MANDATING VACCINATION WITH UNKNOWN INDIRECT EFFECTS

Charles F. Manski

Department of Economics and Institute for Policy Research
Northwestern University


Abstract

Social interactions make communicable disease a core concern of public health policy. A prevalent problem is scarcity of empirical evidence informative about how interventions affect illness. Randomized trials, which have been important to evaluation of treatments for non-infectious diseases, are less informative about treatment of communicable diseases because they do not fully reveal the indirect preventive (herd immunity) effect of vaccination on persons who are not vaccinated or are unsuccessfully vaccinated. This paper studies the decision problem faced by a health planner who observes the illness rate that occurs when persons make decentralized vaccination choices and who contemplates whether to mandate vaccination. The planner's objective is to minimize the social cost of illness and vaccination. Uncertainty about the magnitude of the indirect effect of vaccination implies uncertainty about the illness rate that a mandate would yield. I first study a simple representative-agent setting and derive conditions under which the planner can determine whether mandating vaccination is optimal. When optimal policy is indeterminate, I juxtapose several criteria for decision making—expected utility, minimax, and minimax-regret—and compare the policies they generate. I then extend the analysis to a more general setting in which members of the population may have heterogenous attributes.

The present analysis of mandatory vaccination supercedes that in my working paper "Vaccine Approvals and Mandates under Uncertainty: Some Simple Analytics," NBER Working Paper 20432, 2014. I have benefitted from the opportunity to present this work in seminars at the Booth School of Business, University of Chicago, the Department of Economics, University of California at Santa Barbara, and the Schaeffer Center for Health Policy and Economics, University of Southern California. I have also benefitted from the comments of an anonymous reviewer and associate editor.
1. Introduction

Social interactions in treatment response make communicable disease a core concern of public health policy. Spread of infection creates a negative externality. Preventive administration of vaccines, therapeutic administration of antimicrobial drugs, and separation of infected persons from the general population may reduce disease transmission. In a decentralized health care system, infected and at-risk persons may not adequately recognize the social implications of their actions. Hence, there may be a rationale for government to seek to influence treatment of communicable disease. In practice, policies range from quarantines of infected persons to mandatory vaccination to subsidization of vaccines and drugs.

A prevalent problem in policy choice is scarcity of empirical evidence that are informative about how interventions affect illness. Randomized clinical trials (RCTs), which have been important to evaluation of treatments for non-infectious diseases, are less informative about treatment of communicable diseases. The classical argument for inference from RCTs assumes that the outcome experienced by each person or other treatment unit may vary only with his own treatment, not with those of other members of the population. This assumption—variously called non-interference (Cox, 1958), the stable unit treatment value assumption (Rubin, 1978), or individualistic treatment response (Manski, 2013)—does not hold when treating communicable diseases. RCTs have limited power to identify the effects of treatments for such diseases.

A leading case is the decision problem of a health planner who must choose a vaccination policy for a population. Vaccination of a particular person may benefit him directly by generating an immune response that reduces his susceptibility to the disease. It may also reduce the infectiousness of this person and thereby inhibit transmission of the disease to others who are unvaccinated or unsuccessfully vaccinated. Thus, vaccination may have both a direct preventive effect on the person vaccinated and an indirect preventive effect on other persons. The indirect effect is often called herd or community immunity. See Fine (1993) and Fine, Eames, and Heymann (2011) for discussions of the long history and use of this concept in
epidemiology.\textsuperscript{1}

An RCT that randomly vaccinates a specified fraction of the population may enable evaluation of the direct effect of vaccination on illness, but it does not reveal the indirect effect. The trial only reveals the illness outcomes that occur with the vaccination rate used in the trial. The outcomes that the population would experience with other vaccination rates remain counterfactual. Yet policy choice requires comparison of alternative vaccination rates.\textsuperscript{2}

Attempting to cope with the dearth of empirical evidence, researchers studying vaccination policy have creatively used epidemiological models of disease transmission to forecast the outcomes that would occur with counterfactual policies. See, for example, Brito, Sheshinski, and Intriligator (1991), Becker and Starczak (1997), Ball and Lyne (2002), Scuffham and West (2002), Hill and Longini (2003), Patel, Longini, and Halloran (2005), Boulier, Datta, and Goldfarb (2007), Althouse, Bergstrom, and Bergstrom (2010), and Keeling and Shattock (2012). However, authors typically provide little information that would enable one to assess the accuracy of their assumptions about individual behavior, social interactions, and disease transmission. Hence, I think it prudent to view their forecasts more as informative computational experiments than as accurate predictions of policy impacts.

With this background, I consider the decision problem of a health planner who observes the illness

\textsuperscript{1} While the usual epidemiological presumption is that vaccines have beneficial indirect effects, they can in principle have negative effects. Vaccines based on attenuated live pathogens may infect persons other than the recipient. Vaccination may also encourage selection of pathogens towards variants resistant to the vaccine. These negatives forces are thought to be weak in practice. See Mishra \textit{et al.} (2012).

\textsuperscript{2} Vaccine trials can reveal indirect effects in special cases where the population partitions into many isolated groups (aka clusters) of persons. Then the members of each group may infect one another but not the members of other groups. In such cases, one can define treatment units to be groups rather than persons, randomly assign varying vaccination rates to different groups, and use the trial to learn about illness outcomes under alternative vaccination rates. Hudges and Halloran (2008) develop methodology for analysis of RCTs performed in such settings. Loeb \textit{et al.} (2010) report a trial performed on isolated Hutterite communities in Canada. However, populations rarely partition in modern societies. RCTs have no identifying power in the polar case of a fully connected society, where social interactions are global rather than local (Manski, 2013).
outcomes that occur when persons make decentralized vaccination choices and who contemplates whether to mandate vaccination. In the American federal system, the health planner who decides whether to mandate vaccination typically is a state public health agency. States may mandate that specific populations be vaccinated against specific diseases. The usual alternative to a mandate is decentralization, with vaccination decisions being made by families, schools, care facilities, and employers. While it is natural for economists to think of subsidies as an alternative to mandates, consideration of vaccine subsidies as a practical policy has been rare.

The indirect effect of vaccination is a prevalent consideration when states contemplate institution of a mandate. For example, the state of Washington has listed it as a reason that may motivate mandatory vaccination of children and young adults, stating (Washington State Board of Health, 2006):

"Vaccinating the infant, child, or adolescent against this disease reduces the risk of person-to-person transmission. Having some proportion of the population vaccinated with the antigen helps to stem person to person transmission of the disease (i.e, herd immunity). Even community members who are not vaccinated (such as newborns and those with chronic illnesses) are offered some protection because the disease has less opportunity to spread within the community. Vaccinating children in school and/or child care centers can increase the percentage of children in these groups who are immune and thus reduce the risk of outbreaks of the disease in these groups and in the community at large."

The question studied in this paper is how a state such as Washington might reasonably make mandate decisions when the magnitude of the indirect effects described in the above passage is not known. As far as I am aware, this question has not been studied previously.

Mandatory vaccination has been a subject of considerable controversy, centered on the tension between personal freedom and public health. For example, Vamos, McDermott, and Daley (2008) give arguments for and against mandatory administration of the HPV vaccine to middle-school girls. Thomas

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(2009) and May and Silverman (2005) do likewise for mandatory vaccination of health care workers against influenza and children against multiple diseases. These and other articles in the medical and public health literatures make clear that society has conflicting objectives when contemplating whether to mandate vaccination. They do not address how society might reconcile the conflicting objectives to choose a policy.

The venerable welfare-economic practice of specifying a social welfare (or social cost) function and considering a planner who wants to optimize this function provides a useful normative framework for policy formation. In the context of vaccination policy, the planner's objective presumably is to minimize the social cost of illness and vaccination. Vaccination is socially costly, as is illness. Vaccination is beneficial to public health to the extent that it prevents illness. Mandatory vaccination improves public health relative to decentralized decision making. However, a mandate increases the cost of production and administration of the vaccine, enlarges the side effects of vaccination, and reduces personal freedom. The specified social cost function expresses quantitatively how society evaluates these advantages and disadvantages of a mandate.

Researchers have previously studied choice of vaccination policy as a deterministic planning problem, supposing that the planner knows the outcomes that alternative policies would yield. Brito, Sheshinski, and Intriligator (1991) notably focused on the decision to mandate vaccination. These authors observed that decentralization pareto dominates mandatory vaccination if the vaccine is perfectly effective, the social cost function is utilitarian, and decentralized decision makers have rational expectations. However, they cautioned that (p. 83):

"If there is a probability that the vaccine will not be effective it is possible that the compulsory solution requiring that all individuals be vaccinated is no longer dominated by the market solution."

Further research on this subject ranges from Francis (1997) to Chen and Toxvaerd (2014). Vaccines typically are imperfectly effective in practice, the degree of effectiveness varying with the vaccine. Hence, even if one assumes utilitarian social cost and rational expectations, comparison of decentralization and
mandatory vaccination constitutes a policy-relevant subject for welfare-economic analysis.

The analysis in this paper builds in part on my previous study of the decision problem faced by a planner who chooses a vaccination rate for a population (Manski, 2010). Here, as there, the planner (a) wants to minimize the social cost of illness and vaccination, (b) observes a study population whose vaccination rate has been chosen previously, and (c) does not know the magnitude of the indirect preventive effect of vaccination. Here, as there, I derive explicit conditions under which the planner can determine that a feasible policy alternative is dominated; that is, inferior whatever the indirect effect of vaccination may be. Here, as there, I pose several criteria for choice among undominated alternatives—maximization of expected utility, minimax, and minimax-regret—and compare the policies they generate.

The present analysis differs from the previous one in important respects. The planner of Manski (2010) was empowered to choose any vaccination rate for the population, randomly selecting the persons to vaccinate when the vaccination rate is neither zero nor one. In contrast, the planner of the present paper chooses between two policy alternatives: decentralized and mandatory vaccination. Constraining the planner to two alternatives simplifies the present problem relative to the previous one. However, decentralized decision making is more complex to evaluate because persons may nonrandomly choose to be vaccinated. A further difference between the previous and present analysis is that the earlier one mainly focused on the case of a perfectly effective vaccine. In contrast, I suppose throughout here that a vaccine may be imperfectly effective, which matters considerably when comparing decentralized and mandatory vaccination.

One may ask why state public health agencies commonly choose between decentralization and mandatory vaccination when they might, in principle, choose any vaccination rate for their populations. A possible answer is that having the government establish a fractional vaccination rate and randomly select persons to be vaccinated would violate a version of the ethical principle of "equal treatment of equals." Randomly vaccinating a fraction of the population is consistent with the equal-treatment principle in the ex ante sense that all observationally identical people have the same probability of vaccination. However, it
violates equal treatment in the *ex post* sense that only some persons ultimately are vaccinated. See Manski (2009) for further discussion.

I present the analysis in two stages. Section 2 studies a simple representative-agent setting in which members of the population are assumed to share identical cost of vaccination, cost of illness, probability of vaccine effectiveness, and probability of illness when unvaccinated or unsuccessfully vaccinated. This yields transparent findings showing when, despite lack of knowledge of the illness rate produced by mandatory vaccination, a planner can determine whether it is optimal to institute a mandate. When optimal policy is indeterminate, I compare the policies generated by alternative decision criteria. I use vaccination against influenza to illustrate findings. I also characterize utilitarian planning when persons are risk neutral and have rational expectations.

Section 3 drops the idealized notion of a representative agent and considers the planning problem in a more realistic setting where persons may have heterogeneous costs of vaccination and illness as well as heterogeneous probabilities of vaccine effectiveness and illness. This setting is more complex because decentralized vaccination decisions may be nonrandom. Nevertheless, most of the findings of Section 2 extend in a straightforward manner. An exception is that the decentralized equilibrium when persons are risk-neutral with rational expectations changes qualitatively. Section 4 concludes.

2. Policy Choice with a Representative Agent

The representative-agent setting studied here assumes a large population composed of persons who are identical in all respects relevant to vaccination policy. Consideration of a large population simplifies analysis because the ex ante probability that a person becomes ill is identical to the ex post illness rate in the population. Formally, this holds if the population contain uncountably many members. Standard arguments
using Laws of Large Numbers enable its approximate application to finite populations.

Section 2.1 sets up the optimization problem that I assume the planner would like to solve. Section 2.2 studies policy choice when the planner does not know the indirect effect of vaccination. Section 2.3 examines the special case in which the planner is utilitarian and the representative agent is risk-neutral with rational expectations.

2.1. The Optimization Problem

To begin, suppose that the vaccine under consideration generates a protective immune response in a vaccinated person with probability $\lambda > 0$, preventing the person from becoming ill and from infecting others. Contrariwise, the vaccine confers no immunity with probability $1 - \lambda$. Then $\lambda$ measures the direct effect of vaccination. If fraction $v$ of the population are vaccinated, the effective vaccination rate is $\lambda v$ and the fraction $1 - \lambda v$ of the population are susceptible to the disease.

The indirect-response function $p(\cdot) \colon [0, 1] \rightarrow [0, 1]$ gives the probability that a susceptible person becomes ill when a specified fraction of the population are effectively vaccinated. If vaccination yields an indirect preventive effect, then $p(\cdot)$ is a decreasing function. When the effective vaccination rate is $\lambda v$, the fraction of the population who become ill is $p(\lambda v)(1 - \lambda v)$.

The planner wants to minimize a social cost function with two additive components, the cost of illness and the cost of vaccination. Let $b > 0$ denote the cost resulting from a case of illness and let $c > 0$ denote the cost per vaccination, measured in commensurate units. The cost of vaccination rate $v$ is

\begin{equation}
S(v) = bp(\lambda v)(1 - \lambda v) + cv.
\end{equation}

The first term measures the social cost of illness and the second gives the social cost of vaccination.
A decision to mandate a vaccine poses a constrained optimization problem in which the planner chooses between vaccinating the entire population (the mandate) and the vaccination rate, say $v_d$, generated by decentralized decision making in the absence of the mandate.\(^4\) A mandate is consequential if and only if $v_d < 1$. Given social cost function (1), the optimal decision is

\[
\text{(2) mandate if } \quad \text{bp}(\lambda)(1 - \lambda) + c \leq \text{bp}(\lambda v_d)(1 - \lambda v_d) + cv_d,
\]

\[
\text{do not mandate if } \quad \text{bp}(\lambda)(1 - \lambda) + c \geq \text{bp}(\lambda v_d)(1 - \lambda v_d) + cv_d.
\]

This simple static optimization problem expresses the core tension of vaccination policy: the higher vaccination rate achieved by a mandate reduces illness relative to decentralized decision making but it raises the cost of vaccination.\(^5\)

A special case of this planning problem was studied in Brito, Sheshinski, and Intriligator (1991). However, it has been more common in research on vaccination policy to pose a susceptible-infectious-removed (SIR) or other dynamic model of disease transmission and to assume that the social objective is to keep the transmission rate below the threshold at which an epidemic occurs. See, for example, Ball and Lyne (2002) and Hill and Longini (2003).

The objective of preventing onset of an epidemic differs from minimization of social cost. In

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\(^4\) The maintained assumption that a mandate would result in vaccination of the entire population is a simplifying idealization. In practice, mandates do not require vaccination of immunocompromised persons. States may grant religious exemptions to mandatory vaccination. Moreover, a state may lack the legal power to mandate vaccination of non-residents with whom residents may come into contact.

\(^5\) While this optimization problem expresses the core tension transparently, there are multiple reasons why a planner might want to specify a social cost function more complex than (1). For example, limited availability of medical personnel and capacity constraints in treatment facilities might make the social cost of illness a convex rather than linear function of the illness rate. Limited capacity for production of vaccines might make the social cost of vaccination a convex rather than linear function of the vaccination rate. Or society might consider enactment of a mandate to carry a fixed cost expressing loss of personal freedom, beyond the cost of increasing the vaccination rate per se.
epidemiology, an epidemic is defined to occur when the infected fraction of the population increases with
time. For example, Centers for Disease Control and Prevention (2006) states (p. 1-72) "Epidemic refers to
an increase, often sudden, in the number of cases of a disease above what is normally expected in that
population in that area."

In contrast, social cost function (1) abstracts from the dynamics of illness and considers the
prevalence of illness in the population. From the perspective of dynamic models of disease transmission,
\( p(\lambda)(1 - \lambda v) \) is the steady-state fraction of infectious persons. Classical SIR models imply that \( p(\cdot) \) is a
decreasing function that may attain the value zero at some effective vaccination rate less than one, say \( e^* \).
Hence, the disease is eradicated if the actual vaccination rate is greater than or equal to \( e^*/\lambda \), provided that
\( e^*/\lambda \leq 1 \). In this case, the steady-state fraction of infectious persons is zero. Vaccination rate \( e^*/\lambda \) is called
the critical vaccination or herd immunity threshold. See for example, Anderson and May (1991) and Fine
(1993).

Given its static nature, the analysis of this paper applies most directly to endemic diseases. It applies
as well to epidemic-prone diseases in which vaccination is performed ex ante with the objective of preventing
onset of epidemics and in which social cost depends mainly on the prevalence of illness in the population
rather than on the timing of new cases. A broad direction for extension of the analysis is to consider
epidemic-prone diseases in which social cost does depend on the timing of cases. This requires re-
specification of the social cost function to stipulate how timing matters.

2.2. Policy Choice without Knowledge of the Indirect Effect of a Mandate

A planner can make an optimal decision if he has sufficient knowledge of the social costs of
vaccination and illness, the direct and indirect effects of vaccination, and the vaccination rate that results
from decentralized decision making. Given criterion (2), it suffices to know \([b, c, \lambda, p(\lambda), v_d, p(\lambda v_d)]\). The
criterion is an inequality, so it is not necessary to know the precise values of these quantities. The planner only needs to know enough to determine which inequality holds.

I now assume that the planner knows \(b, c, \lambda, v_d, p(\lambda v_d)\) but not \(p(\lambda)\). The social cost parameters \(b\) and \(c\) express society's evaluation of the social cost of illness and vaccination. An RCT can measure the immune responses that a vaccine generates in vaccinated subjects, which provides evidence on \(\lambda\). In principle, the planner should be able to observe the decentralized vaccination rate \(v_d\) and illness rate \(p(\lambda v_d)\) that occur when decentralization is the status quo policy.

The planner cannot observe \(p(\lambda)\), which would become observable only after a mandate is enacted. In the absence of direct observation, what knowledge might a planner realistically have of \(p(\lambda)\)? A basic assumption of epidemiological analysis of infectious disease has been that raising the vaccination rate increases the indirect protective effect of vaccination.\(^6\) Hence, the indirect-response function \(p(\cdot)\) is decreasing. When combined with available empirical evidence, this monotonicity assumption yields an upper bound on \(p(\lambda)\). Knowing \(p(\lambda v_d)\), a planner can deduce that \(0 \leq p(\lambda) \leq p(\lambda v_d)\). This inequality applies the general idea of monotone treatment response (Manski, 1997).\(^7\)

Beyond the monotonicity assumption, epidemiological analysis yields no clearly credible assumption about the shape of \(p(\cdot)\). Alternative models of disease transmission imply different shapes for the function. Researchers have typically chosen models to be analytically and computationally tractable, with only limited examination of the realism of their assumptions. Hence, a planner may not know much about the shape of

\(^6\) Although this assumption has a strong foundation, it is not above question. Footnote 1 cited biological reasons why increasing the vaccination rate could, in principle, increase rather than decrease illness. There are possible behavioral reasons as well. An increase in the vaccination rate could induce persons to lessen the degree of care they exercise in their social contacts, increasing their susceptibility and infectiousness.

\(^7\) The analysis in this paper assumes that decentralization is the observed status quo policy and that mandatory vaccination is a counterfactual policy. Suppose instead that mandatory vaccination were the observed status quo and that decentralization were a counterfactual. It would then be realistic to assume that the planner knows \([b, c, \lambda, p(\lambda)\]) but not \([v_d, p(\lambda v_d)\]). The monotonicity assumption implies that \(p(\lambda v_d)\) lies in the bound \(p(\lambda) \leq p(\lambda v_d) \leq 1\). Assumption of a behavioral model would be required to predict \(v_d\). Thus, the two possibilities for the status quo policy place the planner in asymmetric informational situations.
p(\cdot) beyond its monotonicity. I assume no further knowledge here.  

2.2.1. Optimization with Partial Knowledge

Although p(\lambda) is unknown, knowledge of the upper bound may suffice to determine whether mandatory vaccination is optimal. A key parameter is \gamma = c/b, the ratio of the social cost per vaccination to the social cost per case of illness. Rewriting criterion (2), a mandate is optimal if

\begin{equation}
(2') \quad p(\lambda)(1 - \lambda) \leq p(\lambda v_d)(1 - \lambda v_d) - \gamma(1 - v_d)
\end{equation}

and is sub-optimal otherwise. The bound 0 \leq p(\lambda) \leq p(\lambda v_d) suffices to determine the optimality of a mandate if either of two conditions hold:

\begin{align}
(3a) & \quad p(\lambda v_d)(1 - \lambda) \leq p(\lambda v_d)(1 - \lambda v_d) - \gamma(1 - v_d) \quad \rightarrow \text{mandate is optimal,} \\
(3b) & \quad 0 > p(\lambda v_d)(1 - \lambda v_d) - \gamma(1 - v_d) \quad \rightarrow \text{mandate is not optimal.}
\end{align}

The inequality on the left-hand side of (3a) implies that inequality (2') holds for all feasible values of p(\lambda); hence, a mandate must be optimal. The inequality on the left-hand side of (3b) implies that (2') holds for no feasible value of p(\lambda); hence, a mandate cannot be optimal. If neither inequality holds, a mandate is optimal.

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8 In some cases, a planner may judge available epidemiological models of disease transmission to be sufficiently credible that he thinks it reasonable to use the models to predict p(\lambda). In the best case, a planner might use a model to make a point prediction and view the prediction as accurate. However, it typically is unrealistic to view any point prediction as accurate. It may be more credible to use epidemiological models to tighten the bound 0 \leq p(\lambda) \leq p(\lambda v_d). One might make predictions using multiple a priori plausible models, determine the range of findings, and use this range as the bound.

Studies of disease transmission sometimes report sensitivity analyses that conjecture multiple parameter values within a specified parametric model. See, for example, Weycker et al. (2005). As far as I am aware, epidemiological research has not sought to generate set-valued predictions conjecturing a broad set of plausible models, as is the norm in partial identification analysis (Manski, 2007).
for some feasible values of \( p(\lambda) \) but not for others.

Conditions (3a) and (3b) show that a planner lacking knowledge of the indirect effect of vaccination may nonetheless be able to determine if a mandate is optimal. One or the other condition necessarily holds if \( \lambda = 1 \). In this case, \( p(\lambda)(1 - \lambda) = 0 \) for all values of \( p(\lambda) \). Hence, indeterminacy of the optimum can happen only if the vaccine has an imperfect direct effect.

To study indeterminate cases when \( \lambda < 1 \), it is instructive to rewrite (3) in the equivalent form

\[
\begin{align*}
(3a') & \quad \gamma \leq \lambda p(\lambda v_d) \quad \rightarrow \text{mandate is optimal}, \\
(3b') & \quad \gamma > p(\lambda v_d)[(1 - \lambda v_d)/(1 - v_d)] \quad \rightarrow \text{mandate is not optimal}.
\end{align*}
\]

Condition (3a') considers the worst-case scenario in which increasing the vaccination rate from \( v_d \) to 1 has no indirect preventive effect. The inequality shows that, all else equal, a mandate is still optimal if the cost of vaccination relative to the cost of illness is sufficiently small. Condition (3b') considers the best-case scenario in which increasing the vaccination from \( v_d \) to 1 has the strongest possible indirect effect. This inequality shows that, all else equal, a mandate cannot be optimal if \( \gamma \) is sufficiently large. The optimality of a mandate is indeterminate if \( \gamma \) has an intermediate value; that is, if

\[
\lambda p(\lambda v_d) < \gamma \leq p(\lambda v_d)[(1 - \lambda v_d)/(1 - v_d)].
\]

### 2.2.2. Decision Making When the Optimal Policy is Indeterminate

It remains to consider decision making when the optimality of a mandate is indeterminate. Decision theory gives no consensus prescription, but it suggests various reasonable criteria.

When economists have studied planning with partial knowledge, it has been standard to assert a
subjective probability distribution over unknown decision-relevant quantities (aka states of nature) and propose choice of an action that maximizes subjective expected utility. Researchers studying vaccination policy have recently begun to use this criterion to cope with uncertainty. Tanner et al. (2008) exposits it under the name stochastic programming.

In the present setting, one would minimize subjective expected cost. Let $S$ denote the set of states of nature, indexing feasible values of the unknown quantity $p(\lambda)$. Let $\pi$ denote the subjective distribution placed on $S$. For $s \in S$, let $p_s(\lambda)$ denote the value of $p(\lambda)$ in state $s$. Let $f(\cdot): \mathbb{R} \to \mathbb{R}$ be a specified increasing function expressing the risk preference of the planner. Then minimization of subjective expected cost means replacement of optimality criterion (2) with the computable criterion

$$\int_S f[p_s(\lambda)(1 - \lambda) + \gamma]d\pi \leq f[p(\lambda v_d)(1 - \lambda v_d) + \gamma v_d],$$  
mandate if

$$\int_S f[p_s(\lambda)(1 - \lambda) + \gamma]d\pi \geq f[p(\lambda v_d)(1 - \lambda v_d) + \gamma v_d],$$  
do not mandate if

Criterion (5) simplifies if $f(\cdot)$ is the identity function, in which case the planner is risk neutral. It reduces to

$$\int_S p_s(\lambda)d\pi(1 - \lambda) + \gamma \leq p(\lambda v_d)(1 - \lambda v_d) + \gamma v_d,$$  
mandate if

$$\int_S p_s(\lambda)d\pi(1 - \lambda) + \gamma \geq p(\lambda v_d)(1 - \lambda v_d) + \gamma v_d,$$  
do not mandate if

Thus, a risk-neutral planner acts as if the unknown $p(\lambda)$ equals its known subjective mean $\int_S p_s(\lambda)d\pi$.

Use of the expected utility criterion to make policy choices with partial knowledge is reasonable when a planner has a credible basis for asserting a subjective probability distribution on unknown quantities. However, a subjective probability distribution is a form of knowledge, and a planner may not have a credible basis for asserting one. I have previously studied various problems of this type, where a planner faces ambiguity; Manski (2011) reviews basic ideas and gives applications. In particular, Manski (2010) studied
choice of a vaccination rate under ambiguity, focusing on the minimax and the minimax-regret criteria.

In my earlier work and here as well, I refer to minimax and minimax-regret as “reasonable” decision criteria because there is no uniquely correct way to choose when optimal policy is indeterminate. After all, the crux of the problem in decision making under ambiguity is that the planner does not know which action is best. Wald (1950), who studied minimax in abstraction, wrote (p. 18): “a minimax solution seems, in general, to be a reasonable solution of the decision problem.” Savage (1951), who introduced the minimax-regret criterion, differed with Wald, writing (p. 63): “Application of the minimax rule . . . is indeed ultra-pessimistic; no serious justification for it has ever been suggested.” Savage observed with favor that the minimax-regret criterion is not similarly “ultra-pessimistic.” Reflecting this distinction, modern writers often use the term "ambiguity averse" when discussing the minimax-regret criterion but the term is less descriptive of the minimax-regret criterion.

The minimax and minimax-regret criteria are complex to evaluate when the planner can choose any vaccination rate, but they simplify when he chooses between decentralized and mandatory vaccination. A planner who uses the minimax criterion and who assumes that $p(\cdot)$ is monotone acts as if he faces the worst-case scenario in which increasing the vaccination rate from $v_d$ to 1 has no indirect preventive effect. Thus, he acts as if $p(\lambda) = p(\lambda v_d)$. This done, he replaces optimality criterion (2) with the computable criterion

\begin{align*}
\text{(7)} & \quad \text{mandate if } \quad p(\lambda v_d)(1 - \lambda) + \gamma \leq p(\lambda v_d)(1 - \lambda v_d) + \gamma v_d, \\
& \quad \text{do not mandate if } \quad p(\lambda v_d)(1 - \lambda) + \gamma \geq p(\lambda v_d)(1 - \lambda v_d) + \gamma v_d.
\end{align*}

This criterion simplifies to

\begin{align*}
\text{(7')} & \quad \text{mandate if } \quad \gamma \leq \lambda p(\lambda v_d), \\
& \quad \text{do not mandate if } \quad \gamma \geq \lambda p(\lambda v_d).
\end{align*}
Thus, the minimax criterion resolves indeterminacy in favor of no mandate.

The minimax-regret criterion differs mathematically from the minimax criterion and it may yield a different policy decision when the optimal policy is indeterminate. To form the criterion, let $S_m$ and $S_d$ respectively be the subsets of $S$ in which a mandate yields strictly lower and higher social cost than decentralized decision making. For states in $S_m$, regret is zero if the planner chooses the mandate, which is optimal in these states. Regret in state $s \in S_m$ takes the positive value $[p(\lambda v_d)(1 - \lambda v_d) + \gamma v_d] - [p_s(\lambda)(1 - \lambda) + \gamma]$ if he chooses not to mandate the vaccine. Analogously, for states in $S_d$, regret is zero if the planner chooses not to mandate the vaccine and takes the positive value $[p_s(\lambda)(1 - \lambda) + \gamma] - [p(\lambda v_d)(1 - \lambda v_d) + \gamma v_d]$ if he choose to mandate it.

Maximum regret in each subset of states is

(8a) $R_m = \max_{s \in S_m} [p(\lambda v_d)(1 - \lambda v_d) + \gamma v_d] - [p_s(\lambda)(1 - \lambda) + \gamma],$

(8b) $R_d = \max_{s \in S_d} [p_s(\lambda)(1 - \lambda) + \gamma] - [p(\lambda v_d)(1 - \lambda v_d) + \gamma v_d].$

The maximum regret of mandate and no-mandate decisions across all states are $R_d$ and $R_m$ respectively.

Hence, a minimax-regret decision is

(9) mandate if $R_d \leq R_m$, 
do not mandate if $R_d \geq R_m$.

To obtain an explicit solution, observe that the maximum in (8a) occurs in the best-case scenario for vaccination, where $p_s(\lambda) = 0$. The maximum in (8b) occurs in the worst-case scenario where $p_s(\lambda) = p(\lambda v_d)$. Hence,
\[ (10a) \quad R_m = p(\lambda v_d)(1 - \lambda v_d) - \gamma(1 - v_d), \]

\[ (10b) \quad R_d = (1 - v_d)[\gamma - \lambda p(\lambda v_d)]. \]

Inserting these values into (9) yields the criterion

\[ (11) \quad \text{mandate if} \quad 2\gamma(1 - v_d) \leq p(\lambda v_d)[1 - 2\lambda v_d + \lambda], \]

\[ \text{do not mandate if} \quad 2\gamma(1 - v_d) \geq p(\lambda v_d)[1 - 2\lambda v_d + \lambda]. \]

Comparison of (11) with (7') shows that the minimax and minimax-regret criteria are broadly similar but differ in their specifics. The numerical example below gives a sense of the specifics.

2.2.3. Example: Seasonal Influenza

To illustrate these findings, consider seasonal influenza in the United States in the 2013-2014 influenza season. The U.S. Centers for Disease Control and Prevention (CDC) reports that 42% of the adult population received the flu vaccine and that the vaccine was 60% effective in preventing illness serious enough to warrant a visit to a physician. These statistics suggest use of the parameter values \( v_d = 0.42 \) and \( \lambda = 0.60 \). I have not been able to find a corresponding estimate of the fraction of American adults who visited a physician to treat influenza in the 2013-2004 season, but the CDC separately reports that each year between 5\% and 20\% of the American population contract the flu. If we use 10\% as the gross rate of influenza serious enough to warrant a physician visit, this implies that \( 0.10 = p(\lambda v_d)(1 - \lambda)v_d + p(\lambda v_d)(1 - v_d) \). The first and second terms on the right-hand side are the illnesses occurring among vaccinated and non-vaccinated

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persons respectively. Inserting the estimates for \((v_d, \lambda)\) into this equation yields the estimate \(p(\lambda v_d) = 0.13\).

Given these estimates, a planner applying (3') can conclude that mandatory vaccination against seasonal influenza in 2013-14 would have been optimal if \(\gamma \leq 0.08\) and not optimal if \(\gamma > 0.17\). Optimal policy is indeterminate if \(0.08 < \gamma \leq 0.17\). Thus, mandatory vaccination is optimal (non-optimal) if society evaluates the social cost of vaccinating a person to be less than 8% (more than 17%) the social cost of a person contracting a case of influenza serious enough to warrant a physician visit.

In cases of indeterminacy, policy choice depends on the decision criterion that the planner uses. The expected utility criterion (5) can yield either policy depending on the transformation \(f(\cdot)\) and subjective distribution \(\pi\). A planner using the minimax criterion (7') would mandate vaccination if \(\gamma \leq 0.08\) but not if \(\gamma > 0.08\). One using the minimax-regret criterion (11) would enact the mandate if \(\gamma \leq 0.12\) but not if \(\gamma > 0.12\).

2.3. Utilitarian Planning When Agents are Risk-Neutral with Rational Expectations

The above analysis required no knowledge of the process that generates the decentralized vaccination rate. An interesting special case occurs when the planner is utilitarian and the representative agent is risk neutral with rational expectations. Then the representative private cost function with decentralized decision making is

\[
\begin{align*}
(12) \quad k(z) &= b(p \lambda v_d)(1 - \lambda z) + cz, \\
\end{align*}
\]

where \(z = 1\) if an agent is vaccinated and \(z = 0\) otherwise. Averaging \(k(z)\) over the population yields social cost function (1), the vaccination rate \(v\) being the fraction of persons who are vaccinated.

In a decentralized decision regime, each agent chooses \(z\) taking the decentralized vaccination rate \(v_d\) as given. An optimal private vaccination choice minimizes \(k(z)\) and, hence, satisfies these conditions:
(13a) \[ bp(\lambda v_d) < bp(\lambda v_d)(1 - \lambda) + c \rightarrow z = 0, \]
(13b) \[ bp(\lambda v_d) > bp(\lambda v_d)(1 - \lambda) + c \rightarrow z = 1. \]

Equivalently,

(13a') \[ \gamma > \lambda p(\lambda v_d) \rightarrow z = 0, \]
(13b') \[ \gamma < \lambda p(\lambda v_d) \rightarrow z = 1. \]

The representative agent is indifferent between the two choices if

(14) \[ \gamma = \lambda p(\lambda v_d). \]

The decentralized vaccination rate \( v_d \) is determined by averaging the choices of population members.

It follows from (13') that

(15a) \[ \gamma > \lambda p(0) \rightarrow v_d = 0, \]
(15b) \[ \gamma < \lambda p(\lambda) \rightarrow v_d = 1. \]

In an intermediate case with \( \lambda p(\lambda) \leq \gamma \leq \lambda p(0) \), there exists a unique equilibrium value of \( v_d \) if the indirect-response function \( p(\cdot) \) is continuous and strictly decreasing on its domain \([0, \lambda]\). Then \( v_d \) solves equation (14). In equilibrium, agents are indifferent between the two choices. The fraction \( v_d \) solving (14) choose to be vaccinated.

It is interesting to examine the optimality of a mandate in the present context. A mandate is inconsequential when (15b) holds so I focus on the other cases.
When (14) holds, combining (2′) and (14) shows that a mandate is optimal if and only if

\[(16) \quad p(\lambda)(1 - \lambda) \leq p(\lambda v_d)(1 - \lambda v_d) - \lambda p(\lambda v_d)(1 - v_d)\]

or, equivalently,

\[(16') \quad p(\lambda) \leq p(\lambda v_d).\]

Given that \(p(\cdot)\) is a decreasing function, inequality (16′) always holds; hence, a mandate is always optimal. This finding is intuitive: agents are privately indifferent regarding vaccination and a mandate yields a positive externality.

When (15a) holds, knowledge that \(p(\cdot)\) is decreasing may not suffice to determine whether a mandate is optimal. In this case, \(v_d = 0\) so (3′) reduces to

\[(17a) \quad \gamma \leq \lambda p(0) \rightarrow \text{mandate is optimal},\]
\[(17b) \quad \gamma > p(0) \rightarrow \text{mandate is not optimal}.\]

Given that (15a) contradicts (17a), the planner cannot be certain that a mandate is optimal. He can conclude that a mandate is not optimal if \(\gamma > p(0)\). The optimality of a mandate is indeterminate if \(\lambda p(0) < \gamma \leq p(0)\).

3. Policy Choice with Heterogeneous Agents

The analysis in this section continues to assume a large population, but now permits persons to have
heterogeneous attributes. To formalize this, let \( J \) denote the set of persons constituting the population. To enable probabilistic statements, I view \( J \) as a probability space \( (J, \Omega, P) \). The assumption of a large population means that \( J \) is uncountable with \( P(j) = 0, j \in J \). Whereas I earlier assumed that \( [\lambda, p(\cdot), b, c] \) are identical across persons, now each \( j \in J \) has person-specific values \( [\lambda_j, p_j(\cdot), b_j, c_j] \). The population distribution of attributes is \( P[\lambda, p(\cdot), b, c] \).

The indirect-response function \( p_j(\cdot): [0, 1] \rightarrow [0, 1] \) gives the probability that person \( j \), if susceptible to disease, becomes ill when a specified fraction of the population are effectively vaccinated. Whereas the effective vaccination rate previously was \( \lambda v \), the present analysis requires attention to the composition of the vaccinated sub-population. Let \( z_j = 1 \) denote that person \( j \) is vaccinated and \( z_j = 0 \) otherwise. Then the vaccination rate is \( E(z) = \int z dP(j) \) and the effective rate is \( E(\lambda z) = \int \lambda z dP(j) \).

A more general specification of indirect response would have \( p_j(\cdot) \) be a function not only of the effective vaccination rate \( E(\lambda z) \) but of the specific configuration \( [(\lambda_k, z_k), k \in J] \) of \( (\lambda, z) \) across the population. This more general specification would enable analysis of situations in which persons have heterogeneous indirect effects on the illness of others, transmitted through their social networks. It would also open the possibility that persons vary their social interactions depending on whether they and others in their networks are vaccinated, as discussed in Fenichel et al. (2011). Although permitting such heterogeneity would further enhance the realism of this analysis, I suppress it because it substantially complicates study of vaccination policy.

The organization of this section emulates Section 2. Section 3.1 sets up the optimization problem. Section 3.2 studies policy choice when the planner does not know the indirect effect of vaccination. Section 3.3 supposes that the planner is utilitarian and that agents are risk-neutral with rational expectations.

3.1. The Optimization Problem
As earlier, the planner wants to minimize a social cost function with two components, the social cost of illness and vaccination. Consider any vaccination configuration \((z_j, j \in J)\). For each \(j\), the social cost of illness is \(b_j\) and vaccination is \(c_j\). The probability that \(j\) becomes ill is \(p_j[E(\lambda z)](1 - \lambda)\) if \(j\) is vaccinated and \(p_j[E(\lambda z)]\) if unvaccinated. Hence, the full contribution of \(j\) to social cost is \(b_j p_j[E(\lambda z)](1 - \lambda z_j) + c_j z_j\).

Averaging the person-specific contributions to social cost across the population yields the aggregate social cost function

\[
S(z_j, j \in J) = \int \{b_j p_j[E(\lambda z)](1 - \lambda z_j) + c_j z_j\} \, dP(j) = E\{b_j[E(\lambda z)](1 - \lambda z_j)\} + E(cz).
\]

A decision to mandate a vaccine poses a constrained optimization problem in which the planner chooses between vaccinating the entire population \((z_j = 1, j \in J)\) and the vaccination configuration \((z_dj, j \in J)\) generated by decentralized decision making. Given social cost function (18), the optimal decision is

\[
\text{mandate if } E\{b_j[E(\lambda)](1 - \lambda)\} + E(c) \leq E\{b_j[E(\lambda z_d)](1 - \lambda z_d)\} + E(cz_d),
\]
\[
do not mandate if \quad E\{b_j[E(\lambda)](1 - \lambda)\} + E(c) > E\{b_j[E(\lambda z_d)](1 - \lambda z_d)\} + E(cz_d).
\]

Rewriting criterion (19), a mandate is optimal if and only if

\[
E[c(1 - z_d)] \leq E\{b_j[E(\lambda z_d)](1 - \lambda z_d)\} - E\{b_j[E(\lambda)](1 - \lambda)\}.
\]

The left-hand side of (19') expresses the disadvantage of a mandate relative to decentralization, namely the marginal social cost of vaccinating persons who choose not to be vaccinated in a decentralized regime. The right-hand side expresses the advantage of a mandate, namely the improvement in public health relative to decentralized decision making.
Derived from a welfare-economic representation of planning as an optimization problem, criterion (19') shows how society might choose a vaccination policy to reconcile the tension between personal freedom and public health. The quantity $E[\varepsilon(1 - z_d)]$ expresses the social valuation of the personal freedom lost when society decides to mandate vaccination. The quantity on the right-hand side of (19') expresses the social valuation of the associated improvement in public health. In utilitarian welfare analysis, the person-specific social costs ($b_j, c_j$) equal the private costs that person $j$ incurs with illness and vaccination. In non-utilitarian analyses, social costs may differ from private ones.

3.1.1. Majority Rule as an Alternative to Utilitarian Planning

As a short digression, it is interesting to compare utilitarian planning with the political economy mechanism in which majority vote determines whether society mandates a vaccine. Society might use a political process to choose policy if personal freedom looms large as an issue when considering mandates.

Suppose that members of the population want to minimize private cost, are risk-neutral, and have rational expectations. The private costs that person $j$ associates with decentralized decision making are

\begin{align}
(20a) & \quad k_{d_{j}}(0) = b_{j}p_{j}[E(\lambda z_{d})], \\
(20b) & \quad k_{d_{j}}(1) = b_{j}p_{j}[E(\lambda z_{d})](1 - \lambda_{j}) + c_{j}.
\end{align}

Here $k_{d_{j}}(1)$ and $k_{d_{j}}(0)$ are the privates cost that $j$ would incur if he would choose to be or not to be vaccinated. The private cost that person $j$ incurs under a mandate is

\begin{align}
(21) & \quad k_{m_{j}} = b_{j}p_{j}[E(\lambda_{j})](1 - \lambda_{j}) + c_{j}.
\end{align}

The monotonicity property of $p_{j}(\cdot)$ implies that $k_{m_{j}} \leq k_{d_{j}}(1)$. Hence, a mandate minimizes private cost
if and only if $k_{mj} \leq k_0(0)$. Aggregating across the population, the majority-vote rule yields a decision to mandate vaccination if

$$P[k_m \leq k_0(0)] > 0.5.$$  

This criterion differs from the utilitarian-planning criterion (19'). Note that the fraction of the population who would vote for a mandate exceeds the fraction who would choose vaccination in a decentralized regime.

3.2. Policy Choice without Knowledge of the Indirect Effect of Vaccination

3.2.1. Optimization with Partial Knowledge

Assume that the planner observes the population in the status quo context of decentralization and learns $P[b, c, \lambda, z_d, E(\lambda z_d)]$. The planner does not know the counterfactual illness rates that susceptible persons would incur under a mandate but, indirect effects being non-negative, he knows that $0 \leq p_j[E(\lambda)] \leq p_j[E(\lambda z_d)], j \in J$. These bounds determine the optimality of a mandate if either of two conditions hold:

$$E[c(1 - z_d)] \leq E\{bp[E(\lambda z_d)]\lambda(1 - z_d)\} \Rightarrow \text{mandate is optimal},$$

$$E[c(1 - z_d)] > E\{bp[E(\lambda)](1 - \lambda z_d)\} \Rightarrow \text{mandate is not optimal}.$$  

The inequality on the left-hand side of (20a) implies that inequality (19') holds for all feasible values of $\{p_j[E(\lambda)], j \in J\}$; hence, a mandate must be optimal. The inequality on the left-hand side of (20b) implies that (19') holds for no feasible value of $\{p_j[E(\lambda)], j \in J\}$; hence, a mandate cannot be optimal.

If neither inequality holds, a mandate is optimal for some feasible values of $\{p_j[E(\lambda)], j \in J\}$ but not for others. Thus, the optimality of a mandate is indeterminate if
3.2.2. Decision Making When the Optimal Policy is Indeterminate

Considered abstractly, minimization of subjective expected cost with a heterogeneous population is a straightforward generalization of the criterion with a representative agent. Let $S$ denote the set of states of nature, now indexing feasible values of the unknown quantities $\{p_j[E(\lambda)], j \in J\}$. Let $\pi$ denote the subjective distribution placed on $S$. Let $\{p_s[E(\lambda)], j \in J\}$ denote the value of $\{p_j[E(\lambda)], j \in J\}$ in state $s$. Let $f(\cdot): \mathbb{R} \rightarrow \mathbb{R}$ be a specified increasing function expressing the risk preference of the planner. Then minimization of subjective expected cost means replacement of optimality criterion (19) with the computable criterion

\begin{equation}
\int_S f[E\{bp_s[E(\lambda)](1 - \lambda)\} + E(c)]d\pi \leq f[E\{bp_s[E(\lambda)](1 - \lambda z_d)\} + E(cz_d)] , \tag{22} \end{equation}

mandate if

\begin{equation}
\int_S f[E\{bp_s[E(\lambda)](1 - \lambda)\} + E(c)]d\pi \geq f[E\{bp_s[E(\lambda)](1 - \lambda z_d)\} + E(cz_d)] , \tag{22} \end{equation}

do not mandate if

Although (22) is analogous to the earlier criterion (5) in abstraction, it is much more complex in practice. Whereas the representative-agent setting only required that one place a subjective distribution on the scalar unknown $p(\lambda)$, the planner now must place one on the collection of unknowns $\{p_j[E(\lambda)], j \in J\}$.

In abstraction and in practice, the minimax criterion with a heterogeneous population is a straightforward generalization of the criterion with a representative agent. Here as before, a planner who uses the minimax criterion acts as if he faces the worst-case scenario in which $E(\lambda) = E(\lambda z_d)$. This done, he replaces optimality criterion (19) with the computable criterion

\begin{equation}
(21) \quad E\{bp[E(\lambda z_d)](1 - z_d)\} < E[c(1 - z_d)] \leq E\{bp[E(\lambda z_d)](1 - \lambda z_d)\}. \end{equation}
mandate if \[
E\{bp[E(\lambda z_d)](1 - \lambda)\} + E(c) \leq E\{bp[E(\lambda z_d)](1 - \lambda z_d)\} + E(c z_d),
\]
do not mandate if \[
E\{bp[E(\lambda z_d)](1 - \lambda)\} + E(c) \geq E\{bp[E(\lambda z_d)](1 - \lambda z_d)\} + E(c z_d).
\]

Equivalently, a mandate is a minimax solution to the planning problem if and only if

\[
(23') \quad E[c(1 - z_d)] \leq E\{bp[E(\lambda z_d)]\lambda(1 - z_d)\}.
\]

As previously, the minimax criterion resolves indeterminacy in favor of no mandate.

To form the minimax-regret criterion, again let \( S_m \) and \( S_d \) be the subsets of \( S \) in which a mandate yields strictly lower and higher social cost than decentralized decision making. For \( s \in S_m \), regret is zero if the planner mandates vaccination and equals \( E\{bp[E(\lambda z_d)](1 - \lambda z_d)\} - E\{bp[E(\lambda)](1 - \lambda)\} - E[c(1 - z_d)] \) if he does not. Analogously, for \( s \in S_d \), regret is zero if the planner does not mandate and equals \( E\{bp_s[E(\lambda)](1 - \lambda)\} - E\{bp[E(\lambda z_d)](1 - \lambda z_d)\} + E[c(1 - z_d)] \) if he does.

Maximum regret in each subset of states is

\[
(24a) \quad R_m = \max_{s \in S_m} E\{bp[E(\lambda z_d)](1 - \lambda z_d)\} - E\{bp_s[E(\lambda)](1 - \lambda)\} - E[c(1 - z_d)],
\]

\[
(24b) \quad R_d = \max_{s \in S_d} E\{bp_s[E(\lambda)](1 - \lambda)\} - E\{bp[E(\lambda z_d)](1 - \lambda z_d)\} + E[c(1 - z_d)].
\]

Mandating is a minimax-regret decision if and only if \( R_d \leq R_m \).

The maximum in (24a) occurs in the best-case scenario for vaccination, where \( p_d[E(\lambda)] = 0 \). The maximum in (24b) occurs in the worst-case scenario where \( p_d[E(\lambda)] = p[E(\lambda v_d)] \). Hence,

\[
(25a) \quad R_m = E\{bp[E(\lambda z_d)](1 - \lambda z_d)\} - E[c(1 - z_d)],
\]

\[
(25b) \quad R_d = E[c(1 - z_d)] - E\{bp[E(\lambda v_d)]\lambda(1 - z_d)\}.
\]
Hence, the minimax-regret criterion is

\[(26)\quad \text{mandate if } \quad 2E[c(1 - z_d)] \leq E\{bp[E(\lambda z_d)](1 - 2\lambda z_d + \lambda)\}.
\]

\[(26)\quad \text{do not mandate if } \quad 2E[c(1 - z_d)] \geq E\{bp[E(\lambda z_d)](1 - 2\lambda z_d + \lambda)\}.
\]

This is a straightforward generalization of the criterion with a representative agent.

### 3.3. Utilitarian Planning When Persons are Risk-Neutral with Rational Expectations

Suppose that the planner is utilitarian and that members of the population are risk neutral with rational expectations. Then the private cost function of person j in the decentralized regime is

\[(27)\quad k_j(z_j) = b_j p_j[E(\lambda z_d)](1 - \lambda_j z_j) + c_j z_j.
\]

Averaging \(k_j(z_j)\) over the population yields social cost function (19).

An optimal private vaccination choice minimizes \(k_j(z_j)\) and, hence, satisfies these conditions:

\[(28a)\quad b_j p_j[E(\lambda z_d)] < b_j p_j[E(\lambda z_d)](1 - \lambda_j) + c_j \quad \rightarrow \quad z_j = 0,
\]

\[(28b)\quad b_j p_j[E(\lambda z_d)] > b_j p_j[E(\lambda z_d)](1 - \lambda_j) + c_j \quad \rightarrow \quad z_j = 1.
\]

Equivalently,

\[(28a')\quad c_j > b_j \lambda_j p_j[E(\lambda z_d)] \quad \rightarrow \quad z = 0,
\]

\[(28b')\quad c_j < b_j \lambda_j p_j[E(\lambda z_d)] \quad \rightarrow \quad z = 1.
\]
A person is indifferent between the two choices if

\[ c_j = b_j p_j [E(\lambda z_d)]. \]  

If the population attribute distribution \( P[\lambda, p(\cdot), b, c] \) is continuous, indifference occurs with probability zero.

The effective vaccination rate \( E(\lambda z_d) \) is determined by averaging the choices of population members. If indifference occurs with probability zero, a rational expectations equilibrium occurs when \( E(\lambda z_d) \) solves the equation

\[ E(\lambda z_d) = E \{ \lambda \cdot 1[c < b\lambda p[E(\lambda z_d)]] \}. \]

To characterize equilibrium, observe that \( E(\lambda z_d) \) ranges over the interval \([0, E(\lambda)]\), with \( E(\lambda z_d) = 0 \) if no one vaccinates and \( E(\lambda z_d) = E(\lambda) \) if everyone vaccinates. As \( E(\lambda z_d) \) increases from 0 to \( E(\lambda) \), the left-hand side of (30) commensurately rises from 0 to \( E(\lambda) \) and the right-hand side falls from \( E \{ \lambda \cdot 1[c < b\lambda p(0)] \} \) to \( E \{ \lambda \cdot 1[c < b\lambda p[E(\lambda)]] \} \). It follows that no one vaccinates in equilibrium if \( P[c < b\lambda p(0)] = 0 \) and everyone vaccinates if \( P[c < b\lambda p[E(\lambda)]] = 1 \). The equilibrium equation has a unique interior solution if \( 0 < P[c < b\lambda p(0)], P[c < b\lambda p[E(\lambda)]] < 1 \), and the distribution \( P[\lambda, p(\cdot), b, c] \) is continuous.

Brito, Sheshinski, and Intriligator (1991) observed that mandates are not optimal if the planner is utilitarian, population members are risk-neutral with rational expectations, and the vaccine is perfectly effective. This holds in the present setting if indifference occurs with probability zero.

Perfect effectiveness means that \( \lambda_j = 1, \) all \( j \in J \). In this case, (19′) shows that a mandate is optimal if and only if

\[ E[c(1 - z_d)] \leq E \{ bp[E(\lambda z_d)](1 - z_d) \}. \]
Condition (31) is not satisfied because, by (28′) and the absence of indifference,

\( E[c(1 - z_d)] > E\{bp[E(\lambda z_d)](1 - z_d) \} \).

Thus, a mandate is not optimal under the assumptions maintained by Brito, Sheshinski, and Intriligator.

4. Conclusion

This paper has developed a welfare-economic framework for choice between decentralized and mandatory vaccination policies when the magnitude of the indirect effect of vaccination on illness is unknown. Study of the representative-agent setting yielded simple findings that should be easy to implement by public health agencies. Analysis of policy choice when population members have heterogeneous attributes required more abstract notation but also yielded explicit findings.

Policymakers may find most congenial the finding that observation of the decentralized regime may suffice to determine optimal policy even in the absence of knowledge of the illness rate that would occur with a mandate. When optimal policy is indeterminate, I did not assert that policymakers should use a specific decision criterion. Instead, I sought to assist decision making by juxtaposing several well-known criteria and comparing the policies they generate. Whether a public health agency should use one of these or some other criterion to cope with uncertainty is ultimately a matter to be decided by the agency.

Beyond the specific analysis performed here, both this paper and my earlier study of a distinct planning problem (Manski, 2010) emphasize that study of vaccination policy should explicitly recognize uncertainty. Predicting the outcomes of alternative policies requires contemplation of counterfactuals. The point predictions of counterfactuals often presented in research on vaccination policy are obtained by
combining available data with untenably strong assumptions about the manner in which biology and behavior interact to determine disease transmission. The value of vaccine research to policy formation would be enhanced if researchers would combine data with weaker assumptions that have greater credibility, present the range of predictions that emerge, and then consider the resulting problem of policy choice under uncertainty. This paper and my earlier study illustrate how such research may be performed.
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