The Zero Bound and Fiscal Policy

Based on work by:
Christiano, Eichenbaum, Rebelo, ‘When is the Government Spending Multiplier Big?’ (JPE, 2011)
Introduction

• The New Keynesian model suggests that an economy may be vulnerable to deep recession when the zero lower bound on the nominal interest rate is binding.

• Fiscal policy could be very effective and desirable in the zero lower bound, though it is relatively less effective in ‘normal’ times.
The ZLB Analysis (Over) Simplified

• Identity:
  \[ \text{expenditures} = \text{GDP} \]

• If one group reduces spending, then GDP must fall unless another group increases.

• Another group increases if real rate drops:
  \[ \frac{R}{\pi^e} \]

• If \( R \) is at lower bound and \( \pi^e \) cannot rise, have a problem.
The ZLB Analysis, cnt’d

- Two reasons people may be reluctant to raise $\pi^e$
  
  - Ex post, monetary authority would not deliver high inflation (Eggertsson).

  - Real-world monetary authorities spent years persuading people they would not use inflation to stabilize economy. Fears consequences of loss of credibility in case they now raise $\pi^e$ for stabilization purposes.
The ZLB Analysis (Over) Simplified

• Recession likely to follow, as real rate fails to drop.

• The recession could be very severe if a deflation spiral occurs.

\[ \frac{R}{\pi^e} \]

– The decrease in spending leads to a fall in marginal cost, which makes firms cut prices.
– When there are price frictions, downward pressure on prices is manifest as a reduction in inflation.
Deflation Cycle in Zero Bound

- Low spending
- Low marginal cost
- Low expected inflation
- High real interest rate
The Whole Analysis, cnt’d

• The preceding indicates that the drop in output might be substantial.

• Options for solving zlb problem
  
  – Direct: by interrupting destructive deflation spiral, increase government spending may have a very large effect on output.

  – Tax credits
    – Investment tax credit
    – ‘cash for clunkers’

  – Increase anticipated inflation
    • Convert to a VAT tax in the future (Feldstein, Correia-Fahri-Nicolini-Teles).

  – Don’t: cut labor tax rate or subsidize employment (Eggertsson)
Outline

• Analysis in ‘normal times’ when zlb constraint on interest rate can be ignored.
  – Show that the government spending multiplier is fairly small.

• Analysis when zlb is binding.
  – Government spending can have a big, welfare-improving impact on output.
Derivation of Model Equilibrium Conditions

• Households
  – First order conditions

• Firms:
  – final goods and intermediate goods
  – marginal cost of intermediate good firms

• Aggregate resources

• Monetary policy

• Three linearized equilibrium conditions:
  – Intertemporal, Pricing, Monetary policy

• Results
Model

• Household preferences and constraints:

\[ E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^\gamma (1-N_t)^{1-\gamma}}{1-\sigma} \right]^{1-\sigma} - 1 + v(G_t) \]

\[ P_t C_t + B_{t+1} \leq W_t N_t + (1 + R_t)B_t + T_t, \quad T_t \sim \text{lump sum taxes and profits} \]

• Optimality conditions

Marginal cost of giving up one unit of consumption to save

\[ u_{c,t} = E_t \beta u_{c,t+1} \]

Marginal benefit tomorrow from saving more today

\[ 1 + R_{t+1} \]

Extra goods tomorrow from saving more today

\[ 1 + \pi_{t+1} \]

Marginal cost (in units of goods) of labor effort

\[ \frac{-u_{N,t}}{u_{c,t}} = \frac{W_t}{P_t} \]

Marginal benefit of labor effort

\[ \pi_{t+1} \]
Linearized Intertemporal Equation

• Inter-temporal Euler equation
\[ E_t \left[ u_{c,t} - \beta u_{c,t+1} \frac{1 + R_{t+1}}{1 + \pi_{t+1}} \right] = 0 \]

• In zero inflation no growth steady state:
\[ 1 = \beta(1 + R) \]

• Totally differentiate:
\[ du_{c,t} - [\beta(1 + R)du_{c,t+1} + \beta u_{c}dR_{t+1} - \beta u_{c}(1 + R)d\pi_{t+1}] = 0 \]

  – Log-differentiation:
\[ u_{c}\hat{u}_{c,t} - \beta(1 + R)u_{c}\left[ \hat{u}_{c,t+1} + \frac{1}{1 + R}dR_{t+1} - d\pi_{t+1} \right] = 0 \]

  – Finally:
\[ \hat{u}_{c,t} - [\hat{u}_{c,t+1} + \beta dR_{t+1} - d\pi_{t+1}] = 0 \]
Linearized intertemporal, cnt’d

- Repeat:

\[ \hat{u}_{c,t} - [\hat{u}_{c,t+1} + \beta dR_{t+1} - d\pi_{t+1}] = 0 \]

\[ u = \frac{[C_t^\gamma (1-N_t)^{1-\gamma}]^{1-\sigma} - 1}{1-\sigma} \rightarrow u_{c,t} = \gamma C_t^{\gamma(1-\sigma)-1}(1 - N_t)^{(1-\gamma)(1-\sigma)} \]

\[ \hat{u}_{c,t} = [\gamma (1 - \sigma) - 1] \hat{C}_t - \frac{(1-\gamma)(1-\sigma)N}{1-N} \hat{N}_t \]
Firms

- Final, homogeneous good

\[ Y_t = \left( \int_0^1 Y_t(i) \frac{\varepsilon-1}{\varepsilon} di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \quad \varepsilon > 1 \]

- Efficiency condition:

\[ P_t(i) = P_t \left( \frac{Y_t}{Y_t(i)} \right)^{\frac{1}{\varepsilon}} \]

- i-th intermediate good

\[ Y_t(i) = N_t(i) \]

- Optimize price with probability \( 1-\theta \), otherwise

\[ P_t(i) = P_{t-1}(i) \]
Intermediate Good Firm Marginal Cost

• Marginal cost:

\[ MC_t = \frac{d\text{Cost}_t}{d\text{Worker}_t} \frac{d\text{Worker}_t}{d\text{Output}_t} = W_t \frac{(1-\nu)}{\text{MP}_{L,t}} \]

subsidy to undo effects of monopoly power \( = (\epsilon-1)/\epsilon \)

household first order condition

\[ = W_t (1-\nu) = P_t \left( \frac{-u_{N,t}}{u_{c,t}} \right) (1-\nu) \]

• Real marginal cost

\[ S_t \equiv \frac{MC_t}{P_t} = \frac{-u_{N,t}}{u_{c,t}} (1-\nu) \]

in steady state

\[ \equiv \frac{\epsilon-1}{\epsilon} \]

marginal cost to household of providing one more unit of labor

\[ \frac{-u_{N,t}}{u_{c,t}} \]

in steady state

marginal benefit of one extra unit of labor

\[ \equiv 1 \]
Aggregate Resources

• Resource relation:
\[ C_t + G_t = Y_t = p_t^* N_t \]
  - \( p_t^* \) is ‘Tak Yun’ distortion
  - recall, distortion = 1 to first order:
    \[ \hat{Y}_t = \hat{N}_t \]

• Log-linear expansion:
\[ (1 - g) \hat{C}_t + g \hat{G}_t = \hat{Y}_t, \quad g \equiv \frac{G}{Y} \]

• Consumption:
\[ \hat{C}_t = \frac{1}{1-g} \hat{Y}_t - \frac{g}{1-g} \hat{G}_t \]
Simplifying Marginal Utility of C

\[
\begin{align*}
\frac{-u_{N,t}}{u_{c,t}} \quad \text{in steady state} \quad 1 & \rightarrow \frac{1-\gamma}{1-N} = \frac{\gamma}{C} \\
\hat{u}_{c,t} &= [\gamma(1 - \sigma) - 1] \hat{C}_t - \frac{(1-\gamma)(1-\sigma)N}{1-N} \hat{N}_t \\
&= [\gamma(1 - \sigma) - 1] \hat{C}_t - \frac{\gamma(1-\sigma)N}{C} \hat{N}_t \\
&= [\gamma(1 - \sigma) - 1] \hat{C}_t - \frac{\gamma(1-\sigma)}{1-g} \hat{N}_t \\
&= [\gamma(1 - \sigma) - 1] \left[ \frac{1}{1-g} \hat{Y}_t - \frac{g}{1-g} \hat{G}_t \right] - \frac{\gamma(1-\sigma)}{1-g} \hat{Y}_t \\
&= -\frac{1}{1-g} \hat{Y}_t - [\gamma(1 - \sigma) - 1] \frac{g}{1-g} \hat{G}_t
\end{align*}
\]
Simplify Intertemporal Equation

• Intertemporal Euler equation:
  \[ \hat{u}_{c,t} = \hat{u}_{c,t+1} + \beta dR_{t+1} - d\pi_{t+1} \]

• Substitute out marginal utility of consumption:
  \[-\frac{1}{1-g} \hat{Y}_t - [\gamma(1 - \sigma) - 1] \frac{g}{1-g} \hat{G}_t \]
  \[= -\frac{1}{1-g} \hat{Y}_{t+1} - [\gamma(1 - \sigma) - 1] \frac{g}{1-g} \hat{G}_{t+1} + \beta dR_{t+1} - d\pi_{t+1} \]

• Rearranging:
  \[\hat{Y}_t + [\gamma(1 - \sigma) - 1]g\hat{G}_t \]
  \[= \hat{Y}_{t+1} + [\gamma(1 - \sigma) - 1]g\hat{G}_{t+1} - (1 - g)[\beta dR_{t+1} - d\pi_{t+1}] \]
Phillips Curve

- Equilibrium condition associated with price setting just like before:

\[
\pi_t = \beta \pi_{t+1} + \kappa \hat{S}_t,
\]

where

\[
\kappa = \frac{(1 - \theta)(1 - \beta \theta)}{\theta}
\]

- Marginal cost:

\[
\hat{S}_t = \frac{(1 - \gamma) C_t}{\gamma (1 - N_t)} = \hat{C}_t - \left(1 - N_t\right) = \hat{C}_t + \frac{N}{1 - N} \hat{N}_t
\]

\[
\left(\hat{C}_t = \frac{1}{1 - g} \hat{Y}_t - \frac{g}{1 - g} \hat{G}_t, \hat{N}_t = \hat{Y}_t\right)
\]

\[
\Rightarrow \left[ \frac{1}{1 - g} + \frac{N}{1 - N} \right] \hat{Y}_t - \frac{g}{1 - g} \hat{G}_t
\]
Monetary Policy

- Monetary policy rule (after linearization)

\[ dR_{t+1} = \rho_R dR_t + (1 - \rho_R) \left[ \frac{\phi_1}{\beta} \pi_{t+k} + \frac{\phi_2}{\beta} \hat{Y}_{t+l} \right] \]

\[ dR_{t+1} \equiv R_{t+1} - R, \quad R = \frac{1}{\beta} - 1 \]

\[ \hat{Y}_t \equiv \frac{Y_t - Y}{Y} \]

\[ k, l = 0, 1. \]
Pulling All the Equations Together

• IS equation:

\[ \hat{Y}_t + [\gamma (1 - \sigma) - 1]g\hat{G}_t = \hat{Y}_{t+1} + [\gamma (1 - \sigma) - 1]g\hat{G}_{t+1} - (1 - g)[\beta dR_{t+1} - d\pi_{t+1}] \]

• Phillips curve:

\[ \pi_t = \beta \pi_{t+1} + \kappa \left[ \left( \frac{1}{1-g} + \frac{N}{1-N} \right)\hat{Y}_t - \frac{g}{1-g} \hat{G}_t \right] \]

• Monetary policy rule:

\[ dR_{t+1} = \rho_R dR_t + (1 - \rho_R) \left[ \frac{\phi_1}{\beta} \pi_{t+k} + \frac{\phi_2}{\beta} \hat{Y}_{t+l} \right] \]
The Equations in Matrix Form

\[
\begin{bmatrix}
-\frac{1}{1-g} & -1 & 0 \\
0 & \beta & 0 \\
l(1 - \rho_R)\frac{\phi_2}{\beta} & k(1 - \rho_R)\frac{\phi_1}{\beta} & 0 \\
\end{bmatrix}
\begin{pmatrix}
\hat{Y}_{t+1} \\
\pi_{t+1} \\
dR_{t+2}
\end{pmatrix}
\]

\[
+ \begin{bmatrix}
\frac{1}{1-g} & 0 & \beta \\
\kappa\left(\frac{1}{1-g} + \frac{N}{1-N}\right) & -1 & 0 \\
(1 - l)(1 - \rho_R)\frac{\phi_2}{\beta} & (1 - k)(1 - \rho_R)\frac{\phi_1}{\beta} & -1 \\
\end{bmatrix}
\begin{pmatrix}
\hat{Y}_t \\
\pi_t \\
dR_{t+1}
\end{pmatrix}
+ \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & \rho_R \\
\end{pmatrix}
\begin{pmatrix}
\hat{Y}_{t-1} \\
\pi_{t-1} \\
dR_t
\end{pmatrix}
\]

\[
+ \begin{pmatrix}
g[\gamma(\sigma - 1) + 1] & 0 & 0 \\
\frac{1-g}{1-g} & 0 & 0 \\
0 & 0 & 0 \\
\end{pmatrix}
\hat{G}_{t+1} + \begin{pmatrix}
\frac{g[\gamma(\sigma - 1) + 1]}{1-g} & 0 & 0 \\
0 & -\frac{\kappa g}{1-g} & 0 \\
0 & 0 & 0 \\
\end{pmatrix}
\hat{G}_t,
\]

- or,

\[
\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t = 0.
\]

\[
s_t = Ps_{t-1} + \varepsilon_t, \quad s_t \equiv \hat{G}_t, \quad P = \rho
\]
Solution:

- Undetermined coefficients, $A$ and $B$:

\[ z_t = Az_{t-1} + Bs_t \]

- $A$ and $B$ must satisfy:

\[ \alpha_0 A^2 + \alpha_1 A + \alpha_2 = 0 \]
\[ \alpha_0 (AB + BP) + \alpha_1 B + \beta_0 P + \beta_1 = 0. \]

- When $\rho_R = 0$, $\alpha_2 = 0 \rightarrow A = 0$ works.
Results

• Fiscal spending multiplier small, but can easily be bigger than unity (i.e., $C$ rises in response to $G$ shock)

• Contrasts with standard results in which multiplier is less than unity
  – Typical preferences in estimated models:
    \[ E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-\sigma}}{1-\sigma} - \psi \frac{N_t^{1+\gamma}}{1+\gamma} + \nu(G_t) \right], \psi, \gamma, \sigma > 0. \]
  – Marginal utility of $C$ independent of $N$ for CGG
  – Marginal utility of $C$ increases in $N$ for KPR.
Simulation Results

- Benchmark parameter values:

\( \kappa = 0.035, \beta = 0.99, \phi_1 = 1.5, \phi_2 = 0, N = 0.23, g = 0.2, \sigma = 2, \rho = 0.8, \rho_R = 0 \)

Multiplier = 1.05, constant.

\[ \frac{G_t - G}{Y} = \frac{G_t - G}{G} \frac{G}{Y} = \hat{G}_t g \]
Multiplier for Alternative Parameter Values

\[ \phi_1 = 1.5, \phi_2 = 0, \rho_R = 0, \rho = 0.8, \kappa = 0.03, \beta = 0.99, \gamma = 0.28571, N = 0.33333, g = 0.2, \sigma = 2 \]

- Results: multiplier bigger
  - the less monetary policy allows \( R \) to rise.
  - the more complementary are consumption and labor (i.e., the bigger is \( \sigma \)).
  - the smaller the negative income effect on consumption (i.e., the smaller is \( \rho \)).
  - smaller values of \( \kappa \) (i.e., more sticky prices)
Multiplier for Alternative Parameter Values

\[ dR_{t+1} = \rho_R dR_t + (1 - \rho_R) \left( \frac{\phi_1}{\beta} \pi_t + \frac{\phi_2}{\beta} \hat{Y}_t \right) \]

- Results: multiplier bigger
  - the less monetary policy allows \( R \) to rise.
Multiplier for Alternative Parameter Values

\[ \phi_1 = 1.5, \phi_2 = 0, \rho_R = 0, \rho = 0.8, \kappa = 0.03, \beta = 0.99, \gamma = 0.28571, N = 0.33333, g = 0.2, \quad \sigma = 2 \]

\[ u(C_t, N_t) = \frac{[C_t^{\gamma} (1 - N_t)^{1-\gamma}]^{1-\sigma} - 1}{1 - \sigma} \]

\[ u_{c,t} = C_t^{[\gamma(1-\sigma)-1]} (1 - N_t)^{(1-\gamma)(1-\sigma)} \]

- Results: multiplier bigger
  - the more complementary are consumption and labor (i.e., the bigger is \( \sigma \)).
Multiplier for Alternative Parameter Values

\[
\hat{G}_t = \rho \hat{G}_{t-1} + \varepsilon_t
\]

- Results: multiplier bigger
  - the smaller the negative income effect on consumption (i.e., the smaller is \( \rho \)).
Multiplier for Alternative Parameter Values

\[ \phi_1 = 1.5, \phi_2 = 0, \rho_R = 0, \rho = 0.8, \kappa = 0.03, \]
\[ \beta = 0.99, \gamma = 0.28571, N = 0.33333, g = 0.2, \quad \sigma = 2 \]

- Results: multiplier bigger
  - smaller values of \( \kappa \) (i.e., more sticky prices)
Analysis of Case when the Non-negativity Constraint on the Nominal Interest Rate is Binding

• Need a shock that puts us into the lower bound.

• One possibility: increased desire to save.
  – Seems particularly relevant at the current time.
  – Other shocks will do it too......

• Discount rate shock.
Monetary Policy

- Monetary policy rule (after linearization)

\[
Z_{t+1} = R + \rho_R (R_t - R) + (1 - \rho_R) \left[ \frac{\phi_1}{\beta} \pi_t + \frac{\phi_2}{\beta} \hat{Y}_t \right]
\]

\[
\hat{Y}_t = \frac{Y_t - Y}{Y}, \quad R = \frac{1}{\beta} - 1
\]

\[
R_{t+1} = \begin{cases} 
Z_{t+1} & \text{if } Z_{t+1} > 0 \\
0 & \text{if } Z_{t+1} \leq 0
\end{cases}
\] nonlinearity
Eggertsson-Woodford Saving Shock

• Preferences:

\[ u(C_0, N_0, G_0) + \frac{1}{1+r_1} E_0 \left\{ u(C_1, N_1, G_1) + \frac{1}{1+r_2} u(C_2, N_2, G_1) + \frac{1}{1+r_2} \frac{1}{1+r_3} u(C_3, N_3, G_3) \ldots \right\} \]

• Before \( t=0 \)

  – System was in non-stochastic, zero inflation steady state,

  \[ r_{t+1} = R = \frac{1}{\beta} - 1 \]

  \[ R_{t+1} = R \]

  \[ \hat{G}_t = 0, \ \text{for all} \ t \]
Saving Shock, cnt’d

• At time $t=0$,

$$r_1 = r^l < 0$$

$$\text{Prob}[r_{t+1} = r | r_t = r^l] = 1 - p$$

$$\text{Prob}[r_{t+1} = r^l | r_t = r^l] = p$$

$$\text{Prob}[r_{t+1} = r^l | r_t = r] = 0$$

• “Discount rate drops in $t=0$ and is expected to return permanently to its ‘normal’ level with constant probability, $1-p$.”
Zero Bound Equilibrium

• simple characterization:

\[ \pi^l, \hat{\pi}^l, R = 0, Z^l \leq 0 \quad \text{while discount rate is low} \]

\[ \pi_t = \hat{\pi}_t = 0, R = r \quad \text{as soon as discount rate snaps back up} \]
Fiscal Policy

• Government spending is set to a constant deviation from steady state, during the zero bound.

• That is,

\[ \hat{G}_t \text{ may be nonzero while } r_{t+1} = r^l, \quad \hat{G}_t = 0 \text{ when } r_{t+1} = r \]
Equations With Discount Shock

• IS equation:

\[ \hat{Y}_t - g[\gamma(\sigma - 1) + 1] \hat{G}_t = -(1 - g)[\beta(R_{t+1} - r_{t+1}) - E_t \pi_{t+1}] + E_t \hat{Y}_{t+1} - g[\gamma(\sigma - 1) + 1]E_t \hat{G}_{t+1} \]

\[ \hat{Y}^l - g[\gamma(\sigma - 1) + 1] \hat{G}^l = -(1 - g)[\beta(0 - r^l) - p \pi^l] + p \hat{Y}^l - g[\gamma(\sigma - 1) + 1]p \hat{G} \]

• Phillips curve:

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa \left[ \left( \frac{1}{1-g} + \frac{N}{1-N} \right) \hat{Y}_t - \frac{g}{1-g} \hat{G}_t \right] \]

\[ \pi^l = \beta p \pi^l + \kappa \left( \frac{1}{1-g} + \frac{N}{1-N} \right) \hat{Y}^l - \frac{g}{1-g} \kappa \hat{G}^l \]

• Monetary Policy:

\[ R_{t+1} = 0 \]

\[ Z_{t+1} = R + \rho_R (R_t - R) + (1 - \rho_R) \left[ \frac{\phi_1}{\beta} \pi_t + \frac{\phi_2}{\beta} \hat{Y}_t \right] \leq 0 \]
Solving for the Zero Bound Allocations

• Is equation:

\[ \hat{Y}^l - g[\gamma(\sigma - 1) + 1]\hat{G}^l = -(1 - g)[\beta(0 - r^l) - p\pi^l] + p\hat{Y}^l - g[\gamma(\sigma - 1) + 1]p\hat{G} \]

• Phillips curve:

\[ \pi^l = \beta p\pi^l + \kappa\left(\frac{1}{1-g} + \frac{N}{1-N}\right)\hat{Y}^l - \frac{g}{1-g}\kappa\hat{G}^l \]

• Two equations in two unknowns!

  – Solve for \( \hat{Y}^l, \pi^l \) and verify that \( Z^l \leq 0 \)
Solution

• Inflation:

\[ \pi^l = \kappa \left( \frac{1}{1-g} + \frac{N}{1-N} \right) \left[ g[\gamma(\sigma-1)+1] \hat{G}^l + \frac{1-g}{1-p} \beta r^l \right] - \frac{g}{1-g} \kappa \hat{G}^l \]

\[ 1 - \beta p - \kappa \left( \frac{1}{1-g} + \frac{N}{1-N} \right) p \frac{1-g}{1-p} \]

• Output:

\[ \hat{Y}^l = g[\gamma(\sigma - 1) + 1] \hat{G}^l + \frac{1-g}{1-p} \left[ \beta r^l + p \pi^l \right] \]
Numerical Simulations

- Results: multiplier 3.7 at benchmark parameter values and may be gigantic.

\[
\frac{\partial Y}{\partial G} = \phi_1 = 1.5, \phi_2 = 0, \rho_R = 0, \rho = 0.8, \\
\kappa = 0.03, \beta = 0.99, \gamma = 0.28571, \\
N = 0.33333, g = 0.2, \hat{G} = 0, s
\]

Problem is worse with more flexible prices!
• As $p$ increases, zero-bound becomes more severe...this is because with higher $p$, fall in output is more persistent and resulting negative wealth effect further depresses consumption.

benchmark parameter values: $\phi_1 = 1.5$, $\phi_2 = 0$, $\rho_R = 0$, $\rho = 0.8$, $\kappa = 0.03$, $\beta = 0.99$, $\gamma = 0.28571$, $N = 0.33333$, $g = 0.2$, $k = 0$, $l = 0$, $Ghat = 0$, $\text{sig} = 2$, $p = 0.8$, $r^l = -0.01$
Fiscal Expansion in Zero Bound Highly Effective, But is it *Desirable*?

- **Intuition:**
  - *Yes*....

  - the vicious cycle produces a huge, inefficient fall in output

  - in the first-best equilibrium, output, consumption and employment are invariant to discount rate shocks

  - If $G$ helps to partially undo this inefficiency, then surely it’s a good thing
Fiscal Expansion in Zero Bound Highly Effective, But is it Desirable?

- Preferences

\[
\sum_{t=0}^{\infty} \left( \frac{p}{1 + r^l} \right)^t \left[ \frac{(C^t)^\gamma (1 - N^l)^{1-\gamma}}{1 - \sigma} \right]^{1-\sigma} - 1 + v(G^l) \]

\[
= \frac{1}{1 - \frac{p}{1 + r^l}} \left[ \frac{(C^t)^\gamma (1 - N^l)^{1-\gamma}}{1 - \sigma} \right]^{1-\sigma} - 1 + v(G^l) \]

\[
= \frac{1}{1 - \frac{p}{1 + r^l}} \left[ \frac{(N(\hat{Y}^l + 1) - Ng(\hat{G}^l + 1))^{\gamma} (1 - N(\hat{Y}^l + 1))^{1-\gamma}}{1 - \sigma} \right]^{1-\sigma} - 1 + v(Ng(\hat{G}^l + 1)) \]

- Compute optimal \( \hat{G}^l \)

  - (i) \( v(G^l) = 0 \),
  
  - (ii) \( v(G) = \psi_g \frac{G^{1-\sigma}}{1 - \sigma} \), \( \psi_g \) chosen to rationalize \( g = 0.2 \) as optimal in steady state
Case Where G is not Valued

Optimal $G$ is substantial, around 5%.

\[
\phi_1 = 1.5, \phi_2 = 0, \rhoR = 0, \rho = 0.8, \kappa = 0.03, \beta = 0.99, \\
g = 0.28571, N = 0.33333, g = 0.2, k = 0, l = 0, G\hat{=} = 0, \sigma = 2, \gamma
\]
Case Where Gov’t Spending is Desirable

\[ \phi_1 = 1.5, \phi_2 = 0, \rho_R = 0, \rho = 0.8, \kappa = 0.03, \beta = 0.99 \]
\[ \gamma = 0.28571, N = 0.33333, g = 0.2, k = 0, l = 0, \hat{G} = 0, \sigma = 2, \psi \]

Optimal \( Y \) higher than before crisis

The high level of output is necessary to get partial recovery in consumption

\[ \psi \]
Introducing Investment

• Inclusion of investment does not have a large, qualitative effect.

• Financial frictions could make things much worse.
  – Deflation hurts net worth of investors with nominal debt, and this forces those agents to cut spending by more.
Conclusion of $G$ Multiplier Analysis

- Government spending multiplier in a neighborhood of unity in ‘normal times’.

- Multiplier can be large when the zero bound is binding (because $R$ constant then).

- Increase in $G$ is welfare improving during lower bound crisis.

- Caveat: focused exclusively on multiplier
  - Increasing $G$ may be bad idea because hard to reverse.
  - May be other ways of accomplishing similar thing (e.g., transition to VAT tax over time).