Financial Frictions Under Asymmetric Information and Moral Hazard

In the past two decades several economies have experienced a ‘Sudden Stop’: over a brief period of time, output collapsed and the current account shifted sharply from negative to positive\(^1\). The observation that these countries exported resources when they hit on hard times seems inconsistent with the notion that international financial markets are efficient. Efficiency, one might imagine, dictates that resources should flow \textit{towards} countries in trouble, and away from countries that are doing fine. It is possible that Sudden Stop is a symptom of financial market inefficiencies. However, a 1991 \textit{Econometrica} paper by Andrew Atkeson raises the possibility that the puzzling Sudden Stop observations may in fact be part of efficient mechanism after all.\(^2\) This note explains the argument in a two period version of Atkeson’s model. A second model is presented which captures the same idea in a different way.

The basic argument is simple. In the model, countries borrow internationally in order to finance investment activities. Investment raises the probability that a high output state will occur in the future. A key feature of the environment is that investment is not observable to foreign lenders. Still, international financial arrangements work best if there is substantial investment, and so loan contracts must be structured to give countries a substantial incentive to invest. Since the effect of investment is to increase the likelihood of high output states, to give countries a strong incentive to invest requires that the level of consumption in the high output state is higher than what it is in the low output state. A point of Atkeson’s paper is that consumption might be so high in the high output state that it exceeds output itself in that state. Similarly, consumption may be so low in the low output state that it is less than output in that state. In this case, we have a current account surplus for countries in bad times (the low output state) and a current account deficit for countries in good times.

\(^1\)Think of the current account simply as exports minus imports of goods.
\(^2\)There is a version of a real business cycle model that may be able to capture the phenomenon. Suppose the drop in output is due to a bad shock to technology that is persistent. It makes sense for the citizens of that country to reduce domestic investment and generate a current account surplus, which corresponds to accumulating financial claims on investment projects in the rest of the world. A drop in domestic employment may be efficient under these circumstances since with the drop in investment there is less to do. Moreover, consumption might optimally fall too if the marginal utility of consumption falls with a rise in leisure.
The first model in these notes is a two period version of Atkeson’s model (you may find it helpful to read about the model in Atkeson’s paper, or in Chapter 15 of the Ljungqvist-Sargent textbook). A shortcoming of the two period version of the Atkeson model is that it is difficult to map such a model to data. Extending the model to multiple periods at first may seem difficult. However, Atkeson shows that the extension to multiple periods is easy if one adopts a functional equation perspective. We do not go into this approach in this handout. Instead, we present a second model. That model can also be studied to determine if the optimal contract displays the Sudden Stop characteristic. An advantage of this model, is that it can be made dynamic simply by repeating it. The reason is that in the one-period contract version of the model, there are no physical state variables. The repeated version of the model presumably has more equilibria than simply the repetition of the equilibria in the one period version. But, that is not of interest at the moment.

A criticism of the moral hazard models described in this handout is that one never sees ‘bankruptcy’ in equilibrium. An attractive feature of the costly state verification models is that those do predict bankruptcies.

1. A one-period version of Atkeson’s Model

1.1. The Environment

There are many ex ante identical, small countries in the world. Each has an endowment in the first period, which is \( Y^H \) in the ‘high state’ and \( Y^L \) in the ‘low state’, where

\[ 0 \leq Y^L < Y^H. \]

Preferences of the typical country are:

\[ u(c) + \beta P(I)u(c^H) + \beta (1 - P(I))u(c^L), \]

where \( c^H \) and \( c^L \) denotes domestic consumption in the high and low states, respectively. Here, the period utility function, \( u \), is strictly concave and continuously differentiable, with

\[ \lim_{c \to 0} u'(c) = \infty, \quad \lim_{c \to \infty} u'(c) = 0. \]

A utility function that satisfies these conditions is:

\[ u(x) = \frac{x^{1-\gamma} - 1}{1 - \gamma}, \quad \gamma > 0. \]
In (1.1), $P(I)$ is the probability of the high state, and $I$ denotes the level of investment in the country. The function, $P : [0, \infty) \rightarrow [0, 1]$ is continuously differentiable and has the following additional properties:

$$P(0) = 0, P'(I) > 0, P(\infty) = 1, \lim_{I \rightarrow \infty} P'(I) = 0, \lim_{I \rightarrow 0} P'(I) = \infty.$$ 

An example which satisfies all but the last condition is:

$$P(I) = 1 - e^{-aI}, a > 0.$$ 

(1.2)

Consumption in each date and state must be non-negative: $c, c^H, c^L \geq 0$.

The country budget constraint in the first period is:

$$c + I \leq b,$$

where $b \geq 0$ denotes period 0 borrowing (on a per country basis) from an international lender described below. The budget constraint in the second period for a small country that is in the high state is:

$$c^H \leq Y^H - d^H,$$

where $d^H$ denotes exports in the high state. Similarly,

$$c^L \leq Y^L - d^L,$$

where $d^L$ denotes exports in the low state. The objects, $d^H$ and $d^L$ may be positive or negative (in which case they would conventionally be called imports).

International lenders are institutions that reside outside of the small countries that we are studying. The lenders finance all the investment and consumption activities of our small countries in the first period. Lenders are uncertain whether any particular country will be in the high state or the low state in period 2. However, because each lender deals with many countries, an individual lender has no uncertainty about how many of the countries it deals with will be in each state in period 2. Each lender knows that precisely $P(I)$ of the countries it lends to will be in the high state and precisely $1 - P(I)$ will be in the low state. As a result, lenders face no uncertainty in terms of total revenues. They know that their revenues will be $P(I) d^H + (1 - P(I)) d^L$, when expressed as an average over the number of countries they deal with. In addition, lenders have access to funds at the rate of interest, $r$. Although each lender lends to many countries, it
is assumed that there are many lenders. As a result, individual lenders take \( r \) as given.

If a lender makes loans in the amount \( b \) to each of the countries it deals with, its profits at the end of period 2 are:

\[
P(I) d^H + (1 - P(I)) d^L - (1 + r) b.
\]

Because there is free entry into the lending business, the above expression cannot be positive. If it were, we suppose that more lenders would enter and the greater number of lenders would push \( r \) up. If profits were negative, lenders would exit and \( r \) would fall. Thus, free entry and exit ensures:

\[
b(1 + r) = P(I)d^H + (1 - P(I))d^L. \tag{1.3}
\]

Individual lenders are free to set the terms, \( (b, I, d^H, d^L) \), of a loan contract. However, competition prevents lenders from setting these terms so that they make positive profits (if they tried, another lender would jump in and offer better terms and draw away all the borrowers) and profit maximization prevents lenders from setting the terms of the loan contract so that lenders make negative profits. We conclude that countries looking for a loan can in effect view (1.3) as defining a menu of loan contracts from which they can choose.

**1.2. Equilibrium Under Full Information**

We begin by considering the full information case where \( I \) and \( c \) are observed by the lender. Thus, if a country takes a particular loan contract, \( (b, I, d^H, d^L) \), the lender has no difficulty verifying that the country has actually implemented the terms.\(^3\) If a country agrees to set \( I \) at a certain level, then the lender can see if it does it or not. We assume that sufficient sanctions exist so that verifiable deviations from the loan contract never occur.

Because countries view the zero profit condition, (1.3), as a menu of available loan contracts, they will pick that contract that maximizes their utility. That is, the loan contract that will exist in equilibrium is a \((b, I, d^H, d^L)\) that maximizes (1.1) subject to (1.3) and the sign restrictions. The Lagrangian representation of this problem is:

\[
\max_{b, d^H, d^L, I} u(b - I) + \beta P(I)u(Y^H - d^H) + \beta (1 - P(I))u(Y^L - d^L) \tag{1.4}
\]

\(^3\)It is obvious that a lender has no problem verifying that a country implements the \( b, d^H \) and \( d^L \) parts of a loan contract.
\[
+\lambda \left[ P(I)d^H + (1 - P(I))d^L - b(1 + r) \right],
\]

where \( \lambda \) is the Lagrange multiplier. To determine the sign of \( \lambda \), suppose that (1.4) were solved with \( \lambda = 0 \). In this case, the optimum is \( I = 0 \) (so that \( P(I) = 0 \)), \( d^L = -\infty \) and (1.3) is violated with

\[
P(I)d^H + (1 - P(I))d^L - b(1 + r) < 0.
\]

The latter indicates that for the solution to (1.4) to correspond to the equilibrium contract requires that \( \lambda \) be strictly positive.

I assume that there exists an interior equilibrium with \( I, b, c^H, c^L > 0 \). The first order necessary conditions associated with the Lagrangian problem are:

\[
\begin{align*}
b & : \ u'(b - I) = \lambda(1 + r) \\
I & : \ u'(b - I) = \beta P'(I) \left[ u(Y^H - d^H) - u(Y^L - d^L) \right] + \lambda P'(I) [d^H - d^L] \\
d^H & : \ \beta u'(Y^H - d^H) = \lambda \\
d^L & : \ \beta u'(Y^L - d^L) = \lambda \\
\lambda & : \ P(I)d^H + (1 - P(I))d^L - b(1 + r) = 0
\end{align*}
\]

This represents 5 equations in the 5 unknowns, \( b, I, d^H, d^L \) and \( \lambda \). The \( d^H \) and \( d^L \) equations imply

\[
c^H = c^L, \quad (1.5)
\]

a key result. According to (1.5), in equilibrium countries with low output receive enough goods from abroad so that their consumption does not suffer.

Result (1.5) implies

\[
Y^H - Y^L = d^H - d^L.
\]

Substituting (1.6) and the \( b \) equation into the \( I \) equation, and rearranging:

\[
P'(I) = \frac{1 + r}{Y^H - Y^L}.
\]

This expression can be solved uniquely for \( I \) strictly positive under our assumptions on \( P \).

Rearranging (1.3) and making use of (1.6), we obtain:

\[
P(I) \left( Y^H - Y^L \right) + d^L = b(1 + r), \quad (1.7)
\]
Substitute out for $\lambda$ in the $b$ equation from the $d^L$ equation:

$$u'(b - I) = \beta u'(Y^L - d^L)(1 + r).$$

Use (1.7) to substitute out for $d^L$ in the previous equation:

$$u'(b - I) = \beta u'(Y^L + P(I)(Y^H - Y^L) - b(1 + r))(1 + r).$$

This represents one equation in the unknown, $b$. Note that the left side $\uparrow +\infty$ as $b \downarrow I$ and is monotonically decreasing towards zero as $b \to \infty$. Turning to the right side, note that it is monotonically increasing in $b$, rising to $+\infty$ as the argument in marginal utility goes to zero. The equilibrium value of $b$ is found as the unique intersection of downward-sloped left side and the upward-sloped right side of (??).\(^4\) In the case where utility is exponential, this intersection may be found analytically.\(^5\) With $b$ in hand, $\lambda$ may be found from the $b$ equation. Given $\lambda$, $d^H$ and $d^L$ may be found from the $d^H$ and $d^L$ equations.

We conclude that the efficient, full information contract implies perfect consumption smoothing across the two states of nature in the second period. This is the sort of efficient insurance arrangement one would expect.

1.3. Equilibrium Under Limited Information

Suppose now that a country’s $I$ and $c$ are not observable to outsiders. The assumption that only verifiable actions can be enforced means that the $I$ stipulated in a loan contract will only be implemented in case countries find it in their direct interest to do so. The $I$ in a full information contract is not implementable because countries do not have the incentive to implement it (the contract is not ‘incentive-compatible’ when there is limited information). Suppose, for example, that a full information contract were offered by lenders. In this case all countries

\(^4\)There is an interior intersection of these lines by the interiority assumption stated at the beginning. This in turn might require adjustment in the parameters of the model, $\beta, Y^H, Y^L, r$ and the parameters of $P(\cdot), u(\cdot)$.

\(^5\)Thus, if $u(x) = (x^{1-\gamma} - 1) / (1 - \gamma)$, then $u'(x) = x^{-\gamma}$ and

$$(b - I)^{-\gamma} = \beta(Y^L + P(I)(Y^H - Y^L) - b(1 + r))^{-\gamma}(1 + r)$$

so that

$$b - I = \beta^{-\frac{1}{\gamma}}(Y^L + P(I)(Y^H - Y^L) - b(1 + r))(1 + r)^{-\frac{1}{\gamma}},$$

which is linear in $b$.\(^6\)
would claim that they implemented the required $I$. However, they would actually set $I = 0$ and cause the low state to occur with unit probability. Lenders could not prove that any particular country in fact deviated from the terms of the contract because low output is always a possibility, even in the absence of a deviation. The reason all countries would implement $I = 0$ is that under the full information contract, countries receive the same level of second period utility regardless of the state (see (1.5)). As a result, there is no point sacrificing consumption in the first period to increase the likelihood of the good state in the second period. But with investment set to zero, all countries end up in the low state and the banks’ zero profit condition is not satisfied. Knowing in advance that countries have no incentive to invest under a full information contract, lenders will not offer such a contract in the first place.

When investment is not verifiable by the lenders, a loan contract must be structured so that countries have the incentive to invest the amount that is required under the contract. That is, the loan contract must be structured so that it is incentive compatible. To understand what constraints incentive compatibility places on a loan contract, we must characterize what exactly a country’s incentives to invest are. A country that receives terms, $d^H$, $d^L$ and $b$ takes these as given and selects $I$ to maximize (1.4) with $\lambda = 0$. The first order necessary condition associated with the optimal choice of $I$ is:

\[ u'(b - I) = \beta P'(I) [u(Y^H - d^H) - u(Y^L - d^L)] . \]  

(1.8)

The object on the left of the equality is the cost of investing. The object on the right is the benefit. The benefit is the increase in the probability of the high state that occurs with an increase in investment, times the improvement in utility in the high state over the low state. It is because utility in the high and low states are the same under the full information contract, that borrowers have no incentive to invest when there is limited information. For any lending contract, $(b, I, d^H, d^L)$, to be incentive compatible requires that (1.8) be satisfied. In particular, if a contract specifies $I > 0$ then to be incentive compatible it must offer a sufficiently big jump in utility from the low to the high state to ensure that countries have the incentive to implement $I$.

We conclude that under limited information, the menu of contracts that will be offered are those that satisfy both the zero profit condition, (1.3), and the incentive constraint, (1.8). Full information contracts are off the menu. The loan contract, $(b, I, d^H, d^L)$ that is traded in equilibrium maximizes (1.1) subject to
(1.3) and incentive (1.8). The Lagrangian representation of this problem is:

$$\max_{b,d^H,d^L,I} u(b - I) + \beta P(I) u(Y^H - d^H) + \beta(1 - P(I)) u(Y^L - d^L)$$

$$+ \lambda \left[ P(I) d^H + (1 - P(I)) d^L - b(1 + r) \right]$$

$$+ \mu \left\{ \beta P'(I) \left[ u(Y^H - d^H) - u(Y^L - d^L) \right] - u'(b - I) \right\}$$

(1.9)

Note that $\mu$, the multiplier on the incentive constraint, must be positive. To see this, imagine solving the Lagrangian problem with $\mu = 0$ (as we did in the previous subsection). The incentive constraint would be violated in this case, since the solution with $\mu = 0$ implies $u'(b - I) > 0$ and a zero value for the object in square brackets in the incentive constraint. With the incentive condition negative, the Lagrangian problem must be adjusted by making $\mu > 0$. Increasing $\mu$ penalizes contracts that violate the incentive constraint. As the value of $\mu$ is raised, the contract that solves (1.9) more closely resembles the equilibrium contract because it places greater and greater weight on respecting the incentive constraint. The standard Lagrangian result is that there exists values of $\mu$ and $\lambda$ such that the solution to (1.9) coincides with the solution to the problem, maximize (1.1) subject to (1.3) and incentive (1.8).

There are six equations associated with the optimal limited information contract: the four first order conditions associated with the Lagrangian problem, plus (1.3) and (1.8). These six equations can be used to determine the unknowns, $b, d^H, d^L, I$, as well as the two multipliers, $\lambda$ and $\mu$.

As noted after (1.8), to provide strong incentives to invest requires that $Y^H - d^H$ be substantially larger than $Y^L - d^L$. This could be achieved by setting $d^H < 0$, and $d^L > 0$. But, $d^H$ is the current account surplus in the high output state and $d^L$ is the current account surplus in the low output state (note, $Y^H - c^H = d^H$). To explore this further requires computing the terms of the optimal contract.

The first order conditions for an interior optimum are:

$$b : \ u'(b - I) = \lambda (1 + r) + \mu u''(b - I)$$

$$I : \ u'(b - I) = \beta P'(I) \left[ u(Y^H - d^H) - u(Y^L - d^L) \right] + \lambda P'(I) (d^H - d^L)$$

$$\quad + \mu \left\{ \beta P'(I) \left[ u(Y^H - d^H) - u(Y^L - d^L) \right] - u'(b - I) \right\}$$

$$d^H : \ \lambda = \left[ 1 + \mu \frac{P'(I)}{P(I)} \right] \beta u'(Y^H - d^H)$$

$$d^L : \ \lambda = \left[ 1 - \mu \frac{P'(I)}{1 - P(I)} \right] \beta u'(Y^L - d^L)$$
\[
\lambda : \quad b(1 + r) = P(I) d^H + (1 - P(I)) d^L \\
\mu : \quad u'(b - I) = \beta P'(I) \left[ u(Y^H - d^H) - u(Y^L - d^L) \right]
\]

There are six unknowns, \(b, I, \lambda, \mu, d^H, d^L\), in six equations. These can be solved once we select functional forms for \(u\) and \(P\), and assign numerical values to the corresponding parameters (the functional forms in section 1.1 can be used). Solving a system of six equations in six unknowns can be quite cumbersome. The problem can be reduced to one of solving two equations in two unknowns as follows.

Fix values of \(d^L < Y^L, Y^H - Y^L > \delta\), where \(\delta \equiv d^H - d^L\), so that \(d^H = d^L + \delta\). Substitute out for \(b\) from the \(\lambda\) equation into the \(\mu\) equation to obtain:

\[
u'(\frac{P(I) (d^H - d^L) + d^L}{1 + r} - I) = \beta P'(I) \left[ u(Y^H - d^H) - u(Y^L - d^L) \right].
\]

This is a non-linear equation in the single variable, \(I\). After \(I\) is computed, \(b\) can be computed from the \(\lambda\) equation. Then, \(\mu\) can be computed by substituting out for \(\lambda\) in the \(d^L\) equation from the \(d^H\) equation and rearranging:

\[
\mu = \frac{1}{P'(I)} \frac{u'(Y^L - d^L) - u'(Y^H - d^H)}{\frac{u'(Y^L - d^L)}{P(I)} + \frac{u'(Y^H - d^H)}{1 - P(I)}}.
\]

The multiplier, \(\lambda\), can now be computed from either the \(d^H\) or the \(d^L\) equation. Now, evaluate the \(b\) and \(I\) equations. If one or both of these are different from zero, try a different value of \(d^L, \delta\). A numerical equation solver can be used to find quickly find values of \(d^L, \delta\) such that the \(b\) and \(I\) equations are satisfied.

Consider the following parameter values:

\[
a = 2, \quad r = 0, \quad \gamma = 2, \quad Y^H = 2, \quad Y^L = 1, \quad \beta = 1.
\]

\(^6\)Write this equation, \(f(I) = 0\). To solve this equation numerically and verify that its solution is unique, fix a grid of values, \(0 < I_1 < I_2 .... < I_N\), where \(N\) is, say, 500. The grid on \(I\) should only contain points that imply non-negative first period consumption given the level of \(b\) implied by the \(\lambda\) equation. Verify that there is only one \(i\) such that \(f(I_i) f(I_{i-1}) < 0\), \(i = 2, 3, ...., N\). Denote that \(i\) by \(i^*\). Then, \(I_{i^*}\) represents a good initial guess for the \(I\) that we seek. Provide this as the initial guess to a numerical zero-finding routine such as MATLAB’s fzerol.m.
Then,

\[ c = 0.68, \ I = 0.23, \ c^H = 0.70, \ c^L = 0.32, \ P(I) = 0.37, \ b = 0.91, \ \mu = 0.29, \ \lambda = 4.06. \]

Note how much higher \( c^H \) is than \( c^L \). Evidently, complete insurance does not occur under the equilibrium loan contract. When \( a \) is increased, so that the return to investing goes up, \( b \) and \( I \) both increase. In addition, the difference between \( c^H \) and \( c^L \) narrow, because incentive compatibility of higher \( I \) is achieved by the higher physical return on investment.

**2. Another Model**

Here is another model of international borrowing. Let the utility function of domestic residents be:

\[ u(c, l) = c - \frac{1}{2}l^2, \]

with budget constraint:

\[ c \leq (1 - \tau)wl + T, \]

where \( w \) denotes the wage rate, \( T \) denotes a transfer from the government, \( \tau \) denotes the labor tax rate, \( l \) denotes labor effort, and \( c \) denotes consumption. For given \( T \) and \( \tau \), the household’s optimum problem is:

\[ \max_l (1 - \tau)wl + T - \frac{1}{2}l^2, \]

which has the following solution:

\[ l = (1 - \tau)w \]
\[ c = [(1 - \tau)w]^2 + T. \]

The production function is:

\[ y = zl, \]

where \( k = 1 \) is fixed. Then, \( w = z \) where \( l \) is equilibrium aggregate employment. The government is benevolent, meaning that its utility corresponds to the utility of the private agents in the economy. It is convenient to express the government’s ‘utility function’ as a function of the government’s own actions. We
do this by replacing consumption and labor by the private sector allocation rules that characterize a private sector equilibrium:

$$u(\tau, T) = [(1 - \tau)z]^2 + T - \frac{1}{2}[(1 - \tau)z]^2$$

$$= \frac{1}{2}[(1 - \tau)z]^2 + T.$$ 

Also, tax revenues to the government are given by:

$$\tau wl = \tau(1 - \tau)z^2.$$ 

Note that this is a unimodal Laffer curve, with maximum at $\tau = 1/2$, where revenues are $1/4z^2$.

The timing in the model works like this. At the beginning of the period, the government borrows $b$ on the international financial market. It splits $b$ between $g$ and $T$, subject to:

$$g + T \leq b.$$ 

Then, the value of $z$ is realized. The variable, $z$, is random with $z \in (z', z^h)$, where $z = z^h$ with probability $p(g)$ and $z = z'$ with the complementary probability, $1 - p(g)$. Suppose that $p$ is concave, with $p \to 1$ as $g \to \infty$ and $p \to 0$ as $g \to 0$. A parametric form with this property is:

$$p(g) = 1 - e^{-ag}, \ a > 0.$$ 

In this setup, $g$ is a form of investment that does not directly give rise to utility. It only gives rise to utility indirectly by increasing the likelihood that a high value of domestic productivity will occur. At the same time, $T$ does directly raise utility.

At this point, the government chooses the labor tax rate, and after this a private sector equilibrium occurs. Finally, at the end of the period, the government pays off its international debt. This gives rise to another restriction:

$$d \leq \tau(1 - \tau)z^2.$$ 

When $g, T$ are not separately observed, there is a moral hazard problem, just like in the previous example. The government may have an incentive to borrow $b$, put all the proceeds into $T$, so that $z = z'$ with probability one. The government could then claim that $z = z'$ because of ‘bad luck’, and collect a high value of $d'$. Whether these incentives are in fact present in some way in the model requires a closer analysis of the model.
A useful benchmark case occurs when $z$, $g$ and $T$ are all separately observed to everyone. Again, imagine a scenario in which we are talking a large number of economies in which $p(g)$ countries experience $z^h$ and $1 - p(g)$ countries experience $z^l$. The lender’s zero profit condition is:

$$b(1 + r) = p(g)d^H + (1 - p(g))d^L, \quad (2.1)$$

where $d^h$ is the amount that countries which experience $z^h$ pay back, and $d^L$ is the amount that countries experiencing $z^L$ pay back.

The optimal contract, $b$, $d^h$, $d^l$, $g$ maximizes expected utility of a country, subject to the zero profit condition of the lending organization. We first compute expected utility conditional on a specific set of values of $b$, $d^h$, $d^l$, $g$. For this to be a relevant set of values, they must satisfy budget feasibility:

$$d^h \leq \frac{1}{4}(z^h)^2, \quad d^l \leq \frac{1}{4}(z^l)^2. \quad (2.2)$$

The problem for the government of choosing the labor tax rate right after the realization of the technology shock is:

$$\max_{\tau^i} \left\{ \left[ \frac{1}{2} \left[ (1 - \tau^i)z^i \right]^2 \right] + T \right\},$$

subject to

$$d^i \leq \tau^i(1 - \tau^i) \left( z^i \right)^2, \quad (2.3)$$

for $i = h, l$. This problem is very simple. Notice that in the range $\tau^i \in (0,1)$ government indirect utility is monotonically falling in $\tau^i$. At the same time, (2.2) guarantees that there are two values of $\tau^i$ that cause (2.3) to be satisfied as an equality. It is optimal for the government chooses the lowest of these two tax rates.\(^7\) Thus, $\tau^h$ and $\tau^l$ are the smaller of the two zeros of the second order polynomials corresponding to (2.3) with equality. Denote these zeros by

$$\tau^h = \tau(z^h, d^h), \quad \tau^l = \tau(z^l, d^l).$$

Substitute the tax rates into the government’s objective just before the realization of $z \in (z^l, z^h)$:

$$p(g) \left[ \frac{1}{2} \left[ (1 - \tau(z^h, d^h))z^H \right]^2 \right] + (1 - p(g)) \left[ \frac{1}{2} \left[ (1 - \tau(z^l, d^l))z^l \right]^2 \right] + b - g. \quad (2.4)$$

\(^7\)The government sets (2.3) to zero for each of $i = l, h$ because there is no gain (only a loss) from raising more revenues than needed.
The optimal contract is found by maximizing (2.4) subject to (2.1), with respect to $b, d^h, d^l, g$.

We next ask whether this is in fact an equilibrium, when $g, T, z$ are not separately observable. This requires that the values of $g$ and $T$ that emerge from the solution to the above full information problem also solve the limited information problem. In particular, for this to be the case, we require that the optimal choice of $g$, denoted $\tilde{g}$, by the government coincide with the value of $g$ implied by the optimal contract. Using the same reasoning applied in the model of the previous section, this corresponds to the requirement that the $g$ in the optimal contract satisfy:

$$p'(g) \left\{ \frac{1}{2} \left[ (1 - \tau(z^h, d^h))z^H \right]^2 - \frac{1}{2} \left[ (1 - \tau(z^l, d^l))z^l \right]^2 \right\} = 1. \quad (2.5)$$

This cannot occur, because when the government chooses $g$, it ignores the zero profit condition. So, the limited information, incentive-compatible contract is computed as the solution to the problem solved in the full information model, with (2.5) added as an extra (binding) restriction.