

# Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy\*

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August 27, 2003

## Abstract

We present a model embodying moderate amounts of nominal rigidities that accounts for the observed inertia in inflation and persistence in output. The key features of our model are those that prevent a sharp rise in marginal costs after an expansionary shock to monetary policy. Of these features, the most important are staggered wage contracts which have an average duration of three quarters, and variable capital utilization.

JEL: E3, E4, E5

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\*The first two authors are grateful for the financial support of a National Science Foundation grant to the National Bureau of Economic Research. We would like to acknowledge helpful comments from Lars Hansen and Mark Watson. We particularly want to thank Levon Barseghyan for his superb research assistance, as well as his insightful comments on various drafts of the paper. This paper does not necessarily reflect the views of the Federal Reserve Bank of Chicago or the Federal Reserve System.

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# 1. Introduction

This paper seeks to understand the observed inertial behavior of inflation and persistence in aggregate quantities. To this end, we formulate and estimate a dynamic, general equilibrium model that incorporates staggered wage and price contracts. We use our model to investigate what mix of frictions can account for the evidence of inertia and persistence. For this exercise to be well defined, we must characterize inertia and persistence precisely. We do so using estimates of the dynamic response of inflation and aggregate variables to a monetary policy shock. With this characterization, the question that we ask reduces to: ‘Can models with moderate degrees of nominal rigidities generate inertial inflation and persistent output movements in response to a monetary policy shock?’<sup>1</sup> Our answer to this question is, ‘yes’.

The model that we construct has two key features. First, it embeds Calvo style nominal price and wage contracts. Second, the real side of the model incorporates four departures from the standard textbook one sector dynamic stochastic growth model. These departures are motivated by recent research on the determinants of consumption, asset prices, investment and productivity. The specific departures that we include are habit formation in preferences for consumption, adjustment costs in investment and variable capital utilization. In addition, we assume that firms must borrow working capital to finance their wage bill.

Our key findings are as follows. First, the average duration of price and wage contracts in the estimated model is roughly 2 and 3 quarters, respectively. Despite the modest nature of these nominal rigidities, the model does a very good job of accounting quantitatively for the estimated response of the US economy to a policy shock. In addition to reproducing the dynamic response of inflation and output, the model also accounts for the delayed, hump-shaped response in consumption, investment, profits, productivity and the weak response of the real wage.<sup>2</sup> Second, the critical nominal friction in our model is wage contracts, not price contracts. A version of the model with only nominal wage rigidities does almost as well as the estimated model. In contrast, with only nominal price rigidities, the model performs very poorly. Consistent with existing results in the literature, this version of the model cannot generate persistent movements in output unless we assume price contracts of extremely long duration. The model with only nominal wage rigidities does not have this problem.

Third, we document how inference about nominal rigidities varies across different specifications of the real side of our model.<sup>3</sup> Estimated versions of the model that do not in-

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<sup>1</sup>This question that is the focus of a large and growing literature. See, for example, Chari, Kehoe and McGrattan (2000), Mankiw (2001), Rotemberg and Woodford (1999) and the references therein.

<sup>2</sup>In related work, Sbordone (2000) argues that, taking as given aggregate real variables, a model with staggered wages and prices does well at accounting for the time series properties of wages and prices. See also Ambler, Guay and Phaneuf (1999) and Huang and Liu (2002) for interesting work on the role of wage contracts.

<sup>3</sup>For early discussions about the impact of real frictions on the effects of nominal rigidities, see Blanchard

corporate our departures from the standard growth model imply implausibly long price and wage contracts. Fourth, we find that if one only wants to generate inertia in inflation and persistence in output with moderate wage and price stickiness, then it is crucial to allow for variable capital utilization. To understand why this feature is so important, note that in our model firms set prices as a markup over marginal costs. The major components of marginal costs are wages and the rental rate of capital. By allowing the services of capital to increase after a positive monetary policy shock, variable capital utilization helps dampen the large rise in the rental rate of capital that would otherwise occur. This in turn dampens the rise in marginal costs and, hence, prices. The resulting inertia in inflation implies that the rise in nominal spending that occurs after a positive monetary policy shock produces a persistent rise in real output. Similar intuition explains why sticky wages play a critical role in allowing our model to explain inflation inertia and output persistence. It also explains why our assumption about working capital plays a useful role: other things equal, a decline in the interest rate lowers marginal cost.

Fifth, although investment adjustment costs and habit formation do not play a central role with respect to inflation inertia and output persistence, they do play a critical role in accounting for the dynamics of other variables. Sixth, the major role played by the working capital channel is to reduce the model's reliance on sticky prices. Specifically, if we estimate a version of the model that does not allow for this channel, the average duration of price contracts increases dramatically. Finally, we find that our model embodies strong internal propagation mechanisms. The impact of a monetary policy shock on aggregate activity continues to grow and persist even beyond the time when the typical contract in place at the time of the shock is reoptimized. In addition, the effects persist well beyond the effects of the shock on the interest rate and the growth rate of money.

We pursue a particular limited information econometric strategy to estimate and evaluate our model. To implement this strategy we first estimate the impulse response of eight key macroeconomic variables to a monetary policy shock using an identified vector autoregression (VAR). We then choose six model parameters to minimize the difference between the estimated impulse response functions and the analogous objects in our model.<sup>4</sup>

The remainder of this paper is organized as follows. In section 2 we briefly describe our estimates of how the U.S. economy responds to a monetary policy shock. Section 3 displays our economic model. In Section 4 we discuss our econometric methodology. Our empirical results are reported in Section 5 and analyzed in Section 6. Concluding comments

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and Fisher (1989), Ball and Romer (1990) and Romer (1996). For more recent quantitative discussions, see Chari, Kehoe and McGrattan (2000), Edge (2000), Fuhrer (2000), Kiley (1997), McCallum and Nelson (1998) and Sims (1998).

<sup>4</sup>Christiano, Eichenbaum and Evans (1998), Edge (2000) and Rotemberg and Woodford (1997) have also applied this strategy in the context of monetary policy shocks.

are contained in Section 7.

## 2. The Consequences of a Monetary Policy Shock

This section begins by describing how we estimate a monetary policy shock. We then report estimates of how major macroeconomic variables respond to a monetary policy shock. Finally, we report the fraction of the variance in these variables that is accounted for by monetary policy shocks.

The starting point of our analysis is the following characterization of monetary policy:

$$R_t = f(\Omega_t) + \varepsilon_t. \quad (2.1)$$

Here,  $R_t$  is the Federal Funds rate,  $f$  is a linear function,  $\Omega_t$  is an information set, and  $\varepsilon_t$  is the monetary policy shock. We assume that the Fed allows money growth to be whatever is necessary to guarantee that (2.1) holds. Our basic identifying assumption is that  $\varepsilon_t$  is orthogonal to the elements in  $\Omega_t$ . Below, we describe the variables in  $\Omega_t$  and elaborate on the interpretation of this orthogonality assumption.

We now discuss how we estimate the dynamic response of key macroeconomic variables to a monetary policy shock. Let  $Y_t$  denote the vector of variables included in the analysis. We partition  $Y_t$  as follows:

$$Y_t = [Y_{1t}, R_t, Y_{2t}]'$$

The vector  $Y_{1t}$  is composed of the variables whose time  $t$  elements are contained in  $\Omega_t$ , and are assumed not to respond contemporaneously to a monetary policy shock. The vector  $Y_{2t}$  consists of the time  $t$  values of all the other variables in  $\Omega_t$ . The variables in  $Y_{1t}$  are real GDP, real consumption, the GDP deflator, real investment, the real wage, and labor productivity. The variables in  $Y_{2t}$  are real profits and the growth rate of M2. All these variables, except money growth, have been logged. We measure the interest rate,  $R_t$ , using the Federal Funds rate. The data sources are in an appendix, available from the authors.

With one exception (the growth rate of money) all the variables in  $Y_t$  are include in levels. Altig, Christiano, Eichenbaum and Linde (2003) adopt an alternative specification of  $Y_t$ , in which cointegrating relationships among the variables are imposed. For example, the growth rate of  $GDP$  and the log difference between labor productivity and the real wage are included. The key properties of the impulse responses to a monetary policy shock are insensitive to this alternative specification.

The ordering of the variables in  $Y_t$  embodies two key identifying assumptions. First, the variables in  $Y_{1t}$  do not respond contemporaneously to a monetary policy shock. Second, the time  $t$  information set of the monetary authority consists of current and lagged values of the variables in  $Y_{1t}$  and only past values of the variables in  $Y_{2t}$ .

Our decision to include all variables, except for the growth rate of  $M2$  and real profits in  $Y_{1t}$ , reflects a long-standing view that macroeconomic variables do not respond instantaneously to policy shocks (see Friedman (1968)). We refer the reader to Christiano, Eichenbaum and Evans (1999) for a discussion of sensitivity of inference to alternative assumptions about the variables included in  $Y_{1t}$ . While our assumptions are certainly debatable, the analysis is internally consistent in the sense that we make the same assumptions in our economic model. To maintain consistency with the model, we place profits and the growth rate of money in  $Y_{2t}$ .

The VAR contains 4 lags of each variable and the sample period is 1965Q3 - 1995Q3.<sup>5</sup> Ignoring the constant term, the VAR can be written as follows:

$$Y_t = A_1 Y_{t-1} + \dots + A_4 Y_{t-4} + C \eta_t, \quad (2.2)$$

where  $C$  is a  $9 \times 9$  lower triangular matrix with diagonal terms equal to unity, and  $\eta_t$  is a 9-dimensional vector of zero-mean, serially uncorrelated shocks with diagonal variance-covariance matrix. Since there are six variables in  $Y_{1t}$ , the monetary policy shock,  $\varepsilon_t$ , is the 7<sup>th</sup> element of  $\eta_t$ . A positive shock to  $\varepsilon_t$  corresponds to a contractionary monetary policy shock. We estimate the parameters -  $A_i$ ,  $i = 1, \dots, 4$ ,  $C$ , and the variances of the elements of  $\eta_t$  - using standard least squares methods. Using these estimates, we compute the dynamic path of  $Y_t$  following a one-standard-deviation shock in  $\varepsilon_t$ , setting initial conditions to zero. This path, which corresponds to the coefficients in the impulse response functions of interest, is invariant to the ordering of the variables within  $Y_{1t}$  and within  $Y_{2t}$  (see Christiano, Eichenbaum and Evans (1999).)

The impulse response functions of all variables in  $Y_t$  are displayed in Figure 1. Lines marked ‘+’ correspond to the point estimates. The shaded areas indicate 95% confidence intervals about the point estimates.<sup>6</sup> The solid lines pertain to the properties of our structural model, which will be discussed in section 3. The results suggest that after an expansionary monetary policy shock there is a:

- hump-shaped response of output, consumption and investment, with the peak effect occurring after about 1.5 years and returning to their pre-shock levels after about three years,
- hump-shaped response in inflation, with a peak response after about 2 years,
- fall in the interest rate for roughly one year,
- rise in profits, real wages and labor productivity, and

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<sup>5</sup>This sample period is the same as in Christiano, Eichenbaum and Evans (1999).

<sup>6</sup>We use the method described in Sims and Zha (1999).

- an immediate rise in the growth rate of money.

Interestingly, these results are consistent with the claims in Friedman (1968). For example, Friedman argued that an exogenous increase in the money supply leads to a drop in the interest rate that lasts one to two years, and a rise in output and employment that lasts from two to five years. Finally, the robustness of the qualitative features of our findings to alternative identifying assumptions and sample sub-periods, as well as the use of monthly data, is discussed in Christiano, Eichenbaum and Evans (1999).

Our strategy for estimating the parameters of our model focuses on only a component of the fluctuations in the data, namely the portion that is due to a monetary policy shock. It is natural to ask how large that component is, since ultimately we are interested in a model that can account for the variation in the data. With this question in mind, the following table reports variance decompositions. In particular, it displays the percent of the variance of the  $k$ -step forecast error in the elements of  $Y_t$  due to monetary policy shocks, for  $k = 4, 8$  and  $20$ . Numbers in parentheses are the boundaries of the associated 95% confidence interval.<sup>7</sup> Notice that policy shocks account for only a small fraction of inflation. At the same time, with the exception of real wages, monetary policy shocks account for a non-trivial fraction of the variation in the real variables. This last inference should be treated with caution. The confidence intervals about the point estimates are rather large. Also, while the impulse response functions are robust to the various perturbations discussed in Christiano, Eichenbaum and Evans (1999) and Altig, Christiano, Eichenbaum and Linde (2003), the variance decompositions can be sensitive. For example, the analogous point estimates reported in Altig, Christiano, Eichenbaum and Linde (2003) are substantially smaller than those reported in Table 1.

### 3. The Model Economy

In this section we describe our model economy and display the problems solved by firms and households. In addition, we describe the behavior of financial intermediaries and the monetary and fiscal authorities. The only source of uncertainty in the model is a shock to monetary policy.

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<sup>7</sup>These confidence intervals are computed based on bootstrap simulations of the estimated VAR. In each artificial data set we computed the variance decompositions corresponding to the ones in Table 1. The lower and upper bounds of the confidence intervals correspond to the 2.5 and 97.5 percentiles of simulated variance decompositions.

### 3.1. Final Good Firms

At time  $t$ , a final consumption good,  $Y_t$ , is produced by a perfectly competitive, representative firm. The firm produces the final good by combining a continuum of intermediate goods, indexed by  $j \in [0, 1]$ , using the technology

$$Y_t = \left[ \int_0^1 Y_{jt}^{\frac{1}{\lambda_f}} dj \right]^{\lambda_f} \quad (3.1)$$

where  $1 \leq \lambda_f < \infty$  and  $Y_{jt}$  denotes the time  $t$  input of intermediate good  $j$ . The firm takes its output price,  $P_t$ , and its input prices,  $P_{jt}$ , as given and beyond its control. Profit maximization implies the Euler equation

$$\left( \frac{P_t}{P_{jt}} \right)^{\frac{\lambda_f}{\lambda_f - 1}} = \frac{Y_{jt}}{Y_t}. \quad (3.2)$$

Integrating (3.2) and imposing (3.1), we obtain the following relationship between the price of the final good and the price of the intermediate good:

$$P_t = \left[ \int_0^1 P_{jt}^{\frac{1}{1-\lambda_f}} dj \right]^{(1-\lambda_f)}. \quad (3.3)$$

### 3.2. Intermediate Good Firms

Intermediate good  $j \in (0, 1)$  is produced by a monopolist who uses the following technology:

$$Y_{jt} = \begin{cases} k_{jt}^\alpha L_{jt}^{1-\alpha} - \phi & \text{if } k_{jt}^\alpha L_{jt}^{1-\alpha} \geq \phi \\ 0 & \text{otherwise} \end{cases} \quad (3.4)$$

where  $0 < \alpha < 1$ . Here,  $L_{jt}$  and  $k_{jt}$  denote time  $t$  labor and capital services used to produce the  $j^{\text{th}}$  intermediate good. Also,  $\phi > 0$  denotes the fixed cost of production. We rule out entry and exit into the production of intermediate good  $j$ .

Intermediate firms rent capital and labor in perfectly competitive factor markets. Profits are distributed to households at the end of each time period. Let  $R_t^k$  and  $W_t$  denote the nominal rental rate on capital services and the wage rate, respectively. Workers must be paid in advance of production. As a result, the  $j^{\text{th}}$  firm must borrow its wage bill,  $W_t L_{jt}$ , from the financial intermediary at the beginning of the period. Repayment occurs at the end of time period  $t$  at the gross interest rate,  $R_t$ .

The firm's real marginal cost is

$$s_t = \frac{\partial S_t(Y)}{\partial Y}, \text{ where } S_t(Y) = \min_{k,l} \{ r_t^k k + w_t R_t l, Y \text{ given by (3.4)} \},$$

where  $r_t^k = R_t^k/P_t$  and  $w_t = W_t/P_t$ . Given our functional forms, we have

$$s_t = \left( \frac{1}{1-\alpha} \right)^{1-\alpha} \left( \frac{1}{\alpha} \right)^\alpha (r_t^k)^\alpha (w_t R_t)^{1-\alpha}. \quad (3.5)$$

Apart from fixed costs, the firm's time  $t$  profits are:

$$\left[ \frac{P_{jt}}{P_t} - s_t \right] P_t Y_{jt},$$

where  $P_{jt}$  is firm  $j$ 's price.

We assume that firms set prices according to a variant of the mechanism spelled out in Calvo (1983). This model has been widely used to characterize price-setting frictions. A useful feature of the model is that it can be solved without explicitly tracking the distribution of prices across firms. In each period, a firm faces a constant probability,  $1 - \xi_p$ , of being able to reoptimize its nominal price. The ability to reoptimize its price is independent across firms and time. If a firm can reoptimize its price, it does so before the realization of the time  $t$  growth rate of money. Firms that cannot reoptimize their price simply index to lagged inflation:

$$P_{jt} = \pi_{t-1} P_{j,t-1}. \quad (3.6)$$

Here,  $\pi_t = P_t/P_{t-1}$ . We refer to this price-setting rule as lagged inflation indexation.

Let  $\tilde{P}_t$  denote the value of  $P_{jt}$  set by a firm that can reoptimize at time  $t$ . Our notation does not allow  $\tilde{P}_t$  to depend on  $j$ . We do this in anticipation of the well known result that, in models like ours, all firms who can reoptimize their price at time  $t$  choose the same price (see Woodford, 1996 and Yun, 1996). The firm chooses  $\tilde{P}_t$  to maximize:

$$E_{t-1} \sum_{l=0}^{\infty} (\beta \xi_p)^l v_{t+l} \left[ \tilde{P}_t X_{tl} - s_{t+l} P_{t+l} \right] Y_{j,t+l}, \quad (3.7)$$

subject to (3.2), (3.5) and

$$X_{tl} = \begin{cases} \pi_t \times \pi_{t+1} \times \cdots \times \pi_{t+l-1} & \text{for } l \geq 1 \\ 1 & l = 0 \end{cases}. \quad (3.8)$$

In (3.7),  $v_t$  is the marginal value of a dollar to the household, which is treated as exogenous by the firm. Later, we show that the value of a dollar, in utility terms, is constant across households. Also,  $E_{t-1}$  denotes the expectations operator conditioned on lagged growth rates of money,  $\mu_{t-l}$ ,  $l \geq 1$ . This specification of the information set captures our assumption that the firm chooses  $\tilde{P}_t$  before the realization of the time  $t$  growth rate of money. To understand (3.7), note that  $\tilde{P}_t$  influences firm  $j$ 's profits only as long as it cannot reoptimize its price. The probability that this happens for  $l$  periods is  $(\xi_p)^l$ , in which case  $P_{j,t+l} = \tilde{P}_t X_{tl}$ . The presence of  $(\xi_p)^l$  in (3.7) has the effect of isolating future realizations of idiosyncratic uncertainty in which  $\tilde{P}_t$  continues to affect the firm's profits.



### 3.3. Households

There is a continuum of households, indexed by  $j \in (0, 1)$ . The  $j^{\text{th}}$  household makes a sequence of decisions during each period. First, it makes its consumption decision, its capital accumulation decision, and it decides how many units of capital services to supply. Second, it purchases securities whose payoffs are contingent upon whether it can reoptimize its wage decision. Third, it sets its wage rate after finding out whether it can reoptimize or not. Fourth, it receives a lump-sum transfer from the monetary authority. Finally, it decides how much of its financial assets to hold in the form of deposits with a financial intermediary and how much to hold in the form of cash.

Since the uncertainty faced by the household over whether it can reoptimize its wage is idiosyncratic in nature, households work different amounts and earn different wage rates. So, in principle, they are also heterogeneous with respect to consumption and asset holdings. A straightforward extension of arguments in Erceg, Henderson and Levin (2000) and Woodford (1996) establish that the existence of state contingent securities ensures that, in equilibrium, households are homogeneous with respect to consumption and asset holdings. Reflecting this result, our notation assumes that households are homogeneous with respect to consumption and asset holdings but heterogeneous with respect to the wage rate that they earn and hours worked.

The preferences of the  $j^{\text{th}}$  household are given by:

$$E_{t-1}^j \sum_{l=0}^{\infty} \beta^{l-t} [u(c_{t+l} - bc_{t+l-1}) - z(h_{j,t+l}) + v(q_{t+l})]. \quad (3.9)$$

Here,  $E_{t-1}^j$  is the expectation operator, conditional on aggregate and household  $j$  idiosyncratic information up to, and including, time  $t - 1$ ;  $c_t$  denotes time  $t$  consumption;  $h_{jt}$  denotes time  $t$  hours worked;  $q_t \equiv Q_t/P_t$  denotes real cash balances;  $Q_t$  denotes nominal cash balances. When  $b > 0$ , (3.9) allows for habit formation in consumption preferences.

The household's asset evolution equation is given by:

$$\begin{aligned} M_{t+1} = & R_t [M_t - Q_t + (\mu_t - 1)M_t^a] + A_{j,t} + Q_t + W_{j,t}h_{j,t} \\ & + R_t^k u_t \bar{k}_t + D_t - P_t (i_t + c_t + a(u_t)\bar{k}_t). \end{aligned} \quad (3.10)$$

Here,  $M_t$  is the household's beginning of period  $t$  stock of money and  $W_{j,t}h_{j,t}$  is time  $t$  labor income. In addition,  $\bar{k}_t$ ,  $D_t$  and  $A_{j,t}$  denote, respectively, the physical stock of capital, firm profits and the net cash inflow from participating in state-contingent securities at time  $t$ . The variable  $\mu_t$  represents the gross growth rate of the economy-wide per capita stock of money,  $M_t^a$ . The quantity  $(\mu_t - 1)M_t^a$  is a lump-sum payment made to households by the monetary authority. The quantity  $M_t - P_t q_t + (\mu_t - 1)M_t^a$ , is deposited by the household with a financial intermediary where it earns the gross nominal rate of interest,  $R_t$ .

The remaining terms in (3.10), aside from  $P_t c_t$ , pertain to the stock of installed capital, which we assume is owned by the household. The household's stock of physical capital,  $\bar{k}_t$ , evolves according to:

$$\bar{k}_{t+1} = (1 - \delta)\bar{k}_t + F(i_t, i_{t-1}). \quad (3.11)$$

Here,  $\delta$  denotes the physical rate of depreciation and  $i_t$  denotes time  $t$  purchases of investment goods. The function,  $F$ , summarizes the technology that transforms current and past investment into installed capital for use in the following period. We discuss the properties of  $F$  below.

Capital services,  $k_t$ , are related to the physical stock of capital by

$$k_t = u_t \bar{k}_t.$$

Here,  $u_t$  denotes the utilization rate of capital, which we assume is set by the household.<sup>8</sup> In (3.10),  $R_t^k u_t \bar{k}_t$  represents the household's earnings from supplying capital services. The increasing, convex function  $a(u_t) \bar{k}_t$  denotes the cost, in units of consumption goods, of setting the utilization rate to  $u_t$ .

### 3.4. The Wage Decision

As in Erceg, Henderson and Levin (2000), we assume that the household is a monopoly supplier of a differentiated labor service,  $h_{jt}$ . It sells this service to a representative, competitive firm that transforms it into an aggregate labor input,  $L_t$ , using the following technology:

$$L_t = \left[ \int_0^1 h_{jt}^{\frac{1}{\lambda_w}} dj \right]^{\lambda_w}.$$

The demand curve for  $h_{jt}$  is given by:

$$h_{jt} = \left( \frac{W_t}{W_{jt}} \right)^{\frac{\lambda_w}{\lambda_w - 1}} L_t, \quad 1 \leq \lambda_w < \infty. \quad (3.12)$$

Here,  $W_t$  is the aggregate wage rate, i.e., the price of  $L_t$ . It is straightforward to show that  $W_t$  is related to  $W_{jt}$  via the relationship:

$$W_t = \left[ \int_0^1 (W_{jt})^{\frac{1}{1-\lambda_w}} dj \right]^{1-\lambda_w}. \quad (3.13)$$

The household takes  $L_t$  and  $W_t$  as given.

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<sup>8</sup>Our assumption that households make the capital accumulation and utilization decisions is a matter of convenience. At the cost of a more complicated notation, we could work with an alternative decentralization scheme in which firms make these decisions.

Households set their wage rate according to a variant of the mechanism used to model price setting by firms. In each period, a household faces a constant probability,  $1 - \xi_w$ , of being able to reoptimize its nominal wage. The ability to reoptimize is independent across households and time. If a household cannot reoptimize its wage at time  $t$ , it sets  $W_{jt}$  according to:

$$W_{j,t} = \pi_{t-1} W_{j,t-1}. \quad (3.14)$$

### 3.5. Monetary and Fiscal Policy

We assume that monetary policy is given by:

$$\mu_t = \mu + \theta_0 \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \dots \quad (3.15)$$

Here,  $\mu$  denotes the mean growth rate of money and  $\theta_j$  is the response of  $E_t \mu_{t+j}$  to a time  $t$  monetary policy shock. We assume that the government has access to lump sum taxes and pursues a Ricardian fiscal policy. Under this type of policy, the details of tax policy have no impact on inflation and other aggregate economic variables. As a result, we need not specify the details of fiscal policy.<sup>9</sup>

### 3.6. Loan Market Clearing, Final Goods Clearing and Equilibrium

Financial intermediaries receive  $M_t - Q_t$  from households and a transfer,  $(\mu_t - 1) M_t$  from the monetary authority. Our notation here reflects the equilibrium condition,  $M_t^a = M_t$ . Financial intermediaries lend all of their money to intermediate good firms, which use the funds to pay for  $L_t$ . Loan market clearing requires

$$W_t L_t = \mu_t M_t - Q_t. \quad (3.16)$$

The aggregate resource constraint is

$$c_t + i_t + a(u_t) \leq Y_t.$$

We adopt a standard sequence-of-markets equilibrium concept. In the appendix we discuss our computational strategy for approximating that equilibrium. This strategy involves taking a linear approximation about the non-stochastic steady state of the economy and using the solution method discussed in Christiano (2003). For details, see the previous version of this paper, Christiano, Eichenbaum and Evans (2001). In principle, the non-negativity constraint on intermediate good output in (3.4) is a problem for this approximation. It turns out that the constraint is not binding for the experiments that we consider and so we ignore it. Finally, it is worth noting that since profits are stochastic, the fact that they are zero, on average,

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<sup>9</sup>See Sims (1994) or Woodford (1994) for a further discussion.

implies that they are often negative. As a consequence, our assumption that firms cannot exit is binding. Allowing for firm entry and exit dynamics would considerably complicate our analysis.

### 3.7. Functional Form Assumptions

We assume that the functions characterizing utility are given by:

$$\begin{aligned} u(\cdot) &= \log(\cdot) \\ z(\cdot) &= \psi_0 (\cdot)^2 \\ v(\cdot) &= \psi_q \frac{(\cdot)^{1-\sigma_q}}{1-\sigma_q} \end{aligned} \quad (3.17)$$

In addition, investment adjustment costs are given by:

$$F(i_t, i_{t-1}) = (1 - S\left(\frac{i_t}{i_{t-1}}\right))i_t. \quad (3.18)$$

We restrict the function  $S$  to satisfy the following properties:  $S(1) = S'(1) = 0$ , and  $\varkappa \equiv S''(1) > 0$ . It is easy to verify that the steady state of the model does not depend on the adjustment cost parameter,  $\varkappa$ . Of course, the dynamics of the model are influenced by  $\varkappa$ . Given our solution procedure, no other features of the  $S$  function need to be specified for our analysis.

We impose two restrictions on the capital utilization function,  $a(u_t)$ . First, we require that  $u_t = 1$  in steady state. Second, we assume  $a(1) = 0$ . Under our assumptions, the steady state of the model is independent of  $\sigma_a = a''(1)/a'(1)$ . The dynamics do depend on  $\sigma_a$ . Given our solution procedure, we do not need to specify any other features of the function  $a$ .

## 4. Econometric Methodology

In this section we discuss our methodology for estimating and evaluating our model. We partition the model parameters into three groups. The first group is composed of  $\beta$ ,  $\phi$ ,  $\alpha$ ,  $\delta$ ,  $\psi_0$ ,  $\psi_q$ ,  $\lambda_w$  and  $\mu$ . We set  $\beta = 1.03^{-0.25}$ , which implies a steady state annualized real interest rate of 3 percent. We set  $\alpha = 0.36$ , which corresponds to a steady state share of capital income equal to roughly 36 percent. We set  $\delta = 0.025$ , which implies an annual rate of depreciation on capital equal to 10 percent. This value of  $\delta$  is roughly equal to the estimate reported in Christiano and Eichenbaum (1992). The parameter,  $\phi$ , is set to guarantee that profits are zero in steady state. This value is consistent with Basu and Fernald (1994), Hall (1988), and Rotemberg and Woodford (1995), who argue that economic profits are close to zero on average. Although there are well known problems with the measurement of profits, we think that zero profits is a reasonable benchmark.

The parameter  $\psi_0$  was chosen to imply a steady state value of  $L$  equal to unity. Similarly, the parameter  $\psi_q$  was set to ensure  $Q/M = 0.44$  in steady state. This value is equal to the ratio of  $M1$  to  $M2$  at the beginning of our sample period. The rationale for using this ratio is that  $M1$  is a measure of money used for transactions, while  $M2$  is a broader monetary aggregate. We re-estimated the model calibrating  $\psi_q$  to different steady state values of  $Q/M$ . The primary impact on our parameter estimates was to change the estimate of  $\sigma_q$ . The impulse response functions were relatively unaffected by different values of  $\sigma_q$ . The parameter  $\mu$  was set to 1.017, which equals the postwar quarterly average gross growth rate of  $M2$ . At our assumed parameter values, the steady state velocity of money is given by:

$$\frac{PY}{M} = (\mu - q)(1 - \alpha) \frac{\beta}{\mu} = 0.36.$$

This ratio is slightly below the average value, 0.44, of  $M2$  velocity in our sample.

We set the parameter,  $\lambda_w$ , to 1.05. In numerical simulations we found that our results are robust to perturbations in this parameter.<sup>10</sup> Our specification of  $z(\cdot)$  implies a Frisch labor supply elasticity equal to unity. This elasticity is low by comparison with the values assumed in the real business cycle literature.<sup>11</sup> However, it is well within the range of point estimates reported in the labor literature (see Rotemberg and Woodford, 1999).

We characterize monetary policy by (3.15), where the  $\theta_i$ 's are the impulse responses implied by our estimated VAR:

$$\mu_t = \tau(I - A_1L - \dots - A_4L^4)^{-1}c\varepsilon_t.$$

Here,  $c$  is the last column of  $C$ , and  $A_1, \dots, A_4, C$  are the estimated parameters of our VAR. In addition,  $\tau$  is a row vector with zeros everywhere, except unity in the last element. The moving average parameter,  $\theta_i$ , is the coefficient on  $L^i$  in the expansion of the polynomial to the right of the equality in the previous expression, for  $i = 0, 1, \dots$ . To incorporate this representation of monetary policy into the model, we use the procedure described by King and Watson (1996). Christiano, Eichenbaum and Evans (1998) show that this representation is not statistically significantly different from the one generated by a first order autoregression with coefficient roughly equal to 0.5.

The third group of model parameters is:

$$\gamma \equiv (\lambda_f, \xi_w, \xi_p, \sigma_q, \varkappa, b, \sigma_a).$$

These parameters were estimated by minimizing a measure of the distance between the model and empirical impulse response functions. Let  $\Psi(\gamma)$  denote the mapping from  $\gamma$  to the

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<sup>10</sup>Holding fixed the other parameter values at their benchmark values reported below, we found that the impulse response functions implied by the model are insensitive to different values of  $\lambda_w$ .

<sup>11</sup>For example, the Frisch elasticity implicit in the 'divisible labor' model in Christiano and Eichenbaum (1992) is roughly 2.5 percent.

model impulse response functions, and let  $\hat{\Psi}$  denote the corresponding empirical estimates. We include the first 25 elements of each response function, excluding those that are zero by assumption. Our estimator of  $\gamma$  is the solution to:

$$J = \min_{\gamma} (\hat{\Psi} - \Psi(\gamma))' V^{-1} (\hat{\Psi} - \Psi(\gamma)). \quad (4.1)$$

Here,  $V$  is a diagonal matrix with the sample variances of the  $\hat{\Psi}$ 's along the diagonal. These variances are the basis for the confidence intervals reported in Figure 1. So, with this choice of  $V$ ,  $\gamma$  is effectively chosen so that  $\Psi(\gamma)$  lies as much as possible inside these confidence intervals.

## 5. Empirical Results

In this section we discuss the estimated parameter values. In addition, we assess the ability of the estimated model to account for the impulse response functions discussed in section 2.

### 5.1. Parameter Estimates

The row labeled ‘benchmark’ in Table 2 summarizes our point estimates of the parameters in the vector  $\gamma$ . With the exception of  $\sigma_a$ , standard errors are reported in parentheses.<sup>12</sup> We do not report a standard error for  $\sigma_a$  because our estimation procedure drives that parameter towards zero, at which point the algorithm breaks down. As a result, we simply set  $\sigma_a = 0.01$  and optimize the estimation criterion over the remaining elements in  $\gamma$ . To interpret this low value of  $\sigma_a$ , consider the Euler equation associated with the household’s capital utilization decision:

$$E_{t-1} \psi_t [r_t^k - a'(u_t)] = 0. \quad (5.1)$$

According to this expression, the expected marginal benefit of raising utilization must equal the associated expected marginal cost. After linearizing this expression about nonstochastic steady state, we obtain:<sup>13</sup>

$$E_{t-1} \left[ \frac{1}{\sigma_a} \hat{r}_t^k - \hat{u}_t \right] = 0. \quad (5.2)$$

>From this expression we can see that  $1/\sigma_a$  is the elasticity of capital utilization with respect to the rental rate of capital. So, a small value of  $\sigma_a$  corresponds to a large elasticity. Below, we provide evidence on the empirical plausibility of our model’s implications for capacity utilization.

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<sup>12</sup>Standard errors were computed using the asymptotic delta function method applied to the first order condition associated with (4.1).

<sup>13</sup>Here, we have used the fact that we impose  $r^k = a'$ , where  $r^k$  denotes the rental rate on capital in steady state. This rental rate is determined solely by  $\beta$  and  $\delta$ .

We now discuss the remaining parameters in Table 2. Our point estimate of  $\xi_w$  implies that wage contracts last, on average, 2.8 quarters. Our point estimate of  $\xi_p$  implies that price contracts last, on average, 2.5 quarters. While the standard errors on  $\xi_p$  and  $\xi_w$  are small, later we will see that, in fact, sticky wages play a more important role in model fit than sticky prices.

To interpret the point estimate of  $\sigma_q$ , it is useful to note that the household's first order condition for cash balances,  $Q_t$ , is:

$$v'(q_t) + \psi_t = \psi_t R_t, \quad (5.3)$$

where  $q_t = Q_t/P_t$ . Here,  $\psi_t$  is the marginal utility of  $P_t$  units of currency. That is,  $\psi_t = v_t P_t$ , where  $v_t$  is the Lagrange multiplier on the household's budget constraint, (3.10). According to (5.3) the marginal utility of a dollar allocated to cash balances must equal the marginal utility of a dollar allocated to the financial intermediary. To interpret  $\psi_t$ , note that the household's optimization problem implies:

$$E_{t-1} u_{c,t} = E_{t-1} \psi_t. \quad (5.4)$$

Here,  $u_{c,t}$  is the realized value of the marginal utility of consumption at date  $t$ :

$$u_{c,t} = \frac{\partial u(c_t - bc_{t-1})}{\partial c_t} - bE_t \frac{\partial u(c_{t+1} - bc_t)}{\partial c_t}. \quad (5.5)$$

According to (5.4) and (5.5), in the absence of uncertainty  $\psi_t$  would be the marginal utility of consumption. So, (5.3) relates real cash balances to the nominal interest rate as well as to consumption flows.

Log-linearizing (5.3) and imposing (3.17), we obtain:

$$\hat{q}_t = -\frac{1}{\sigma_q} \left[ \frac{R}{R-1} \hat{R}_t + \hat{\psi}_t \right]. \quad (5.6)$$

Here and throughout the paper, a hat over a variable denotes the percent deviation from its steady state value. Equation (5.6) implies that, holding  $\hat{\psi}_t$  constant, the interest semi-elasticity of money demand is

$$-\frac{\partial \log q_t}{\partial R_t} = \frac{1}{4\sigma_q (R-1)}.$$

This expression takes into account that the time period of the model is quarterly and the elasticity is measured with respect to the annualized rate of interest. Our parameter estimates imply that this elasticity is 0.96, i.e., a one percentage point rise in the annualized rate of interest leads to a 0.96 percent reduction in real balances. This elasticity is considerably smaller than standard estimates reported for static money demand equations. For example,

the analogous number in Lucas (1988) is 8.0. We found that our estimate of  $\sigma_q$  is driven primarily by the model's attempt to replicate the initial responses of the interest rate to a monetary policy shock. Consequently, we interpret our interest semi-elasticity as pertaining to the short-run response of money demand. This elasticity is often estimated to be quite small (see Christiano, Eichenbaum and Evans, 1999).

To interpret the point estimate of  $\varkappa$ , it is useful to consider the household's first order condition for investment:

$$E_{t-1}\psi_t = E_{t-1} [\psi_t P_{k',t} F_{1,t} + \beta \psi_{t+1} P_{k',t+1} F_{2,t+1}]. \quad (5.7)$$

Here,  $F_{j,t}$  is the partial derivative of the investment adjustment cost function,  $F(i_t, i_{t-1})$ , defined in (3.18), with respect to its  $j^{\text{th}}$  argument,  $j = 1, 2$ . Also,  $P_{k',t}$  is the shadow value, in consumption units, of a unit of  $\bar{k}_{t+1}$  as of the time that the household makes its period  $t$  investment and capital utilization decision. The variable  $P_{k',t}$  is what the price of installed capital would be if there were a market for  $\bar{k}_{t+1}$  at the beginning of period  $t$ .

The left side of (5.7) is the marginal cost, in utility terms, of a unit of investment goods. The right side of (5.7) is the associated marginal benefit. To understand the benefit, note that an extra unit of investment goods produces  $F_{1,t}$  extra units of  $\bar{k}_{t+1}$ . The value of these goods, in utility terms, is  $P_{k',t} F_{1,t} E_{t-1} \psi_t$ . An increase in  $i_t$  also affects the quantity of installed capital produced in the next period by  $F_{2,t+1}$ . The last term in (5.7) measures the utility value of these additional capital goods.

Log-linearizing (5.7) about steady state, we obtain:

$$\hat{P}_{k',t} = \varkappa E_{t-1} \{ \hat{i}_t - \hat{i}_{t-1} - \beta [\hat{i}_{t+1} - \hat{i}_t] \},$$

so that:

$$\hat{i}_t = \hat{i}_{t-1} + \frac{1}{\varkappa} \sum_{j=0}^{\infty} \beta^j E_{t-1} \hat{P}_{k',t+j}.$$

According to this expression,  $1/\varkappa$  is the elasticity of investment with respect to a one percent temporary increase in the current price of installed capital. Our point estimate implies that this elasticity is equal to 0.40. A more persistent change in the price of capital induces a larger percentage change in investment because adjustment costs induce agents to be forward looking. For example, a permanent one-percent change in the price of capital induces a  $1/[\varkappa(1-\beta)] = 55$  percent change in investment.

The literature on Tobin's  $q$  also reports empirical estimates of investment elasticities. It is difficult to compare these estimates with ours because the Tobin's  $q$  literature is based on a first order specification of adjustment costs. That is, one in which adjustment costs depend only on the current level of investment, i.e., the first derivative of the capital stock. Given this specification, only the current value of  $\hat{P}_{k',t}$  enters into  $\hat{i}_t$ . In contrast, we assume



a second order specification of adjustment costs, i.e., one in which the costs depend on the second derivative of the capital stock. From our perspective, the elasticities reported in the Tobin’s  $q$  literature represent a combination of  $\varkappa$  and the degree of persistence in  $\hat{P}_{k',t}$ . Later, we document why it is important to allow for second order rather than first order adjustment costs in our analysis.

Our point estimate of the habit parameter  $b$  is 0.65. This value is close to the point estimate of 0.7 reported in Boldrin, Christiano and Fisher (2001). Those authors argue that the ability of standard general equilibrium models to account for the equity premium and other asset market statistics is considerably enhanced by the presence of habit formation in preferences. Below we discuss the role that habit formation plays in the fit of our model. Finally, the estimated value of  $\lambda_f$ , 1.20, is close to the values used in the literature (see, e.g., Rotemberg and Woodford, 1995).

## 5.2. Properties of the Estimated Model

The impulse response functions of the estimated model are represented by the solid lines in Figure 1. A number of results are worth emphasizing here. First, the model does well at accounting for the dynamic response of the US economy to a monetary policy shock. Most of the model responses lie within the two-standard deviation confidence interval computed from the data. Second, the model succeeds in accounting for the inertial response of inflation. Indeed, there is no noticeable rise in inflation until roughly three years after the policy shock. This result is particularly notable, since firms and households in our model change prices and wages on average only once every 2.5 quarters.

Third, the model generates a very persistent response in output. The peak effect occurs roughly one year after the shock. The output response is positive for 9 quarters, during which the cumulative output response is 3.14 percent. Over 78 percent of this cumulative rise occurs after the typical wage and price contract in effect at the time of the shock has been reoptimized. The part of the output response that occurs beyond the length of the typical contract reflects, to a first approximation, the staggering of wages and prices. A different way to quantify the effect of staggering is to consider a statistic that is analogous to the one proposed by Chari, Kehoe and McGrattan (2001) (CKM). This statistic is constructed as follows. First, calculate the amount of time until the output expansion caused by a positive policy shock goes to zero. Then, calculate the ratio of this number to the number of periods in a typical contract. Following CKM, we call this ratio the contract multiplier. In our benchmark model, this statistic is 3.7.<sup>14</sup> So, both of our statistics indicate that staggering

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<sup>14</sup>Our measure of the contract multiplier differs from the one in Chari, Kehoe and McGrattan (2001). Theirs is based on a measure of the half life of a shock, namely the number of periods when the response of a variable is one-half its response in the initial period of the shock. We cannot apply this half-life measure

of contracts contributes substantially to the propagation of monetary policy shocks. As we show below, this property depends critically on the real frictions embedded in our benchmark model.

Figure 2 provides a different way of illustrating the persistent output response and the inertial inflation response to a monetary policy shock. There, we display the response of the price level, the money stock and output in the model. Each is expressed as a percentage of its level along the unshocked growth path. Notice how the money stock rises to its peak level by the third quarter after the shock and is roughly back to where it started by the middle of the third year. Despite the prolonged rise in the money stock, there is essentially no change in the price level. At the same time, there is a prolonged boom in output that lasts even after the boom in the money supply is over. The peak in output is almost twice as big as the peak in the money supply, with the former occurring one-half year after the latter.

Returning to Figure 1, notice that the model is able to account for the dynamic response of the interest rate to a monetary policy shock. Consistent with the data, an expansionary monetary policy shock induces a sharp decline in the interest rate which then returns to its pre-shock level within a year. It is interesting that a policy shock induces a more persistent effect on output than on the interest rate. Indeed, the peak effect on output occurs one quarter after the policy variable has returned to its steady state value. So, regardless of whether we measure policy by the money stock or the interest rate, the effects of a policy shock on aggregate variables persist beyond the effects on the policy variable itself. This property reflects the strong internal propagation mechanisms in the model.

Next note that, as in the data, the real wage rises by a small positive amount in response to the policy shock. In addition, consumption and investment exhibit persistent, hump-shaped rises that are consistent with our VAR-based estimates. Figure 1 also shows that both productivity and profits rise in response to a monetary policy shock. While the extent in the rise is not as large as our VAR-based estimates, it is interesting that there is any rise at all. We discuss this further below.

We conclude by assessing the model's implications for capital utilization. In practice, there are various competing measures of capital utilization.<sup>15</sup> We considered three alternatives and compared their response to a policy shock with the implications of our model. The first two are the Federal Reserve Board's time series on capacity utilization and electricity consumption in the industrial production sector. The third measure of capital utilization, developed in Basu, Fernald and Shapiro (2001), pertains to the economy as a whole. The first series measures the intensity with which all factors of production are used in the indus-

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to output because the initial response of output to a policy shock is zero.

<sup>15</sup>For surveys of different approaches to measuring capacity utilization, see Christiano (1984) and Shapiro (1989).

trial production sector (see Shapiro, 1989). The second series is a useful measure of capital utilization under the assumption that capital services and electricity are used in fixed proportions (see Burnside, Eichenbaum and Rebelo, 1995). The third series measures capital utilization under the assumption that capital services are tied to the workweek of capital, as measured by average hours worked.

Figure 3 displays the dynamic response of capacity utilization in our estimated model, along with the corresponding empirical estimates, based on our three measures of capacity utilization. These estimates were obtained by constructing three different VARs. Each augments our benchmark VAR with an alternative measure of capacity utilization. Consistent with our model, we assume that capacity utilization does not respond contemporaneously to a monetary policy shock. A number of results are worth noting. First, consistent with the model, all three capacity utilization measures rise in a hump-shaped manner after an expansionary policy shock. Second, the model does very well at matching the response of the Basu-Fernald-Shapiro measure. Third, the model does somewhat less well at matching the quantitative responses of the other two capacity utilization measures. Its responses lie below the empirical 95 percent confidence interval at several lags.

To understand the significance of these results, recall from above that our estimation procedure drives  $\sigma_a$  to its lower bound in an attempt to increase the elasticity of capital services supply. Below, we explain why the resulting strong response of capital utilization to a monetary policy shock plays an significant role in the model's performance. So, it is important to determine whether the model relies too heavily on a counterfactually strong response of utilization. The results in Figure 3 indicate that, if anything, the model understates the response of utilization.

## 6. Understanding the Key Features of the Model

This section is organized in two parts. First, we provide intuition for how the different features of our model contribute to our results. Second, we illustrate this intuition through a series of quantitative exercises.

### 6.1. Qualitative Considerations

To describe the intuition for the monetary transmission mechanism in our model, it is useful to proceed in two steps. First, we provide intuition for why consumption, investment, output, employment, productivity, profits and capital utilization rise, while the interest rate falls. This discussion takes as given the inertial behavior of prices and wages. In our second step, we provide intuition for why prices and wages respond slowly to a monetary policy shock. Of course, in general equilibrium all effects occur simultaneously. Still, to highlight the different

frictions in our model, we find it useful to proceed in this sequential manner.

To understand the contemporaneous effect of a policy shock, it is useful to focus on the money market clearing condition, (3.16), and the household's first order condition, (5.3), for cash balances,  $Q_t$ . Given our assumptions, the full amount of a policy shock-induced money injection must be absorbed by household cash holdings,  $Q_t$ . Firms do not wish to absorb any part of a cash infusion because  $W_t L_t$  does not respond to a policy shock. The wage rate,  $W_t$ , is predetermined because the  $W_{jt}$ 's are predetermined by assumption. Employment,  $L_t$  is predetermined because of our assumption that consumption, investment and capital utilization are predetermined. It follows from (3.16) that a period  $t$  money injection must be accompanied by an equal increase in  $Q_t$ .

To understand the impact of the rise in  $Q_t$  on  $R_t$ , suppose for the moment that  $\psi_t$  is constant. Since  $P_t$  is predetermined, the rise in  $Q_t$  corresponds to a rise in real balances. According to (5.3),  $R_t$  must fall to induce households to increase real cash balances,  $Q_t/P_t$ . In practice, we found that  $\psi_t$  falls, but by only a relatively small amount. Finally, since  $R_t$  falls and the firm's wage bill and revenues are unaffected by the shock, profits must rise.

We now turn to the dynamic effects of a monetary shock on  $R_t$ ,  $C_t$ ,  $I_t$ ,  $Y_t$ ,  $L_t$ ,  $u_t$ , productivity and profits. The persistent drop in  $R_t$  reflects the slow adjustment of  $Q_t/P_t$  relative to its high value in the period of the shock. In part, this sluggish adjustment is due to the inertia in  $P_t$ . But it is also the case that households are slow to reduce their cash holdings from its high level in the impact period of the shock. This slow response in  $Q_t$  reflects money market clearing, (3.16), the inertial behavior of  $W_t$  and the slow rise of  $L_t$  after a policy shock. The slow expansion in hours worked occurs because household demand for goods rises slowly, in a hump-shaped manner, reflecting habit formation in preferences and second order adjustment costs in investment. These considerations provide intuition for the persistent rise in  $Q_t/P_t$  after an expansionary monetary policy shock. It then follows from (5.3) that  $R_t$  must be low for a prolonged period of time.<sup>16</sup> The hump-shaped response in  $C_t$  and  $I_t$  implies that output, employment and capital utilization rates also rise in a hump-shaped manner. Finally, the rise in productivity reflects the effects of capital utilization, as well as the presence of the fixed cost in the production function.

Note that our mechanism for generating a persistent liquidity effect contrasts with the approach in the recent literature, which emphasizes frictions in the adjustment of financial portfolios (see, for example, Christiano and Eichenbaum, 1992a, and Alvarez, Atkeson and Kehoe, 2002). These types of frictions are absent from our model, where the liquidity effect is an indirect consequence of non-financial market frictions.

The previous intuition takes as given the inertial responses in prices and wages to a

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<sup>16</sup>As in the impact period, the movements in  $\psi_t$  are sufficiently small that they can be ignored for purposes of intuition.

monetary policy shock. We now discuss the features of our model which allow it to generate inertial price and wage behavior.

To understand price inertia, recall that the firm chooses  $\tilde{P}_t$  to maximize (3.7) subject to (3.2), (3.5) and (3.8). The log-linearized first order condition associated with this problem is:

$$\widehat{p}_t = E_{t-1} \left[ \hat{s}_t + \sum_{l=1}^{\infty} (\beta \xi_p)^l (\hat{s}_{t+l} - \hat{s}_{t+l-1}) + \sum_{l=1}^{\infty} (\beta \xi_p)^l (\hat{\pi}_{t+l} - \hat{\pi}_{t+l-1}) \right]. \quad (6.1)$$

Here,  $\tilde{p}_t = \tilde{P}_t/P_t$  and recall that a hat over a variable indicates the percent deviation from its steady state value. That is,  $\widehat{p}_t = (\tilde{p}_t - \tilde{p})/\tilde{p}$ . A variable without a hat or a subscript indicates its nonstochastic steady state value. Several features of (6.1) are worth noting. First, if inflation and real marginal cost are expected to remain at their time  $t$  levels, then the firm sets  $\widehat{p}_t = E_{t-1} \hat{s}_t$ . Second, suppose the firm expects real marginal costs to be higher in the future than at time  $t$ . Anticipating those future marginal costs, the firm sets  $\widehat{p}_t$  higher than  $E_{t-1} \hat{s}_t$ . It does so because it understands that it may not be able to raise its price when those higher marginal costs materialize. We refer to this type of forward looking behavior as ‘front loading’. Third, suppose firms expect inflation in the future to rise above  $E_{t-1} \hat{\pi}_t$ . The one period lag in the dynamic price-setting rule, (3.6), implies that the firm’s relative price would fall.

It follows from well-known results in the literature that (3.3) can be expressed as:

$$P_t = \left[ (1 - \xi_p) \left( \tilde{P}_t \right)^{\frac{1}{1-\lambda_f}} + \xi_p (\pi_{t-1} P_{t-1})^{\frac{1}{1-\lambda_f}} \right]^{1-\lambda_f}. \quad (6.2)$$

Dividing by  $P_t$ , linearizing and rearranging, we obtain:

$$\widehat{p}_t = \frac{\xi_p}{1 - \xi_p} (\hat{\pi}_t - \hat{\pi}_{t-1}). \quad (6.3)$$

Relations (6.1) and (6.3) imply:

$$\hat{\pi}_t = \frac{1}{1 + \beta} \hat{\pi}_{t-1} + \frac{\beta}{1 + \beta} E_{t-1} \hat{\pi}_{t+1} + \frac{(1 - \beta \xi_p)(1 - \xi_p)}{(1 + \beta) \xi_p} E_{t-1} \hat{s}_t. \quad (6.4)$$

Imposing  $E_{t-1} \beta^j (\hat{\pi}_{t+j} - \hat{\pi}_{t+j-1}) \rightarrow 0$ , (6.4) is equivalent to:

$$\hat{\pi}_t - \hat{\pi}_{t-1} = \frac{(1 - \beta \xi_p)(1 - \xi_p)}{\xi_p} E_{t-1} \sum_{j=0}^{\infty} \beta^j \hat{s}_{t+j}. \quad (6.5)$$

Four features of (6.5) are worth noting. First, consistent with our timing assumptions, (6.5) implies that  $\hat{\pi}_t$  does not respond to a period  $t$  monetary policy shock. Second, inflation depends on expected future marginal costs. So, other things equal, the more inertial marginal

costs are, the more inertial inflation is. Relation (3.5) implies that marginal cost is an increasing function of the wage rate, the rental rate on capital and the interest rate. From this perspective, a key function of nominal wage rigidities is to induce inertia in inflation. Variable capital utilization, by increasing the elasticity of supply of capital services, dampens the rise in the rental rate of capital. For this effect to be operative,  $u_t$  must rise in the wake of an expansionary policy shock. Our assumption that the costs of increased capital utilization take the form of consumption goods plays a role in ensuring that this rise occurs. If, for example, the cost were a higher capital depreciation rate, then  $u_t$  could actually fall. To see this, note that after an expansionary policy shock, investment increases. In the presence of investment adjustment costs, this implies that the marginal cost of physical capital rises. This increase in turn leads to a rise in the cost of  $u_t$ , which could lead to a fall in the utilizing of capital.

Third, the interest rate appears in marginal cost because firms must borrow the wage bill in advance. Since the interest rate drops after an expansionary monetary policy shock, the model embeds a force that pushes marginal costs down for a period of time. Indeed, in the estimated benchmark model the effect is strong enough to induce a transient fall in inflation.

Fourth, according to (6.5) current inflation depends on lagged inflation. This induces an extra source of inertia in the rate of inflation, that is not present in the standard formulation of Calvo-style pricing frictions. For example, Yun (1996) and others assume that firms which do not reoptimize their price, reset it according to  $P_{jt} = \pi P_{jt-1}$ . Here,  $\pi$  is the steady state inflation rate. With this static indexation formulation, (6.4) is replaced by

$$\hat{\pi}_t = \beta E_{t-1} \hat{\pi}_{t+1} + \frac{(1 - \beta \xi_p)(1 - \xi_p)}{\xi_p} E_{t-1} \hat{s}_t, \quad (6.6)$$

and (6.5) holds without the lagged inflation term. Authors like Fuhrer and Moore (1995) and Gali and Gertler (1999) argue on empirical grounds that lagged inflation belongs in an expression like (6.6). Our lagged inflation indexation pricing rule provides one way to rationalize the presence of such an inflation term.

## 6.2. Quantitative Considerations

We now analyze how the various features of the model contribute quantitatively to its performance. We do this by considering two sets of model perturbations. The first pertains to the nominal part of the model, and focusses on the role of price and wage frictions, as well as the representation of monetary policy. The second focuses on the role of several features of the real economy.

### *Nominal Side of the Model Economy*

It is commonplace in the literature to represent monetary policy as a parsimonious Taylor rule. To assess the model's properties when we represent policy this way, we replace the money growth rule, (3.15), with the following Taylor rule:

$$\hat{R}_t = \rho \hat{R}_{t-1} + (1 - \rho) [a_\pi E_{t-1} \hat{\pi}_{t+1} + a_y \hat{y}_t] + \varepsilon_t.$$

Here,  $\varepsilon_t$  is a shock to monetary policy. In addition, we chose values for the parameters consistent with the post-1979 era estimates reported by Clarida, Gali and Gertler (1999):  $\rho = 0.80$ ,  $a_\pi = 1.5$  and  $a_y = 0.1$ . Row 1 of Figure 4 displays the dynamic response of inflation and output in this version of the model, along with the benchmark responses. Notice that the response functions are similar in the two versions of the model. The main difference is that the overall rise in output is smaller in the Taylor rule version. Still, inflation exhibits substantial inertia and output exhibits substantial persistence.

Next, we consider the role of sticky prices in the performance of the benchmark model. Row 2 in Figure 4 displays the impulse response functions for a version of the model in which we set  $\xi_p = 0$  and hold the remaining parameters at their benchmark values. Notice that the response of inflation and output are not substantially affected by this change. The main impact is that inflation falls more and there is a larger rise in output in the immediate aftermath of a shock relative to the benchmark model. The fall in inflation reflects the impact of the fall in the interest rate on marginal cost. Somewhat paradoxically, sticky prices play the role of muting the fall in prices that would otherwise occur after an expansionary monetary policy shock. In any event, even without sticky prices, inflation continues to display substantial inertia and output continues to rise in a hump-shaped manner.

A different way to assess the impact of sticky prices is to reestimate our model, subject to the constraint,  $\xi_p = 0$ . The resulting point estimates are reported in Table 2. Interestingly, inference about model parameters is quite robust to imposing the restriction  $\xi_p = 0$ . Notice in particular that our point estimate of  $\xi_w$  is virtually unaffected. The impulse response functions for inflation and output are very similar to what is displayed in row 2 of Figure 4. So, with  $\xi_p = 0$ , the model still generates large, persistent increases in output and an inertial response in inflation. These observations substantiate our claim that sticky prices play a limited role in accounting for the good fit of the benchmark model.

We now turn to sticky wages, which turn out to play a crucial role in the model's performance. Row 3 in Figure 4 displays the impulse response functions for a version of the model in which we set  $\xi_w = 0$  and hold the remaining parameters at their benchmark values. Now inflation surges in the aftermath of a policy shock. This surge reflects a sharp, persistent rise in real wages (not displayed). Notice also that output rises by only a small amount in the first period after the shock and then quickly returns to its pre-shock growth path. When we attempted to estimate the model with  $\xi_w = 0$ , the estimate of  $\xi_p$  is driven to unity.

Evidently, the estimation criterion prefers extreme degrees of price stickiness when there are no sticky wages. Clearly, sticky wages play a crucial role in allowing the model to account for the effects of a monetary policy shock.

Next, we turn to the role of lagged inflation indexation versus static inflation indexation. Row 4 in Figure 4 displays the impulse response functions for a version of the model in which prices are set according to the static inflation indexation scheme. Notice is that the model properties are not substantially affected by this change. Inflation continues to be inertial and there is still a persistent rise in output after a monetary policy shock. Another way to assess the impact of the price setting scheme is to reestimate the model under static inflation indexation. The parameter estimates are reported in Table 2. Two things are worth noting. First, consistent with the discussion in section 6.1, the degree of price stickiness required to match the empirical impulse response functions is greater under the static price-updating scheme. For example, the average duration of price contracts rises from 2.5 quarters to almost a year. In contrast, the average duration of wage contracts declines from roughly 2.8 quarters to about 2 quarters. Of course, once sampling uncertainty is taken into account, the differences between the two models are less dramatic. Second, the estimated degree of market power rises from 1.20 to 1.36 in the static price-updating version of the model. Again, taking into account sampling uncertainty, the differences are not significant. So, while there are marginal improvements with our lagged inflation indexation scheme, they are not critical to the model's performance.

### *The Role of Timing Assumptions in The Model*

Our benchmark model incorporates various timing assumptions that ensure it is consistent with the assumptions used to identify a monetary policy shock in our VAR. These assumptions do not have a substantial impact on the dynamic properties of the model. Here, we illustrate this claim by examining the impact of timing on the output and inflation response to a policy shock. The solid line in Figure 5, row 1 displays the response of inflation and output when we drop the assumption that consumption, investment and capital utilization are predetermined. For convenience, the dashed line reproduces the responses in the benchmark model. While output now rises in the impact period of the shock, the magnitude and persistence of the response, as well as its hump-shaped pattern is similar across the two models. Notice also that the properties of inflation are virtually unaffected.

The solid line in Figure 5, row 2 displays the response of inflation and output when we drop the assumption that wages and prices are set before the realization of the policy shock. With this change, inflation drops by a small amount in the impact period of a shock, reflecting the drop in the interest rate. Still, the dynamic response of inflation in the two



models is very similar. Moreover, the response of output is virtually identical in the two models.

### *Real Side of the Model Economy*

In this section, we evaluate the role of the different real frictions embedded in our benchmark model. Our primary conclusions are as follows. The key real friction that allows the model to generate an inertial inflation response and a persistent output response to a policy shock is variable capital utilization. The primary role of the other frictions - investment adjustment costs, habit formation in consumption and working capital - is to enable the benchmark model to account for the response of other variables to a policy shock.

Row 1 in Figure 5 reports the effects of a monetary policy shock on inflation and output when we do not allow for variable capital utilization ( $\sigma_a = 100$ ). The remaining model parameters are fixed at their benchmark values. Notice that the output effect of a monetary shock is roughly cut in half when this feature is dropped from the model. Also inflation rises substantially more in the immediate aftermath of a monetary policy shock. A different way to assess the importance of variable capital utilization is to consider the results in Table 2. There we report the results of reestimating the parameters of the benchmark model, fixing  $\sigma_a = 1000$ . Note that our point estimate of  $\xi_p$  is now 0.92, implying an average duration of price contracts equal to a little over three years. This is clearly inconsistent with existing microeconomic evidence. See, for example, Bils and Klenow (2002). In addition, our point estimate for  $\lambda_f$  jumps to 1.85, implying a markup well above standard estimates. We conclude that variable capital utilization plays a critical role in the model's performance.

Row 2 in Figure 5 reports the effects of a monetary policy shock on inflation and output when we eliminate habit formation (i.e.,  $b = 0$ ) and hold all other parameters at their benchmark values. Now, a policy shock leads to a larger initial rise in output and a slightly larger rise in inflation than in the benchmark model. The increase in output reflects the way that consumption responds to the policy shock. In results not displayed here, we found that the maximal impact on consumption occurs immediately in the period after the shock. After that, consumption slowly declines back to its pre-shock level. This temporal pattern can be understood as follows. When  $b = 0$ , households relate the growth rate of consumption to the level of the interest rate, which is low after a shock. Intertemporal budget balance requires that this downward-sloped consumption profile starts from an initially high level of consumption.

An implication of the previous argument is that, with  $b = 0$ , there is no way to reconcile a hump-shaped response of consumption with a low interest rate. With habit formation ( $b > 0$ ), it is possible to do this. Roughly speaking, with habit formation households relate the change in the growth rate of consumption to the interest rate. So, with a low interest

rate, the household chooses a consumption profile characterized by a declining growth rate of consumption. Intertemporal budget balance implies that this profile begin with a positive growth rate. This explains why the benchmark model with  $b > 0$  generates a hump-shaped consumption profile in conjunction with a low interest rate.

Table 2 reports the results of re-estimating the parameters of the benchmark model, subject to  $b = 0$ . Notice that inference about parameters is sensitive to this change in the real side of the model economy. For example, our estimate of the parameter  $\varkappa$  drops from 2.48 in the benchmark model to 0.91 in the model with  $b = 0$ . In an experiment not reported here, we found that the drop in  $\varkappa$  encourages a stronger investment response and a weaker consumption response. The latter effect moves the no habit formation model into closer conformity with the data.

Row 3 in Figure 5 reports the effects of a monetary policy shock on inflation and output when investment adjustment costs are very small (i.e.,  $\varkappa = 0.5$ ) and we hold all other parameters at their benchmark values.<sup>17</sup> Now, a policy shock leads to a substantially larger initial rise in output and inflation relative to the benchmark model. This pattern reflects the fact that investment surges more than in the benchmark model. To understand why, it is useful to consider the rate of return on capital. Abstracting from capital utilization, we have:

$$\frac{r_{t+1}^k + P_{k',t+1}(1 - \delta)}{P_{k',t}},$$

where  $P_{k',t}$  is defined after equation (5.7). Our model implies that an expansionary monetary policy shock leads to a persistent fall in the real interest rate,  $R_t/\pi_{t+1}$ . Abstracting from risk considerations, equilibrium requires that the rate of return on capital fall with the real interest rate. Suppose there are no adjustment costs, so that  $P_{k',t} = P_{k',t+1} = 1$ . In this case, the rate of return formula reduces to  $r_{t+1}^k + 1 - \delta$ . Leaving aside movements in the markup, the only way the rate of return on capital can fall is if its rental rate and marginal product fall. This fall in turn requires a surge in investment. This rise in investment is what accounts for the strong rise in output observed in row 3 of Figure 5. When there are adjustment costs in investment, it is possible for the rate of return on capital to fall without any counterfactually large rise in investment, as long as there is an appropriate intertemporal pattern in  $P_{k',t}$ . Table 2 reports the results of re-estimating the parameters of the benchmark model subject to  $\varkappa = 0.5$ . Interestingly, here there is very little sensitivity in parameters.

We now consider the impact of the way we modeled adjustment costs in investment. Recall that in the benchmark model, firms face second order costs of changing investment. It is more typical in the business cycle literature to work with the first order adjustment

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<sup>17</sup>We encountered difficulties with our solution algorithm when we tried to solve the model setting  $\varkappa$  equal to zero. This is why we do not report results for the case  $\varkappa = 0$ .

costs. For early references, see Eisner and Strotz (1963), Lucas (1967,a,b), and Lucas and Prescott (1971). For more recent examples, see CKM and McCallum and Nelson (1998). To assess the performance of the model with first order adjustment costs, we replace (3.11) with the adjustment cost formulation in Christiano and Fisher (1998):

$$\bar{k}_{t+1} \leq Q((1 - \delta)\bar{k}_t, i_t).$$

Here

$$Q(x, z) = [a_1 x^\nu + a_2 z^\nu]^{1/\nu},$$

and  $\nu \leq 1$ . In this expression,  $x$  denotes previously installed capital after depreciation and  $z$  denotes new investment goods. The scalars  $a_1, a_2 > 0$  were chosen to guarantee that  $Q_x = Q_z = 1$  in nonstochastic steady state.<sup>18</sup> When  $\nu = 1$ , the above technology corresponds to the conventional linear capital accumulation equation. The case of adjustment costs corresponds to  $\nu < 1$ . Here, the marginal product of new investment goods is decreasing in the flow of investment.

We re-estimated all the parameters of this model, including  $\nu$ . We refer to this model as the *alternative adjustment cost model*. Our results are reported in Table 2. Two results are of interest. First, apart from  $\nu$ , the estimated parameters of this model are very similar to those of the benchmark model. Second, our point estimate of  $\nu$  is equal to  $-0.47$ . This parameter estimate implies an elasticity of Tobin's  $q$  with respect to the price of installed capital equal to roughly 0.70. This value is well within the range of estimates reported in the literature (see Christiano and Fisher (1998)).<sup>19</sup>

Row 4 of Figure 5 displays the response of inflation and output in the estimated version of this model. Two results are worth noting. First, the implications of this model and the benchmark model for the response of inflation are very similar. Second, the response of output in this model over the first two years is somewhat weaker than it is in the benchmark model. The output response is also less persistent. These problems with persistence and magnitude reflect the model's counterfactual implications for investment. In particular, in results not reported here, we found that the alternative adjustment cost model does not match the strong, hump-shaped response of investment. Instead, investment peaks in the period after the shock and then quickly reverts to its pre-shock level. The overall magnitude of the response of investment is far weaker than either the benchmark model or the VAR

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<sup>18</sup>In steady state,  $x = (1 - \delta)\bar{k}$  and  $z = \delta\bar{k}$ . It is straightforward to confirm that, with  $a_1 = (1 - \delta)^{1-\psi}$  and  $a_2 = \delta^{1-\psi}$ ,  $Q_x = Q_z = 1$  in steady state.

<sup>19</sup>We identify  $P_{k'}$  with Tobin's  $q$ . Household optimization implies that the marginal cost of a unit of investment - unity in our model - equals the marginal benefit,  $P_{k'}/Q_z$ . Following Christiano and Fisher (1998), we identify the elasticity of investment with respect to Tobin's  $q$  as the percent change in household investment associated with a percent change in Tobin's  $q$ , holding the stock of capital,  $x$ , fixed. It is straightforward to confirm that this elasticity, evaluated in steady state, is  $1/((1 - \psi)(1 - \delta))$ .

based estimates. The model is still able to produce a reasonably large response of output, but it leads to a counterfactually large surge in consumption. We conclude that the second order adjustment cost assumption leads to a significantly better overall account of the response of the economy to a monetary policy shock.

Row 5 in Figure 5 reports the effects of a monetary policy shock on inflation and output when we drop the assumption that firms must borrow their wage bill in advance. The model's parameters are fixed at their benchmark values. The key results to note are as follows. First, consistent with our previous discussion, inflation no longer declines after a monetary policy shock. The absence of a decline reflects the fact that, without a working capital channel, a drop in the interest rate does not reduce firms' marginal costs. Nevertheless, inflation still displays substantial inertia. Second, the rise in output is very similar to what it is in the benchmark model.

These results suggest that the role of the working capital assumption in our model is relatively minor. A different picture emerges when we reestimate the model, dropping this feature. Table 2 reports the results. Notice is that our point estimate of  $\xi_p$  rises to 0.89, which corresponds to an average duration of price contracts equal to 2.5 years. This value is implausible in the light of the available microeconomic evidence. We conclude that the working capital channel plays an important role in the benchmark model's performance.

Finally, row 6 in Figure 5 shows what happens when we drop all the real frictions discussed in this subsection. In particular, we make investment adjustment costs very small ( $\varkappa = .5$ ), set the habit formation parameter  $b$  to zero, and drop the assumption that firms must borrow their wage bill in advance. The remaining parameters are set at their benchmark values. For convenience we refer to this version of our model as the *no real frictions model*. Notice that this model's properties are very different from those of the benchmark model. There is no inertia in inflation and the output effects are not persistent. For example, the contract multiplier drops from its value of 3.7 in the benchmark model to 2.3 in the no real frictions model. Recall that, in the benchmark model, over 78 percent of the increase in output occurs after the typical wage and price contract in effect at the time of the shock is reoptimized. In contrast, the corresponding statistic in the no real frictions model is 46 percent.

Not surprisingly, other properties of the no real frictions model also differ sharply from those of the benchmark model. In results not displayed here, we found that the model does not generate a persistent drop in the interest rate. This result is not surprising in light of sharp rise in inflation. In addition, investment surges in the wake of the shock, but exhibits little persistence. There is a notable drop in productivity. This decline is to be expected, in view of the fact that utilization cannot rise. Finally, consumption and profits exhibit essentially no response to the policy shock. When we attempted to re-estimate the parameters of the no real frictions model,  $\xi_w$  and  $\xi_p$  were driven to unity. This result is

a dramatic illustration of our claim that inference about nominal rigidities is sensitive to getting the real side of the model right. Of course, the no real frictions model with only sticky wages and flexible prices would do quite poorly as well.

In the previous discussion, we evaluated the real frictions in our model by deleting different frictions from the benchmark model. An alternative is to add features to the no real frictions model. When we did this, we obtained several results, which complement the experiments reported above. First, if one is only interested in inflation inertia and output persistence, then the key real-side friction that must be included is variable capital utilization. To reach this conclusion we took the no real frictions model, added variable capital utilization to it, and then calculated the consequences of a monetary policy shock. The resulting inflation path lies within the confidence intervals associated with our estimated VAR. While the model's performance with respect to output is not as good as the benchmark model's, the output response still exhibits substantial persistence. For example, the contract multiplier is equal to 3. In addition, 65 percent of the increase in output after a policy shock occurs after the typical wage and price contract in effect at the time of the shock has been reoptimized. Second, we found that habit formation and investment adjustment costs play a much smaller role than variable capital utilization in promoting inflation inertia and output persistence. Indeed, when we add these features to the no real frictions model, the response of output is actually weaker and less persistent. Consistent with the results discussed above, the primary role of these two features is to help account for the temporal response of other variables in the model, such as consumption, investment and the interest rate.

We conclude by briefly discussing the relationship of our results to those in CKM. They pose a challenge they call *the persistence problem*. This is the problem associated with the work of Mankiw (1985), Akerlof and Yellen (1985) and others, which seeks to identify small frictions that can account for the observed degree of inertia in prices and persistence in output. The frictions CKM explore are staggered price contracts, as modeled in Taylor (1980). They think of these frictions as small if contracts have a duration of one quarter or less. CKM perform a series of computational exercises using models with various frictions on the real side of the economy. They find that, for plausible parameterizations of the real frictions, small nominal frictions do not solve the persistence problem. Moreover, they argue that even with contracts with longer duration one cannot account for the persistence in output. Seemingly, this stands in contrast to the claim that our model can account for the dynamic response of output to a monetary policy shock.

There are three key differences between our analysis and that of CKM. First, we take a different position on what constitutes a reasonable contract length to use in a model. Second, we have a different way of assessing whether a model matches the persistence of the response

of output to a monetary policy shock. Third, they consider a different set of real frictions.

Regarding the first difference, our position is that a reasonable contract length is one that matches the duration of contracts found in survey evidence. In this respect, we follow the empirical literature on wage and price frictions.<sup>20</sup> If we instead proceed as suggested by CKM, by adopting contracts of duration one quarter, then our model also generates little persistence in output. So, in this respect our conclusion is not in conflict with that of CKM.

The results in CKM imply that even if they had adopted our estimated contract length of 2.5 quarters, their model still would not have matched their estimate of the degree of output persistence. This brings us to the other two differences between our analysis and that of CKM. Consider the measurement of output persistence. CKM's measure is based on an estimate of the dynamic response of output to the innovation in its estimated univariate representation. This representation is obtained by fitting a second order autoregression to quadratically detrended, log output. Their measure of persistence is the half life of a shock in this autoregression. They estimate that the half life of a policy shock on output is 10 quarters, i.e., it takes 10 quarters for the output response to a shock to be one-half its value in the impact period. CKM define the contract multiplier to be the ratio of the half-life of a monetary policy shock to what the half-life of the shock would have been if contracts were fully synchronized. They approximate the latter by one-half the duration of the contract. They conclude that the largest multiplier that can be rationalized with a model that does not have extraordinarily implausible parameter values is 4.17. This multiplier implies that with contracts of duration 2.5 quarters, the half-life of a monetary shock is a little over 5 quarters. This is far short of their target of 10 quarters.

The apparent difference in our results is due to one or both of two factors: the different frictions in our respective models, or the different targets. Disentangling the role played by these two factors is beyond the scope of this paper. To this end, it would be of interest to evaluate the robustness of their conclusions and ours to the presence of different sets frictions.

At the same time, we are skeptical about CKM's measure of persistence. Recall that their measure is a particular function of the dynamic response of output to the shock in a univariate autoregression for detrended output. One interpretation of this shock is that it is proportional to a monetary policy shock. This interpretation requires that disturbances to monetary policy are the only shocks driving aggregate output. This conflicts with results in the literature which suggest that aggregate data are driven by several shocks (see, for example, Quah and Sargent (1993) and Uhlig (2002)) as well as our own variance decomposition results. A different interpretation of CKM's procedure is that the disturbance term in the autoregression is a combination of various shocks. But, this interpretation is also

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<sup>20</sup>See, for example, Gali and Gertler (1999, page 209) and Taylor (1980, 1999).

unappealing because then their model based and data based experiments are different.

A natural question is whether the apparent conflict in results simply reflects that our impulse response function exhibits less persistence than CKM's. It is not possible to answer this using their measure of persistence. This is because their half life measure is not defined for an impulse response function like ours in which the initial output response is zero. An alternative approach is based on the persistence measure discussed above: the percent of the positive output response that occurs after the typical contract in place at the time of the shock has been reoptimized. This is 90 percent in our VAR-based estimates, for contracts of length 2.5 quarters. The analogous number for CKM is only 71 percent.<sup>21</sup> By this measure, our impulse response function for output exhibits more persistence than CKM's.

## 7. Conclusion

We present a model embodying moderate amounts of nominal rigidities, which accounts well for our estimate of the dynamic response of the U.S. economy to a monetary policy shock. Specifically, the model generates an inertial response in inflation and a persistent, hump-shaped response in output after a policy shock. In addition, the model generates hump-shaped responses in investment, consumption, employment, profits and productivity as well as a small response in the real wage. Also, the interest rate and the money growth rate move persistently in opposite directions after a monetary policy shock. A key finding of the analysis is that stickiness in nominal wages is crucial for the model's performance. Stickiness in prices plays a relatively small role.

An important direction for future research is to incorporate additional shocks into the analysis. A test of the model structure developed here is whether it can account for the consequences of other shocks. Preliminary results in Altig, Christiano, Eichenbaum and Linde (2003) suggest that the answer to this question is yes, at least for aggregate technology shocks.

We conclude by remarking on our decision to model nominal rigidities using the familiar Calvo framework. We think of the Calvo model as a useful reduced form for capturing the factors that contribute to nominal sluggishness. Given the key role of wages in our results, this suggests the importance of modeling these factors in a structural manner and integrating them into dynamic, general equilibrium models. This conclusion is consistent with a long strand in the macroeconomics literature.

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<sup>21</sup>The time series representation for output that they estimate is  $(1 - 1.03L + 0.38L^2)y_t = \varepsilon_t$ , where  $y_t$  denotes log, detrended output and  $\varepsilon_t$  denotes the shock. The total output response to a unit shock in  $\varepsilon_t$  is  $1/(1 - 1.03 + .38)$ . The percent of this response occurring after the third period is obtained by subtracting the first three elements in the impulse response function.

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	4 Quarter Ahead	8 Quarters Ahead	20 Quarters Ahead
Output	15 (4,26)	38 (15,48)	27 (9,35)
Inflation	1 (0,8)	4 (1,11)	7 (3,18)
Consumption	14 (4,26)	21 (5,37)	14 (4,26)
Investment	10 (2,21)	26 (7,39)	23 (6,32)
Real Wage	2 (0,8)	2 (0,14)	4 (0,15)
Productivity	15 (3,25)	14 (3,26)	10 (3,20)
Federal Funds Rate	32 (18,44)	19 (8,27)	18 (5,27)
M2 Growth	19 (8,29)	19 (8,26)	19 (8,24)
Real Profits	13 (5,25)	18 (6,31)	7 (2,20)

Model	$\lambda_f$	$\xi_w$	$\xi_p$	$\sigma_q$	$\varkappa$	$b$	$\sigma_a$	$\nu$
Benchmark	1.20 (.06)	.64 (.03)	.60 (.08)	10.62 (.67)	2.48 (.43)	.65 (.04)	.01	NA
Flexible Prices	1.11 (.05)	.65 (.02)	0	8.63 (.63)	3.24 (.47)	.66 (.04)	.01	NA
Unconditional								
Indexation	1.36 (.09)	.49 (.07)	.72 (.16)	11.09 (.67)	1.92 (.35)	.63 (.05)	.01	NA
No Variable								
Capital Utilization	1.85 (.13)	.42 (.05)	.92 (.02)	10.83 (.67)	1.58 (.28)	.62 (.05)	100	NA
No habit	1.01 (.04)	.80 (.02)	.28 (.15)	10.12 (.70)	.91 (.18)	0	.01	NA
Small Adjustment								
Costs in Investment	1.06 (.04)	.76 (.03)	.64 (.08)	10.92 (.70)	.5	.52 (.11)	.01	NA
Lucas-Prescott								
Investment Adj. Costs	1.08 (.06)	.62 (.03)	.53 (.23)	10.60 (.60)	NA	.71 (.03)	.01	-.47 (.22)
No Working Capital	1.25 (.06)	.46 (.05)	.89 (.02)	10.85 (.67)	1.89 (.37)	.62 (.05)	.01	NA

Figure 1: Model- and VAR-Based Impulse Responses

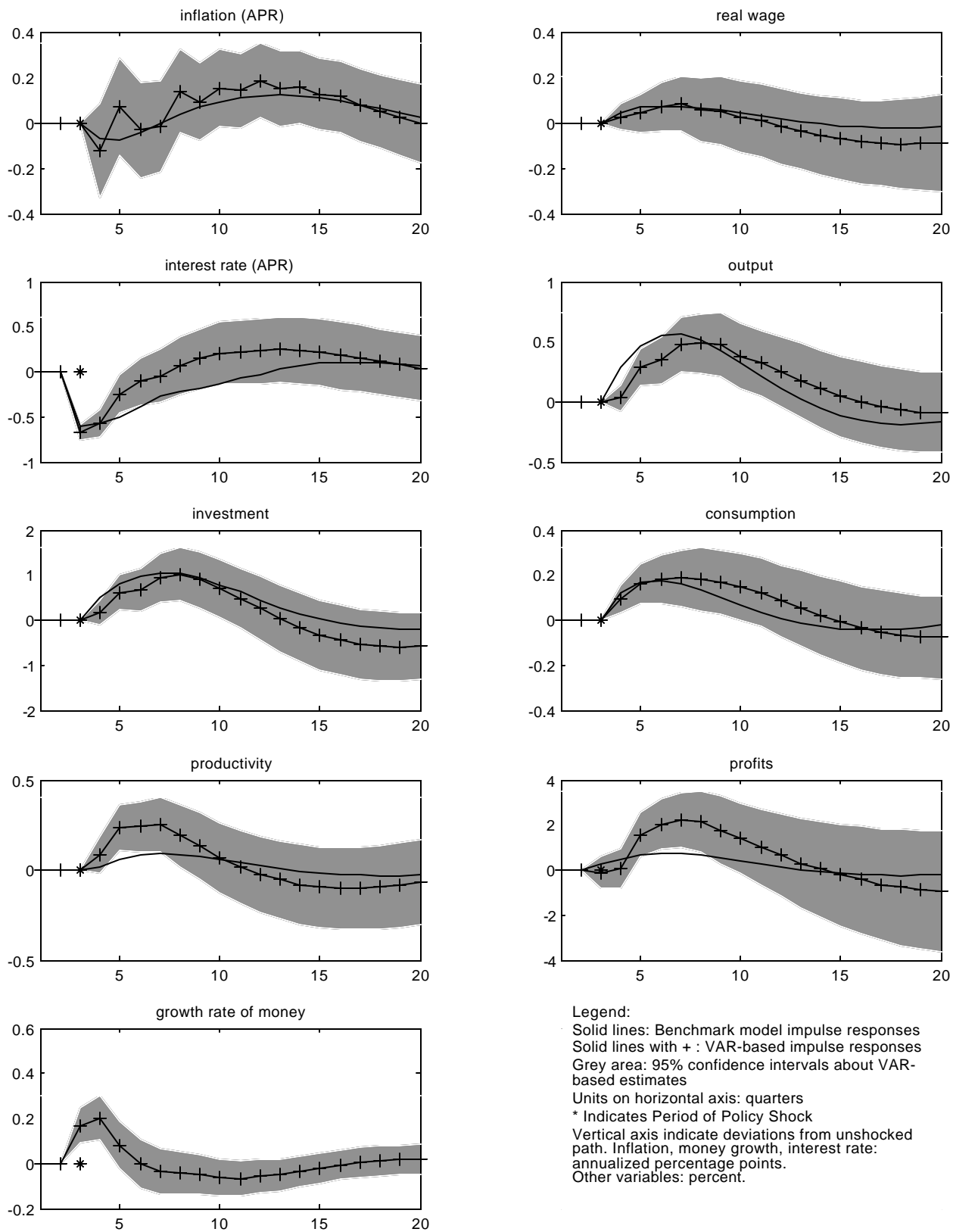


Figure 2: Response of Price Level, Output, and Money Stock to an expansionary Monetary Policy Shock, In the Benchmark Model

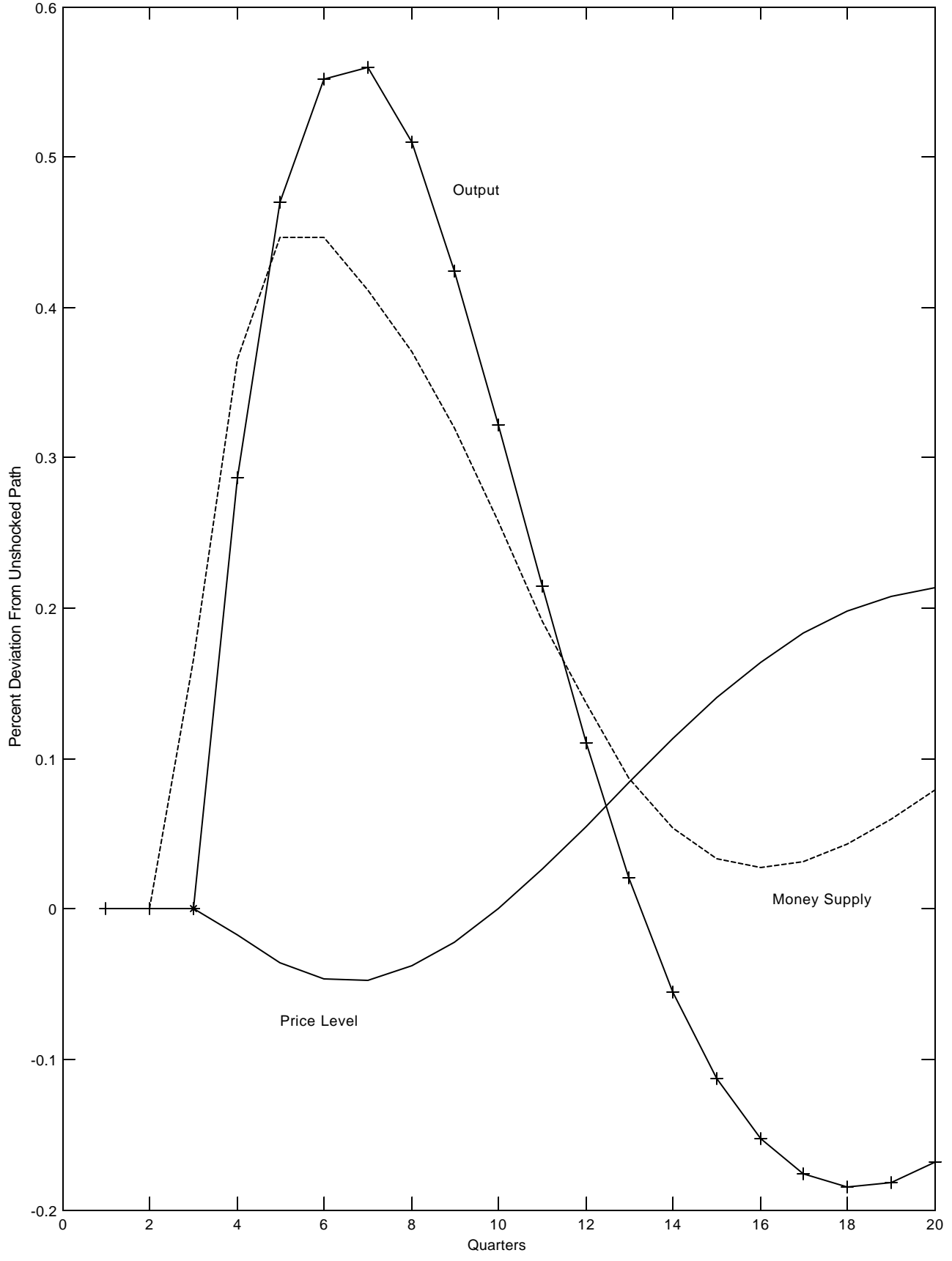
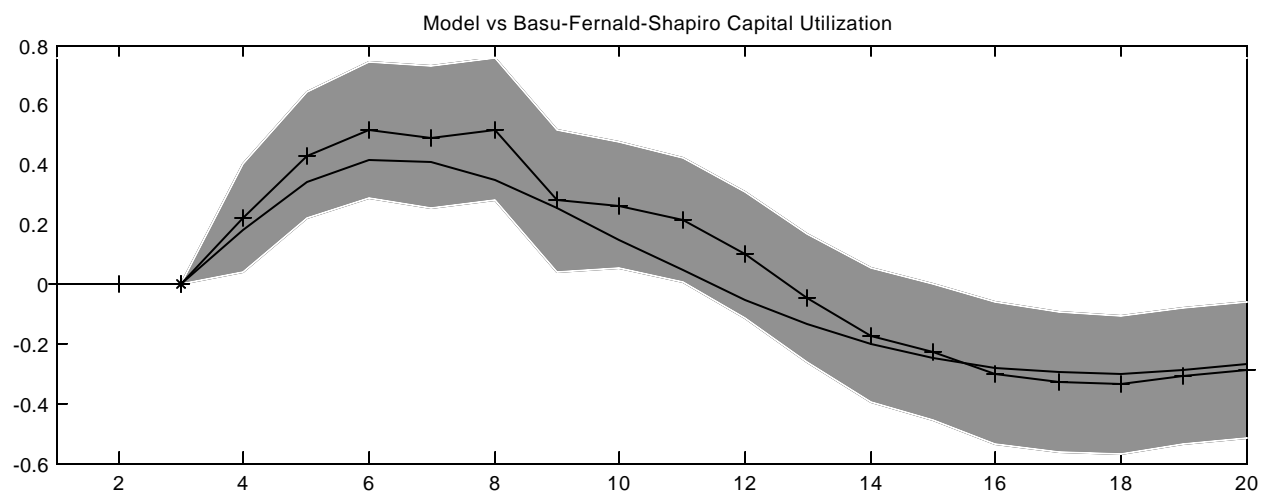
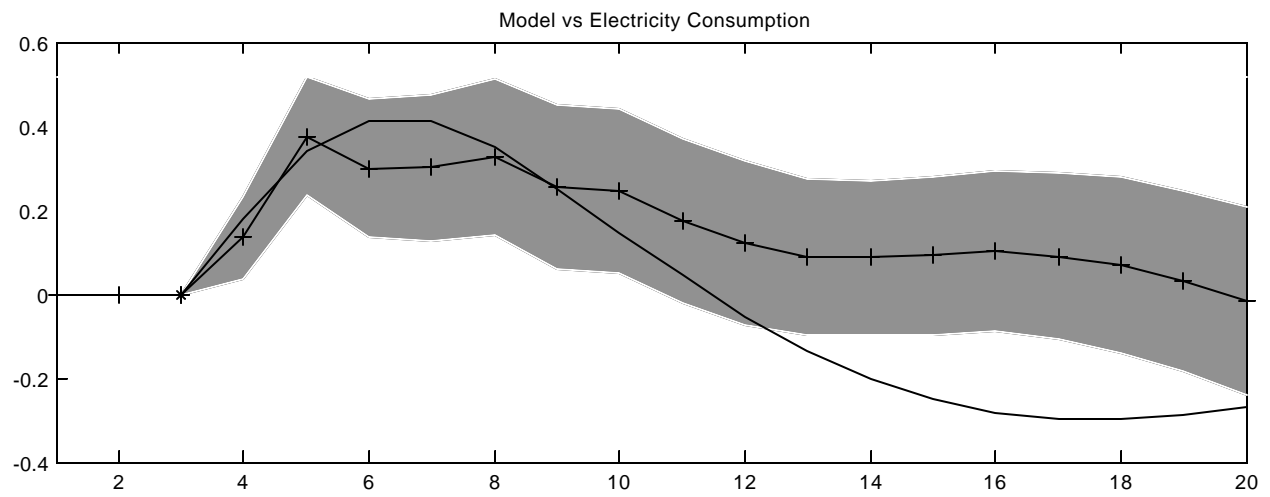
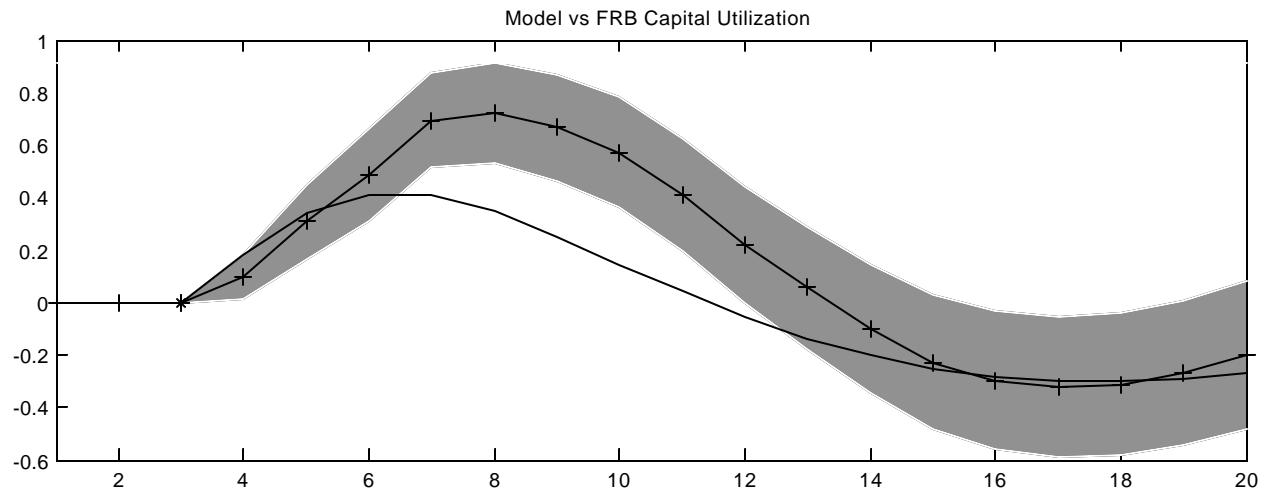


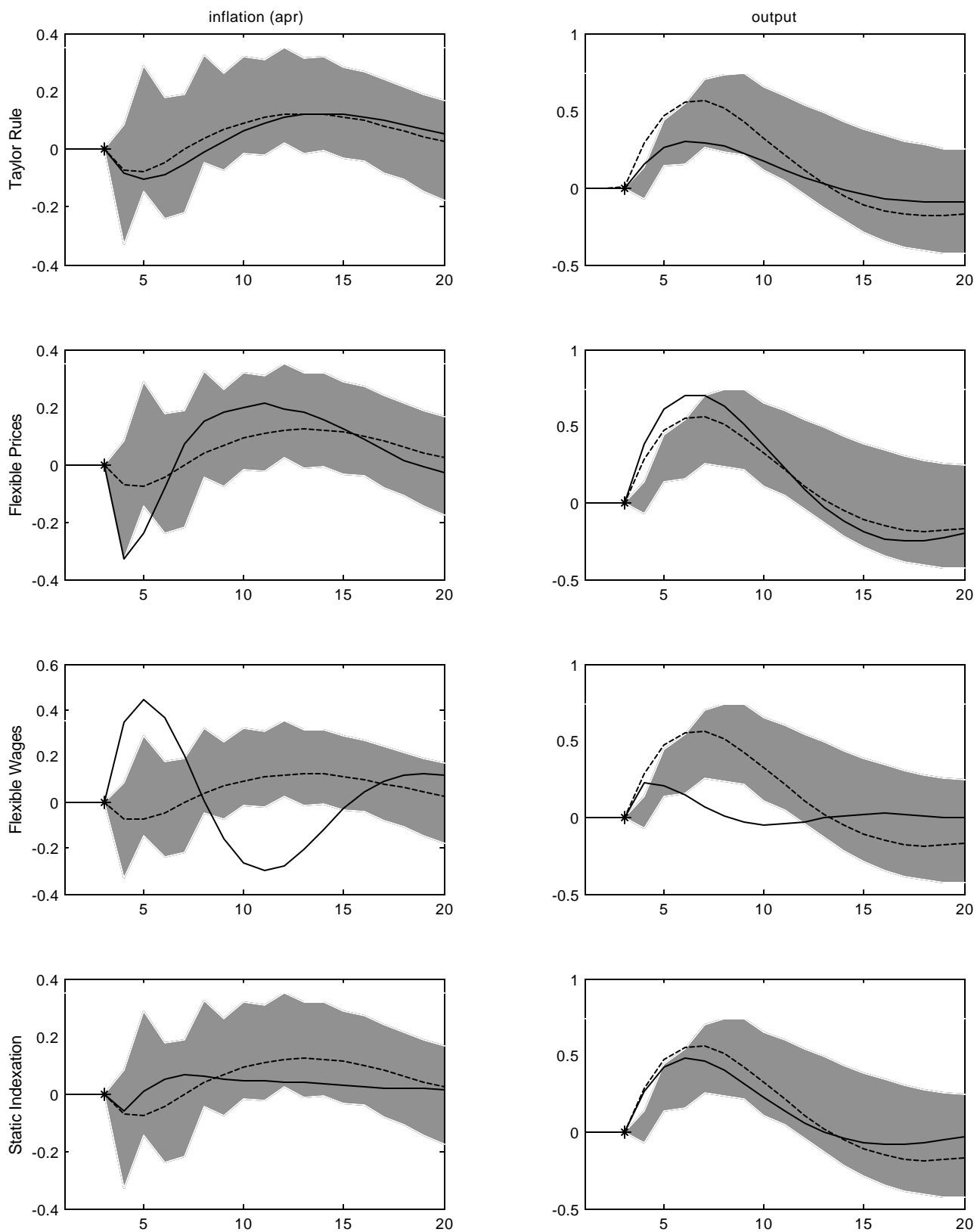


Figure 3: The Response of Capital Utilization to Monetary Policy Shocks



Legend: Solid lines -- model impulse responses; Dashed lines - VAR-based impulse responses;  
 Grey Area 95% confidence intervals about VAR-based impulse responses

Figure 4. Variants of the Benchmark Model: Perturbing the Policy and the Nominal Side



Legend: Solid lines: impulse responses for model indicated on vertical axis; Dashed lines: benchmark model impulse responses;  
 Grey Area 95% confidence intervals about VAR-based impulse responses

Figure 5. Role of Timing in Model Dynamics

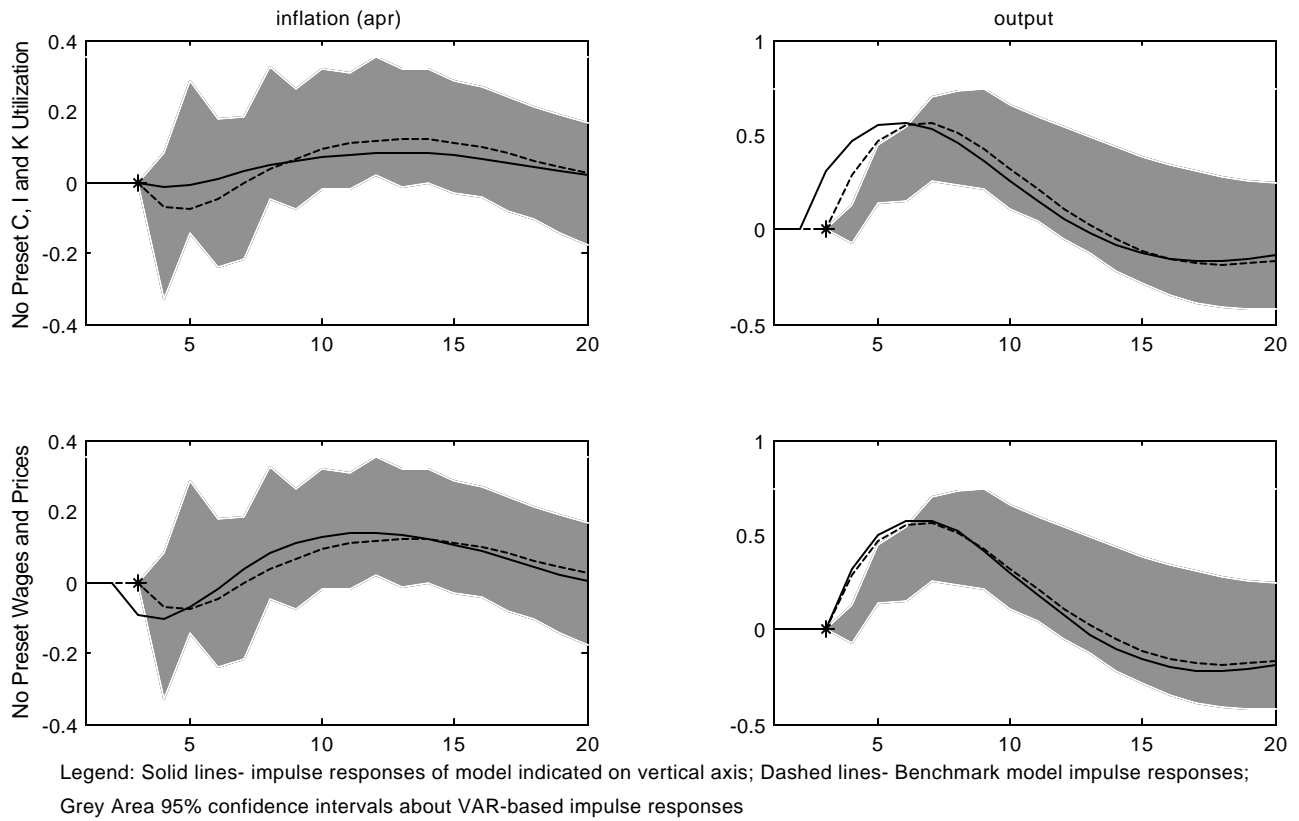
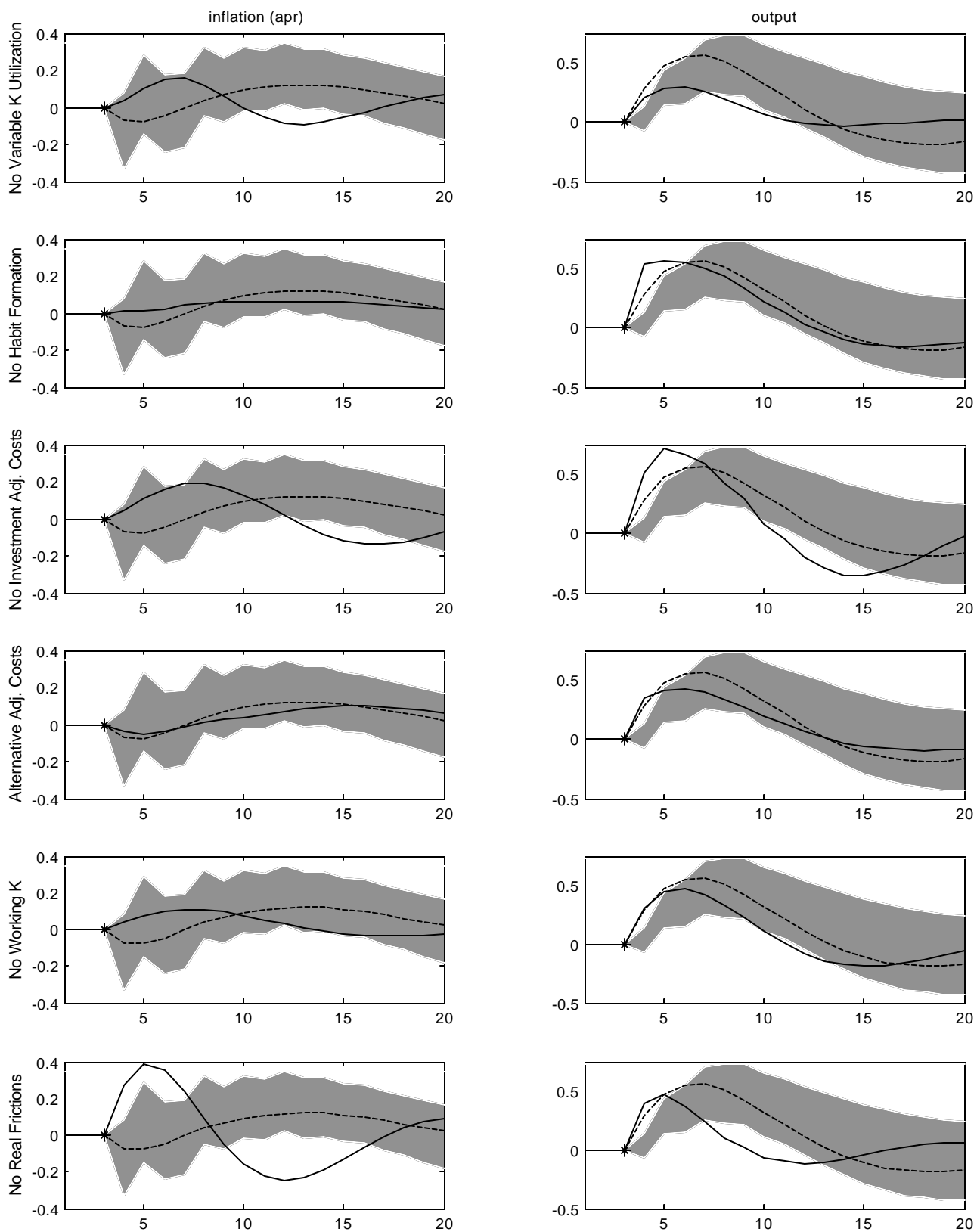


Figure 6. Variants of the Benchmark Model: Perturbing the Real Side of the Model Economy



Legend: Solid lines- impulse responses of model indicated on vertical axis; Dashed lines- Benchmark model impulse responses; Grey Area 95% confidence intervals about VAR-based impulse responses