The Macroeconomics of Testing and Quarantining*†

Martin S. Eichenbaum‡ Sergio Rebelo§ Mathias Trabandt¶

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Abstract

Much of the economics literature on epidemics assumes that people know their current health state. Under this assumption, there is no role for testing. To study the general equilibrium effects of testing on economic outcomes, we develop a model of epidemics in which people who are not tested are uncertain about their health state. We find that, when combined with quarantines, testing dramatically reduces the economic costs of the epidemic. This reduction is particularly dramatic when people who recover from an infection acquire only temporary immunity to the virus.

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†Matlab/Dynare replication codes are available at the following URL:
https://sites.google.com/site/mathiastrabandt/home/research

‡Northwestern University and NBER. Address: Northwestern University, Department of Economics, 2211 Campus Dr, Evanston, IL 60208. USA. E-mail: eich@northwestern.edu.

§Northwestern University, NBER, and CEPR. Address: Northwestern University, Kellogg School of Management, 2211 Campus Dr, Evanston, IL 60208. USA. E-mail: s-rebelo@kellogg.northwestern.edu.

¶Freie Universität Berlin, School of Business and Economics, Chair of Macroeconomics. Address: Boltzmannstrasse 20, 14195 Berlin, Germany, German Institute for Economic Research (DIW) and Halle Institute for Economic Research (IWH), E-mail: mathias.trabandt@gmail.com.

1 Introduction

The initial response of most governments to the COVID-19 epidemic was to implement simple containment measures that don't condition on people's health state. These policies imply a sharp, negative trade-off between the level of economic activity and the health consequences of an epidemic (see, for example, Alvarez, Argente, and Lippi (2020) and Eichenbaum, Rebelo and Trabandt (2020)).

There is widespread agreement that testing-based containment policies have important health benefits (see The Rockefeller Foundation Report (2020) and the references therein). In this paper, we show that these policies can also dramatically reduce the economic costs of the epidemic. Our results provide strong support for policies like those advocated by Romer and Garber (2020) and Romer (2020).

There are two reasons to engage in testing. The first reason is to obtain better estimates of how many people have been exposed to the virus and refine estimates of key parameters in epidemiology models. The second reason is to reduce transmission rates by quarantining infected people. We focus on the second reason because testing alone does not resolve a key market failure associated with epidemics: people do not internalize the infection externality associated with their economic activities. Quarantining people who test positively for infection corrects this externality in a way that minimizes damage to the economy as a whole.

Much of the economics literature on epidemics assumes that people know their current health state.[1] Under this assumption, there is no role for testing. To study the general equilibrium effects of testing on economic outcomes, we develop a model of epidemics in which people who are not tested are uncertain about their health state.

In our model testing has two effects. First, it reveals information to people about their health state. Second, it allows the government to condition quarantines on those states.

Analyzing these effects is difficult because there is a natural tendency for the number of people with different probabilities across health states to increase over time. As otherwise identical people receive test results, they become different from each other. To limit this

[1] We discuss some exceptions in our literature review. Examples of COVID models in which people know their health status include Alvarez, Argente, and Lippi (2020), Eichenbaum, Rebelo, and Trabandt (2020) Glover, Heathcote, Krueger, and Rios-Rull (2020), Jones, Philippon, and Venkateswaran (2020). In the context of the HIV/AIDS epidemic, Greenwood, Kircher, Santos, and Tertilt (2019) develop a model in which people know their current health status. In a similar context, Greenwood, Kircher, Santos, and Tertilt (2013) consider a model where people are uncertain about their health status.

source of heterogeneity, we make the following assumptions in our benchmark analysis. First, tests perfectly reveal a person's health state.[2] Second, infected people are asymptomatic. Third, the population is divided into a testing pool and a non-testing pool. Those in the testing pool are tested every period until they recover or die. While stark, these assumptions allow us to highlight the key mechanisms through which testing affects the economy.

Consider first the information revelation effect. We show that this effect is an important source of heterogeneity in people's economic behavior. A person who knows they are susceptible (infected) acts more (less) cautiously than a person who does not know their true health state. So, testing creates dispersion in people's consumption and work decisions. This dispersion plays a central role in the equilibrium effects of testing.

Consider next the use of testing in quarantine policies. To be concrete we consider the following "smart containment" policy. Initially, the government tests α percent of the population. In each subsequent period, the government adds another α percent of the population to the testing pool. People who test positive for infection are not allowed to work or go shopping but receive consumption goods from the government in a way that bypasses social interactions.

We quantify the equilibrium effects of smart containment using a calibrated version of our model. The benefits of smart containment rise sharply as α increases from zero. When α equals 2 percent, the impact of smart containment is very large. For the U.S., it would save roughly a quarter-of-million lives relative to the competitive equilibrium without smart containment. This benefit, in conjunction with a smaller epidemic-induced recession, translates into a present value of roughly 1.7 trillion U.S. dollars. The marginal benefit of increasing α beyond 6 percent is relatively small. In addition, we find that the benefits of smart containment are disproportionately larger the earlier the policy is introduced. An alternative measure of the gains from smart containment is the compensating variation associated with this policy. This variation is the percentage of annual consumption that would make a person in an economy without smart containment have the same lifetime utility of a person in an economy with smart containment. When $\alpha = 0.02$, the annual compensating variation is 0.44 percent of consumption.

Ferguson et al. (2006) argue that a substantial fraction of virus transmissions do not

[2] There are two types of tests for SARS-CoV-2, the virus that causes COVID-19: reverse transcriptase polymerase chain reaction (RT-PCR) tests and serological or blood tests. RT-PCR detect whether a person is currently infected with the virus. Serological tests determine whether a person has been exposed to the virus.

occur as a result of economic activity. This observation suggests that there are large gains from preventing infected people from engaging in all social interactions, not just those related to economic activity. We refer to such a policy as "strict containment." According to our model, the benefits of strict containment are substantially larger than those of smart containment. Relative to the competitive equilibrium, strict containment saves half a million lives, generating a present value of benefits equal to 3.7 trillion U.S. dollars. For $\alpha = 0.02$, the annual compensating variation for this policy is 1 percent of consumption. While the benefits are very large, strict containment is likely to be difficult to implement in practice.

Finally, we study the benefits of both smart and strict containment when people who survive an infection acquire only temporary immunity against the virus. This analysis is of interest because the World Health Organization (2020) cautions that there is no hard evidence to support the assumption that people who recover from an infection acquire permanent immunity. In our model, when immunity is only temporary, an epidemic generate multiple waves of infections that are accompanied by recurrent declines in economic activity. The benefits of both smart and strict containment are much larger than in our benchmark model.

Viewed as a whole, our results are strongly supportive of testing and quarantining policies. We understand that the benefits of these policies are smaller the more altruistic people are and the less informative are test results. So we view our results as providing an upper bound on the welfare gains of smart and strict containment policies. Still, this upper bound illustrates in a concrete way the potential of these policies for reducing the economic costs and health consequences of an epidemic.

Our paper is organized as follows. In Section 2, we describe our benchmark economy. In Section 3, we study the impact of testing alone, smart containment and strict containment. In Section 4, we present our quantitative results. In Sections 3 and 4, we assume that people who recover from an infection acquire permanent immunity to the virus. Section 5 contains a version of the model in which these people acquire only temporary immunity. We review the macroeconomics literature related to our work in Section 6. Section 7 provides some conclusions.

2 Economy with no testing

2.1 The pre-infection economy

The economy is populated by a continuum of ex-ante identical people with measure one. Prior to the start of the epidemic, people are identical and maximize the objective function:

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t, n_t).$$

Here, $\beta \in (0, 1)$ denotes the discount factor and c_t and n_t denote consumption and hours worked, respectively. For simplicity, we assume that momentary utility takes the form

$$u(c_t, n_t) = \ln c_t - \frac{\theta}{2} n_t^2.$$

The budget constraint of the representative person is:

$$c_t = w_t n_t. \tag{1}$$

Here, w_t denotes the real wage rate.

The first-order condition for the representative-person's problem is:

$$\theta n_t = c_t^{-1} w_t.$$

There is a continuum of competitive representative firms of unit measure that produce consumption goods (C_t) using hours worked (N_t) according to the technology:

$$C_t = A N_t.$$

The firm chooses hours worked to maximize its time-t profits Π_t:

$$\Pi_t = A N_t - w_t N_t.$$

In equilibrium, $n_t = N_t$ and $c_t = C_t$.

2.2 The outbreak of an epidemic

Much of the new economic literature on epidemics assumes that people know their health state. In contrast, we assume that individuals do not know their true health state. This state is influenced both by economic and non-economic based social interactions.

As in Kermack and McKendrick (1927), the population consists of four groups: susceptible (people who have not yet been exposed to the virus), infected (people who were infected by the virus), recovered (people who survived the infection and acquired immunity), and deceased (people who died from the infection). The fractions of people in these four groups are denoted by S_t, I_t, R_t and D_t, respectively. People don't know which group they belong to. We denote the state of being alive by a_t. People's time-t subjective probabilities about whether they are susceptible, infected or recovered are given by $p(s_t|a_t)$, $p(i_t|a_t)$, and $p(r_t|a_t)$, respectively

In every period, a fraction π_r of infected people recover and a fraction π_{dt} die. We assume that π_{dt} is time varying to allow for the possibility that the efficacy of the healthcare system deteriorates when a substantial fraction of the population becomes infected. A simple way to model this possibility is to assume that the mortality rate depends on the number of infected people, I_t:

$$\pi_{dt} = \pi_d + \kappa I_t^2. \tag{2}$$

This functional form implies that the mortality rate is a convex function of the fraction of the population that becomes infected.

The timing of events within each period is as follows. Social interactions, including consumption- and work-related activities, happen in the beginning of the period. Then, changes in health states unrelated to social interactions (recovery or death of infected people) occur. Finally, the consequences of social interactions materialize and some susceptible people become infected.

At time zero, a fraction ε of the population becomes infected:

$$I_0 = \varepsilon,\, S_0 = 1 - \varepsilon.$$

This information is public and is used by people to form their time-zero health-state subjective probabilities:

$$p(s_0|a_0) = 1 - \varepsilon,\, p(i_0|a_0) = \varepsilon,\, p(r_0|a_0) = 0.$$

People meet in one of three ways: purchasing consumption goods, working, and engaging in non-economic activities. Meetings occur randomly in all social interactions.

The representative person's subjective probability that the virus is transmitted to him or her is

$$\tau_t = \pi_1 c_t \left(I_t C_t\right) + \pi_2 n_t \left(I_t N_t\right) + \pi_3 I_t. \tag{3}$$

Here, I_tC_t and I_tN_t are the aggregate consumption and hours worked of infected. The terms $\pi_1 c_t (I_tC_t)$ and $\pi_2 n_t (I_tN_t)$ reflects transmissions that result from consumption- and work-related interactions, respectively. The parameter π_1 reflects both the amount of time spent shopping and the probability that the virus is transmitted as a result of that activity. The parameter π_2 reflects the probability that the virus is transmitted as a result of work interactions. The term $\pi_3 I_t$ reflects transmissions that result from non-economic interactions.

Infected or recovered people are unaffected if the virus is transmitted to them. Only susceptible people can become infected by the virus. The representative person's subjective probability of becoming infected is:

$$\tau_t p(s_t|a_t) + \tau_t p(i_t|a_t) \times 0 + \tau_t p(r_t|a_t) \times 0.$$

The subjective probability of being infected at time $t+1$, conditional on being alive at time t, is

$$p(i_{t+1}|a_t) = \tau_t p(s_t|a_t) + (1 - \pi_r - \pi_{dt})p(i_t|a_t). \tag{4}$$

Here, $(1-\pi_r-\pi_{dt})p(i_t|a_t)$ is the subjective probability that a person who is infected at time t survives until time $t+1$ but does not recover. In addition, $\tau_t p(s_t|a_t)$ is the subjective probability of being susceptible at time t and becoming infected at time $t+1$. The representative person's subjective probability of being susceptible at time $t+1$ conditional on being alive at time t is

$$p(s_{t+1}|a_t) = (1 - \tau_t)p(s_t|a_t). \tag{5}$$

The subjective probability of being recovered at time $t+1$, conditional on being alive at time t is

$$p(r_{t+1}|a_t) = p(r_t|a_t) + \pi_r p(i_t|a_t). \tag{6}$$

Using the following conditions

$$p(s_{t+1}|a_{t+1}) = \frac{p(s_{t+1}|a_t)}{1 - \pi_{dt}p(i_t|a_t)},$$

$$p(i_{t+1}|a_{t+1}) = \frac{p(i_{t+1}|a_t)}{1 - \pi_{dt}p(i_t|a_t)},$$

$$p(r_{t+1}|a_{t+1}) = \frac{p(r_{t+1}|a_t)}{1 - \pi_{dt}p(i_t|a_t)},$$

we can rewrite equations (4), (5), and (6) as

$$p(i_{t+1}|a_{t+1}) \left[1 - \pi_{dt}p(i_t|a_t)\right] = \tau_t p(s_t|a_t) + (1 - \pi_r - \pi_{dt})p(i_t|a_t), \tag{7}$$

$$p(s_{t+1}|a_{t+1})\left[1-\pi_{dt}p(i_t|a_t)\right] = p(s_t|a_t)(1-\tau_t), \tag{8}$$

$$p(r_{t+1}|a_{t+1})\left[1-\pi_{dt}p(i_t|a_t)\right] = p(r_t|a_t) + \pi_r p(i_t|a_t). \tag{9}$$

2.3 The problem of the representative person

Since everybody has the same subjective probabilities about their health state, everyone chooses the same level of consumption (c_t) and hours worked (n_t). The lifetime utility of the representative person at time t, U_t, is given by

$$U_t = \sum_{j=0}^{\infty} \beta^j p(a_{t+j}|a_t) u\left(c_{t+j}, n_{t+j}\right),$$

where $p(a_{t+j}|a_t)$ is the probability of being alive at time $t+j$ given that the person is alive at time t. We can rewrite U_t as

$$U_t = u(c_t,n_t)+\beta\left[1-\pi_{dt}p(i_t|a_t)\right]u(c_{t+1},n_{t+1})+\beta^2\left[1-\pi_{dt}p(i_t|a_t)\right]\left[1-\pi_{dt+1}p(i_{t+1}|a_{t+1})\right]U_{t+2}. \tag{10}$$

The problem of the representative person is to maximize (10) subject to the budget constraint, (1), the transmission function, (3), and the probability equations (7) and (8).[3]

The first-order conditions with respect to c_t, n_t, τ_t, $p(i_{t+1}|a_{t+1})$, and $p(s_{t+1}|a_{t+1})$ are given by

$$u_1\left(c_t,n_t\right) - \lambda_t^b + \lambda_t^\tau \pi_1\left(I_t C_t\right) = 0,$$

$$u_2\left(c_t,n_t\right) + \lambda_t^b A + \lambda_t^\tau \pi_2\left(I_t N_t\right) = 0,$$

$$-\lambda_t^\tau + \lambda_t^i p(s_t|a_t) - \lambda_t^s p(s_t|a_t) = 0,$$

$$-\beta^2\pi_{dt+1}U_{t+2} - \lambda_t^i + \beta\lambda_{t+1}^i[1-\pi_r-\pi_{dt+1}(1-p(i_{t+2}|a_{t+2}))] + \beta\lambda_{t+1}^s\pi_{dt+1}p(s_{t+2}|a_{t+2}) = 0,$$

$$\beta\lambda_{t+1}^i\tau_{t+1} - \lambda_t^s + \beta\lambda_{t+1}^s(1-\tau_{t+1}) = 0.$$

Here, $\lambda_{t+j}^b\beta^j p(a_{t+j}|a_t)$, $\lambda_{t+j}^\tau\beta^j p(a_{t+j}|a_t)$, $\lambda_{t+j}^i\beta^j p(a_{t+j}|a_t)$, and $\lambda_{t+j}^s\beta^j p(a_{t+j}|a_t)$ denote the Lagrange multipliers associated with constraints (1), (3), (7), and (8), respectively.

[3] Equation (9) is redundant since $p(s_{t+1}|a_{t+1}) + p(i_{t+1}|a_{t+1}) + p(r_{t+1}|a_{t+1}) = 1$.

Equilibrium In equilibrium, each person solves their maximization problem. In addition, the goods and labor markets clear:

$$(S_t + I_t + R_t)\, c_t = AN_t,$$

$$(S_t + I_t + R_t)\, n_t = N_t.$$

Given rational expectations, the subjective and objective probabilities of different health states coincide:

$$\begin{aligned} S_t &= p(s_t|a_0), \\ I_t &= p(i_t|a_0), \\ R_t &= p(a_t|a_0) - p(s_t|a_0) - p(i_t|a_0), \\ D_t &= 1 - p(a_t|a_0). \end{aligned}$$

where

$$\begin{aligned} p(s_t|a_0) &= p(s_t|a_t)p(a_t|a_{t-1})p(a_{t-1}|a_{t-2})...p(a_1|a_0), \\ p(i_t|a_0) &= p(i_t|a_t)p(a_t|a_{t-1})p(a_{t-1}|a_{t-2})...p(a_1|a_0), \\ p(a_t|a_0) &= p(a_t|a_{t-1})p(a_{t-1}|a_{t-2})...p(a_1|a_0), \end{aligned}$$

and

$$p(a_t|a_{t-1}) = 1 - \pi_{dt-1}p(i_{t-1}|a_{t-1}).$$

Herd immunity Herd immunity is a term used in the epidemiology literature to refer to situations in which the number of susceptible people is sufficiently low so that the number of infected people cannot rise, i.e. $I_{t+1} < I_t$. In the standard SIR model ($\pi_1 = \pi_2 = 0$, π_d constant), the highest value of S_t consistent with herd immunity is $(\pi_r + \pi_d)/\pi_3$.[4] The decomposition of non-susceptible people between recovered and infected is irrelevant.

In our model, herd immunity depends on both the number of susceptible and infected people. The reason is as follows. The number of infected people determines the risk of infection from engaging in economic activities. This risk affects the level of consumption and hours worked by the representative person which, in turn, influences the likelihood of new infections (equation (3)).

[4]See Fernández-Villaverde and Jones (2020) who use an estimated version of the SIR model to assess whether different countries have achieved herd immunity.

For our model, we define: (i) herd immunity as the set of pairs $\{S_t, I_t\}$ such that $I_{t+1} < I_t$ and (ii) "steady-state herd immunity" as the highest level of S_t such that $I_{t+1} < I_t$ when per capita consumption and hours worked are equal to their pre-epidemic steady state levels. The second concept of immunity applies when I_t is arbitrarily close to zero, so the risk of infection from engaging in economic activities is negligible. In general, herd immunity obtains for higher values of S_t than is required for steady-state herd immunity. The reason is that, during an epidemic, consumption and hours worked are below their steady-state levels, exerting downwards pressure on the number of new infections.

3 Model with testing

Two critical issues facing policy makers are as follows. First, how widespread should testing be in a world where people are uncertain about their health state? Second, how should containment measures be conditioned on the results of such tests?

To highlight the key mechanisms through which testing affects the economy, we assume that tests perfectly reveal people's health state. In addition, we suppose that in each period the government tests an additional α percent of the population that has not yet been tested. A person who enters the testing pool gets tested in every period. Taken together, our simplifying assumptions bound the degree of heterogeneity in the economy because the timing of entry into the testing pool does not affect current consumption or work decisions. All that matters for these decisions is a person's current health state.

We now discuss the maximization problem of people inside and outside the testing pool. We use the superscripts u and k to denote variables that pertain to people with unknown and known health states, respectively.

3.1 People outside the testing pool

People outside the testing pool are uncertain about their current health state. Those who survive, enter the testing pool at time $t + 1$ with probability α and will, at each point in time, learn their heath state.

We assume that testing starts in period 0, so the initial conditions for the different groups in the population are:

$$I_0^u = \varepsilon,\ S_0^u = 1 - \varepsilon, \text{ and } S_0^k = I_0^k = R_0^u = R_0^k = 0.$$

The probabilities that a given person outside the testing pool is susceptible, infected or recovered at time zero are given by

$$p(s_0|a_0) = 1-\varepsilon,\ p(i_0|a_0) = \varepsilon,\ p(r_0|a_0) = 0.$$

The lifetime utility of a person who is outside the testing pool, U_t^u, is given by

$$\begin{aligned} U_t^u &= u(c_t^u, n_t^u) + (1-\alpha)\beta\left[1-\pi_{dt}p(i_t|a_t)\right]U_{t+1}^u \\ &\quad +\alpha\beta\left[1-\pi_{dt}p(i_t|a_t)\right]\left[p(s_{t+1}|a_{t+1})U_{t+1}^s + p(i_{t+1}|a_{t+1})U_{t+1}^i + p(r_{t+1}|a_{t+1})U_{t+1}^r\right] \end{aligned} \tag{11}$$

The variables U_{t+1}^s, U_{t+1}^i, and U_{t+1}^r denote the lifetime utility of a person who is susceptible, infected and recovered at time $t+1$, respectively.

In deriving the first-order conditions of a person's maximization problem, it is useful to write U_{t+1}^u as

$$\begin{aligned} U_{t+1}^u &= u(c_{t+1}^u, n_{t+1}^u) + (1-\alpha)\beta\left[1-\pi_{dt+1}p(i_{t+1}|a_{t+1})\right]U_{t+2}^u \\ &\quad \alpha\beta\left[1-\pi_{dt+1}p(i_{t+1}|a_{t+1})\right]\left[p(s_{t+2}|a_{t+2})U_{t+2}^s + p(i_{t+2}|a_{t+2})U_{t+2}^i + p(r_{t+2}|a_{t+2})U_{t+2}^r\right] \end{aligned}$$

The problem of a person outside the testing pool is to maximize (11) subject to the budget constraint, the transmission function, and the laws of motion for the probability of being infected and susceptible:

$$c_t^u = An_t^u + \Gamma_t^u, \tag{12}$$

$$\tau_t^u = \pi_1 c_t^u\left(I_t^u C_t^u + I_t^k C_t^i\right) + \pi_2 n_t^u\left(I_t^u N_t^u + I_t^k N_t^i\right) + \pi_3\left(I_t^u + I_t^k\right), \tag{13}$$

$$p(i_{t+1}|a_{t+1})[1-\pi_{dt}p(i_t|a_t)] = \tau_t^u p(s_t|a_t) + (1-\pi_r-\pi_{dt})p(i_t|a_t), \tag{14}$$

$$p(s_{t+1}|a_{t+1})[1-\pi_{dt}p(i_t|a_t)] = p(s_t|a_t)(1-\tau_t^u). \tag{15}$$

In the budget constraint (12), Γ_t^u denotes a lump-sum transfer from the government. The first-order conditions with respect to c_t^u, n_t^u, τ_t^u, $p(i_{t+1}|a_{t+1})$, and $p(s_{t+1}|a_{t+1})$ are given by

$$u_1\left(c_t^u, n_t^u\right) - \lambda_{bt}^u + \lambda_{\tau t}^u\pi_1\left(I_t^u C_t^u + I_t^k C_t^i\right) = 0,$$

$$u_2\left(c_t^u, n_t^u\right) + \lambda_{bt}^u A + \lambda_{\tau t}^u\pi_2\left(I_t^u N_t^u + I_t^k N_t^i\right) = 0,$$

$$-\lambda_{\tau t}^u + \lambda_{it}^u p(s_t|a_t) - \lambda_{st}^u p(s_t|a_t) = 0,$$

$$\frac{dU_t^u/dp(i_{t+1}|a_{t+1})}{1-\pi_{dt}p(i_t|a_t)} - \lambda_{it}^u + \beta\lambda_{it+1}^u \pi_{dt+1} p(i_{t+2}|a_{t+2}) +$$
$$\beta\lambda_{it+1}^u(1-\pi_r-\pi_{dt+1}) + \beta\lambda_{st+1}^u \pi_{dt+1} p(s_{t+2}|a_{t+2}) = 0,$$

$$\frac{dU_t^u/dp(s_{t+1}|a_{t+1})}{1-\pi_{dt}p(i_t|a_t)} + \beta\lambda_{it+1}^u \tau_{t+1}^u - \lambda_{st}^u + \beta\lambda_{st+1}^u(1-\tau_{t+1}^u) = 0.$$

Here, $\lambda_{bt+j}^u \beta^j p(a_{t+j}|a_t)$, $\lambda_{\tau t+j}^u \beta^j p(a_{t+j}|a_t)$, $\lambda_{it+j}^u \beta^j p(a_{t+j}|a_t)$, and $\lambda_{st+j}^u \beta^j p(a_{t+j}|a_t)$ denote the Lagrange multipliers associated with constraints (12), (13), (14), and (15), respectively.

The aggregate distribution of people outside the testing pool, according to health states is given by

$$\begin{aligned}
S_{t+1}^u &= p(s_{t+1}|a_0)(1-\alpha)^t, \\
I_{t+1}^u &= p(i_{t+1}|a_0)(1-\alpha)^t, \\
R_{t+1}^u &= [p(a_{t+1}|a_0) - p(s_{t+1}|a_0) - p(i_{t+1}|a_0)](1-\alpha)^t.
\end{aligned}$$

3.2 People inside the testing pool

People inside the testing pool know whether they are susceptible, infected or recovered at time t. People who are susceptible and infected face uncertainty about their future health state.

A person of type $j \in \{s, i, r\}$ has the budget constraint

$$c_t^j = w_t n_t^j + \Gamma_t^j, \tag{16}$$

where Γ_t^j is a lump sum transfer from the government. The indexes s, i, and r, denote infected, susceptible and recovered, respectively.

We now describe the optimization problem of the different people inside the testing pool.

Susceptible people The lifetime utility of a susceptible person, U_t^s, is

$$U_t^s = u(c_t^s, n_t^s) + \beta\left[(1-\tau_t^s)U_{t+1}^s + \tau_t^s U_{t+1}^i\right]. \tag{17}$$

Here, the variable τ_t^s represents the probability that a susceptible person becomes infected:

$$\tau_t^s = \pi_1 c_t^s \left(I_t^u C_t^u + I_t^k C_t^i\right) + \pi_2 n_t^s \left(I_t^u N_t^u + I_t^k N_t^i\right) + \pi_3 \left(I_t^u + I_t^k\right). \tag{18}$$

Critically, susceptible people understand that consuming and working less reduces their probability of becoming infected.

The first-order conditions for consumption and hours worked are

$$u_1(c^s_t, n^s_t) - \lambda^s_{bt} + \lambda^s_{\tau t}\pi_1 \left(I^u_t C^u_t + I^k_t C^i_t\right) = 0,$$

$$u_2(c^s_t, n^s_t) + A\lambda^s_{bt} + \lambda^s_{\tau t}\pi_2 \left(I^u_t N^u_t + I^k_t N^i_t\right) = 0.$$

Here, λ^s_{bt} and $\lambda^s_{\tau t}$ are the Lagrange multipliers associated with constraints (16) and (18), respectively.

The first-order condition for τ^s_t is

$$\beta \left(U^i_{t+1} - U^s_{t+1}\right) - \lambda^s_{\tau t} = 0. \tag{19}$$

Infected people The lifetime utility of an infected person, U^i_t, is

$$U^i_t = u(c^i_t, n^i_t) + \beta \left[(1 - \pi_r - \pi_{dt})\, U^i_{t+1} + \pi_r U^r_{t+1}\right]. \tag{20}$$

The expression for U^i_t embodies a common assumption in macro and health economics that the cost of death is the foregone utility of life.

The first-order conditions for consumption and hours worked are given by

$$u_1(c^i_t, n^i_t) = \lambda^i_{bt},$$

$$u_2(c^i_t, n^i_t) = -A\lambda^i_{bt},$$

where λ^i_{bt} is the Lagrange multiplier associated with constraint (16).[5]

Recovered people The lifetime utility of a recovered person, U^r_t, is

$$U^r_t = u(c^r_t, n^r_t) + \beta U^r_{t+1}. \tag{21}$$

The first-order conditions for consumption and hours worked are

$$u_1(c^r_t, n^r_t) = \lambda^r_{bt}$$

$$u_2(c^r_t, n^r_t) = -A\lambda^r_{bt}$$

where λ^r_{bt} is the Lagrange multiplier associated with constraint (16).

[5] We assume that infected people are as productive as other people. Absent this assumption people could learn whether they are infected based on their productivity.

Equilibrium In equilibrium, group-specific aggregates and individual levels of consumption and hours worked coincide:

$$c_t^j = C_t^j,\ n_t^j = N_t^j,$$

where $j \in \{s, i, r, u\}$.

The government budget constraint holds:

$$\Gamma_t \left(S_t^k + R_t^k + S_t^u + I_t^u + R_t^u\right) + \Gamma_t^i I_t^k = 0,$$

where Γ_t^i is a positive lump-sum transfer that finances the consumption of the infected and quarantined. The variable $\Gamma_t = \Gamma_t^j$ for $j = s, r, u$ is a negative lump-sum transfer on everybody else. In equilibrium, each person solves their maximization problem and the government budget constraint is satisfied. In addition, the goods and labor markets clear:

$$\left(S_t^k C_t^s + I_t^k C_t^i + R_t^k C_t^r\right) + \left(S_t^u + I_t^u + R_t^u\right) C_t^u = AN_t,$$

$$\left(S_t^k N_t^s + I_t^k N_t^i + R_t^k N_t^r\right) + \left(S_t^u + I_t^u + R_t^u\right) N_t^u = N_t.$$

Population dynamics We now describe how the size of different groups in the economy evolve over time. The aggregate number of new infections among people outside the testing pool (T_t^u) is equal to the number of viral transmissions (τ_t^u, defined in equation (13)) times the fraction of people outside the testing pool that survived from period zero to period t and are susceptible ($p(s_t|a_0)$)

$$T_t^u = \tau_t^u p(s_t|a_0).$$

The aggregate number of new infections among people inside the testing pool (T_t^k) is equal to:

$$T_t^k = \pi_1 S_t^k C_t^s \left(I_t^u C_t^u + I_t^k C_t^i\right) + \pi_2 S_t^k N_t^s \left(I_t^u N_t^u + I_t^k N_t^i\right) + \pi_3 S_t^k \left(I_t^u + I_t^k\right). \tag{22}$$

This equation is an aggregate, equilibrium version of equation (18) taking into account that there are S_t^k susceptible people in the testing pool.

Recall that social interactions which occur during period t lead to changes in the health state of susceptible people at the end of time t. So, the number of susceptible people at the end of period t inside and outside of the testing pool is $S_t^k - T_t^k$ and $S_t^u - T_t^u$, respectively.

The number of susceptible people in the testing pool at time $t+1$ is equal to the number of susceptible people in the testing pool at the end of time t ($S_t^k - T_t^k$), plus the number of people outside the testing pool who got tested for the first time in the beginning of period $t+1$ and learned they are susceptible $(\alpha(S_t^u - T_t^u))$:

$$S_{t+1}^k = S_t^k - T_t^k + \alpha(S_t^u - T_t^u). \tag{23}$$

The number of susceptible people outside the testing pool at the beginning of $t+1$ is equal to the number of susceptible people who were outside of the pool at the end of period t and did not get tested in the beginning of time $t+1$:

$$S_{t+1}^u = (1-\alpha)(S_t^u - T_t^u). \tag{24}$$

The number of infected people in the testing pool at the beginning of time $t+1$ is equal to the number of newly infected people (T_t^k) in the testing pool, plus the number of infected people in the testing pool at the beginning of time t (I_t^k), minus the number of infected people in the testing pool that either recovered $(\pi_r I_t^k)$ or died $(\pi_{dt} I_t^k)$, plus the number of people outside the testing pool who got tested for the first time at the beginning of time $t+1$ and learned that they are infected $(\alpha\left[T_t^u + (1-\pi_r-\pi_{dt})\,I_t^u\right])$:

$$I_{t+1}^k = T_t^k + (1-\pi_r-\pi_{dt})\,I_t^k + \alpha\left[T_t^u + (1-\pi_r-\pi_{dt})\,I_t^u\right].$$

The number of infected people outside the testing pool at the beginning of time $t+1$ is equal to the number of infected people who were outside of the pool at the end of time t $(T_t^u + (1-\pi_r-\pi_{dt})\,I_t^u)$ and did not get tested at the beginning of time $t+1$:

$$I_{t+1}^u = (1-\alpha)[T_t^u + (1-\pi_r-\pi_{dt})\,I_t^u].$$

The number of recovered people in the testing pool at time $t+1$ is the number of recovered people in the testing pool at beginning of time t (R_t^k), plus the number of infected people in the testing pool who just recovered $(\pi_r I_t)$, plus the number of people outside the testing pool who got tested for the first time at the beginning of period $t+1$ and learned they are recovered $(\alpha\,(R_t^u + \pi_r I_t^u))$:

$$R_{t+1}^k = R_t^k + \pi_r I_t^k + \alpha\,(R_t^u + \pi_r I_t^u)\,. \tag{25}$$

The number of recovered people outside the testing pool at the beginning of time $t+1$ is equal to the number of recovered people who were outside the pool at the end of time t and did not get tested at the beginning of time $t+1$:

$$R_{t+1}^u = (1-\alpha)(R_t^u + \pi_r I_t^u). \tag{26}$$

Finally, the number of deceased people at time $t+1$ is the number of deceased people at time t plus the number of new deaths ($\pi_{dt}\left(I_t^u + I_t^k\right)$):

$$D_{t+1} = D_t + \pi_{dt}\left(I_t^u + I_t^k\right).$$

The number of tests administered at time t is given by

$$\begin{aligned}\text{Test}_t &= S_t^k + I_t^k + \alpha(S_t^u - T_t^u) + \alpha\left[T_t^u + (1 - \pi_r - \pi_{dt})\, I_t^u\right] + \alpha\left(R_t^u + \pi_r I_t^u\right) \\ &= S_t^k + I_t^k + \alpha\left[S_t^u + (1 - \pi_{dt})\, I_t^u + R_t^u\right].\end{aligned}$$

To interpret this equation, recall that we test all the people in the testing pool who are not recovered or dead. In addition, we test a fraction α of the people outside the testing pool.

4 Quantitative results

In this section we discuss our choice of parameter values and our quantitative results.

4.1 Parameter values

A unit of time in the model corresponds to one week. To choose the case mortality rate, π_d, in equation (2), we use data from the South Korean Ministry of Health and Welfare from April 21, 2020. These data are relatively reliable because, as of late April, South Korea had one of the world's highest per capita test rates for COVID-19. Estimates of mortality rates based on data from other countries are probably biased upwards because the number of infected people is likely to be underestimated. We compute the weighted average of the mortality rates using weights equal to the percentage of the U.S. population for different age groups. If we exclude people aged 65 and over, because their labor-force participation rates are very low, we obtain an average mortality rate of 0.2 percent. We assume that it takes on average 14 days to either recover or die from the infection. Since our model is weekly, we set $\pi_r + \pi_d = 7/14$. A 0.2 percent mortality rate for infected people implies $\pi_d = 7 \times 0.002/14$.

We use the method described in Eichenbaum, Rebelo, and Trabandt (2020) to choose π_1, π_2, and π_3. This method combines information on the modes of transmission of respiratory diseases obtained from Ferguson et al. (2006) with information from the Bureau of Labor Statistics 2018 Time Use Survey. In addition, we consider the so-called "Merkel scenario" implied by the simple SIR of Kermack and McKendrick (1927). This scenario, described by Angela Merkel in her March 11, 2020 speech, implies that 60 percent of the population either recover from the infection or die.[6]

The initial population is normalized to one. The number of people that are initially infected, ε, is 0.001. We choose $A = 39.835$ and $\theta = 0.001275$ so that in the pre-epidemic steady state the representative person works 28 hours per week and earns a weekly income of $\$58,000/52$. We obtain the per-capita income in 2019 from the U.S. Bureau of Economic Analysis and the average number of hours worked from the Bureau of Labor Statistics 2018 time-use survey. We set $\beta = 0.96^{1/52}$ so that the value of a life is 9.3 million 2019 U.S. dollars in the pre-epidemic steady state. This value is consistent with the economic value of life used by U.S. government agencies in their decisions process.[7] Below, we also consider the value of life proposed by Hall, Jones and Klenow (2020): 3.5 million U.S. dollars.

We fix κ, the parameter in equation (2) that controls the impact of changes in the aggregate level of infections on the mortality rate to 0.3.

4.2 Model without testing

An important role of testing is information revelation. To isolate this role, we consider two extreme versions of our model. In the first, no one is ever tested ($\alpha = 0$), so everybody's health state is uncertain. In the second, everyone is tested in every period and infected people are not quarantined.[8] The latter is a version of the model in Section 3 where $I_0^k = \varepsilon$, $I_0^u = 0$, $S_0^k = 1 - \varepsilon$, $S_0^u = 0$, and $\alpha = 1$. For simplicity, we refer to the economies in which people know and don't know their health state as the $\alpha = 1$ and $\alpha = 0$ economies, respectively.

Figure 1 displays the consumption and hours worked for people in these economies. When $\alpha = 0$, no one knows for sure what their health state is and everybody attaches the same probabilities to being in different health states. So, in response to the epidemic, everybody cuts consumption and hours worked by the same amount. They do so because they are worried about being susceptible and getting infected.

In the $\alpha = 1$ economy, susceptible, infected, and recovered people behave very differently from each other. Infected and recovered people do not reduce consumption and hours worked

[6] The values of π_1, π_2 and π_3 are as follows: $\pi_1 = 1.00423 \times 10^{-7}$, $\pi_2 = 1.59356 \times 10^{-4}$, and $\pi_3 = 0.49974$.

[7] See U.S. Environmental Protection Agency (2010) and Moran (2016). See Viscusi and Aldy (2003) for a review of the literature on the value of a statistical life.

[8] The economy where people know their health status corresponds to the one considered in Eichenbaum, Rebelo, and Trabandt (2020)

relative to the pre-epidemic steady state at all because they suffer no additional negative effects from further exposure to the virus. Susceptible people reduce their consumption and hours worked more than people in the $\alpha = 0$ economy because they know for sure that they are susceptible. This information revelation effect is quite strong. People in the $\alpha = 1$ economy drop their consumption by 10 percent from peak to trough. The analogous drop in consumption in the $\alpha = 0$ economy is 16 percent.

Figure 2 shows that the *net* effect of information revelation on aggregate consumption and hours worked is small. Susceptible people in the $\alpha = 1$ economy reduce their consumption and hours worked by much more than people in the $\alpha = 0$ economy because they know with certainty that they are susceptible .But infected and recovered in the $\alpha = 1$ economy respond by much less than people in the $\alpha = 0$ economy. The latter effect partially offsets the former effect.

To understand why there are more infections and deaths in the $\alpha = 1$ economy, recall that new infections depend on the interaction between the economic activities of infected and susceptible people. Infected people consume and work more in the $\alpha = 1$ economy than in the $\alpha = 0$ economy. Other things equal, this effect leads to more infections in the $\alpha = 1$ economy. Susceptible people consume less than people in the $\alpha = 0$ economy. Other things equal, this effect leads to less infections in the $\alpha = 1$ economy. For our parameter values, the first effect dominates the second effect, resulting in higher infections and deaths in the $\alpha = 1$ economy.

In our benchmark calibration, we assume that the value of life is 9.3 million 2019 U.S. dollars. There is substantial disagreement about this estimate. In a recent paper, Hall, Jones and Klenow (2020) argue that, taking demographics into account, a more appropriate value of life for a representative-agent model is 3.5 million U.S. dollars. To assess the robustness of our results to using this value of life, we follow Hall and Jones (2007) and add a constant, b, to momentary utility:

$$u(c_t, n_t) = b + \ln c_t - \frac{\theta}{2} n_t^2.$$

We set $b = -4.05$ which, given the unchanged parameters of the model, implies that the value of life for a representative-agent model is 3.5 million U.S. dollars. Figure 3 is the analog of Figure 2 for this lower value of life. Qualitatively, the two figures are very similar. Quantitatively, the economy with a lower value of life has a smaller contraction in economic activity because people have less to lose by engaging in consumption and work activities. Nevertheless, the epidemic induces a steep decline in economic activity, with a 5 percent drop in consumption from peak to trough, and a large death toll.

We conclude that the effect of information revelation is to increase the dispersion of economic behavior across agents with different health states. We believe these qualitative results is robust to a variety of perturbations such as imperfect testing and partial altruism. That said, the less informative is testing and the more altruistic people are the smaller are the effects of information revelation.

4.3 Model with smart containment

We now consider an economy with testing. For expository purposes, we set the testing rate to 2 percent per week ($\alpha = 0.02$). Figure 4 displays our results. The blue line corresponds to the competitive equilibrium without testing. The red line corresponds to the equilibrium under smart containment.

Because known infected people do not work or directly engage in consumption we set C_t^i and N_t^i to zero in the transmission functions (13) and (18):

$$\tau_t^u = \pi_1 c_t^u (I_t^u C_t^u) + \pi_2 n_t^u \left(I_t^u N_t^u\right) + \pi_3 \left(I_t^u + I_t^k\right),$$

$$\tau_t^s = \pi_1 c_t^s \left(I_t^u C_t^u\right) + \pi_2 n_t^s \left(I_t^u N_t^u\right) + \pi_3 \left(I_t^u + I_t^k\right).$$

Equation (22), which determines the aggregate number of new infections amongst people inside the testing pool (T_t^k) takes the form

$$T_t^k = \pi_1 S_t^k C_t^s \left(I_t^u C_t^u\right) + \pi_2 S_t^k N_t^s \left(I_t^u N_t^u\right) + \pi_3 S_t^k \left(I_t^u + I_t^k\right).$$

The government finances consumption of quarantined infected people with a lump-sum tax on other people in the economy. Because the equilibrium number of infected people is small (roughly 3.5 percent at the peak), the lump-sum tax is also small, amounting to roughly 1 percent of the level of consumption in the pre-epidemic steady state.

The government budget constraint is given by

$$I_t^k \Gamma_t^i + \Gamma_t \left(S_t^k + R_t^k + S_t^u + I_t^u + R_t^u\right) = 0,$$

where $\Gamma_t < 0$ and $\Gamma_t^i = C_t^i = \bar{c}^r$. For now, we abstract from the resource costs associated with testing.

Two key results emerge from Figure 4. First, relative to the equilibrium without testing, smart containment cuts peak infection rates from 5.7 to 3.6 percent and reduces death rates from 0.17 to 0.10 of the initial population. For the U.S., this reduction represents roughly a quarter of million lives saved. Second, smart containment reduces the severity of the recession associated with the epidemic. In the equilibrium with $\alpha = 0$, the peak-to-trough drop in consumption is 10.2 percent. Under smart containment, the peak-to-trough drop in consumption is reduced to 4.2 percent. So, smart containment improves both health and economic outcomes. With simple containment measures that don't condition on people's health states, there is an extremely painful trade-off between the severity of a recession and the health consequences of an epidemic (see, for example, Alvarez, Argente, and Lippi (2020) and Eichenbaum, Rebelo and Trabandt (2020)). According to our results, policies that combine testing and quarantining infected people dramatically improve this trade-off.

To understand the mechanisms underlying the impact of smart containment, Figure 5 displays consumption and hours worked by different types of people. The first and second rows correspond to the competitive equilibrium and the economy with smart containment, respectively.

Our key results are as follows. First, consumption of all people except for the recovered drops by much less under smart containment. The reason is that quarantining infected people removes them from social interactions related to consuming and working. The resulting reduction in the risk of being infected leads to higher consumption and work by everyone who is at the risk of being infected. Second, consumption of recovered people falls slightly because of the lump sum tax that they pay to finance the consumption of known infected people.[9]

A natural question is: what fraction of the population is tested when $\alpha = 0.02$. The number of tests that is administered rises gradually over time. Within one year, 38 percent of the population is tested every week. By two years, that fraction rises to roughly 50 percent. The latter level is consistent with the scale of testing advocated by Romer (2020).[10]

In our model, the gains from testing diminish rapidly after roughly one year because the population develops steady-state herd immunity by that time. This immunity is attained for two reasons. First, because testing ramps up gradually, many infected people who are not quarantined continue to spread the virus during the first year. Second, during the same time period infected people who are quarantined continue to transmit the virus through

[9] Absent this effect, consumption would be equal to its level in the pre-epidemic steady state.

[10] Romer (2020) proposes dividing the population into two groups and testing each group in alternating weeks. While the Romer proposal is likely to be more efficient than the policy we consider, it is less tractable to model in general equilibrium.

non-economic social interactions. Both forces reduce the pool of susceptible people to the point where steady-state herd immunity is obtained.

We now discuss how the gains from smart containment depend on the fraction of the population that is tested. Figure 6 displays, for various values of α, the peak-to-trough change in consumption, the death toll from the epidemic, as well as peak infection and mortality rates. The solid blue line depicts outcomes if smart containment is implemented at the beginning of the epidemic. The dashed black line depicts the corresponding outcomes if smart containment begins only in week 17.

Consider first the solid blue line. As α rises, both the economic and the health costs of the epidemic decline. The economic cost declines quite steeply as α rises from zero. A rise in α from zero to 2 percent cuts the peak-to-trough change in consumption in half. Further rises in α continue to reduce the economic costs of the epidemic but at a slower rate, with very small reductions beyond $\alpha = 0.06$. A similar but less stark pattern emerges regarding the death toll from the epidemic. For example, a rise in α from zero to 2 percent cuts the death toll from 0.17 to 0.10 percent of the initial population. For the U.S. this amounts to about a quarter of million lives saved. Further rises in α continue to reduce the death toll but at a slower rate.

Another way to evaluate the gains from smart containment is to compute the compensating variation associated with this policy. This variation is the percentage of annual consumption that would make a person in an economy without smart containment have the same lifetime utility of a person in an economy with smart containment.

The first column of Figure 8 displays the compensating variation associated with smart containment for different values of α. The variation is increasing in α, rising sharply as α increases from zero. To be concrete, suppose that $\alpha = 0.02$, then the annual compensating variation is 0.44 percent of consumption which, for the U.S., amounts to 66 billion U.S. dollars per year. For an annual discount rate of 4 percent, the associated present value is 1.7 trillion U.S. dollars.

Consider next the results of starting smart containment in week 17. From Figure 6 we see that the qualitative impact of the policy remains unchanged. However, the effects are much weaker. So, delaying the policy by four months substantially raises the economic and health costs of the epidemic. Even so, there are substantial gains from implementing smart containment.

4.4 Strict containment

In the previous section, we considered quarantine policies that apply to the work and consumption activities of people who have been identified as infected. A natural question is: what if policy also succeeds in minimizing the non-economic interactions of those people. We refer to this policy as "strict containment." As a practical matter, it might be very difficult to enforce strict containment. So, we view this part of the analysis as providing an upper bound on the gains from minimizing the non-economic interactions of infected people.

Because known infected people do not work or directly engage in consumption, or in non-economic social interactions we set C_t^i, N_t^i and I_t^k to zero in the transmission functions (13) and (18)

$$\tau_t^u = \pi_1 c_t^u \left(I_t^u C_t^u\right) + \pi_2 n_t^u \left(I_t^u N_t^u\right) + \pi_3 I_t^u,$$

$$\tau_t^s = \pi_1 c_t^s \left(I_t^u C_t^u\right) + \pi_2 n_t^s \left(I_t^u N_t^u\right) + \pi_3 I_t^u.$$

Equation (22), which determines the aggregate number of new infections amongst people inside the testing pool (T_t^k), is now given by

$$T_t^k = \pi_1 (S_t^k C_t^s) \left(I_t^u C_t^u\right) + \pi_2 (S_t^k N_t^s) \left(I_t^u N_t^u\right) + \pi_3 S_t^k I_t^u.$$

Figure 7 displays our results. The dashed-dotted grey line corresponds to the behavior of the economy under strict containment. The solid blue line and dashed red line correspond to the behavior of the economy with no testing and the economy with testing and smart containment, respectively. Strict containment dramatically reduces the economic and health costs of the epidemic. The reason is straightforward. In our calibration, 2/3 of virus transmissions result from non-economic social interactions. So, a policy which minimizes those interactions has a dramatic effect on economic and health outcomes.

The second column of Figure 8 displays the compensating variation associated with strict containment for different values of α. The variation is increasing in α, rising sharply as α increases from zero. Indeed, the gains rise even more sharply than under smart containment. These gains stabilize at values of α greater than 0.03.

The gains from strict containment are clearly larger than those associated with smart containment. For example, a rise in α from zero to 2 percent cuts the death toll from 0.17 to 0.017 percent of the initial population. For the U.S. this amounts to half a million people instead of the roughly quarter of million lives saved under smart containment. For $\alpha = 0.02$, the annual compensating variation is 1 percent of consumption which, for the U.S., amounts to 150 billion U.S. dollars per year. For an annual discount rate of 4 percent, the associated present value is 3.8 trillion U.S. dollars instead of the 1.7 trillion U.S. dollars under smart containment.

In terms of testing strict containment differs from smart containment in two important ways. First, it requires testing a much higher percentage of the population. For example, by the end of the first year, under strict and smart containment, 59 and 38 percent of the population is tested every week, respectively. The analog numbers for end of year two are 80 and 51 percent. Second, under strict containment, the economy never reaches steady-state herd immunity. So, testing and quarantining policies have to be deployed on a permanent basis until effective treatments or vaccines are developed. As we saw, under smart containment steady-state herd immunity is reached after one year so testing and quarantining can be ended at that point without risk of a surge in infections.

5 What if immunity is temporary?

A key maintained assumption of the economics literature on epidemics is that people who have recovered from the disease can't be reinfected. According to the World Health Organization (2020), there is no hard evidence in favor of this assumption for SARS-CoV-2, the virus that causes COVID-19. Indeed, there is evidence that people do not acquire permanent immunity after exposure to other corona viruses (see, e.g., Shamanand and Galanti (2020)). Wu et al. (2007) report that antibodies for the severe acute respiratory syndrome virus (SARS-COV), a type of corona virus, last on average for two years.

In this section, we accomplish two objectives. First, we extend our model to allow for the possibility that recovered people can be reinfected. Second, we examine the efficacy of smart and strict containment under those circumstances.

5.1 People outside the testing pool

People outside the testing pool maximize their lifetime utility, (11), subject to the budget constraint, (12), the transmission function, (13), and the probability of being infected, (14). The equation for the probability of being susceptible, (15), is replaced by the following equation

$$p(s_{t+1}|a_{t+1})[1-\pi_{dt}p(i_t|a_t)] = p(s_t|a_t)(1-\tau_t^u) + \pi_s p(r_t|a_t).$$

Here, π_s denotes the probability that a recovered agent becomes susceptible again. In the standard SIR model $\pi_s = 0$. We add the following equation for $p(r_{t+1}|a_{t+1})$[11]

$$p(r_{t+1}|a_{t+1})[1-\pi_{dt}p(i_t|a_t)] = p(r_t|a_t)(1-\pi_s) + \pi_r p(i_t|a_t).$$

The term $p(r_t|a_t)(1-\pi_s)$, reflects the probability that a person who is recovered does not lose immunity and remains recovered at time $t+1$.

The first-order conditions for the problem of a person outside the testing pool are displayed in the appendix.

5.2 People inside the testing pool

The problem of people inside the testing pool remains the same as before with one important exception. The lifetime utility of a recovered person now takes into account the probability of becoming susceptible

$$U_t^r = u(c_t^r, n_t^r) + \beta(1-\pi_s)U_{t+1}^r + \beta\pi_s U_{t+1}^s. \tag{27}$$

A recovered person maximizes (27) subject to the budget constraint (16). The first-order conditions for consumption and hours worked for a recovered person are the same as in the problem without reinfections.

5.3 Population dynamics

The equations governing population dynamics are the same as in the model without reinfections with the following exceptions. Equations (23), (24), (25), and (26) are replaced by

$$\begin{aligned}
S_{t+1}^k &= S_t^k - T_t^k + \pi_s R_t^k + \alpha(S_t^u - T_t^u + \pi_s R_t^u), \\
S_{t+1}^u &= (1-\alpha)(S_t^u - T_t^u + \pi_s R_t^u), \\
R_{t+1}^k &= R_t^k + \pi_r I_t^k - \pi_s R_t^k + \alpha\left(R_t^u + \pi_r I_t^u - \pi_s R_t^u\right), \\
R_{t+1}^u &= (1-\alpha)(R_t^u + \pi_r I_t^u - \pi_s R_t^u).
\end{aligned}$$

The economy converges asymptotically to a steady state in which the number of susceptible people and the ratio of infected people to recovered people are constant. Asymptotically, the number of new deaths from infection converges to zero.

[11] In the version of the model without reinfections, we replaced $p(r_{t+1}|a_{t+1})$ by $1-p(s_{t+1}|a_{t+1})-p(i_{t+1}|a_{t+1})$ instead of imposing the equation for $p(r_{t+1}|a_{t+1})$ as a constraint.

5.4 Quantitative results

As far as we know, there are no reliable estimates of the rate at which recovered people get reinfected by SARS-CoV-2. For this reason, we rely on estimates of reinfection rates for the severe acute respiratory syndrome (SARS) to calibrate our model. Wu et al. (2007) report that SARS antibodies last on average for two years. So, we choose $\pi_s = 1/104$.

Figure 9 displays our results. The blue line, reproduced from Figure 2, corresponds to the model in which people do not know their health states and the probability of reinfection is zero. The black dashed line corresponds to the model with reinfections. The key result is that, when π_s is positive, there are waves of infections that dampen over time. These waves are accompanied by recurrent recessions. The asymptotic number of susceptible people is roughly forty percent higher than in the no-reinfection economy. Critically, over a ten-year period the cumulative death toll is more than double in the reinfection economy.[12]

Figure 10 displays the dynamics of the epidemic with no interventions (blue line), with smart containment (dashed red line), and with strict containment (dashed grey line).

Smart containment substantially reduces the peak level of infections during the first outbreak of the epidemic. Moreover, it eliminates all future outbreaks. The net effect is that the death toll of the epidemic is capped at 0.1 percent of the initial population. This result stands in sharp contrast to the death toll in the economy without containment, which exceeds 0.4 percent in the first decade of the epidemic.

The benefits of smart containment in terms of lives saved are clearly enormous. But the benefits are also very large in terms of economic activity. Smart containment dramatically reduces the severity of the recession caused the first outbreak of the epidemic. And it also eliminates all of the subsequent recessions that would occur absent containment.

Figure 10 shows that strict containment generates even larger benefits than smart containment. Indeed, it eliminates almost all of the deaths and output losses caused by the epidemic.

Viewed as a whole, the results in this section are very supportive of the idea that society ought to invest in the required infrastructure to engage in continuous testing of the population and quarantining of those infected.

[12] The number of deaths rises over a long time period before it stabilizes. The point at which the death toll stabilizes is not shown in the figure.

6 Related literature

There is a fast-growing literature on the macroeconomic impact of the COVID-19 epidemic. See, for example, Bodenstein, Corsetti, and Guerrieri (2020), Buera, Fattal-Jaef, Neumeyer, and Shin (2020), Eichenbaum, Rebelo, and Trabandt (2020), Farboodi, Jarosch, and Shimer (2020), Faria-e-Castro (2020), Glover, Heathcote, Krueger, and Rios-Rull (2020), Gonzalez-Eiras and Niepelt (2020), Guerrieri, Lorenzoni, Straub, and Werning (2020), Jones, Philippon, and Venkateswaran (2020), Kapicka and Rupert (2020), Kaplan, Moll, and Violante (2020), Krueger, Uhlig, and Xie (2020), Stock (2020), and Toxvaerd (2020). Below, we briefly summarize the branch of this literature focused on the role of testing as part of an optimal containment policy.

Alvarez, Argente, and Lippi (2020) use a variant of the SIR model reviewed by Atkeson (2020) to study the lockdown policy that maximizes the present value of output. They consider a scenario where antibody tests allow people who recover to receive an immunity card and go back to work. In contrast to these authors, we study the competitive equilibrium of our model economy as well as the effects of smart and strict containment. In addition, our model allows for a two-way interaction between the dynamics of the epidemic and the level of economic activity. The epidemic affects people's economic decisions and these decisions, in turn, affect the rate at which the epidemic unfolds.

Piguillem and Shi (2020) consider a planning problem in which the objective function is the discounted utility of aggregate output minus a penalty function for infection-related deaths. They use this framework to study the efficacy of lockdown policies along with random testing. Holtemöller (2020) embeds a version of the SIR model into the Solow (1956) model. He analyses the combinations of lockdowns, testing and quarantines that maximize the discounted utility of aggregate consumption associated with an exogenous savings rule. The key differences between our analysis and these two papers are as follows. First, we study a competitive equilibrium as well as the effects of smart and strict containment. Second, we allow for an interaction between people's economic decisions, testing, and the dynamics of the epidemic.

Berger, Herkenhoff, and Mongey (2020) study the importance of randomized testing in estimating the health states of the population and designing optimal mitigation policies. In contrast to our model, their framework abstracts from interactions between the state of the economy and the number of infections.

Chang and Velasco (2020) consider a two-period model in which there is potentially multiple equilibrium in people's decision to go to work during an epidemic. They discuss the effect of testing and quarantining on the labor supply.

Two recent papers consider models in which people are uncertain about their health states. In Farboodi, Jarosch, and Shimer (2020) people choose their level of social activity without knowing whether they are susceptible or infected. In contrast to these authors, we consider the impact of the epidemic on production and consumption decisions. In addition, we explicitly analyze the impact of testing on the economy.

Brotherhood et al. (2020) study the efficacy of different health policies focusing on age heterogeneity and allowing for partial altruism. In their model, a subset of people don't know their health state. These are the people who develop a fever which could be due to either a viral infection or a common cold. Absent testing, people discover the source of their fever after one week. Testing provides that information a week earlier. In contrast, no one in our model knows their true health state unless they are tested. In addition, we consider the impact of policies that test broad sections of the population, not just those who have fever.

Finally, in contrast to all of the papers cited above, we consider the possibility that people who recover from an infection acquire only temporary immunity to the virus.

7 Conclusion

In this paper, we develop a SIR-based macroeconomic model where people do not know their true health state. In this environment, testing allows the government to identify infected people and quarantine them. We argue that the potential social gains from such a policy are very large. Non-test-based policies like lockdowns and other restrictions to economic activity improve upon the competitive equilibrium. But test-based quarantines ameliorate the sharp trade-off between declines in economic activity and health outcomes that are associated with broad-based containment policies. This amelioration is particularly dramatic when people who recover from an infection acquire only temporary immunity to the virus.

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Appendix A Equilibrium Equations

This appendix provides the equilibrium equations for the model with unknown and known health states due to testing. We consider the model with temporary immunity. The model with permanent immunity is a special case where $\pi_s = 0$.

A.1 Equilibrium equations for people with unknown health states

Present value utility of people with unknown health states:

$$\begin{aligned} U_t^u &= u(c_t^u, n_t^u) + (1-\alpha)\beta \left[1 - \pi_{dt} p(i_t|a_t)\right] U_{t+1}^u \\ &\quad + \alpha\beta \left[1 - \pi_{dt} p(i_t|a_t)\right] \left[p(s_{t+1}|a_{t+1}) U_{t+1}^s + p(i_{t+1}|a_{t+1}) U_{t+1}^i + p(r_{t+1}|a_{t+1}) U_{t+1}^r\right]. \end{aligned}$$

Transmission function, budget and probability transition functions:

$$\tau_t^u = \pi_1 c_t^u \left(I_t^u C_t^u + I_t^k C_t^i\right) + \pi_2 n_t^u \left(I_t^u N_t^u + I_t^k N_t^i\right) + \pi_3 \left(I_t^u + I_t^k\right),$$

$$c_t^u = A n_t^u + \Gamma_t,$$

$$p(i_{t+1}|a_{t+1})[1-\pi_{dt}p(i_t|a_t)] = \tau_t^u p(s_t|a_t) + (1-\pi_r-\pi_{dt})p(i_t|a_t),$$

$$p(s_{t+1}|a_{t+1})[1-\pi_{dt}p(i_t|a_t)] = p(s_t|a_t)(1-\tau_t^u) + \pi_s p(r_t|a_t),$$

$$p(r_{t+1}|a_{t+1})[1-\pi_{dt}p(i_t|a_t)] = p(r_t|a_t)(1-\pi_s) + \pi_r p(i_t|a_t).$$

First-order condition for c_t^u:

$$u_1\left(c_t^u, n_t^u\right) - \lambda_{bt}^u + \lambda_{\tau t}^u \pi_1 \left(I_t^u C_t^u + I_t^k C_t^i\right) = 0.$$

First-order condition for n_t^u:

$$u_2\left(c_t^u, n_t^u\right) + \lambda_{bt}^u A + \lambda_{\tau t}^u \pi_2 \left(I_t^u N_t^u + I_t^k N_t^i\right) = 0.$$

First-order condition for τ_t^u:

$$-\lambda_{\tau t}^u + \lambda_{it}^u p(s_t|a_t) - \lambda_{st}^u p(s_t|a_t) = 0.$$

First-order condition for $p(i_{t+1}|a_{t+1})$

$$\begin{aligned}
&\frac{dU_t^u}{dp(i_{t+1}|a_{t+1})}\frac{1}{1-\pi_{dt}p(i_t|a_t)} - \lambda_{it}^u + \lambda_{it+1}^u \beta p(i_{t+2}|a_{t+2})\pi_{dt+1} \\
&+\lambda_{it+1}^u \beta(1-\pi_r-\pi_{dt+1}) + \lambda_{st+1}^u \beta \pi_{dt+1} p(s_{t+2}|a_{t+2}) \\
&+\lambda_{rt+1}^u \beta \pi_{dt+1} p(r_{t+2}|a_{t+2}) + \lambda_{rt+1}^u \beta \pi_r.
\end{aligned}$$

First-order condition for $p(s_{t+1}|a_{t+1})$

$$\frac{dU_t^u/dp(s_{t+1}|a_{t+1})}{1-\pi_{dt}p(i_t|a_t)} + \lambda_{it+1}^u \beta \tau_{t+1}^u - \lambda_{st}^u + \lambda_{st+1}^u \beta(1-\tau_{t+1}^u) = 0.$$

First-order condition $p(r_{t+1}|a_{t+1})$

$$\frac{dU_t^u}{dp(r_{t+1}|a_{t+1})}\frac{1}{1-\pi_{dt}p(i_t|a_t)} + \lambda_{st+1}^u \beta \pi_s - \lambda_{rt}^u + \lambda_{rt+1}^u \beta(1-\pi_s) = 0.$$

The relevant derivatives of lifetime utility are given by

$$\begin{aligned}
&\frac{dU_t^u}{dp(i_{t+1}|a_{t+1})} = \alpha\beta\left[1-\pi_{dt}p(i_t|a_t)\right]U_{t+1}^i - \left[(1-\alpha)\beta\right]^2\left[1-\pi_{dt}p(i_t|a_t)\right]\pi_{dt+1}U_{t+2}^u \\
&-\pi_{dt+1}\alpha(1-\alpha)\beta^2\left[1-\pi_{dt}p(i_t|a_t)\right]\times\left[p(s_{t+2}|a_{t+2})U_{t+2}^s + p(i_{t+2}|a_{t+2})U_{t+2}^i + p(r_{t+2}|a_{t+2})U_{t+2}^r\right],
\end{aligned}$$

$$\frac{dU_t^u}{dp(s_{t+1}|a_{t+1})} = \alpha\beta\left[1-\pi_{dt}p(i_t|a_t)\right]U_{t+1}^s,$$

$$\frac{dU_t^u}{dp(r_{t+1}|a_{t+1})} = \alpha\beta\left[1-\pi_{dt}p(i_t|a_t)\right]U_{t+1}^r.$$

A.2 Equilibrium equations for people with known health states after testing

$$c_t^s = An_t^s + \Gamma_t,$$

$$c_t^i = An_t^i + \Gamma_t^i,$$

$$c_t^r = An_t^r + \Gamma_t,$$

$$U_t^s = u(c_t^s, n_t^s) + \beta \left[(1 - \tau_t^s) U_{t+1}^s + \tau_t^s U_{t+1}^i \right],$$

$$\tau_t^s = \pi_1 c_t^s \left(I_t^u C_t^u + I_t^k C_t^i \right) + \pi_2 n_t^s \left(I_t^u N_t^u + I_t^k N_t^i \right) + \pi_3 \left(I_t^u + I_t^k \right),$$

$$u_1(c_t^s, n_t^s) - \lambda_{bt}^s + \lambda_{\tau t}^s \pi_1 \left(I_t^u C_t^u + I_t^k C_t^i \right) = 0,$$

$$u_2(c_t^s, n_t^s) + A\lambda_{bt}^s + \lambda_{\tau t}^s \pi_2 \left(I_t^u N_t^u + I_t^k N_t^i \right) = 0,$$

$$\beta \left(U_{t+1}^i - U_{t+1}^s \right) - \lambda_{\tau t}^s = 0,$$

$$U_t^i = u(c_t^i, n_t^i) + \beta \left[(1 - \pi_r - \pi_{dt}) U_{t+1}^i + \pi_r U_{t+1}^r \right],$$

$$u_1(c_t^i, n_t^i) = \lambda_{bt}^i,$$

$$u_2(c_t^i, n_t^i) = -A\lambda_{bt}^i,$$

$$U_t^r = u(c_t^r, n_t^r) + \beta(1 - \pi_s) U_{t+1}^r + \beta \pi_s U_{t+1}^s,$$

$$u_1(c_t^r, n_t^r) = \lambda_{bt}^r,$$

$$u_2(c_t^r, n_t^r) = -A\lambda_{bt}^r.$$

A.3 Population dynamics

The equations for the population dynamics are as follows

$$S_{t+1}^u = p(s_{t+1} | a_{t+1}) M_{t+1}^*,$$

$$I_{t+1}^u = p(i_{t+1} | a_{t+1}) M_{t+1}^*,$$

$$R_{t+1}^u = (1 - p(s_{t+1} | a_{t+1}) - p(i_{t+1} | a_{t+1})) M_{t+1}^*,$$

$$D_{t+1}^u = D_t^u + \pi_{dt} I_t^u,$$

$$T_t^u = \tau_t^u p(s_t | a_t) M_t^*,$$

$$M_{t+1}^* = M_t^* [1 - \pi_{dt} p(i_t | a_t)] (1 - \alpha),$$

$$T_t^k = \pi_1 S_t^k C_t^s \left(I_t^u C_t^u + I_t^k C_t^i \right) + \pi_2 S_t^k N_t^s \left(I_t^u N_t^u + I_t^k N_t^i \right) + \pi_3 S_t^k \left(I_t^u + I_t^k \right),$$

$$S_{t+1}^k = S_t^k - T_t^k + \pi_s R_t^k + \alpha(S_t^u - T_t^u + \pi_s R_t^u),$$

$$I_{t+1}^k = T_t^k + (1 - \pi_r - \pi_{dt}) I_t^k + \alpha \left[T_t^u + (1 - \pi_r - \pi_{dt}) I_t^u\right],$$

$$R_{t+1}^k = R_t^k + \pi_r I_t^k - \pi_s R_t^k + \alpha \left(R_t^u + \pi_r I_t^u - \pi_s R_t^u\right),$$

$$D_{t+1}^k = D_t^k + \pi_{dt} I_t^k.$$

A.4 Government budget and equilibrium

$$\left(S_t^k + R_t^k + S_t^u + I_t^u + R_t^u\right) \Gamma_t + I_t^k \Gamma_t^i = 0,$$

$$c_t^j = C_t^j, n_t^j = N_t^j.$$

A.5 Aggregate variables

$$\begin{aligned} C_t &= \left(S_t^k C_t^s + I_t^k C_t^i + R_t^k C_t^r\right) + \left(S_t^u + I_t^u + R_t^u\right) C_t^u, \\ N_t &= \left(S_t^k N_t^s + I_t^k N_t^i + R_t^k N_t^r\right) + \left(S_t^u + I_t^u + R_t^u\right) N_t^u, \end{aligned}$$

$$\begin{aligned} D_t &= D_t^u + D_t^k, \\ R_t &= R_t^u + R_t^k, \\ I_t &= I_t^u + I_t^k, \\ S_t &= S_t^u + S_t^k. \end{aligned}$$

A.6 Numerical algorithm

We use a time-stacking algorithm together with a gradient-based method to solve for the equilibrium paths of all endogenous variables for $t = 0, ..., 500$.

Figure 1: Model with Unknown and Known Health Status

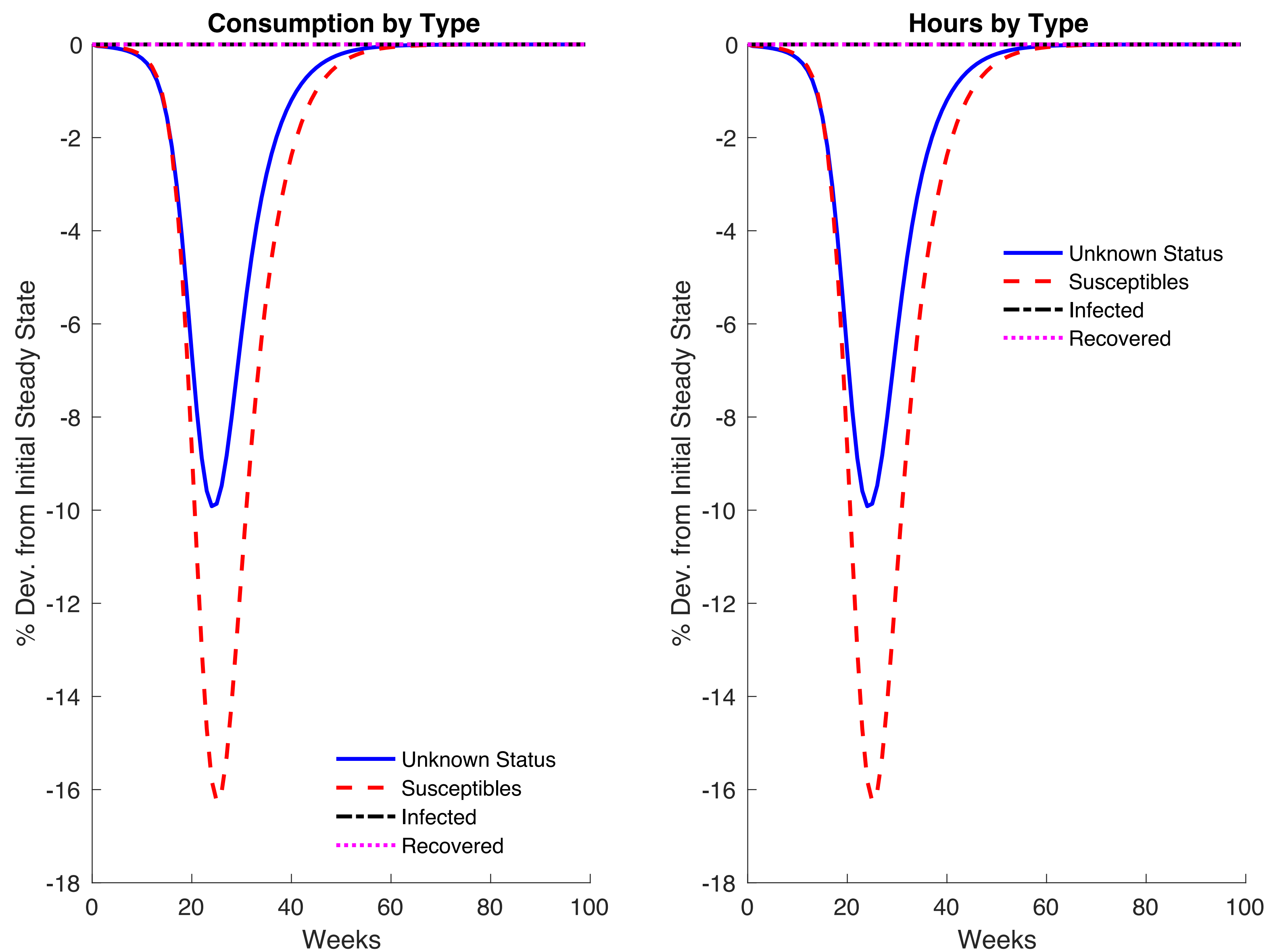


Figure 2: Model with Unknown and Known Health Status

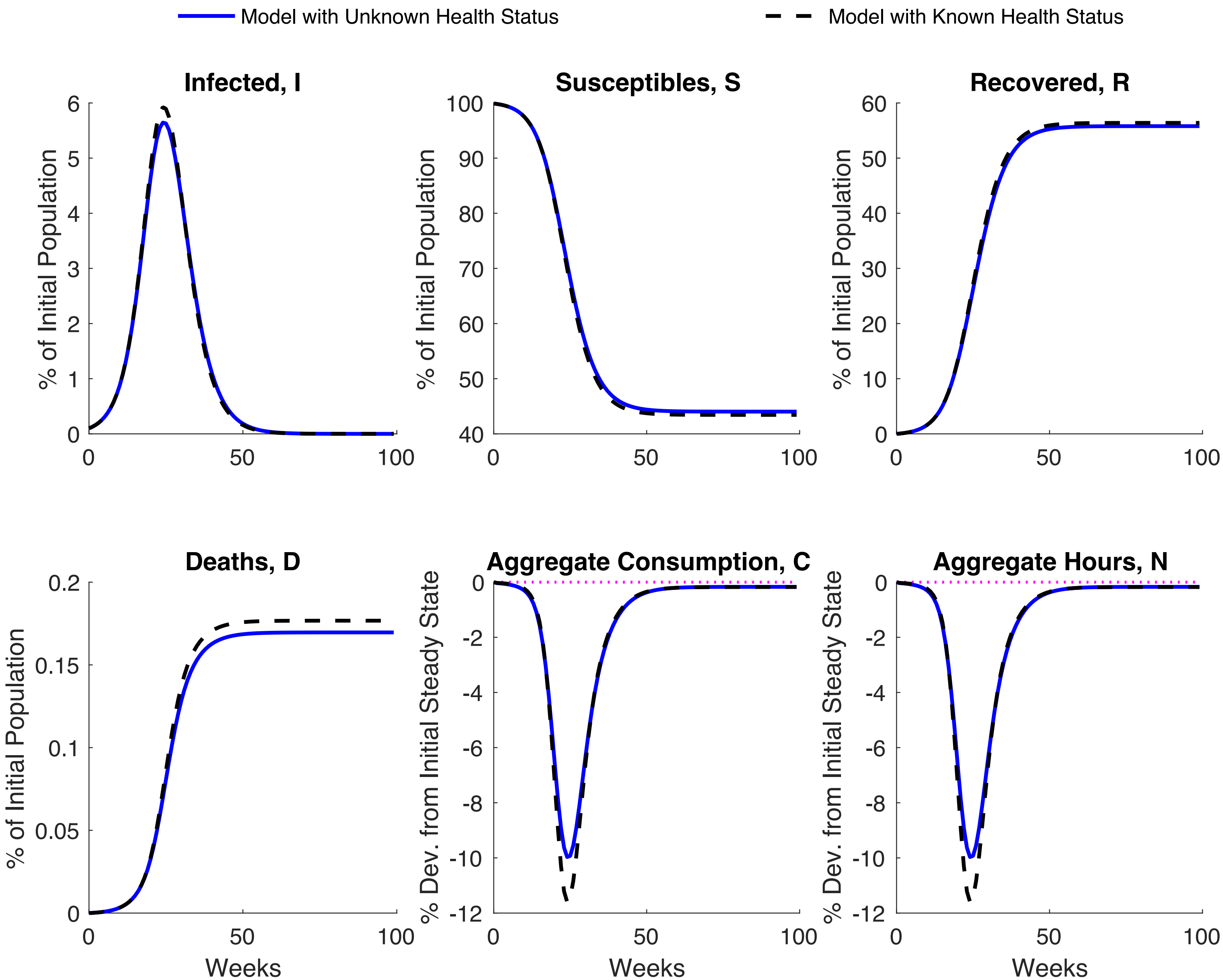


Figure 3: Model with Unknown and Known Health Status (Lower Value of Life)

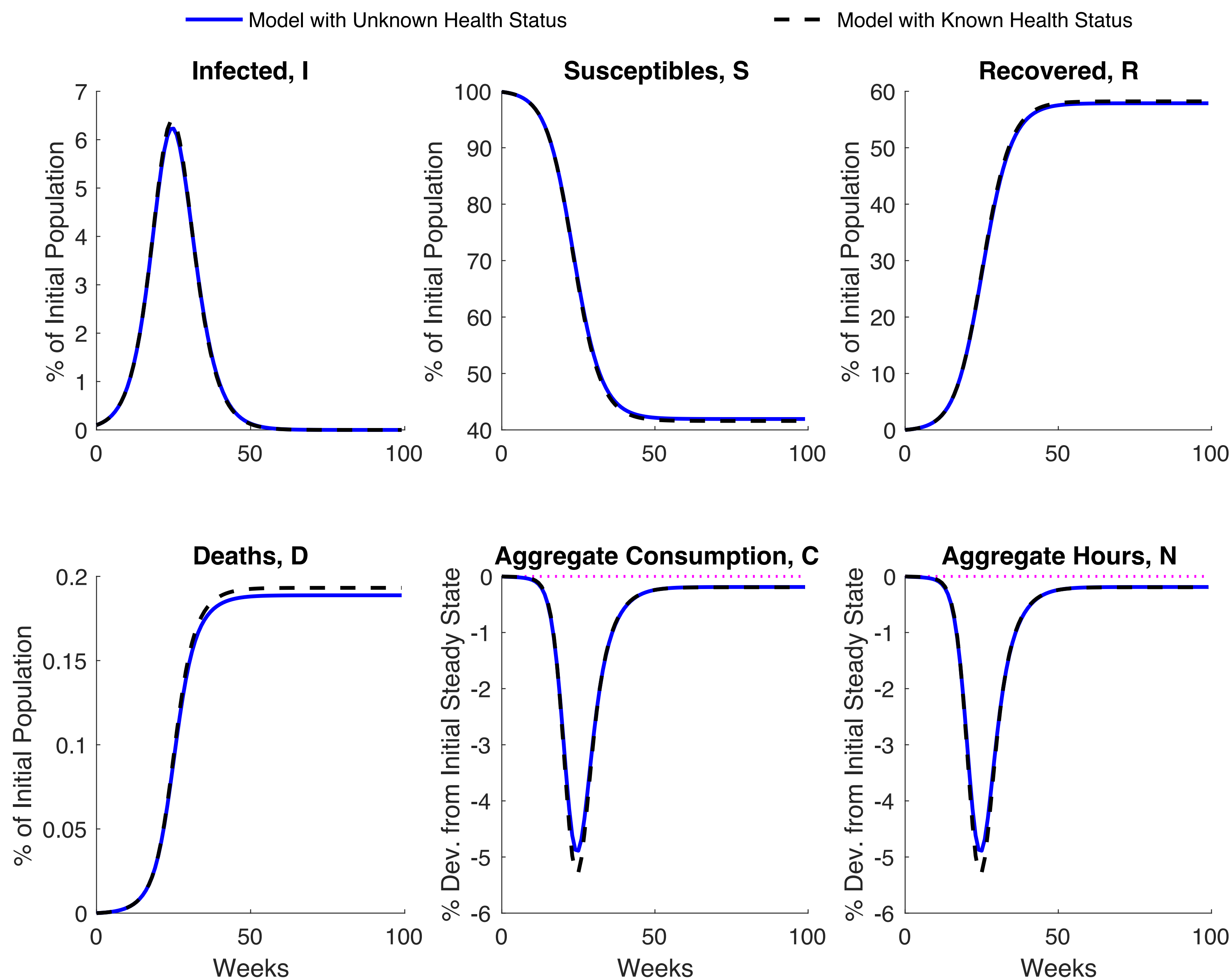


Figure 4: Model with Testing and Smart Containment

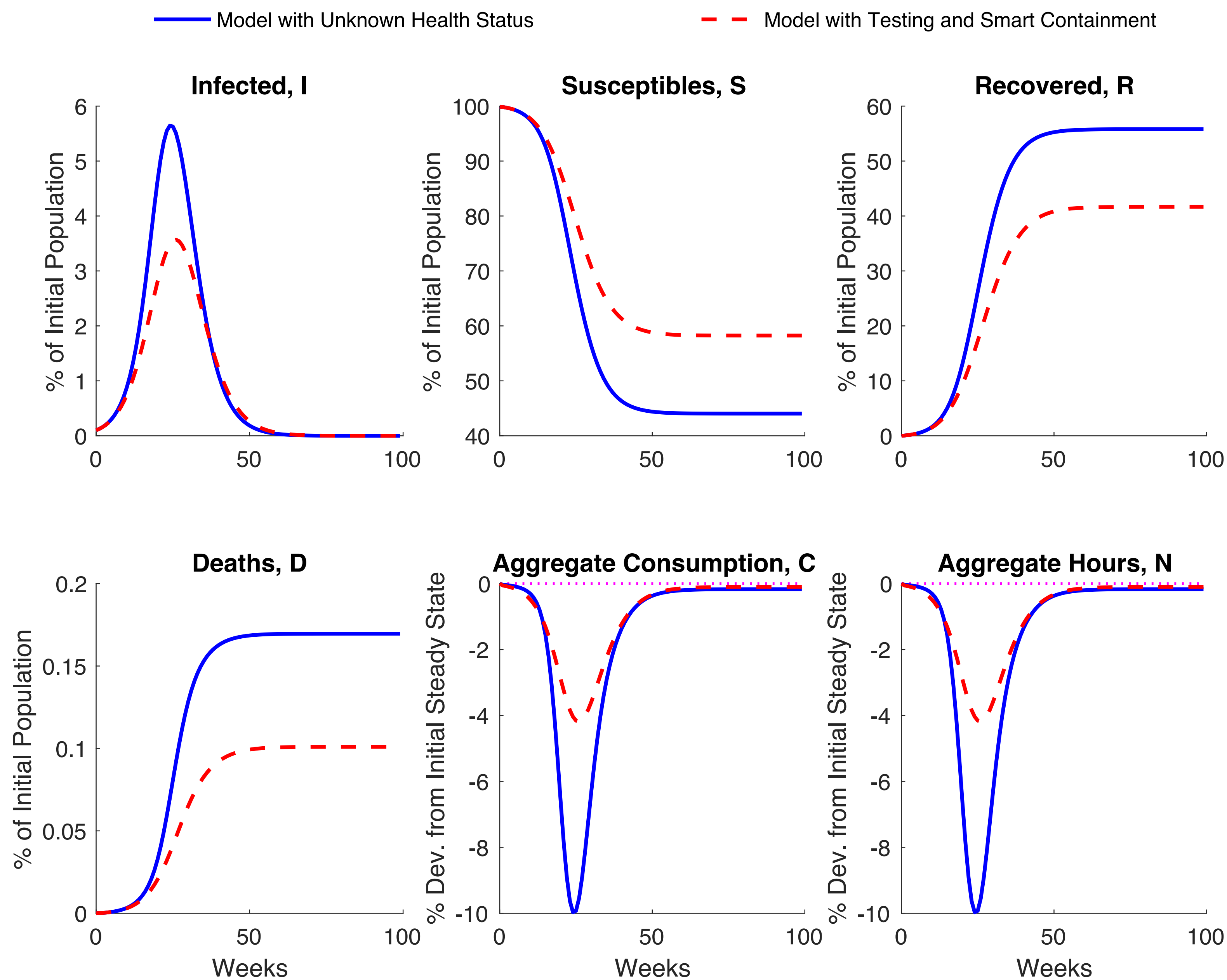
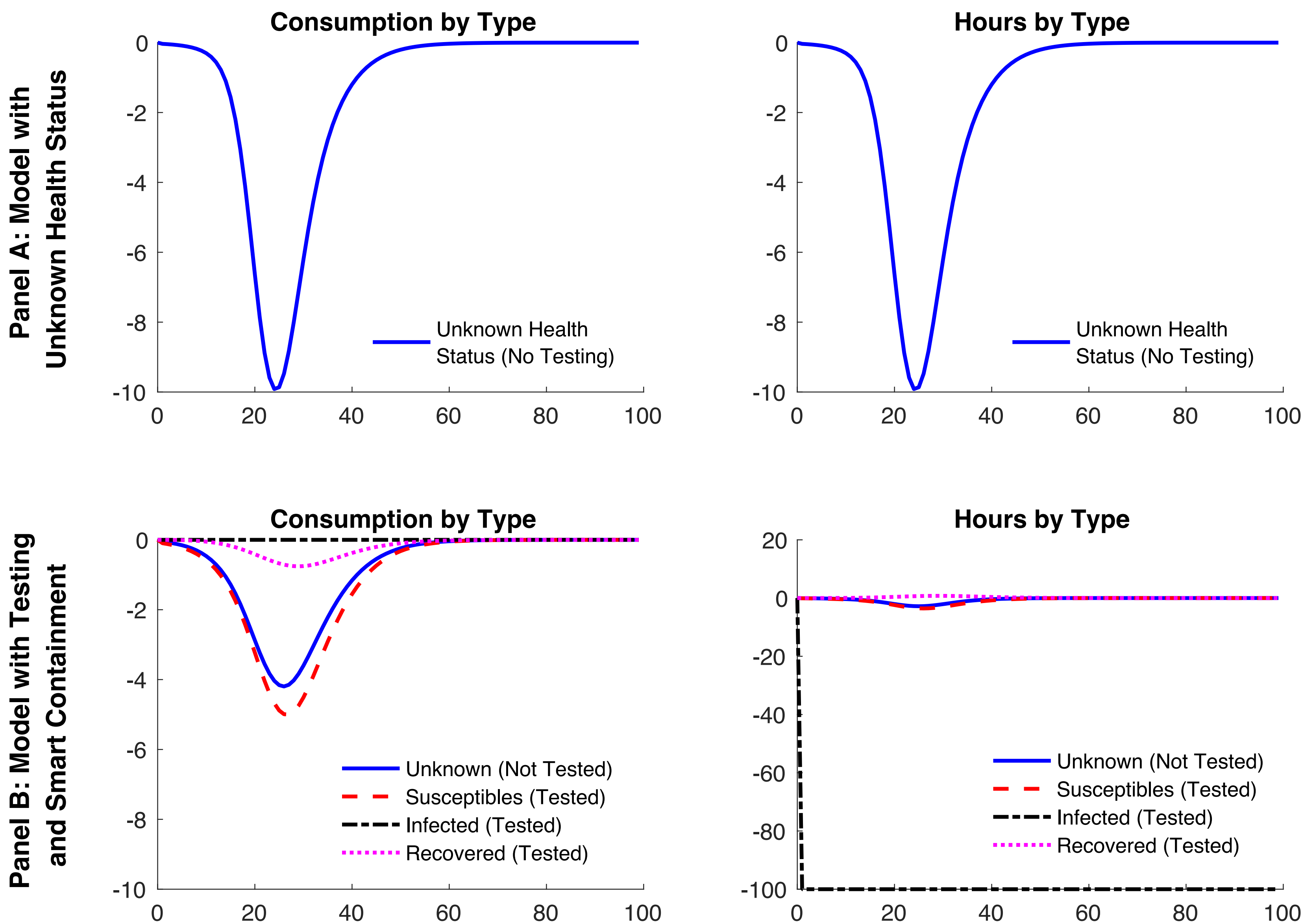


Figure 5: Model with Testing and Smart Containment



Notes: x-axis in weeks. y-axis in percent deviations from pre-infection steady state.

Figure 6: Model With Testing and Smart Containment

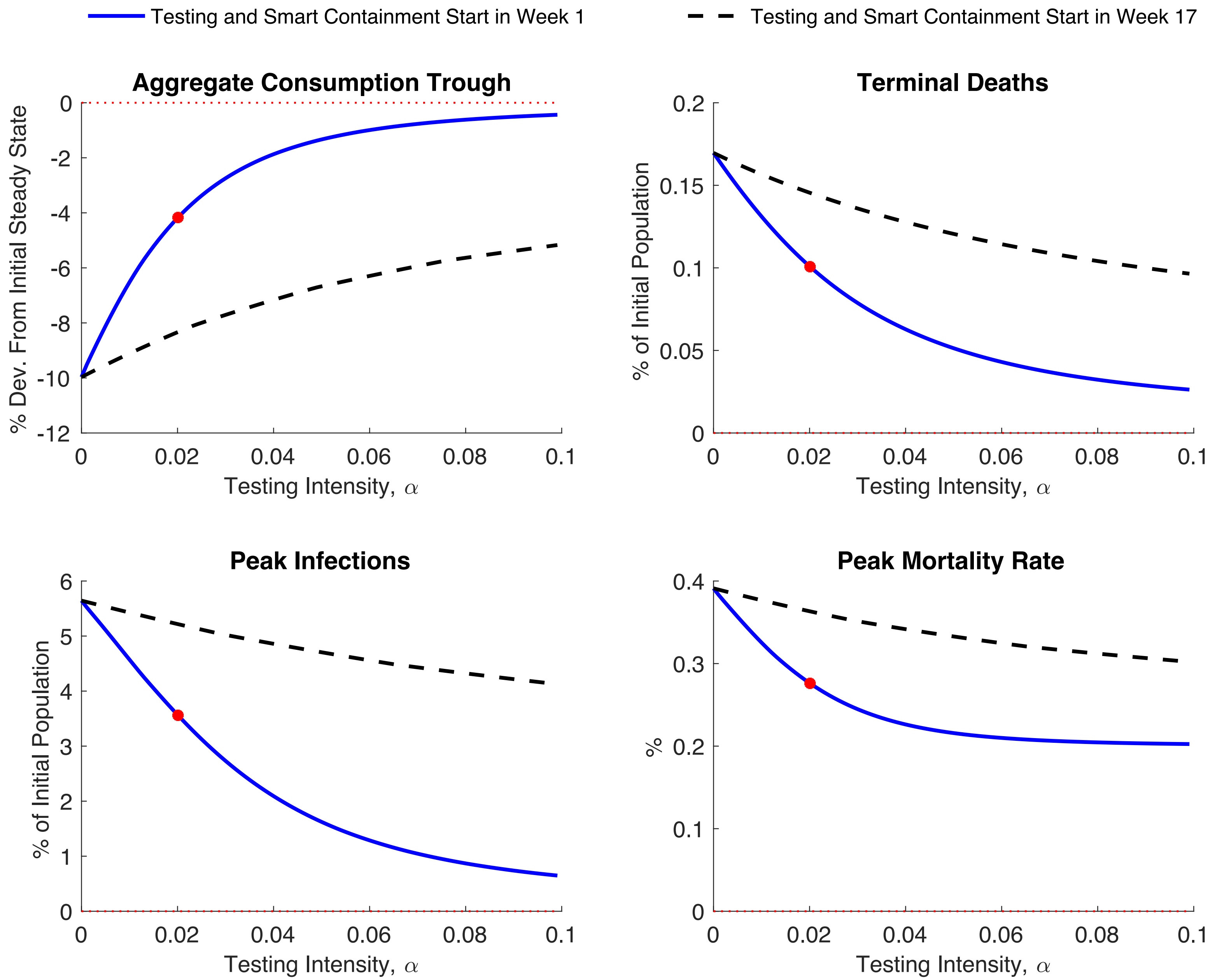


Figure 7: Model with Testing and Strict Containment

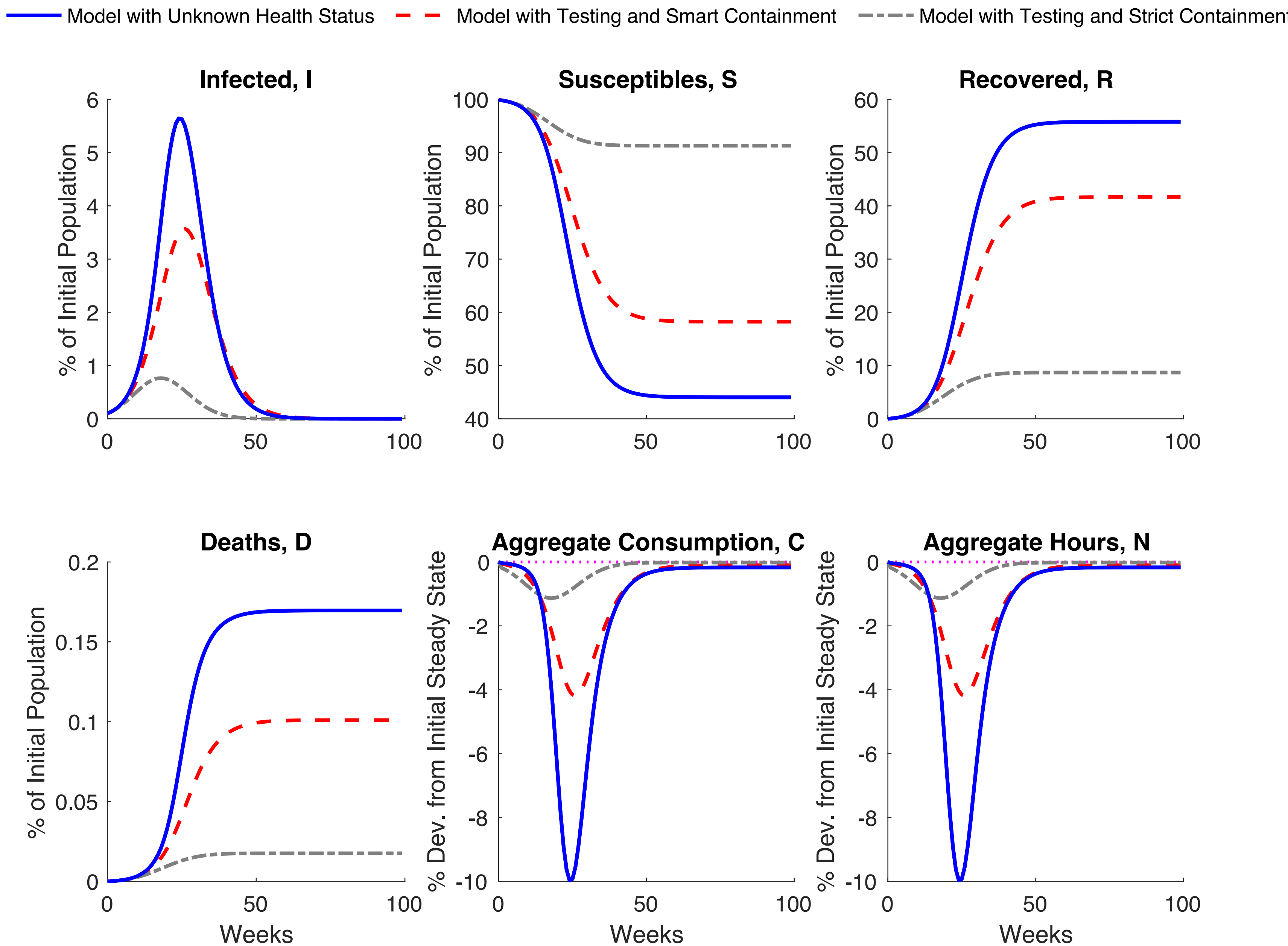


Figure 8: Welfare Gains of Smart Containment vs. Strict Containment

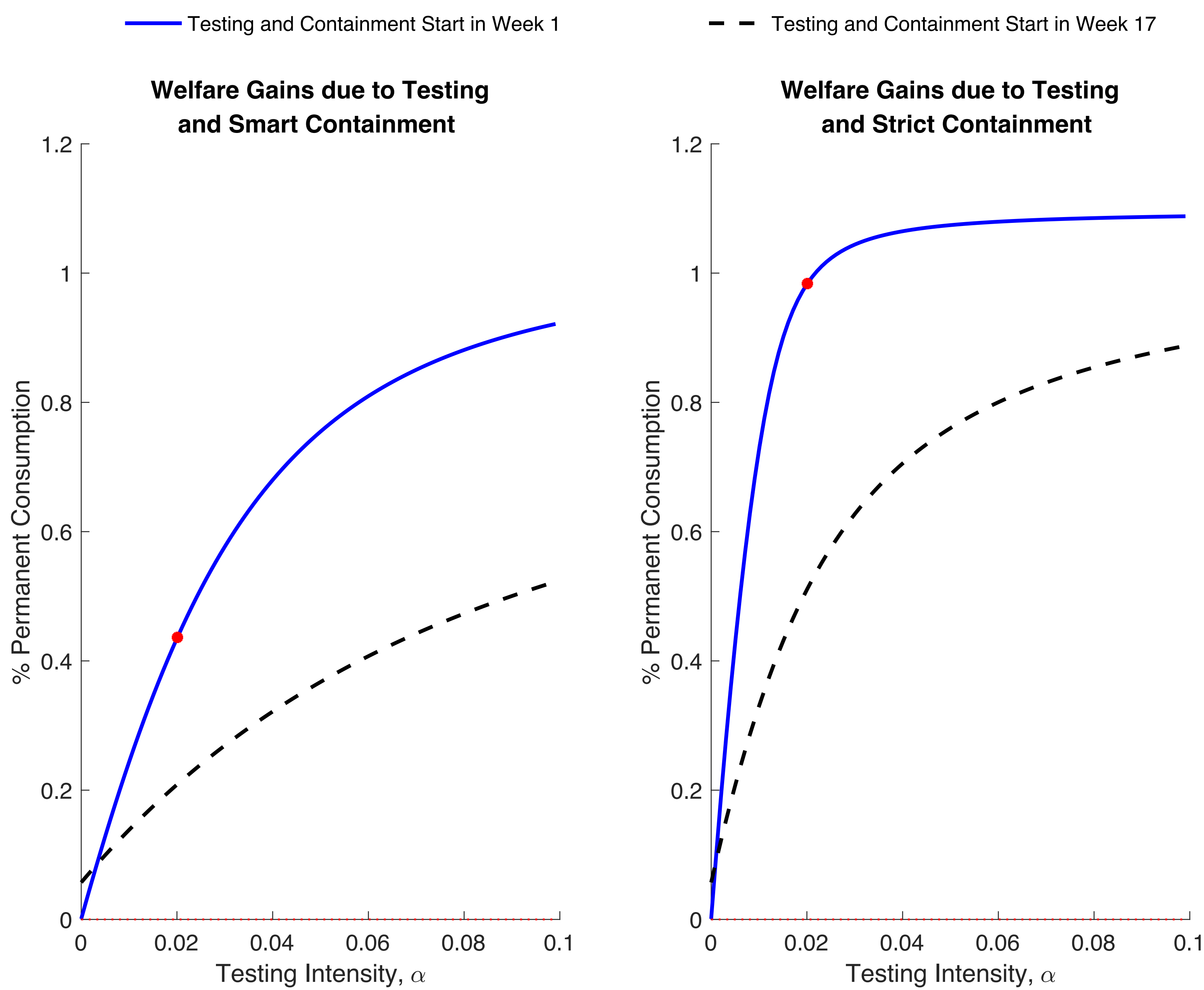


Figure 9: Model with Re-infections

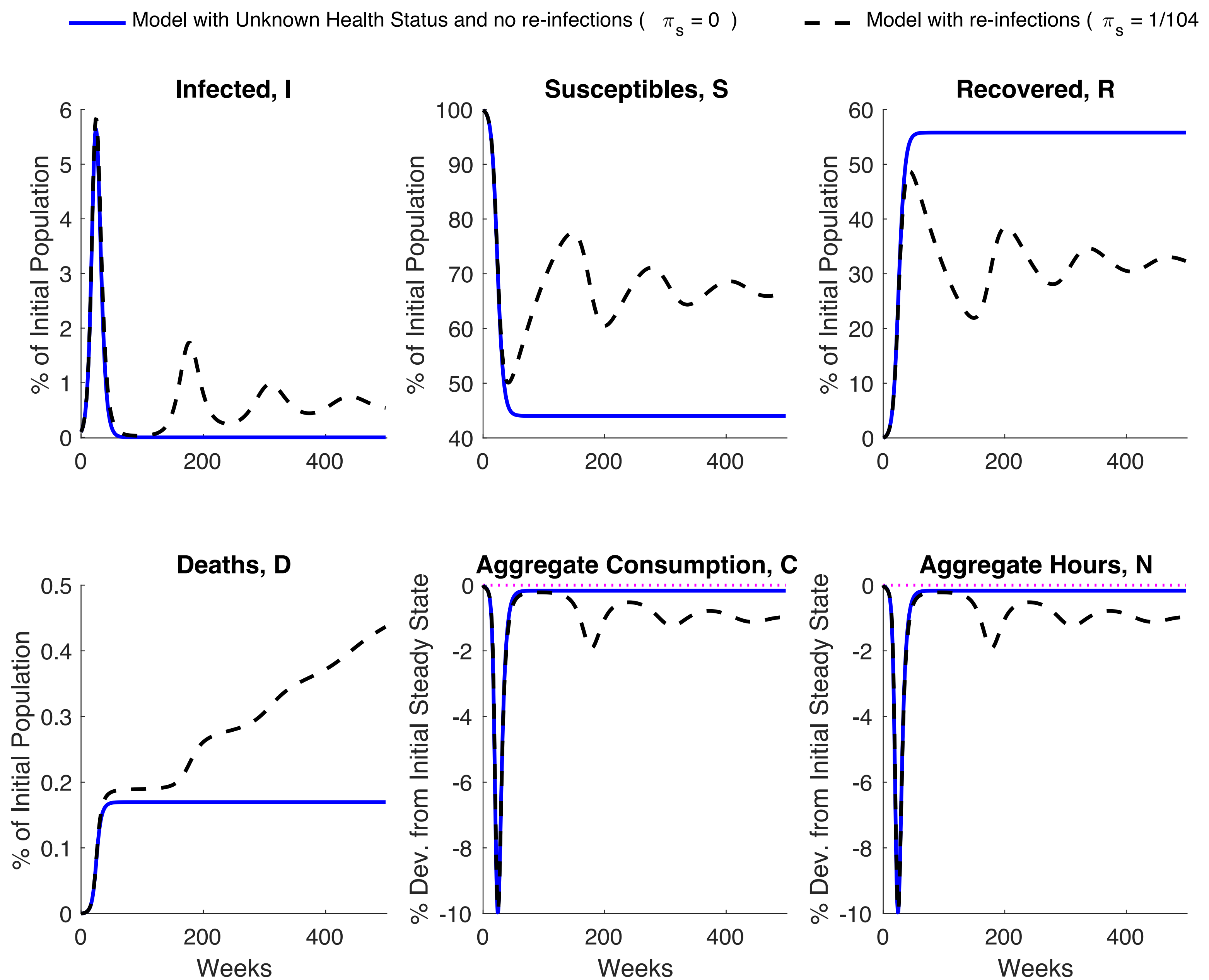


Figure 10: Model with Re-infections, Testing and Containment