Simplest New Keynesian Model without Capital

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Objective

• Describe the model sufficiently, so that ‘homework #9’ can be done.

• Define the model, display its linearized equilibrium conditions.

• Define a model ‘solution’.
Clarida-Gali-Gertler Model

• Households maximize:

\[ E_0 \sum_{t=0}^{\infty} \left( \log C_t - \exp(\tau_t) \frac{N_t^{1+\phi}}{1+\phi} \right), \quad \tau_t = \lambda \tau_{t-1} + \varepsilon_t^\tau, \quad \varepsilon_t^\tau \sim iid, \]

• Subject to:

\[ P_tC_t + B_{t+1} \leq W_tN_t + R_{t-1}B_t + T_t \]

• Intratemporal first order condition:

\[ C_t \exp(\tau_t)N_t^{\phi} = \frac{W_t}{P_t} \]
Household Intertemporal FONC

- Condition:

$$1 = \beta E_t \frac{u_{c,t+1}}{u_{c,t}} \frac{R_t}{1 + \pi_{t+1}}$$

- or

$$1 = \beta E_t \frac{C_t}{C_{t+1}} \frac{R_t}{1 + \pi_{t+1}}$$

$$= \beta E_t \exp[\log(R_t) - \log(1 + \pi_{t+1}) - \Delta c_{t+1}]$$

$$\approx \beta \exp[\log(R_t) - E_t \pi_{t+1} - E_t \Delta c_{t+1}], \ c_t \equiv \log(C_t)$$

- take log of both sides:

$$0 = \log(\beta) + r_t - E_t \pi_{t+1} - E_t \Delta c_{t+1}, \ r_t = \log(R_t)$$

- or

$$c_t = -\log(\beta) - [r_t - E_t \pi_{t+1}] + c_{t+1}$$
Firms

• Competitive final good firms:

\[ Y_t = \left[ \int_0^1 Y_{i,t}^\frac{\varepsilon-1}{\varepsilon} \right]^\frac{\varepsilon}{\varepsilon-1} di, \ \varepsilon > 1, \]

— First order condition:

\[ Y_{i,t} = Y_t \left( \frac{P_t}{P_{i,t}} \right)^\varepsilon. \]

• Intermediate good producer (monopolist in output, competitive in labor market):

\[ Y_{i,t} = \exp(a_t)N_{i,t}, \ \Delta a_t = \rho\Delta a_{t-1} + \varepsilon_t^a, \]

— Calvo price frictions

\[ P_{i,t} = \begin{cases} \tilde{P}_t \text{ with probability } 1 - \theta \\ P_{i,t} \text{ with probability } \theta \end{cases}, \]
Marginal Cost

\[ s_t = \left( 1 - v \right) \frac{W_t}{P_t} \exp(a_t) \]

\[ = \frac{1}{\lambda_f} \left( 1 - v \right) C_t \exp(\tau_t) N_t^\phi \]

### Household Efficiency Condition

\[ \Rightarrow \]

- In steady state marginal cost and product of labor equal (‘first-best’):

\[ s = \frac{1}{\lambda_f} = \frac{1}{\lambda_f} \frac{C_t \exp(\tau_t) N_t^\phi}{\exp(a_t)} \rightarrow \frac{C_t \exp(\tau_t) N_t^\phi}{\exp(a_t)} = 1 \]
Optimal Monetary Policy

• Properties of (Ramsey-) optimal monetary policy in CGG model when effects of monopoly power are extinguished with an employment subsidy to monopolists:

  – Inflation is zero for all $t$ and for all realizations of shocks.
  – Allocations coincide with allocations in first-best (‘natural’) equilibrium.
First Best Allocations

• Maximize:

\[ E_0 \sum_{t=0}^{\infty} \left( \log C_t - \exp(\tau_t) \frac{N_t^{1+\phi}}{1 + \phi} \right), \quad \tau_t = \lambda \tau_{t-1} + \varepsilon_t^\tau, \quad \varepsilon_t^\tau \sim iid, \]

• Subject to \( C_t = \exp(a_t)N_t \)

• Intratemporal first order condition:

\[ \frac{\text{marginal utility of leisure}}{\text{marginal utility of consumption}} = C_t \exp(\tau_t)N_t^\phi = \exp(a_t) = \text{marginal product of labor} \]

– natural employment and consumption:

\[ \log(N_t^*) = -\frac{\tau_t}{1+\phi}, \quad \log(C_t^*) = a_t - \frac{\tau_t}{1+\phi} \]
Natural Rate of Interest

• Given natural consumption, intertemporal Euler equation defines natural rate of interest

\[ 1 = \beta E_t \frac{u^{*\text{c},t+1}}{u^{*\text{c},t}} \frac{R^*_t}{1+\pi^*_t} \]

• Applying the same log as before:

\[ c^*_t = -\log(\beta) - [r^*_t - E_t \pi^*_{t+1}] + c^*_{t+1} \]

\[ c^*_t = \log(C^*_t), \quad r^*_t = \log R_t, \quad \pi^*_t = 0 \]

• The natural rate:

\[ r^*_t = -\log(\beta) + E_t[c^*_{t+1} - c^*_t] \]
Key Features of First-Best

• Employment does not respond to technology
  – Improvement in technology raises marginal product of labor and marginal cost of labor by same amount.

• First best consumption not a function of intertemporal considerations
  – Discount rate irrelevant.
  – Anticipated future values of shocks irrelevant.

• Natural rate of interest steers consumption and employment towards their natural levels.
Back to Actual Economy

• Output gap, $x_t$

\[ x_t = c_t - c_t^* \]

• Intertemporal conditions in natural and actual equilibrium:

\[ c_t = -\log(\beta) - [r_t - E_t \pi_{t+1}] + E_t c_{t+1} \]

\[ c_t^* = -\log(\beta) - r_t^* + E_t c_{t+1}^* \]

• Subtract, to obtain familiar IS equation:

\[
x_t = E_t x_{t+1} - [r_t - E_t \pi_{t+1} - r_t^*] \]
Actual Economy

• Marginal cost:

\[
S_t = \frac{1}{\lambda_f} C_t \exp(\tau_t) N_t^\phi \over \exp(a_t) \]

\[\approx \frac{1}{\lambda_f} C_t \exp(\tau_t) \left[ \frac{C_t}{\exp(a_t)} \right]^\phi \exp(a_t)
\]

\[
= \frac{1}{\lambda_f} C_t^{1+\phi} \left[ \frac{\exp\left(\frac{\tau_t}{1+\phi}\right)}{\exp(a_t)} \right]^{(1+\phi)} = \frac{1}{\lambda_f} \left( \frac{C_t}{C_t^*} \right)^{1+\phi}
\]

• Then,

a hat indicates log-deviation from steady state

\[
\hat{S}_t \equiv \log(s_t \lambda_f) = (1 + \phi)(c_t - c_t^*)
\]

\[= (1 + \phi)x_t
\]
Actual Economy

- Phillips curve summarizes price setting by intermediate good firms:

\[ \pi_t = \beta E_t \pi_{t+1} + \frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p} \hat{s}_t \]

- or, substituting from previous slide

\[ \pi_t = \beta E_t \pi_{t+1} + \frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p} (1 + \varphi)x_t \]
Equations of Actual Equilibrium Closed by Adding Policy Rule

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t$$ (Calvo pricing equation)

$$x_t = -[r_t - E_t \pi_{t+1} - r^*] + E_t x_{t+1}$$ (intertemporal equation)

$$r_t = \alpha r_{t-1} + (1 - \alpha)(\phi \pi \pi_t + \phi_x x_t) + u_t$$ (policy rule)

$$r^*_t = E_t(y^*_{t+1} - y^*_t) = E_t \left( \Delta a_{t+1} - \frac{1}{1 + \phi} \Delta \tau_{t+1} \right)$$ (natural rate)

$$y^*_t = a_t - \frac{1}{1 + \phi} \tau_t$$ (natural output), \quad x_t = y_t - y^*_t$$ (output gap)

also, time series representations for shocks listed above
Solving the Model

• Express the equations in matrix form:

\[
\begin{align*}
  z_t &= \begin{pmatrix} \pi_t \\ x_t \\ r_t \\ r^*_t \end{pmatrix}, \\
  s_t &= \begin{pmatrix} \tau_t \\ a_t \\ u_t \end{pmatrix}, \\
  \varepsilon_t &= \begin{pmatrix} \varepsilon^\tau_t \\ \varepsilon^a_t \\ \varepsilon^u_t \end{pmatrix}
\end{align*}
\]

\[(*) \quad \alpha_0 E_t z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 E_t s_{t+1} + \beta_1 s_t = 0\]

\[s_t = Ps_{t-1} + \varepsilon_t\]

• Solution: \(A\) and \(B\) matrices such that \((*)\) is satisfied and

\[z_t = Az_{t-1} + Bs_t\]