On the Fundamentals of Self-Fulfilling Speculative Attacks

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Abstract

Thailand is not Switzerland.

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1. Introduction

The past decade has witnessed dramatic collapses of fixed exchange rate regimes in countries as diverse as Sweden, Mexico, Russia, Thailand and Korea. This has led to a resurgence of interest in the causes of currency crises. While there is disagreement about the source of these crises, there is widespread agreement that banking crises have become increasingly linked to currency crises. This is the ‘twin crises’ phenomenon emphasized by Kaminsky and Reinhardt (1999).

This paper proposes a theory of such crises in which both fundamentals and self-fulfilling beliefs play crucial roles.\footnote{The recent literature emphasizes the distinction between fundamental and multiple equilibrium explanations of ‘twin crises’. For examples of papers that emphasize fundamentals see Corsetti, Pesenti and Roubini (1997), Bordo and Schwartz (1998), and Burnside, Eichebaum and Rebelo (1998). For papers that stress the importance of multiple equilibrium considerations see, for example, Chang and Velasco (1997), Aghion, Bacchetta, and Banerjee (1999) and Krugman (1999).} Fundamentals determine whether crises will occur. Self-fulfilling beliefs determine when they will occur.\footnote{See Cole and Kehoe (1996) for a theory of debt crises in which both fundamentals and self-fulfilling beliefs play an important role.} The fundamental that causes ‘twin crises’ is government guarantees to domestic banks’ foreign creditors. When these guarantees are in place, twin crises inevitably occur. But their timing is a multiple equilibrium phenomenon that depends on agents’ beliefs. So while self-fulfilling beliefs have an important role to play, twin crises do not happen just anywhere. They don’t happen in Switzerland. They happen in countries where there are fundamental problems. Countries like Thailand where the government provided implicit guarantees to banks’ foreign creditors.

In our model the government guarantees the repayment of bank’s foreign loans in the event of a devaluation. These guarantees lead banks to expose themselves to exchange rate risk and to declare bankruptcy when a devaluation occurs. Consequently, a devaluation transforms potential government liabilities into actual...
liabilities. This transformation is the key mechanism by which government guarantees create the possibility of self-fulfilling currency crises. To understand our basic argument, suppose market participants believe that a devaluation is imminent and that the government will finance bank bailouts, at least in part, via seignorage revenues. Then, anticipating future inflation, private agents will exchange domestic money for foreign reserves. If there is a limit on the amount of reserves that the government is willing to lose in defense of the currency, then, depending on the magnitude of agents’ switch into foreign reserves, the fixed exchange rate regime will be abandoned. The resulting devaluation leads banks to declare bankruptcy and activates the government’s obligations to foreign creditors. As a consequence, the government will validate agents’ expectations by partially financing the bailout with seignorage revenues. Thus government guarantees trigger a self-fulfilling rational run on the domestic currency, a devaluation and a banking crisis.

How can the government eliminate these self-fulfilling twin crises? The two most obvious routes are: eliminate government guarantees or (somehow) credibly commit to financing post-devaluation bank bailouts without recourse to seignorage revenues. Our analysis suggests a third route related to a recent proposal by Feldstein (1999): the government must obtain and be willing to use a ‘sufficient’ amount of reserves to fend off a speculative attack. But what does ‘sufficient’ mean? In our model it means a fraction of the money supply that is an increasing function of the inflation rate that would result if a speculative attack succeeded.

The remainder of this paper is organized as follows. Section 2 presents a model of a small open economy which is populated by four different sets of agents: banks, firms, households, and a government. The banking sector is a simplified version of the one in Burnside, Eichenbaum and Rebelo (1999). Banks borrow dollars from abroad and make loans to domestic firms. By assumption these domestic
loans are denominated in local currency, so that banks face foreign exchange rate risk. This risk can be hedged in forward currency markets. We characterize banks' optimal hedging strategy when the government guarantees that foreign creditors will be repaid in the event of a devaluation. In addition we consider the case in which these guarantees are absent. Firms borrow funds to hire labor and produce output using a constant returns to scale technology. Households supply labor inelastically and derive utility from consumption and domestic real balances. Because they have access to international capital markets they have a non-trivial forward looking portfolio problem. In particular, the amount of domestic real balances that they hold depends on their beliefs about the longevity of the fixed exchange rate regime. The government faces an intertemporal budget constraint which must hold for every realization of the state of the economy. To simplify the analysis we assume, as in Krugman (1979), that the government follows a threshold rule according to which it abandons fixed exchange rates when its reserves reach a certain lower bound. The only source of uncertainty in this economy is agents’ beliefs about the collapse of the fixed exchange rate regime. We model these beliefs by assuming that agents coordinate on an exogenous signal which takes on the value one with probability $q$ and zero with probability $(1 - q)$. When the signal equals one agents believe that the exchange rate regime will collapse before the end of the period. When it equals zero, they believe that the fixed exchange rate will persist for at least one more period.

Section 3 displays the competitive equilibrium when self-fulfilling speculative attacks are ruled out by assumption. Section 4 analyzes the conditions under which these attacks can occur. Section 5 contains concluding remarks.
2. The Basic Model

We consider a simple general equilibrium model of a small open economy. By assumption there is a single consumption good and no barriers to trade, so that purchasing power parity holds:

\[ P_t = S_t P_t^*. \]  

(2.1)

Here \( P_t \) and \( P_t^* \) denote the domestic and foreign price level respectively, while \( S_t \) denotes the exchange rate defined as units of domestic currency per unit of foreign currency. For convenience we normalize the foreign price level to one: \( P_t^* = 1 \) for all \( t \).

The economy is initially in a fixed exchange rate regime with \( S_t = S^I \). To allow for the possibility of a self-fulfilling speculative attack, we suppose that agents coordinate on some signal, observed at the beginning of each period. The signal takes on the values zero or one with probabilities \( 1 - q \) and \( q \), respectively. When the signal is equal to zero, agents believe that the fixed exchange rate will endure for at least one more period. If the signal equals one, then agents believe the fixed exchange rate regime will collapse before the end of the period, with the exchange rate initially depreciating to an endogenously determined value \( S^D \) and then depreciating at the rate \( \gamma \) per unit of time.

The key question is whether agents’ beliefs about a devaluation can be self-fulfilling. We denote by \( T \) the random time of a (possible) self-fulfilling speculative attack. It is useful to distinguish between three types of time periods in the life of our model economy.

- **Fixed Exchange Rate Regime**: here \( S_t = S^I \) for all \( t < T \) and the supply of money is determined by the central bank’s need to fix the nominal exchange rate.
• **Devaluation Period**: this is the time period $T$ in which the fixed exchange rate is abandoned. To simplify our analysis, we adopt the standard assumption of the speculative attack literature regarding the behavior of the monetary authority.\(^3\) Specifically, we assume that the central bank defends the fixed exchange rate $S^I$ by selling its reserves at that price until reserves fall by an amount $\chi$. Once this happens, the central bank floats the exchange rate, and allows the money supply to grow at the rate $\gamma$ forever.

• **Floating Exchange Rate Regime**: this obtains for all $t > T$. The growth rate of money is equal to $\gamma$. We consider two separate cases. In the first case the government does not change its tax and spending policy in the aftermath of the devaluation. Here $\gamma$ is determined by the magnitude of the bank bailout and the government’s intertemporal budget constraint. In the second case $\gamma$ is given exogenously and taxes adjust so that the government’s intertemporal budget constraint holds.

The economy is populated by four sets of agents: perfectly competitive banks, good producing firms, households, and a government. In the following subsection we provide a detailed analysis of the banking sector. We then discuss the problems of the other agents in the economy.

**2.1. The Banking Sector**

In this subsection we analyze a simplified version of the banking model in Burns, Eichenbaum and Rebelo (1999) in which banks are exposed to exchange rate risk. The focus of our analysis is on banks’ optimal hedging strategies when the economy is operating under the fixed exchange rate regime discussed above, i.e. at time $t < T$. We show that: (i) it is optimal for a bank to fully hedge exchange

\(^3\)See, for example, Krugman (1979) and Flood and Garber (1984).
rate risk when there are no government guarantees to foreign creditors; and (ii) it is not optimal for a bank to hedge exchange rate risk in the presence of government guarantees. In the latter case, it is optimal for banks to declare bankruptcy when the currency is devalued. Their optimal hedging strategy has the property that when a bank declares bankruptcy, its residual value, net of bankruptcy costs is zero.

We assume that banks are perfectly competitive. Individual banks borrow foreign currency at a gross interest rate $R^b$, and issue non-indexed loans to domestic firms. These loans to firms are to be repaid in local currency units at a gross interest rate $R^a$. When firms repay these loans, the exchange rate $S$ is either $S^I$ or $S^D$, depending on whether the fixed exchange rate regime has been abandoned.

To simplify the analysis we assume that banks do not borrow funds from domestic residents. Instead banks finance themselves entirely by borrowing $L$ dollars in the international capital market. These funds are converted into units of local currency at the prevailing exchange rate, $S^I$. Banks can hedge exchange rate risk by entering into forward contracts. Let $F$ denote the one-period forward exchange rate defined as units of local currency per dollar. By assumption these contracts are priced in a risk neutral manner, so that:

$$\frac{1}{F} = (1-q)\frac{1}{S^I} + q\frac{1}{S^D}. \quad (2.2)$$

This condition states that the expected real rate of return of purchasing a forward contract, denominated in units of the consumption good, is equal to zero. Relation (2.2) implies that forward contracts make a profit in the devaluation state since $S^D > S^I$. To lend $L$ units of output banks must incur transactions costs of $\delta L$. Dollar-denominated profits from these loans are:

$$\pi^L(S, R^b) = \frac{R^a S^I L}{S} - R^b L - \delta L.$$
Dollar-denominated profits from hedging activities are given by:
\[
\pi^H(S) = x \left( \frac{1}{F} - \frac{1}{S} \right) .
\]  
(2.3)

Here \( x \) denotes the number of units of local currency sold by the bank in the forward market. We assume that there is full information about the values of \( L \) and \( x \) chosen by the banks. Notice that the expected value of the bank’s profits from hedging is \( E(\pi^H) = 0 \). Total dollar-denominated profits, \( \pi \), are given by
\[
\pi(S) = \pi^L(S) + \pi^H(S).
\]  
(2.4)

Banks can default on loans contracted in the international capital market. It is optimal for banks to default in states of the world where \( \pi \) is negative. The expected profit of a bank that defaults whenever \( \pi(S) < 0 \) is
\[
V = (1 - q) \max \{ \pi(S^I), 0 \} + q \max \{ \pi(S^D), 0 \}. 
\]  
(2.5)

When a bank defaults it has gross assets with a residual value given by
\[
V^R(S) = \frac{R^n S^I L}{S} - \delta L + x \left( \frac{1}{F} - \frac{1}{S} \right) .
\]  
(2.6)

These assets, net of bankruptcy costs, are distributed to the bank’s international creditors. We assume that bankruptcy costs are given by
\[
C^B(S) = \omega + \lambda V^R(S).
\]  
(2.7)

Here \( \omega \) represents fixed costs associated with bankruptcy and \( \lambda \) represents the fraction of the bank’s gross assets that are dissipated upon default. Note that we have included the bank’s net profits from hedging in \( V^R \). This means we are assuming that there is no default on forward contracts—these contracts must be settled before the bank’s foreign creditors are paid. This implies that if the bank defaults, its residual value must be sufficiently large to pay its bankruptcy costs:
\[
V^R(S) \geq C^B(S).
\]  
(2.8)
Using (2.7) the previous condition can be written as

$$(1 - \lambda) \left[ \frac{R^a S^i L}{S} - \delta L + x \left( \frac{1}{F} - \frac{1}{S} \right) \right] \geq \omega. \quad (2.9)$$

For a given value of $L$, this imposes a restriction on the values of $x$ that an individual bank can choose.

We consider two scenarios. In the first scenario there are no government guarantees to foreign creditors. If banks default foreign creditors receive the residual value of the bank net of bankruptcy costs, $V^R - C^B$. In the second scenario the government guarantees foreign creditors against default by domestic banks in the event of a devaluation.

It is straightforward to show that the bank’s expected profit can be expressed as:

$$V = \left( \frac{S^i}{F} R^a - \delta \right) L - ECB(x, L) \quad (2.10)$$

where $ECB$, the expected cost of borrowing, is given by,

$$ECB(x, L) = \Pr(\text{no default}) \times R^b L + \Pr(\text{default}) \times V^R. \quad (2.11)$$

Notice that in equation (2.10) $x$ only affects $ECB(x, L)$. So for any given $L$, it is optimal for a bank to choose $x$ to minimize $ECB(x, L)$.

**No Government Guarantees**

Absent government guarantees, $R^b$ is determined by the condition that the expected return to international creditors equals $R$:

$$RL = \Pr(\text{no default}) \times R^b L + \Pr(\text{default}) \times (V^R - C^B) \quad (2.12a)$$

**Proposition 1** In an economy with no guarantees where $\omega > 0$ and $0 < \lambda < 1$ it is optimal for banks to fully hedge exchange rate risk. When $\lambda = \omega = 0$, the Modigliani-Miller theorem applies and the bank is indifferent between hedging and not hedging.
Proof: Using (2.11), we can write (2.12a) as:

\[ RL = ECB(x, L) - \Pr(\text{default}) \times C^B, \]

so that

\[ ECB(x, L) = RL + \Pr(\text{default}) \times C^B. \]

The bank can avoid paying bankruptcy costs by choosing \( x \) so that \( \pi(S) \geq 0 \) for all \( S \). This strategy is optimal because it minimizes \( ECB(x, L) \). This establishes that full hedging is optimal for a bank in the absence of government guarantees. Thus banks never go bankrupt.

For future reference it is useful to note that given a full hedging strategy, the bank’s first order condition for \( L \) is:

\[ \frac{R^a S^I}{F} = R + \delta. \quad (2.13) \]

This expression equates the expected real returns to lending to the real cost of borrowing plus the marginal cost of producing a loan.

Government Guarantees

In the presence of government guarantees \( R^b \) is given:

\[ RL = [\Pr(\text{no default}) + \Pr(\text{default in state 2})] \times R^b L + \]

\[ \Pr(\text{default in state 1}) \times (V^R - C^B) \quad (2.14) \]

Proposition 2 Consider an economy in which the government guarantees the repayment of bank’s foreign loans in the event of a devaluation. In addition suppose that: \( 0 < \lambda < 1 \) and \( 0 < \omega < (1 - \lambda) RL \). Then fully hedging is not optimal and the optimal strategy is to set \( x \) to its lowest permissible value.
**Proof:** A bank whose \((x, L)\) is such that it defaults only in the devaluation state \((S = S^D)\), can borrow at the risk free rate: \(R^b = R\). This fact and (2.11) imply that

\[ ECB(x, L) = (1 - q)RL + qV^R(S^D) \]  
\[ = RL + q \left[ V^R(S^D) - RL \right]. \]  

Consider a bank that decided to default in the devaluation state. Its optimal strategy is to choose \(x\) to minimize \(ECB(x, L)\), defined in (2.15). This requires setting \(x\) to its lowest feasible value as determined by (2.9) which in turn implies that \(V^R(S^D) = C^B(S)\). It follows that when the bank pursues this strategy,

\[ ECB(x, L) = (1 - q)RL + q \frac{\omega}{1 - \lambda}. \]

In contrast, suppose that the bank chooses a hedging strategy such that it is never optimal to default. Then \(R^b = R\), and

\[ ECB(x, L) = RL. \]

Suppose that the fixed costs associated with bankruptcy are small enough that repaying a loan is more costly than defaulting,

\[ \frac{\omega}{1 - \lambda} < RL. \] (2.16)

Then the optimal strategy for a bank is the first one, namely set \(x\) to its lowest feasible bound and default whenever the devaluation state occurs. Following an argument similar to that used in the proof of Proposition 1, one can show that it is not optimal to default in the no-devaluation state, since government guarantees do not apply in that state.
Note that condition (2.16) implies that bankruptcy costs are small enough that the residual value of the bank that defaults in the devaluation state is less than the payment owed to foreign creditors. We assume that this condition holds in the remainder of our analysis. Also note that the optimal value of $x$ can be negative, so that banks make hedging profits during the fixed exchange regime and lose money when the currency is devalued. It is the latter feature that allows them to minimize their residual value in bankruptcy states, so that $V^R(S^D) = C^B(S)$. As a consequence, there are no assets for the government to seize in bankruptcy to offset their liabilities to banks’ foreign creditors.

It is useful to note for future reference that given the bank’s optimal hedging strategy when there are government guarantees, its first order condition for $L$ is:

$$\frac{R^a S^I}{F} = (1 - q) R + \delta.$$  \hspace{1cm} (2.17)

This expression equates the expected real returns to lending to the real cost of borrowing plus the marginal cost of producing a loan.

2.2. The Firm’s Problem

Output ($y$) is produced by perfectly competitive firms which use labor ($h$) according to the technology:

$$y = Ah.$$  

Firms must borrow their wage bill from the banks at the gross interest rate $R^a$, so their expected profits are given by $Ah - R^a \frac{W}{S} h$. Here $W$ denotes the nominal wage rate. The first order condition for $h$ is:

$$\frac{W}{S} = \frac{A}{R^a}.$$  \hspace{1cm} (2.18)
2.3. The Household Problem

The representative households inelastically supplies one unit of labor in each period and maximizes expected lifetime utility, which depends on consumption \(c_t\) and real balances \((M_t/S_t)\):

\[
U = E_0 \sum_{t=0}^{\infty} \beta^t \left( \log c_t + \phi \log \frac{M_t}{S_t} \right), \quad 0 < \beta < 1.
\]

To abstract from trends in the economy’s current account we assume that \(\beta = 1/R\). The sequential budget constraint faced by the household depends on the time period under consideration. During floating exchange rate periods \((t > T)\) it is given by:

\[
a_{t+1} = Ra_t + w_t - \tau - c_t - \frac{M_{t+1} - M_t}{S_t},
\]

(2.19)

The variable \(a_t\) represents beginning of period \(t\) net foreign assets, \(M_t\) represents beginning of period \(t\) nominal money balances, \(w_t\) is the real wage rate, \(\tau\) represents constant lump-sum taxes.

In the devaluation period \((t = T)\) the household’s sequential budget constraint is given by:

\[
a_{T+1} = Ra_T + w_T - \tau - c_T - \frac{M_{T+1} - M^D}{S^D} + \chi + x^h_T \left( \frac{1}{F} - \frac{1}{S^D} \right),
\]

(2.20)

\[
M^D = M_T - \chi S^I.
\]

(2.21)

At the time of the devaluation the household redeems \(\chi S^I\) units of local currency in exchange for foreign reserves. This is why its initial money holdings in (2.20) equals \(M^D\) (defined in (2.21)) and the term \(\chi\) appears as an asset. The variable \(x^h_T\) denotes the number of units of local currency sold by the household in the forward market in the previous period. The household has an incentive to enter these
contracts during the fixed exchange rate regime. By entering the forward market it can insure against the effect of a devaluation on the value of its real balances. We will see later that allowing households to hedge implies that consumption is constant over time. This greatly simplifies the analysis, enabling us to characterize analytically the equilibrium of the economy.

During the fixed exchange regime \((t < T)\) the budget constraint is:

\[
a_{t+1} = Ra_t + w_t - \tau - c_t - \frac{M_{t+1} - M_t}{S^f_t} + x_t \left( \frac{1}{F^0} - \frac{1}{S^f_t} \right),
\]

where \(a_0\) and \(M_0\) are given.

Finally we impose the no-Ponzi game condition on the household:

\[
E_0 \lim_{t \to \infty} \frac{a_{t+1}}{R^t} = 0. \quad (2.22)
\]

### 2.4. The Government

During the fixed exchange rate regime \((t < T)\) the government’s flow budget constraint is:

\[
f_{t+1} = Rf_t + \frac{M^S_{t+1} - M^S_t}{S^f_t} + \tau - g. \quad (2.23)
\]

Here \(f_t\) denotes the government’s net foreign assets at the beginning of the time period, \(g\) is the constant level of real government purchases and \(M^S_t\) denotes the endogenous level of the money supply that is consistent with a fixed exchange rate regime at time \(t\). Not surprisingly, \(M^S_t\) is constant for \(t < T\).

The government’s flow budget constraint during the devaluation period \((t = T)\) is:

\[
f_{T+1} = Rf_T - \chi - \Gamma + \frac{M_{T+1} - M^D}{S^D} + \tau - g, \quad (2.24)
\]

Here \(\Gamma\) represents the cost of honoring guarantees to bank’s foreign creditors.
From Proposition 2 we know that $\Gamma = RL$. For convenience we assume that the government repays $\Gamma$ in the devaluation period $T$.

For now we confine ourselves to the ‘no fiscal reform case’ where the government does not adjust taxes or government purchases after the collapse of the fixed exchange rate regime. Under this assumption, the government’s flow budget constraint during the floating exchange rate period is:

$$f_{t+1} = Rf_t + \frac{M_{t+1}^S - M_t^S}{S_t} + \tau - g. \quad (2.25)$$

We impose the condition:

$$E_0 \lim_{t \to \infty} \frac{f_{t+1}}{R_t} = 0. \quad (2.26)$$

Note that once the fixed exchange rate is abandoned, there is no uncertainty in the economy. This fact together with equations (2.24), (2.25) and (2.26) imply that at time $T$ the government’s intertemporal budget constraint is:

$$\Gamma + \chi = \frac{M_{T+1} - M^D}{S^D} + \sum_{j=1}^{\infty} \frac{1}{R^j} \left( \frac{M_{T+j+1} - M_{T+j}}{S_{T+j}} \right) \quad (2.27)$$

This equation simply says that seignorage revenues must equal the value of the bailout, $\Gamma$, plus the loss of reserves incurred during the attack.

2.5. The Competitive Equilibrium

We conclude this section with a definition of the competitive equilibrium that applies to economies with and without government guarantees to foreign creditors.

**Definition 2.1.** A competitive equilibrium for this economy is a set of stochastic processes for quantities $\{c_t, x_t, x_t^b, M_t, M_t^S, a_{t+1}, f_{t+1}, \Gamma\}$ and prices $\{W_t, R_t^a, R_t^b, P_t, S_t, F_t\}$ such that: (i) $c_t, M_{t+1}, a_{t+1}$ solve the household’s problem given the stochastic process for prices; (ii) the government’s intertemporal budget constraint (2.23) -
(2.27) holds; (iii) the money market clears with $M_t^S = M_t$; (iv) the labor market clears with $h_t = 1$; and (v) the loan market clears with $S_t L_t = W_t$.

3. A Sustainable Fixed Exchange Rate Regime

In this section we describe a version of our economy in which self-fulfilling currency attacks are ruled out by assumption. The purpose is to demonstrate that there exists a sustainable equilibrium with a fixed exchange rate. The existence of this equilibrium follows from two basic assumptions: (i) agents assign probability zero to a devaluation, so that there is no uncertainty in the economy; (ii) the government does not require seignorage revenues to satisfy its intertemporal budget constraint.

Since there is no exchange rate uncertainty banks can borrow from foreigners at the risk free rate, $R^b = R$. Absent the possibility of a devaluation, government guarantees are irrelevant, so that $\Gamma = 0$. Also, hedging plays no role in the analysis, so $x_t = x^h_t = 0$. Finally, the forward rate coincides with the spot rate $F = S^I$.

It follows from (2.13) that:

$$R^a = R + \delta.$$ 

In addition, equation (2.18) implies that the market clearing real wage rate is given by:

$$\frac{W}{S} = \frac{A}{R + \delta}$$

This completes the description of the equilibrium prices.

To determine the household’s consumption and real balances we write its value function as:
\[ V(a_t, M_t) = \max_{c_t, a_{t+1}, M_{t+1}} \left\{ \log c_t + \phi \log \frac{M_t}{S^t} + \beta V(a_{t+1}, M_{t+1}) \right\} \]

subject to (2.19). The first order conditions for this problem are:

\[
\frac{1}{c_t} = \beta \left( \frac{1}{c_{t+1}} + \frac{\phi}{M_{t+1}/S^t} \right),
\]

\[
\frac{1}{c_t} = \beta R \frac{1}{c_{t+1}}.
\]

Our assumption about \( \beta \) implies that consumption, net foreign assets and money holdings are constant over time:

\[
c_t = (R - 1)a_0 + w - \tau, \quad (3.1)
\]

\[
a_t = a_0, \quad (3.2)
\]

\[
\frac{M}{S^t} = \frac{\beta \phi c}{1 - \beta}. \quad (3.3)
\]

Since real balances are constant and the inflation rate is zero, the government collects no seignorage revenues. Consequently, its intertemporal budget constraint is given by:

\[(R - 1)f_0 = g - \tau. \quad (3.4)\]

By construction the previous analysis demonstrates that there is a unique fixed exchange rate competitive equilibrium with \( c_t, a_t \) and \( M_t \) constant over time, given by (3.1) - (3.3). In addition \( f_t \) is constant: \( f_t = f_0 \). This completes the description of the equilibrium quantities.

Throughout the paper we assume that (3.4) holds, so that, in the absence of self-fulfilling speculative attacks, the fixed exchange rate regime is sustainable.
4. Self-Fulfilling Currency Attacks

In this section we turn to the question: Can self-fulfilling speculative attacks occur? To answer this question we begin by assuming that such an attack can exist. We then construct candidate equilibrium price and quantity allocations for the three types of time periods in our model. By construction these allocations satisfy the optimization problems of the different agents in the model and the market clearing conditions. The key condition that must be verified is whether $S^D/S^I > 1$, i.e. the exchange rate actually devalues in the proposed equilibrium. Whether or not this is true depends on the nature of monetary and fiscal policy after the devaluation. Here we consider two cases. First, we analyze the ‘no fiscal reform’ case. Here the government finances the costs associated with a devaluation entirely via seignorage revenues. For the sake of simplicity we confine ourselves to an endogenously determined constant growth rate of money. Proposition 3 establishes that, subject to a regularity condition, self-fulfilling speculative attacks will occur. Second, we analyze the ‘fiscal reform’ case. Here the government commits to expanding the growth rate of money at an exogenous rate $\gamma$ and adjusts lump sum taxes to fulfill its intertemporal budget constraint. Proposition 4 provides conditions on the quantity of reserves that the government is prepared to lose, the growth rate of money, and the size of the bailout for which a self-fulfilling attack will occur.

In the Appendix we solve our model in three stages working backward in time. First, we study the dynamic programming problem that agents face during the floating exchange rate regime. Then using this solution, we analyze agents’ dynamic programming problem during the devaluation period. Finally, we solve the dynamic programming problem that agents face during the fixed exchange rate regime. Below we summarize the key features of the economy during the
different time periods, assuming that a self-fulfilling speculative attack exists.

Throughout our discussion we use the fact, proved in the Appendix, that consumption and the household’s beginning of time $t$ real assets $a_t$ are constant for all $t$:

$$
c = (R - 1)a_0 + y - \tau - \frac{(R - 1)q}{q + R - 1}\gamma, \quad (4.1)
$$

$$
a_t = a_0.
$$

The fact that $c_t$ and $a_t$ are constant hinges on our assumption that during the fixed exchange rate period households hedge exchange rate risk through forward contracts. According to (4.1), consumption is equal to the household’s permanent income, where the latter is defined to take account of the annuitized expected present value of the bank bailout, $(R - 1)\lambda\gamma/(\lambda + R - 1)$.

4.1. The Floating Exchange Rate Peg Regime ($t \geq T + 1$)

With no exchange rate uncertainty, banks can borrow from foreigners at the risk free rate, $R^b = R$. The presence of guarantees is irrelevant and hedging plays no role in the analysis, so $x_t = x^h_t = 0$. Finally, the forward rate coincides with the spot rate, so $F_t = S_{t+1}$ and $F_t/S_t = \gamma$. The law of motion for the exchange rate and the money supply are:

$$
S_t = S^D\gamma^{t-T}, \quad t \geq T
$$

$$
M_t = M^D\gamma^{t-T}, \quad t \geq T \quad (4.2)
$$

Recall that $S^D$ is the exchange rate that prevails when the government abandons fixed exchange rates in period $T$. The variable $M^D$ represents the level of the money supply after the speculative attack.
The banks’ first order condition for $L$ during the floating exchange rate regime implies:

$$R^a = \gamma (R + \delta), \quad (4.3)$$

while equation (2.18) implies that the market clearing real wage rate is given by:

$$\frac{W}{S} = \frac{A}{\gamma (R + \delta)}. \quad (2.18)$$

During the floating exchange rate regime expected inflation is constant and equal to $\gamma - 1$. The demand for real balances is given by:

$$m^F = \frac{M_t}{S_t} = \frac{\phi c}{\gamma / \beta - 1}. \quad (4.4)$$

Relation (4.4) implies that real balances are a decreasing function of the nominal interest rate since the latter equals $\gamma / \beta - 1$ during the flexible exchange rate regime.

4.2. The Fixed Exchange Regime ($t < T$)

At any time period $t < T$ there is exchange rate uncertainty, since a devaluation may occur next period with probability $q$. In the presence of government guarantees banks behave as described in Proposition 2 and $R^b = R$. In addition, banks set $x_t$ to the lowest value consistent with (2.9). The exchange rate is equal to $S^f$ and the forward rate is given by (2.2). Denote the inflation rate between time $t$ and $t + 1$ by $\pi_t$. Then, the time $t$ equilibrium interest rate at which banks lend to firms, given by (2.17), can be written as:

$$R^a = \frac{(1 - q)R + \delta}{E_t[1/(1 + \pi_t)]} \quad (4.5)$$
where $E_t[1/(1 + \pi_t)] = (1 - q) + qS^f/S^D$. The numerator of (4.5) reflects the fact that the bank only pays off its loans if there is no devaluation, a state that occurs with probability $(1 - q)$. If we were to make the approximation (which we don’t) that $1/E_t[1/(1 + \pi_t)] \approx E_t(1 + \pi_t)$, then we would obtain a version of the Fisher equation for the nominal interest rate, $R_a = E_t(1 + \pi_t)(1 - q)R + \delta$.

Comparing (4.3) and (4.5), we see that the interest rate for domestic loans is lower in the fixed exchange rate regime than in the floating exchange rate regime for two reasons. First, guarantees, which lower banks’ borrowing costs from $R$ to $(1 - q)R$, no longer apply once the currency is devalued. Second, actual inflation in the floating exchange rate regime, $\gamma$, exceeds expected inflation in the fixed exchange rate regime.

Real wages are given by the firm’s first order condition, (2.18):

$$\frac{W}{S} = \frac{A}{R^a}$$

(4.6)

It follows from the previous remarks that the real wage is lower in the floating exchange rate regime.

The solution to the household problem yields a constant value for $x^h_t$ which must satisfy:

$$x^h \left( \frac{1}{F} - \frac{1}{S^f} \right) = \left[ x^h \left( \frac{1}{F} - \frac{1}{S^D} \right) - \Gamma \right] \frac{R - 1}{R}$$

Since $1/F - 1/S^f$ is negative, the right hand side of this equation is the loss associated with forward contracts in states of the world where the exchange rate does not devalue. The right hand side is the annuity value of the bailout minus the profits from forward contracts at the time of the devaluation. By choosing this value of $x^h$ the household can perfectly smooth consumption across the devaluation and non-devaluation states of the world.

During the fixed exchange rate regime expected inflation is constant and equal
to $q(S^D/S^T - 1)$. In the Appendix we show that households choose real balances so that

$$m^I = \frac{\phi c \beta}{1 - \beta + \beta q (1 - S^T/S^D)}$$

(4.7)

Note that real balances are decreasing in expected inflation. During this regime the endogenous money supply must be consistent with the real balances demanded by households and the fixed exchange rate. Therefore:

$$M_t^S = S^I m^I \text{ for all } t < T.$$

4.3. The Devaluation Period ($t = T$)

At the beginning of period $T$, prior to the realization of the stochastic process signalling the onset of a devaluation, banks borrow from abroad and lend to domestic firms at the value of $R^a$ that prevails in the fixed exchange rate regime. Proposition 2 implies that, in an economy with government guarantees, once the currency is devalued the banks renege on their foreign debt and declare bankruptcy. Due to the banks’ hedging strategy, their residual value, net of bankruptcy costs, is equal to zero. Thus the total realized liability of the government, $\Gamma$, is $RL$.

Since firms make their hiring decisions at the beginning of the period prior to the devaluation, the real wage is the same as in the fixed exchange rate regime and is given by (4.6).

Recall that households enter the period with $M_T = m^I S^I$ units of the local currency. Once the random variable signaling the onset of a devaluation is realized, agents redeem $\chi S^I$ units of local currency in exchange for foreign reserves. The exchange rate rises from $S^I$ to $S^D$, at which point agents are left holding $M^D = M^T - \chi S^I$ units of local currency. In the Appendix we prove the following lemma which reflects our assumption that money grows at rate $\gamma$ starting from the level $M^D$ (see equation 4.2).
Lemma The rate of inflation from the onset of the devaluation to the first period of the floating exchange rate regime \((T + 1)\) is: \(S_{T+1}/S^D = \gamma\). The level of real balances at the onset of the devaluation is:

\[
\frac{M^D}{S^D} = m^F = \frac{\phi c}{\gamma/\beta - 1}.
\]

(4.8)

Integrating over the previous results we have constructed all of the endogenous variables in an equilibrium where a self-fulfilling currency attack occurs at the random date \(T\) as a function of three unknowns: \(\gamma, S^D\) and \(m^I\). We now solve for these three variables and verify whether \(S^D\) exceeds \(S^I\), i.e. whether a self-fulfilling currency attack actually occurs at \(T\).

No Fiscal Reform

We first consider the case in which there is no fiscal reform, i.e. the government finances all of the costs associated with a devaluation via seignorage revenues.

Proposition 3 Suppose that in the event of a devaluation the government chooses \(\gamma\) so that the present value of government liabilities after a devaluation, \(\Gamma + \chi\), is fully financed by seignorage. Then, as long as \(\Gamma > 0\) and \(\Gamma + \chi\) is smaller than the maximum present value of seignorage, \(\phi c/(R - 1)\), a self-fulfilling speculative attack exists.

Proof: We first solve for the value of \(\gamma\) that satisfies the government’s intertemporal budget constraint at the onset of the devaluation, (2.27). Using (4.8), and (4.4) this equation can be rewritten as:

\[
\Gamma + \chi = \frac{m^F(\gamma - 1)R}{R - 1} = \frac{\phi c(\gamma - 1)}{(\gamma - \beta)(R - 1)}.
\]

(4.9)

Since \(c\) is invariant to \(\gamma\) (see (4.1)), this equation implies that the equilibrium value of \(\gamma\) is:

\[
\gamma = \frac{\phi c - (R - 1)\beta(\Gamma + \chi)}{\phi c - (R - 1)(\Gamma + \chi)}
\]

23
Note that as long as $\Gamma + \chi > 0$, then $\gamma > 1$. The value of $m^F$ associated with this value of $\gamma$ is:

$$m^F = \frac{\beta \phi c}{1 - \beta} - (\Gamma + \chi). \quad (4.10)$$

It follows that for $m^f$ to be positive, we require

$$\frac{\beta \phi c}{1 - \beta} > (\Gamma + \chi) \quad (4.11)$$

Equation (2.21), which characterizes the level of the money supply at the instant of devaluation ($M^D$), can be rewritten as:

$$m^I = m^F \frac{1}{S^I/S^D} + \chi \quad (4.12)$$

Note that $m^F$ is determined by equations (4.10) and (4.1), and can be treated as known. Also recall that $m^I$ must satisfy (4.7), which we repeat for the reader’s convenience.

$$m^I = \phi c \frac{\beta}{1 - \beta + \beta q (1 - S^I/S^D)}$$

Since $c$ is given by (4.1), equations (4.12) and (4.7) form a system of two equations in two unknowns, $m^I$ and $S^I/S^D$. Figure 1 displays these two equations for $m^I$ as a function of $S^I/S^D$. Equation (4.12) is a downward sloping rectangular hyperbola which asymptotes to $\chi$ and infinity. Equation (4.7) is upward sloping with positive intercept. Also $m^I$ asymptotes to infinity at $S^I/S^D = 1 + (1 - \beta)/\beta q$. This implies that the two equations have a unique intersection. We now show that this intersection occurs at $S^I/S^D > 1$. Note that (4.7), evaluated at $S^I/S^D = 1$, is equal to $\phi c \beta/(1 - \beta)$, while (4.12), evaluated at the same point, is $\phi c \beta/(1 - \beta) - \Gamma$ (see equation (4.10)). Since the former value of $m^I$ exceeds the latter, the intersection of the two curves must occur at $S^I/S^D < 1$. 

24
The previous proposition implies that as long as $q > 0$, there are government guarantees and it is feasible to finance the obligations associated with the devaluation via seignorage, a self-fulfilling speculative attack will almost surely occur. This is true regardless of the reserves that the government is willing to spend in defense of the currency ($\chi$). The reason for this is that the higher $\chi$ is, the higher are the government’s losses during the speculative attack and the more seignorage that needs to be collected after the fixed exchange rate regime is abandoned.

At first glance, Proposition 3 might suggest that a self-fulfilling currency attack can arise if $\Gamma = 0$, i.e. there are no government guarantees. This is incorrect. Figure 1 shows that when $\Gamma = 0$ the unique intersection of the two equations for $m^I$ occurs for $S^I/S^D = 1$. So subject to the regularity condition in Proposition 3, for the class of monetary and fiscal policies under consideration a self-fulfilling currency attack exists if and only if the government guarantees loans to banks’ foreign creditors.\(^4\)

To understand regularity condition (4.11), note that $\bar{\phi}c/(1 - \beta)$ equals the maximal present value of seignorage that the government can extract from the economy. This follows from the fact that the right hand side of (4.9), the present value of seignorage for a given value of $\gamma$, is strictly increasing in $\gamma$. It follows that maximal seignorage equals $\phi c/(R - 1)$ corresponding to $\gamma = \infty$. So according to (4.11), in order for a self fulfilling currency attack to occur in the ‘no fiscal reform’, $\Gamma + \chi$, must be smaller than the maximum present value of seignorage. The reason this condition is necessary is that agents are expecting that the costs associated with a devaluation, $\Gamma + \chi$, will be fully financed with seignorage. Obviously, if this is not possible, such an expectation cannot be self-fulfilling.

\textit{Fiscal Reform}

\(^4\)Results in Obstfeld (1986) suggest that for non-constant growth rate of money policies self-fulfilling speculative attacks may occur even when $\Gamma = 0$, i.e. there are no government guarantees.
We now turn to the case in which the devaluation is accompanied by a fiscal reform. Specifically, we assume that the law of motion for money is given by (4.4) where $\gamma$ is now an exogenous parameter. Since the government’s intertemporal budget constraint does not hold for an arbitrary $\gamma$, we suppose that a devaluation is followed by a fiscal reform in which lump sum taxes are adjusted to ensure that the intertemporal budget constraint, (2.27), holds. The following proposition characterizes the necessary and sufficient conditions for a self-fulfilling currency attack to occur under these circumstances.

**Proposition 4** Suppose that in the event of a devaluation the government finances the present value of government obligations associated with the devaluation, $\Gamma + \chi$, by choosing a fixed value of $\gamma$ and financing the remainder with post devaluation taxes, $\tau^D$. Then a self-fulfilling speculative attack exists if:

$$\chi \leq \frac{\beta c}{1 - \beta} \left( \frac{\gamma - 1}{\gamma - 1 + \beta \rho} \right)$$

(4.13)

where $c$ is the level of consumption under a sustainable fixed exchange rate regime ($q = 0$). The value of $\tau^D$ is given by:

$$\tau^D = \chi + \Gamma - \phi c \frac{\beta}{\gamma - 1 + \beta}$$

**Proof:** With $\gamma$ exogenous, $m^F$ is given by (4.4). Since the level of consumption is independent of $\gamma$ and $\tau^D$ (it only depends on the size of the bailout $\Gamma$), we can solve (4.12) and (4.7) to obtain $m^I$ and $S^I/S^D$. The solution for $S^D/S^I$ is given by $\alpha_2(S^D/S^I)^2 + \alpha_1(S^D/S^I) + \alpha_0 = 0$, where $\alpha_2 = (\phi \beta c)(1 - \beta + \beta q)$, $\alpha_1 = (\gamma - \beta) \chi (1 - \beta + \beta q) - (\phi \beta c)(\gamma - \beta + \beta q)$, and $\alpha_0 = - (\gamma - \beta) \chi \beta q$. In order for $S^D/S^I \geq 1$ it is necessary that $\alpha_0 + \alpha_1 + \alpha_2 \geq 0$. Replacing the $\alpha$ parameters and rearranging the resulting equation we obtain the following condition for the existence
of self-fulfilling speculative attacks:

\[ \chi \leq \frac{\beta \phi c \gamma - 1}{1 - \beta \gamma - \beta}. \quad (4.14) \]

To rule out speculative attacks for all values of \( q \) we evaluate this condition for a value of \( c \) given by (4.1) when \( q = 0 \). This corresponds to the consumption in a fixed exchange rate regime where self-sustainable currency attacks cannot take place (see 3.1).

To interpret (4.14) recall from (3.3) that real balances in a sustainable fixed exchange rate regime are \( \beta \phi c/(1 - \beta) \). So according to (4.14), the government must be able to buy at least a fraction \((\gamma - 1)/(\gamma - \beta)\) of these real balances to avoid a self-fulfilling speculative attack. Note that if the government sets \( \gamma \) to infinity the right hand side of (4.14) converges to \( \beta \phi c/(1 - \beta) \). So, in this case the government must be able to buy back all outstanding real balances to avoid a speculative attack. Finally, if the government can credibly commit to not using seignorage revenues to finance an eventual bank bailout (i.e. it sets \( \gamma \) to one), then self-fulfilling speculative attacks do not exist.

5. Conclusion

This paper developed a theory of ‘twin crises’ in which both fundamentals and self-fulfilling beliefs play an important role. The presence of government guarantees to banks’ foreign creditors implies that a ‘twin crises’ will inevitably occur. In this sense fundamentals matter. This makes us optimistic about the prospect of identifying countries in which crises will occur. However, the timing of the crises in our model depends subtly on agent’s self-fulfilling beliefs about when the fixed exchange rate regime will collapse. This makes us pessimistic about the prospect of forecasting the precise time at which ‘twin crises’ will occur.
References


6. Appendix

To be added.