Simple New Keynesian Model without Capital

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What’s It Good For?

• Conveying basic principles of macroeconomics -
  – Concept and measurement of output gap:
    • ‘difference between the actual economy and where would be if policy was managed as well as possible’.
  – Importance of aggregate demand.
    • problems when it goes awry.
  – Important policy objective: assuring the right level of aggregate demand.

• Thinking through the operating characteristics of policy rules:
  – Inflation targeting, Tax/spending rules, Leverage restrictions on banks.

• Can even use it to learn econometrics
  – how well do standard econometric estimators work?
  – how good is HP filter at estimating output gap?
Our Approach to NK Model

• We will derive the familiar ‘three equation NK model’, but they will not be our starting point.
  – Start with households, firms, technology, etc....

• Necessary to build the model from scratch -
  – need this to uncover the principles hiding inside it
  – needed to know how to ‘go back to the drawing board’ and modify the model so it can address interesting questions:
    • how should macro prudential policy be conducted?
    • how might currency mismatch problems affect the usual transmission of exchange rate depreciation to the economy?
    • what should the role of inflation, labor markets, credit growth, stock markets, etc., be in monetary policy?
    • how does an expansion of unemployment benefits in a recession affect the business cycle?
Households

- Problem:

\[
\max E_0 \sum_{t=0}^{\infty} \beta^t \left( \log C_t - \exp(\tau_t) \frac{N_t^{1+\varphi}}{1+\varphi} \right), \quad \tau_t = \lambda \tau_{t-1} + \varepsilon_t^\tau
\]

s.t. \( P_tC_t + B_{t+1} \leq W_tN_t + R_{t-1}B_t + \text{Profits net of taxes}_t \)

- First order conditions:

\[
\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}}
\]

\[
\exp(\tau_t) C_t N_t^\varphi = \frac{W_t}{P_t}
\]
Goods Production

- A homogeneous final good is produced using the following (Dixit-Stiglitz) production function:

\[ Y_t = \left[ \int_0^1 Y_{i,t} \frac{\varepsilon-1}{\varepsilon} d\varepsilon \right]^{\frac{\varepsilon}{\varepsilon-1}}. \]

- Each intermediate good, \( Y_{i,t} \), is produced as follows:

\[
Y_{i,t} = \sqrt[\varepsilon-1]{\text{exp}(a_t) A_t N_{i,t}}, \quad a_t = \rho a_{t-1} + \varepsilon_t^a
\]

- Before discussing the firms that operate these production functions, we briefly investigate the socially efficient (‘First Best’) allocation of labor across \( i \), for given \( N_t \):

\[
N_t = \int_0^1 N_{it} di
\]
Efficient Sectoral Allocation of Labor

- With Dixit-Stiglitz final good production function, there is a socially optimal allocation of resources to all the intermediate activities, $Y_{i,t}$
  - It is optimal to run them all at the same rate, i.e., $Y_{i,t} = Y_{j,t}$ for all $i, j \in [0,1]$.
- For given $N_t$, it is optimal to set $N_{i,t} = N_{j,t}$ for all $i, j \in [0,1]$
- In this case, final output is given by
  \[ Y_t = e^{a_t} N_t. \]

- Best way to see this is to suppose that labor is *not* allocated equally to all activities.
  - But, this can happen in a million different ways when there is a continuum of inputs!
  - Explore one simple deviation from $N_{i,t} = N_{j,t}$ for all $i, j \in [0,1]$. 
Suppose Labor Not Allocated Equally

• Example:

\[
N_{it} = \begin{cases} 
2\alpha N_t & i \in \left[0, \frac{1}{2}\right] \\
2(1-\alpha)N_t & i \in \left[\frac{1}{2}, 1\right] 
\end{cases}, \quad 0 \leq \alpha \leq 1.
\]

• Note that this is a particular distribution of labor across activities:

\[
\int_0^1 N_{it} di = \frac{1}{2} 2\alpha N_t + \frac{1}{2} 2(1-\alpha)N_t = N_t
\]
Labor Not Allocated Equally, cnt’d

\begin{align*}
Y_t &= \left[ \int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} \, di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= \left[ \int_0^{\frac{1}{2}} Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} \, di + \int_{\frac{1}{2}}^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} \, di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t} \left[ \int_0^{\frac{1}{2}} N_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} \, di + \int_{\frac{1}{2}}^1 N_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} \, di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t} \left[ \int_0^{\frac{1}{2}} (2\alpha N_t)^{\frac{\varepsilon-1}{\varepsilon}} \, di + \int_{\frac{1}{2}}^1 (2(1-\alpha)N_t)^{\frac{\varepsilon-1}{\varepsilon}} \, di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t} N_t \left[ \int_0^{\frac{1}{2}} (2\alpha)^{\frac{\varepsilon-1}{\varepsilon}} \, di + \int_{\frac{1}{2}}^1 (2(1-\alpha))^{\frac{\varepsilon-1}{\varepsilon}} \, di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t} N_t \left[ \frac{1}{2} (2\alpha)^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{2} (2(1-\alpha))^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= e^{a_t} N_t f(\alpha)
\end{align*}
Efficient Resource Allocation Means Equal Labor Across All Sectors

\[ f(\alpha) = \left[ \frac{1}{2} (2\alpha)^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{2} (2(1 - \alpha))^\frac{\varepsilon-1}{\varepsilon} \right]^{\frac{\varepsilon}{\varepsilon-1}} \]
Final Goods Production

- Final good firms:
  - maximize profits:
    
    \[ P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} dj, \]

    subject to:
    
    \[ Y_t = \left[ \int_0^1 Y_{i,t}^\frac{\varepsilon-1}{\varepsilon} dj \right]^\frac{\varepsilon}{\varepsilon-1}. \]

- Foncs:
  
  \[ Y_{i,t} = Y_t \left( \frac{P_t}{P_{i,t}} \right)^\varepsilon \rightarrow P_t = \left( \int_0^1 P_{i,t}^{(1-\varepsilon)} di \right)^{\frac{1}{1-\varepsilon}} \]
Intermediate Goods Production

- Demand curve for $i^{th}$ monopolist:

$$Y_{i,t} = Y_t \left( \frac{P_t}{P_{i,t}} \right)^\varepsilon.$$

- Production function:

$$Y_{i,t} = \exp(a_t) N_{i,t}, \quad \Delta a_t = \rho \Delta a_{t-1} + \varepsilon^a_t$$

- Calvo Price-Setting Friction:

$$P_{i,t} = \begin{cases} \tilde{P}_t & \text{with probability } 1 - \theta \\ \frac{P_{i,t-1}}{P_{i,t}} & \text{with probability } \theta \end{cases}.$$

- Real marginal cost:

$$S_t = \frac{d\text{Cost}}{d\text{worker}} = \frac{\left(1 - \nu \right)}{\exp(a_t)} \frac{W_t}{P_t}$$

minimize monopoly distortion by setting $= \frac{\varepsilon - 1}{\varepsilon}$
Optimal Price Setting by Intermediate Goods Producers

- Let

\[ \tilde{p}_t \equiv \frac{\bar{p}_t}{P_t}, \quad \bar{\pi}_t \equiv \frac{P_t}{P_{t-1}}. \]

- First order condition implied by optimal price setting:

\[ \tilde{p}_t = \frac{K_t}{F_t}, \]

where

\[ K_t = \frac{\varepsilon}{\varepsilon - 1} s_t + \beta \theta E_t \bar{\pi}_{t+1}^{\varepsilon} K_{t+1} \] \hspace{1cm} (1)

\[ F_t = 1 + \beta \theta E_t \bar{\pi}_{t+1}^{\varepsilon-1} F_{t+1}. \] \hspace{1cm} (2)

- Note:

\[ K_t = \frac{\varepsilon}{\varepsilon - 1} s_t + \beta \theta E_t \bar{\pi}_{t+1}^{\varepsilon} \frac{\varepsilon}{\varepsilon - 1} s_{t+1} \]

\[ + (\beta \theta)^2 E_t \bar{\pi}_{t+2}^{\varepsilon} \frac{\varepsilon}{\varepsilon - 1} s_{t+2} + \ldots \]
Price Equilibrium Conditions

- Cross-price restrictions imply, given the Calvo price-stickiness:

\[ P_t = \left[ (1 - \theta) \tilde{P}_t^{(1-\epsilon)} + \theta P_{t-1}^{(1-\epsilon)} \right]^{\frac{1}{1-\epsilon}}. \]

- Dividing latter by \( P_t \) and solving for \( \tilde{p}_t \):

\[ \tilde{p}_t = \left[ \frac{1 - \theta \bar{\pi}_t^{\epsilon-1}}{1 - \theta} \right]^{\frac{1}{1-\epsilon}}. \]

- Combining with the first order condition for \( \tilde{p}_t \):

\[ \frac{K_t}{F_t} = \left[ \frac{1 - \theta \bar{\pi}_t^{\epsilon-1}}{1 - \theta} \right]^{\frac{1}{1-\epsilon}} \quad (3) \]
Aggregate Inputs and Aggregate Output

- Tack Yun argument:

\[ Y^*_t \equiv \int_0^1 Y_{i,t} di \left( = \int_0^1 e^{a_t N_{i,t}} di = e^{a_t N_t} \right) \]

\[ \text{demand curve} \]

\[ \equiv \int_0^1 Y_t P_t^\varepsilon P_{i,t}^{-\varepsilon} di = Y_t P_t^\varepsilon \int_0^1 P_{i,t}^{-\varepsilon} di \]

\[ \equiv P_t^* \]

\[ \rightarrow Y_t = \left( \frac{P_t^*}{P_t} \right)^\varepsilon e^{a_t N_t} \]

\[ P_t^* = \left[ (1 - \theta) \tilde{P}_t^{-\varepsilon} + \theta P_{t-1}^{-\varepsilon} \right]^{-\frac{1}{\varepsilon}} \]

\[ \rightarrow p_t^* = \left[ (1 - \theta) \tilde{p}_t^{-\varepsilon} + \theta \frac{\bar{\pi}_t^\varepsilon}{p_{t-1}^*} \right]^{-1} \]
Goods Equilibrium Conditions

- Relationship between aggregate output and aggregate inputs:

\[ C_t = p_t^* A_t N_t, \quad (6) \]

where (‘Tack Yun distortion’)

\[ p_t^* = p^* (\bar{\pi}_t, p_{t-1}^*) \equiv \left[ (1 - \theta) \left( \frac{1 - \theta \bar{\pi}_t^{(\varepsilon - 1)}}{1 - \theta} \right)^\frac{\varepsilon}{\varepsilon - 1} + \theta \frac{\bar{\pi}_t^\varepsilon}{p_{t-1}^*} \right]^{-1} \]

(7)
Tack Yun Distortion: a Closer Look

- Distortion:

\[
\begin{aligned}
p_t^* &= p^* (\bar{\pi}_t, p_{t-1}^*) \\
&\equiv \left[ (1 - \theta) \left( \frac{1 - \theta \bar{\pi}_t^{(\varepsilon-1)}}{1 - \theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \theta \frac{\bar{\pi}_t^\varepsilon}{p_{t-1}^*} \right]^{-1}
\end{aligned}
\]

- Distortion, \( p_t^* \), increasing function of lagged distortion, \( p_{t-1}^* \).

- Current shocks affect current distortion via \( \bar{\pi}_t \) only.

- Derivatives:

\[
\begin{aligned}
p_1^* (\bar{\pi}_t, p_{t-1}^*) &= - (p_t^*)^2 \varepsilon \theta \bar{\pi}_t^{\varepsilon-2} \left[ \frac{\bar{\pi}_t}{p_{t-1}^*} - \left( \frac{1 - \theta \bar{\pi}_t^{(\varepsilon-1)}}{1 - \theta} \right)^{\frac{1}{\varepsilon-1}} \right] \\
p_2^* (\bar{\pi}_t, p_{t-1}^*) &= \left( \frac{p_t^*}{p_{t-1}^*} \right)^2 \theta \bar{\pi}_t^\varepsilon.
\end{aligned}
\]
Linear Expansion of Tack Yun Distortion in Undistorted Steady State

- Linearizing about $\bar{\pi}_t = \bar{\pi}, p^*_t = p^*$:

  $$dp^*_t = p^*_1 (\bar{\pi}, p^*) d\bar{\pi}_t + p^*_2 (\bar{\pi}, p^*) dp^*_{t-1},$$

  where $dx_t \equiv x_t - x$, for $x_t = p^*_t, p^*_t, \bar{\pi}_t$.

- In an undistorted steady state (i.e., $\bar{\pi}_t = p^*_t = p^*_t = 1$):

  $$p^*_1 (1, 1) = 0, p^*_2 (1, 1) = \theta.$$

  so that

  $$dp^*_t = 0 \times d\bar{\pi}_t + \theta dp^*_{t-1}$$

  $$\rightarrow p^*_t = 1 - \theta + \theta p^*_{t-1}$$

- Often, people that linearize NK model ignore $p^*_t$.
  - Reflects that they linearize the model around a price-undistorted steady state.
Current Period Tack Yun Distortion as a Function of Current Inflation

Graph conditioned on two alternative values for $p_{t-1}^*$ and $\theta = 0.75$, $\varepsilon = 6.00$

lagged Tack Yun distortion = 1 (i.e., no distortion)
lagged Tack Yun distortion = 0.9
Ignoring Tack Yun Distortion, a Mistake?

Tack Yun Distortion and Quarterly US CPI Inflation (Gross)
First Best Consumption, Employment

- ‘First Best Consumption and Employment’ useful concepts in simple NK model.
  - Ramsey is the appropriate benchmark, but Ramsey and first best coincide in simple NK model.
- Explained above that with socially efficient sectoral allocation of labor,
  \[ Y_t = \exp(a_t) N_t. \]
- First best level of employment and consumption is solution to
  \[ N_{t}^{\text{best}} = \arg \max_{N} \left\{ \log \left[ \exp(a_t) N \right] - \exp(\tau_t) \frac{N^{1+\varphi}}{1+\varphi} \right\} \]
  so,
  \[ N_{t}^{\text{best}} = \exp \left(-\frac{\tau_t}{1+\varphi}\right), \quad C_{t}^{\text{best}} = \exp \left(a_t - \frac{\tau_t}{1+\varphi}\right) \]
Linearizing around Efficient Steady State

• In steady state (assuming $\bar{\pi} = 1, 1 - \nu = \frac{\varepsilon - 1}{\varepsilon}$)

$$p^* = 1, \quad K = F = \frac{1}{1 - \beta\theta}, \quad s = \frac{\varepsilon - 1}{\varepsilon}, \quad \Delta a = \tau = 0, \quad N = 1$$

• Linearizing the Tack Yun distortion, (4):

$$p_t^* = 1, \quad t \text{ large enough}$$

• Denote the output gap in ratio form by $X_t$:

$$X_t \equiv \frac{C_t}{\exp \left( a_t - \frac{\tau_t}{1+\varphi} \right)} = p_t^* N_t \exp \left( \frac{\tau_t}{1+\varphi} \right),$$

where the denominator is the socially efficient (‘First Best’) level of consumption.

• Then, with $x_t \equiv \hat{X}_t$ and $\hat{p}_t^* = 0$:

$$x_t = \hat{N}_t + \frac{d\tau_t}{1+\varphi}$$
The intertemporal Euler equation, (5), after substituting for $C_t$ in terms of $X_t$:

\[
\frac{1}{X_t \exp \left( a_t - \frac{\tau_t}{1+\varphi} \right)} = \beta E_t \frac{1}{X_{t+1} \exp \left( a_{t+1} - \frac{\tau_{t+1}}{1+\varphi} \right)} \frac{R_t}{\bar{\pi}_{t+1}}
\]

\[
\frac{1}{X_t} = E_t \frac{1}{X_{t+1} R_{t+1}^*} \frac{R_t}{\bar{\pi}_{t+1}},
\]

where

value of $R_t$ in ‘first best’

\[
\hat{R}_{t+1}^* \equiv \frac{1}{\beta} \exp \left( a_{t+1} - a_t - \frac{\tau_{t+1} - \tau_t}{1+\varphi} \right)
\]

then, (using $\hat{z}_t u_t = \hat{z}_t + \hat{u}_t, \left( \frac{u_t}{z_t} \right) = \hat{u}_t - \hat{z}_t$):

\[
\hat{X}_t = E_t \left[ \hat{X}_{t+1} - (\hat{R}_t - \hat{\pi}_{t+1} - \hat{R}_{t+1}^*) \right]
\]
NK IS Curve, Baseline Model

- Note:

\[ Z_t = \exp(z_t), \text{ where } z_t \equiv \log Z_t \]

\[ \hat{Z}_t = \frac{dZ_t}{Z} = \frac{d \exp(z_t)}{Z} = \frac{Zdz_t}{Z} = dz_t = \log Z_t - \log Z. \]

- Use this to establish, when the steady state is efficient:

\[ E_t (\hat{R}_t - \hat{\pi}_{t+1} - \hat{R}^*_{t+1}) \]

\[ = \log R_t - E_t \pi_{t+1} - E_t \log R^*_{t+1} \]

\[ = r_t - E_t \pi_{t+1} - r^*_t \]

where

\[ r_t \equiv \log R_t, \quad r^*_t \equiv E_t \log R^*_{t+1}, \quad \pi_{t+1} \equiv \log \bar{\pi}_{t+1} \]

- and, in efficient steady state:

\[ \log R^* = \log R, \quad \log \bar{\pi} = 0. \]
NK IS Curve, Baseline Model

- Substituting

\[ \hat{X}_t = E_t [\hat{X}_{t+1} - (\hat{R}_t - \hat{\pi}_{t+1} - \hat{R}^*_t)] , \ x_t \equiv \hat{X}_t, \]

we obtain NK IS curve:

\[ x_t = E_t x_{t+1} - E_t [r_t - \pi_{t+1} - r^*_t] \]

- Also,

\[ r^*_t = -\log (\beta) + E_t \left[ a_{t+1} - a_t - \frac{\tau_{t+1} - \tau_t}{1 + \varphi} \right]. \]
Linearized Marginal Cost in Baseline Model

- Marginal cost (using $da_t = a_t, d\tau_t = \tau_t$ because $a = \tau = 0$):

$$s_t = (1 - \nu) \frac{\bar{w}_t}{A_t}, \quad \bar{w}_t = \exp(\tau_t) N_t^\varphi C_t$$

$$\rightarrow \hat{w}_t = \tau_t + a_t + (1 + \varphi) \hat{N}_t$$

- Then,

$$\hat{s}_t = \hat{w}_t - a_t = (\varphi + 1) \left[ \frac{\tau_t}{\varphi + 1} + \hat{N}_t \right] = (\varphi + 1) x_t$$
Linearized Phillips Curve in Baseline Model

- Log-linearize equilibrium conditions, (1)-(3), around steady state:

\[
\hat{K}_t = (1 - \beta\theta) \hat{s}_t + \beta\theta (\epsilon \hat{\pi}_{t+1} + \hat{K}_{t+1}) \quad (1)
\]
\[
\hat{F}_t = \beta\theta (\epsilon - 1) \hat{\pi}_{t+1} + \beta\theta \hat{F}_{t+1} \quad (2)
\]
\[
\hat{K}_t = \hat{F}_t + \frac{\theta}{1 - \theta} \hat{\pi}_t \quad (3)
\]

- Substitute (3) into (1)

\[
\hat{F}_t + \frac{\theta}{1 - \theta} \hat{\pi}_t = (1 - \beta\theta) \hat{s}_t + \beta\theta \left(\epsilon \hat{\pi}_{t+1} + \hat{F}_{t+1} + \frac{\theta}{1 - \theta} \hat{\pi}_{t+1}\right)
\]

- Simplify the latter using (2), to obtain the NK Phillips curve:

\[
\pi_t = \frac{(1 - \theta)(1 - \beta\theta)}{\theta} \hat{s}_t + \beta \pi_{t+1}
\]
Nonlinear Private Sector Equilibrium Conditions

\[ K_t = \frac{\varepsilon}{\varepsilon - 1} s_t + \beta \theta E_t \bar{\pi}^{\varepsilon}_{t+1} K_{t+1} \quad (1) \]

\[ F_t = 1 + \beta \theta E_t \bar{\pi}^{\varepsilon-1}_{t+1} F_{t+1} \cdot (2) \]

\[ \frac{K_t}{F_t} = \left[ \frac{1 - \theta \bar{\pi}^{\varepsilon-1}_t}{1 - \theta} \right]^{\frac{1}{\varepsilon - 1}} \quad (3) \]

\[ p_t^* = \left[ (1 - \theta) \left( \frac{1 - \theta \bar{\pi}^{(\varepsilon-1)}_t}{1 - \theta} \right)^{\frac{\varepsilon}{\varepsilon - 1}} + \theta \frac{\bar{\pi}^\varepsilon_t}{p_{t-1}^*} \right]^{-1} \quad (4) \]

\[ \frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}} \quad (5) \]

\[ C_t = p_t^* A_t N_t. \quad (6) \]
The Linearized Private Sector Equilibrium Conditions

\[
x_t = E_t x_{t+1} - [r_t - E_t \pi_{t+1} - r^*_t]
\]

\[
\pi_t = \frac{(1-\theta)(1-\beta\theta)}{\theta} \hat{s}_t + \beta E_t \pi_{t+1}
\]

\[
\hat{s}_t = (\varphi + 1) x_t
\]

\[
r^*_t = -\log(\beta) + E_t \left[ a_{t+1} - a_t - \frac{\tau_{t+1} - \tau_t}{1+\varphi} \right]
\]

Monetary policy rule:

\[
r_t = \alpha r_{t-1} + (1 - \alpha) \left[ r + \phi_{\pi} \pi_t + \phi_x x_t \right]
\]
Solving the Model

- Vision about evolution of actual data:
  - Nature draws the exogenous shocks.
  - The economy transforms exogenous shocks into realization of endogenous variables, inflation, output, unemployment, etc.

- ‘Solving the model’:
  - Determining the model’s implications for the mapping from exogenous shocks to endogenous variables.
  - Potentially massive problem: current value of endogenous variables a function of past data and expected future value of endogenous variables.

- Primary strategy for solving a model:
  - Find a linear representation (‘policy rule’) of the endogenous variables, $z_t$, in terms of current and past data only:
    $$ z_t = Az_{t-1} + Bs_t $$
  
  such that the equilibrium conditions (after linearization) are satisfied.
• Exogenous shocks:

\[ s_t = \begin{pmatrix} \Delta a_t \\ \tau_t \end{pmatrix} = \begin{bmatrix} \rho & 0 \\ 0 & \lambda \end{bmatrix} \begin{pmatrix} \Delta a_{t-1} \\ \tau_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_t^a \\ \varepsilon_t^\tau \end{pmatrix} \]

\[ s_t = P_{s_{t-1}} + \epsilon_t \]

• Equilibrium Conditions:

\[
\begin{bmatrix}
\beta & 0 & 0 & 0 \\
1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{pmatrix}
\pi_{t+1} \\
x_{t+1} \\
r_{t+1} \\
r_{t+1}^*
\end{pmatrix} + 
\begin{bmatrix}
-1 & \frac{(1-\theta)(1-\beta\varphi)}{\theta} & (1+\varphi) & 0 & 0 \\
0 & -1 & -1 & 1 & 1 \\
(1-\alpha)\phi_\pi & (1-\alpha)\phi_x & -1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{pmatrix}
\pi_t \\
x_t \\
r_t \\
r_t^*
\end{pmatrix} \\
+ 
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & \alpha & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{pmatrix}
\pi_{t-1} \\
x_{t-1} \\
r_{t-1} \\
r_{t-1}^*
\end{pmatrix} + 
\begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
-1 & \frac{1}{1+\varphi}
\end{bmatrix}
\begin{pmatrix}
s_{t+1} \\
0 \\
0 \\
0 - \frac{1}{1+\varphi}
\end{pmatrix}
\]

\[ E_t[\alpha_0z_{t+1} + \alpha_1z_t + \alpha_2z_{t-1} + \beta_0s_{t+1} + \beta_1s_t] = 0 \]
• Collecting:

\[
E_t \left[ \alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t \right] = 0
\]

\[
s_t - Ps_{t-1} - \epsilon_t = 0.
\]

• Policy rule:

\[
z_t = Az_{t-1} + Bs_t
\]

• As before, want \( A \) such that

\[
\alpha_0 A^2 + \alpha_1 A + \alpha_2 I = 0,
\]

• Want \( B \) such that:

\[
(\beta_0 + \alpha_0 B)P + [\beta_1 + (\alpha_0 A + \alpha_1)B] = 0
\]

• Note: if \( \alpha = 0 \), then \( A = 0 \) is one solution (there is another one!).
$\phi_x = 0$, $\phi_\pi = 1.5$, $\beta = 0.99$, $\varphi = 1$, $\rho = 0.2$, $\theta = 0.75$, $\alpha = 0$, $\delta = 0.2$, $\lambda = 0.5$. 

Dynamic Response to a Technology Shock

- Inflation
- Output gap
- Nominal rate
- Natural real rate
- Natural output
- Natural employment
- Actual output
- Actual employment