1 Overvaluation and multiple equilibria

• We build on the model developed in Notes 2
• Suppose the central bank is committed to a fixed exchange rate
  \[ E_t = \overline{E} \]

• We want to study how this commitment can come under attack, if inflation expectations are out of line
• Consider a version of the model with two groups of firms
  • A mass \( \alpha \) cannot change price, price is pre-set at \( \bar{P} \)
  • A mass \( 1 - \alpha \) (flex price firms) can change price at date 0

Game at date 0:
  – Flex price firms set price \( \hat{P}_{h0} \) forming expectations about \( C_0 \) and \( N_0 \)
  – Central bank sets \( i_0 \) and \( E_0 \) and quantities are determined

• When setting \( \hat{P}_{h0} \) firms are also forming expectations about other firms’ prices

1.1 Equilibrium

• Backward induction, given \( \hat{P}_{h0} \) solve the central bank problem
• Price of home good is
  \[ P_{h0} = \left( \alpha \bar{P}_h^{1-\varepsilon} + (1 - \alpha) \hat{P}_{h0}^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}} \]

• Given total demand \( Y_0 \) for home goods the demand for the goods produced by fix and flex firms are
  \[ \left( \frac{\bar{P}}{P_{h0}} \right)^{-\varepsilon} Y_0 \text{ and } \left( \frac{\hat{P}_{h0}}{P_{h0}} \right)^{-\varepsilon} Y_0 \]
• So aggregating and using linearity of the technology we have that total labor demand is

\[ N_0 = J_0 Y_0 \]

where

\[ J_0 \equiv \alpha \left( \frac{\hat{P}_h}{P_{h0}} \right)^{-\varepsilon} + (1 - \alpha) \left( \frac{\hat{P}_{h0}}{P_{h0}} \right)^{-\varepsilon} \]

• By choosing the nominal interest the central bank can choose any triple \( C_0, p_0 \) and \( Y_0 \) that satisfies

\[ C_0 = p_0^{-\omega} \]
\[ Y_0 = p_0^{-1} \]

exactly as in Notes 2

• Moreover the value of \( B_1 \) and the continuation welfare are independent of central bank policy so we can focus on welfare at date 0

\[ U_0 = \log C_0 - \frac{\psi}{1 + \phi} N_0^{1+\phi} \]

• Expressing it in terms of \( Y_0 \) we have

\[ \omega \log Y_0 - \frac{\psi}{1 + \phi} (J_0 Y_0)^{1+\phi} \]

• If the central bank decides to float, its optimality condition is

\[ \frac{\omega}{Y_0} = \psi J_0^{1+\phi} Y_0^{-\phi} \]

• That is, the central bank best response is

\[ Y_0 = (\omega/\psi)^{1+\phi} J_0^{-1} \]

• If central bank sticks to peg then

\[ p_0 = \frac{P_{h0}}{\bar{E}} \]

• Gain from floating

\[ \Delta W(\hat{P}_h) = \max_Y \left\{ \omega \log Y - \frac{\psi}{1 + \phi} \left( J(\hat{P}_h)Y \right)^{1+\phi} \right\} - \left[ \omega \log \hat{Y}(\hat{P}_h) - \frac{\psi}{1 + \phi} \left( J(\hat{P}_h)\hat{Y}(\hat{P}_h) \right)^{1+\phi} \right] \]
• Go backward to price setters optimality
• Price setters choose prices in anticipation of $C_0, N_0, E_0$
• Optimality of price setters, together with equilibrium wages
\[ \hat{P}_{h0} = P_0 C_0 N_0^\phi \]
where
\[ P_0 = P_{h0}^\omega e_0^{1-\omega} \]
• Assume
\[ \omega = \psi \]
so if $\hat{P}_{h0} = \hat{P}_h = P_{h0}$ it is optimal for the central bank to implement the flexible price allocation
\[ Y_0 = C_0 = p_0 = 1 \]
• Assume
\[ \bar{P}_h/\bar{E} > 1 \]
so currency is initially overvalued

1.2 Multiple equilibria
• Conjecture: equilibrium with
\[ \hat{P}_{h0} = \hat{P}_h = P_{h0} \]
• Then $J_0 = 1$ and gain from floating is
\[ \Delta W_{float} = \omega \log 1 - \frac{\psi}{1 + \phi} - \left[ \omega \log \frac{\bar{E}}{\bar{P}_h} - \frac{\psi}{1 + \phi} \left( \frac{\bar{E}}{\bar{P}_h} \right)^{1+\phi} \right] \]
• Price setters optimality holds because they expect $C_0 = N_0 = 1$ and $E_0 = P_{h0} = \hat{P}_h$
\[ \hat{P}_{h0} = P_0 C_0 N_0^\phi \]
where
\[ P_0 = P_{h0}^\omega e_0^{1-\omega} = \hat{P}_h \]
• Suppose cost of floating is $\kappa$ and satisfies
\[ \kappa < \Delta W_{float} \]
then we have an equilibrium
• Can we have also an equilibrium with fixed exchange rates?
• Now price setters anticipate

\[ C_0 = \left( \frac{\hat{\xi}}{\hat{P}_{h0}} \right)^{\omega} \]

\[ Y_0 = \frac{\hat{\xi}}{\hat{P}_{h0}} \]

and

\[ J_0 = \left[ \alpha \hat{P}_{h0}^{-\epsilon} + (1 - \alpha) \hat{P}_{h0}^{-\epsilon} \right] P_{h0}^{\epsilon} \]

and

\[ P_0 = P_{h0}^{\omega} \hat{\xi}^{1-\omega} \]

• So we have

\[ \hat{P}_{h0} = P_0 C_0 N_0^\phi = P_{h0}^{\omega} \hat{\xi}^{1-\omega} \left( \frac{\hat{\xi}}{\hat{P}_{h0}} \right)^{\omega} \left( \left[ \alpha \hat{P}_{h0}^{-\epsilon} + (1 - \alpha) \hat{P}_{h0}^{-\epsilon} \right] P_{h0}^{\epsilon} \frac{\hat{\xi}}{\hat{P}_{h0}} \right)^{\phi} = \]

\[ = \hat{\xi}^{1+\phi} \left( \frac{\alpha \hat{P}_{h0}^{-\epsilon} + (1 - \alpha) \hat{P}_{h0}^{-\epsilon}}{\alpha \hat{P}_{h0}^{1-\epsilon} + (1 - \alpha) \hat{P}_{h0}^{1-\epsilon}} \right)^{\phi} \]

• Graphically we can see this has unique fixed point and

\[ \hat{P}_{h0} < \hat{\xi} < \hat{P}_h \]

which implies

\[ \frac{P_{h0}}{\hat{\xi}} < \frac{\hat{P}_{h0}}{\hat{\xi}} \]

• So output if fixed expected and fixed is realized is higher than output if float is expected and fixed is realized

• If fixed is expected there is some internal devaluation that helps

• This suggests that \( \Delta W_{fix} \) will be lower than \( \Delta W_{float} \)

• There are added complications in proving this inequality, due to the presence of \( J \)

• But numerically I always got \( \Delta W_{fix} < \Delta W_{float} \)

• Moreover the distance between the two depends on the initial degree of overvaluation, if \( \frac{\hat{P}_{h0}}{\hat{\xi}} = 1 \) then \( \Delta W_{fix} = \Delta W_{float} = 0 \)

• So it’s possible to find a \( \kappa \) such that

\[ \Delta W_{fix} < \kappa < \Delta W_{float} \]

so we have two equilibria