1. Consider the Pareto distribution, with density function,

\[ f(\omega) = \alpha \frac{d^{\alpha}}{\omega^{\alpha+1}}, \alpha > 2, \omega \geq d. \]

(a) Show that

\[ E\omega = \frac{\alpha d}{\alpha - 1}, \sigma^2 = E[\omega - E\omega]^2 = \left( \frac{d}{\alpha - 1} \right)^2 \frac{\alpha}{\alpha - 2}. \]

(b) Evaluate \( \Gamma(\bar{\omega}) \) and \( G(\bar{\omega}) \), where these objects are defined in the handout on financial frictions under costly state verification and asymmetric information. In addition, display expressions for

\[ s \left[ \frac{1 - F(\bar{\omega})}{1 - \Gamma(\bar{\omega})} - \frac{\mu F'(\bar{\omega})}{\mu G'(\bar{\omega})} \right] \]

where

\[ s = \frac{1 + R^k}{1 + R}. \]

(c) Set \( \mu = 0.33, \sigma = 0.8216, s = 1.0126, \) and \( E\omega = 1. \)

i. Solve for \( \alpha \).
ii. Compute leverage, \( L \), and the risk spread, \( Z/(1 + R) \).
iii. Double \( \sigma \). What happens to leverage and to the risk spread in the standard debt contract? Can you provide an economic interpretation of the results?

2. The overheads on costly state verification that were distributed only sketch the equations that complete the model of costly state verification.
(a) Write those equations out for the case of no uncertainty (you could have a look at the handout on costly state verification on the course website\(^1\)).

(b) Describe a straight neoclassical model whose equilibrium allocations coincide exactly with those of the economy with the financial frictions. In this economy, the household accumulates capital on its own, but the government taxes the income from capital earnings and then spends at least a part of it. The resource constraint in this economy will look as follows:

\[ c_t + I_t + g_t \leq y_t, \]

where \( y_t \) is output and government spending is \( g_t \). The neoclassical model should have the property that an outsider looking at data on investment, capital, consumption, and output cannot tell whether the data were generated by the model with financial frictions or by the neoclassical model with taxes on capital income and government spending.

3. Consider example 1 in the notes on “Financial Frictions Under Asymmetric Information and Moral Hazard”. Consider the model parameter values listed at the end of that section.

(a) How does the contract compare for \( a = 2 \) and \( a = 3 \)? What happens to the multiplier on the incentive compatibility constraint? Provide intuition.

(b) How does the contract compare for \( \gamma = 2 \) and \( \gamma = 1.5 \). Provide intuition.

(c) How does the contract compare for \( \beta = 1 \) and \( \beta = 0.98 \). Provide intuition.

\(^1\)http://faculty.wcas.northwestern.edu/~lehrist/d11/d1108/patrick.pdf