourage agents from saving. There are two ways in which such a fall can occur: a large fall in the nominal interest rate and/or a substantial rise in the expected inflation rate. The extent to which the nominal interest rate can fall is limited by the zero bound. In our sticky-price economy a rise in the rate of inflation is associated with a rise in output and marginal cost. But a transitory increase in output is associated with a further increase in the desire to save, so that the real interest rate must rise by even more. Given the size of our shock to the discount factor there is no equilibrium in which the nominal interest rate is zero and inflation is positive. So the real interest rate cannot fall by enough to

[3]For the zero bound to be binding in this case the discount rate shock must rise from a steady state value of four percent (APR) to minus five percent (APR).

[4]For the zero bound to be binding in this case the discount rate shock must rise from a steady state value of four percent (APR) to minus 15 percent (APR).

reduce desired savings to zero. Instead, the equilibrium is established by a large, temporary fall in output, deflation, and a rise in the real interest rate.

Figure 4 displays a stylized version of this economy. Savings (S) are an increasing function of the real interest rate. Since there is no investment in this economy savings must be zero in equilibrium. The initial equilibrium is represented by point A. But the increase in the discount factor can be thought of as inducing a rightward shift in the savings curve from S to S'. When this shift is large, the real interest rate cannot fall enough to re-establish equilibrium because the lower bound on the nominal interest rate becomes binding prior to reaching that point. This situation is represented by point B.

To understand the mechanism by which equilibrium is reached after the shift in the savings consider equation (3.7). This equation shows how the rate of inflation, π^l, depends on the discount rate and on government spending. In the region where the zero bound is binding the denominator of this equation is positive. Since r^l is negative, it follows that π^l is negative and so too is expected inflation, $p\pi^l$. Since the nominal interest rate is zero and expected inflation is negative, the real interest rate (nominal interest rate minus expected inflation rate) is positive. Both the increase in the discount factor and the rise in the real interest rate increase agents' desire to save. There is only one force remaining to generate zero savings in equilibrium: a large, transitory fall in income. Other things equal this fall in income reduces desired savings as agents attempt to smooth the marginal utility of consumption over states of the world. Because the zero bound is a transitory state of the world this force leads to a decrease in agents desire to save. This effect has to exactly counterbalance the other two forces which are leading agents to save more. This reasoning suggest that there is a very large decline in income when the zero bound is binding. In terms of Figure 4 we can think of the temporary fall in output as inducing a shift in the savings curve to the left.

From equation (3.7) we see that the rate of deflation is increasing in the degree of price flexibility as summarized by κ. Other things equal, a larger κ is associated with a larger rise in the real interest rate, as long as the zero bound is binding. To compensate for this effect the fall in output must be even larger.

To understand why the multiplier is so large in the zero bound note that a temporary rise in government purchases induces a fall in private consumption. Other things equal, the contemporaneous fall in private consumption is smaller than the rise in government spending because agents want to smooth their consumption over time. So a rise in government purchases is associated with a fall in aggregate savings. This effect offsets the rise in desired private savings that sent the economy into the zero bound to begin with. The government spending multiplier is large precisely because output falls so much when the zero bound is binding. An additional complementary effect arises if σ is greater than one. Equation (3.7) implies that π^l is increasing in $\hat{G}^l$. Other things equal this effect reduces the rise in the real interest rate that occurs when the zero bound is binding. For the reasons discussed above this effect reduces the fall in output that occurs when the zero bound is binding.

Sensitivity to the timing of government spending In practice there is likely be a lag between the time at which the zero bound becomes binding and the time at which additional government purchases begin. A natural question is: how does the economy respond at time t to the knowledge that the government will increase spending in the future? Consider the following scenario. At time t the zero bound is binding. Government spending does not change at time t, but it takes on the value $G^l > G$ for all future periods as long as the economy is in the zero bound. Under these assumptions equations (2.13) and (3.4) can be written as:

$$\pi_t = \beta p \pi^l + \kappa \left(\frac{1}{1-g} + \frac{N}{1-N} \right) \hat{Y}_t, \tag{3.9}$$

$$\hat{Y}_t = (1-g)\,\beta r^l + p\hat{Y}^l - g\left[\gamma\,(\sigma - 1) + 1\right] p\hat{G}^l + (1-g)p\pi^l. \tag{3.10}$$

Here we use the fact that $E_t\,(\pi_{t+1}) = p\pi^l$, $E_t(\hat{G}_{t+1}) = p\hat{G}^l$, and $E_t(\hat{Y}_{t+1}) = p\hat{Y}^l$. The values of π^l and $\hat{Y}^l$ are given by equations (3.7) and (3.5), respectively. Using equation (3.5) to replace $\hat{Y}^l$ in equation (3.10) we obtain:

$$\frac{dY_{t,1}}{dG^l} = \frac{1-g}{g} \frac{1}{1-p} \frac{d\pi^l}{d\hat{G}^l}. \tag{3.11}$$

Here the subscript 1 denotes the presence of a one period delay in implementing an increase in government spending. So, $dY_{t,1}/dG^l$ represents the impact on output at time t of an increase in government spending at time $t+1$. One can show that the multiplier is increasing in the probability, p, that the economy remains in the zero bound. The multiplier operates through the effect of a future increase in government spending on expected inflation. If the economy is in the zero bound in the future, an increase in government purchases increases future output and therefore future inflation. From the perspective of time t this effect leads to higher expected inflation and a lower real interest rate. This lower real interest rate reduces desired savings and increases consumption and output at time t.

Evaluating equation (3.11) at the benchmark values we obtain a multiplier equal to 2.4. While this multiplier is lower than the benchmark multiplier of 3.7, it is still large. Moreover, this multiplier pertains to an increase in today's output in response to a *possible* increase in future government consumption.

Suppose that it takes two periods for government purchases to increase in the event that the zero bound is binding. It is straightforward to show that the impact on current output of a potential increase in government spending that takes two periods to implement is given by:

$$\frac{dY_{t,2}}{dG^l} = p\frac{1-g}{g}\left[\frac{d\pi_{t,1}}{d\hat{G}^l} + \frac{1}{1-p}\frac{d\pi^l}{d\hat{G}^l}\right].$$

Here the subscript 2 denotes the presence of a two period delay. Using our benchmark parameters the value of this multiplier is 2.38, so the rate at which the multiplier declines as we increase the implementation lag is relatively low.

The usual objection to using fiscal policy as a tool for fighting recessions is that there are long lags in gearing up increases in spending. Our analysis indicates that the key question is: in which state of the world does additional government spending come on line? If it comes on line in future periods when the zero bound is binding there is a large effect on current output. If it comes on line in future periods where the zero bound is not binding the current effect on government spending is smaller. Suppose, for example, that at time t the government promises to implement a persistent increase in government spending at time $t+1$, if the economy emerges from the zero bound at time $t+1$. This increase in government purchases is governed by: $\hat{G}_{t+j} = 0.8^{j-1}\hat{G}_{t+1}$, for $j \geq 2$. In this case the value of the multiplier, dY_t/dG_{t+1}, is only 0.5 for our benchmark values.

Suppose, finally that government spending increases both in periods in which the zero bound is binding and in periods in which it is not. Then the value of the government spending multiplier will be in between the two polar cases discussed above.

Optimal government spending The fact that the government spending multiplier is so large in the zero bound raises the following question: taking as given the monetary policy rule described by equation (2.6) what is the optimal level of government spending when agent's discount rates are high? In what follows we use the superscript L to denote the value of variables in states of the world where the discount rate is r^l. In these states of the world the zero bound may or may not be binding depending on the level of government spending. From equation (3.7) we anticipate that the higher is government spending, the higher is expected inflation, and the less likely the zero bound is to bind.

We choose G^L to maximize the expected utility of the consumer in states of the world in which the discount factor is high and the zero bound is binding. For now we assume that in other states of the world $\hat{G}$ is zero. So, we choose G^L to maximize:

$$\begin{aligned} U^L &= \sum_{t=0}^{\infty}\left(\frac{p}{1+r^l}\right)^t \left[\frac{\left[\left(C^L\right)^{\gamma}(1-N^L)^{1-\gamma}\right]^{1-\sigma}-1}{1-\sigma}+v\left(G^L\right)\right], \quad (3.12) \\ &= \frac{1+r^r}{1+r^l-p}\left[\frac{\left[\left(C^L\right)^{\gamma}(1-N^L)^{1-\gamma}\right]^{1-\sigma}-1}{1-\sigma}+v\left(G^L\right)\right]. \end{aligned}$$

To ensure that U^L is finite we assume that $p < (1+r^l)$.

Note that:

$$\begin{aligned} Y^L &= N^L = Y\left(\hat{Y}^L+1\right), \\ C^L &= Y\left(\hat{Y}^L+1\right) - G\left(\hat{G}^L+1\right). \end{aligned}$$

Substituting these expressions into equation (3.12) we obtain:

$$\begin{aligned} U^L &= \frac{1+r^r}{1+r^l-p}\left[\frac{\left(\left[N\left(\hat{Y}^L+1\right)-Ng\left(\hat{G}^L+1\right)\right]^{\gamma}(1-N\left(\hat{Y}^L+1\right))^{1-\gamma}\right)^{1-\sigma}-1}{1-\sigma}\right] \\ &\quad +\frac{1+r^r}{1+r^l-p}v\left[Ng\left(\hat{G}^L+1\right)\right]. \end{aligned}$$

We choose the value of $\hat{G}^L$ that maximizes U^L subject to the intertemporal Euler (equation (2.13)), the Phillips curve (equation (2.14)), and $\hat{Y}_t = \hat{Y}^L$, $\hat{G}_t = G^L$, $E_t(\hat{G}_{t+1}) = pG^L$, $\pi_{t+1} = \pi^L$, $E_t(\pi_{t+1}) = p\pi^L$, and $R_{t+1} = R^L$, where

$$R^L = \max\left(Z^L, 0\right),$$

and

$$Z^L = \frac{1}{\beta} - 1 + \frac{1}{\beta}\left(\phi_1 \pi^L + \phi_2 \hat{Y}^L\right).$$

The last constraint takes into account that the zero bound on interest rates may not be binding even though the discount rate is high.

Finally, for simplicity we assume that $v(G)$ is given by:

$$v(G) = \psi_g \frac{G^{1-\sigma}}{1-\sigma}.$$

We choose ψ_g so that $g = G/Y$ is equal to 0.2.

Since government purchases are financed with lump sum taxes equation (2.20) implies that, in the steady state, the optimal level of G has the property that the marginal utility of G is equal to the marginal utility of consumption:

$$\psi_g G^{-\sigma} = \gamma C^{\gamma(1-\sigma)-1} N^{(1-\gamma)(1-\sigma)}.$$

This relation implies:

$$\psi_g = \gamma\left([N(1-g)]\right)^{\gamma(1-\sigma)-1} N^{(1-\gamma)(1-\sigma)} (Ng)^{\sigma}.$$

Using our benchmark parameter values we obtain a value of ψ_g equal to 0.015.

Figure 5 displays the values of U^L, $\hat{Y}^L$, Z^L, $\hat{C}^L$, R^L, and π^L as a function of $\hat{G}^L$. The '*' indicates the level of a variable corresponding to the optimal value of $\hat{G}^L$. The 'o' indicates the level of a variable corresponding to the highest value of $\hat{G}^L$ that satisfies $Z^l \leq 0$. A number of features of Figure 5 are worth noting. First, the optimal value of $\hat{G}^L$ is very large: roughly 30 percent (recall that in steady state government purchases are 20 percent of output). Second, for this particular parameterization the increase in government spending more than undoes the effect of the shock which made the zero bound constraint bind. Here, government purchases rise to the point where the zero bound is marginally non binding and output is actually above its steady state level. These last two results depend on the parameter values that we chose and on our assumed functional form for $v(G_t)$. What is robust across different assumptions is that it is optimal to substantially increase government purchases and that the government spending multiplier is large when the zero-bound constraint is binding.

Fear of deflation In this subsection we show that deflation expectations can be self-fulfilling, making the zero bound binding and leading to a situation where the government multiplier is large.[5] Consistent with the intuition discussed in the introduction, suppose that agents expect inflation to be $\pi^l < 0$ with probability one. Recall that $\hat{Y}^l$ and π^l are given by equations (3.5) and (3.7). By construction there are no fundamental shocks to the economy so the discount factor r^l is equal to its steady state value , $1/\beta - 1$. Suppose that $\hat{G}^l = 0$. Equations (3.5) and (3.7) imply that π^l and $\hat{Y}^l$ are given by:

$$\pi^l = \beta - 1,$$

$$\hat{Y}^l = \frac{-(1-\beta)^2}{\kappa\left(\frac{1}{1-g} + \frac{N}{1-N}\right)}.$$

A self-fulfilling fear of deflation equilibrium exists as long as π^l and Z^l are negative. Here, Z^l is the nominal interest rate implied by equation (2.6), which is given by:

$$Z_l = \frac{1}{\beta} - 1 + \left(1 - \frac{1}{\beta}\right)\left[\phi_1 + \phi_2 \frac{-(1-\beta)}{\kappa\left(\frac{1}{1-g} + \frac{N}{1-N}\right)}\right].$$

[5] It is well known that Taylor rules can generate multiple equilibrium. For a recent analysis that emphasizes the potential importance of liquidity traps see Schmitt-Grohé and Uribe (2009). See Adão, Correia, and Teles (2009) and the references therein for a discussion of alternative monetary policy rules that achieve a unique equilibrium in an interesting class of monetary models.

The variable Z_l is negative as long as $\phi_1 > 1$ and $\phi_2 \geq 0$. Since π^l is always negative a self-fulfilling fear of deflation equilibrium always exists. At our benchmark values the annualized value of π^l is minus four percent, $\hat{Y}^l = -0.002$. Note that the value of $\hat{Y}^l$ is relatively small compared to the drop in output that occurs in our benchmark case in response to a fall in the discount rate. The reason is that desired savings shifts by less in response to the self-fulfilling rise in the real interest rate than the shift induced by the benchmark discount rare shock. Because the decline in output is relatively small the multiplier is also modest in this case (0.72).

The zero bound and interest rate targeting In this subsection we consider the relation between our analysis of the zero bound and the impact of fiscal policy when the monetary authority deviates from the Taylor rule and sets the interest rate to zero for some amount of time. Here the interest rate does not fall to zero in response to a shock that makes the zero bound binding because the monetary authority is committed to a Taylor rule. Instead, the zero interest rate reflects the desire of the monetary authority to embark on unusual expansionary monetary policy.

Suppose that, starting from the non-stochastic steady state, the monetary authority sets the nominal interest rate to zero and keeps it there with probability p. It is straightforward to show that, while R is equal to zero, the rate of inflation and the percentage deviation of output from the steady state, π^l and Y^l, are given by equations (3.5) and (3.6). Here r^l is equal to the steady state value of the discount factor, $1/\beta - 1$. If $\hat{G}^l$ is equal to zero then π^l and $\hat{Y}^l$ are given by:

$$\pi^l = \frac{\kappa\left(\frac{1}{1-g} + \frac{N}{1-N}\right)(1-\beta)}{(1-\beta p)(1-p)/(1-g) - p\kappa\left(\frac{1}{1-g} + \frac{N}{1-N}\right)},$$

$$\hat{Y}^l = \frac{1-g}{1-p}\left(1-\beta+p\pi^l\right).$$

Under our assumptions π^l and $\hat{Y}^l$ are both positive so the zero interest rate experiment is associated with inflation and an increase in output.

Conditional on the zero interest rate policy being analyzed, the government spending multiplier is given by equation (3.8). The fiscal multiplier is exactly the same as in the case in which the zero bound is binding. The key difference is that in our zero bound experiment the fiscal stimulus is designed to overcome strong deflationary pressures and a decline in output. In the current experiment the fiscal stimulus adds to the inflationary pressures and the expansion in output is created by an exogenous shift to expansionary monetary policy.

Finally, suppose that for some reason the monetary authority is unwilling to lower rates below R^*. All of our results, including the value of the multiplier in the log-linearized version of the model (equation (3.8)), would go through for shocks that are sufficiently large to make the lower bound binding. Zero is, of course, a natural value for R^* but there is nothing special about zero per se.

4. A model with capital and multiple shocks

In the previous section we use a simple model without capital to argue that the government spending multiplier is large whenever the zero bound on the nominal interest rate is binding. Here we show that this basic result extends to a generalized version of the previous model in which we allow for capital accumulation. In addition, we consider three types of shocks: a discount-factor shock, a neutral technology shock, and a capital-embodied technology shock. These shocks have different effects on the behavior of the model economy. But, in all cases, the government spending multiplier is large whenever the zero bound is binding.

The model The preferences of the representative household are given by equations (3.1) and (3.2). The household's budget constraint is given by:

$$P_t\left(C_t + I_t e^{-\psi_t}\right) + B_{t+1} = B_t\left(1+R_t\right) + W_t N_t + P_t r_t^k K_t + T_t, \tag{4.1}$$

where I_t denotes investment, K_t is the stock of capital, and r_t^k is the real rental rate of capital. The capital accumulation equation is given by:

$$K_{t+1} = I_t + (1-\delta)\,K_t - \frac{\sigma_I}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t. \tag{4.2}$$

The variable ψ_t represents a capital-embodied technology shock. The price of investment goods in units of consumption is equal to $\exp(-\psi_t)$. A positive shock to ψ_t is associated with a decline in the price of investment goods. The parameter $\sigma_I > 0$ governs the magnitude of adjustment costs to capital accumulation. As $\sigma_I \to \infty$, investment and the stock of capital become constant. The resulting model behaves in a manner very similar to the one described in the previous section.

The household's problem is to maximize life-time expected utility, given by equations (3.1) and (3.2), subject to the resource constraints given by equations (4.1) and (4.2) and the condition $E_0 \lim_{t\to\infty} B_{t+1}/[(1+R_0)(1+R_1)...(1+R_t)] \geq 0$.

It is useful to derive an expression for Tobin's q, i.e. the value in units of consumption of an additional unit of capital. We denote this value by q_t. Equation (4.1) implies that the real cost of increasing investment by one unit is $e^{-\psi_t}$. Equation (4.2) implies that increasing investment by one unit raises K_{t+1} by $1-\sigma_I\left(\frac{I_t}{K_t}-\delta\right)$ units. It follows that the optimal level of investment satisfies the following equation:

$$e^{-\psi_t} = q_t\left[1 - \sigma_I\left(\frac{I_t}{K_t} - \delta\right)\right]. \tag{4.3}$$

Firms The problem of the final good producers is the same as in the previous section. The discounted profits of the i^{th} intermediate good firm are given by:

$$E_t \sum_{j=0}^{\infty} \beta^{t+j} \upsilon_{t+j} \left\{ P_{t+j}(i) Y_{t+j}(i) - (1-\nu) \left[W_{t+j} N_{t+j}(i) + P_{t+j} r^k_{t+j} K_{t+j}(i) \right] \right\}. \tag{4.4}$$

Output of good i is given by:

$$Y_t(i) = e^{a_t} \left[K_t(i) \right]^{\alpha} \left[N_t(i) \right]^{1-\alpha},$$

where $N_t(i)$ and $K_t(i)$ denote the labor and capital employed by the i^{th} monopolist. The variable a_t represents a neutral technology shock that is common to all intermediate goods producers.

The monopolist is subject to the same Calvo-style price-setting frictions described in Section 2. Recall that $\nu = 1/\varepsilon$ denotes a subsidy that is proportional to the costs of production. This subsidy corrects the steady-state inefficiency created by the presence of monopoly power. The variable υ_{t+j} is the multiplier on the household budget constraint in the Lagrangian representation of the household problem. Firm i maximizes its discounted profits, given by equation (4.4), subject to the Calvo price-setting friction, the production function, and the demand function for $Y_t(i)$, given by equation (2.4).

The central bank follows the same Taylor rule described in Section 2. We compute the government spending multiplier assuming that government consumption increases by one percent over its steady state value, for as long as the zero bound is binding.

Equilibrium The economy's resource constraint is:

$$C_t + I_t e^{-\psi_t} + G_t = Y_t. \tag{4.5}$$

A 'monetary equilibrium' is a collection of stochastic processes,

$$\{C_t, I_t, N_t, K_t, W_t, P_t, Y_t, R_t, P_t(i), r_t^k, Y_t(i), N_t(i), \upsilon_t, B_{t+1}, \pi_t\},$$

such that for given $\{d_t, G_t, a_t, \psi_t\}$, the household and firm problems are satisfied, the monetary policy rule given by equation equation (2.6) is satisfied, markets clear, and the aggregate resource constraint holds.

Experiments At time zero the economy is in its non-stochastic steady state. At time one agents learn that one of the three shocks (r^L, a_t, or ψ_t) differs from its steady state value for T periods and then returns to its steady state value. We consider shocks that are sufficiently large so that the zero bound on the nominal interest rate is binding between two time periods that we denote by t_1 and t_2, where $1 \leq t_1 \leq t_2 \leq T$. The values of t_1 and t_2 are different for different shocks.[6] We solve the model using a shooting algorithm. In practice the key determinants of the multiplier are t_1 and t_2. To maintain comparability with the previous section we keep the size of the discount factor shock the same and choose $T = 10$. In this case t_1 equals one and t_2 equals six. Consequently, the length for which the zero bound is binding after a discount rate shock is roughly the same as in the model without capital.

With the exception of σ_I and δ all parameters are the same as in the economy without capital. We set δ equal to 0.02. We choose the value of σ_I so that the elasticity of I/K with respect to q is equal to the value implied by the estimates in Eberly, Rebelo, and Vincent (2008).[7] The resulting value of σ_I is equal to 17.

[6] The precise timing of when the zero bound constraint is binding may not be unique.

[7] Eberly, Rebelo and Vincent (2008) obtain a point estimate of b equal to 0.06 in the regression $I/K = a + b\ln(q)$. This estimate implies a steady state elasticity of I_t/K_t with respect to Tobin's q of $0.06/\delta$. Our theoretical model implies that this elasticity is equal to $(\sigma_I \delta)^{-1}$. Equating these two elasticities yields a value of σ_I of 17.

As a reference point we note that when the zero bound is not binding the government spending multiplier is roughly 0.9. This value is lower than the value of the multiplier in the model without capital. This lower value reflects the fact that an increase in government spending tends to increase real interest rates and crowd out private investment. This effect is not present in the model without capital.

A discount factor shock We now consider the effect of an increase in the discount factor from its steady state value of four percent (APR) to -1 percent (APR). Figure 6 displays the dynamic response of the economy to this shock. The zero bound is binding in periods one through six. The higher discount rate leads to substantial declines in investment, hours worked, output, and consumption. The large fall in output is associated with a fall in marginal cost and substantial deflation. Since the nominal interest rate is zero, the real interest rate rises sharply. We now discuss the intuition for how investment affects the response of the economy to a discount rate shock. We begin by analyzing why a rise in the real interest rate is associated with a sharp decline in investment. Ignoring covariance terms, the household's first-order condition for investment can be written as:

$$E_t\left(\frac{1+R_{t+1}}{P_{t+1}/P_t}\right) = \frac{1}{q_t}E_t e^{a_t}\alpha K_{t+1}^{\alpha-1}N_{t+1}^{1-\alpha}s_{t+1} + \tag{4.6}$$
$$\frac{1}{q_t}E_t\left\{q_{t+1}\left[(1-\delta)-\frac{\sigma_I}{2}\left(\frac{I_{t+1}}{K_{t+1}}-\delta\right)^2+\sigma_I\left(\frac{I_{t+1}}{K_{t+1}}-\delta\right)\frac{I_{t+1}}{K_{t+1}}\right]\right\},$$

where s_t is the inverse of the markup rate. Equation (4.6) implies that in equilibrium the household equates the returns to two different ways of investing one unit of consumption. The first strategy is to invest in a bond that yields the real interest rate defined by the left-hand side of equation (4.6). The second strategy involves converting the consumption good into $1/q_t$ units of installed capital. The returns to this capital has three components. The first component is the marginal product of capital (the first term on the right-hand side of equation (4.6)). The second component is the value of the undepreciated capital in consumption units ($q_{t+1}(1-\delta)$). The third component is the value in consumption units of the reduction in adjustment costs associated with an increase in installed capital.

To provide intuition it is useful to consider two extreme cases, infinite adjustment costs ($\sigma_I = \infty$) and zero adjustment costs ($\sigma_I = 0$). Suppose first that adjustment costs are infinite. Figure 7 displays a stylized version of this economy. Investment is fixed and savings are an increasing function of the real interest rate. The increase in the discount factor can be thought of as inducing a rightward shift in the savings curve. When this shift is very large, the real interest rate cannot fall by enough to re-establish equilibrium. The intuition for this result and the role played by the zero bound on nominal interest rates is the same as in the model without capital. That model also provides intuition for why the equilibrium is characterized by a large, temporary fall in output, deflation, and a rise in the real interest rate.

Suppose now that there are no adjustment costs ($\sigma_I = 0$). In this case Tobin's q is equal to $e^{-\psi}$ and equation (4.6) simplifies to:

$$E_t \frac{1+R_{t+1}}{P_{t+1}/P_t} = E_t \left[e^{a_t} \alpha K_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha} s_{t+1} + (1-\delta) \right].$$

According to this equation an increase in the real interest rate must be matched by an increase in the marginal product of capital. In general the latter is accomplished, at least in part, by a fall in K_{t+1} caused by a large drop in investment. In Figure 7 the downward sloping curve labeled 'elastic investment' depicts the negative relation between the real interest rate and investment in the absence of any adjustment costs. As drawn the shift in the savings curve moves the equilibrium to point C and does not cause the zero bound to bind. So, the result of an increase in the discount rate is a fall in the real interest rate and a rise in savings and investment.

Now consider a value of σ_I that is between zero and infinity. In this case both investment and q respond to the shift in the discount factor. For our parameter values the higher the adjustment costs the more likely it is that the zero bound is binding. In terms of Figure 7 a higher value of σ_I can be thought of as generating a steeper slope in the investment curve, thus increasing the likelihood that the zero bound binds.

Suppose that the zero bound is binding. Other things equal, a higher real interest rate increases desired saving and decreases desired investment. So the fall in output must be larger to equate the two. This larger fall in output is undone by an increase in government purchases. Consistent with this intuition Figure 6 shows that the government spending multiplier is very large when the zero bound binds (on impact dY/dG is roughly equal to four). This multiplier is actually larger than in the model without capital.

A natural question is what happens to the size of the multiplier as we increase the shock. Recall that in the model without capital, as long as the zero bound is binding, the size of the shock does not affect the size of the multiplier. The analogue result here, established using numerical methods, is that the size of the shock does not affect the multiplier as long as it does not affect t_1 and t_2. For a given t_1 the size of the multiplier is decreasing in t_2. For example, suppose that shock is such that t_2 is equal to four instead of the benchmark value of six. In this case the value of the multiplier falls from 3.9 to 2.3. The latter value is still much larger than 0.9, the value of the multiplier when the zero bound is not binding.

A neutral technology shock We now consider the effect of a temporary, three-percent increase in the neutral technology shock, a_t.[8] Figure 8 displays the dynamic response of the economy to this shock. The zero bound is binding in periods one through eight.[9] Strikingly, the positive technology shock leads to a decline in output, investment, consumption, and hours work. The shock also leads to a sharp rise in the real interest rate and to substantial deflation. To understand these effects it is useful to begin by considering the effects of a technology shock when we abstract from the zero bound. A transitory technology shock triggers a relatively small rise in consumption and a relatively large rise in investment. Other things equal, the expansion in output leads to a rise in marginal cost and the rate of inflation. However, the direct impact of the technology shock on marginal cost dominates and generates strong deflationary pressures. A Taylor rule with a large coefficient on inflation relative to output dictates that the central bank lower real rates to reduce the rate of deflation. If the technology shock is large enough, the zero bound becomes binding. At this point the real interest rate may simply be too high to equate desired savings and investment. The intuition for what happens when the zero bound is binding is exactly the same as for the discount factor shock. The key point is that the only way to reduce desired savings is to have a temporary large fall in output. As with the discount rate shock once the zero bound binds, the government spending multiplier rises dramatically (see Figure 8).

We find that the lowest value of σ_I for which a neutral technology shock renders the zero bound binding is 5.5. Keeping σ_I at its benchmark value the smallest

[8] Our formulation of the Taylor rule assumes that the natural rate of output is constant and equal to the level of output in the steady state. In reality, the monetary authority could well revise its estimate of the natural rate in response to a persistent technology shock. For simplicity we abstract from this possibility.

[9] In computing the government spending multiplier we set $\hat{G}^L$ to 0.5 percent in periods one through eight.

neutral technology shock for which the zero bound binds is 2.3 percent (APR). Again, as long as the zero bound binds, the multiplier is relatively insensitive to the size of the shock.

An investment-specific shock We now consider the effect of a temporary eight percent increase in the price of investment goods (i.e. an eight percent fall in ψ_t). Figure 9 displays the dynamic response of the economy to this shock. Even though the shock that we consider is very large, the zero bound binds only in periods one through three.[10] In addition, the effects of the shock on the economy are small relative to the effects of the other shocks that we discussed. So, while the multiplier is certainly large when the zero bound binds, it is much smaller than in the cases that we have already analyzed.

The shock leads to a decline in output, investment, consumption, and hours worked. It is also associated with deflation and a rise in the real interest rate. To understand how the zero bound can become binding in response to this shock, consider the impact on the economy when the zero bound does not bind. In this case output falls. This fall occurs because the investments become more expansive, reducing the incentive to work. Consumption also falls because of the negative wealth effect of the shock. Other things equal, the fall in output is associated with strong deflationary pressures. Suppose that these deflationary pressures predominate. The fall in output and deflation lead the central bank to lower nominal interest rates. For a sufficiently large shock, the zero bound becomes binding. The intuition for what happens when the zero bound is binding is exactly the same as for the discount factor shock and the neutral technology shock.

We find that zero bound continues to bind even for very low values of σ_I (e.g.

[10] In computing the government spending multiplier we set $\hat{G}^L$ to one percent in periods one, two, and three.

$\sigma_I = 1$). Keeping σ_I at its benchmark value the smallest investment-specific shock for which the zero bound binds is six percent. Once again, as long as the zero bound binds, the multiplier is relatively insensitive to the size of the shock.

5. Robustness analysis

In previous sections we emphasize the response of aggregate demand to shocks as a key mechanism which makes the zero-bound constraint binding. In general the larger is the fall in aggregate demand, the larger is the fall in output and the larger is the government spending multiplier when the zero bound constraint binds. In this section we modify our benchmark analysis to consider various sources of inertia in aggregate demand.

5.1. An alternative form of investment adjustment costs

In this subsection we document the sensitivity of our calculations to the form of investment adjustment costs proposed by CEE. This specification is often used in the macroeconomics literature because it generates impulse responses to monetary policy shocks that are consistent with those estimated using vector autoregressions.

According to this specification the law of motion for investment is given by:

$$K_{t+1} = (1-\delta)K_t + \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right]. \tag{5.1}$$

The function S is increasing, convex, and satisfy the following conditions: $S(1) = S'(1) = 0$, and $S''(1) > 0$. We assume that $S''(1) = 5$ which is close to the value estimated by Smets and Wouters (2007).

Figures 11, 12 and 13 report the response of the economy when the zero bound is binding to a discount factor shock, a neutral technology shock, and a capital embodied shock, respectively. The precise experiments are the same as those underlying Figures 7, 9 and 10.

According to Figure 10, the zero bound is binding in periods one through six after a discount factor shock. Comparing Figures 7 and 11 we see that the main impact of moving to the CEE adjustment cost specification is to reduce the fall in investment and output that occurs after the shock. Not surprisingly, the government spending multiplier is smaller than the one we obtain using the benchmark adjustment cost specification (2.6 versus 3.8 on impact). Nevertheless, the multiplier is much larger than it would be if the zero bound were not binding. To establish this result we calculate the government spending multiplier setting $\hat{G}_t = \hat{G}^l$ in periods one through six after the shock occurs. The resulting impact multiplier is only 0.85.

Figure 11 shows that after a neutral technology shock the zero bound is binding in periods one through seven. Again, the main impact of moving to the CEE adjustment cost specification is to reduce the fall in investment and output after the shock occurs. The government spending multiplier on impact is smaller than we obtain with the benchmark adjustment cost specification (3.9 versus 5.1). But, again, the multiplier is much larger than it would be if the zero bound were not binding. The multiplier associated with setting $\hat{G}_t = \hat{G}^l$ in periods one through seven after the shock occurs is only 0.77.[11]

Figure 12 shows that a different pattern emerges after an investment-specific shock. Here the zero bound is not immediately binding but does become binding in periods four through eight. In this case the multiplier actually rises from 2.3 to 3.3 when we move to the CEE adjustment cost specification.

[11] The reason why the non-zero bound technology shock multiplier is lower than the analogue discount rate shock multiplier (0.85) is that we are increasing $\hat{G}_t$ for one more period in the first case, so there is a stronger negative wealth effects on consumption which drives down the multiplier.

In summary, the results of this subsection show that our basic conclusion about the multiplier is robust: government spending has a much bigger impact on output when the economy is in the zero bound than under normal circumstances.

5.2. The multiplier in a medium-size DSGE model

In this subsection we discuss the size of the multiplier implied by the model in Altig, Christiano, Eichenbaum, and Lindé (2005). This model is estimated to match the estimated impulse response function of ten aggregate U.S. time series to three identified shocks: a neutral technology shock, a capital-embodied technology shock and a monetary policy shock. The ten aggregate time series include measures of the change in the relative price of investment, the change in average productivity, the change in the GDP deflator, capacity utilization, per capita hours worked, the real wage, the shares of consumption and investment in GDP, the Federal Funds Rate and the velocity of money. The sample period is 1959.Q2 to 2008.Q3. For comparability we set the value of $S''(1) = 5$, the value used in the previous subsection.

The model includes a variety of frictions that are useful to match the estimated impulse response functions. These frictions include: sticky wages, sticky prices, variable capital utilization, and the CEE adjustment-cost specification. The representative agent's momentary utility is given by equation (2.22), modified to include internal habit formation in consumption. Both habit formation and the CEE specification for adjustment costs slow down the response of aggregate demand to shocks.

Our key finding is that in order for the zero bound to be binding, shocks must be larger than in our benchmark specification. But, once the economy is in the zero bound the government spending multiplier is much larger than its normal value.

For example, suppose that the discount rate moves from 4 percent to -10.5 percent on an annual basis for ten periods. Then the zero bound is binding between periods two and 11 and the government spending multiplier is 3.4 on impact. The analogue multiplier when the zero bound is not binding is 0.8. The latter multiplier is computed by raising $\hat{G}_t$ to $\hat{G}_t^l$ in periods two to 11. Suppose that the neutral technology shock falls by 4.5 percent for ten periods. Then the zero bound is binding from period three to nine and the government spending multiplier is 1.3 on impact. This multiplier is still substantially larger than the multiplier outside of the zero bound, which is 0.7. The latter multiplier is computed by raising $\hat{G}_t$ to $\hat{G}_t^l$ in periods six to nine. Interestingly, we could not find a reasonable capital-embodied technology shock that makes the zero bound binding. This result reflects the weak direct effect of this shock on aggregate demand coupled with two features that mute the impact of a given shock on aggregate demand: habit formation and the CEE specification for investment adjustment costs.

Using a model similar to the one in Altig et al. (2005), Cogan, Cwik, Taylor, and Wieland (2009) report values of the multiplier that are around one for experiments in which monetary policy is very accommodative, namely the nominal interest rate is set to zero for one or two years. There are three key differences between their experiment and ours. First, in their experiment the nominal interest rate falls to zero due to an expansionary shift in monetary policy. Second, in their main experiments the increase in government spending is permanent. Third, since the shift in monetary policy is temporary (one or two years), the bulk of the increase in government spending occurs when the nominal interest rate is no longer zero. The results in Section 3 imply that the first difference is not important for the size of the multiplier. The second difference implies that their fiscal policy experiment is associated with a stronger negative wealth effect on consumption than ours. So, other things equal, their multiplier should be smaller than ours (see Section 2). Finally, our discussion of the sensitivity of the multiplier to the timing of government spending implies that, because of the third difference in our experiments, their multiplier should be smaller than ours (see Section 3).

6. Conclusion

In this paper we argue that the government spending multiplier can be very large when the zero bound on nominal interest rates is binding. We obtain this conclusion in a variety of models where the government spending multiplier is quite modest when the zero bound is not binding.

Our analysis abstracts from a host of political economy considerations which might make an increase in government spending less attractive than our analysis indicates. We are keenly aware that it is much easier to start new government programs than to end them. It remains very much an open question whether, in the presence of political economy considerations, tax policy of sort emphasized by Eggertson (2009) is a better way of responding a zero bound episode than an increase in government purchases. What our analysis does indicate is that measures designed to increase aggregate demand are particularly powerful during such episodes.

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Figure 1: Effect of an increase in government spending when the zero bound is not binding (model with no capital).

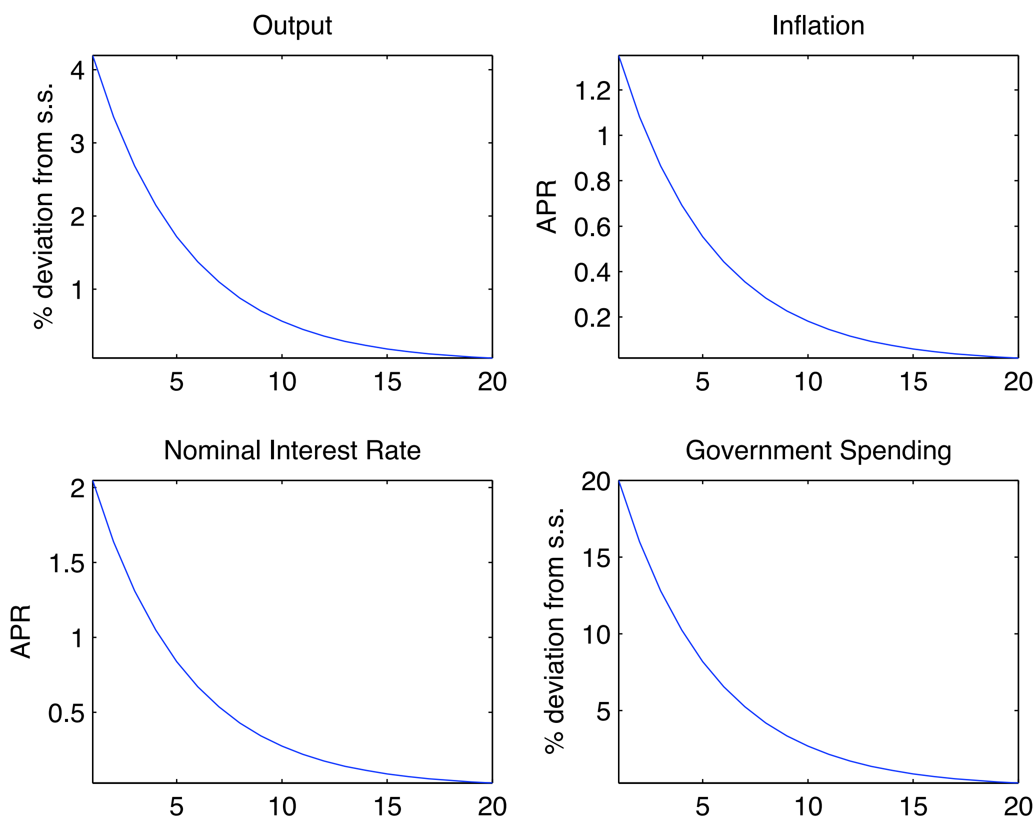

Figure 2: Government spending multiplier when the zero bound is not binding (model with no capital).

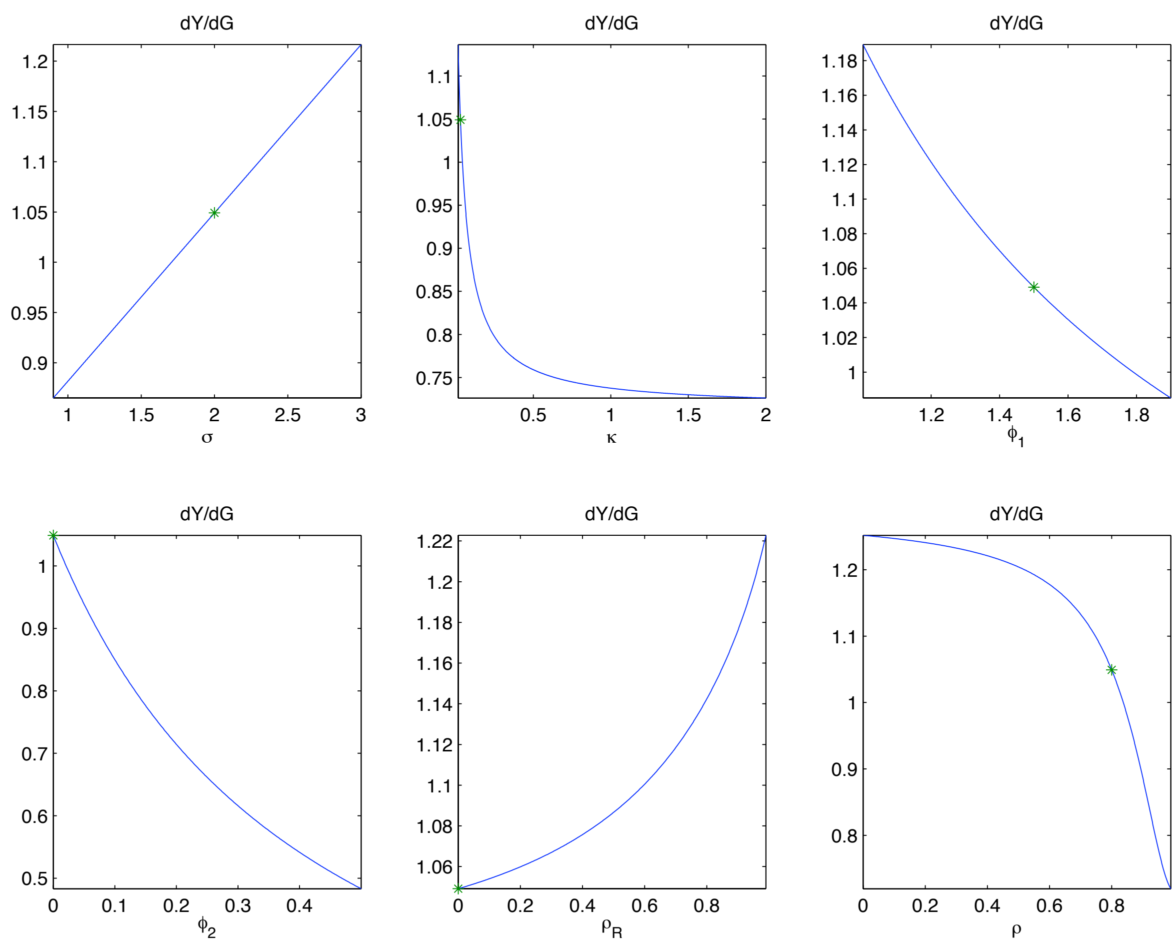

Figure 3: Government spending multiplier
when the zero bound is binding
(model with no capital).

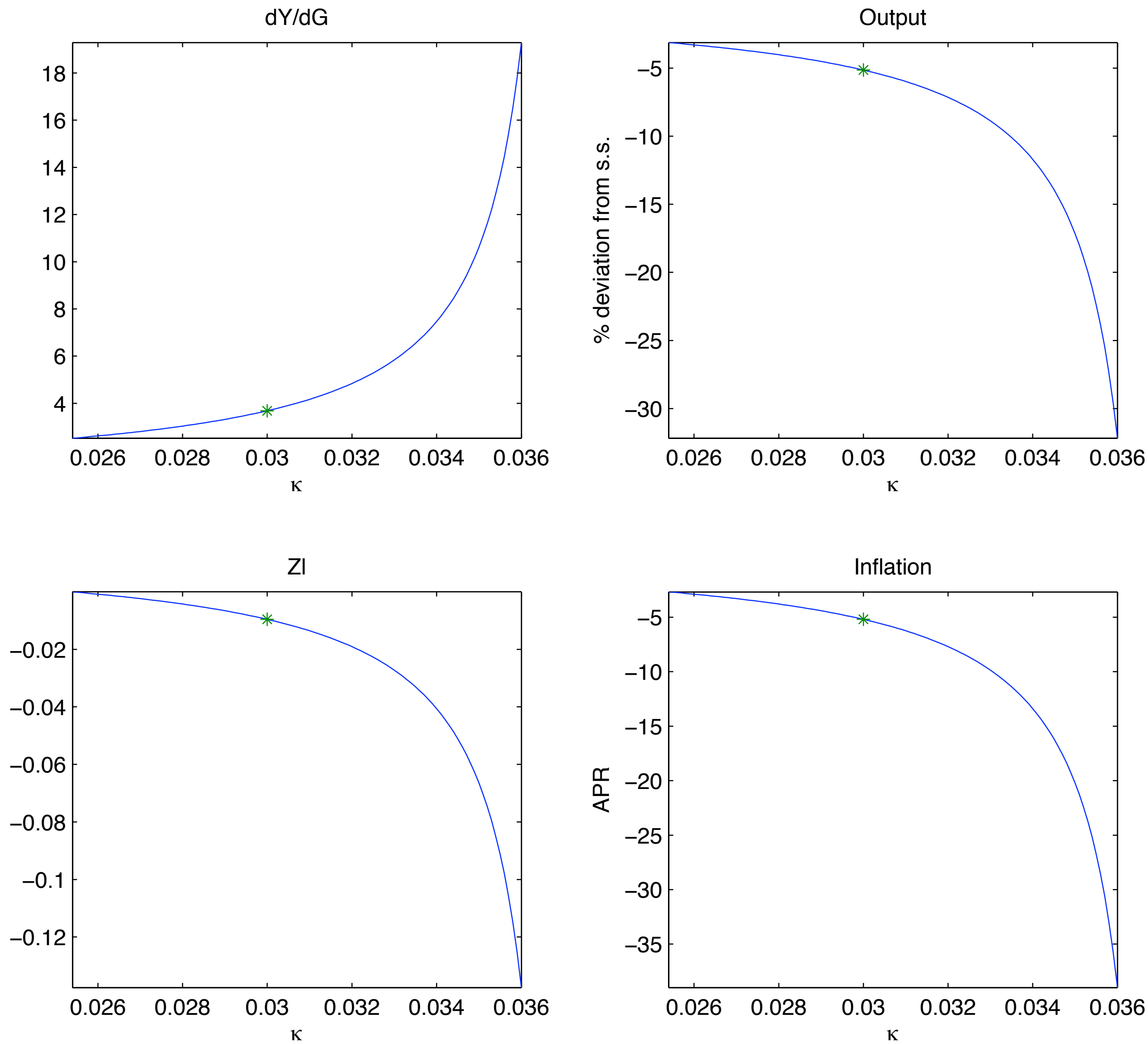

Figure 4: Simple diagram for model with no capital.

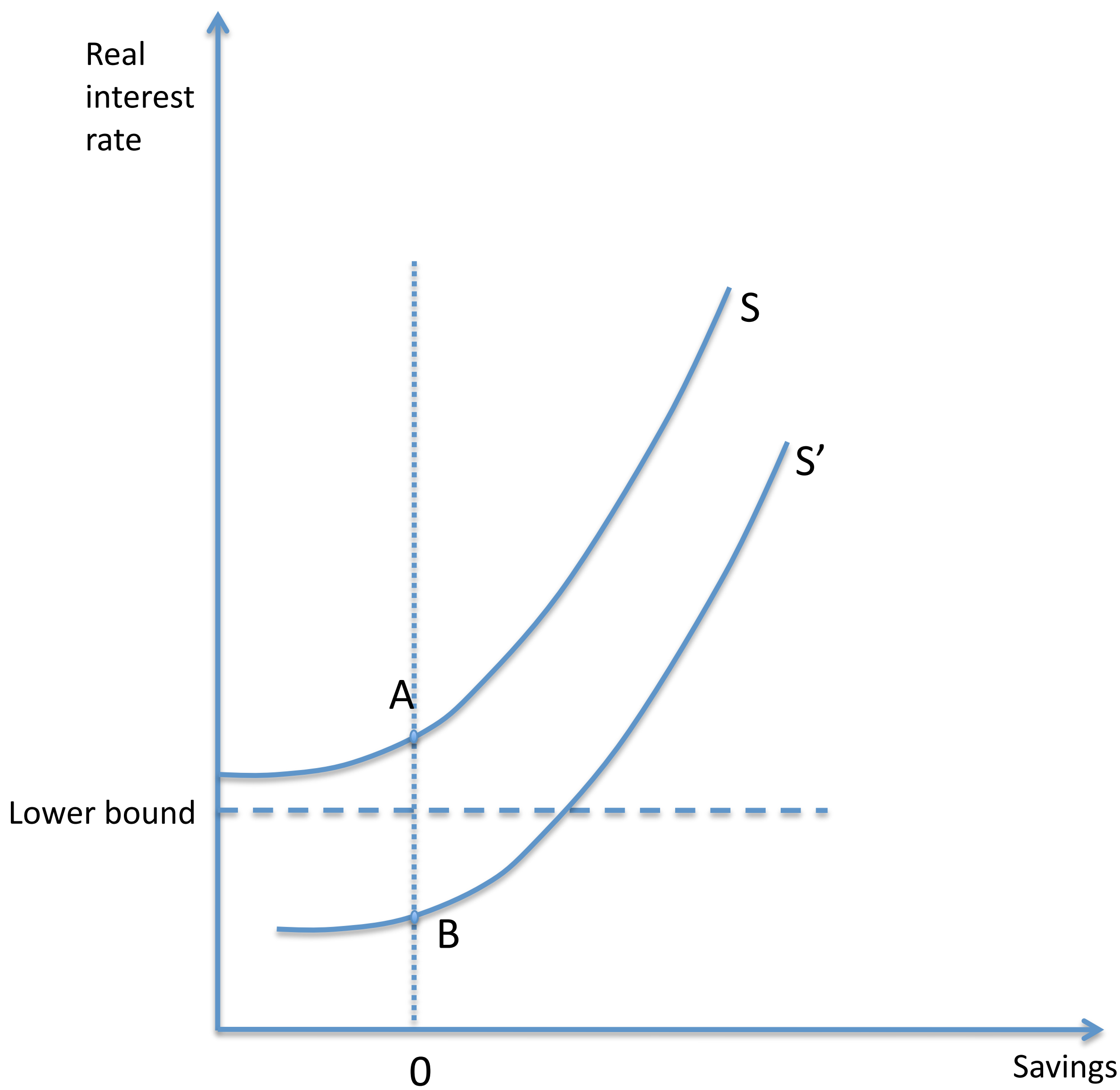

Figure 5: Optimal level of government spending in the zero bound.

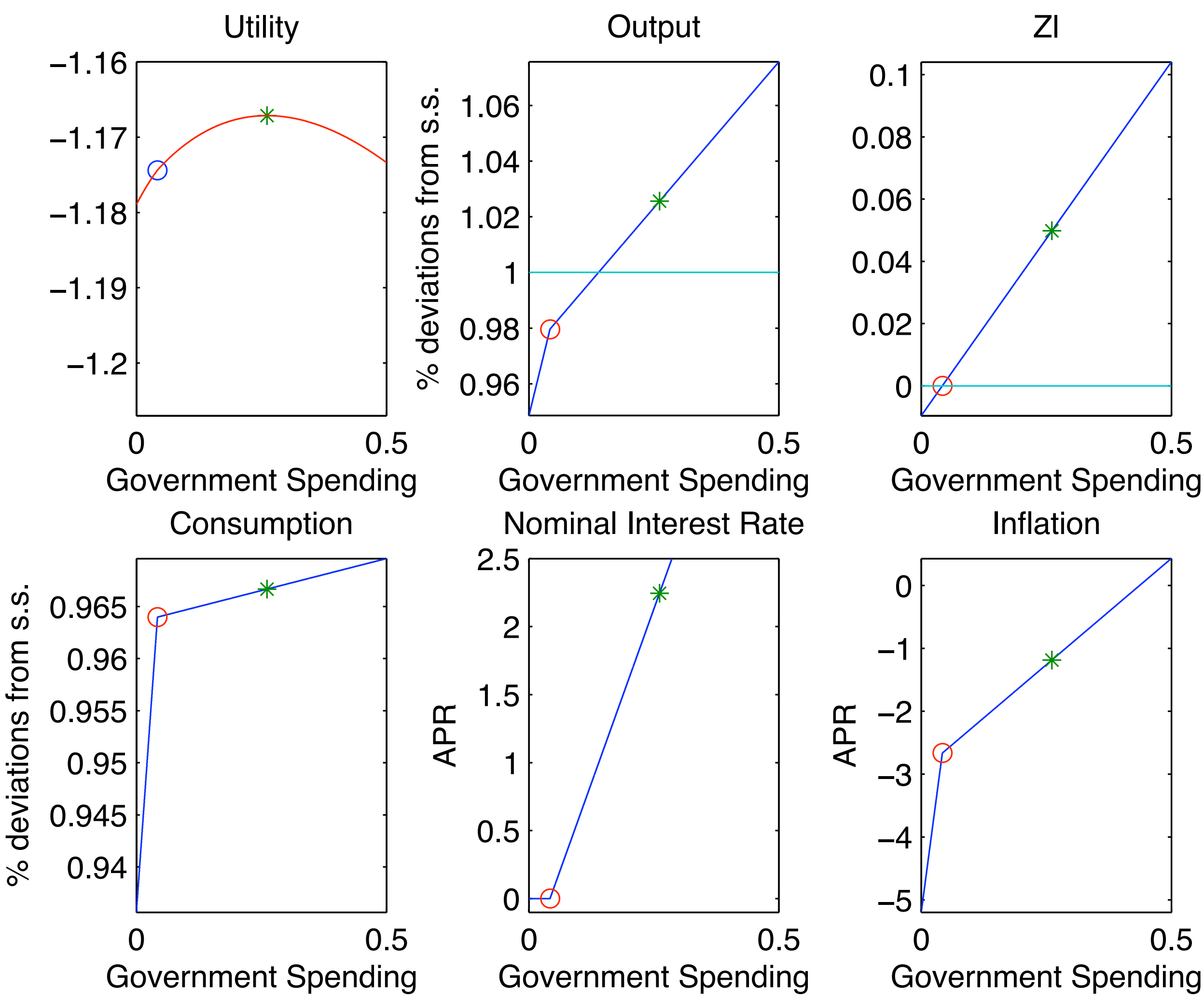

Figure 6: Effect of discount rate shock
when the zero bound is binding
(model with capital, ‘standard’ adjustment costs).

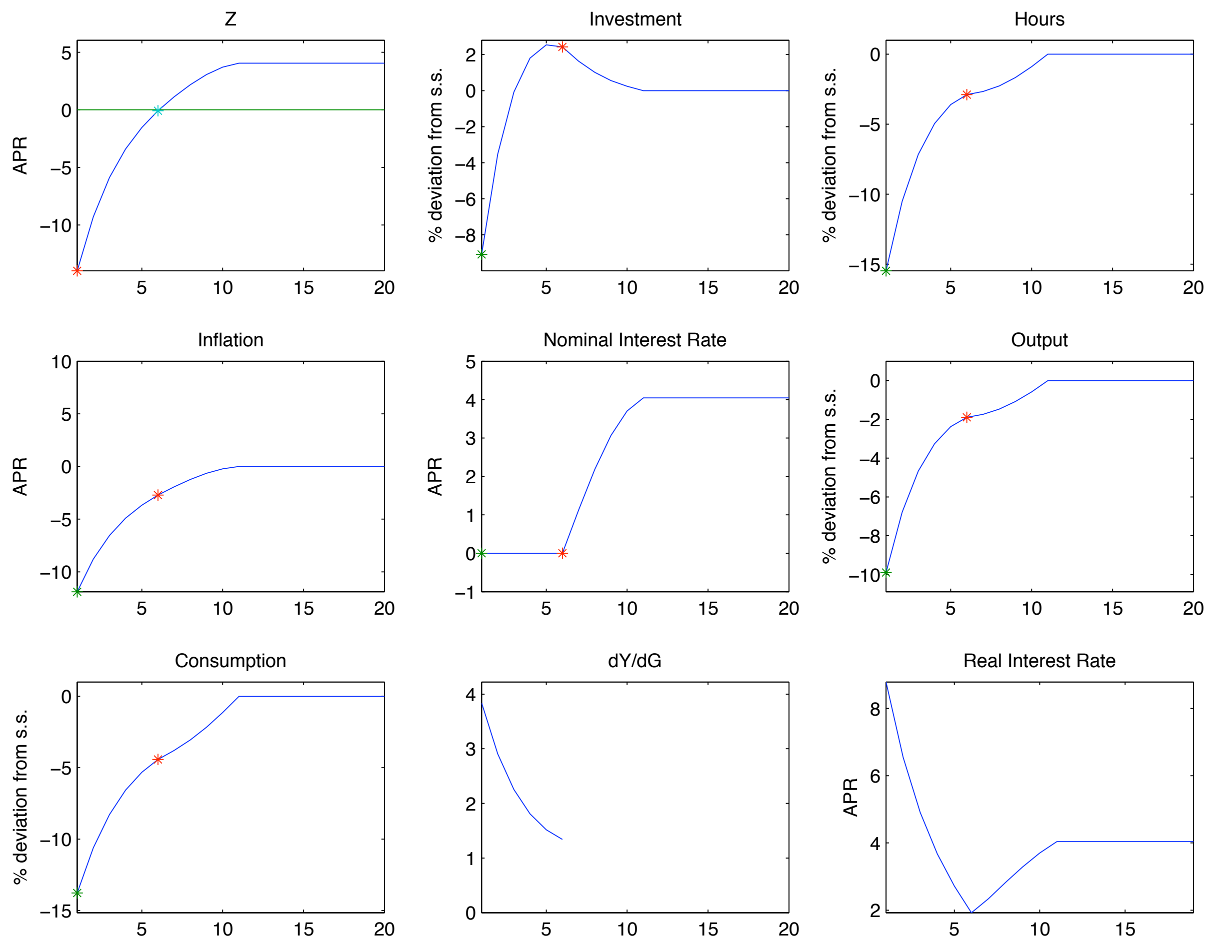

Figure 7: Simple diagram for model with capital.

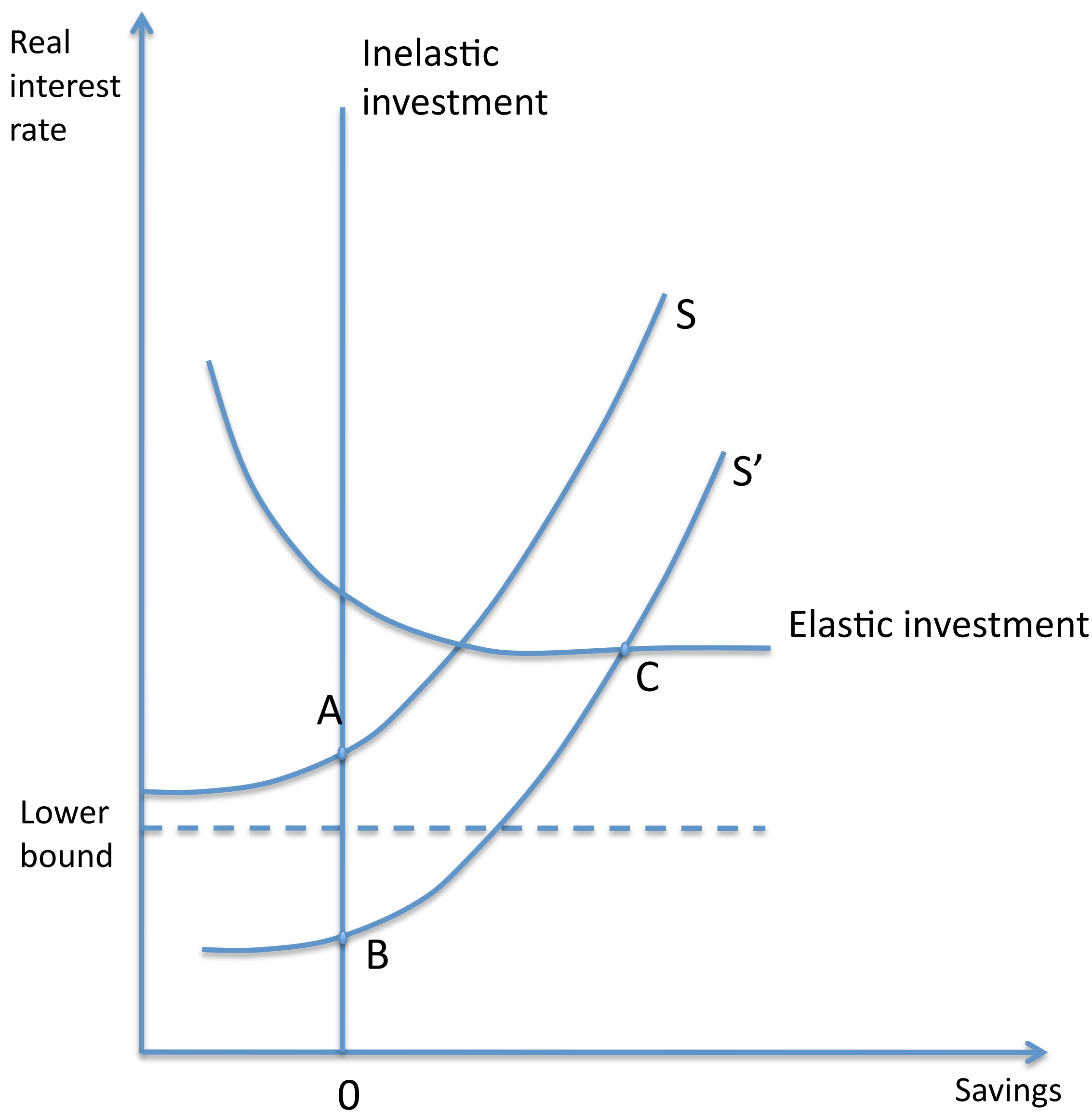

Figure 8: Effect of neutral technology shock
when the zero bound is binding
(model with capital, 'standard' adjustment costs).

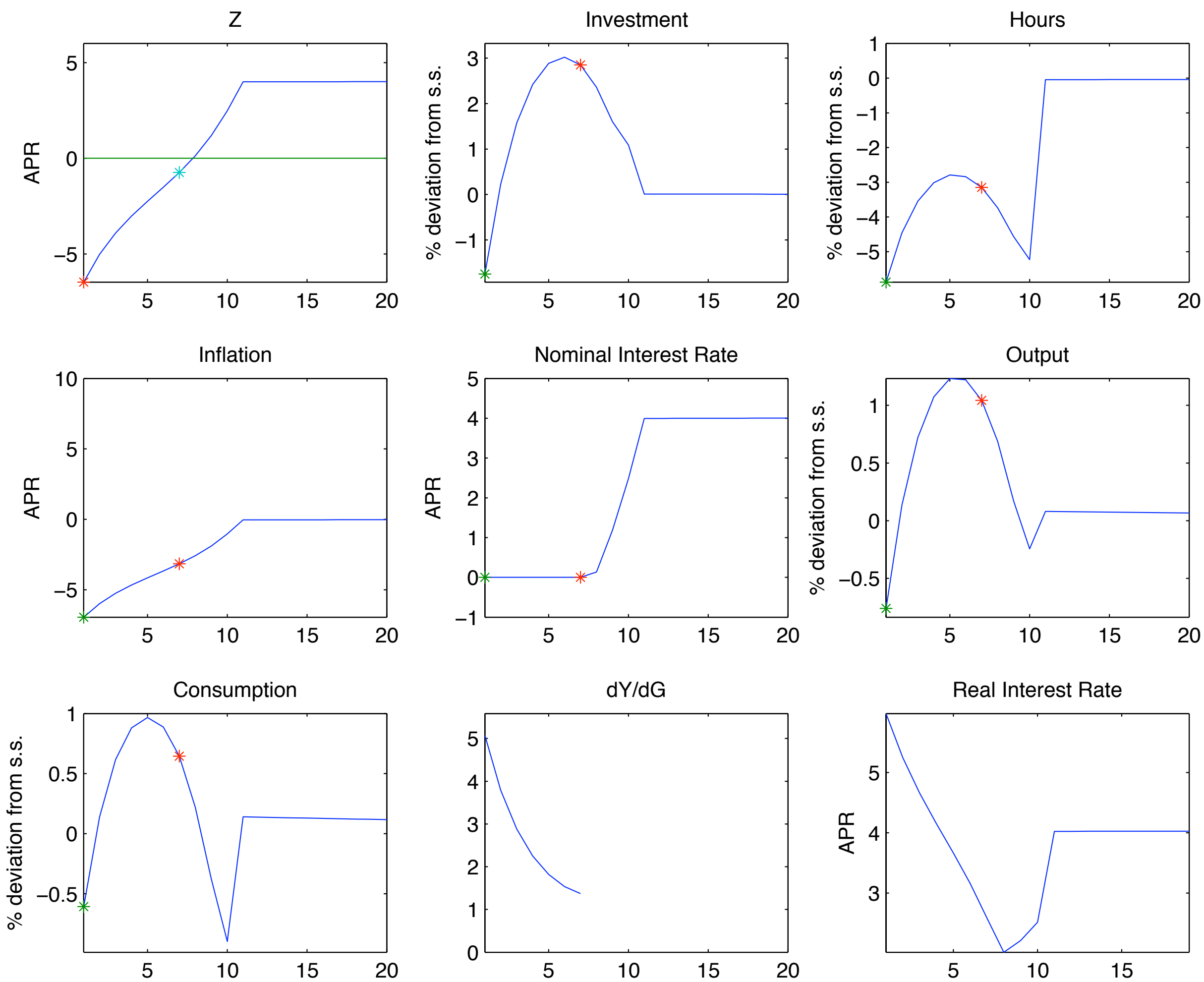

Figure 9: Effect of investment-specific shock
when the zero bound is binding
(model with capital, 'standard' adjustment costs).

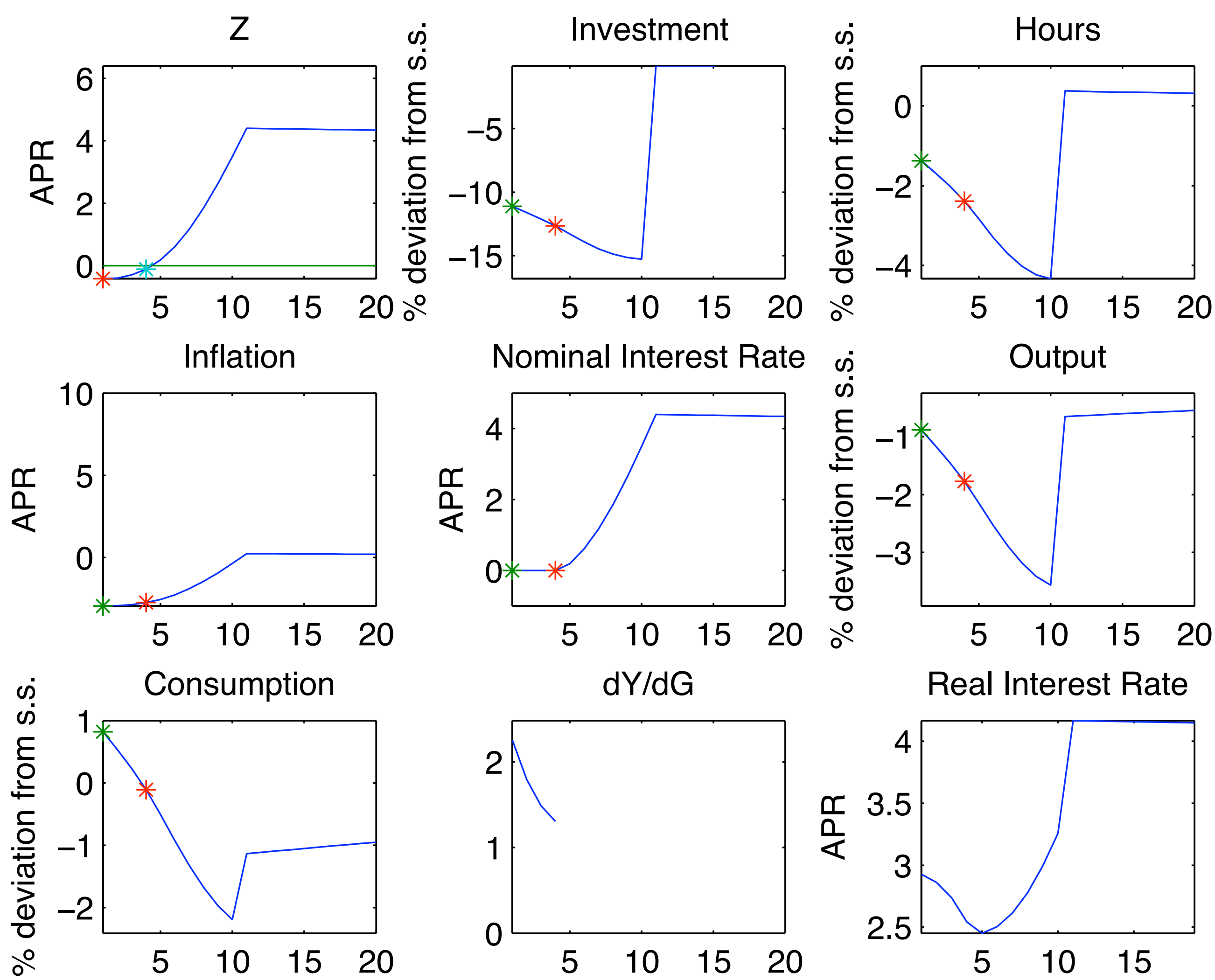

Figure 10: Effect of discount rate shock
when the zero bound is binding
(model with capital, CEE adjustment costs).

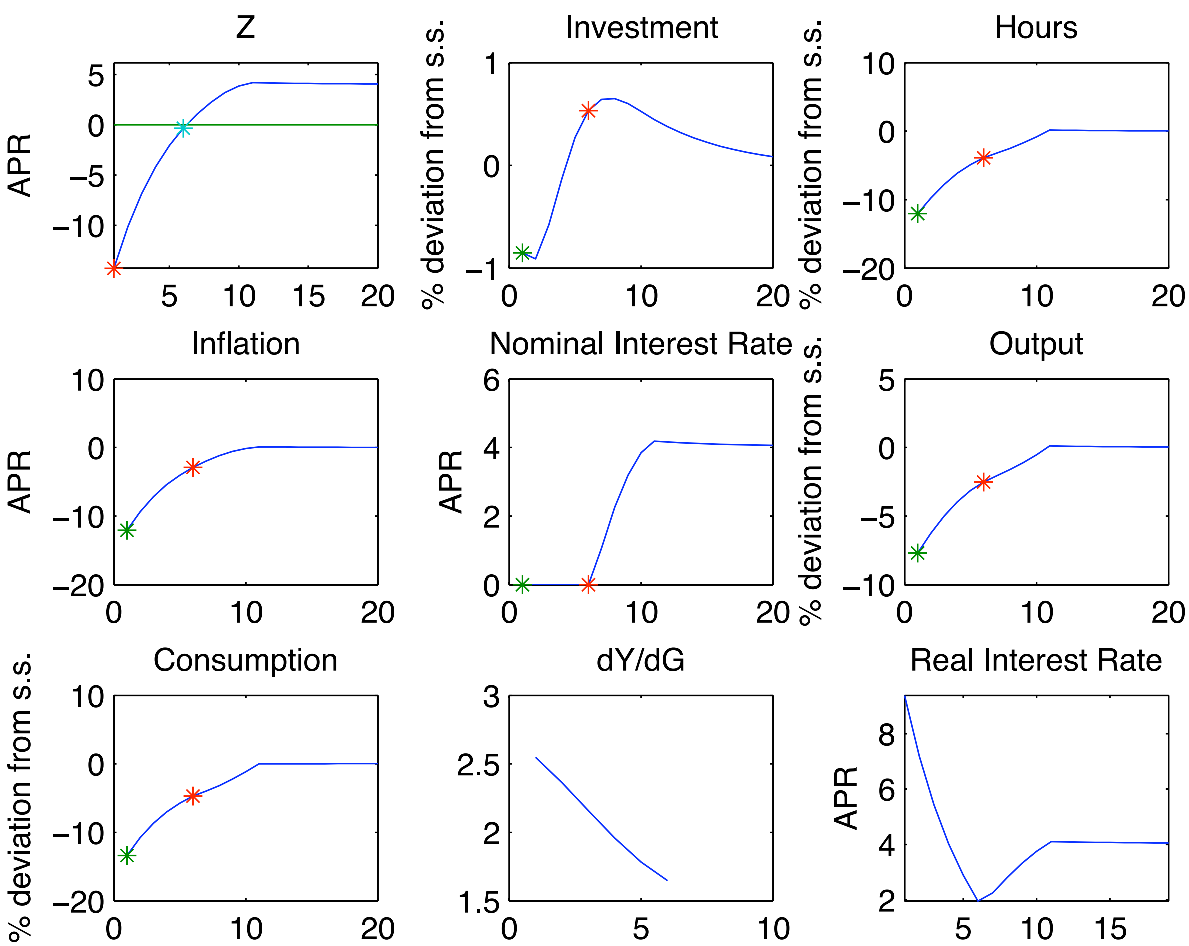

Figure 11: Effect of neutral technology shock
when the zero bound is binding
(model with capital, CEE adjustment costs).

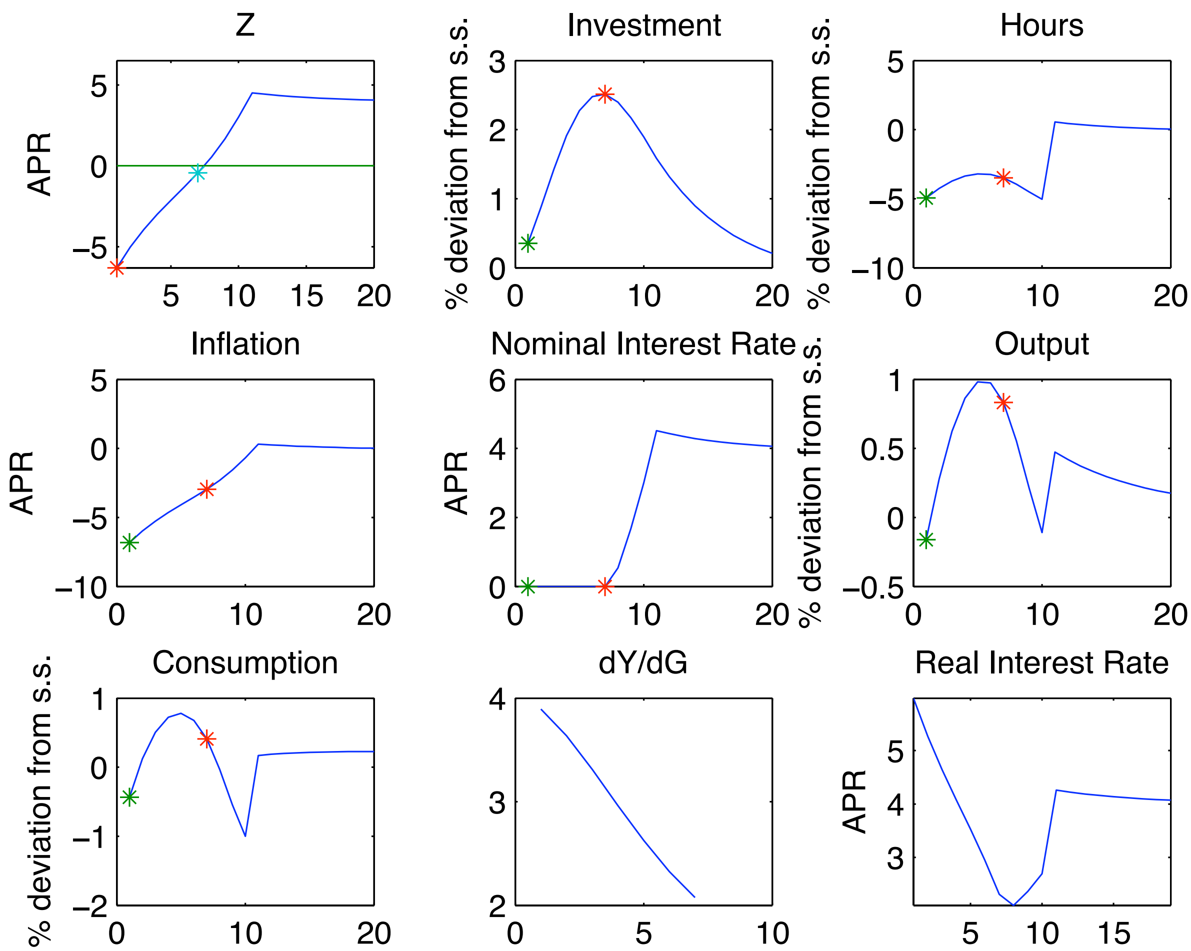

Figure 12: Effect of investment-specific shock
when the zero bound is binding
(model with capital, CEE adjustment costs).