Financial Frictions Under Asymmetric Information and Costly State Verification
General Idea

• Standard dsge model assumes borrowers and lenders are the same people..no conflict of interest.

• Financial friction models suppose borrowers and lenders are different people, with conflicting interests.

• Financial frictions: features of the relationship between borrowers and lenders adopted to mitigate conflict of interest.
Discussion of Financial Frictions

• Simple model to illustrate the basic costly state verification (csv) model.
  – Original analysis of Townsend (1978), Gale-Helwig.

• Then: integrate the csv model into a full-blown dsge model.
  – Follows the lead of Bernanke, Gertler and Gilchrist (1999).

• After fitting model to data, find that a new shock, ‘risk shock’, appears to be important in business cycles.
Simple Model

• There are entrepreneurs with all different levels of wealth, $N$.
  – Entrepreneur have different levels of wealth because they experienced different idiosyncratic shocks in the past.

• For each value of $N$, there are many entrepreneurs.

• In what follows, we will consider the interaction between entrepreneurs with a specific amount of $N$ with competitive banks.

• Later, will consider the whole population of entrepreneurs, with every possible level of $N$. 
Simple Model, cont’d

• Each entrepreneur has access to a project with rate of return,
\[(1 + R^k)\omega\]

• Here, \(\omega\) is a unit mean, idiosyncratic shock experienced by the individual entrepreneur after the project has been started,
\[\int_0^\infty \omega dF(\omega) = 1\]

• The shock, \(\omega\), is privately observed by the entrepreneur.

• \(F\) is lognormal cumulative distribution function.
Banks, Households, Entrepreneurs

$$\omega \sim F(\omega), \int_{0}^{\infty} \omega dF(\omega) = 1$$
• Entrepreneur receives a contract from a bank, which specifies a rate of interest, \( Z \), and a loan amount, \( B \).
  
  – If entrepreneur cannot make the interest payments, the bank pays a monitoring cost and takes everything.

• Total assets acquired by the entrepreneur:

\[
\hat{A} = \hat{N} + \hat{B}
\]

• Entrepreneur who experiences sufficiently bad luck, \( \omega \leq \bar{\omega} \), loses everything.
• Expected return to entrepreneur, over opportunity cost of funds:

\[
\int_{\bar{\omega}}^{\infty} \left[ (1+R^k)\omega A - ZB \right] dF(\omega) \over N(1+R)
\]

Expected payoff for entrepreneur

For lower values of \( \omega \), entrepreneur receives nothing ‘limited liability’.  

opportunity cost of funds
• Rewriting entrepreneur’s rate of return:

\[
\int_{\bar{\omega}}^{\infty} \frac{[(1 + R^k)\omega A - ZB]dF(\omega)}{N(1 + R)} = \int_{\bar{\omega}}^{\infty} \frac{[(1 + R^k)\omega A - (1 + R^k)\bar{\omega}A]dF(\omega)}{N(1 + R)}
\]

\[
= \int_{\bar{\omega}}^{\infty} [\omega - \bar{\omega}]dF(\omega) \left( \frac{1 + R^k}{1 + R} \right) L
\]

\[
\bar{\omega} = \frac{Z}{(1 + R^k)} \frac{L - 1}{L} \rightarrow L \rightarrow \infty \quad \frac{Z}{(1 + R^k)}
\]

• Entrepreneur’s return unbounded above
  – Risk neutral entrepreneur would always want to borrow an infinite amount (infinite leverage).
• If given a fixed interest rate, entrepreneur with risk neutral preferences would borrow an unbounded amount.

• In equilibrium, bank can’t lend an infinite amount.

• This is why a loan contract must specify both an interest rate, $Z$, and a loan amount, $B$. 
‘Indifference Curves’ of Entrepreneurs

- Think of the loan contract in terms of the loan amount (or, leverage, \((N+B)/N\)) and the cutoff, \(\bar{\omega}\)

\[
\int_{\bar{\omega}}^{\infty} \frac{[(1+R^k)\omega A-ZB]}{N(1+R)} dF(\omega) = \int_{\bar{\omega}}^{\infty} [\omega - \bar{\omega}] dF(\omega) \left( \frac{1+R^k}{1+R} \right) L
\]

\[
L = \frac{A}{N} = \frac{N+B}{N}
\]
Banks

• Source of funds from households, at fixed rate, $R$

• Bank borrows $B$ units of currency, lends proceeds to entrepreneurs.

• Provides entrepreneurs with standard debt contract, $(Z,B)$
Banks, cont’d

• Monitoring cost for bankrupt entrepreneur with $\omega < \bar{\omega}$

\[ \mu(1 + R^k)\omega A \]

• Bank zero profit condition

\[
\begin{align*}
\text{fraction of entrepreneurs with } \omega > \bar{\omega} & = \frac{1 - F(\bar{\omega})}{[1 - F(\bar{\omega})]} \\
\text{quantity paid by each entrepreneur with } \omega > \bar{\omega} & = ZB \\
\text{quantity recovered by bank from each bankrupt entrepreneur} & = (1 - \mu) \int_{0}^{\bar{\omega}} \omega dF(\omega)(1 + R^k)A \\
\text{amount owed to households by bank} & = (1 + R)B
\end{align*}
\]
Banks, cont’d

• Simplifying zero profit condition:

\[
[1 - F(\bar{\omega})]ZB + (1 - \mu) \int_{0}^{\bar{\omega}} \omega dF(\omega)(1 + R^k)A = (1 + R)B
\]

\[
[1 - F(\bar{\omega})]\bar{\omega}(1 + R^k)A + (1 - \mu) \int_{0}^{\bar{\omega}} \omega dF(\omega)(1 + R^k)A = (1 + R)B
\]

\[
[1 - F(\bar{\omega})]\bar{\omega} + (1 - \mu) \int_{0}^{\bar{\omega}} \omega dF(\omega) = \frac{1 + R}{1 + R^k} \frac{B/N}{A/N} = \frac{1 + R}{1 + R^k} \frac{L - 1}{L}
\]

• Expressed naturally in terms of \((\bar{\omega}, L)\)
Bank zero profit condition, in (leverage, $\bar{\omega} - \text{bar}$) space

- Free entry of banks ensures zero profits

- Zero profit curve represents a ‘menu’ of contracts, $(\bar{\omega}, L)$, that can be offered in equilibrium.

- Only the upward-sloped portion of the curve is relevant, because entrepreneurs would never select a high value of $\bar{\omega}$ if a lower one was available at the same leverage.
Some Notation and Results

• Let

\[ G(\bar{\omega}) = \int_{0}^{\bar{\omega}} \omega dF(\omega) \quad , \quad \Gamma(\bar{\omega}) = \bar{\omega}[1 - F(\bar{\omega})] + G(\bar{\omega}), \]

• Results:

\[ G'(\bar{\omega}) = \frac{d}{d\bar{\omega}} \int_{0}^{\bar{\omega}} \omega dF(\omega) \quad \xrightarrow{\text{Leibniz’s rule}} \quad \bar{\omega}F'(\bar{\omega}) \]

\[ \Gamma'(\bar{\omega}) = 1 - F(\bar{\omega}) - \bar{\omega}F'(\bar{\omega}) + G(\bar{\omega}) = 1 - F(\bar{\omega}) \]
Moving Towards Equilibrium Contract

• Entrepreneurial utility:

\[
\int_{\bar{\omega}}^{\infty} (\omega - \bar{\omega})dF(\omega) \frac{1 + R^k}{1 + R} L
\]

\[
= (1 - G(\bar{\omega}) - \bar{\omega}[1 - F(\bar{\omega})]) \frac{1 + R^k}{1 + R} L
\]

share of entrepreneur return going to entrepreneur

\[
= \left[1 - \Gamma(\bar{\omega})\right] \frac{1 + R^k}{1 + R} L
\]
Moving Towards Equilibrium Contract, cn’t

- Bank profits:

\[
(1 - F(\bar{\omega}))\bar{\omega} + (1 - \mu) \int_{0}^{\bar{\omega}} \omega dF(\omega) = \frac{1 + R}{1 + R^k} \frac{L - 1}{L}
\]

\[
\Gamma(\bar{\omega}) - \mu G(\bar{\omega}) = \frac{1 + R}{1 + R^k} \frac{L - 1}{L}
\]

\[
L = \frac{1}{1 - \frac{1+R^k}{1+R} [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]}
\]
Equilibrium Contract

• Entrepreneur selects the contract is optimal, given the available menu of contracts.

• The solution to the entrepreneur problem is the \( \bar{\omega} \) that solves:

\[
\log \left\{ \int_{\bar{\omega}}^{\infty} [\omega - \bar{\omega}]dF(\omega) \frac{1 + R_k}{1 + R} \times \frac{1}{1 - \frac{1+R_k}{1+R} [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]} \right\}
\]

\[
= \log [1 - \Gamma(\bar{\omega})] + \log \frac{1 + R_k}{1 + R} - \log \left( 1 - \frac{1 + R_k}{1 + R} [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})] \right)
\]

Higher \( \bar{\omega} \) drives share of profits to entrepreneur down (bad!)

Higher \( \bar{\omega} \) drives leverage up (good!)
Computing the Equilibrium Contract

- Solve first order optimality condition uniquely for the cutoff, \( \bar{\omega} \):

  
  \[
  \frac{1 - F(\bar{\omega})}{1 - \Gamma(\bar{\omega})} = \frac{\frac{1 + R^k}{1 + R} [1 - F(\bar{\omega}) - \mu F'(\bar{\omega})]}{1 - \frac{1 + R^k}{1 + R} [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]}
  \]

- Given the cutoff, solve for leverage:

  
  \[
  L = \frac{1}{1 - \frac{1 + R^k}{1 + R} [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]}
  \]

- Given leverage and cutoff, solve for risk spread:

  
  \[
  \text{risk spread} \equiv \frac{Z}{1 + R} = \frac{1 + R^k}{1 + R} \bar{\omega} \frac{L}{L - 1}
  \]
Result

• Leverage, \( L \), and entrepreneurial rate of interest, \( Z \), **not a function of net worth, \( N \).**

• Quantity of loans proportional to net worth:

\[
L = \frac{A}{N} = \frac{N + B}{N} = 1 + \frac{B}{N}
\]

\[
B = (L - 1)N
\]

• To compute \( L, \frac{Z}{1 + R} \), must make assumptions about \( F \) and parameters.

\[
\frac{1 + R^k}{1 + R}, \mu, F
\]
The Distribution, $F$

Log normal density function, $E_\omega = 1$, $\sigma = 0.82155$
Results for log-normal

• Need: \[ G(\bar{\omega}) = \int_{0}^{\bar{\omega}} \omega dF(\omega), \ F'(\omega) \]

Can get these from the pdf and the cdf of the standard normal distribution.

These are available in most computational software, like MATLAB.

Also, they have simple analytic representations.
Results for log-normal

• Need: \( G(\tilde{\omega}) = \int_{0}^{\tilde{\omega}} \omega dF(\omega), F'(\omega) \)

\[
\int_{0}^{\tilde{\omega}} \omega dF(\omega) = \frac{1}{\sigma_x \sqrt{2\pi}} \int_{-\infty}^{\log \tilde{\omega}} e^x e^{-\frac{(x-Ex)^2}{2\sigma_x^2}} \, dx
\]

\( E\omega=1 \) requires \( Ex=-\frac{1}{2} \sigma_x^2 \)

\[
= \frac{1}{\sigma_x \sqrt{2\pi}} \int_{-\infty}^{\log \tilde{\omega}} e^x e^{-\frac{(x+\frac{1}{2} \sigma_x^2)^2}{2\sigma_x^2}} \, dx
\]

combine powers of \( e \) and rearrange

\[
= \frac{1}{\sigma_x \sqrt{2\pi}} \int_{-\infty}^{\log \tilde{\omega}} e^{-\frac{(x-\frac{1}{2} \sigma_x^2)^2}{2\sigma_x^2}} \, dx
\]

change of variables, \( v = \frac{x-\frac{1}{2} \sigma_x^2}{\sigma_x} \)

\[
= \frac{1}{\sigma_x \sqrt{2\pi}} \int_{-\infty}^{\log(\tilde{\omega}) + \frac{1}{2} \sigma_x^2} \exp \left( -\frac{v^2}{2} \right) \sigma_x \, dv
\]

\( = \text{prob} \left[ v < \frac{\log(\tilde{\omega}) + \frac{1}{2} \sigma_x^2}{\sigma_x} - \sigma_x \right] \) ← cdf for standard normal
Results for log-normal, cnt’d

• The log-normal cumulative density:

\[
F(\bar{\omega}) = \int_{0}^{\bar{\omega}} dF(\omega) = \frac{1}{\sigma_x \sqrt{2\pi}} \int_{-\infty}^{\log \bar{\omega}} e^{-\frac{(x+\frac{1}{2}\sigma_x^2)^2}{2\sigma_x^2}} dx
\]

• Differentiating (using Leibniz’s rule):

\[
F_{\bar{\omega}}(\omega; \sigma) = \frac{1}{\bar{\omega} \sigma} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{\log(\bar{\omega}) + \frac{1}{2} \frac{1}{\sigma}^2}{\frac{1}{2} \frac{\sigma}{\sigma}}\right]^2
\]

\[
= \frac{1}{\bar{\omega} \sigma} \text{Standard Normal pdf}\left(\frac{\log(\bar{\omega}) + \frac{1}{2} \frac{1}{\sigma}^2}{\frac{1}{\sigma}}\right)
\]
Figure: Impact on standard debt contract of a 5% jump in $\sigma$

- Risk spread = 2.67
  - Leverage = 1.12
- Risk spread = 2.52
  - Leverage = 1.13

Leverage, $L = (B+N)/N$
Put this Into DSGE Model
Standard Model

Firms

\[ Y_t = \left[ \int_0^1 Y_{jt} \frac{1}{\lambda_{jt}} \, dj \right]^\lambda_{jt}, \quad 1 \leq \lambda_{jt} < \infty, \]

\[ Y_{jt} = e_t K_{ji}^\alpha (z_t l_{jt})^{1-\alpha} \]

Households

Backyard capital accumulation:

\[ \bar{K}_{t+1} = (1 - \delta)\bar{K}_t + G(\zeta_{i,t}, I_t, I_{t-1}) \]

\[ u_{c,t} = E_t \beta_{c,t} u_{c,t+1} \frac{p_{t+1}^k}{\pi_{t+1}} \]

\[ R_{t+1}^k = \frac{u_{t+1}^k r_{t+1}^k + (1 - \delta)P_{k',t+1}^k}{P_{k',t}} \]
Standard Model

Firms

Labor market

household

Market for Physical Capital

L

C

I

K
Financing

• In the standard model, already have borrowing by firms for working capital.
  – will now have banks intermediate this borrowing between households and firms.

• In standard model, ‘putting capital to work’ is completely straightforward and is done by households. They just rent capital into a homogeneous capital market.

• Now: ‘putting capital to work’ involves a special kind of creativity that only some households – entrepreneurs – have.
  – Entrepreneurs finance the acquisition of capital in part by themselves, and in part by borrowing from regular ‘households’.
  – Conflict of interest, because there is asymmetric information about the payoff from capital.
  – Standard sharing contract between entrepreneur and household not feasible.
Financial Frictions with Physical $K$

- Labor market
- Household
- Capital Producers
- Entrepreneurs

Entrepreneurs sell their $K$ to capital producers.
Financial Frictions with Physical $K$

- **Firms**
- **Entrepreneurs**
- **Capital Producers**
- **Banks**
- **Household**
- **Labor market**

Arrow $K'$ from Capital Producers to Entrepreneurs, arrow from households to banks labeled "Loans".
Banks, Households, Entrepreneurs

\[ \omega \sim F(\omega, \sigma_t), \quad E\omega = 1 \]

Accounts for nearly 50% of GDP

Households

Bank

entrepreneur

entrepreneur

entrepreneur

entrepreneur

entrepreneur

Standard debt contract
• Net worth of an entrepreneur who goes to the bank to receive a loan in period $t$:

\[
n_t = P_{k',t}(1 - \delta)\omega\bar{K}_t + r^k_t \omega\bar{K}_t - B_{t-1} \frac{Z_{t-1}}{\pi_t}
\]

An entrepreneur who bought capital in $t-1$ experienced an idiosyncratic shock, $\omega$.

This log-normal shock has mean unity across all entrepreneurs, $\omega \sim F(\omega, \sigma_t)$.
Five Adjustments to Standard DSGE Model for CSV Financial Frictions

• Drop: household intertemporal equation for capital.

• Add: characterization of the loan contracts that can be offered in equilibrium (zero profit condition for banks).

• Add: efficiency condition associated with entrepreneurial choice of contract.

• Add: Law of motion for entrepreneurial net worth (source of accelerator and Fisher debt-deflation effects).

• Introduce: bankruptcy costs in the resource constraint.
Risk Shock and News

• Assume

\[ \hat{\sigma}_t = \rho_1 \hat{\sigma}_{t-1} + u_t \]

iid, univariate innovation to \( \hat{\sigma}_t \)

• Agents have advance information about pieces of \( u_t \)

\[ u_t = \xi_t^0 + \xi_t^1 + \ldots + \xi_{t-8} \]

\[ \xi_{t-i} \sim iid, \ E(\xi_{t-i})^2 = \sigma_i^2 \]

\[ \xi_{t-i} \sim \text{piece of } u_t \text{ observed at time } t - i \]
Economic Impact of Risk Shock

Lognormal distribution:
20 percent jump in standard deviation

Larger number of entrepreneurs in left tail problem for bank
Banks must raise interest rate on entrepreneur
Entrepreneur borrows less
Entrepreneur buys less capital, investment drops, economy tanks
Monetary Policy

• Nominal rate of interest function of:
  – Anticipated level of inflation and change.
  – Slowly moving inflation target.
  – Deviation of output growth from ss path.
  – Monetary policy shock.
Estimation

• Use standard macro data: consumption, investment, employment, inflation, GDP, price of investment goods, wages, Federal Funds Rate.

• Also some financial variables: BAA-AAA corporate bond spreads, value of DOW, credit to nonfinancial business.

• Data: 1985Q1-2008Q4
Key Result

• Risk shocks:
  – important source of fluctuations.

• Out-of-Sample evidence suggests the model deserves to be taken seriously.
Risk Shocks

• Important

• Why are they important?

• What shock do they displace, and why?
Role of the Risk Shock in Macro and Financial Variables

A. GDP growth (y-o-y %)

B. Equity (log-level)

C. Premium

D. Credit growth (y-o-y %)

E. Slope ($R_{Long} - R_{Short}$)

Notes: The grey solid line represents the (two-sided) fitted data. The dotted black line is the model simulations.
Variance Decomposition In Business Cycle Frequencies

<table>
<thead>
<tr>
<th>Risk shock, $\sigma_t$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>49</td>
</tr>
<tr>
<td>Credit</td>
<td>63</td>
</tr>
<tr>
<td>Slope of Term Structure</td>
<td>33</td>
</tr>
<tr>
<td>Risk spread</td>
<td>98</td>
</tr>
<tr>
<td>Real Value of Stock Market</td>
<td>76</td>
</tr>
</tbody>
</table>
Why Risk Shock is so Important

• A. Our econometric estimator ‘thinks’ risk spread ~ risk shock.

• B. In the data: the risk spread is strongly negatively correlated with output.

• C. In the model: bad risk shock generates a response that resembles a recession.

• A+B+C suggests risk shock important.
Figure 6: Dynamic Responses to Two Shocks

A: interest rate spread (Annual Basis Points)

B: credit

C: investment

D: output

E: net worth

F: consumption

G: inflation (APR)

unanticipated risk shock, $\xi_{0,0}$
What Shock Does the Risk Shock Displace, and why?

• The risk shock crowds out some of the role of the marginal efficiency of investment shock.
Figure 6: Dynamic Responses to Two Shocks

A: interest rate spread (Annual Basis Points)
B: credit
C: investment
D: output
E: net worth
F: consumption
G: inflation (APR)

- unanticipated risk shock, $\xi_{0,0}$
- innovation in marginal efficiency of investment, $\zeta_R$
Why does Risk Crowd out Marginal Efficiency of Investment?

Price of capital

Demand shifters: risk shock, $\sigma_t$;

Supply shifter: marginal efficiency of investment, $\zeta_{i,t}$

Quantity of capital
• Marginal efficiency of investment shock can account well for the surge in investment and output in the 1990s, as long as the stock market is not included in the analysis.

• When the stock market is included, then explanatory power shifts to financial market shocks.
CKM Challenge

• CKM argue that risk shocks (actually, any intertemporal shock) cannot be important in business cycles.

• Idea: a shock that hurts the intertemporal margin will induce substitution away from investment and to other margins, such as consumption and leisure.

• CKM argument probably right in RBC model.

• Not valid in New Keynesian models.
Failure of Comovement Between C & I in RBC Models With Risk Shocks

• In RBC model, jump in risk discourages investment.

• Reduction in demand leads to reduction of price of current goods relative to future goods, i.e., real interest rate.

• Real interest rate decline induces surge in demand, partially offsetting drop in investment.

• This Mechanism does not necessarily work in NK model because real rate not fully market determined there.
‘Out of Sample Evidence’

- Out of sample forecasting performance good.
- Predictions for aggregate bankruptcy rate good.
- Correlates well with Bloom evidence on cross-sectional uncertainty.
Conclusion

• Much of the dynamics of past data can be explained as reflecting a risk shock.

• In this analysis, shock is treated as exogenous.

• Interesting to investigate mechanisms that make that ‘shock’ endogenous.